If Music Be The Food of Love, Play On

Chapter Three

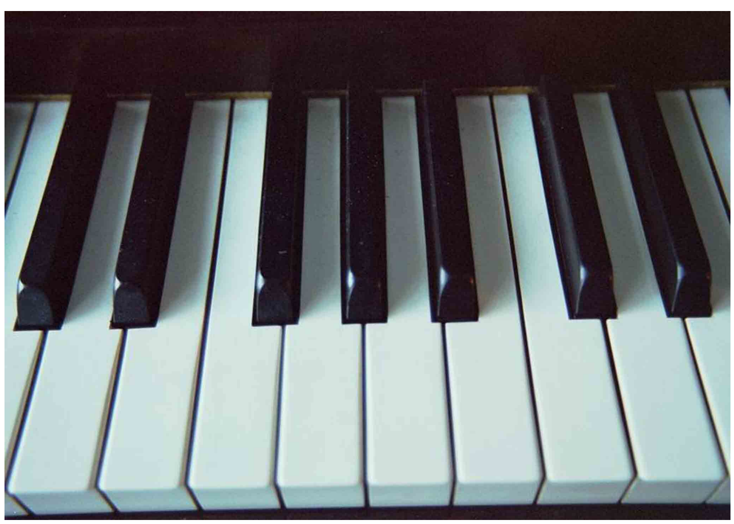
Jacob Ochterveldt (1634-1682), *The Music Lesson* 1671
The Art Institute, Chicago

Chapter Three

Am I In Tune?

In the first two chapters we've discussed the general issues of music's definition and its importance. Here we turn to a topic of more specifically musical importance and of interest to musicians themselves. It is the matter of tuning. Have you ever gone to a concert and during the intermission over-heard someone complain about the soloist, saying "Oh, she's out of tune!" Or have you played in a band, or perhaps sat-in on a rehearsal, and heard the conductor say to someone, "You're flat! Tune up!"? How did these people know that there was a problem? What does it mean to be "in tune"? "In tune" with what? Who decides? And how do they know?

Look at the keyboard reproduced below (Ex. 3.1). Haven't you wondered about that configuration? There are long and short levers, and white leavers and black levers, and the white leavers partially separated by black leavers and white leavers adjacent to other white levers. Doesn't that look like a rather whimsical design? Is there a reason for that pattern?



Example 3.1
Piano keyboard

And consider the way we talk about musical pitches. We give them letter names: A, B, C, D, E, F, G (and if you're German there's a 'H' that you use for B natural; the 'B' in Germany is what we call a 'B flat'). As you look at that series wouldn't it be reasonable to think that since these are all regular letter names (meaning that there's nothing here like a 'B plus x' or an 'D minus' or a 'Fq2'—but instead just plain 'B' and 'D' and a plain 'F'), that the relationship between adjacent letters would be the same? Shouldn't the relationship between the adjacent letters of 'D' and 'E' be the same as between 'B' and 'C'? But this isn't the case at all, is it? In that series of 'A, B, C, D, E, F, G' adjacent pitches are divided by the distance of a whole step except in the cases of 'B' to 'C' and 'E' to 'F' in which cases the dividing distance is a half step. But isn't this odd? Why are there different-sized spaces, some half-steps and others whole-steps? Why aren't they all whole-steps or all half-steps? And what's a whole-step anyway? How big it is? And who says so?

While we are thinking about the way we name pitches and that odd pattern of half and whole steps, consider this. Is there a difference between C-sharp and D-flat? If you go to the piano and play a C-sharp you depress the same lever that you'd depress to play a D-flat (it's the short black lever that half way separates the white C and D keys). But if both notes are produced by the same mechanical device why are there two names? Isn't this redundant? But perhaps you play violin or cello. You know that on your instrument C-sharp is played in a different position than D-flat. But if you can play both pitches on the piano by depressing the same lever why can't you do the same thing on the violin? Why isn't C-sharp fingered exactly the same way as D-flat?

Try this experiment. Go to the piano and play a perfect fifth (you can play C and the G above it but any Perfect fifth will do). After you've played it, sing the major third above your lowest note without striking it on the keyboard (in other words, if you've playing that 'C' and 'G' perfect fifth try to sing the 'E' above the 'C' without first playing the 'E' on the piano). Then, after you've sung the pitch, strike the note on the keyboard. What do you hear? Does the pitch you've played match the pitch that you've just sung? If you listen very carefully it's likely that they didn't. The pitch you sang probably was a bit higher than the pitch the piano produced. Why? Were you 'out of tune'? Or perhaps the piano is out of tune? How can you tell?

This returns us to our first question, the notion of being "in tune." If there are such things as "half-steps" and "whole-steps" who says how big a "whole-step" must be? And, whatever the whole-step is, is the "half-step" then automatically exactly the size of one half of a whole-step or is it perhaps something else (yes, I know that sounds like a silly question but it's not)? When someone says that I'm "out of tune"? Does that mean that my whole step not big enough, or too small? And what kind of comment is that? Is that a scientific remark, a fairly neutral observation concerning an acoustic measurement, similar to saying that my yardstick was a quarter inch too short? Or is that a musical remark about an aesthetic observation? Or perhaps it's something else altogether.

Perhaps it's really an act of aggression, an attempt to subjugate me to the constraints of a tuning system perpetuated by power structures—political, sociological, sexual—mindful of the necessity of maintaining their hegemony required for their survival? Is "correct tuning" and being "in tune" really simply an act of dominance? In that band when the conductor yells at me that my major second is "flat", is she "right" and I'm "wrong" simply because she yells louder than I can (and do I "correct" my out of tune interval, and play it the way she wants it, only because if I don't she'll fire me?). Fundamentally, is being "in tune" just her opinion against mine? Just how big is a major second?

In 1896 Richard Strauss composed his tone poem *Also Sprach Zarathustra* inspired by Friedrich Nietzsche's book of the same name (we've come across Strauss before: he wrote the opera aria sung by Pavarotti that we talked about in Chapter One). Strauss' tone poem has an introduction that was popularized by Stanley Kubrick when the director decided to make it the main music in his movie *2001* (you know it: dah, dah, DAAAH!!..[drums: DUM dum DUM dum DUM dum DUM dum], repeat) This introduction ends with the full orchestra holding a C Major chord along with an organ but Strauss has the organ hold the chord one measure longer than the orchestra; the orchestra drops out while the organ continues to play it. If you listen carefully to that final organ chord you'll notice that it sounds a bit sour. We might say that the organ sounds "flat." But is the organist perhaps playing the wrong notes? No, they are the right notes, the chord when played on the organ just sounds different than the chord the orchestra played before. Maybe the orchestra is "out of tune" and the organ's right? Or is it the other way around? Why is this sounding the way it does?

Here's another example. Orchestras and bands today have instruments called "French horns." They were called "French" beginning in the nineteenth century to distinguish them from traditional horns that didn't have valves (if you go to page 32 in the first chapter you can see a Meissen porcelain goblet with a figure playing a traditional horn)—and the new valves were largely developed in France. If you listen to a piece played on natural horns the instruments will sound beautiful together—most of the time. But occasionally there are some surprisingly sour notes. Are the notes out of tune because the players are incompetent or is there something else going on making them so distasteful to our ears (and that word, "distasteful" is very important)?

Many of us enjoy working with various kinds of electronic instruments interfaced with our computers (called "MIDI" for "musical instrumental digital interface"). These MIDI stations allow us to alter pitch easily and subtly. We can "re-tune" keyboards and other electronic instruments to frequencies and even timbers different from those set by the instrument manufacturers. What guides our decisions when we do this kind of re-tuning? Again, do we just raise or lower a pitch capriciously or can there be a logic behind our decisions? And is there a tradition behind that logic?

What about the cultural aspect of music? We have already seen that all music is a function of the culture of which it is a part. To understand music fully we need to understand it within its cultural milieu. Does this mean that cultural notions influence what's heard as a "correct" tuning? Is there such a thing as a South Indian "Perfect Fifth" that's different from a Portuguese "Perfect Fifth"? And finally, what does "science" have to do with tuning? We're probably all heard "A 440". What does that mean? And can a "scientific" notion of what's in tune trump a culture's notion of what's in tune?

These questions are not at all tangential to our lives as either performing musicians or music lovers. For those of us who are performers they bring us directly to both a central question of our art and to an issue which has commanded the attention of thinkers about music for at least two and a half millennia: this is the issue of "pitch" and how pitches relate to one another within the art of music. More simply put, it addresses the question, "Am I in tune?" And why.

I

We need to return to the story of Pythagoras that I introduced to you in the previous chapter. You will remember that charming story of the philosopher at the smithy, initially told by the first-century Neo-Pythagorean writer Nicomachus (ca 60 AD to ca 120 AD) in his *Harmonium enchiridium* or "Handbook on Harmony" and apparently repeated in almost every discussion of Pythagoras written since.[1] He found that the blacksmith's twelve and six pound hammers sounded the interval of the perfect octave between them, that the blacksmith's nine and eight pound hammers sound the interval of the major second and that the twelve and eight pound hammers sounded the interval of the perfect fifth. We know that hammers of different weight really don't work this way (the example was probably invented for its easily manipulated numbers) but the story does describe a valid acoustic principle that actually can be demonstrated by marking off lengths of a vibrating string.

This vibrating string (which the Greeks called the *kanon* and the Latins the *monochord*) became the most important pedagogical device in the history of music. Consisting of a single string stretched across a sounding board, the relationships between different intervals could be demonstrated by moving the monochord's bridge and marking the different lengths of the string. When the monochord's string is plucked unfettered, meaning that its whole length vibrates, it produces a particular tone. It can be

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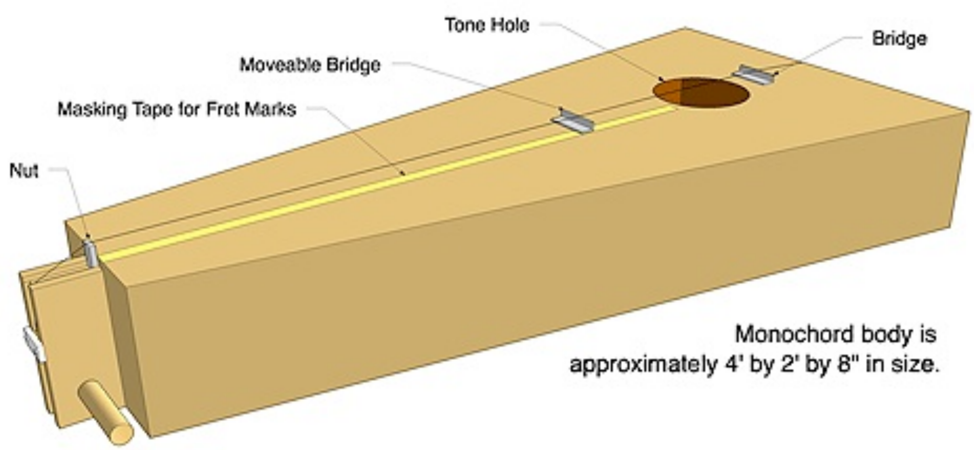
[1] Nicomachus is best know for his "Introduction to Arithmetic." Translated from Greek into Latin by none other than Apuleius, his "Introduction" was still used by schoolboys in the Renaissance, probably setting a record for the longevity of a textbook.

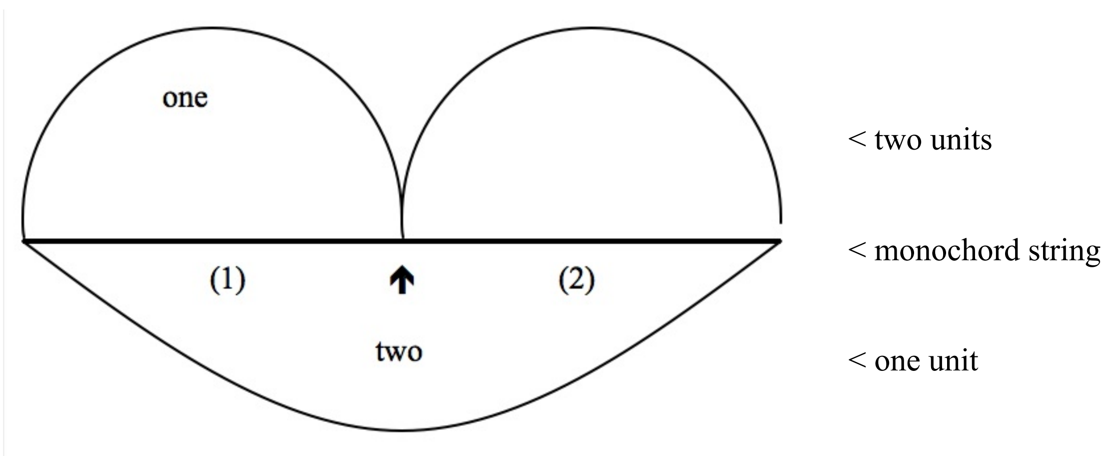
The full page of the thirteenth century manuscript Bayerische Staatsbibliothek clm 259, fl 96v, showing Pythagoras at his forge, but notice the student in the lower left hand corner working with a monochord.

Below: a modern plan for constructing a monochord.



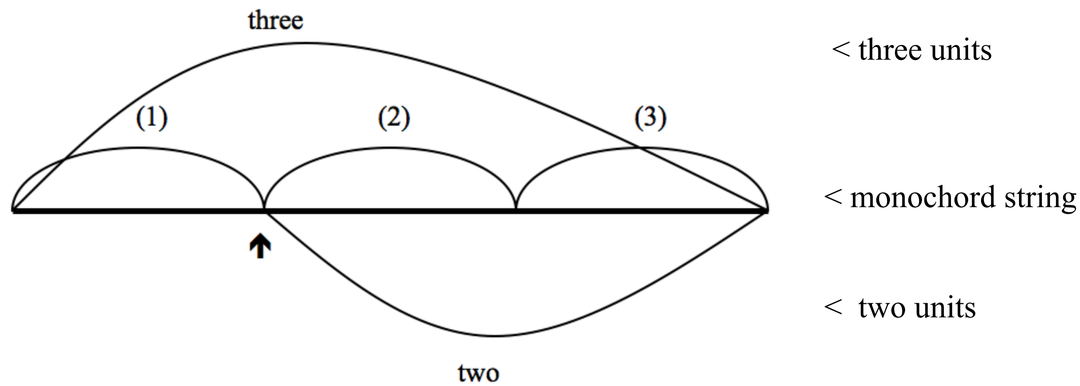
any tone, but for our ease we're going to refer that tone, which we'll call the "fundamental", as "C".

If we take the string and divide it into two equal parts and then pluck the full string (which is of course both parts vibrating together), we'll get a particular pitch. And if we then pluck just one of those two parts having dampened the string at the midpoint, we'll get a second pitch. We can name the first pitch "two" because it's created by two units vibrating together. And we can name the second pitch "one" because it's created by just one of those parts vibrating by itself. The relationship between those two pitches we can precisely describe by the ratio 2:1.



If, again only for convenience sake, we think of the string vibrating undamped (that is as two units together) as producing the pitch "C", one of the divisions of the string will produce the "C" an octave above that.

But let's divide our same string into three units. Now the "name" for our string isn't "two" but instead "three" (and remember "three" still produces our same "C" as did "two" before). Should we now dampen our string at the end of two of our new divisions and then pluck that section which is two-thirds of the full string's length we'll get another pitch. This pitch is the "G" above our original "C", or a perfect fifth higher (if you've forgotten whey it's called "perfect" look back to footnote 15 in the previous chapter). The distance of the perfect fifth, or the interval (and interval is the way musicians describe distances) we can describe by the ratio 3:2, or the relationship between all of the string vibrating (which is named “three”) and just two parts of that string vibrating (which is “two”).

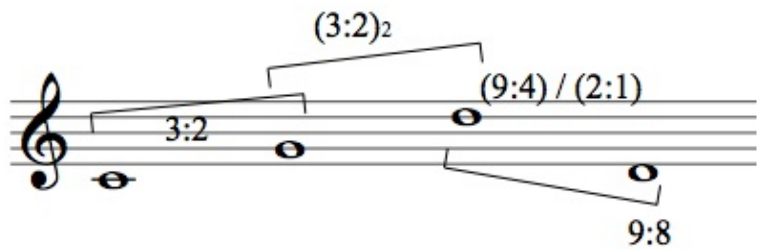


Using this procedure of dividing our monochord we have found two intervals: the octave above our original “C” and the perfect fifth above that fundamental. And we have found that these two intervals can be defined by their relationships to the string’s total length, a relationship described by ratios. The octave is defined by the ratio 2:1 and the perfect fifth by the ratio 3:2.

But what happens when we begin manipulate these relationships? First, what happens when we take the ratio of the perfect octave and multiply by itself? We get $(2:1)^2$. The math isn’t too hard, working out to 4:1. If we take our string and now divide it into four equally sized units and then pluck just one of those units, we’ll produce the C two octaves above our fundamental C. And should we do this again, and multiply $(2:1)^2$ by the octave we’ll get $(2:1)^3$ which works out to 8:1. Should we divide our monochord into eight units, pluck one of the divisions, we’ll produce the C three octaves above our fundamental pitch. If we multiply the octave by itself we produce ever higher octaves or our original pitch; one octave higher, then two, then three, then four, and so on

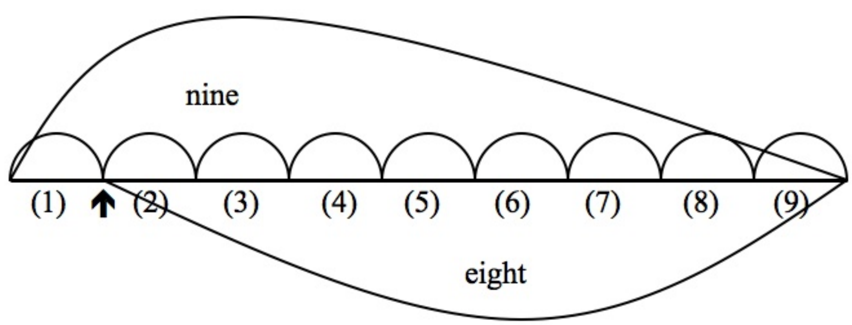
Second, something similar happens when we multiply the perfect fifth by itself: the process produces ever ascending perfect fifths. And we can use these ascending perfect fifths to find other intervals too. Take a perfect fifth and multiply it by a perfect fifth, or $(3:2)^2$. This gives us the ratio of 9:4. If we divide our monochord into nine units, strike where four of these units can vibrate freely, we will produce the interval of two ascending perfect fifths or, since our fundamental is “C”, the “D” one octave and a step above that original “C”.

But if we take that $(3:2)^2$ and then reduce it by an octave we can find the interval of the major second. It works like this: take $(3:2)^2$, divide it by the ratio of the octave, which is 2:1, we get the ratio of 9:8.[2] The process we just went through looks like this:



We constructed one perfect fifth (3:2), then built another one on top of that $(3:2)^2$, and then dropped that pitch by an octave by dividing it by 2:1.

And we can see this on the monochord. We take our compass and divide our string into nine equally sized units (this was basically done through trial and error because the only thing the ancients used to determine measure was the compass). The

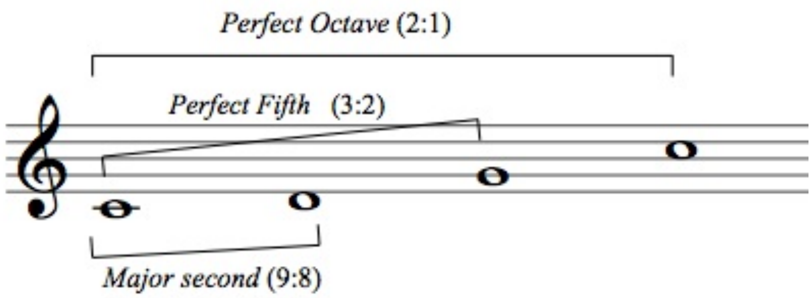


"name" for the full string is now "nine." If we count up eight of our units, dampen the string at that point, pluck the longer string, we'll get a new pitch and the relationship between our new pitch and our original pitch can be called 9:8, the relationship between the full string and its partial. Again, if our full string vibrates at the pitch "C" the new pitch we have is "D" and the distance between the pitches is a major second.

Given that the fundamental pitch generated by our monochord's string is C, here are the intervals and pitches we've discovered so far (again, it's important to emphasize

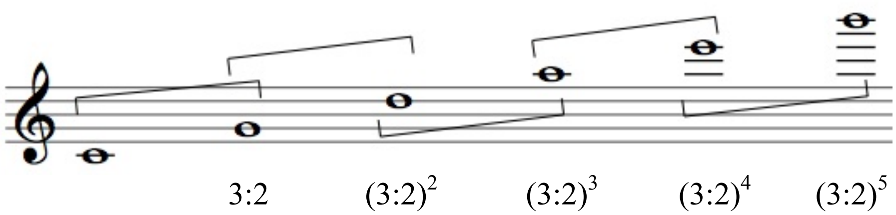
[2] $(9:4) / (2:1) = (9:4) \times (1:2) = 9:8$

that what's important here are the intervals, not the particular pitches. It's like trying to find the definition of a "yard;" our interest is in the unit of measure not whether or not we're measuring bolts of wool or cotton).



The octave can described by the ration 2:1, the perfect fifth by the ratio 3:2 and the major second by the ratio 9:8, or $(3:2)^2$.

Just as we found the perfect fifth above our first perfect fifth by squaring the ratio of 3:2, we can find the perfect fifth's about that by the same process. $(3:2)^3$, $(3:2)^4$, $(3:2)^5$, etc.



If we reduce each of these successive fifth to pitches within our first "C" to "C" octave we find that have the diatonic scale with which we are so familiar – or almost. Here's how it works.[3]

For the perfect fifth above "D", we calculate $(3:2)^3$, which works out to 27:8. This means that if we divide our monochord into twenty-seven equal divisions, dampen the point between the eighth section and the ninth and pluck the string we would, at least theoretically, sound the pitch "A" one octave and a major sixth above our fundamental

[3] The arithmetic given here isn't the way these matters were done in either Antiquity or the Middle Ages (for one thing these theorists didn't have Arabic numerals). For an introduction to the way the Greeks themselves manipulated these figures see: Richard Crocker, "Pythagorean Mathematics and Music," *The Journal of Aesthetics and Art Criticism*, Vol. 22 (1963), pp. 1898-198 (Part One), pp. 325-335 (Part Two).

"C." But, since our interest is in finding the intervals *within* the octave we must reduce, or bring down, that "A" by one octave. This we can do by dividing the ratio 27:8 by the ratio of the octave, which you remember is 2:1. Work out the arithmetic and you get 27:16. This is the ratio of the Major Sixth.[4]

$(3:2)^4$ give us the next ascending perfect fifth (in our example, from "A" up to "E"), which works out to 81:16. But to find this pitch within our original "C" to "C" octave we must reduce it. When we reduced $(3:2)^2$ to find our major second we reduced it by one octave, or 2:1. But here we must bring down our "E" by two octaves and the ratio for two octaves is $(2:1)^2$ or 4:1. Do the arithmetic and we get 81:64, which is the ratio of the major third.[5]

Next on our spiral of fifths (and *spiral* is a very important word that I'll come back to) is $(3:2)^5$. In our example beginning on "C" this pitch is "B." But as before, this "B" is two octaves higher than our original octave and we must reduce it by $(2:1)^2$ for it to serve our current purposes. Work out the arithmetic and we get 256:243. This is the ratio of the Pythagorean major seventh. If we go back to our monochord, use our compass to divide our string precisely into two hundred and fifty six equal units, count up two hundred and forty three of them, dampen the string at that point and pluck the string's shorter section, we'll produce the "B" a major seventh above our fundamental "C".[6]

If we continue with the spiral of perfect fifths our next fifth is $(3:2)^6$. Work-out the arithmetic and reduce by three octaves (8:1) we get the ratio for the augmented fourth, 729:512, which in our example is "F sharp."[7] That means to find this interval, we need to take our compass, find exactly seven hundred and twenty nine divisions of our string, count up five hundred and twelve of them, mark it and pluck. The complication of the calculations probably more than the sound itself helps us understand why the augmented forth from an early period acquired the reputation as "the devil in music."

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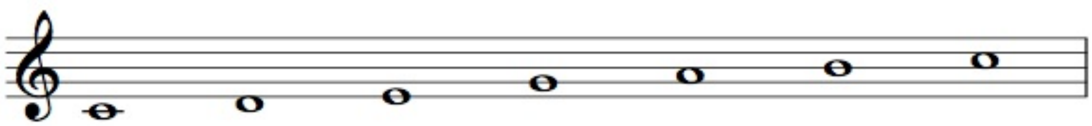
[4] $\frac{27}{8} : \frac{2}{1} = \frac{27}{8} \times \frac{1}{2} = \frac{27}{16}$ = Pythagorean Major Sixth

[5] $(3:2)^4 = 81:16$ $\quad \frac{81}{16} : \frac{4}{1} \quad \frac{81}{16} \times \frac{1}{4} \quad \frac{81}{64}$ = Pythagorean Major Third

[6] $(3:2)^5 = 243:32$ $\quad \frac{243}{32} : \frac{4}{1} = \frac{243}{32} \times \frac{1}{4} = \frac{243}{128}$ = Pythagorean Major Seventh

[7] $(3:2)^6 = \frac{729}{64} : \frac{8}{1} = \frac{729}{64} \times \frac{1}{8} = \frac{729}{512}$ = Pythagorean augmented fourth

If we look over what we've done up to this point and refer back to our original "C" we can see that through manipulating perfect fifths and octaves we've come pretty close to generating the major scale that's so familiar to Westerners (at this point it's important to note that historically this isn't the way the modern scale came into being but we'll discuss that more later).



Perfect Octave:	2:1	(C to C)
Major Second:	9:8	(C to D)
Major Third:	81:64	(C to E)
Perfect Fifth	3:2	(C to G)
Major Sixth:	27:16	(C to A)
Major Seventh:	243:128	(C to B)

But you're probably already noticed that there's something important missing here. We don't have a perfect fourth; there's no "F" and you would expect there to be one (and I'll come back to that "expect" also). If we continue the ascending progression of perfect fifth's we'll never reach an "F" because that progression is a spiral, it continues without ever returning to its origin. Our progression moves from "F sharp" to "C sharp", to "G sharp", to "D sharp", to "A sharp" and then to "E sharp" (and then "B sharp", "F double sharp", etc.). But this "E sharp" is not the same pitch as "F". The "E sharp" is $(3:2)^{11}$ which works out to 177,147: 2,048. Reduce this by $(2:1)^{6}$ and we get 177,147:131,072. This in turn can be reduced to 4.43:3.28 which is the ratio of the augmented third.

To find the interval of the perfect fourth we must abandon our spiral of perfect fifths and instead find the difference between the perfect fifth and the octave; that difference is the perfect fourth, 4:3 (and I'm going to come back to the difference between the augmented third and the perfect fourth a bit later).[8]

Now, with the addition of our perfect fourth of 4:3 we have all the intervals of what we call today a major scale and we can answer at least one of the questions with which be began this chapter: why do we have this odd configuration of half and whole

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[8] $(2:1) : (3:2) = \frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$ = Pythagorean Perfect Fourth

steps between adjacent letters? The development of our system scales and the naming of notes is rich and very complex and while this is a great simplification it's largely true: that pattern grows out of a tradition of creating "right" intervals through Pythagorean principles.

You will remember from the previous chapter our introduction to Pythagoras and our addition to that theorem we all learned: in the matter of a right triangle $A^2 + B^2 = C^2$ *always*. It's that *always* that's so important and testified to the Greeks of the changeless, and therefore divine, character of numbers.

I'll expand on this in a bit later, but for now it's important to note that we known when a major second is in tune when the interval is defined by the ratio 9:8. We know when a perfect fifth is in tune when the interval is defined by the ratio 3:2. It's when the numbers are right that we know the interval is "right" because, by their very nature, the numbers are changeless and hence, again in this Pythagorean tradition, divine. And these numbers do seem to be remarkably consistent. We've already seen that we can construct a major third by reducing $(3:2)^4$ by two octaves, giving us the ratio of 81:64. But if we stack one major second on top of another major second, that also should make a major third, and it does: 9:8 multiplied by 9:8 equals 81:64. Similarly, should we stack a major third on top of a perfect fifth we should be able to create a major sixth. And it does: 3:2 multiplied by 9:8 equals 27:16. What is the difference between a perfect fifth and a perfect fourth? A major second. Do the arithmetic as we get 9:8, just as we should. And should we stack perfect fifth on top of a perfect fourth we create an octave. Do the ratios work? Yes they do, 4:3 x 3:2 reducing to 2:1.

We can figure the minor second by finding the difference between the perfect octave and the major seventh. When we do the arithmetic we get the ratio of 256:243.[9] But there is also the interval of a minor second between a major third and a perfect fourth. Is the difference between these intervals the same ratio as between the perfect octave and the major seventh (as it should be)? Do the arithmetic and yes, it is: 256:243.[10] We can also find this minor second by creating a major third by stacking two major seconds and then subtracting that interval from the perfect fourth.[11]

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[9] (2:1) : (243:128) $= \frac{2}{1} : \frac{243}{128} \quad \frac{2}{1} \times \frac{128}{243} = \frac{256}{243}$ = Pythagorean minor second (*diesis* or *limma*)

[10] (4:3) : (81:64) $= \frac{4}{3} : \frac{81}{64} = \frac{4}{3} \times \frac{64}{81} = \frac{256}{243}$ = Pythagorean minor second (*diesis* or *limma*)

[11] (4:3) : $(9:8)^2$ = 254:243 = Pythagorean minor second (*diesis* or *limma*)

Pythagoras and Musica (holding a monochord)
From the Munich State Library Cod. Lat 2599, fol 103, c. 1300

The Greek work Euclid uses for “ratio” is “logos” (λόγος) a word with a rich, and deeply complex, tradition in western philosophy. The Stoics used it to mean the reason uniting the universe, a notion we see reflected by the writer of the Gospel of John were he began his account with the line, “In the beginning was the *logos* and the *logos* was with God and the *logos* was God." Logos here isn't just a fundamental principle, it is the personality of God incarnated in Jesus of Nazareth (in most English Bibles "logos" is translated as "word" following Martin Luther's lead when he translated the text into German). The consistency of the Pythagorean ratios testifies to the existence fundamental principles that unite all existence and harmonizes of the cosmos into a rational whole.

Well, almost but not quite. If you divide an orange in half, what do you get? Two halves of an orange. If you take the orange you've divided and end up with one part three quarters the size of your original orange and a second part one quarter the size, you've made a mistake. You've split your orange but you haven't divided in half. And if you're hawking both parts as half an orange and selling them for the same price somebody is going to be cheated.

Lets to back to the Pythagorean half-step we created by taking the major seventh away from the octave. That process gave us the ratio 256:243. And we already know that our whole step is 9:8. Now, quickly, what ratio should we get if we take a half step away from a whole step? No, don't do any arithmetic. Just think. Shouldn't half of a whole step be a half step? Shouldn't the ratio be 256:243? Right? Of course. But it isn't. The difference between 9:8 and 256:243 is 2187:2048.

Bluntly, this ratio shouldn't exist, but it does and this has been one of the most controversial and problematic matters in the history of music. In Pythagorean tuning we find ourselves confronted by two different half steps. The half step we calculated by finding the difference between the perfect octave and the major seventh (and the difference between the perfect fourth and the major third), is called the "diatonic" minor second or the "Pythagorean diesis" (which means "difference") or the "limma" (which means "remnant"). It's the ratio 256:243. "Cent" is a modern unit of acoustic measurement. One "cent" is one-one hundredth of an equally tempered half-step (I know that we haven't discussed equal temperament yet but we will; for now just think of a "cent" as like a millimeter, a simple unit of measure).[12] One equally tempered half step is 100 cents. The diesis minor second is 90.22 cents, meaning that it's slightly less then ten percent smaller than the half step we're accustomed to hearing on the piano. The other half step, the one with the ratio 2187:2048 is called the "apotome". It is difference between the major second and the diesis and is 114 cents, making it significantly larger than the equally tempered half step of 100 cents. The difference by which the larger

[12] The "cent" was developed by the English polymath Alexander Ellis (1814-1890) who not only wrote on comparative musicology but also mathematics and philology and was George Bernard Shaw's model for Professor Henry Higgins in his1912 play *Pygmalion* (which served as the basis for the music *My Fair Lady*).

apotome exceeds the smaller diesis is called the "Pythagorean comma" and is 531,441 : 524,288 which is 23.5.

Twenty three and a half cents is a significant size and can be easily heard and will be a significant issue for us. But we have to remember that for the Pythagoreans of Antiquity (and some their disciples even into the Renaissance) "music" was not so much about sound that strikes the ear but instead a calculation that strikes the mind. The music of musicians, with some important exceptions (such as the story about Alexander the Great we read in the previous chapter), plays generally small role in Pythagorean thought. Hellenistic culture had a rich and vibrant musical tradition and you can't help but be struck by the fact that the intonation of such importance to the Pythagoreans and classical Neo-Platonists had very little to do with the scales musicians were using to make their music.[13]

But this issue wasn't missed by the ancient musicians themselves. Aristoxenus (c. 375 – 360 B.C.), a pupil of Aristotle's in Athens and the greatest theorist of antiquity, argued against the Pythagoreans by insisting that the trained ear, and not the compass and monochord, was the final authority in all arguments of intonation. He insisted that all musical intervals should be measured not by ratios but instead by numbers of tones and fractions of a tone. The perfect fourth should be defined by the sum of two and a half whole tones and the musician--not the philosopher--was the one who decided what size the semi-tone was.[14] Aristoxenus was not alone in championing the trained ear. Apparently there was a whole school of philosophers holding that position, a school of thought that lasted well into the Roman period. Theophrastus (c. 371 – c 286 BC), Aristotle's successor in Athens, called such thinkers *harmonikoi*, or "those who judge by sense perception" as opposed to those who give a mathematical account of intervals.[15] There were even attempts at reconciling the two positions, as evidenced by the fragmentary *Pythagorean Elements of Music* by the lady philosopher Ptolemais of Cyrene and *On the Difference between the Aristoxenians and the Pythagoreans* by Didymus, a theorist writing during the reign of the Roman emperor Nero. [16]

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[13] Greek scales could take on at least three forms: a diatonic, a chromatic, or an enharmonic. The difference between these three can be demonstrated in the varying construction of their initial tetrachords. The intervals of the diatonic tetrachord descended in the order: tone, tone, semi-tone. The descending order of the chromatic: three semi-tines, semi-tone, semi-tone. And the enharmonic, again descending: two whole tones, quarter-tone, quarter-tone. These three genera could be further altered through forms called "soft," "hemiolic," "tonal," or "tense," But Pythagorean calculations only produce the diatonic scale, ignoring all other forms. See: R. P. Winnington-Ingram, "Greece, I" *The New Grove* (1980).

[14] M. L. West, *Ancient Greek Music*, pp. 167, 233

[15] *Ibid*., p. 281 (note 1)

[16] *Ibid*., p. 239

II

We will return to the business of these numbers in a moment but before we do I'd like to take some time and look at the influence those numbers have had on Western Civilization. We touched on this in our previous chapter but now, after having looked at the ratios we're equipped to look at the mater a little more closely.

We often forget by how slender a thread ancient literature was preserved into the Middle Ages. The poem of Catullus (c. 84 – 54 BC), who was the greatest lyric poet of Rome, escaped destruction in only one manuscript. Once held in the Italian city of Verona, that manuscript was itself lost in the twelfth century but not until after two copies had been made that now serve as our only sources for this great poetry. And while the case of Catullus may be extreme it serves as a reminder that every civilization is only one generation old. The accomplishments and monuments of thousands of years can be completely annihilated should only one generation choose not to preserve them. And once gone they can be recovered only by the greatest toil, it at all. Look at the Mayans.

Anicius Manlius Severinus Boethius (c. 480 – c 524 AD) was acutely aware of this responsibility.[17] Living in an age where fewer and fewer of his fellow Romans knew Greek and when his German conquerors were beginning to transform his native Latin into what would become Italian, Boethius resolved to translate into Latin some of the great intellectual accomplishments of the Greeks and in that way preserve them. Since adequate Latin translations and commentaries already existed for the disciplines of grammar, rhetoric, and dialectic, (called the *Trivium*), Boethius chose to focus upon what were thought of as the "mathematical disciplines:" arithmetic, music, geometry, and astronomy, a quartet of subjects Boethius called the *Quadrivium.*[18]

Boethius' books on geometry and astronomy were lost apparently fairly soon after they were written. And even though his work on music was only partially preserved into the Carolingian period (the extant text breaks off in midsentence near the beginning of book five), *De institutione musica* ("The Principles of Music) became the most important text on music throughout the Middle Ages and Renaissance. At Oxford, it was still used as a text at the end of the eighteenth century.

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[17] For an introduction to Boethius' life and work see: Henry Chadwick*: Boethius, The Consolations of Music, Logic, Theology, and Philosophy* (Oxford: Clarendon Press, 1981).

[18] Boethius apparently borrowed this division of learning from Martinus Minneus Felix Capella's *The Marriage of Mercury and Philosophy*, a work usually dated between 460 and 470. Martianus had taken the division from the second century BC *Disciplinarum libri IX* of Varro, although that earlier work had also included the "utilitarian arts" of medicine and architecture. See Henry Chadwick, *op ci*t, p. 21

from *On the Consolation of Philosophy* by Boethius, Italian MS ca. 1385
Boethius teaching (above) and imprisoned awaiting execution (below)
the Glasgow University Library MS Hunter 374 (V.1.11), 4r

In *De institutione musica* Boethius draws heavily on Ptolemy's (ca 90 – ca 168 AD) *Harmonica* and Nicomachus' (c. 60 – c. 120 AD) *Handbook on Harmonics*. He references both Plato's *Timaeus* and *Republic* in his discussion of the affect that music has upon morals and character. He divides music into three kinds: cosmic (*musica mundana*), human (*musica humana*), and instrumental (*musica instrumentalis*). Instrumental music, by which Boethius means singing and performing music on instruments, is the most base type. "Cosmic" music is the "music of the spheres," or the relationship between the elements of the cosmos expressible in harmonies and intervallic ratios. "Human music" is the joining together of the disparate elements which constitute the "single consonance' of the human being. Boethius repeats the tale of Pythagoras at the smithy, presents the calculations of Pythagorean ratios (and includes discussions of the Pythagorean comma and dieses) and, while acknowledging the existence of the chromatic and enharmonic forms of the scale, opines that the Pythagorean diatonic scale is superior to them on the authority of *Timaeus*. Boethius thus bequeaths to modern Europe a highly Pythagorean and Neo-Platonic musicology, a bequest that profoundly affected our music in at least four ways.

First, and probably of the greatest importance for us as musicians, is the standardization of Pythagorean intonation throughout Western Europe where it remained largely unchallenged for almost 1,000 years. This was not accomplished for aesthetic reasons but rather rested upon the enormous prestige which Pythagoras, Plato, Ptolemy and Boethius commanded with intellectuals in Western Christendom. Generally, medieval authors had a profound respect for the culture of Classical Antiquity and a great love (I really don't think that any other term will do) for the writing of ancient authors. They loved books, treasured them (literally, they were regarded as treasure), believed almost anything in them, and did almost anything they could to preserve them, copying and re-copying ancient texts at a cost of great toil and expense.[19] Taking their cue from "the master" Boethius, writer after writer begins their treatises on music with discussions of the divisions of the monochord. This dialogue, attributed to Odo of Cluny and preserved in the tenth-century *Enchiridion musices* ("Handbook on Music") is typical of many.

Disciple:	What is the monochord?
Master:	It is a long rectangular wooden chest, hollow within like a kithara; upon it is mounted a string, by the sounding of which you easily understand the varieties of sounds. . .
Disciple:	How does the string produce many different sounds?
Master:	The letters, or notes, used my musicians are placed in order on the line beneath the string, and when the bridge is moved between the line and the string, shortening or lengthening it, the string

[19] For more on the respect Medieval writer had for Greco-Roman authors see: C.S. Lewis, *The Discarded Image: An Introduction to Medieval and Renaissance Literature* (Cambridge: 1967), Chpt. 1.

from a twelfth-century copy of Boethius's *De Musica,* showing the philosopher seated with a monochord in the upper left-hand corner with other figures representing Pythagorean intervals (with hammers and bells) and *musica mundana* (seated on a celestial orb) and *musica instrumentalis* below.
Cambridge University Library, MS Ii.3.12, ff 73v-74

marvelously reproduces each melody by means of these letters. When any antiphon is marked with the same letters, the boys learn it better and more easily from the string than if they heard someone sing it, and after a few months' training, they are able to discard the string and sing by sight alone, without hesitation, music that they have never heard.

Disciple: What you say is very marvelous. Our singers, indeed, have never aspired to such perfection.

Master: Instead, brother, they missed the right path, and failing to ask the say, they labored all their life in vain.

Disciple: How can it be true that a string teaches more than a man?

Master: A man sings as he will or can, but the string is divided with such art by very learned men, using the aforesaid letters, that if it is diligently observed or considered, it cannot mislead.

Disciple: What is this art, I inquire.

Master: The measurement of the monochord, for if it is well measured, it never deceives.

Disciple: Can I perchance learn the exact measurements, simply and in a few words?

Master: Today, with God's help, only listen diligently.

(here follows a demonstration of the monochord)[20]

Odo is absolutely clear. Correct intonation is shown by the monochord, not by singers who have "labored all their life in vain." The monochord "cannot mislead" with the clear implication that instruction lacking it will. Writing about two hundred years later, John (we know little about him apart from his name) concludes his own discussion of the monochord with the line, "As we said, the monochord serves to silence their wrong-headedness, so that those who still not trust the words of a musician are refuted by the testimony of the sound itself."[21] Here again, intonation is "proven" by the monochord. In both cases, it is not the ear that establishes intonation but rather calculation, arithmetic, and the prestige of Boethius's model. The Medieval view, taken from Boethius, is a rigidly Pythagorean method if tuning; Aristoxenus' view that the ear alone is the only valid judge is forgotten, or ignored.

But not only Pythagorean intonation became the standard tuning in the Middle Ages but also what we may call "Pythagorean/Ptolemaic cosmology" became a

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[20] *Enchiridion musices,* attributed to Odo of Cluny, edited and translated by Oliver Strunk, *Source Readings in Music History* (New York: W. W. Norton & Company, 1950), p. 105.

[21] The "they" to whom John refers are bad students (he actually calls them "devil possessed," *energumenos*) who, when corrected when they misperform a chant, "get angry and make a shameless uproar and are unwilling to admit the truth but defend their error with the greatest effort." John, *De Musica*, translated by Warren Babb, *Huchbald, Guido, and John on Music, Three Medieval Treatises* (New Haven: Yale University Press, 1978), p. 110.

fundamental component of the Medieval world view. This is the second part of Boethius' legacy. We already know the basic outlines of this cosmology from classical Pythagoreanism. In the late Classical period aspects of Plato and Ptolemy are added to it creating the idea that music is a fundamental aspect of the physical universe. But writers in the Middle Ages and the Renaissance took the mythology to new lengths. They believed that the orbits of the planets around the earth could be described by ratios. Those rations also described musical intervals and even pitches. These pitches, in turn, could be organized into modes. Thus the cosmos could be described, literally, as music, with specific pitches assigned to the heavenly bodies and the relationships between them described by modes. In 1496, Franchinus Gafurious (1451-1522) included a woodcut of this musical cosmos as the front piece of his great treatise, *Practica musicae.* At the bottom of the page is the earth (Terra). Above the earth we see the orbiting planets, each with its own muse, pitch, and mode. Clio, muse of the moon, "sings" an "A." Calliope,

"By virtue of his intellect has Apollo set [the cosmos] moving."

The Three Graces	Apollo enthroned		
(Muse)	*(Planet)*	*(Pitch)*	*(Mode)*
Stellatum	Urania	A	Hypermixolydian
		(tone)	
Polihymnia	Saturn	G	Mixolydian
		(tone)	
Euterpe	Jupiter	F	Lydian
		(semitone)	
Eratho	Mars	E	Phrygian
		(tone)	
Melpomene,	Sun	D	Dorian
		(tone)	
Terpsicore	Venus	C	Hypolydian
		(semitone)	
Calliope	Mercury	B	Hypophrygian
		(tone)	
Clio	the Moon	A	Hypodorian
Thalia	Terra (earth)		

schemata of the of the *Practica musicae* front piece, opposite

From the *Practica musicae*, 1496, Milan: Guilllermus Le Signerre, printer for Mercury, sings a “B”. Terpsicore, for Venus, sings a “C”. Melpomene, on the Sun, sings a “D.” Eratho, on Mars, sings an “E.” Euterpe, on Jupiter, sings a “F” and Polihymnia, on Saturn, sings a “G.” In the sphere of the stellatum, Urania completes the octave and sings an “A.” Above the stellatum, in the *caelum ipsum*, sits Apollo, enthroned, surmounted by the motto: “By virtue of his intellect has Apollo set [the cosmos] moving.” The life-blood of the cosmos which animates it with movement, is music.[22]

Even after the Ptolemaic model had been abandoned in favor of Copernicus' (1473-1543) heliocentric universe (where the earth and the other planets revolve around the sun), the old cosmology still exercised a powerful pull on thinkers' imaginations. We've already come across Shakespeare's speech in the fifth act of *The Merchant of Venice* where Lorenzo echoes Plato and Cicero's reasons for our inability to hear the music of the spheres.

> How sweet the moonlight sleeps upon this bank!
> Here will we sit and let the sounds of music
> Creep in our ears: soft stillness and the night
> Become the touches of sweet harmony.
> Sit, Jessica. Look how the floor of heaven
> Is thick inlaid with patines of bright gold:
> There's not the smallest orb which thou behold'st
> But in his motion like an angel sings,
> Still quiring to the young-eyed cherubins;
> Such harmony is in immortal souls;
> But whilst this muddy vesture of decay
> Doth grossly close it in, we cannot hear it.[23]

Johannes Kepler (1571–1630), whose three laws of planetary motion validated Copernicus' cosmology, spent much of his life trying to reconcile the heliocentric universe with the old Neo-Platonic/Pythagorean mythology. In his *De Harmonice Mundi*, published in Augsburg in 1619, he argued that the pitches of the planets depended upon their relative velocities (which change as each planet progresses through its

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[22] Gafurius' woodcut is actually a Renaissance attempt at re-paganizing a pagan antique cosmology that had been substantially Christianized in the Middle Ages. In Medieval iconography, God the Father sits enthroned in the *caelum ipsum*, surrounded by His angels and saints. By the power of His love, God sets the primum mobile into motion, i. e. , he gives life to the universe. Apparently trying to present a model of Plato's cosmos form the Timaeus—in other words a Pythagorean/Neo-Platonic pre-Christian cosmos—Gafurius' artist takes that Christianized model, puts Apollo on the heavenly throne, substitutes the three graces for the cherubim, and replaces God's love as the primum mobile by Apollo's intellect. But in the Timaeus, Plato stubbornly refuses to identify God, calling him only “the god who is forever”, and while the direction of the primum mobile's movement if clear, its cause isn't.

[23] Shakespeare, *The Merchant of Venice*, Act V, scene i.

elliptical orbit), pitches which are heard only by the "soul" that animates the sun. John Milton (1608-1674), writing a generation later, has Adam hail the first day of creation in his 1667 *Paradise Lost* as that "mystic dance, not without song."

> These are thy glorious works, Parent of good,
> Almightie, thine this universal Frame,
> Thus wondrous fair; thy self how wondrous then!
> Unspeakable, who sitst above these Heavens
> To us invisible or dimly seen
> In these thy lowest works, yet these declare
> Thy goodness beyond thought, and Power Divine:
> Speak yee who best can tell, ye Sons of Light,
> Angels, for yee behold him, and with songs
> And choral symphonies, Day without Night,
> Circle his Throne rejoycing, yee in Heav'n,
> On Earth joyn all ye Creatures to extoll
> Him first, him last, him midst, and without end.
> Fairest of Starrs, last in the train of Night,
> If better thou belong not to the dawn,
> Sure pledge of day, that crownst the smiling Morn
> With thy bright Circlet, praise him in thy Spheare
> While day arises, that sweet hour of Prime.
> Thou Sun, of this great World both Eye and Soule,
> Acknowledge him thy Greater, sound his praise
> In thy eternal course, both when thou climb'st,
> And when high Noon hast gaind, and when thou fallst.
> Moon, that now meetst the orient Sun, now fli'st
> With the fixt Starrs, fixt in thir Orb that flies,
> And yee five other wandring Fires that move
> In mystic Dance not without Song, resound
> His praise, who out of Darkness call'd up Light.[24]

The "harmony" of the heavens (which is the balance between extremes, not the concord between different music pitches with which we associate the term today) was another aspect of Boethius' *musica mundana*. As well as describing the delicate balance between elements observed in the heavens it was also was reflected in the harmony of the human being, or the *musica humanis*, something we discussed in the previous chapter.

Music can restore health, or *harmonia,* because music and the harmonies of the cosmos are one in the same: they are number. And number is an object of thought and contemplation, not something experienced through the senses. This is our third legacy from Boethius: the division of those who "think" about music from those who "merely" perform it and write it, or what was known in the Middle Ages as the division between

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[24] John Milton, *Paradise Lost,* Book V, lines 153-179

musici and *cantors*, “musicians” and “singers.” Guido d’Arezzo (991? – 1050) is unusually blunt with he begins his *Prologus antiphonariis sui* with “In our times, of all men, singers are the most foolish,”[25] but his comment testifies to the sharp division drawn between the superior contemplator of music and the lowly performer of musical pieces. Singers (*canti*) and not musicians (*musici*). We have already seen where the origins of this notion lie in Antiquity. Those precedents are taken-up by Augustine when he writes that the proper object of thought is not “the art but the science of music.”[26]

Boethius classifies three types of “musicians.”

> He however is a musician who on reflection has taken to himself the science of singing, not by the servitude of work but by the rule of contemplation – a thing that we see in the work of buildings and wars, namely in the opposite conferring of the name. For the buildings are inscribed and the triumphs held in the names of those by whose rule and reason they were begun, not of those by whose labor and servitude they were completed.
>
> Thus there are three classes concerned with the musical art. One class has to do with instruments, another invents songs, a third judges the work of instruments and the song. But that class which is dedicated to instruments and there consumes its entire efforts (as for example the players of the kithara and those who show their skill on the organ and other musical instruments) are separated from the intellect of musical science, since they are servants, as has been said, nor do they bear anything of reason, being wholly destitute of speculation. The second class having to do with music is that of the poets, which is born to song not so much by speculation and reason as by a certain natural instinct. Thus the class also is to be separated from music. The third is that which assumes the skill of judging, so that it weights rhythms and melodies and the whole of song. And seeing that the whole is founded in reason and speculation, this class is rightly reckoned as musical, and that man as a musician who possesses the faculty of judging, according to speculation or reason, appropriate and suitable to music, of modes and rhythms and of the classes of melodies and their mixtures and of all those things about which there is to be discussion later and of the songs of the poets.[27]

Performers on instruments cannot understand the “reason” of music and are therefore “destitute of speculation” which, I think, means that they aren’t able to juggle the math of the ratios. Without understanding they are but servants. They have a “certain natural instinct” for invention but little else, which means that they’re a bit like animals. A twelfth-century writer compares a singer with a drunk, “who does indeed get home but does not in the least know by what path he returns.” Such a singer only holds the road by

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[25] Guido of Arezzo, *Prologus antiphonarii sui*, in *Source Reading in Music History*, p. 117. But to be fair to Guido (as well as to other medieval writers), I should point out that his purpose in writing is not to ridicule the singers but rather to make them into musicians, not always an easy task.

[26] Quoted by Henry Chadwick, *Boethius*, p. 87.

[27] Boethius, *De institutione musica*, *Source Readinsg in Music History*, p. 86.

habit, whereas “the musician always proceeds correctly and by calculation. . .”[28] Musicians are those who can think about the music, who know it by *calculation*. And it’s through calculation that they can grasp the “whole” of music.

Of course there is a good deal of simple social snobbery here, as there was in Classical Antiquity too. For the most part performing musicians came from the lower classes, philosophers from the upper (Boethius counted two Roman emperors among his ancestors). But there is more at work here besides aristocratic disdain. It isn’t only that the lower orders play music and the upper ponder it but rather the kind of thing the philosophers tended to think about and call “music” was really a very different sort of thing than what the lower orders composed, sang, and danced to. For Boethius and his intellectual descendants—as it was for the earlier Pythagoreans and Neo-Platonists—music is something which elevates the soul to divine realms, an ascent possible because music and the heavens share a common language: number. The pure contemplation of music is the contemplation of the divine in the form of numerical relationships: 3:2, 4:3, 81:64, etc.

When we read medieval authors writing about the importance of music we must be careful not to misunderstand them. Sometimes, as when they are writing handbooks for training boys to sing (as in the case of Guido d’Arezzo), they are indeed talking about what we today would recognize as “music.” But frequently what they are referring to as “music” we would either call “geometry” or even perhaps “number games.” Today, when we talk to someone and hear him say that he found a particular musical performance “heavenly” or that he was in some way “moved” by a particular piece we understand him to mean that the piece of music or the particular performance had a powerful *emotional* impart upon him. He was moved *emotionally*, and this is what we talked about at some length in the previous chapter. But apparently for most Medieval writers it was the abstract importance of the numbers that interested them and that interest was intellectual, not emotional.

But there is at least one very important exception to this rule: the case of St. Augustine (354-430). Augustine was the bishop of the North African city of Hippo and his extensive writings had an enormous influence in the development of Western Europe. Although elsewhere he attested to the intellectual importance of music in a typically Pythagorean fashion, in his *Confessions* (which is his autobiography) he wrote of music’s powerful emotional impact upon him, apparently driving him near to what he feared was idolatry.

> How much I wept at your hymns and canticles, deeply moved by the voices of your sweetly singing Church. . . But when I recall the tears which I shed at the song of the Church in the first days of my recovered faith, and even now as I am moved not by

[28] John, *On Music* (trans. by Warren Babb), pl 105

> the song but by the things which are sung, when sung with fluent voice and music that is most appropriate, I acknowledge again the great benefit of this practice . . . Yet when it happens to me that the song moves me more than the thing which is sung, I confess that I have sinned shamefully and then prefer not to hear the singer. Look at my condition! Weep with me and weep for me, you who so control your inner feelings that only good comes forth. And you who do not behave thus, these things more you not. You however, O Lord my God, give ear, look and see, have pity and heal me, in whose sight I have become an enigma unto my self; and this itself is my weakness.[29]

This passage is remarkable on so many levels that it probably deserves its own book (and so much has been written on Augustine that it probably has had one) but for our purposes here it's enough to note that despite the silence of so many early writers on the matter, the strongly emotional nature of music which allows us to order our emotional lives – which you will remember I thought was music's fundamental character at the end of the last chapter – was not unknown in late Antiquity; it was at least known to Augustine and he wrote about it thinking that his readers would understand too.

The fourth part of Boethius' legacy is institutional: because of him music is part of the Liberal Art's Quadrivium. Because of the ratios of its intervals, Boethius included it with the three other mathematical arts: arithmetic, geometry, and astronomy. And it's through its inclusion in the Quadrivium that music becomes a founding part of the universities that grew up in Europe beginning in the twelfth-century. It is also because of its inclusion in the Quadrivium that medieval thinkers came to understand music as one of the primary components of civilization. The designers of Chartres Cathedral chose to give this last notion a particularly dynamic expression when, in the voussiors of south portal on the cathedral's great western façade, they surround the infant Christ, enthroned on the Virgin Mary, with personifications of the liberal arts. There, along with the other members of the Quadrivium and Trivium (grammar, rhetoric, and dialectic) we see the muse of Music, with a psaltery on her lap and a viol hanging behind her, striking three bells (perhaps for the perfect consonances of the fifth, fourth, and octave), with Pythagoras, working at a lap desk, beneath her.

[29] Augustine, *Confessions*, IX, 14; X, xxxiii, 50.

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Previous page: Chartres Cathedral, west façade, largely constructed between 1193 and 1250. This page: left: tympanum above the south door (far right in the picture of the façade) showing the infant Jesus on the lap of Mary with scenes of the Nativity, and in the voussiors figures representing the Liberal Arts Music (left) and Grammar (right) with philosophers below; right: close up of Music and Pythagoras below her.

III

By now you have probably wondered what all of this has to do with the questions I raised when we began this chapter. The sculptures on the Cathedral of Chartres, Boethius, the "reason" of music (meaning the ratios), etc., it's all very interesting but what does this have to do with our modern keyboard, the way we name notes, and most importantly, how we know then we're in tune and when we're not?

A lot. I touched on this earlier but for one thing we appear to have answered the question of why our notes are made-up of that odd pattern of different-sized steps. That pattern of step, step, half-step, step, step, step, half-step (which you might know from the C major scale)—which we today call *diatonic* (meaning there are no sharps or flats added to it), is simply the kind of scale you produce if you calculate intervals in the Pythagorean method, that is by projecting perfect fifths (with of course one subtraction to get the perfect fourth). The structure of the keyboard, at least in regards to the "white notes" and within one octave, reproduces that fundamental pattern. The modern design testifies to the enormous prestige the Pythagorean/Platonic legacy has commanded in our culture for the last two thousand and three hundred years.

Second, it would also seem that we've answered the question of how the space between pitches is determined. It's not an arbitrary decision but based upon mathematical logic. How big is a whole step and how do we know? It's an interval defined by the ratio 9:8. How big is a Perfect Fifth? It's defined by the ratio 3:2. The major third? Defined by the ration 81:64. For as long as eighteen hundred years students were taught to recognize the distance between notes by constructing intervals on the monochord and learning the sound. In school at least, the monochord came first, sound second.

That answer (and there's a problem with it that I'll return to later) can help us understand another question raised at this chapter's beginning: the relationship between "C sharp" and "D flat," or between "G flat" and "A sharp," or between "E sharp" and "F", and many others—all what we call, somewhat mistakenly, *enharmonic* pitches. On the piano they are all played by pressing the same lever. But are they the same pitches? Do they really make the same sound, or better, are they both the same frequency? According to what we've just learned, no. Let's return the issue of "E sharp" and "F natural." In our example, where 2:1 gives us the perfect octave "C" to "C", the ratio for the perfect fourth, which will give us the pitch "F", is 4:3. But the ratio that will produce for us "E sharp", the ratio for the augmented third, is $(3:2)^{11}$ which, when we reduce it down to our original octave, is 177,147:131,072. Simplify that ratio to the more usable 4.43:3.28 we can immediately see that this ratio is substantially larger than 4:3, the ratio of the perfect fourth. And when "E sharp" is played by violinist, or cellist, or trombonist, it's actually higher than when that same musician plays a "F" (especially if it's the leading tone to "F sharp"). So, "E sharp" is not the same ratio as "F" and the pitches are different.

But we know this isn't the case when we play both notes on the piano. They are the same. Both notes are produced by the depressing the same lever that hits the same course of strings that produces exactly the same sound. But is this a mistake? Is the piano built wrong?

This is a very important question that we will spend some time on later, but now I have to return to an issue that I dealt with rather cursorily earlier. You will remember that we were able to calculate all the intervals of our basic diatonic scale (minor second, major second, major sixth, etc.) by squaring the ratio of the perfect fifth (3:2) and then reducing that ratio by the appropriate octave (2:1, 4:1, etc.)—all the intervals, that is, **except the perfect fourth**. That interval we had to calculate by finding the difference between the Perfect Octave and the Perfect Fifth.

I have that "except. . . ." in **bold** because it's really important. It's a mathematical slight of hand that points to a very important matter. If that perfect fourth is not part of the spiral of perfect fifths, if it does not appear "naturally" in that mathematical process, why did the Pythagoreans need to find a way to calculate it? Why didn't they just ignore it? Or why didn't they simply include the interval of the augmented fourth (which would, in our example, give us a "F sharp") that appears in the spiral of perfect fifths and make that the "diatonic" model? Why did they feel compelled, at least on this occasion, to negate their process of ascending perfect fifths and instead obtain the perfect fourth by subtraction? Why the wink and the nod?

The reason is simple—and memorable. The Greeks had to include the perfect fourth because it already existed in their musical culture. And it existed in that music culture because (1) the Greeks heard it and (2) they liked it. Indeed, the *tetrachord* (the name the Greeks gave to a unit of four adjacent pitches spanning the interval of a perfect fourth) was the fundamental unit of Greek scales and the "Greater Perfect System" (or the full spectrum of pitches available to the Greeks) was describes as consisting of four tetrachords. Just as today most western musicians think in units of tertian chords (or chords built out of three notes arranged a third apart), the Greeks thought in tetrachords.

The decisions to hear the perfect fourth as a discrete interval and to recognize it as a "harmonious" sound and to shape a musical culture in which it is the primary structural interval (and not the perfect fifth or the octave) are all fundamentally *aesthetic decisions.* The orbits of the planets, the "perfection" of some numbers (and the "imperfection" of others), the immutability of divinity, the correspondence between the World Soul and the make-up of the souls of men—all the stuff of the Pythagoreans and Neo-Platonists—can add nothing to somebody's decision to *like* a particular sound just because he likes it. Back at smithy, Pythagoras heard the interval of the perfect fourth and *liked* it. He heard the interval of the perfect fifth and *liked* it. He heard the perfect octave and he *liked* it. He called them *concords*. He liked the *sounds.* He heard the major second and didn't like it as well, calling it a *dissonance*. We don't "like" or "dislike" numbers or ratios or the relationships between the orbits of planets. We "like" certain people, or colors, or particular activities like golf or tennis or Thursday night bingo. At the smithy, Pythagoras liked the sounds before he ran the numbers. "Like" came first. Ratio second.

The phenomenon of the perfect fourth helps us understand the fact that tuning **is primarily an aesthetic activity and as an aesthetic activity it is culturally dependent.** Why did Pythagoras "like" the concords? Iamblichus writes that upon hearing them, Pythagoras "recognized in these sounds the *concord* of the octave, the fifth, and the fourth" (my italics).[30] "Recognized." That's an important word. Pythagoras did not discover these sounds, and in telling the story Iamblichus doesn't expect his readers to think of them as exotic either. As a member of a culture where these sounds were part of its musical fabric, Pythagoras heard them and knew already that they were concords. That's just the way he'd been brought up; he didn't need to be reminded of that. Who says that the fifth, fourth, and octave were concords? Well, just about everybody. Ask any Greek. It was a part of their culture.

But cultures change. Pythagorean intonation formed the foundation of Western musical practice not as long as the intellectuals could justify it philosophically but instead as long as musicians liked the way it sounded. Until the end of the fourteenth century most musicians considered only the Pythagorean "perfect" intervals for harmonic consonances. While second, thirds, sixths and sevenths could be found between voices in their polyphonic works composers accepted such intervals only as dissonances with which they enlivened their music. At both internal and final cadences they used only the perfect intervals. But beginning as early as 1350 some composers began to use thirds as "imperfect" consonances in their works' interior phrases. By the death of the English composer John Dunstable in 1453 the third was firmly established as an aesthetically acceptable interval for interior cadences. By the next generation's close composers regularly ended their works with triads containing major or minor thirds.

But these thirds were increasingly not Pythagorean thirds. Tastes had changed. In the context of a triad many Renaissance musicians developed a distaste for the Pythagorean intervals, finding the major third of 81:64 "too sharp" and the minor third of 32:27 too flat for their taste. So, they began to "fudge" the intervals. The minor third they apparently preferred to sing a bit sharper, close to the ratio of 6:5, and the major third they began to sing a bit flat, close to the ratio 5:4. This process of adjusting intervals is called "tempering" and the scale which resulted from this process, where

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[30] Iamblichus, *The Life of Pythagoras*, p. 86

perfect intervals kept their Pythagorean sizes but where thirds were adjusted), became known as "just intonation."

Originally singers did this fudging by ear, something the English Benedictine Water Odington described as early as 1300. But in 1482 the Italian theorist Bartolomé Ramos de Pareja (c. 1440- c. 1522) described a division of the monochord that resulted in ratios yielding a just tempered scale.[31] It was one thing for singers to intuitively adjust pitches but it was another thing altogether for an intellectual to suggest an alternative to the tuning system of Pythagoras, Plato, and Boethius – and in so doing calling into question the whole baggage of Pythagorean metaphysics along with it. As could be expected, Ramos' position was aggressively attacked. Yet, while writers who wrote about music continued to defend the Pythagorean tradition, musicians continued to listen, and to innovate. Even Gafurius, who strongly opposed Ramos' ideas about tuning, mentioned his own acquaintance with organists who customarily flattened fifths when they tuned their instruments.[32] This method of temperament, in which fifths were flattened but thirds remained unaltered, we call today "meantone" temperament.

For the next several hundred years musicians experimented with various schemes of tuning.[33] Although just intonation was advocated by Ludovico Fogliano in 1539 and by Martin Agricola ten years later, by the middle of the sixteenth century meantone tunings had gained favor with many of Europe's most important musicians. Pietro Aaron (c. 1480-1550) advocated for meantone temperament in his *Toscanello in musica* (1523), Gioseffo Zarlino (1517-1590) in his *Dimostrationi armoniche* (1571), and Michael Praetorius (?1571-1621) in his *Syntagma musicum* (1618). Although he argued for what we will call equal temperament (in certain circumstances), Marin Mersenne (1588-1648) gave directions for constructing several meantone scales in his *Harmonie universelle* (1635-1636) and in his 1666 *Propositiones mathematico-musical*, Otto Gibelius (1612-1682) gave instructions for constructing a meantone monochord with fourteen pitches to the octave.[34] But even with the tuning's enthusiasts there was little agreement. In a letter around 1600, the Dutchman Abraham Verheyen wrote to Simon Stevin that he knew of at least four different kinds of meantone temperament. There were actually more, one modern scholar has found evidence for at least seventeen. Besides these, many musicians invented the own "irregular" temperaments which, while derivative of meantone practices, incorporated idiosyncratic refinements, the most famous of which was the great German organ builder Gottfried Silbermann's (1683-1753) taste for flattening all his fifths by 1/6 of a comma. Tuning was so controversial that during much of the

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31 For Ramos' calculations see the selection translated from his *Musica practica* included in Oliver Strunk's *Source Readings in Music History*, pp. 200-204.

32 Franchinus Gafurius*, Practica musica*, Book 2, Chapter 3. See Barfour: *Tuning*, p. 5

33 What follows is only the barest bones summary of the development of various temperaments. For a thorough overview of this subject see: J. Murray Barbour: *Tuning and Temperament: A Historical Survey* (East Lansing, Michigan: Michigan State College Press), 1951.

34 See: Barbour, *Tuning*, Chapt. III

seventeenth century it was assume that groups of differing instruments (such as keyboard instruments and lutes, or strings with woodwinds) could not play together, the groups were impossible to tune.

The second change that influenced tuning was sociological as well as aesthetic. Although certainly used in the Middle Ages, instruments played only a secondary role in most Medieval music. Vocal composition and performance dominated the musical culture – so much so that composers only infrequently even bothered to write-out any instrumental music at all. But in the Renaissance this began to change. Any man who aspired to civility was expected to play the lute and to play it fairly well. Performing on an instrument became an accoutrement of a gentleman, like the ability to fence, ride, dance, and to manipulate that new invention at the table, the fork. Young women and girls of rank were expected to be musically competent on an instrument; in England so frequently was the clavier their choice that the instrument became knows as a "virginal." Eventually fretted instruments (such as the lute and guitar) and keyboards became the most popular instruments in Europe and became the foundation of the continent's musical culture.

But tuning these instruments posed serious problems. Consider the following example.

Let's imagine clavier builder has built an instrument of four octaves with the traditional keyboard pattern that we saw at this chapter's beginning. And let's suppose that he's going to tune it by simply keeping the perfect fifths and octaves in tune with each other (that is with the intervals defined by the ratios 3:2 and 2:1). He begins by tuning "F" and then tunes all the "F's" up and down the keyboard, keeping those courses of strings perfectly in tune with each other. Then he goes up a perfect fifth, tuning "C". He makes sure that the "C" is a pure 3:2 ratio above his first "F" and then, when that's done, he tunes all the "C's" on the keyboard to be perfectly in tune with each other by the

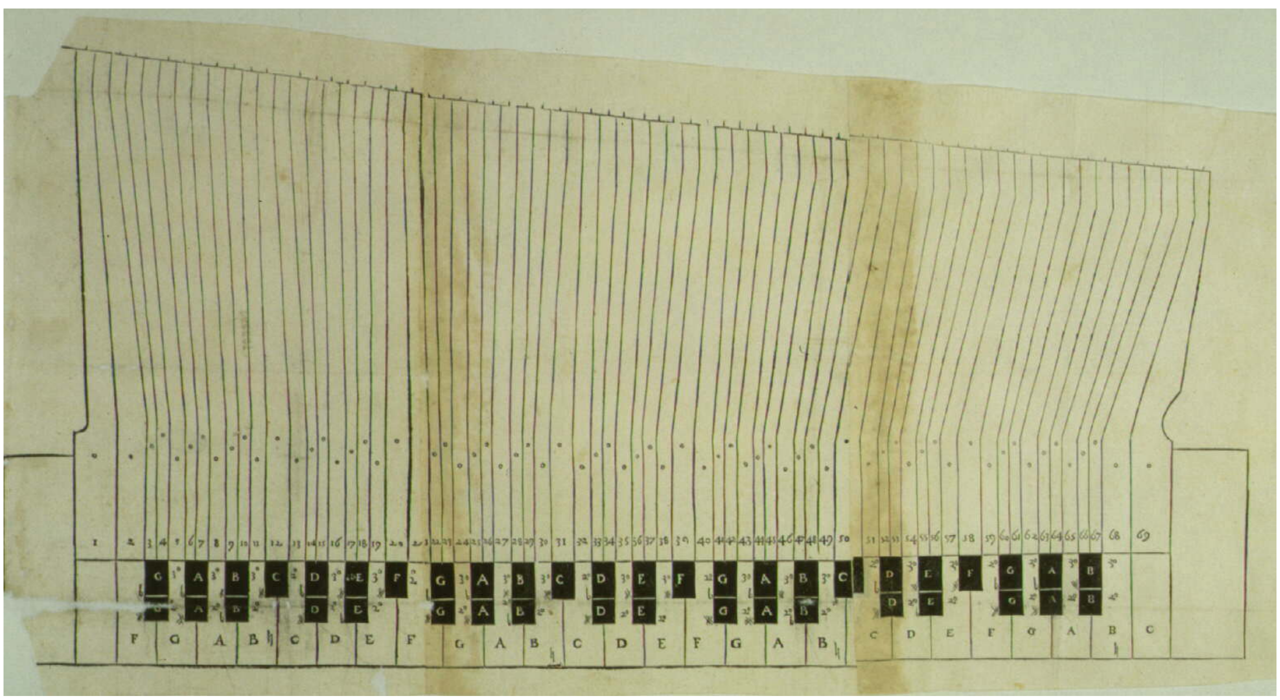
F C G D A E B F# C# G# D# A# E#?

Pythagorean ratio of 2:1. Then he goes up another perfect fifth, tuning the course of strings sounding "G" to be a pure 3:2 above C and then tunes all the "G" octaves to be purely in tune with each other. And he continues the process, each time ascending perfect fifths, "G" to "D" to "A" to "E" to "B" to "F sharp" to "C sharp" to "G sharp" to "D sharp" to "A sharp and then to "E sharp".

But here he has a problem. We already know that a projection of twelve perfect fifths exceeds the projection of seven octaves by the Pythagorean comma, or 23.5 cents.

In-other-words, his “E sharp” is almost a quarter tone higher than the “F” that he began with.

Our instrument builder has several solutions. First, he can simply let his “F’s” stand as they are and ignore the “E sharp”. This would result in a keyboard that would sound lovely in what today we would call the keys of C Major, G Major, and D Major and a minor. But other keys would be problematic and some impossibly out of tune. F Major would be problematic because the “A sharp” that our builder has tuned would much sharper than the “B flat” required in F. The keys of c minor and f minor would be problematic because the thirds of their tonic chords would excessively sharp (“D sharp” instead of “E flat” in the case of c minor and “G sharp” instead “A flat” in the case of f minor). And any chord containing “A sharp” would sound so out of tune that pitches like this were called “wolf tones” because musicians thought that they sounded like the howling of a wolf.



Design for the lower keyboard of Vicentino’s archicembalo
Printed by Antonio Barre, Rome, 1555
Vatican Library Casimiri II 173 foldout music28 NB.23

Our instrument maker has a second solution. For the “E sharp” he could add another lever to the keyboard and one more course of strings to the instrument’s harp. This would take care of the “wolf” since he’d have a pure fifth between “A sharp” and “E sharp” but it would add at least two additional problems. First, once our instrument

Two views of the *Clavemusicum Omnitonum Modulis Diatonicis Cromaticis et Enearmonicis* exemplifying Vicentino's principles, built by Vito Trasuntino, Venice, 1606. Bologna

maker started adding courses of strings and new levers to the original twelve courses and keys it gets hard to know when to stop. Since, in our example, we don't have a pure fourth between "F" and "B flat" (we have instead an "A sharp"), should our instrument maker add a lever and course of strings here too, giving us a true "B flat"? And should we now have a "B flat" does that require our instrument maker to add a "E flat" to correct the problems we would have between his new "B flat" and the "D sharp" that has already been built? Yes, he should and he could. And instruments accommodating the extra keys were built for just this purpose. The most famous musician to argue for this was Nicola Vicentino (1511-1576) who designed an "archicembalo", or large harpsichord, with thirty-six keys to the octave divided between two keyboards (there are fewer than thirty-six different pitches since some of the keys sound courses of strings that are duplicated between the keyboards). A design for the lower keyboard exists in the Vatican library, published by the Roman printer Antonio Barre in 1555 and the Museo Internazionale e Biblioteca della Musica in Bologna preserves an archicembalo built after Vicentino's death by the Venetian instrument builder Vito Trasuntino. But Vincentino was not alone in his interest in creating complex keyboard instruments. In his 1558 *Istitutioni armoniche*, Gioseffo Zarlino describes a cembalo that "Master Domenico Pesarese" made for him, an instrument with nineteen notes to the octave. Experimen - tation continued even into the 19th Century; the Englishman Robert Holford Macdowall Bosanquet (1841-1912) developed instruments with the octave divided into eighty-four , forty-eight, and twenty-two pitches to the octave.

But these proved to be highly unsatisfactory solutions to our tuning problem because the keyboards were just too hard to play. Nobody used them. Clearly a third solution had to be sought and it was first shown by lute players.

Because of the frets across its neck, a player on the lute can produce several pitches by his same finger position (a fret is a raised piece of bone or metal imbedded in the neck of a string instrument that stops the sting. Guitars, lutes, banjos and mandolins have frets; violins, viola, celli and double basses don't). For instance, "E flat" and "D sharp" had to be playable by the same finger position while at the same time "E flat" had to form an acceptable fifth both with the string potentially producing a "B flat" and with the string producing an "A flat." Neither Pythagorean tuning, meantone or just temperaments could accomplish this. In 1533 Giovanni Lanfranco in his *Scintille di musica* proposed the following solution: the player should lower his instrument's fifths "so flat that the ear is not well pleased with them" and raise the thirds "as sharp as can be endured."[35] This is our first description of what became known as "equal temperament." In 1577, Francisco Salinas (1513-1591) argued that all fretted instruments should be tuned in octaves made-up of twelve equally sized semi-tones.[36] In 1581, Vincenzo

[35] Barbour, *Tunings*, p. 50. This Giovanni Lanfranco is not to be confused with the painter of the same name who lived a generation later, between 1581-1647.

[36] Francisco Salinas, *De musica libri septem*, quoted in Barbour, *Tuning*, p. 50.

French composer and lutenist Charles Mouton (c. 1626-1710) engraved by
Gerard Edelinck (1640-1707) after a painting by François de Troy (1645-1730).
Notice that the frets across the lute's neck are clearly visible.

Galilei (c 1520-1591), the father of the astronomer, gave more specific instructions for lute tuning. If one used the ratio of 18:17 for the semi-tone and then apply that ratio twelve times, Galilei found that he could arrive at the interval of the perfect octave.[37] Here, for the first time, a tempering system was proposed in which each interval -- half steps, whole steps, major and minor thirds – were all the same size across the musical spectrum regardless of what pitches they were calculated from. Galilei's reckoning isn't quite right (the twelfth fret would not mark the string's exact midpoint but would fall short by twelve cents, or slightly more than on tenth of a modern half step), Galilei's work would serve as the focus of corrections by several generations of musicians.

Girolamo Frescobaldi (1583-1644) was the first major composer to endorse equal temperament for keyboards as well as fretted instruments and, while he described just intonation as the basis of the modern major scale, Mersenne argued for the adoption of equal temperament as a practical necessity, at least with lutes. Godfrey Keller in his *A Compleat Method for attaining to Play a Thorough Bass upon either Organ, Harpsichord, or Theorbo-lute, by the late famous Mr. Godfrey Keller. With a variety of proper Lessons and Fugues and a Scale for Tuning the Harpsichord or Spinnet, all taken from his own copies which he did design to print* (it's too marvelous a title not to cite it in full), published three years after his death in 1704, advocated for equal temperament. Jean Philippe Rameau in his 1737 *Génération Harmonique, ou Traité de Musique Théorique et Pratique*, F. W. Marpurg in his 1776 *Versuch ueber die musikalische Temperation*, and Johann Philippe Kirnberger in his 1779 *Die Kunst des reined Satzes in der Musik* all wrote about of equal temperament.

Although Farthold Fritz's 1780 *Rules for Equal Temperament* gained the praise of Johann Sebastian Bach's son Emanuel, the elder Bach probably did not share his son's enthusiasm for the tuning. Since "well tempered" (*wohltemperite*) was frequently used in Germany not to describe equal temperament but rather any temperament in which all keys could be used with equal facility, it is misleading to argue that Bach intended his famous collections of preludes and fugues as written specifically for an equally tempered instrument. It probably wasn't. Since Johann Sebastian Bach once instructed his pupil Johann Kirnberger (1721-1783) to temper all thirds higher than pure ("pure" meaning "just" thirds), it can be argued that Bach probably preferred some irregular form of meantone temperament.[38]

Johann Sebastian Bach's opposition not withstanding, through the course of the latter eighteenth century equal temperament slowly displaced other temperaments so that by the close of the nineteenth century other tuning systems had become largely historic curiosities.[39] When Richard Wagner (1813-1883) wrote *Tristan und Isolde* in 1865 and

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[37] *Dialogo della musica antica e moderna* (Florence, 1581), p. 49. Galilei did not publish any calculations for his temperament, a deficiency corrected by Johannes Kepler when he included calculations for this kind of equally tempered scale in his *Harmonices mundi* in 1619. See: Barbour, *Tunings*, p. 57.

[38] See "Well Tempered Clavier," *The New Groves*.

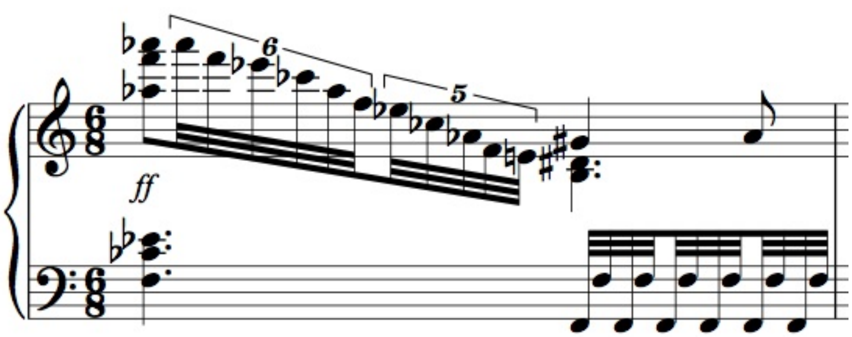
[39] But in some areas adherents of the old systems proved to be remarkably tenacious. English cathedral organists refused to abandon meantone for equal temperament until well into the 1870's. For a discussion

moved between two spellings of his famous "Tristan Chord" he was fully intending his musicians to be using equal temperament. "G sharp" and "A flat" were to be exactly the same pitch, as well as "B natural" and "C flat."

m. 2 m.81

First appearance of the Tristan Chord in m.2 and a simplification
of the chord as it is found in m. 81

m. 83



Both spellings of the Tristan Chord in the same measure

Simply stated, in equal temperament the octave is divided into twelve equally sized semi-tones. Each of these semi-tones is one hundred cents apart, the distance between octaves being 1200 cents. This produces uniformly sized intervals regardless of pitch and allows for easy modulation between distantly related keys. Going pack to the problem confronted by our instrument builder with his "E sharp" and "F", here this is no problem because, at least within the same octave, they are both exactly the same pitch. But to accomplish this uniformity two prices had to be paid. First, in equal temperament

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of the history of Pythagorean tuning into the modern area see: J. Murray Barbour, "The Persistence of Pythagorean Tuning Systems," *Scripta Mathematica I*, 1933, pp. 286-304.

Comparison of Tuning Systems

Pythagorean Tuning				Just Intonation[40]		Meantone Intonation[41]	Equal Temperament
Interval	*Pitch*	*Ratio*	*Cents*	*Ratio*	*Cents*	*Cents*	*Cents*
Unison	C	1:1	0	1:1	0	0	0
Minor 2nd	Db	256:243	90.2	16:15	111.7	- - -	100
Aug. prime	C#	2187:2048	113.7	25:24	70.7	76.1	100
Major 2nd	D	9:8	203.9	9:8	203.9	193.2	200
Minor 3rd	Eb	32:27	294.1	6:5	315.6	310.3	300
Major 3rd	E	81:64	407.8	5:4	386.6	386.3	400
Perfect 4th	F	4:3	498.0	4:3	498.0	503.4	500
Aug. 4th	F#	729:512	611.7	45:32	590.2	579.5	600
Perfect 5th	G	3:2	702.0	3:2	702.0	696.6	700
Aug. 5th	G#	6561:4096	815.6	25:16	772.6	776.6	800
Minor 6th	Ab	128:81	792.2	8:5	813.6	- - -	800
Major 6th	A	27:16	905.9	5:3	884.4	889.7	900
Minor 7th	Bb	19:16	996.1	9:5	1017.6	1006.9	1000
Major 7th	B	243:128	1109.8	15:8	1088.3	1082.9	1100
Octave	C	2:1	1200.0	2:1	1200.0	1200.0	1200.0

[40] These figures are from *Marpurg's Versuch ueber die musikalische Temperature*, Monochord 1
[41] These figures are from Aaron's *Toscanello in musica* (1532), one-quarter comma temperament.

all the intervals are slightly "out of tune" with the exception of the octave which maintains the pure Pythagorean tuning of 2:1. Compared with Pythagorean tunings, fifths are just a bit flat and fourths a bit sharp but not so much that the difference is heard (both are off by two cents). The difference is more noticeable where the equally tempered third is almost ten percent lower than the Pythagorean. And the Pythagorean augmented fourth is almost twelve percent higher than the equally tempered interval.

Second, with equal temperament the strong distinctions between keys that characterized other tuning systems is homogenized. In just or meantone temperaments the "A flat" characteristic of f minor gives the key a strident quality (because of the typical "wolf" the "A flat" caused) and distinguishes it from the less dramatic g minor (with the much less problematic "B flat" for it's minor third). But with equal temperament these distinctions, and many others, are flattened-out as every key sounds pretty much like every other key except for its range. It was this lack of tonal character that probably caused Johann Sebastian Bach's preference for some sort of "well tempered" system over equal temperament. More recently it has lead to a renewed interest in the older tunings both for the performance of earlier music (even up to the piano works of Schubert) and for new compositions.

The differences between the tunings we've discussed are summarized in the table *Comparison of Tuning Systems* on the previous page. We'll return to this table in a moment but before we do we must consider two remaining issues: "standard pitch" and the "harmonic series."

IV

I expect that through the preceding discussion you've been wanting to ask two simple and quite reasonable questions: Why all this *fal-dal-rahl* about ratios and wolf tones and perfect intervals? Isn't there a standard for pitches? Isn't the pitch a^1 (the first "A" above "middle C") 440 Hz? Isn't that an objective standard, like the number of feet in a mile? And what about the physical nature of music? The business of wavelengths and harmonic and envelopes, don't these things influence, and indeed set, tuning systems? Haven't we just wasted our time?

No, we haven't, but before I answer that more fully we have to do a little house keeping.

You've already learned one unit used for measuring the distance between pitches, the *cent* (which is one one-hundredth of an equally tempered half step). We need to learn another: the *hertz*. Named after the German physicist Heinrich Hertz (1857-1894), the hertz (abbreviated Hz) measures cycles per second. One hertz means one cycle per second. Sound is carried in waves and one "cycle" is the appearance of one full wave, with both its peak and trough. The more waves per second the higher the pitch, the fewer waves the lower the pitch. Adults can generally hear between 20 Hz and 20,000 Hz. Whereas cent is used as an objective measurement of the distance between pitches, we use hertz as an objective identifier of pitch.

"A 440" precisely identifies a pitch, it's the "A" a major sixth above what we call "middle C" sounding precisely at four hundred and forty cycles per second. Yes, it's used in many cases as an objective standard for being "in tune" but it's relatively new and in any case of more help for engineers than musicians.

Before the 19th Century, pitch was relative. In the Middle Ages, mode, or the pattern of half and whole steps that characterized the structure of a chant, might be described as having a range of "D to D" or "F to F" but those pitches themselves were of little interest, it was the pattern of steps and half steps that characterized those particular octaves that was important. A chant in Mode One (which was the mode that began on "D") could be pitched high or low, perhaps beginning on what we today would call a "B flat" or even a "F"; it didn't matter as long as the pattern of half and whole steps was correct. The range of the cantor (and whether or not he had a cold that morning), the acoustics of the building, the time of day all could have an impact on the pitch of the chant.

But instruments are not as flexible as singers, they can't be continually rebuilt to play higher or lower (although modest adjustments can be made). And as instrumental music became a more important part of the musical culture in the Renaissance local standardizations of pitch came into practice. This standard was generally set by the parish organ. Not only was it the biggest instrument in town (and also the culture's most sophisticated piece of technology), once pitched and tuned it was the most labor intensive to alter. So, for instance, whatever was "A" on the parish organ became "A" for that area, at least generally speaking—there were always exceptions. But the pitching of one organ in one town would be different from the pitching of the organs in neighboring towns and of course even here the pitch of the organ would change depending on the season. Seeking to set a standard of uniformity, Michael Pratorius suggested a basic pitch for Germany of about A 430. Nothing came of it.

John Shore (c 1662-1752) was one of the greatest trumpet virtuosos in England and his playing was admired by both Henry Purcell and George Frideric Handel. But we remember him today for the tuning fork which he invented in 1711. Because it produced a pure tone and its metal construction was unaffected by changes of temperature and humidity it proved much more reliable in producing a constant pitch than the wooden pipes that had been used previously as ways of checking pitch.

late 19th century tuning fork on a resonator base by Max Kohl, Chemnitz, Germany

In 1751 Handel had a tuning fork that produced A 422.5 Hz. But the forks were difficult to manufacture so that they produced exactly the same pitch and we have tuning forks from the period producing "A's" at 419, 422, 424, 427, 430, 438, and 454.[42] Through the Baroque period pitch continued to differ widely depending on region. Some German organs were known to be pitched as high as A 506 while the French had organs pitched at 399 Hz. In England, Worchester Cathedral's organ was pitched at an almost unbelievably low A 360 Hz.

This lack of uniformity is probably best documented in Alexander J Ellis' *The History of Musical Pitch*, published in 1880 in London. After an extensive introduction to the matter of pitch, Ellis, in a table of several hundred entries, lists different pitches for "A" in the countries of Europe and the United States from the fourteenth-century to his own day. A selection can give you an idea of the variety Ellis documented.[43]

Date	Place	Source	A (in Hz)
1361	Saxony	Halberstadt organ	505.8
1543	Hamburg	St. Catherine organ	480.8
1619	Brunswick	Praetorius's "Suitable Pitch"	424.2
c.1640	Vienna	Large Franciscan Organ	457.6
1625	Lavenham (UK)	Church bell	431.3
1648	Paris	Mersenne's foot pipe	373.7

42 Alexander J. Ellis, *The History of Musical Pitch* (London, 1889) p. 334.
43 Alexander J. Ellis, *op cit.,* pp. 333 ff.

Date	Place	Source	A (in Hz)
1648	Paris	Mersenne's spinet	402.9
1714	Freiberg (GR)	Cathedral organ (Silbermann)	419.5
1715	London	Tuning fork (found "buried at Brixton")	454.2
1711(?)	London	Tuning fork	424.3
1720	Rome	Pitch-pipes	395.2
1722	Dresden	St Sophie church organ	415.5
1730	Padua	Colbacchini low pitch pipe	403.9
1740	Gr. Yarmouth	Church organ	437.7
1748	London	St. George's church organ	474.1
1749	Hamburg	Lehnert's positiv organ	455.2
1754	Lille	Francois's tuning fork	433.6
1759	Cambridge	Trinity College organ	395.2
1760	Berlin	Flute by Floth (Naeke)	418.0
1780	Vienna	Piano tuned for Mozart	421.6
1781	St Pteresburg	Court church organ	421.2
1785	Madrid	T Bosch organ	419.6
1788	Windsor (UK)	St. George's Chapel organ	427.8
1789	Versailles	Palace chapel, conservatoire fork	395.8
1812	Eutin	von Weber's tuning fork	421.1
1820	Westminster	Abbey organ	422.5
1829	Paris	Piano at the opera	425.5
1829	Paris	Grand opera orchestral pitch	440.0
1845	Florence	Opera pitch under Marloye	436.7
1854	Paris	Pleyel's pianos	443.3
1857	Milan	La Scala pitch	451.7
1859	Brussels	Military Band (of Guides)	455.5
1859	Paris	Ciapason Normal (Conservatoire)	435.4
1862	Vienna	Sellner's Oboe	435.0
1869	Leipzig	Gewandhaus concert pitch	448.2
1878	Vienna	St. Stephen's Cathedral Organ	443.2
1879	Hamburg	St Jacobi church organ	494.5
1879	Hamburg	Opera pitch under Krebs	448.0
1879	New York	Steinway piano	457.2

The French physicist Joseph Sauveur (1653-1716) suggested a universal pitch for France, an A 427 Hz, although no more came of his suggestion than had Pratorius's a century earlier. Generally speaking, through the eighteenth and into the nineteenth century pitch rose. In 1815 the Dresden opera house used an A 432.2 Hz. In 1855 the Paris Opera was using an A 449 Hz. By 1856 Milan's La Scala opera was using an "A" pitched at almost 452 Hz. The reason for this change was tied to the growing commercialization of opera and the change in way tenors sang. For the most part, opera began as a court entertainment and whether actually housed in a court (as at Versailles) or run as private enterprise (as in London and Venice) it was patronized by the aristocracy and the form remained an aristocratic entertainment through the eighteenth century. But with the French Revolution and the growing wealth and political power of the upper middle class that came in its wake, opera attracted a new, and wider audience. This audience had little taste for the castrati who were so much a feature of the opera of the *ancien régime*, preferring more natural voices (if anything about opera can be considered "natural"). And they liked these voices virtuosic, high, and loud. The bravura works of Gaetano Donizetti (1797-1848) and Vincenzo Bellini (1801-1835) were written for them. And in the interest in creating a more exciting (and profitable) sound, tenors began to abandon their previous practice of singing their top notes in a cultivated falsetto in favor of singing them fully in their chest voice. Donnizetti's 1840 *La fille du régiment* (*The Daughter of the Regiment*) with its tenor aria *"Ah! mes amis, quel jour de fête!"* was written for tenors performing its nine high "C's" full voice.

Luciano Pavarotti (1935-2007) taking a curtain call at the Metropolitan Opera after a performance of Donizetti's *La fille du regiment*

It was a thrilling sound. And thrilling sounds sell seats. And by raising the pitch their orchestras tuned to and increasing the tension of the strings of their musicians' instruments, impresarios could make their ensembles sound both louder and more exciting too. But by the middle of the century the practice was getting out of hand and the higher pitch was putting a serious strain on singers' vocal health. In 1858 the French established a governmental commission to study the matter and the next year, in a typically federalist manner, the French passed a law establishing "A" at 435 Hz. Tuning forks were manufactured and distributed throughout the country for the enforcement of what was known as *diapason normal* (It didn't work. The tuning forks sent out couldn't all produce the same pitch; the technology to manufacture them with that kind of precision didn't yet exist). Giuseppe Verdi (1813-1901) liked an even lower "A" and in 1884 was instrumental in getting a bill through the Italian parliament mandating an "A" of 432 Hz in Italy. A year later an international conference gathered in Vienna and established the concert pitch of continental Europe at A 440 Hz, although the policy had no enforcement capability. With the advent of radio, the need developed for a standard pitch to be used in calibrating acoustic equipment and A 440 Hz began to serve that engineering function. In the later 1920's, American instrument makers reached a general agreement of setting "A" at 440 Hz, an agreement that was formalized in 1936 by the American Standards Association, an organization that sets voluntary consensus standards for products and services. Two years later A 440 Hz was adopted for the United Kingdom as the "standard tuning frequency" and the following spring an international conference, meeting in London, passed the same recommendation. In 1955 the Geneva based International Organization of Standardization passed the A 440 Hz standard. They reaffirmed the decision in 1975.

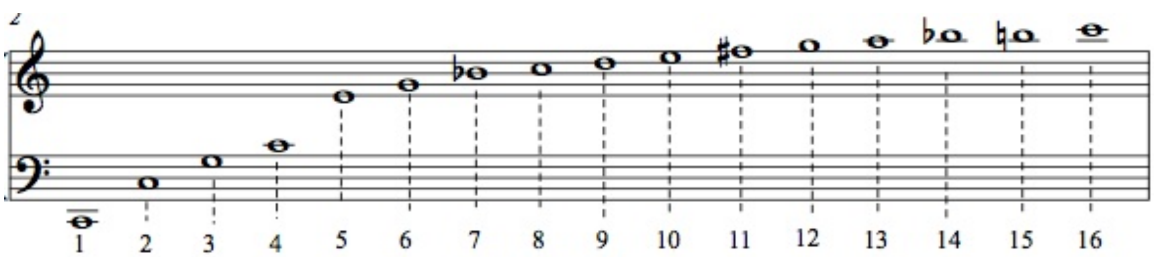
While today there is general agreement on the A 440 Hz standard there are so many exceptions to it that, at least as far as musicians are concerned, it's a standard more in theory than practice. Orchestras tune to a variety of A's. The New York Philharmonic, under Zubin Mehta, tuned to an A at 442 Hz, as did the Chicago Symphony under Georg Solti and the Boston Symphony under Seiji Ozawa. At one time the symphony in Moscow tuned to an "A" near 450 Hz although Russian orchestras now tend to tune to A 443 Hz. The Vienna Philharmonic and the Leipzig Gewandhaus Orchestra tune to a A 443 Hz while the Bayreuth Festspielhaus tunes to A 444 Hz. Ensembles specializing in Baroque music tune to much lower A's: the English Baroque Soloists, the London Classical Players, Les Musiciens du Louvre of Grenoble and the Salzburg Chamber Orchestra all tune to an A 430 Hz.

We began this chapter with the question "Am I in tune?" But the question of "standard pitch" and whether or not "A" should be 440 Hz, or 442 Hz, or 338 Hz doesn't seem to help us very much. Not only is the "standard" illusive (there seems to be a lot of "standards" and each with their reasonable justification) but our concern as musicians isn't so much with exact pitch (in cycles per second) but rather in the relationships between pitches. Any pairing of cycles per second will make an interval. Our interest is just what sort of interval is acceptable. Having a standard pitch might be required by engineers to calibrate their equipment and could be profitable to instrument makers as they more easily satisfy the requirements of larger markets, but here we're not concerned with engineering or business. We're concerned with art. And we'll get back to that "art" part.

But this business of cycles per second and frequency does bring us to the physics of sound and the phenomenon known as the "harmonic series" or "the overtone series" (musicians use the terms interchangeably and the call the pitches "partials," "harmonics," or "overtones" but physicists are more particular and use "partials"). A physical property of any object is that it will resonate. The bell "Big Ben" in London, you mother's saucepan, my head—hit them and they will resonate, they will produce a standing wave and that wave can be measured. Even the earth and its atmosphere resonate; it's called the "Schumann resonance" and it's a very big wave, equal to the circumference of the Earth and has a frequency of about 7.83 Hz. But not only does every physical object resonate, it produces a series of waves in a predictable and replicable pattern. This pattern is the "overtone" series.

Let's go back to that string on the monochord with which we began this chapter. It's an object that will resonate. And again let's assume that when it vibrates as a whole it produces a "C". And we'll let this "C" be two octaves below "middle C." That low "C" is called the fundamental and is numbered as the "first partial." But at the same time as that "C" is sounding a number of other pitches are sounding too, faint but audible and becoming fainter as they move further away from the fundamental.

The first new pitch sounded is the perfect octave above our fundamental. This is another "C" and is called the "second partial." It can be pretty clearly heard. Less strong but still audible is the "third partial" which is a perfect fifth above the second partial. With our example the pitch is "G." The fourth partial is the fourth above that, or the "C" two octaves above our fundamental. The fifth partial is a major third above the fourth. Here the pitch is "E." The sixth partial is the "G" above the "E" and the seventh partial is the minor third above the sixth. With our example that pitch is a "B flat". From this point the partials ascend by major and minor seconds, as seen in the example below.[44]



Overtone Series on "C" to the sixteenth partial

The overtone series can be easily demonstrated on the piano by playing the fundamental pitch (in our case "C") while raising the dampers off the strings of various overtones, being careful not to allow the hammers to strike the strings. If you depress the "C" an octave above our fundamental and thereby raise the damper off that string,

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44 There's an easy way to remember the order of the first sixteen pitches in the overtone series. Once a pitch is introduced it's replicated at the octave above by the ratio 2:1. So, if our fundamental is "C", the second overtone will the be "C" above that and the fourth overtone will be the "C" above that and the eighth overtone will be the "C" above that. If our third overtone is the perfect fifth above the second overtone and is "G" in our example, then the sixth overtone will be the "G" above (2x3=6) and the twelfth overtone will be the "G" above that. If the fifth over tone is the major third above the fourth, which in our example is "E", then the tenth overtone will be the "E" above that and the twentieth overtone the "E" above that. The seventh overtone is the minor third above the sixth, which is "B flat" so the fourteenth overtone is the "B flat" the octave above that. At this point we can remember that the sixteenth, fifteenth, fourteenth and thirteenth overtones are half-steps apart (roughly) and we can easily fill them in. This leaves only two pitches, the ninth overtone and the eleventh. Both of these are a major second above the pitches that precede them. Overtones four, five, and six spell a major chord in root position and four, five, six and seven together spell a dominant seventh

allowing it to vibrate feely, and then loudly play the fundamental (which is the "C" an octave below the "C" your allowing to vibrate undamped) you'll clearly hear the "C" that you didn't play, sound. Again, if you depress the "G" key of the third partial and play the fundamental you will hear that "G" as well as the original C but this "G" will be slightly fainter than the second partial you demonstrated earlier (as you continue this experiment for higher harmonics they will grow increasingly fainter). Because the fundamental produces partials that are the same wave lengths as the fundamentals of these other strings, part of its energy is transposed to them, causing them to vibrate too, even if weakly. These other strings are said to vibrate "in sympathy" with partials of the lower note.

Well, almost. The piano we're playing is equally tempered. The pitches of the overtone series aren't. Below is that overtone series on "C" again but now the various partials are labeled with plus or minus numbers, the "plus" numbers showing how many cents the particular partial is higher than the equally tempered pitch and the "minus" numbers showing how much lower.

The "C" octaves are all in tune. The "G's" are slightly sharp, but only by two cents. The fifth partial, which produces the major third above the fundamental, is fourteen cents lower than the equally tempered major third. The "Bb," the seventh partial, is almost one third of a half step lower than the equally tempered third but most problematic is the eleventh partial which, if we label it an "F sharp", is fifty-one cents higher than the equally tempered pitch. That eleventh partial is a significant tuning problem.

We can see the problem most dramatically when we look at the way brass instruments produce their pitches. A column of air, encapsulated in a tube or bottle, is a physical thing that can resonate. And when it resonates that column of air produces the harmonic series germane to that column's length (actually the dimension of the tube enters into the equation too but to simplify matters we'll pretend it doesn't). A trumpet, trombone, tuba, or horn are instruments that encapsulate a column of air and when that column of air is caused to vibrate by the player buzzing through a mouth piece attached to the column's end, that column of air will produce the pitches of that column's harmonic series.

The photograph above is of a particularly impressive hunting horn now in the collection of the National Museum of Music on the campus of University of South Dakota at Vermillion. Made by Wolf Wilhelm Hass in Nuremberg between 1754 and 1759, it's made out of solid silver with a gold plated mouthpiece and garland around its bell. It was probably used to signal during the hunt, in a scene very much like that shown on the Meissen goblet that's on page 32 of the first chapter.

The tube of this horn encompasses a column of air and for convenience sake let's say that the fundamental of this column of air is the same "C" as the overtone series we've been discussing. This means that the performer has access to the pitches of the "C" harmonic series: C, C, G, C, E, G, Bb, C, D, E, F#, G, A, Bb, B, C and etc. Depending on the way the performer chooses to blow on that pipe he can "pull out" different pitches of that series *but no others*. So, should the performer want to produce a "B flat" he can do so easily (although it will be "out of tune" and we have to come back to that). But if he wants to produce an "A flat" or a "C sharp" he can't. These pitches are not acoustically possible. He can try some tricks to "lip up" a "C" to a "C sharp", changing the way he blows upon the mouthpiece, but the result isn't very satisfactory. Similarly, should he want to produce a perfect fourth above "C", a "F", he can't. The overtone series natural to that horn produces an out of tune "F sharp" and our performer is basically stuck with that.

In order to produce pitches outside of that list I gave, the length of the column of air has to change. To do this instrument makers began to make "crooks" for their horns. These are lengths of tubing, called "crooks" because they were frequently bent and looked like curved end of a shepherd's staff, that can be added to the horn to increase its length and thus change the instrument's fundamental. If the performer of our instrument were playing in an ensemble and the composer needed the horn to play a "F", the horn player could add a crook to his instrument, possibly a "F" crook where the pitch would be the new fundamental or even "G" crook where the "F" could be the seventh partial, and in that way play the passage, but the composer would have to be sure to allow the horn player plenty of rests before and after that passage so the he had time to shove

the new plumbing in and pull it out – it was a bit of a process. Above we have a picture of a "natural" horn with eight crooks. Including the pitch of the unaltered instrument, these crooks allow the instrument to play the harmonics of nine different fundamentals.

The crooks made the instrument more versatile but they were a bit of a problem to tote around and neither performers nor composers found the multiplication of miscellaneous tubing a very satisfactory solution to the problem created by the limited pitches possible above a fundamental. In the last half of the eighteenth century horn performers changed the way they played the instrument. Instead of holding the bell over their shoulder (as they would outside in the hunt--and as shown on the Meissen cup), they tucked the bell under their arm and supported it with their fist held inside the bell. In this position they found that they could moderately change pitch by changing their hand position and positioning it at different depths within the bell. Called “hand stopping,” the technique gave performers a way to approach playing chromatic tones, or tones “outside” of the natural harmonic series of their instruments’ lengths as well as correcting natural pitches that were out of tune (such as the very out of tune eleventh partial and the more modestly flat seventh partial) but the different hand positions strongly changed the tonal quality of the various pitches and made a uniform sound though out the instrument’s range impossible.

In the nineteenth century a number of instrument makers began to incorporate valves along with a variety of lengths of tubing in their instruments. One instrument might have the lengths of tubing pitched at the fundamentals of Eb, Bb, and D coiled in its interior. By changing valves the performer could quickly and easily access these differing columns of air. When the player needed a pitch not accessible in the overtone series of one column of air (or badly out of tune in it), he could press a valve and access that same pitch from another. Crooks and “hand stopping” were unnecessary and the horn player now had a fully chromatic instrument. This instrument became known as the “French” horn familiar to us today and pictured above. The instrument without valves came to be known as the “natural” horn.

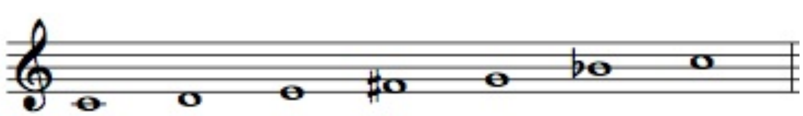
Trombones, because their length varied with the extension of their slide, were always fully chromatic, but valves made the rest of the brass family capable of full, and in tune, chromaticism. But we’re suddenly back to where we started, that business of being “in tune.”

Several pages back I passed over pretty casually the business of the overtone series producing pitches that were “sharp” or “flat”, or at least sharp or flat in comparison with equal temperament. I need to return to that.

The overtone series is a fact of nature. It simply exists. In that sense it is like that part of the electromagnetic spectrum that we call "light." At sometime in school you've probably taken a prism, held it up to the light, and seen the light refracted into colors: red, orange, yellow, green, cyan, blue, and violet (other colors, such as magenta, are created by mixed wave lengths). Those colors are a fact of nature, they simply exist, and they from the basic material artists use for their painting.

When we say that the eleventh partial of the overtone series (in our example, "F sharp") is "out of tune" it's as if we were painters and saying, "Yellow is wrong. We don't like it and we need to fix it." That would be a remarkable thing to say but that's very much what we say when we talk about the ways we have to adjust the overtone series to make it useful in our music. We're saying the overtone series is wrong.

You'll remember our earlier discussion about the perfect fourth and how it was calculated in Pythagorean tuning and why it needed to be calculated in the first place: the interval was a part of the Greek musical culture and because it would never appear in the ascending spiral of perfect fifths its ratio had to be reckoned as the difference between the perfect octave and the perfect fifth. Now, notice that the perfect fourth above the fundamental doesn't exist in the overtone series either. And if we think that the strongest overtones would naturally become a universal musical scale, which wouldn't be an unreasonable idea, this is the scale we would have (made up of the fundamental and the third, fifth, seventh, and ninth partials):



But this isn't a traditional scale at all. We can call it a hybrid of the Lydian and Mixolydian modes and it's been perfectly useful thing to make music out of in the in the last century or so, but it's not like scales that were traditional in Western music earlier and we certainly can't say that it is the foundation of our music making.

Bluntly put, nature is wrong, or at least wrong in important aspects. And this again returns us to the fact that "music" does not grow out of nature but is something that we impose upon nature. It is an aesthetic construct in which we make nature "better." The fourth doesn't exist in nature but we like it and we invent it. And as in the matter of the existence of the perfect fourth, being "in tune" is an aesthetic quality and not a physical characteristic. In regards to the highly problematic eleventh partial, physically the pitch exists. It is an objective physical thing. But within our musical culture it must be changed to be made useable. The partials that create octaves above our fundamental and the fifths above that we find acceptable and usually use them either unaltered or minimally changed, but the other partials we almost always fudge. We prefer them that way. We think they sound better.

And what guides our preference? It's our artistry as musicians. Musicians, and music lovers, make aesthetic judgments about pitch and our judgments are not based upon physics or arithmetic or tape measures held against monochords or myths about the divinity of numbers but upon our ears and our purposes.

Remember the experiment that I asked you to do at this chapter's beginning when I asked you to play an open perfect fifth and then sing the major third above your bottom pitch? And, after you sang the pitch, I asked you to play it on the piano and then compare the two pitches? And I told you that probably the pitch you sang was higher than the pitch the piano produced? The third that you played on the piano you now know was an equally tempered major third, which was 400 cents above the lower note on the piano (you can check the size at the chart *Comparison of Tuning Systems* earlier in this chapter). But the third you sang was probably a Pythagorean major third which was close to nine cents higher than the one you played because singers tend to prefer large Pythagorean thirds to the smaller equally tempered thirds. If you were a string player and I asked you do to the same thing except play the missing third rather than sing it you'd probably play a similarly high third because string players too tend to Pythagorean intervals. But, if you were a brass player and I asked you to play the missing third you'd probably play a much lower pitch. Brass players tend to prefer the intervals of the overtone series since those intervals produce the pitches that are natural to the harmonic series exploited by their instruments (these tunings are also physically easier to play than others). The overtone third (sometimes called the "pure" third) is the same as the third of just and meantone temperaments, which is 386.6 cents, which is over forty cents lower than the Pythagorean major third.

Which third is right? Which do we prefer? It depends. If a brass ensemble is playing alone they will probably gravitate to the low thirds of the overtone series while correcting the naturally flat seventh partial. But if a trumpet player is playing a recital with a pianist he will have to adjust the natural thirds of his instrument to match the equally tempered thirds of the piano (and if the composer is smart he will have written the two parts to avoid unisons of thirds there the tuning differences would be the most pronounced). Choirs tend to sing toward Pythagorean intervals, with large bright thirds. And almost every choral singer has experienced a kind of disappointment when a piece, which she has sung without organ or piano accompaniment, is performed accompanied with the resulting flattening of pitch and diminishing of brilliance that comes with the combination. String ensembles also tend to Pythagorean intonation with its high thirds. Should brass perform with a chorus and strings the brass players are continually adjusting their tuning, altering their thirds to come closer to the tuning of the other musicians. And should choirs, brass, and strings perform together with an equally tempered organ all of the musicians are adjusting their thirds to match the organ's pitch, some raising theirs while others lowering theirs. And sometimes, even with the best musicians, it just doesn't work. Remember that final chord in the first part of Richard Strauss' *Also Sprach Zarathustra*? I'll come back to that.

Musicians are not only virtuoso players, they are also virtuoso listeners. And this is particularly true of our finest orchestral players. No orchestra plays in equal temperament although in highly chromatic late nineteenth-century works and atonal pieces they come close. Instead the musicians in the ensemble are continually retuning themselves with each other individually and together as members of a section. This tuning will generally lean to a modified meantone temperament although sustained chords frequently come close to Pythagorean tuning with high and bright thirds and pure fifths. And this is one reason why orchestral musicians complain so bitterly when they are required to perform in a hall where the acoustics don't allow them to hear each other. Certainly they can still play the notes but they will not be able to play them in tune as an ensemble. They can't play them as musical *artists*.

As we saw in the discussion of the Pythagorean perfect fourth, the presence of this interval as a fundamental part of the make-up of Western music helps to emphasize music's primarily aesthetic character. Music is not something reproduced after having observed nature. Music is nature made better. Although exploitative of the physical phenomenon of sound, music is a cultural expression that is not dependant upon physics.

Michelangelo Mersis da Caravaggio (1571-1610) *The Musicians* c, 1595
New York, the Metropolitan Museum of Art

V

A scholar friend of mine made a formidable reputation for herself by attending meetings of learned societies of which she was a member where, after having heard a particularly erudite paper, she would simply ask the speaker, “So what?” More than one academic career shriveled in the heat of that honest question. Information for the sake of information, even if it is correct information, isn’t always all that helpful. Like barnacles on a ship, sometimes it impedes progress, not hastens it. Better to scrape it off and move on.

What is the use of this discussion of tuning, temperaments, and the overtone series? How can it help music speak to me and what can it do to make me a better performer or composer? So what?

We began this chapter with the question, "Am I in tune?" Both the overtone series and our tradition give some answers. "In tune" octaves and perfect fifths are both intervals that are found naturally in the overtone series and were recognized by the Pythagoreans as expressible by the ratios 2:1 and 3:2. Thus these intervals seem to have objective definitions in observable science and tradition and appear to be the same in all society's musics. But the issue for other intervals is much more complex. The perfect fourth neither appears in the overtone series as an audible partial nor is it constructed as part of a Pythagorean spiral of perfect fifths. It's apparently a fully a human construct reflecting an interval within the overtone series (between the third and fourth partial) but not found above the fundamental. Simply put, the pitch is invented. The major third appears in both the overtone series and the Pythagorean spiral yet the size of the interval that is acceptable for music depends upon taste. This appears to be the case for all the other intervals, each of which has been adjusted at some point in our tradition's history. While there is agreement among cultures and between styles that fifths are larger than fourths and minor thirds smaller than major thirds just what constitutes a musically useful major third, or minor third, or minor second, and so on, is determined culturally. "Being in tune" is thus primarily a matter of the culture's artistic standards. It might be better to ask the question, "Who should I be in tune with?" With tuning we again face what we saw in the first chapter: music—even the business of what is considered "in" our "out" of tune—is a cultural construct. It is neither the reflection of a "natural order" (in other words, the overtone series) nor is it a pure mathematical construct.

The business of mathematics brings us to another point. We frequently hear of "music" and "mathematics" being described as sister disciplines and there are books written with titles such as "The Mathematics of Music", "Music and Math" and the like. But it's all nonsense. Music is no more like mathematics than it reproduces, in numerical form, the relationship between the planets.

The confusion about the relationship between music and mathematics rests in the confusion about what music and mathematics are. As we saw in Chapter Two, as it exists as an art in our tradition, music is about *feelings* and while mathematics may be about many things it isn't primarily about feelings (at least I've never attended a "recital" of a mathematician where a hall of several thousand people, from all walks of life, left the performance wiping away tears because of how they have been moved emotionally). Mathematics is about relationships expressed numerically. Since musical intervals *could* be expressed by numbers, many philosophers thought that they *ought* to be expressed by numbers and thought their sound was of secondary importance (if of any at all). This led Boethius to mis-catagorize music as a numerical science, placing it as part of the Quadrivium. But music is the sister of poetry, not astronomy, arithmetic and geometry, and as such should be included in the Trivium as an aspect of Rhetoric (as was instruction in poetry). Before Plato, the Greeks were quite clear on this and for many generations refused to clearly differentiate between music and poetry; the oldest Greek expression for "poetry" was not "making" but rather "singing" and the poet was first called *aoidos*, or "singer."[45] Augustine, with his great fear of the expressive power of music, seems to have understood this better than either Plato or Boethius.

The confusion about the relationship between music and mathematics also spills over into what we can call the sociology of music. We have seen that there is an ancient tradition that holds that people who talk about music have a superior understanding of it to people who actually make music. This has reflected differences in class origins between aristocratic intellectuals and more humble musicians but it also has been a perceived difference in intelligence and intellectual sophistication. But that is a bit like claiming that professors who write essays about virtue are more virtuous than people who actually turn the other cheek when struck or when asked for their coat give their cloak also. Putting it another way, would any sane person argue that people who write about basketball know it better than Magic Johnson? Despite the position's blatant stupidity it has a long tradition, running from Plato through Boethius to Nietzsche and has even been institutionalized in the organization of some American universities. The structure at Yale is not unusual. "Music" as a thing to be researched and written about is part of the university's liberal arts graduate school. But "music" as something to be actually performed and composed is part of the university's "school of music." These divisions are separate entities with their own faculties, facilities, budgets and endowments. That this tradition is still very much with us can be seen in most parent's respective reactions upon hearing of their children's wish to become surgeons or singers.

Because we are a young county, we Americans tend to think that we are also members of a young civilization. But we are not. As musicians, we are members of a living artistic tradition with ancient roots. We did not invent ourselves. Our concepts of intervals, the grouping of those intervals into scales, the organization of those scales into keys or modes—in all of these things we are heirs of a long and beneficent heritage. And should we chose to significantly change those things we have inherited, inventing new scales, using intervals of different sizes, and proposing new theories of pitch generation, even here we will be acting traditionally for innovation is one of the principle characteristics of our culture. Indeed, "revolution" in the West is traditional. But not only should knowledge of our heritage encourage our own invention, it should also teach us humility before that tradition and gratitude for it, which in turn ought to make us artists of better character.

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[45] For a discussion the relationship between poetry and music see: Walter Theodore Watts-Dutton, "Poetry," *The Encyclopaedia Britannica*, 11th ed. 1911

Our knowledge of tradition also should help us perform traditional music more accurately. If we have as a goal the performance of a piece of music in a way as close to the composer's intentions as possible—with the composer's correct notes, tempi, phrasing, ornamentation, and so forth—certainly too we should be committed to performing it in the tuning or temperament the composer was known to have favored. Since a strong case can be made that the Baroque notion of the "Doctrine of Affections" was intimately related to the idiosyncratic character of certain keys in particular tunings (and especially in French keyboard literature), this becomes particularly important when performing seventeenth and eighteenth-century literature. To perform it in equal temperament (or in a just intonation when it would be performed in an irregular meantone, etc.) is perhaps to miss the music's whole point.

My other points are more technical and involve "part-writing", the importance of the overtone series in composition, and the compositional use of pitch.

Beginning music students traditionally spend a good deal of time completing exercises in an eighteenth-century vocal style. These kinds of compositions are called "part-writing" exercises. To assist this task, several generations of well meaning pedagogues have written manuals full of rules. There are rules about thirds and rules about leading tones and rules about fifths, etc., etc. Students are told if they obey the rules they can write reasonably stylistic music, or so the line goes. But music is not about rules, it's about life, and generations of music students have watched their love of music die under their own hands as they dutifully obey rules and write-out miserable but correct little exercises.

What is important to us as musicians are not the "rules" of eighteenth-century vocal composition—rules which apply only to a dead style and even in the eighteenth century weren't "obeyed" by artists but rather formulated by music grammarians who wished to codify the style long after composers had moved on to other things—but rather the artist logic behind the rules. The rules are ephemeral, applicable only to a certain time, but the artistic logic that propagated those rules can and must enliven our own creative work.

Here is a "rule": "In a four-voce texture, the third of a tertian chord must always be present but it is acceptable, on occasion, to omit the fifth." This means that when we have a chord such as a "C—E—G" the "C" (which we call the chord's root) must always be present along with the "E" (the chord's third) but we can omit the "G" (the chord's fifth).

1 2 3 4 5 6

C Major Chords

We can see this rule exemplified in the example above. Here we have six C major chords in a variety of inversions and voicings ("inversion" refers to the member of the triad that is in the bass and "voicing" refers to how the pitches of the triad are divided between the basses, tenors, altos, and sopranos). In chords two and six there's no "G" in the chord, the fifth is omitted. Why is this permissible? It's permissible because of the overtone series.

What is the strongest partial, apart from replications of the fundamental? It's the fifth, which appears as the third, six, and twelfth partials. The composer can omit the fifth from the chord for the simple reason that the pitch will always be heard, at least very slightly, because of its acoustic prominence. In some performance spaces that harmonic will be very faint, in others, such as large vaulted spaces like New York's Grand Central Station and St. Paul's Cathedral in London, the partial will be brilliantly present. The "rule" about omitting the fifth is actually a reminder to the composer never to forget the basic characteristics of his medium: sound. We're always dealing with the overtone series and we do well to remember that.

Another rule: "Leading tones in exterior voices tend to resolve up." Why?

1 2 3 4 5 6

In the example given above, between chords 2 and 3 the soprano moves from a "B" up to a "C". But between chords 5 and 6 the soprano leaps down from the "B" to a "G" (everything else between the two measures is the same).

Our example is in the key of C major and the "B" in this case is the *leading tone*. Although "leading tone" is frequently defined as simply the seventh degree of a major scale, its name means more than that. "Leading tone" points out an important function of the pitch: it *leads* to the tonic and has a crucial function is creating the tonic. And the movement of the soprano, from the "B" up to the "C" helps to reinforce that function while the movement down from the "B" to the "G" weakens it.

It works like this. A sensitive musician, looking at the soprano line between chords 1, 2, and 3 will notice the movement in that line and automatically, because of its ascent, sing the "B" slightly sharp. This will create a small, or diatonic (or *diesis*) minor second of about ninety cents between the "B" and the "C". This smaller minor second helps propel the phrase forward. In other words, the melodic movement up to the "C" promotes correct tuning (and even if we're writing for an equally tempered keyboard instrument it promotes correct music thinking). Movement from the "B" down to the "G" encourages a lower leading tone, which somewhat defeats the pitch's purpose

In that same chord we have an "F" in the alto. That "F" is the seventh of the dominant chord (the chord build on "G" in the bass). We have another rule: "In chords including sevenths, all sevenths resolve down." So, according to this rule, the "F", which is the seventh of this chord, must resolve down to the "E", which is what it does. Why?

As in the case of the soprano movement up to the "C", here a sensitive musician would look at the whole alto line and seeing it move from the "F" down to the "E" would sing the "F" slightly flat. This would ensure that between the "F" and the "E" there was again a small Pythagorean minor second of about ninety cents. Not only would this low "F" help propel the music toward the tonic but along with the high "B" in the sopranos it would increase the dissonance, and instability, of the dominant chord. And since the whole purpose of the dominant chord is to be dissonant, this voice leading, and the tuning it encourages, would strengthen the music.

Finally, take this rule and the examples below: "It is forbidden to double the third in a tertian chord." Below, example A is acceptable, example B is not (because in the second example's second chord both the tenor and the soprano have the third of the chord, "E").

You will remember that the major third is one of the most controversial intervals in our tradition with at least three possible sizes. There is the "pure" third of the overtone series (and both meantone and just tunings) of about 386 cents. There is also the

Pythagorean major third of almost 409 cents and then the equally tempered major third of 400 cents. Which of these thirds is the right one?

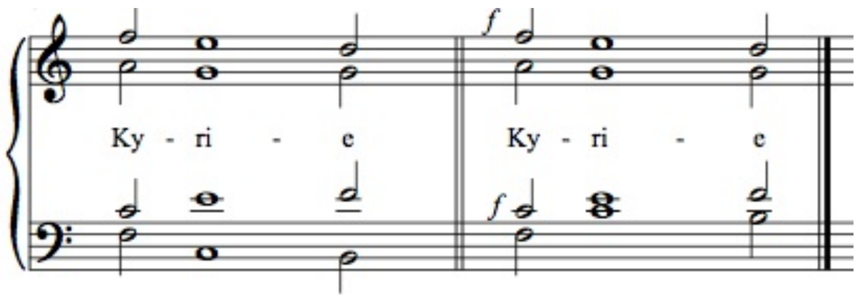
A
acceptable third

B
unacceptable third

It depends. The reason that composers rarely double the third in a tertian chord is because that the interval presents unique tuning problems. Including the third in two voices compounds the difficulty for besides having the problem of tuning the third against the fundamental--and the question which third?--we have the added problem of tuning the third as an octave with itself. And that difficulty is increased by melodic considerations. The sopranos in the case of our example B above will tend to sing their third on beat two slightly flat because of the direction of their melodic line. At the same time, because of the direction of their line, tenors will tend to sing their “E” higher than the sopranos in anticipation of the their own movement up to the “F”. Neither of these probable behaviors is wrong, neither is unmusical. Indeed, both are just what a composer would expect a choir of experienced musicians to do. But by placing the two “E’s” where he has, the composer has at least created a rehearsal problem that will require some time to correct (that is if he wants the two “E’s” to be in tune, meaning a 2:1 octave).

But a text complicates, and enriches, things. Let’s image this same passage but now as a part of a longer “Kyrie” movement (shown on the next page). We can add the text and slightly change the rhythm.

We still have the doubled thirds (the “E’s”) on the second chord and they still will be slightly out of tune but here the discord can serve an artistic purpose. The harsh, and frequently difficult to sing ‘e’ vowel of the “ky – *re* –a” (pronounced “keh-ree-eh”), is made more harsh by the potentially out of tune octaves between the soprano and tenor that we pointed out before. But that harshness can be appropriate for the penitential and emotional character of the text. That harshness is further strengthened by the fact that the “E” itself is a problematic pitch for tenors since it tends to lie in their voices’ “break.”



"Kyrie" added

But the potential expressiveness of the "Kyrie" needs to be further enhanced, and I've done that in the second example, "B". The tenor "E" to "F" is somewhat high in the tenors' range and unless the bass movement "C" to "B" is moved up an octave the chord will be unbalanced. Add the dynamic of *forte* to the passage and lengthen the time devoted to the second chord and the "mistake" of doubling that third makes artistic sense. Together, all of these things strengthen the passage's tension and heighten its drama. The music is made more beautiful, coherent, and purposeful by these inclusions, not less. The "broken rule" of doubling the third is artistically superior to the ruled "obeyed;" the "wrong" thing is exactly the "right" thing to do.

Two things are important to remember about this example. First, each of these separate compositions decision contributes to the other. None of them work as well without the rest. This is an aspect of "concinnity" or "the purposeful reinforcement of the elements of a work of art" that we will discuss at length later (it's the mechanics of what makes a great work of art great and it's a very important matter to take up). Second, it helps us understand "rules." In art there is only one rule: the art must be deeply beautiful. What generally passes for "rules" are guidelines for getting there.

In all these cases, the "rules" are encouragements for sensitive tuning. Of course musicians don't ponder how they will perform these tones, carefully calculating just how low or high they will perform them. The musician doesn't think to herself, "Oh, this 'E', because it resolves up to an 'F' I'll sing ten cents sharper than I would if it went down to a 'D'". That is hardly the case. No, musicians perform things like these "musically" which is a way of saying that they, through experience, know how an idiom is supposed to sound and perform as artists within it.[46]

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[46] Beginning keyboard players may complain that all of this has little to do with them since they perform on equally tempered instruments where nuances of tuning are impossible to achieve. This is true, but only to a point. First, it is crucial to remember that while Chopin, Liszt, and Brahms indeed couldn't shift between tunings as they performed their piano works, they thought and composed as if they could. The

We also need to consider the overtone series. Certainly the instrument maker must recognize the properties of the overtone series if he is to build an instrument successfully. And the performer must know the characteristics of the series if she is going to be at all articulate about why she must alter some tones while other tones she plays unchanged. But the composer too must keep the series in mind. Remember the cadence with which Richard Strauss ended that first section of *Also Sprach Zarathustra*? Strauss has the orchestra and the organ both hold a C Major triad but he has the orchestra drop out while the organist continues to hold it. Remember how sour that final sonority sounded? The organ sounded flat.

But the organ was flat, or at least flat when compared to the orchestra which had played the same chord just a seconds before. The orchestra, with its strings, brass and woodwinds and percussion (here the tympani), is not an equally tempered instrument. Because of their heavy predominance, orchestras tend to be pulled by their string sections into something approaching Pythagorean intonation with high, Pythagorean thirds, and pure perfect fifths, and this is especially true of long, sustained chords. But the organ is equally tempered, with equally tempered thirds and all other intervals, with the exception of the octave, adjusted too. The "sourness" that we heard when the orchestra drops-out and the organ remains in *Zarathustra* is the difference between the equally tempered organ and the "brighter" (i.e. near Pythagorean) sounds of the orchestra. We're hearing the organ's slightly flat perfect fifths and lower major thirds. By forgetting this difference in tunings, Richard Strauss here wrote one of the most colossal orchestration blunders in music literature, which is surprising since his only rivals as a composer for the modern orchestra are Ravel and Stravinsky. But it's a blunder Strauss didn't repeat when he again wrote for his Festliches Präludium op.61 for organ and orchestra in 1913.

We tend to think of composers moving around pitches a bit like a painter selects certain paints for her canvas while rejecting others. She squeezes the paint out of a tube, mixes it with some other paints on the pallet, and places the color on her canvas. Apart from thinning it, we typically don't think of the painter changing the character of the paint itself (perhaps by mixing it wilt sand, although there are painters who do this). But pitch itself is not just an element that a composer can use to his purposes. It is also something that the composer can adjust and alter. Pitch can be bent. It can be raised or lowered. It can be emphasized (by holding it out over a period of time) or de-emphasized to where it almost vanishes (a "snap" is a high pitch that has a very short duration).

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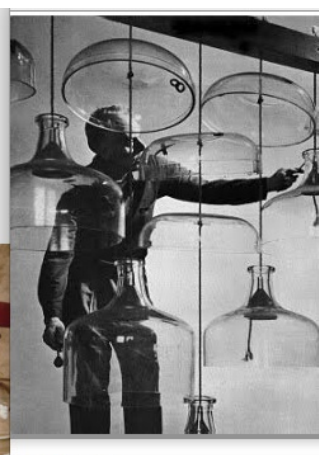
foundation of their music isn't keyboard technique or the fancy stuff that fingers can do with levers but rather voice leading and harmonic movement between consonance and dissonance. Second, by nuanced shading of tempo, dynamic, and touch the pianist can achieve affects similar to the effect that choirs and orchestras achieve by musical tuning. When keyboard players do these kinds of things we say that they are playing "musically."

"Bending" pitches is also an important part of interpretation. To lend particular emphasis to a word, or to highlight a spot in a phrase, or to add intensity to an appoggiatura (and other non-chord tones), a performer may deliberately bend his or her pitch. The great jazz singer Billie Holiday (1915-1959) was a master of this art and the classical singers Jessye Norman (b. 1945) and Peter Schreier (b. 1935) use the technique to great affect in their performances of German song (called *Leider*). The practice is both the lifeblood of Carnatic performance and of jazz instrumental improvisation.

Musicians can even so "bend" their pitches that they inaugurate new tuning systems. We've already seen how a late medieval change in aesthetic led musicians to "bend-down" thirds to suite their new taste—a decision that lead to just intonation. Because we know that our current equal temperament has not always characterized our musical culture there is little reason to believe that it will always be used by our musical descendants. It is likely that should some other temperament be found to be more useful than equal temperament—and great music written using the tuning—equal temperament will be abandoned in favor of the new system.



And composers are increasingly interested in using new tuning systems. Probably the best known of these is the American composer Harry Partch (1901-1976). Dissatisfied with traditional western tunings, Partch devised a forty-three-note scale and composed a body of theater pieces using it. Equally unhappy with traditional instruments (Partch was generally dissatisfied with anything traditional), he invented and built a large body of instruments and used them in performances of his works. His instruments are as interesting as pieces of sculpture as they are useful as music making devices (he is shown at the left playing his "Cloud Chamber Bowls" an instrument he completed in 1951) and they are a delight to see as well as hear.

But Partch's innovations have, thus far, failed to be influential and this failure points to the difficulties faced by innovators. First, since it cannot be performed on traditional instruments, Partch's music is dependent upon his own constructions for its performance. No instruments, no performance. And no performance the music goes unheard. And if the music can't be heard, and heard widely, the public is unable to make a judgment as to whether or not Partch's art merits the unusual tuning system and instrumentation it requires. If the composer's art makes a strong impact then other composers will adopt aspects of it for their own purposes. Aspects which were at one time unique to one composer now become aspects of a larger "style" shared by many composers. But if the public finds the music itself wanting then the tuning system upon which it is based will provoke no extended interest. In Partch's case the physical difficulty of getting the instruments needed to perform his music has crippled his artistic influence.

But more important than this is the fact that "tuning" is a *cultural* construct. It's not enough for an artist to invent a new tuning system and to write pieces using it. That tuning must be understood by the composer's listeners. It must become a shared language. It if isn't, if the composer invents the tuning system in isolation then even a correct performance of the composer's works in that new tuning system will only sound like a series of mistakes to the composer's listeners. Pythagorean tuning, meantone and just temperaments and equal temperament were not the inventions of individuals. They were developed slowly by communities of musicians to better serve the best literature they were composing, performing, and preserving.

Today it is possible that such a community of musicians is growing up around MIDI stations. MIDI's are quickly replacing pianos as the most common instrument in homes. They are more versatile than pianos, far less expensive, and—in this age of mobility—they are easily portable. They also have a shorter learning curve then piano. These are all aspects that, to the contemporary amateur musician, overshadow MIDI's far more clumsy live performance ability when compared with the piano. But a MIDI's tuning system can be easily changed in any number of ways. If a great literature becomes written for MIDI using a variety of non-traditional tunings (or even traditional but little used tunings, such as meantone temperament), as this literature becomes known and imitated, I think that eventually we could see a new tuning system develop around MIDI. Just as equal temperament arose with the development of the clavier and keyboard literature to answer the needs of that instrument and its music, so too this new temperament could fulfill the needs of MIDI composers and their listeners.

But that is all speculation. This isn't. Temperaments may change as one aesthetic replaces another and opinions of being properly "in tune" will similarly shift as the age and artistic climates mutate, but one thing will remain inviolate. The final authority in all musical matters is the musician's ear. This is my last, and most important, point. Music is about sound. And sound is about making it and listening to it. It isn't about reading, or weighing, or measuring, or pondering the orbits of the planets or squinting up at the stars. Music isn't justified by geometry or judged by pointing to the chalk marks on a monochord or citing the similarities between the ratios of musical intervals and the character of the planets. It isn't defended by physics or the overtone series. Plato was completely muddle-headed about this. Aristoxenus was right. Something is in tune not when the intellectual *says* that it's in tune but instead when the musician *hears* it's in tune. Musicians began to flatten Pythagorean major thirds when they began to hear them as too sharp (much to the complaining squawks of the philosophers). Musicians gradually began to favor equal temperament over other systems as they began to hear its pan-chromatic advantages. Today a musician is in tune when she hears that she's in tune. The singer in a choir, the soloist in a recital, the bassoonist in the orchestra, the cellist in the string quartet and the trumpeter in the band—all of these musicians continually re-tune themselves as they listen to themselves and to each other. In this piece, in this hall, with these performers—even in this phrase and in this part of the phrase—the musician is always critically listening to judge whether or not he is appropriately in tune. And to be "appropriately" in tune doesn't mean that the same pitch will always have the same frequency. Sometimes to be "in tune" the pitch must be high. Sometimes to be "in tune" it must be low. Sometimes it will be the same.

In the final analysis, tuning—in the art of music—is an aspect of interpretation. What tuning system do you use? Pythagorean? Meantone? Just? How do you perform that pitch? What member of the triad is it and what is the harmonic function of the triad (if the harmonic syntax is functional)? If the pitch is a major third should it be a high or a low third? And how high and how low? Perhaps the pitch is a harmonic dissonance that can be strengthened through the subtlest bending of the pitch. Perhaps the pitch is the octave above the fundamental in which case it should be exactly the ratio 2:1 from that fundamental. The musician has no interest in mechanically reproducing frequencies. That's the job of a tuning fork. The musician's task is to interpret musical works of art. And that requires a skilled listener and an imaginative interpreter.

So, to return to our original question: "Am I in tune?" What can we say? "Well, it depends."