Forgotten Algebra

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1 Introduction

A self-teaching book intended for people who are going to be taking a standardized test such as the GRE or GMAT. Quick-paced, with lots of worked examples and sample problems. I liked this book, and I would recommend it. There were come complaints about typos in the second edition, so it is probably worthwhile to get the 3rd edition.

These are some notes I jotted down as I worked my way through the book as an aid to memory later. Use it for a quick refresher, but get the book if you need help, because algebra is a skill to be mastered through repetition. This file is available from http://www.benespen.com/

2 Definitions

- First degree equation-a first degree equation has the following characteristics:
  1. There is only one variable
  2. The variable is involved in one or more of only the four fundamental operations
  3. The variable is never multiplied by itself
  4. The variable does not appear in any denominator
- Conditional equation-an equation that is true only for certain values
- Identity-an equation that is true for all values of the variable
- Fractional equation-an equation in which the variable appears in a denominator
  - Note: multiplying or dividing both sides of an equation by an expression containing the variable may result in a different equation
- Literal equation-an equation that contains letters in addition to variables and numbers(used as constants or other variables)

- Rational number- let a and b be integers with $b \neq 0$, then $\frac{a}{b}$ is called a rational number (usually referred to as a fraction). a is the numerator and b is the denominator.
- Complex fractions-fractions in which there are one or more fractions in the numerator or denominator
- Fractional exponent-an exponent of the form $a^{\frac{n}{d}} = (\sqrt[d]{a})^n = \sqrt[d]{a^n}$
- Monomial-expressions with one term
- Binomial-expressions with two terms
- Trinomial-expressions with three terms
- Polynomial-expressions with more than one term
- Standard form-a polynomial is said to be in standard form if
    1. All parentheses are removed
    2. Like terms are combined
    3. The terms are arranged in order of descending powers of x
- Polynomial degree-the highest power of x
- Logarithm-x is called the logarithm of N to the base b if $b^n = N$, where N and b are both positive numbers, $b \neq 1$

3 Key Terms

- Signed number
- Absolute value
- Term
- Variable
- Coefficient
- Expression
- Like terms
- Cross-multiply
- Base
- Exponent
- Factor
- Lowest terms

4 Algebraic Laws

- Laws of exponents
  1. Multiplication $a^n \cdot a^m = a^n = m$
  2. Power of a power $(a^n)^m = a^{nm}$
  3. Power of a product $(ab)^n = a^n \cdot b^n$
  4. Power of a fraction $(\frac{a}{b})^n = \frac{a^n}{b^n}$
  5. Division $\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$
- Laws of fractions
  1. Addition of rationals $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
  2. Subtraction of rationals $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$
  3. Multiplication of rationals $\frac{a}{b} \cdot \frac{c}{d} = \frac{ad+bc}{bd}$
  4. Division of rationals $\frac{a}{b} \div \frac{e}{f} = \frac{a}{b} \cdot \frac{f}{e} = \frac{af}{be}$
- Distributive law $a(b + c) = ab + ac$
- Laws of logarithms
  1. Product $log_b AC = log_b A + log_b C$
  2. Quotient $log_b \frac{A}{C} = log_b A - log_b C$
  3. Power $log_b A^k = k log_b A$
  4. Root $log_b \sqrt[k]{A} = \frac{1}{k} log_b A$

5 Formulas

- Difference of two squares $x^2 - y^2 = (x + y)(x - y)$
- Sum two cubes $x^2 + y^2 = (x + y)(x^2 - xy + y^2)$
- Difference of two cubes $x^3 - y^3 = (x + y)(x^2 + xy + y^2)$

6 Procedure for Solving Algebraic Equations

1. Simplify
2. Transpose
3. Simplify again
4. Divide by coefficient
5. Check by substituting tentative answer into the original equation

7 Procedure for Factoring

1. Remove any common monomial factor
2. If there are two terms, check for the difference of squares, difference of cubes, or sum of cubes, and factor accordingly
3. If there are three terms, try factoring into two binomials.
4. If there are four terms, try grouping them into pairs that have some common variable.