COLORADO MATHEMATICS TEACHER - SPRING 2016

The Official Publication of Colorado Council of Teachers of Mathematics

CCTM
COLORADO COUNCIL OF TEACHERS OF MATHEMATICS

A growth mindset for students and teachers

CONTENTS & FEATURES

From The Editor's Desk

Sandie Gilliam, CCTM Editor

It's that time of year again to learn that last little tidbit of math, wrap up the 2015–2016 school year, reflect on the journey we've been on with our students, and begin to (1) think about our own summer professional development, (2) start our planning for summer school teaching, and/or (3) dream about that upcoming vacation!

This edition of the CMT presents one more idea from *Principles to Actions: Ensuring Mathematical Success for All* —**Support productive struggle in learning mathematics:** *"Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships"* (NCTM, 2014).

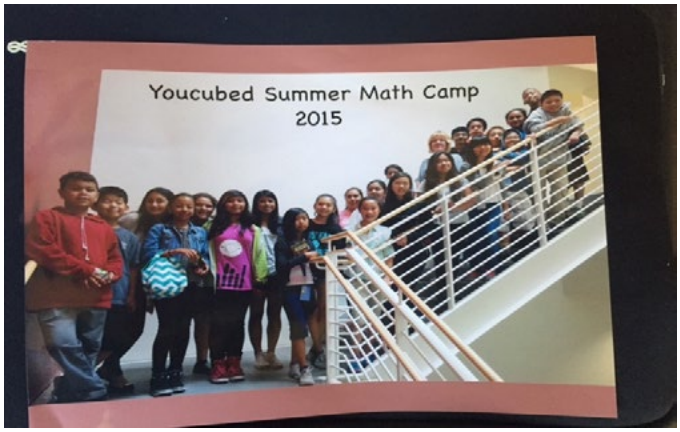
Encouraging productive struggle has been ingrained in my practice since reading NCTM's *Professional Standards for Teaching Mathematics* in 1991. In the book, Standard 6: Promoting Mathematical Dispositions states that the teacher "promotes students' confidence, flexibility, perseverance, curiosity, and inventiveness in doing mathematics through the use of appropriate tasks and by encouraging students in mathematical discourse." Over the years, what I continued to realize about this standard is that:

- Non-judgmental verbal cues by the teacher are an important part of students being able to clarify their process to solution and correct any mistakes, rather than the teacher saying, "Here, let me show you."
- The classroom culture should support students raising questions and challenging ideas generated by other students as well as by the teacher, rather than students feeling afraid or embarrassed to speak up, just in case they are wrong.
- Students should embrace mistakes as a visible and natural part of the learning process.
- Ample time given for students to be active participants in engaging tasks both encourages and supports productive struggle.

Two years ago, *productive struggle* became my concern for students in a Math for Elementary Teachers class I taught at Colorado College. Students came face-to-face with the conceptual understanding of the base-10 system versus their previously learned systematic routine of how one adds, subtracts, multiplies, and divides. Their own frustration looking outside their once comfortable box of knowing math procedures led them to previous memories of "failing math so many times." Somehow, *struggle* suggests the words: hard, impossible, failure—negative words—and since they already knew how to do arithmetic, they were unclear as to why they must be subjected to struggling.

On the flip side, when teachers initially think of helping students to have a *growth mindset*, they may mistakenly think this has everything to do with *praise* for one's efforts. This was clearly the interpretation my college students had: "*I was praised by my teachers and parents for both my effort and my academic achievement*"; "*My teachers were rewarding outcomes*"; and "*I had not seen any positive outcomes, and was thus putting in no effort.*" As a teacher of these pre-service elementary teachers of math, I was trying to model and support productive struggle, and due to memories, they wanted little of it.

Last summer, Jo Boaler asked me to teach one of four math classes of middle schoolers at the four-week YouCubed Summer Math Camp at Stanford University. My awesome experiences there provided me with both new ideas and realizations on growth mindset. Recently, I went back to interview some of my students for Jo's continuing research. What a difference from my college students. These middle-schoolers told me: "*I know there are many ways to solve math problems*"; "*I know that mistakes grow your brain*"; and "*This year, I'm not afraid to speak up in class, even if I'm wrong.*" Whereas, struggle might suggest negative words, *growth mindset* for these adolescents was purely positive! Each still has a can-



do attitude and a positive mathematical disposition.

As you read the *President's Message* and *Why Zombies Love ME: Shifting Mathematical Mindsets*, examine your own thoughts on productive struggle and growth mindset. Then peruse the book review for Jo Boaler's *Mathematical Mindsets,* and perhaps read the book to learn more.

The second foci of the **Spring CMT is: Summer: Professional Development or Summer School**. Last summer, I participated in all three of the suggested summer activities. In addition to teaching the YouCubed Summer Math Camp, the experiences there provided me with the collegial planning and discussion time that is rarely afforded to teachers in the regular school year, as well as new ideas on growth mindset. I plan to turn my learnings into conference presentations and an action research paper. Professional development (PD) and summer school teaching all in one! The work at Stanford was followed by horseback riding, hiking, fishing, and good food at a Wyoming dude ranch; and a visit to Grand Teton National Park.

What are your summer plans? Opportunities abound. Read about past activities from *Harrison District 2*, *Transition Summer Camp*, and *NCTM's 2015 Summer Institute*. They might whet your appetite. *Fast Connections* and *NCTM Professional Learning Opportunities* give you already calendared PD opportunities.

Perhaps you're the teacher who: 1) wants an unstructured summer of reflecting on your journey with students, and 2) likes to read about ideas and activities you could put into practice next year? If you want to learn more about *productive struggle*, go to: Principles to Actions Professional Learning Toolkit. A K–12 grade-band range of tasks in the toolkit helps teachers understand, and be able to use in their classrooms, the Mathematics Teaching Practices (from *Principles to Actions*) that continue to be foci of CMT issues. The Case of Jeffrey Ziegler and the S-Pattern Task is a high school example that supports productive struggle. While some tasks in each grade-band or teaching practice are available to everyone, the entire collection is available to NCTM members.

The foci of the Fall 2016 CMT are:

- Facilitate meaningful math discourse
- Build procedural fluency from conceptual understanding

The CMT Editorial Panel is looking for articles on each focus that clarify/explain research, and/or demonstrate and connect to classroom practice. Questions and article submissions can be sent to sandie.gilliam@coloradocollege.edu. Articles are due by July 15.

Whatever summer plans you choose, go and recharge your batteries for a productive new school year!

President's Message

Joanie Funderburk, CCTM President

A COWORKER OF MINE creates puzzles and posts them for us to solve. Here's a recent one:

ipip ipip ipip
ipip ipip ipip
ipip ipip ipip
ipip ipip ipip

Invariably, someone in the office gets an answer really quickly, and you'll hear a chorus of "Wait! Don't tell me!"

This phrase is one of my favorites as a math teacher – when my students would say, "Wait! Don't tell me," I knew they were hooked. They wanted to figure things out for themselves, and it was exciting to struggle the right amount. They preferred a bit of struggle over just being told how to solve a problem or find an answer. In *Principles to Actions*, NCTM names "supporting productive struggle in learning mathematics" as one of the Mathematics Teaching Practices.

As teachers, there is a temptation to keep our students from struggling. The word itself brings a negative connotation, and many of us became teachers because we want to help, not because we want students to struggle! Many of us have seen students struggle to the point of frustration, causing them to give up, shut down, or become disruptive to the learning of others. However, *productive* struggle doesn't create those negative feelings or behaviors. The good kind of struggle feels more like wanting to solve that puzzle myself, without someone just telling me the answer. Accomplishing something that is difficult is far more rewarding than accomplishing something that was made easy.

So how do we keep struggle from becoming *unproductive*? In this article, "Support Struggling Students with Academic Rigor," published by ASCD (formerly doing business as the Association for Supervision and Curriculum Development), Robyn Jackson contrasts productive struggle with *destructive* struggle. She suggests that learning happens when students struggle in ways that lead to understanding, that make their effort feel worthwhile, and that leave them feeling empowered and efficacious. Alternately, learning is blocked when students' struggle leaves them feeling frustrated, abandoned, and inadequate. Teachers can plan for productive struggle by providing rich tasks and problems for students to engage with, anticipating student difficulties, and planning questions and supports that don't remove all opportunity to struggle.

For instance, when fifth grade students begin adding and subtracting fractions with unlike denominators (CCSSM 5.NF.A.1), a problem such as 2/3 + 5/4 might be a struggle. If directed to reason about the relative size of each fraction in order to estimate the value of their sum, students can move past an initial block and begin to consider their previous work with fractions as inroads to understanding the problem. They might consider using fraction manipulatives, a number line model, or their own sketch to conclude that the sum is close to two. A teacher might suggest a strip diagram to help students consider the different size parts of each fraction, and a well-formulated question can activate students' background knowledge about equivalent fractions, allowing them to reason their way to the sum of 23/12 without being told a specific algorithm or approach for calculating this answer. Teachers planning together in a PLC or grade-level team can support one another in designing the right questions, suggestions, and tools that keep students in the sweet spot of struggle that is productive.

As you grapple with creating productive struggle for your own students, you might consider Carol Dweck's book *Mindset*, or Jo Boaler's article "Unlock-

ing Students' Math Potential" (found here), or Triumph Learning's research summary white paper "Productive Struggle for Deeper Learning" (found here), all of which enhance the ideas in *Principles to Actions*. I'd love to hear what other resources you've found helpful, or what you've learned about how to foster productive struggle in your classroom. Write me and tell me about it (joaniefunderburk@gmail.com)!

When I first began my teaching career, I thought my measure of success would be my students telling me that math was easy. Now, however, I would prefer my students say that math is sometimes hard, that it is worth working at, and that math always makes sense.

Still wondering about the ipip puzzle? Here's a hint: it's a great riddle for a third grade math student!

PRODUCTIVE STRUGGLE

Why Zombies Love ME: Shifting Mathematical Mindsets

Stephanie Hammes, Lemon Avenue Elementary, La Mesa, California

"I guess you are just not really a math person. That's OK—your strengths lie more in the arts and humanities." —My father, circa 1980 (sophomore year), in response to my C in Algebra 2.

I AM NO MATH EXPERT, and I have no math pedigree to recommend me. I am a liberal arts/humanities major. If someone had told me at the beginning of my teaching career that I would end up teaching mainly math (and loving it!), submitting an article to a math journal for publication, and having students with the top math scores in my district, I would have fallen down laughing.

Only rarely did I feel like much of a mathematician during my own schooling. Admittedly, I was what I would now call a "Betty Crocker mathematician." I was a great little cook *if* I had the right recipes from the textbook and my instructors. Memorizing, replicating, and performing on tests was what math seemed to be about in most of my school experiences. I can almost hear you *real* math teachers and mathematicians gasp. You are most likely wondering why you should even read on. Rest assured, I now proudly count myself among you—a confident and competent math educator. I have come to see the beauty, creativity, and connectedness of mathematics, and I strive to help my students do the same. Yet, my greatest strength as an educator, in mathematics and any subject I teach, stems more from my math journey from a fixed to a growth mindset.

Since 2013, I have overtly made it my number one mission to help my tween students understand and shift their own mindsets. It is this growth mindset journey—mine and theirs— I am excited to share with you. Never has there been such a wealth of supporting research in cognitive science to back up my long held belief, and that of so many educators, that ***all* students have the ability to learn in *all* subject areas.**

Early in my teaching career, I was keenly aware that I did not want to infect my students with my

own math fears and insecurities. I believe they can *all* succeed with effort, education, and practice. I had to come to believe the same of myself—even in math. For example, I would not say, "It's OK that math is not your thing; it wasn't mine either," as so many well-meaning parents and teachers do. (Research now clearly shows what my instincts were telling me, that this kind of talk propels learners into a fixed mindset rather than soothing their anxiety, as intended.) I now had to model a positive attitude about and relationship to math if I wanted my students to believe they could succeed. I set about the task of deepening my own understanding of key math concepts and shifting my beliefs about my own math abilities—my math mindset—in order to become a better math teacher. This process involved a great deal of professional development and also collaboration with respected colleagues (and admitting when I needed help).

Every math teacher knows only too well that mathematics is a discipline with which learners often come to have an anxiety-based relationship. No matter how much we project a love for math (or fake it till we make it) and plan amazing math lessons and experiences, we still see the fear in far too many of our students' eyes. Sadder yet is the dull look of students who have moved beyond fear to apathy, especially prevalent in upper elementary grades and middle school. During early adolescence, test scores and grades in mathematics usually show marked declines. This begs the question as to what may be done to help educators and students heed the call to greater math literacy in the 21st century.

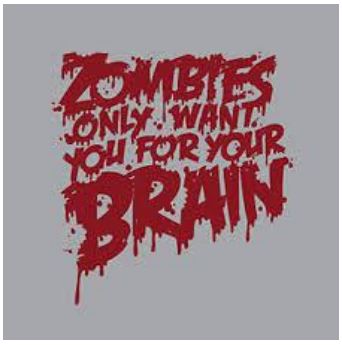
More depth, coherence, and rigor in the standards and testing, even when accompanied by major paradigm shifts in instructional approaches, are not enough. It will take more for students to become the nimble and confident "Top Chef mathematicians" these new standards and tests—and more importantly, 21st century careers—demand them to be. It is more crucial than ever for educators—especially math educators—to understand and explicitly address academic mindsets. There is no greater gift we can give a student than the knowledge and experience that she/he can learn math, and any other subject—with hard work, practice, and a growth mindset.

"Students with no experience of examinations and tests can score at the highest levels because the most important preparation we can give students is a growth mindset, positive beliefs about their own ability, and problem-solving mathematical tools to equip them for any mathematical situation." **from "Aligning Assessment to Brain Science" by Jo Boaler.**

I credit Professor Jo Boaler's Stanford online course, EDU115 "How to Learn Math for Teachers and Students" which I took in 2013, with first opening my eyes to Carol Dweck's mindset research, especially as it relates to math. In reflecting on my teaching career during this Stanford course, I experienced so many "ahas". Looking back through the lens of mindset research, I clearly see that behind every student turnaround, every light coming back into dull eyes that often seemed so magical and mysterious, was a shift to a growth mindset.

I want each and every one of my students to experience this mindset magic at some point in our time together, whether in math, or in some other area of study or aspect of their lives. I openly and often declare to my students and their parents, from the beginning of the year to the end, that I am on a mission to shift fixed mindsets toward growth mindsets. Before I elaborate on how I go about achieving this mindset mission, let me share with you student reactions to a year of mindset lessons and messages. I am sure you noticed by this article's title that I chose to put a pop-culture zombie spin on my mindset mission.

Let's dig into some tween braaaaaiiins! At the end of the 2015 school year, I asked my fifth and sixth grade students to respond to this prompt (note that it does *not* mention mindset):



Why Zombies Love Me!
In what academic area (this school year) have you grown and stretched your brain the most and created the most NEW synapses (neural connections – oh, ZAP!)? Write a TEEEEEC paragraph (at least 7 excellent sentences). PROOFREAD for clarity, flow, and mechan-

ics, please.

- Topic Sentence: Address the prompt using key words from it.
- Example Sentences: How have you accomplished this? Be specific. What measure(s) could teachers, parents, and/or classmates see to show this growth?
- Conclusion Sentence: How do you feel about your achievement(s) in this subject area and why?

All but a few of our 67 students said math was the academic area in which they had grown and stretched their brains the most. Overwhelmingly, they credited learning about mindsets and neuroscience with their successes. The errors, grammatical, mechanical, and typos, are all theirs—as is the spirit and content. (All revision and editing is their own and was done without the aid of word-processing tools because Juno assignments are designed for assessment purposes.) Enjoy these glimpses into the amazing, colorful, juicy zombilicious brains of fifth and sixth graders following a year with growth mindset lessons and messages. I have introduced their responses with some brief observations and included some of their mindset brain collages.

This learner took to heart messages from number talks along with the growth mindset messages:

> I have grown in math the most this year. I have grown so much because before 6th grade, I thought there was only one way to do a problem. I also thought that some people were not that much of a math person. But, now I know that there are multiple ways to solve problems. I also know now that sometimes there are multiple answers. I now know there isn't a math person or non-math person because everyone can be good at what they try hard at (practice makes permanant). I now feel that I have a growth mindset in math and I have improved a lot.

There will be no more Betty Crocker recipe math for this student—she will be a Top Chef!

> The subject I feel that I have grown the most in is math. I have learned that math isn't just memberising alot of formulas but it is actually taking time to read a problem and work your way to finding an answer. I have accomplished growing my brain in math by using a growth mindset on math. I love the idea of a growth mindset because you can put a growth mindset on anything not just that but anyone can have a growth mindset on anything. Mrs. Hammes really helped me with this and really encouraged me not to give up and to not be afraid to make a mistake doing anything. When imput a Growth mindset on math I grew alot and I mean alot. With all the support Mrs. Hammes gave me through out the year I really appricaite her. I feel that with the fundation Mrs. Hammes gave me I really do get what math is and how it math works.

This next response is from a student on an IEP who *was* apathetic about math. He shifted his mindset about two-thirds of the way into the year, and his achievement followed. It is never too late to shift a mindset. Just when you think a student has not listened to all the mindset messages, he/she may surprise you. This young man received one of nine "Growth Mindset Awards" we gave at sixth grade promotion for a 26% improvement in his overall math grade from trimester 1 to trimester 3!

> I feel awsome when I get something done I feel exsided.I improved on geting someing done becuase at the begining of the year I was not geting stuff done and I was getting frustraded and stressed and starting to go into a fixed mindset but somehow after spring break I was getting so much work done and not being frustrated and stressed and I went into a groth mindset. My predidickson for next year in middle school I am

going to have a great year.

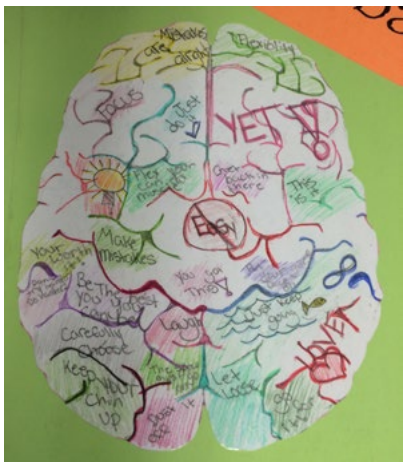
This response is from a "gifted" student—the other end of the spectrum in terms of prior achievement in math. She came in with excellent math skills, and she learned to stretch, to take risks, and to dig deeper. Like Dweck, I find that students labeled "gifted" are even more likely to have fixed mindsets than students who struggle with math anxieties and/or skills.

> An academic area where I have grown and stretched my brain the most is math. I have accomplished this by turning my fixed mindset closer to a growth mindset. Another way I have accomplished this is by becoming a little more positive. You can see me do this by looking at my survey results. A final way to do this is by adding yet when I say I can't do something. You can hear me adding yet if I say I can't. I feel proud of my achievements in this subject area because I had a very fixed mindset about math.

Note that she is highly aware that she is not quite fully in growth mindset—yet.

This next learner has also internalized some good math strategies and our math class norms (adapted from Professor Boaler's suggested norms, available at Youcubed.org).

> The academic area I have stretched my brain the most was science and math because its where Mrs. Hammes taught us that you can grow your brain no matter what. We watched a video where people interviewed other people that said their hardest subject was math because they didn't get it but thats probably because they were older and hadn't practiced it much lately. Like Mrs. Hammes says "Practice makes permanent." Also if you don't get things you should probably raise your hand. Also like Mrs. Hammes says "The smart people are the ones that raise their hands." I used to think that you were born with your smarts and you couldn't increase your smarts in any area no matter what, so when Mrs. Hammes said that my peers and I could I was totally shocked. Now that I know that I think about things differently then how I used to. Also I use one of Mrs. Hammes's methods: When you get stuck on a problem take a break and come back with fresh eyes don't go panicking and banging your head against the wall. All in all I think that my efforts have been better then earlier in the year.



I am so proud of my students, their growth mindsets, and their academic progress in math. They constantly teach me new ways of seeing the world of mathematics (and the world, in general). It is an honor to get to peek into their dynamic growing and changing brains and glean insights. I have seen firsthand how powerful a shift from a fixed to a growth mindset can be—with my own and with my students' math mindset shifts. In addition to this compelling anecdotal evidence in the students' own words, **this group of students had the top Smarter Balanced Assessment Consortium (SBAC) scores in our district last spring!**

The math scores for my two sixth grade math classes were significantly higher (over 10%) than the school site that usually has the top scores in our district—a site with higher API rankings and higher socioeconomic status. (Our fifteen fifth graders also helped raise the bar for that grade level in math.) The next highest scores were those of my friend, colleague, and math mentor, who teaches math at the aforementioned site. Her scores were also quite impressive—the second highest at our grade level and *well above* district averages (again, over 10%) — not only at our grade level, but at *every* grade level tested.

We both took the online Stanford course in 2013, and we discussed it throughout the summer. Both of us started the year with the article "You Can Grow Your Intelligence." (available at mindsetworks.com). Both of us piloted new Common Core aligned textbooks last year and sat on the district's math text adoption committee. Our multiple site Professional Learning Community meets monthly, so we share a great deal, including SBAC style assessments created by members of our PLC—most often her. What was the big difference between our courses then?

My friend and colleague is an example of an educator with a growth mindset and a generous spirit. She called to tell me about the scores—excited for my students' success, and even more excited about the potential for integrating more mindset lessons and messages into her own math classes. Remember that people with growth mindsets find inspiration in the success of others. She knew that we used common assessments and materials, and we have similar philosophies and teaching styles. **The most significant difference in our math courses last year was that I'd gone further with mindset lessons and reinforcement,** at the expense of some of the textbook lessons her students made it through.

I worried that I had not covered enough, and was thankful that it was just a baseline year for SBAC testing. During the online testing, I noticed my students were confident and they said felt well prepared. They did not stress out when confronted with problems types they'd never seen, and even glitchy iPads did not ruffle them. Jo Boaler is so right that "the most important preparation we can give students is a growth mindset . . ."

Like my friend and mentor, I, too, am inspired by the success of others and always looking to grow and improve. I am so grateful to all the generous teachers, researchers, and companies who make mindset materials and lessons for both educators and students so readily available. There is so much great material from which to choose!

For those of you interested in implementing your own "Mission Mindset," I encourage you to make whatever mindset materials you use your own. You saw from the "Why Zombies Love Me" prompt, that you can put your own spin on mindset lessons in order to best connect with *your* students. I went with zombie pop culture and the "ick" factor, and my tween students ate it up. (Pun completely and unapologetically intended–6th grade humor.)

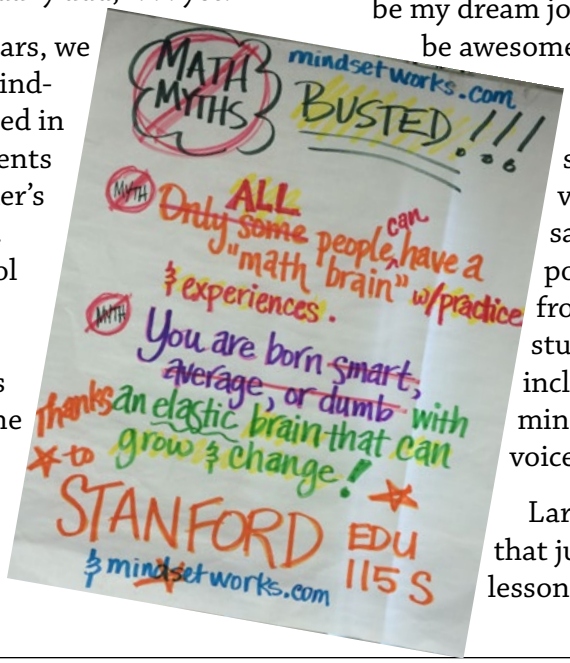
One of my students was so inspired by the mindset lessons and messages that she baked me a very special red velvet brain cake right after conferences this fall. My teammates and I ate it "zombie style."

Consider teaming up with a science teacher if you teach math exclusively and having him/her present more information on brain

structure and function and neuroscience. Don't give away *all* the yummy mindset lessons though—research and experience inform us that math is definitely the subject area in which the most students have fixed mindsets, so take the time to present at least a couple lessons. My team even carries the "zombies only want you for your brains" theme out to PE and our weekly mile run. Good cardio can prevent you from becoming zombie chow *and* help you succeed in school.

I begin on day two of school with the Mindset Works article: "You Can Grow Your Intelligence." My students first listen to the article as I read it aloud, and then complete the review activity that goes along with it as a listen and sketch. Their homework is to read the article aloud to a parent/guardian (or older sibling, in a pinch) and to do the listen and sketch activity with him/her. Thus, parents are primed and ready when Back to School Night rolls around and I share the information on mindsets and my mission. I also give parents an article about resilience that quotes Carol Dweck and discusses how to praise your child for a growth mindset along with other math support materials. This article, "The Fourth R: Resilience" is available online. (See references.) I continually reinforce growth mindset messages, especially when writing descriptive feedback on formative assessments and in daily discourse throughout the year. When I hear a student say, "I can't . . . I don't know how . . ." I pause for a couple of breaths, and they eventually add, ". . . yet."

The last two academic years, we delved more deeply into mindsets than when I first started in 2013. I now show my students the first module of Jo Boaler's online course for students. The first lessons share Carol Dweck's mindset research and engage in math myth busting. Our "Math Myth's Busted" poster hangs on the wall all year. Students also complete the "Effective Effort Rubric" (also from Mindset Works) at the start of the year and again at the end of the year. Eduardo Briceño's TEDxtalk "The Power of Belief—Mindset and Success" gives us a mindset booster (linked on Mindset Works, and also available on YouTube, 2012).



Students utilize a viewing guide, which I adapted from the PERTS & Khan Academy growth mindset lesson plan, and take notes. As homework, they are asked to share their notes with a parent/guardian and/or watch the talk and discuss it with him/her. Last year, I showed the video at the end of the year as a refresher. This year I showed it right before student-led conferences. Most of our students chose to watch the talk with their parents, and I received a lot of positive notes from parents on the viewing guides. My dozen returning students from our 5th/6th grade combination class remembered the video fondly and enjoyed seeing it again. They are our best growth mindset advocates. Both years, a number of parents expressed that they felt somewhat guilty over praising their kids for being smart (fixed mindset praise) and would make a shift to praising effort and hard work (growth mindset praise).

As I checked in the viewing guide work, one of my boys excitedly told me: "My mom really loved this video! Our whole family watched it, and she said she wishes *every* teacher in the district would teach about mindsets." I must admit that I wish the same thing. I shared with him that being a traveling mindset coach for grades 5-8 in our district would be my dream job. "Yeah! That's a great idea—You'd be awesome at that!" he responded. (I am considering asking him to write me a letter of recommendation!) Having students. and most of their parents, view this inspirational mindset message just before conferences led to powerful discussions about shifting from fixed to growth mindsets. Many students wrote trimester 2 goals that included "talking back to their fixed mindset voice with a growth mindset voice," as Briceño suggests.

Large scale research definitively shows that just two forty-five minute mindset lessons—including teaching about neural

plasticity and the central idea that **you can grow and change your brain throughout your life**—can have a significant positive impact on math achievement for up to two years, especially among students most at risk to stereotype threat (which includes girls in math). Academic mindset interventions, praise, messages and feedback may not be a magic potion that will cure *every* student's math woes immediately, but they are a math teacher's best bet for maximum impact with a minimal investment of time. I hope you will join me, and educators across the country and world, on this mindset mission. Spend at least a couple of precious instructional days teaching your math students about the power and promise of mindsets. Then let your students—and their improved scores—speak for themselves!

In grades 5 and up, mindset interventions and growth mindset feedback is particularly important. This is where much of the research has shown students to be the most vulnerable to negative academic impacts from fixed mindsets. Prior to this, differences in mindset in early elementary school do not seem to have much impact on academic success. Still, it is never too early to begin with growth mindset praise and feedback, even if you do not present formal lessons.

There are now many high quality, solidly research-based resources and references available. Some great resources for mindset materials, most of which are free, include:

- Khan Academy's Growth Mindset Lesson Plan – created in collaboration with PERTS www.khanacademy.org
- Mindset Works Educator Kit www.mindsetworks.com
- PERTS – Project for Education Research that Scales www.perts.net
- YouCubed at Stanford University – Professor Jo Boaler's site that also has a wealth of math resources, articles, videos, and a link to the free "How to Learn Math for Students" online Stanford course www.youcubed.org/think-it-up/

If you are interested in further reading, the best of the myriad of resources I have explored is Jo Boaler's latest book: *Mathematical Mindsets*: *Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching*. This book wraps up the main ideas presented in Professor Boaler's online course beautifully in a neat little package. Admittedly, I am a huge fan of *all* her work. My edition is already well worn and dog-eared, and I've only had it since November. (Be sure to check out Professor Boaler's mention of our very own CMT editor and stellar math teacher on page 19 of the book.) Here's to being lifelong learners with growth mindsets, generous spirits, and juicy braaaaaiiins! Get infected!

(Editors note: Stephanie Hammes is a former Colorado teacher from Cañon School in Cheyenne Mountain District 12, and recently graduated with a Masters of Arts in Teaching from Colorado College.

For another example of how teachers are using pop culture to engage students in the sciences, see the Living Dead in Fact and Fiction course offering at the University of Nebraska at Kearney http://unknews.unk.edu/2015/10/28/zombies-vampires-find-their-way-to-unk-classroom/.)

References

Blackwell, L., Trzesniewski, K., & Dweck, C. (2007). Implicit Theories of Intelligence Predict Achievement Across an Adolescent Transition: A Longitudinal Study and an Intervention. *Child Development, 78*(1), 246-263.

Boaler, J. (n.d.). Aligning assessment to brain science. Retrieved February 13, 2016. YouCubed.org. www.youcubed.org/think-it-up/aligning-assessment-brain-science/

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco: Jossey-Bass.

Boaler, J. (2013, July - September). *EDUC 115: How to learn math: For teachers and parents*. Online course & video lectures conducted from Stanford University.

Boaler, J. (2014, July - August). *EDUC 115S: How to learn math: For students*. Online course & video lectures conducted from Stanford University.

Dweck, C. (2006). *Mindset: The new psychology of*

success. New York: Random House.

Mindset Works®: Student Motivation through a Growth Mindset, by Carol Dweck, Ph.D. Retrieved July 5, 2015, from mindsetworks.com

Paunesku, D., Walton, G., Romero, C., Smith, E., Yeager, D., & Dweck, C. (2015). Mind-Set Interventions Are a Scalable Treatment for Academic Underachievement. *Psychological Science*, 784-793.

Ricci, M. (2013). *Mindsets in the classroom building a culture of success and student achievement in schools*. Waco: Prufrock Press.

Schipani, D. (2013, September). The Fourth R: Resilience. *Better Homes and Gardens*. 214-223. http://deniseschipani.com/wp-content/uploads/2013/09/2013_09-BHG-The-Fourth-R-feature.pdf

Verstynen, T., & Voytek, B. (2014). *Do zombies dream of undead sheep?: A neuroscientific view of the zombie brain*. Princeton, N.J.: Princeton University Press.

Willis, J. (2010). *Learning to love math: Teaching strategies that change student attitudes and get results*. *Alexandria:* Association for Supervision and Curriculum Development.

Wilson, D., & Conyers, M. (2013). *Five big ideas for effective teaching: Connecting mind, brain, and education research to classroom practice*. New York, NY: Teachers College Press.

Yeager, D., & Walton, G. (2011). Social-Psychological Interventions in Education: They're Not Magic. *Review of Educational Research,* 267-301.

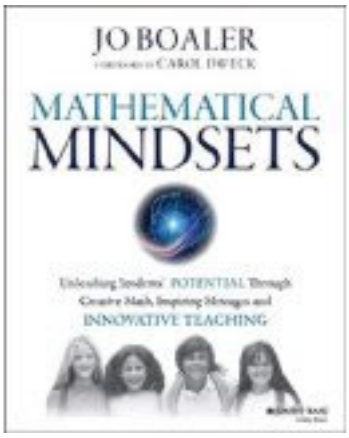
YouCubed: Inspiring Students to Math Success and a Growth Mindset. (n.d.). Retrieved July 5, 2015, from Youcubed.org

PRODUCTIVE STRUGGLE

Mathematical Mindsets

Reviewed by Leigh Ann Kudloff, CCTM Regional Representative

With a forward by Carol Dweck, the author of *Mindset: The New Psychology of Success*, to set the stage, *Mathematical Mindsets* by Jo Boaler applies the concept of mindset to the mathematics classroom and challenges current practices. Starting with Dweck's work and what she has learned from their collaboration, Boaler details how the idea of mindset not only applies in a math classroom, but also about how teachers can build their students' mindsets to improve math achievement. Referencing her online class (taken by more than 40,000 people), Boaler compels the reader to join those teachers and commit to changing mathematics classroom practices. An overview of recent brain research and connections to learning math is presented. Boaler provides examples and explanations of the power of mistakes and struggle, while defining both growth and fixed mindsets. Our own CMT editor, Sandie Gilliam, is highlighted early in the book for the way she encourages her students to share mistakes openly and how practice not only builds growth mindsets but also creates a collaborative learning environment. Boaler is clear throughout about her goal for writing *Mathematical Mindsets*: "Wherever you are on your own mindset journey, whether these ideas are new to you or you are a mindset expert, I hope that the data and ideas I share in this book will help you and your students see mathematics—any level of mathematics—as both reachable and enjoyable."



Next, Boaler challenges the status quo of mathematics education today and encourages us to see the creativity and beauty in mathematics. She highlights the widespread misconceptions held by students, teachers, parents, and the community. She suggests that by changing our focus from calculating to making connections, thinking logically, and using space, data, and numbers creatively, we can make real mathematics a creative, visual, connected, and living subject. Using the definition of a growth mindset—the belief that intelligence grows the more you learn—Boaler outlines several changes to school mathematics to create mathematical mindsets. First and foremost, mathematics should be viewed as a flexible conceptual subject that is focused on thinking and sense making rather than procedures. She stresses the importance of flexibility with numbers and asking questions about math facts, math practice, homework, math apps, and games.

Two of the most valuable aspects of this book are the selection of rich mathematical tasks provided and Boaler's explanation of how to find, design, and adapt other tasks to create mathematical excitement. She suggests that the following reflective questions should be used in this process:

- Can you open the task to encourage multiple methods, pathways, and representations?
- Can you make it an inquiry task?
- Can you ask the problem before teaching the method?
- Can you add a visual component?
- Can you make it low floor and high ceiling?
- Can you add the requirement to convince and reason?

Boaler also includes a list of websites to support the use of rich tasks. She suggests that when such mathematical tasks provide different ways of seeing, different methods and pathways, and different representations, the mathematics changes for children,

parents, and teachers. The math becomes transformational.

Boaler raises more issues with the status quo in mathematics education by pointing out "the elitist construction of math" and "the myth of the mathematically gifted child." She explains how our current practices limit achievement and participation of students of different ethnicities, genders, and socioeconomic income levels—more than any other subject taught! Additionally she illustrates when math inequalities in course placement can become illegal. After painting this bleak picture, Boaler goes on to provide strategies to move along the path of equity with these strategies:

- Offer all students high-level content
- Work to change ideas about who can achieve in mathematics
- Encourage students to think deeply about mathematics
- Teach students to work together
- Give girls and students of color additional encouragement to learn math and science
- Eliminate (or at least change the nature of) homework

She summarizes by empowering teachers to provide positive messages about success and the value of persistence and hard work for disengaged students. Teachers are responsible for and have the opportunity to make mathematics accessible to all students. Further Boaler recommends "de-tracking" in favor of "growth mindset grouping," and offers ways to teach heterogeneous groups effectively. Boaler acknowledges that equitable, growth mindset teaching is more difficult that traditional teaching, but also states that it is more important and fulfilling for teachers as students achieve more.

Mathematical Mindsets not only addresses instruction but also provides strategies for assessment with a growth mindset. The book encourages developing student self-awareness and responsibility in the following ways: self-assessment, peer assessment, reflection time, traffic lighting, jigsaw groups, exit tickets, online forms, doodling, student-written questions and tests. These strategies help students know what they are learning and where they should be in their learning. Boaler goes on to explain the importance of teacher feedback to help students understand the gap between where they are and where they should be. Additionally, she explains how teacher feedback is more valuable than grading, while providing advice about grading to teachers that have to give grades. The change from grading to feedback has a positive, powerful impact on student self beliefs, motivation, and future learning pathways.

The book also includes two appendices with very helpful information. Appendix A includes supporting materials and mathematical tasks that can be used in classrooms. Appendix B outlines classroom norms that nurture mathematical mindsets.

Mathematical Mindsets provides a plethora of material for use by new teachers, experienced teachers, administrators, coaches, parents, and higher education faculty, and strongly supports rich discussion for book study groups.

Personally, I have already recommended it to many of my colleagues and plan to use it in my math methods class in January. Used in conjunction with the online course, *How to Learn Math*, this book can be a catalyst for great changes in mathematics classrooms. Teachers have the opportunity to examine all aspects of their practice while building growth mindsets. I can hardly wait to get started!

(Editors note: If you are interested in participating in an online summer book study on *Mathematical Mindsets*, send an email to sandie.gilliam@comcast.net.)

ASSESSMENT

New Assessments for Colorado 10th and 11th Grades: PSAT and SAT

Catherine Martin, CCTM Past President

In place of PARCC testing in 2016, Colorado tenth grade students will be assessed using the Preliminary Scholastic Aptitude Test (PSAT) while eleventh grade students will be assessed on the ACT. However, from spring 2017 on, eleventh grade students will be assessed on the SAT and tenth graders will continue to be assessed on the PSAT. This change in assessments will create a coherent and seamless assessment system for Colorado students in grades 3 through 11 since the new PSAT and SAT are strongly aligned with the Colorado Academic Standards (Common Core).

Overview of New PSAT/SAT

The new assessments will measure students' engagement in the instructional shifts of conceptual understanding, procedural skills and fluency, and application as called for in our new standards. On the assessments, students will be asked to:

- Demonstrate their understanding of math concepts, operations, and relations. For instance, they might be asked to make connections between properties of linear equations, their graphs, and the contexts they represent;
- Carry out procedures flexibly, accurately, efficiently, and strategically and solve problems by identifying and using the most efficient solution approaches; and
- Solve problems grounded in the real world, directly related to work performed in college and career. Multi-step applications include problems in science, social science, career scenarios, and other real-life situations. These real-world problems ask student to analyze a situation, determine the essential elements required to solve the problem, represent the problem mathematically, and carry out a solution.

The new PSAT and SAT are divided into two sections: Math Test – Calculator (31 questions) and the Math Test—No Calculator (17 questions). Thus, it will be important for students in the calculator portion to determine whether it's best to use a calculator or whether they should make use of structure or reasoning to solve the problem.

Although most of the questions on the Math Test are multiple-choice questions, 22 percent are student-produced response questions, also known as grid-ins. Instead of choosing a correct answer from a list of options, students solve problems and enter their answers in the grids provided on the answer sheet.

Finally, there is no penalty for guessing on the PSAT and SAT, so students should give their best effort on all questions and not leave any questions blank.

Focus of Math on PSAT/SAT

The Math Test will focus in depth on four areas of math that play the biggest role in a wide range of college majors and careers:

- **Heart of Algebra,** focusing on the mastery of linear equations and systems (33%);
- **Problem Solving and Data Analysis,** focus on quantitative literacy (29%);
- **Passport to Advanced Math,** featuring questions that require the manipulation of complex equations (28%); and
- **Additional Topics in Math,** including geometry, trigonometry, and other topics relevant to college and career readiness (10%).

For a complete description of these four areas, see pages 50–57 in The Redesigned SAT® Teacher Implementation Guide. This guide is available for down-

load at: https://collegereadiness.collegeboard.org/pdf/redesigned-sat-k12-teacher-implementation-guide.pdf

Some Key Instructional Strategies for Teachers to Support Students on PSAT and SAT

The Redesigned SAT® Teacher Implementation Guide further describes many strategies that teachers can use in the classroom to support students' readiness for the new assessments. A few examples of these instructional strategies are:

- Ensure that students routinely practice solving multi-step problems.
- Vary the types of problems in homework assignments so that students do not always use the same strategy to solve every problem. It will help students to practice choosing the right mathematical strategy as well as solving the problem correctly.
- Include problems in which students may use the calculator and ones in which calculators are not allowed. Have students discuss how they used calculators as a tool and when they used other more expedient strategies.
- Provide frequent opportunities for students to interpret and apply mathematical skills and concepts in real-world contexts, particularly in the sciences and social studies.
- Provide students with explanations and/or equations that incorrectly describe a graph. Ask students to identify the errors, provide corrections, and cite their reasoning.

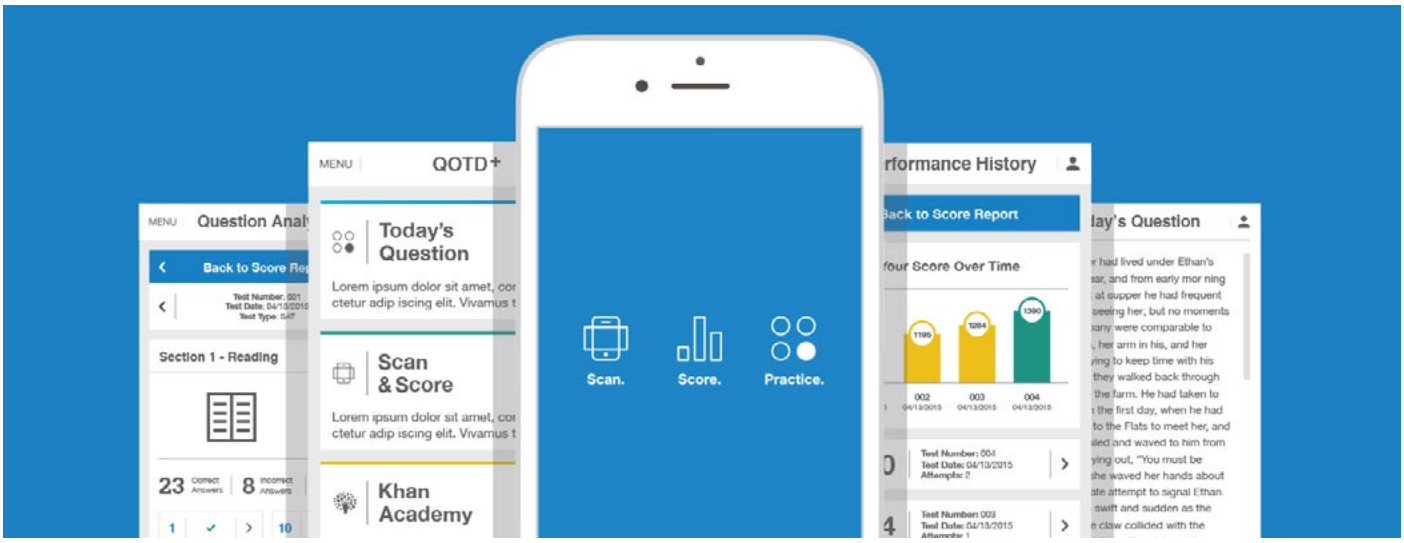
In addition, the Guide also describes strategies to share with students and with families.

Key Resources

In addition to all of the resources on the College Board website, Khan Academy, in its partnership with the College Board, has full practice tests and individual practice items for all of the skills in the four math areas. Videos are also included for extra support for students.

Final Thoughts

After perusing all of the free resources from College Board and Khan Academy, keep in mind and remind students that the best test prep is engagement in rigorous, standards-based mathematics classes.



CCTM

Members at a Glance

Laurie Hillman, Membership Chair

Have you and your colleagues ever wanted to reach out to others in order to explore new activities, new methods for teaching mathematics, or just invigorate creativity? Through the understanding of CCTM membership and the region representatives, you can reach out and attempt just this.

CCTM has seven different regions within our state boundaries with a combined membership total of 680 math educators. Regions, membership totals, school districts, and region representatives are listed below:

Region 1 - (144 members)

Englewood, Littleton, Mapleton, Sheridan, and school districts in Denver County. (Ann Summers)

Region 2 - (45 members)

Boulder Valley, and school districts in the counties: Clear Creek, Gilpin, and Jefferson. (Leigh Ann Kudloff)

Region 3 - (83 members)

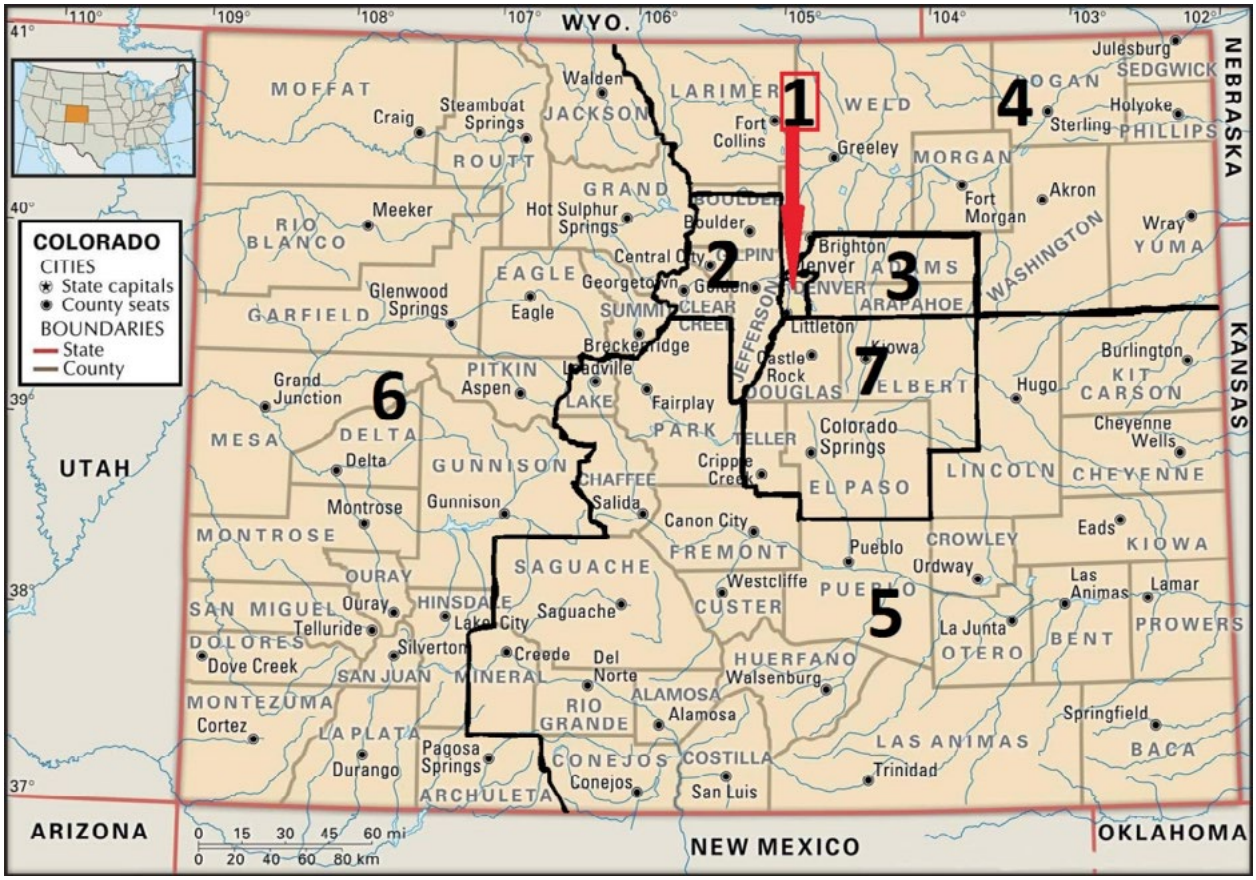
Adams 12, Adams 14, Adams 50, Aurora, Bennett, Byers, Cherry Creek, Deer Trail, and Strasburg. (Jen Overley)

Region 4 - (103 members)

Brighton, St. Vrain Valley, and school districts in the northeast counties: Larimer, Logan, Morgan, Phillips, Sedgwick, Yuma, Washington, and Weld (Cindy Ritter)

Region 5 - (80 members)

School districts in the southern counties: Alamosa, Baca, Bent, Chaffee, Cheyenne, Conejos, Costilla, Crowley, Custer, Fremont, Huerfano, Kiowa, Kit Carson, Lake, Las Animas, Lincoln, Mineral, Otero,



Park, Prowers, Pueblo, Rio Grande, and Saguache. (Jammie Brian)

Region 6 - (127 members)

School districts in the Western Slope counties: Archuleta, Delta, Dolores, Eagle, Garfield, Grand, Gunnison, Hinsdale, Jackson, La Plata, Mesa, Moffat, Montezuma, Montrose, Ouray, Pitkin, Rio Blanco, Routt, San Juan, San Miguel, and Summit (Nicole Wimsatt)

Region 7 - (98 members)

School districts in Douglas, El Paso, Elbert, and Teller counties. (Cassie Gannett)

The remaining 17 members consist of out-of-state speakers or vendors at the conference.

We are very excited to have each and every one of you and look forward to seeing you at an upcoming workshop or the conference again next fall. If you have any membership questions, be sure to send me an email at laurie.hillman@weldre4.org.

(Editor's note: Laurie Hillman lives and teaches in Windsor, CO. She has been our membership chair for the past five years.)

CCTM

Spring Elections

Stacy Larson, Elections Chair

The time has come for CCTM elections! Many great candidates are vying for the six open positions:

President

Joanie Funderburk

Vice President

Kevin Duren

Heather Lynn Johnson

Michelle "Shelly" Ray Parsons

Treasurer

Cassie Gannett

Secretary

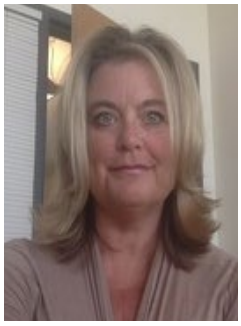
Julie Schmalz

Jennifer Yacoubian

Region 3 Representative

Cassie Harrelson

Gretchen Hazelwood



Jennifer Overley

Region 6 Representative

Jill Buecking

Nicole Wimsatt

To learn more about each candidate, please see the CCTM website and click on their photo. Each candidate provided information about themselves, their experiences, and why they want to be part of CCTM.

Elections will run from April 1st to April 20th. Voting will be done on the website. You will need your login information to be able to vote. Please go to www.cctmath.org for all the candidate information and voting.

NCTM CORNER

News from NCTM

Catherine Martin, CCTM Past President and NCTM Board Member

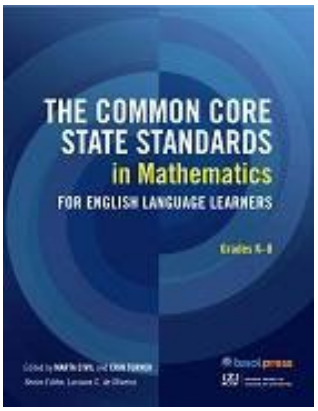
NCTM is offering professional learning opportunities through the interactive institutes this summer that will jumpstart your instructional practice for the 2016–17 school year. In addition, new publications are being released that would complement your summer professional reading. If you are not currently a member of NCTM and would like to join, you may use the code, BCM0616, to receive a discount on your membership.

Interactive Institutes

- **Engaging Students in Learning: Mathematical Practices (Grades K–8 and 9–12).** July 14–16, 2016 in Atlanta, GA. This professional learning experience will help you transform your classroom into an environment where students will learn how to examine, interpret, and think critically about math concepts.
- **Connecting Number and Operations in the Classroom (Grades Pre K–5).** July 21–23, 2016, in Denver, CO. This professional learning experience will give you the skills in number and operations to help your students develop number sense with a particular focus on conceptual understanding, procedural fluency, and applications. (See NCTM Professional Learning Opportunities, pp. 27–28, for more detailed information.)
- **Algebra Readiness for Every Student (Grades 6–8)** July 18–20, 2016, in Denver, CO. This professional learning experience will focus on techniques to develop algebraic reasoning skills in your students, best practices aligned with the Common Core State Standards, and students' preparation for college and career. Leave with practical methods to prepare your students for success. (See NCTM Professional Learning Opportunities, pp. 27–28, for more detailed information.)

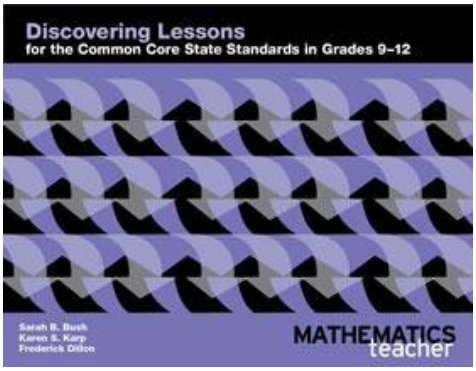
New Books from NCTM

- *The Common Core State Standards in Mathematics for English Language Learners: Grades K–8.*



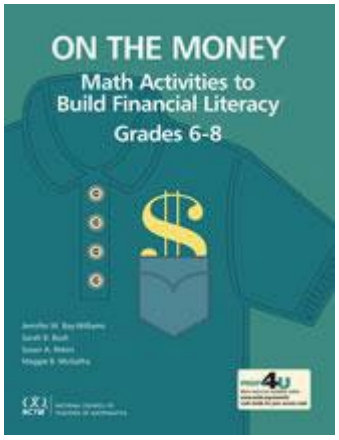
This book addresses a gap acknowledged in the Common Core State Standards for Mathematics (CCSSM): "It is … beyond the scope of the Standards to define the full range of supports appropriate for English language learners." Authors of the ten chapters that compose this collection focus on pedagogical practices that can support ELLs and their teachers in meeting the content and language demands of CCSSM. Each chapter features detailed classroom-based vignettes that highlight specific pedagogical practices that teachers can use to support ELLs in building their skill with the practices identified in the Standards for Mathematical Practice, and each concludes with questions for reflection and suggestions for action plans.

- *(eBook) Discovering Lessons for the Common Core State Standards in Grades 9–12 (PDF).* This practical guide aligns high-quality, engaging activities with specific content and



practice standards in the Common Core State Standards for Mathematics (CCSSM). Drawing on their work with preservice and in-service teachers, the authors have selected peer-reviewed articles from NCTM's award-winning journal for high school teachers, Mathematics Teacher, for this rich assortment of classroom activities aligned directly with CCSSM.

- *On the Money: Math Activities to Build Financial Literacy Grades 6–8.* The fourteen chapters in *On the Money* provide lessons and activities on such financial topics as jobs and net pay, sales discounts, credit card management, car insurance, and paying for college. The related mathematical skills include determining percentages, analyzing and representing data, evaluating probabilities, creating functions, and more.



Coming Soon!

NCTM will publish a series of three books around the teaching practices in *Principles to Actions: Ensuring Mathematical Success for All.* These books, along with a research companion for *Principles to Actions,* will be released in April 2016. Additional new releases at the Annual Meeting in April include: *Mathematical Modeling and Modeling Mathematics;* and *Building a System of Tens*.

CDE CORNER

New Mathematics Content Specialist

Raymond Johnson, Colorado Department of Education

This is my first opportunity to write in the CMT Journal as CDE's new mathematics content specialist, and I'd like to introduce myself by way of acknowledging some of the people and places that have helped bring me here. Like my predecessor, Mary Pittman, I enjoyed an excellent undergraduate teaching program at the University of Northern Iowa. There I worked with Ed Rathmell and Bonnie Litwiller, both of whom helped with the 1989 NCTM Standards, and introduced me to curriculum policy and the important role to be played by professional organizations.

I've enjoyed two great opportunities to teach high school in Colorado. The first was at Florence High School in Fremont County, where I got great support from my mentor, Bob Anderson, and the other math teachers there. I moved on to become the only full-time math teacher at South Park High School in Park County, where my math colleagues were the middle and elementary school math teachers on our small campus. At both schools, I learned a lot about the challenges faced by rural educators, and as I gained experience, I increasingly turned to CCTM and NCTM to find ways to improve my practice.

My students were often my best teachers. They showed me where in my practice I made progress, as well as the areas where I tended to struggle. Eventually, I went to graduate school to explore these and other questions I had about teaching and education. I owe a lot to my advisor, David Webb, for bringing me to CU-Boulder and into the math education program, where I've learned so much from the faculty and my fellow graduate students. The opportunities I've had to work with Cathy Martin and to do research in Denver Public Schools have helped balance my rural roots with a big-district perspective.

Thanks to the internet, I continue to learn every day from a wider world of math educators. Whether I find them on Twitter, Google+, blogs, or other virtual spaces, my perspectives are shaped by the teachers in these communities who reach out in search of ideas, resources, feedback, and support. If you can imagine taking some of what makes a CCTM conference valuable and repackaging it into an always-on, digital form, then you can get an idea of what I gain from my online colleagues in Colorado and beyond.

Now, as I work to complete my Ph.D. and transition into my new role at CDE, I'm relishing the variety of opportunities and challenges before me. I get to play the roles of student, teacher, researcher, and policy maker, all in the service of helping other mathematics educators. I especially look forward to the new connections and colleagues I'll make through CCTM and all the learning we'll do together.

SUMMER PROFESSIONAL DEVELOPMENT

Harrison District 2 New Staff Institute

Cassie Gannett, CCTM Regional Representative

Teaching is hard. Throughout our nation the statistics are staggering as to the number of teachers who leave the profession within three years. In districts like Harrison School District 2, with a diverse student population and 75% of the students receiving free or reduced lunch, the challenges are even greater. Therefore, we have created a "New Staff Institute" with the goal of supporting and growing teachers in this challenging profession.

In an effort to strengthen this program and meet the needs of our teachers, a team of dedicated teacher leaders and Instructional Coordinators recently applied to attend the Teach to Lead Summit in Baltimore, Maryland. Teach to Lead is a joint partnership of the U.S. Department of Education, ASCD (doing business as the Association for Supervision and Curriculum Development) , and the National Board for Professional Teaching Standards with a focus on advancing change through teachers. Our application was one of 29 teams accepted out of 150. At the summit, we met other state, district, and non-profit groups with the same goal—to improve education. We were the only group from Colorado. Other states that were represented included: Louisiana, Arizona, Illinois, Connecticut, Maryland, Michigan, Pennsylvania, Ohio, New York, Wisconsin, New Jersey, Florida, California, South Carolina, Arkansas, Virginia, and Alabama.

To launch our work at the summit, acting Secretary of Education, Dr. John King, gave an inspiring speech about the "need to invest in teachers and the teaching profession," stating that "it is important to the President that we lift up teachers." With news reports in recent years about the failure of education and teachers, it was refreshing to hear that our President and Secretary of Education both believe in the hard work that is being done on a daily basis.

Dr. John King

Our work towards enhancing our New Staff Institute to grow and retain teachers proved both difficult and rewarding. We were provided a short training on a Logic Model, in which we

clarified a problem statement and goal, and began work on inputs and activities that would lead to the desired outcomes. In addition, we were assigned a "critical friend" who had both experience with the process and a perspective as a teacher in-service coordinator from another state. Our "critical friend" provided us with support and encouragement throughout the process. An ASCD representative in our group provided insight and ideas from a national perspective that proved helpful. Overall, the experience was invigorating and inspiring.

The team with their "critical friend"

As compared with other district new staff induction programs, our New Staff Institute is unique; it is almost a month long and involves professional development, teaching experience and teacher observations during summer school, and support of a mentor. We have found that although new teachers come away exhausted, they are excited that they have found a support system within their new district and have had the opportunity to get to know our students. The goal is to keep that feeling of support and excitement for working in our district sustained throughout the year. Teaching is hard, but through the work that we did in Baltimore, we hope to help our teachers grow and stay in the profession by providing increased and more continuous support.

SUMMER PROFESSIONAL DEVELOPMENT

NCTM Professional Learning Opportunities

Janet Oien, NCTM Representative

The **NCTM Annual Meeting and Exposition** is right around the corner. This year it will be held **April 13–16 in San Francisco, California**.

Although early bird registration has closed, there are still discounts available for groups of five or more. Search the program online to see highlighted speakers for the conference. Matt Larson, who led the pre-session at our CCTM Annual Conference last September, is President-Elect. He will be speaking, along with many others including Diane Briars, Jo Boaler, Peg Smith, and Dan Meyer.

Have you ever considered speaking at the **Annual Meeting**? If so, now is the time to start planning. Each year, this conference has over 700 sessions. Speaker proposals for next year's conference in **San Antonio, Texas** are not on the website yet, but will be due May 1, 2016. If your proposal is accepted, the lead speaker will receive free conference registration.

If the National Conference in San Francisco doesn't work out for you this year, we have another opportunity for professional development through NCTM that is a little closer to home! This summer we will have two **NCTM Interactive Institutes in Denver.** Each institute is 2 ½ days of professional development provided by experts in mathematics education. **Register by May 13th** to take advantage of early bird specials.

The **Algebra Readiness Institute (6–8)** will be **July 18–20**. Peg Smith and Diane Briars are keynote speakers. In this institute participants will:

- Gain strategies to build the foundation of knowledge and skills to support students' future success in algebra.
- Use *Principles to Actions* as a tool for learning new instructional techniques for posing purposeful questions, engaging students in productive discourse, and building student responsibility within the community of learners.
- Learn strategies for implementing tasks that promote reasoning and problem solving and provide all students with opportunities to develop strong algebraic reasoning skills, including the development of conjectures and generalizations.
- Determine the role of the Standards for Mathematical Practice in instructional strategies and assessments.

- Understand how concepts within multiple domains of the Common Core State Standards support algebraic reasoning.

The **Number and Operation Institute (Pk–5)** will be **July 21–23**. Jennifer Bay-Williams and Max Ray-Riek are keynote speakers. In this institute participants will:

- Understand the importance of number as a critical foundation for college and career readiness.
- Acquire instructional strategies that provide all students with an opportunity to develop a sense of number.
- Determine the role of the Standards for Mathematical Practice as they impact number-related content domains and topics.
- Increase your understanding of the mathematical content of the Common Core domains that emphasize number.

Consider how NCTM's publication *Principles to Action: Ensuring Mathematical Success for All* will regularly impact implementation of the CCSS-M when considering access and equity, learning, teaching, curriculum, assessment and professional development.

SUMMER PROFESSIONAL DEVELOPMENT

NCTM 2015 Summer High School Institute

Jennifer Perego, Richmond, VA and Mary E Pilgrim, Colorado State University

What do we as teachers need to do in order to prepare lessons that will engage our students in the eight Common Core State Standards for Mathematical Practice (SMPs)? The goal of NCTM's High School Interactive Institute is to enable teachers to dive into the lessons and strategies that make for effective mathematics teaching. Through keynote speeches, interactive workshops, and facilitated task discussion groups, teachers have the opportunity to reflect on current and best practices, participate in lessons that they can take back into their classrooms, and work with other educators in their specific content areas on tasks and strategies that will enhance student learning.

NCTM's publication, *Principles to Actions: Ensuring Mathematical Success for All* outlines the role teachers, specialists, and school leaders need to play in supporting mathematics students in the classroom (NCTM, 2014). The High School Interactive Institute brings these principles to life and gives teachers a firsthand experience with implementing the mathematics teaching practices. Participants work on establishing goals to focus learning and implementing tasks that promote reasoning and problem solving. During this process, participants use and connect mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning, and elicit and use evidence of student thinking (NCTM, 2014).

The Geometry Group

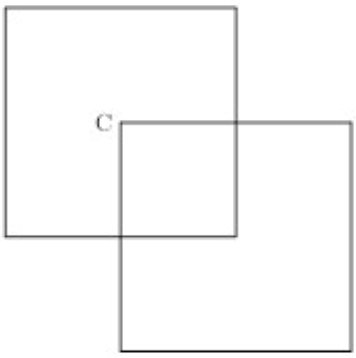
As the geometry task-group facilitators, our role at the institute was a dynamic one. Prior to the event, we worked together to select and prepare tasks that would be useful in communicating the mathematics practices and could be used in a high school level geometry classroom. Preparing the tasks involved considering the best strategy for posing the initial problem. As we did this, we asked ourselves: *What prior knowledge did we want to activate? What would draw students [participants] into the task?* This enabled us to break down each task into manageable pieces that promoted active engagement and discussion. We wanted to allow participants to explore tasks as their students would.

The facilitated task discussion groups met once each day for three days, so that participants could network and unpack their learning from the workshops and keynote sessions that they attended individually. At each meeting, we highlighted the mathematical practices that were the focus for the selected task. Participants worked individually on parts of their task, but were frequently asked to talk at their tables of 6–10 people, or with the entire group, about teaching and questioning strategies for their students. We found it was best to have participants wear their "student hat" and then switch to their "teacher hat" as we worked on each task. This process allowed the participants to actually engage in productive struggle and think critically about how their students would approach a problem and where their frustrations and breakthroughs would occur. While wearing their "teacher hat," participants could consider what kind of purposeful questions they would provide, what help and tools they would offer to their students, and what their "look fors" would be—as evidence for student learning.

Geometry Tasks

The geometry tasks drew upon a variety of NCTM resources including the journal *The Mathematics Teacher*, and NCTM's *Illuminations* website. We also encouraged participants to explore the *Mathematics Assessment Resource Service* website (http://map.mathshell.org) to see what available tasks and lessons could be implemented in their own classrooms. The tasks and SMPs for each day are summarized in the following tables:

Task 1: Rotating Square



This task involves two congruent overlapping squares (*n* units by *n* units). The center of one square is the vertex of the other square. Students [participants] are asked about the shared area if the square with vertex *C* is rotated about *C*. (A full description of the task can be found in *Focus in high school mathematics: Reasoning and sense making in geometry* (King, Orihuela, & Robinson, 2010)

SMPs Emphasized	Facilitating Questions	Connecting to Principles to Actions
SMP 2: Reason abstractly and quantitatively SMP 3: Construct viable arguments and critique the reasoning of others. SMP 6: Attend to precision	*- How would you present this task to your students?* *- How might your students approach this problem?* *- What might students' answers look like?* *- What mathematical questions might arise as they work?* *- How might SMPs 2, 3, and 6 be demonstrated in this task?* *- What are possible extensions for this task?*	Reasoning and Sense-Making *How can students get engaged in the process of Conjecture -> Justification -> Generalization?* *In what kind of meaningful mathematical discourse can students engage?* *What purposeful questions can be posed and how can they be posed?*

Task 2: Soda Cans
Resource: NCTM's *Illuminations* website

SMPs Emphasized	Facilitating Questions	Connecting to Principles to Actions
SMP 4: Modeling with mathematics SMP 7: Look for and use structure SMP 8: Look for and express regularity in repeated reasoning	*- How might your students approach this problem?* *- What might students' answers look like?* *- What mathematical questions might arise as they work?* *- How might SMPs 4, 7, and 8 be demonstrated in this task?* *- How can we get students engaged in this task?* *- What are the multiple entry points?*	*How will you ensure that there are opportunities for reasoning and sense-making by all students every day in class?* *What specific actions will you take during the first few weeks of school to create such a culture in your classroom?*

Task 3: Geometry Tools

For the last task session, we provided a variety of construction tools (straightedge, compass, protractor, Mira™) and asked participants to find as many ways as they could to complete several geometry constructions, such as bisecting a line segment, constructing an equilateral triangle, constructing the center of a circle, etc. The emphasis of this task was on geometry tools and appropriate use. In addition, we spent a significant amount of time on Miras™ - what they are, their use, and how to incorporate them in the geometry classroom. We drew upon the Mira™ activities described in an article in *The Mathematics Teacher* by Powell, Anderson, & Winterroth (1994).

SMPs Emphasized	Facilitating Questions	Connecting to Principles to Actions
SMP 1: Make sense of problems and persevere in solving them SMP 3: Construct viable arguments and critique the reasoning of others. SMP 5: Use appropriate tools strategically SMP 6: Attend to precision	*- For each task, which tool did you find most appropriate? Why?* *- Would you let students choose their tool?* *- How do these tools impact testing?* *- What about technology such as GeoGebra?*	*How can providing different mathematical tools help students learn?* *How do the tools we choose for our students influence their mathematical reasoning and making sense of situations?* *How do mathematical tools help students to communicate and justify their thinking and reasoning?*

Through exploration with tasks, taking part in facilitated discussions, and hearing ideas presented in workshops and speeches, the participants of the 2015 High School Interactive Institute were able to engage in and implement each of the Mathematics Teaching Practices outlined in *Principles to Actions*. As task group facilitators, we not only chose specific tasks to highlight the SMPs, but also paced and structured our sessions so that teachers could both work on the task and think critically about how they would bring these ideas and strategies back to their classrooms.

While we discussed the Geometry group at the high school institute, NCTM offers institutes for elementary and middle grade levels as well as statistics, Algebra I & II for high school level. In addition, the regional as well as annual NCTM conferences provide ample opportunities for teachers to share, discuss, explore, and engage in the teaching and learning of mathematics.

References

National Council of Teachers of Mathematics (NCTM). (n.d.). *Illuminations*. Retrieved from https://illuminations.nctm.org/Default.aspx.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

King, J., Orihuela, Y., & Robinson, E. (2010). *Focus in high school mathematics: Reasoning and sense making in geometry*. Reston, VA: National Council of Teachers of Mathematics.

Powell, N. N., Anderson, M., & Winterroth, S. (1994). Reflections on Miniature Golf. *The Mathematics Teacher*, 87(7), 490–495.

SUMMER PROFESSIONAL DEVELOPMENT

Connecting Algebra to Geometry: A Transition Summer Camp for At-risk Students

Mary E. Pilgrim and John Bloemker, Colorado State University

In the summer of 2013, we were tasked with creating and implementing an eight-day algebra camp for students who had just completed eighth grade Algebra I and were entering ninth grade Geometry. The students lived in a district comprised of high percentages of Latino students and students who qualify for free and reduced lunches. Participants for the algebra camp were selected based on poor eighth grade algebra exit exam scores.

The Algebra I exam that the students took at the end of their eighth grade year identified several areas of weakness. Therefore, the content of the camp focused on concepts within these areas that exemplified algebraic thinking, specifically emphasizing algebraic topics that would be utilized in the students' upcoming geometry course. Furthermore, we conveyed content using inquiry and problem-based learning mechanisms as opposed to the traditional lecture method. The purpose of this article is to share how we designed camp curriculum and the impact the algebra camp had on both confidence in mathematics and mathematical ability.

Designing the Curriculum: Connecting Algebra and Geometry

The participating high school wanted to implement a summer camp targeting students who needed to improve upon their algebra skills prior to beginning ninth grade geometry. In the short period of time that we had with the students, it was important to address students' weaknesses on algebraic concepts that would be directly applicable in Geometry. Specifically, we assessed the literature to identify links between algebraic and geometric thinking patterns.

Thinking Algebraically

Before discussing components of algebra that are present in geometry as well as thinking geometrically, it is useful to have an understanding of what it means to think algebraically. Algebraic thinking is recognized as having interconnected components: (1) the use of variables/symbols, (2) the exploration of patterns and relationships, and (3) the use of models and multiple representations (Burrill, 1992; Friel, Rachlin, & Doyle, 2001; Herbert & Brown, 1997; Lee & Freiman, 2006; Usiskin, 1988).

Variables. The word *variable*, though a central concept in algebraic thinking, can be difficult to concisely define, as its purpose can vary depending on context. Usiskin (1988) discusses variables as taking on four different roles:

1. Variables as unknown quantities.
2. Variables as part of "relationships among quantities" (p. 10), such as the relationship described between the area and radius of a circle in the equation $A=\pi r^2$.
3. Variables as part of algebraic structures. For example factorization of $x^2+2xy+y^2$ involves utilizing operations that can be used upon real numbers and polynomials.
4. Variables as "pattern generalizers" (p. 9).

This subtle change of the variable's role naturally leads to confusion in students (Schoenfeld & Arcavi, 1988). Rosnick (1981) found, and we have all certainly seen this, that students struggle to recognize when a variable is used in different contexts such as a parameter versus an unknown quantity. The idea that a variable can take on subtly different roles can be difficult for students to understand.

Patterns. Though pattern finding does not have to be associated with algebra, mathematical patterns are foundational in algebraic thinking (Herbert & Brown, 1997; Lee & Freiman, 2006). By asking the right kinds of questions, pattern recognition can lead to thinking about generalized algebraic concepts and abstract reasoning. Generalizing patterns

often requires one to utilize algebraic expressions or equations together with variables. Lee and Freiman argue that through the guidance of "scaffolded questioning, pattern explorations can lead to some very rich algebraic thinking about variables and unknowns, equivalence of algebraic expressions, symbol manipulation, domain and range of expressions and equations, and solving for the unknown" (p. 433). Students often engage in algebra via patterns by first writing about what they "see" and then attempting to formalize their thinking with algebraic notation, which often involves variables.

Multiple representations. When students begin to formalize patterns with algebraic expressions or equations, they can represent the patterns they identify in various ways. Depending on how a student describes a pattern, different students may develop multiple, yet equivalent, representations for the pattern (Lee & Freiman, 2006). When speaking about multiple representations, however, there is no restriction on speaking only about equivalent expressions such as $3n + 1$ and $3(n + 1) - 2$. Multiple representations can refer to different presentations of mathematical information, such as graphical, numerical, symbolic, verbal, etc. The various forms represent the same mathematical information, but in certain situations, one representation is more advantageous to use than the others (Burrill, 1992).

To put it briefly, one explores patterns and analyzes relationships and structures. Variables are used to represent quantities in relationships and structures. In turn, multiple representations of such quantities are used to create models of relationships and structures. These ideas are most certainly present in geometry as well as in algebra.

Connecting Algebra and Geometry

Although there are those who see Algebra and Geometry as two separate courses, algebra and geometry are deeply connected. While the word *geometry* often brings to mind shapes and pictures, there are components of geometry present in algebra. For one, as Banchoff (2008) states, "the geometric demonstration can show why an algebraic argument works" (p.107). Both algebra and geometry allow for multiple representations of concepts, which researchers agree add to the development of conceptual understanding, provide an opportunity to tie the symbolic to the real world, and in turn allow for flexibility in solving mathematical problems (Douglas, 1986; Duval, 2002; Gehrke & Pengelley, 1996; Griffin & Case, 1997; Heinze, Star, & Verschaffel, 2009). The use of multiple representations, a recognized component of algebraic thinking, is clearly tied to geometric thinking, and in fact provides an opportunity to link together concepts from both algebra and geometry.

The use of variables, another component of algebraic thinking, is also prevalent in geometry (Dindyal, 2004, 2007; Schoenfeld & Arcavi, 1988). It is not uncommon for students to have to determine the values of unknown sides or angles of a geometric shape or to set up equations involving angles or sides, thus incorporating the use of variables. Algebra abounds in geometry. As students do in algebra, geometry students also have to explore, understand, and model relationships between variable quantities (Dindyal 2004, 2007). To do this, they explore patterns and generalize results. In fact, one could look at algebra as a tool for exploring within the context of geometry. It is upon these ideas that we built a curriculum for the summer algebra camp.

Algebra Camp Curriculum

When choosing which topics to cover during camp, we first consulted the eighth grade algebra exit exam scores. Looking at student performance on each question, we narrowed our focus on questions on which fewer than 40% of students answered correctly. However, this left a significant amount of material to consider, which was far too much for an eight-day camp. We further pared down content by considering which topics reflected the three key aspects of algebraic thinking, and would be pertinent for students to know entering Geometry. With that in mind, we chose to focus our work on: (1) discovering and generalizing patterns, (2) simplifying expressions and combining like terms, (3) solving equations, (4) expressing word problems with an equation, (5) finding the area and circumference of a circle, (6) representing a line as an equation and a graph, and (7) multiplying binomials.

Some of the materials used for the camp were developed, while others came from NCTM publications. In addition, the Common Core State Standards for Mathematical Practice were always

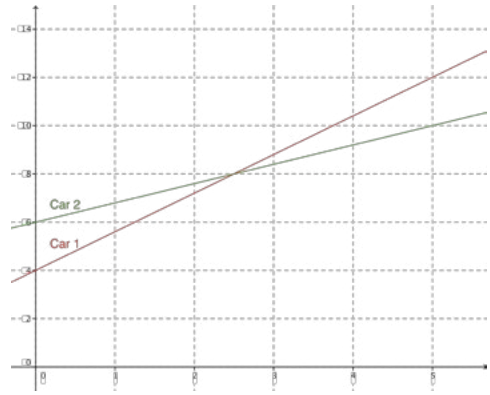
considered when designing the curriculum. A few examples of activities will be briefly discussed in the following sections.

Linear Equations and Race cars. We felt it was important to start the camp with a fun, interactive activity. Thus, an activity that tied in technology via the Texas Instruments CBR2™ data collection device was developed. Time was spent at the beginning showing students how to use the technology and set up the race cars. They learned how to interpret the information gathered from the CBR2™, recognizing that the slope of the race cars was always positive because the distance between the car and the CBR2™ grew as time passed, and the y-intercept was the starting location of the race car relative to the CBR2™. Once students understood these concepts and how they connected to the data, they could then explore the impact a weight had when added to a race car, as well as what happened when the cars had different starting points.

Students used the CBR2™ to collect position data from toy race cars, and the activity tied together patterns and multiple representations (physical model/toy race cars, scatter plot/graphical, table/numerical, and linear equation/symbolic). As they worked on questions, students explored and discussed the concepts of slope and the y-intercept of lines:

- Students compared the data between a race car with and without a weight attached to it (slope).
- Students compared the data between race cars that had different starting points. (y-intercept).

A brief example of combining concepts is given. The graph depicts the position of two race cars with respect to time.



After the ideas solidified, questions were posed, such as:

- Describe how the race started. Did either of the cars get a head start? How much of a head start did they have? Explain how you know.
- Was one of the cars faster than the other? Explain how you know.
- At what ordered pair do the two graphs intersect? What are the units of the first coordinate of the ordered pair? What are the units of the second coordinate of the ordered pair? What does this intersection point mean in terms of the race?

The race cars activity not only gave students a simple real-world application of slope and y-intercept, but was also something that they created and could visualize: they set up the race cars, watched them move, and then saw the position data displayed on their calculators. Students connected the multiple representations that position data could have, whether through a physical model or displayed as a discrete point in a scatter plot (or other representation). Connecting through multiple representations provided an opportunity to develop a deeper understanding of the concepts of slope and y-intercept.

Patterns and Polygons. Some of the activities that were created connected to geometry and were done toward the end of camp. One such activity had students explore polygons and the relationship between the number of sides and the sum of the interior angles. Students used sidewalk chalk and drew three different types of triangles on the concrete. For each triangle, they used a protractor to measure the interior angles, recorded data in a table,

and used the data to form a hypothesis about the sum of the interior angles of a triangle. Following this exploration with triangles, students repeated the activity for quadrilaterals and pentagons. Pre-created polygons were then used for a similar exploration.

Students compared findings from the sidewalk-drawn shapes to those that were pre-drawn and encouraged to look for patterns. Patterns that were discovered included that a three-sided figure has a sum of interior angles of 180°, while for four-sided figures the sum is 360°, and five-sided figures the sum is 540°.

The next question to explore was: *What if the shape has n sides?* This pushed students to examine their patterns for structure. When patterns were established, students sometimes had different expressions that depicted those patterns. After comparing answers, students used simplification and like terms to determine whether or not they had equivalent expressions.

In addition to exploring patterns and simplifying algebraic terms, the activity provided an opportunity to discuss possible drawing imperfections and measurement errors, and how these contributed to the accuracy of their approximations for the sum of the measures of the interior angles. Although students had errors, their data was accurate enough to make and test these conjectures and justify their results. Justification and asking students *"How do you know?"* was very common throughout camp.

NCTM Patterns Activities.

"Building with Toothpicks" from NCTM's *Navigating through Algebra in Grades 6-8* (Friel, Rachlin, & Doyle, 2001), and a "growing T" pattern exploration from NCTM's *Mathematics Teaching in the Middle School* (Lee & Freiman, 2006) connected patterns to algebra, explored equivalent symbolic representations, and connected algebra to geometry.

Delivery of Algebra Camp Content

In light of the connections between algebraic and geometric thinking, the content of the summer algebra camp emphasized the three pieces of algebraic thinking identified: the use of variables, the generalization and discovery of patterns, and the use of multiple representations. Problem-Based Learning (PBL) and Inquiry-Based Learning (IBL) mechanisms were implemented to encourage critical thinking.

Students were encouraged to ask themselves questions relating to the three key points of algebraic thinking:

- What I am considering in this problem?
- Are there any patterns that I can identify and use to solve this problem?
- Can I approach this problem differently to find an easier solution or verify my solution?

While some may view PBL and IBL as teaching strategies with minimal guidance, this is not the case. There is an important presence of scaffolding and instructor guidance involved in PBL and IBL (Hmelo-Silver, Duncan, & Chinn, 2006), and appropriately facilitated questions were key in pushing students to explore ideas and concepts.

Accountability and engagement are highly encouraged when participating in PBL and IBL environments. Therefore students become active learners. As active learners, students involved in inquiry develop deeper understandings and problem solving techniques (Kuhn et al., 2000). Thus, students not only leave with a conceptual understanding of the topic they were studying, they are also better prepared to explore other topics which they did not explicitly study.

In the context of the summer camp, PBL and IBL strategies were used to develop skills in algebraic thought and problem solving, avoiding memorization of steps and procedures. Students were able to leave with a conceptual understanding of the algebraic topics covered throughout camp. Furthermore, students also gained the problem solving and inquiry skills necessary to apply these algebraic concepts in a geometry setting.

What Happened?

The intent of the summer camp was to teach

concepts of algebraic thinking to students who were lower-performing in algebra, yet proceeding on to take geometry. Therefore, we wanted to know if students (1) had more confidence in doing mathematics after participating in the camp, (2) had increased algebra skills at the end of the camp, and (3) would be successful in geometry in the upcoming year. We had positive results in all three areas.

Confidence, which significantly increased, was measured using the Confidence construct from the Indiana Mathematics Beliefs Scales (Kloosterman & Stage, 1992). A shortened version of the same exit algebra exam was given, and students performed significantly higher on this as well. However, after working on math intensively for eight days, one might expect such results.

Based on midterm grades, 18 of the 19 students who participated in the summer camp continued on to Geometry. Though non-participants performed significantly better at the end of the first semester, there was no statistical difference between non-participants and participants at the end of the second semester. At the end of the academic year, all 18 students that had attended the summer camp were passing with a grade of D or higher, and 12 of the 18 students received a grade of C or better. This is notable, as all of these students prior to entering the summer camp had low scores in algebra, weak mathematical skills, and were lacking confidence in their mathematical ability.

Conclusion

The summer algebra camp was an overall success. Students improved their algebra skills and gained confidence in their ability to solve math problems. In addition, these students succeeded in their ninth grade geometry course, a course many of them might have otherwise failed.

Beyond the data collected, there were anecdotal signs of student improvement. The very first day of the camp, none of the students wanted to attend. However, by the end of the second week, most of the students were disappointed that the camp was over. Furthermore, the chaperone who rode with the students from the participating high school made note that as the camp progressed, the students talked about math subjects more often on the bus ride. During the camp, the improvement in the student's problem solving ability was apparent, as they were able to move through topics quicker as the week progressed. All of this indicates that our task was accomplished, and the students gained skill sets in solving mathematical problems with an unintended side effect of improved confidence and success overall.

References

Banchoff, T. (2008). Algebraic thinking and geometric thinking. *Algebra and Algebraic Thinking in School Mathematics,* 70, 99–110.

Burrill, G. (1992, August). *Algebra for the twenty-first century.* Lecture presented for the National Council of Teachers of Mathematics Annual Meeting, Nashville, TN.

Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Dindyal, J. (2004). Algebraic thinking in geometry at high school level: Students' use of variables and unknowns. Retrieved from http://www.merga.net.au/documents/RP202004.pdf

Dindyal, J. (2007). The need for an inclusive framework for students' thinking in school geometry. *The Montana Mathematics Enthusiast,* 4(1), 73–83.

Douglas, Ronald G. (Ed.). (1986) Toward a Lean and Lively Calculus. MAA Notes 6. Washington, DC: Mathematical Association of America.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of Mathematics. *Educational studies in mathematics* 61(1–2), 103–131.

Friel, S., Rachlin, S., & Doyle, D. (2001). *Navigating through algebra in grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

Gehrke, M., & Pengelley, D. (1996). Towards active processes for teaching and learning. In A. W. Roberts (Ed.), *Calculus: The Dynamics of Change. MAA Notes (39)*. 20–23. Washington DC: Mathematical Association of America.

Griffin, S., & Case, R. (1997). Re-thinking the primary school math curriculum: An approach based on cognitive science. *Issues in Education* 3(1), 1–49.

Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM*, *41*(5), 535-540.

Herbert, K., & Brown, R. H. (1997). Patterns as Tools for Algebraic Reasoning. *Teaching Children Mathematics* 3(6), 340–345.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and Achievement in Problem-based and Inquiry Learning: A Response to Kirschner, Sweller, & Clark (2006). *Educational Psychologist* 42(2), 99–107.

Kuhn, D., Black, J., Keselman, A., & Kaplan, D. (2000). The Development of Cognitive Skills to Support Inquiry Learning. *Cognition and Instruction* 18(4), 495–523.

Kloosterman, P., & Stage, F. K. (1992). Measuring Beliefs about Mathematical Problem Solving. *School Science and Mathematics* 92(3), 109–115.

Lee, L., & Freiman, V. (2006). Developing Algebraic Thinking through Pattern Exploration. *Mathematics Teaching in the Middle School* 11(9), 428–433.

Quintana, C., Reiser, B. J., Davis, E. A., Krajcik, J., Fretz, E., Duncan, R. G., Kyza, E., Edelson, D., & Soloway, E. (2004). A scaffolding design framework for software to support science inquiry. *The Journal of the Learning Sciences*, 13(3), 337–386.

Rosnick, P. (1981). Some Misconceptions Concerning the Concept of Variable. *The Mathematics Teacher* 47(6), 418–420.

Schoenfeld, A. H.., & Arcavi, A. (1988). On the Meaning of Variable. *The Mathematics Teacher*, *81*(6), 420–427.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. F. Clark (Ed.), *The Ideas of Algebra, K–12* (pp. 8–19). Reston: National Council of Teachers of Mathematics.

SUMMER PROFESSIONAL DEVELOPMENT

Fast Connections

Sandie Gilliam, Editor

Are you looking to find some summer professional development offerings? On my listserves, I receive short blurbs of summer opportunities. So far:

NCTM Summer Institutes

See "News from NCTM" on page 22 and "NCTM Professional Learning Opportunities" on page 27.

The Rocky Mountain Math Teachers' Circle

<http://rmmtc.ucdenver.edu/> is preparing for a big summer, including:

- HS Teacher Workshop: Active Learning Activities for Teaching Precalculus (June 13–17)
- HS Teacher Workshop: Active Learning Activities for Teaching Calculus (June 20–24)
- Summer Workshop in Winter Park with Northern Colorado Math Teachers' Circle (July 11–15)
- Additionally, the Rocky Mountain Math Teachers' Circle and the Southwest Math Teachers' Circle are jointly offering a workshop from August 8–11 at Fort Lewis College in Durango. Graduate credit or continuing education units will be available, and information about scholarships to pay for those credits is forthcoming. Applications<https://docs.google.com/forms/d/1xpr4FE4cW0sxnByILiZRYWDcuArbzOzhcaKHLDDtSzg/viewform> are accepted until June 15.

Online course from Jo Boaler

"How to Learn Math: for Teachers and Parents."
https://www.youcubed.org/category/mooc/