FLUIDS

$F = m \cdot a$ $\quad g = 9.807\ m/s^2 = 32.174\ ft/s^2$ $\quad gpm = 0.002228\ cfs$

$W = m \cdot g$ $\quad 1N = 1kg \cdot m/s^2$ SI $\quad 1cfs = 449\ gpm$

$1\ slug = lb/ft/s^2 = lb \cdot s^2/ft$ British

$$1\ lbf = \frac{1\ lbm \cdot 32.2 ft/s^2}{32.2 ft \cdot lbm/lbf s^2}$$ Engi.

$= 1\ slug \cdot 1\ ft/s^2$

Density $\rho = \frac{m}{V}$ $\quad \frac{kg}{m^3}$ $\quad \frac{slug}{ft^3}$ $\quad \frac{lbm}{ft^3}$ $\quad$ mass in slugs $= \frac{\text{mass in } lb_m}{g_c}$

Unit weight $\boxed{\gamma = \rho g}$ fluid or solid $= \frac{weight}{volume}$ $\quad \frac{N}{m^3}$ $\quad \frac{lb}{ft^3}$ $\quad \frac{lbf}{ft^3}$ $\quad \gamma = \rho(g/g_c)$ in eng.

$\gamma_{fluid} = SG \cdot \gamma_{water}$

$\rho_{water} = 1000\ kg/m^3 = 1.94\ slugs/ft^3$
$= 62.4\ lbm/ft^3$

Specific Gravity $SG = \frac{\gamma_s}{\gamma_{water}} = \frac{\rho_s}{\rho_w = 1}$ $\quad \frac{ft^3}{slug}$ $\quad \frac{m^3}{kg}$

$\gamma_{water} = \frac{62.4\ lb}{ft^3} = 9.80\ kN/m^3$
$= 62.4\ lbf/ft^3$

Viscosity $\quad \frac{Ns}{m^2}$ $\quad \mu = 1\ \frac{lb_f\ sec}{ft^2} = 479$ Poise

$\mu_{water} = 1.307 \times 10^{-3}\ N \cdot s/m^3 = 2.735 \times$
$lb \cdot s/ft$

Kinematic viscosity $\quad \nu = \mu/\rho$ $\quad \frac{ft^2}{sec}$ $\quad \frac{m^2}{s}$

$\nu_{water} = 1.306 \times 10^{-6}\ m^2/s = 1.410 \times 10^{-5}\ ft^2$

$P_{abs} = P_{atm} \pm P_g$ — gage or -vacuum

absolute — atmospheric

suction side of pump $\quad P < 0$ absolute

$P_a = \frac{N}{m^2}$ $\quad$ psi

Pressure of liquid $\quad P = \gamma h = \gamma(z_2 - z_1)$ — gage; FT

Pressure at given depth in psi $\quad P = \frac{\gamma h}{144}$

$F = PA = \gamma h_c A$

→ vertical depth of area's centroid from liquid surface

momentum

$\Sigma F_x = \rho Q (V_{2x} - V_{1x})$ — volume out; volume in

$\Sigma F_y = \rho Q (V_{2y} - V_{1y})$

$F = \sqrt{F_x^2 + F_y^2}$

force on pipe due to a bend

Submerged Surfaces

plate $F = \gamma h_c A$ — vertical depth to centroid (magnitude)

vert. distance to pressure acting $\cancel{y_p} - \cancel{y_c} = \dfrac{I_c}{l_c A}$ — inertia (location)

$I_c = \dfrac{\pi}{64} D^4$ circular p

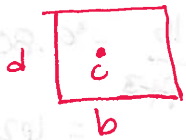
$= \dfrac{bd^3}{12}$ rect. plate

curved/plane $F_h = \gamma h_c A$, $F_v = \gamma V$ $\Big]$ $F = \sqrt{F_H^2 + F_V^2}$

bouyant $F = \gamma V$ $=W$ — volume displaced

weight of body = weight of volume displaced

center of pressure $y_p = y_c + \dfrac{I_c}{y_c A}$ — moment inertia of plate about centroidal axis



distance of plate centroid from pt where intersects liquid surface

continuity

$$Q = A_1 V_1 = A_2 V_2$$

$$A_1 V_1 = A_2 V_2$$

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{(for gases)}$$

$$H = z + \frac{P}{\gamma} + \frac{V^2}{2g}$$

Energy

$$I_1 + e_h + e_m + z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L + I_2 + \text{losses}$$

$h_L = \dfrac{P}{\gamma}$ energy lost per unit we

$I_1 = I_2$ if no temp change

$e_h = 0$ if no heat added/lost

$e_m = 0$ if no pump

major and minor losses!

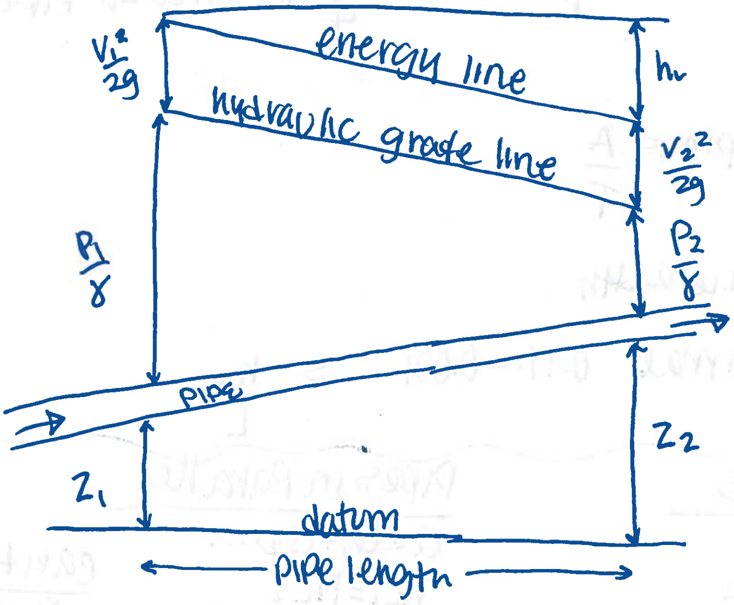
HYDRAULICS

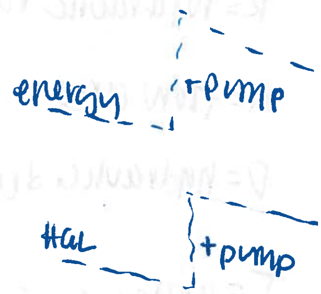
$\frac{1\,ft^3}{sec} = 449\,gpm$

2.31 ft head = 1 psi
1 ft water = 0.433 psi

$P = \frac{\gamma}{h} \Rightarrow h = \frac{p}{\gamma}$

$g = 32.2\,ft/s^2$
$= 9.81\,m/s^2$





head = energy / weight

S = slope of energy line = $\frac{h_L}{L}$

Darcy Weisbach

$h_L = \underbrace{\boxed{\frac{fL}{D}}}_{K} \frac{V^2}{2g} = KQ^n = \frac{P}{\gamma}$ associated pressure drop

where n = 1.85 (HW)
n = 2 (DW)

MOODY DIAGRAM

f - find $\frac{\epsilon}{D}$; use Re for laminar
no Re for turbulent

$K = f\left(\frac{L}{D}\right)\left(\frac{1}{2gA^2}\right)$

$Q = f\left(\frac{L}{D}\right)\left(\frac{Q^2}{2gA^2}\right)$

Reynolds $Re = \frac{VD}{\nu} = \frac{VD\rho}{\mu}$

$f = \frac{64}{Re}$ for laminar (Re < 2000)

Hazen Williams

$h_L = \frac{4.73\,L\,Q^{1.852}}{C^{1.852}\,D^{4.87}}$ (L – FT, Q – cfs, D – FT) eng.

pg. 17 chart

$= \frac{10.675\,L\,Q^{1.852}}{C^{1.852}\,D^{4.87}}$ metric

$V = 1.318\,C\,R^{0.63}\,S^{0.54}$ (ft/s) eng.

$V = 0.85\,C\,R^{0.63}\,S^{0.54}$ SI

LOSSES & FITTINGS

equivalent length $L_{eq} = \frac{KD}{f}$ (L_{eq} – FT, D – FT, f – darcy)

$h_L = \boxed{L + L_{eq}}$ into Hazen Williams equation

$\boxed{\text{* take into account entrance/exit losses}}$

minor losses $h_L = \frac{kV^2}{2g}$

FLOW

P = wetted perimeter *

R = hydraulic radius = $\frac{A}{P}$ $\qquad$ $R = \frac{D}{4}$ for circular pipes $\qquad$ hydraulic diameter $= 4R$

A = flow area → look at chart in back of here

D = hydraulic depth = $\frac{A}{T}$

T = water surface width

S = slope in decimal $0.1\% = 0.001$ $\qquad$ $S = \frac{h_L}{L}$ $\qquad$ in-mi = pipe diam. (in) X length (mi)

Pipes in Series

$Q = Q_1 = Q_2$

$H_{L1} = H_{L2} = H_{L3}$

Flow into junction = flow out

$R = \frac{D}{4}$ pipe full or half full

equiv. pipe lengths $K_E Q^n = K_1 Q^n + K_2 Q^n + \ldots$

Pipes in Parallel

$Q = Q_1 + Q_2 + \ldots$

$h_{L1} = h_{L2}$

$h_L = KQ^n$

$Q = \left(\frac{h_L}{K}\right)^{1/n}$

cavitation

P is vapor press. on suction side of pump low pressure

→ lower elevation to fix or increase P

high points in pipeline

$P_{atm} = 13.8$ psia = 1987 lb/ft²

manning's formula

cfs – $Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$ eng. (A: ft², R: ft, S: decimal)

$V = \frac{1.486}{n} R^{2/3} S^{1/2}$ (ft/s) OR $Q = VA$

$\frac{m^3}{s}$ – $Q = \frac{1}{n} A R^{2/3} S^{1/2}$ SI (S: decimal)

$V = \frac{1}{n} R^{2/3} S^{1/2}$

PUMPS:

AFFINITY LAWS:

$$\frac{Q_2}{Q_1} = \frac{D_2}{D_1} \qquad \frac{Q_2}{Q_1} = \frac{n_2}{n_1}$$ — speed of motor

$$\frac{H_2}{H_1} = \frac{(D_2)^2}{(D_1)^2} \qquad \frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{Q_2}{Q_1}\right)^2$$

$$\frac{P_2}{P_1} = \frac{(D_2)^3}{(D_1)^3} \qquad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^3 = \left(\frac{Q_2}{Q_1}\right)^3$$

(BHP) $$hp = \frac{\overset{lb/ft^3}{\gamma}\, \overset{cfs}{Q}\, H_p}{EFF\ 550} = \frac{E_1 - E_2}{EFF\ 550} = \frac{\gamma Q(e_1 - e_2)}{EFF\ 550}$$

H_p — FT; $e_1 - e_2$ — sp energy

$$e = \frac{\text{output}}{\text{input}}$$

$$= \frac{\overset{gpm}{Q}\, \overset{FT}{H} S}{EFF\ 3956}$$ S — SP. grav.

S=1 FOR WATER

EFF = (pump EFF)(motor EFF)

$$1\ hp = 0.746\ kW = 550\ lb{\cdot}ft/s$$

$$1\ kW = 1\ kN{\cdot}m/s$$

$$W = \frac{\overset{kN/m^3}{\gamma}\, \overset{m^3/s}{Q}\, H}{e}$$ H — m

W — power (kW)

CAVITATION: P is vapor pressure on suction side of pump { suction side of pump, high point in pipe, low pressures

- INSTALL pump near upper reservoir to avoid cavitation
- TO REDUCE CAVITATION DO THE FOLLOWING:
  1. increase suction diameter
  2. decrease elevation
  3. decrease discharge diameter

MANNING'S FORMULA

$$Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$$

english

cfs, FT^2, FT, decimal

$$V = \frac{1.49}{n} R^{2/3} S^{1/2}$$

BC Q=VA

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

SI

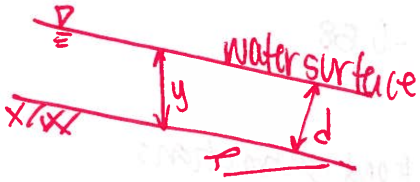
$\frac{m^3}{s}$, dec.

OPEN CHANNEL FLOW

LOOK AT MANNING'S EQUATION

wetted perimeter P

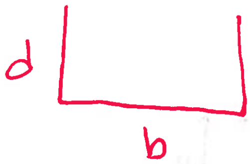
hydraulic radius $R = \frac{A}{P}$



flow area A

hydraulic depth $D = \frac{A}{T}$

Water surface T

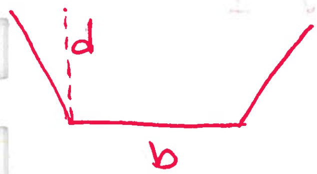


$A = bd = \frac{d^2}{2}$ rectangle

$A = bd + d\sqrt{b^2 - d^2}$ trapezoidal

$P = 3b$

$R = \frac{A}{3b}$



$A = \pi r^2$ circular

CHEZY'S FORMULA:

$Q = CA\sqrt{RS}$ — Chezy's coeff.

Specific energy:

$e = y + \frac{V^2}{2g}$

total energy $E = \gamma Q\left(y + \frac{V^2}{2g}\right)$

FROUDE NUMBER:

rect. $F = \frac{V}{\sqrt{gy}}$

F=1 critical
F<1 subcritical
F>1 supercritical

non-rect. trapez. $F = \frac{V}{\sqrt{gy_h}}$ $\quad y_h = \frac{A}{T}$

Best hydraulic cross section

$d = \frac{b}{2}$ rect. channel

b = side width
R = radius

semi circle any shape channel $A = \frac{1}{2}\pi R^2$

half hexagon $A = 1.3b^2$ trapez. channel

CRITICAL DEPTH:

F=1 sp energy = min

rect. $y_c = \left[\frac{q^2}{g}\right]^{1/3}$ $\quad q = \frac{Q}{b - \text{width}}$ cfs

non rect. $\frac{Q^2}{g} = \frac{A^3}{T}$ — product of y_c if y_c of unk $y_c \cdot w = A$

width water surface

trapez. $A = wy_c + \sqrt{\text{slope}}$

$T_{top} = w + 3y_c$

HYDRAULIC JUMP:

supercritical to subcritical
velocity decreases, depth increases

$y_2 = \ominus \frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2V_1^2 y_1}{g}}$ conjugate depths relationship

$h_j = \frac{(y_2 - y_1)^3}{4y_1y_2}$ headloss in hyd. jump

$hp = \frac{\gamma Q h_j}{550} = \frac{E_1 - E_2}{\text{eff } 550} = \frac{\gamma Q(e_1 - e_2)}{\text{eff } 550}$ $\quad hp = \frac{\text{lb} \cdot \text{ft}}{s}$

$V_1 = \sqrt{\frac{gy_2}{2y_1}(y_1 + y_2)}$

NORMAL DEPTH:

$d = 0.79\left[\frac{QA}{\sqrt{S}}\right]^{3/8}$

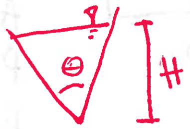
flow in steady slope — depth doesnot change

slope changes from steep to low = hy. jump

WEIRS:

v-notch triangular $Q = C\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2}$

b = width of weir/channel FT
H = head over weir FT
C = 0.61 – 0.58



90° triang $Q = 2.5H^{2.5}$ (FT) $C = 0.59$

CFS

$= 1.4H^{5/2}$ (meters)

rectan. weir $Q = \frac{2}{3}C(b-0.2H)\sqrt{2g}\,H^{3/2}$

N = # end contractions, if none (0.2H)

0.1N w/ N=2 → 0.1N for end contrac.

cfs

$= 3.33bH^{3/2}$ (FT) when weir extends entire length of channel

$= 1.84bH^{3/2}$ (meters)

trapezoid. (Cipoletti)
4:1 sides AKA 14° to vertical
4V:1H

$Q = \frac{2}{3}Cb\sqrt{2g}\,H^{3/2}$

$= 3.367bH^{3/2}$ for $C = 0.63$

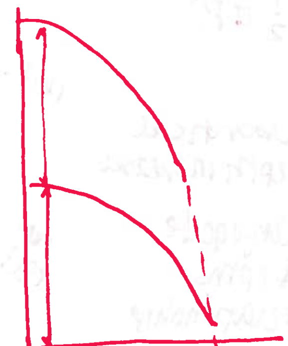
$= 1.86bH^{3/2}$ for meters

broad crest (weir crest width is greater than 1/2 head over weir)

$Q = CbH^{3/2}$ $2.63 < C < 3.33$

PUMPS:

SERIES: $H = H_A + H_B + \dots$
$Q = Q_A = Q_B$



eff $\eta = \dfrac{H_A + H_B + \dots}{H_A/\eta_A + H_B/\eta_B}$

$P = \dfrac{\gamma Q(H_A + H_B + \dots)}{550\eta}$

$\frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^2$ if changing impeller

$\frac{Q_2}{Q_1} = \frac{D_2}{D_1}$ "

PARALLEL: $H = H_A = H_B$

$Q = Q_A + Q_B + \dots$

$\eta = \dfrac{Q_A + Q_B + \dots}{Q_A/\eta + Q_B/\eta}$

$P = \dfrac{\gamma H(Q_A + Q_B + \dots)}{550\eta}$

Reduce cavitation: ↑ suct. Ø ↓ elev. ↓ dis Ø

Reduce pump speed: ↑ elev. ↓ pump rotation ↓ impellor diameter

THEORETICAL QUESTIONS:

To measure pipe flow
- venturimeter] $C_v = 0.98$
- nozzle]
- orifice $C_v = 0.61$

To measure open channel flow
- weirs
  - sharp crested → triang, rectang, trapez., V-notch for small Q
  - broad crested
- Parshall flume for wastewater/deposition of solids

Best hyd. cross section

Rectang $d = b/2$

trapez. ½ hexagon

any shape semicircle

Mannings: $Q = \frac{1.486}{n} A R^{2/3} S^{1/2}$ (cfs)

$A = \frac{\pi d^2}{4}$

$V = \frac{1.486}{n} R^{2/3} S^{1/2}$

1 gal = 8.34 lbm

Wastewater Treatment

I/I = Inflow/Infiltration

Formula Sheet

Wastewater flow rates

Average flows

$$Q_{Average} = \frac{Annual\ flow\ volume}{365\ days}$$

$$Q_{Per\ Capita} = \frac{Q_{Average}}{Population}$$

$$Q_{Design\ average} = Q_{Per\ Capita} \times Population_{Design\ Year}$$

Peak Flows

$$Q_{Peak} = [Q_{Average} \times F_P] + I/I$$

$$F_P = 1 + \frac{14}{4 + \sqrt{P}}$$

Where:

F_p = peaking factor or ratio of peak flow to average flow
P = population in thousands

Harmon's equation is just one equation commonly used to determine peaking factors. Other equations and charts can be used to determine peak flows and peaking factors.

Collection System Design flows

$$Q_{Design} = Q_{Peak} + Allowance_{I/I}$$

typical Allowance: 500gpd/ac ; 10 gpm/in.mi

I and I Allowance:

$$Allowance_{I/I} = Area\ allowance \times Service\ Area$$

Or

$$Allowance_{I/I} = Pipe\ allowance \times \sum Pipe\ diameter \times Pipe\ length$$

Collection Systems

Lift stations

Wet Well design

$$V = \frac{T_{min} q_P}{4}$$

If Q is given as system, don't double even if there are 2 pumps. system incap. 2 pumps already

Where:

V = volume of the wet well between pump on and pump off, gallons.
T_{Min} = Minimum cycle time, minutes.
q_P = the pump's average flow rate, gallons per minute.
↳ of 1 pump even if there are 3 pumps

Unit Operations and Processes

Typical Unit Process

A mass balance around a unit process will follow the laws of conservation of mass:

$$Influent\ Mass = Effluent\ Mass + Waste\ Mass$$

$S = Q \times Concent. \times 8.34$ (6 units)

[illegible] $= (mgd) \times mg/L \times 8.34\ lb/Mgal \cdot mg/L$

Likewise, the flows around a unit process will be conserved as well:

$$Influent\ Flow = Effluent\ Flow + Waste\ Flow$$

Removal Efficiency

removal eff. needed

$$E = \frac{S_{in} - S_{out}}{S_{in}} \times 100\%$$

BOD you have → BOD you want to have

$E = \frac{waste}{S_{in}} \times 100\%$ → taking out of system

Where:

E = removal efficiency of unit process in percent.
S_{in} = mass of contaminant entering unit process, Influent mass.
S_{out} = mass of contaminant leaving unit process, Effluent mass.

$$S_{in} - S_{out} = Waste\ Mass$$

Primary Sedimentation (Clarification)

design
SH = radius height
HRT - [illegible] depth
HLR/SLR - diameter

Settling velocity:

FOR Re < 1

$$v = \frac{g(\rho_p - \rho)d^2}{18\mu}$$

Where:

v = Settling Velocity of the Particle
g = Gravitational Acceleration
d = Diameter of the Particle
ρ_p = Density of the Particle
ρ = Density of Water
μ = Dynamic Viscosity of Water

Surface overflow rate:

$$v_0 = SOR = \frac{Q}{A_S}$$

Dissolved Air Flotation

$$A/S = \frac{1.3 s_a (fP - 1)}{S_c}$$

Where:

A/S = air to solids ratio, mL/mg
s_a = air solubility, mL/L
f = fraction of air dissolved at pressure P, usually 0.5
P = pressure, atm
p = gage pressure, lb/in²
S_c = sludge solids, mg/L

Note that:

$$P = \frac{p + 14.7}{14.7}$$

And that:

Temp °C	0	10	20	30
s_a, mL/L	29.2	22.8	18.7	15.7

For systems that pressurize a recycle flow to cause flotation the following equation may be used:

SLUDGE

use in digester problems: $\frac{FT^3}{d} = \frac{\frac{lb}{d}}{SG\ \gamma_w (1-W\%_s)}$? $\frac{lb}{FT^3}$ decimal % solids

$$A/S = \frac{1.3 s_a (fP-1)R}{S_c Q}$$

Where:

R = pressurized recycle, MGD
Q = influent flow, MGD.

Adsorption

Freundlich isotherm:

$$\frac{x}{m} = kC^{1/n}$$

Where:

x = the mass of compound adsorbed onto the carbon, lb.
m = the mass of activated carbon, lb. ; bulk density lb/ft³
C = the concentration of compound, mg/L.
k and n = isotherm constants determined experimentally for the specific compound and for the specific activated carbon.

Langmuir isotherm:

$$\frac{x}{m} = \frac{akC}{1+kC}$$

Where:

x = the mass of compound adsorbed onto the carbon.
m = the mass of activated carbon.
a = mass of adsorbed solute required to completely saturate a unit mass of adsorbent.
C = the concentration of compound, mg/L.
k = isotherm constant determined experimentally for the specific compound and for the specific activated carbon. The units of the constant determine the units of the isotherm.

Granular Activated Carbon (GAC)

Empty Bed Contact Time (HRT):

$$EBCT = \frac{contactor\ volume}{flow} = \frac{V_C}{Q}$$

Where:

$EBCT$ = Empty Bed Contact Time
V_C = Contactor volume
Q = Flow through contactor

Approach Velocity (HLR):

$$v_a = \frac{flow}{contactor\ area} = \frac{Q}{A_S}$$

Where:

v_a = HLR or approach velocity.
Q = Flow through contactor.
A_S = Surface area of the GAC bed.

Empty Bed Volumes:

$$EBV = \left(\frac{x}{m}\right)\frac{\rho_B}{(C_0 - C)} \times 16{,}000$$

Where:

EBV = The number of empty bed volumes of water that can be treated.
$\frac{x}{m}$ = The isotherm capacity of the GAC in LB/LB, may use the design isotherm $\left(\frac{x}{m}\right)_d$ instead to include a safety factor.
ρ_B = The bulk density (unit weight) of the GAC in LB/FT³.
C_0 = The influent compound concentration in mg/L.
C = The effluent compound concentration in mg/L, usually = 0.
16,000 = unit conversion factor = $\frac{1{,}000{,}000\frac{mg}{kg}}{62.4\frac{LB}{FT^3}}$.

Time to Exhaustion:

$$T_E = \left(\frac{x}{m}\right)\frac{\rho_B}{(C_0 - C)} \times EBCT \times 11.1$$

Where:

TE = Time to exhaustion in days.
$EBCT$ = Empty bed contact time in minutes.
11.1 = unit conversion factor = $\frac{1{,}000{,}000\frac{mg}{kg}}{62.4\frac{LB}{FT^3}24\ HR\ 60\ min}$.
Others as defined above.

Depth of the Adsorptive Zone or Mass Transfer Zone:

$$Z_S = Z\left[\frac{V_Z}{V_T - 0.5V_Z}\right]$$

Where:

Z_S = Depth of the adsorptive zone, FT.
Z = Total depth of GAC bed, FT.
V_T = Total volume treated at bed Exhaustion, FT³ ($C=0.95C_0$).
$V_Z = V_T - V_B$
V_B = Total volume treated at bed breakthrough, FT³ ($C=0.05C_0$).

Biological Treatment

Biochemical Oxygen Demand

Unseeded Test:

$$BOD_t = \frac{D_i - D_t}{p}$$

Seeded Test:

$$BOD_t = \frac{(D_i - D_t) - (B_i - B_t)f}{p}$$

Where:

BOD_t = the biochemical oxygen demand at time t, (mg/L)
D_i = the initial dissolved oxygen concentration, (mg/L)
D_t = the dissolved oxygen concentration at time t, (mg/L)
p = the decimal fraction of sample water in test
B_i = the initial dissolved oxygen concentration of the seed control, (mg/L)
B_t = the dissolved oxygen concentration of the seed control at time t, (mg/L)
f = the ratio of seed in sample to seed in control

$$BOD_t = BOD_u(1 - e^{-kt})$$

Where:

BOD_u = the ultimate BOD, (mg/L)
k = a decay constant determined experimentally, (d⁻¹)
t = time, days

$$k_T = k_{20}\theta^{T-20}$$

Where:

k_T = BOD decay constant at temperature, T, d⁻¹.
k_{20} = BOD decay constant at standard temperature 20°C, d⁻¹.
θ = Constant, usually 1.047 for BOD decay.
T = Temperature, °C.

Vol BOD = 300 mL bottles

ACTIVATED SLUDGE BASIN:

The BOD can adjusted for temperature as follows:

$$BOD_T = BOD_{20°C}(0.02T + 0.6)$$

Where:
BOD_T = BOD at Temperature T, mg/L.
T = Temperature, °C.

Activated Sludge

Hydraulic Retention Time:

$$\theta = \frac{V}{Q10^6}$$

V(gal) = Q t d. 10^6 days mgd

Where:

θ = Hydraulic retention time, days.
V = volume of the aeration tank, Gallons.
Q = influent wastewater flow, MGD.

Organic Loading Rate:

$$OLR = \frac{QS_0 8.34}{V10^{-3}}$$

Where:

OLR = organic loading rate, lb/d/ 1,000 FT^3.
Q = influent wastewater flow, MGD.
S_0 = influent organic concentration, BOD_5, mg/L.
V = volume of the aeration tank, FT^3.

$$U = \frac{Q(S_0 - S_e)}{VX}$$

Mean Cell Retention Time, Solids Retention Time, Sludge Age:

$$\theta_c = \frac{VX}{Q_w X_r + (Q - Q_w)X_e}$$

Where:

U = specific substrate utilization rate, d.
θ_c = mean cell residence time, d.
S_0 = Influent organic concentration, BOD_5, mg/L.
S_e = Effluent organic concentration, BOD_5, mg/L.
X = mixed liquor suspended solids (MLSS) concentration in the aeration tank, mg/L. (biomass)
X_e = MLSS concentration in the effluent, mg/L.
X_r = Return Activated Sludge (RAS) concentration, mg/L.
Q = influent wastewater flow rate, MGD.
Q_w = Waste Activate Sludge (WAS) flow rate, MGD.
V = volume of the aeration tank, MG.

Mass of microorganisms removed to maintain steady state conditions:

$$\left(\frac{lb}{d}\right) \quad P_x = Q_w X_r 8.34 = \frac{YQ(S_0 - S_e)8.34}{1 + \theta_c k_d}$$

Where:

Px = mass of waste activated sludge(WAS), lb/d.
Y = maximum yield coefficient (microbial mass synthesized / mass of substrate utilized), lb/lb.
θ_c = mean cell residence time, d.
k_d = endogenous decay rate, d^{-1}.
S_0 = Influent organic concentration, BOD_5, mg/L.
S_e = Effluent organic concentration, BOD_5, mg/L.
Q = influent wastewater flow rate, MGD.
Q_w = Waste Activate Sludge (WAS) flow rate, MGD.
X_r = Return Activated Sludge (RAS) concentration, mg/L.

$$\frac{1}{\theta_c} = YU - k_d$$

Food to Microorganism Ratio: MGD mg/L

$$F/M = \frac{QS_0}{XV} \approx U = \frac{Q(S_0 - S_e)}{XV}$$

MG

Oxygen Required: mg/L

$$O_2 = \frac{Q(S_0 - S_e)8.34}{f} - 1.42P_x \quad \frac{lb}{d}$$

Where:

O2 = Oxygen required, LB/d.
Q = influent wastewater flow rate, MGD.
S_0 = Influent organic concentration, BOD_5, mg/L.
S_e = Effluent organic concentration, BOD_5, mg/L.
P_x = mass of waste activated sludge, LB/d.
f = factor to convert BOD5 to ultimate BOD, unitless (usually between 0.45 and 0.68).

Plug Flow Reactor

$$r_{su} = \frac{kS_e\bar{X}}{K_s + S_e}$$

Where:

r_{su} = rate of substrate (BOD_5) utilization, d^{-1}.
k = maximum specific substrate utilization rate per unit mass of microorganisms, d^{-1}.
S_e = effluent substrate (BOD_5) concentration, mg/L.
K_s = half-velocity constant, mg/L.
$\bar{X}$ = average biomass concentration in the reactor, mg/L.

$$\frac{1}{\theta_c} = \frac{Yk(S_0 - S_e)}{(S_0 - S_e) + (1 + \alpha)K_s \ln(S_i/S_e)} - k_d$$

Where:

θ_c = mean cell retention time, d.
Y = yield coefficient, LB biomass/LB BOD_5.
k = maximum specific substrate utilization rate per unit mass of microorganisms, d^{-1}.
K_s = half-velocity constant, mg/L.
k_d = endogenous decay rate, d^{-1}.
S_0 = Influent organic concentration, BOD_5, mg/L.
S_e = Effluent organic concentration, BOD_5, mg/L.
α = recycle ratio = $\frac{Q_r}{Q}$.
S_i = influent concentration to reactor after dilution with return activated sludge (RAS) = $\frac{S_0 + \alpha S_e}{1+\alpha}$.

Complete Mix Reactor

Monod Expression for Reactor microorganism concentration: /biomass concent.

$$X = \frac{\theta_c Y(S_0 - S_e)}{\theta(1 + \theta_c k_d)}$$

HRT days

And Effluent BOD concentration:

$$S_e = \frac{K_s(1 + \theta_c k_d)}{\theta_c(Yk - k_d) - 1}$$

Where:

X = concentration of microorganisms (biomass)in the reactor, mg/L.
S_e = Effluent organic concentration, BOD_5, mg/L.

S_0 = Influent organic concentration, BOD_5, mg/L.
θ = hydraulic retention time in reactor, V/Q, d.
θ_c = mean cell retention time, d.
Y = yield coefficient, LB biomass/LB BOD_5.
k = maximum specific substrate utilization rate per unit mass of microorganisms, d^{-1}.
K_s = half-velocity constant, mg/L.
k_d = endogenous decay rate, d^{-1}.

$$k = \frac{\mu_m}{Y} \text{ and } \mu_m = Yk$$

Where:

µm = maximum specific substrate utilization rate, d-1.
Y = Growth yield coefficient, LB biomass/LB BOD5.
k = maximum specific substrate utilization rate per unit mass of microorganisms, d-1.

Trickling Filters

NRC Equations

$$E = \frac{100}{1 + 0.0561\sqrt{\frac{W}{VF}}}$$

1000 FT³

Where:

E = efficiency of BOD removal for process at 20°C, including recirculation and sedimentation, percent
W = BOD loading to filter, lb/d
V = volume of filter media, 10^3 ft³ , # in // multiply by 2
F = recirculation factor

$$F = \frac{1 + R}{(1 + R/10)^2}$$

F=1 NO RECIRC.

Where:

R = recirculation ratio = Q_r/Q
Q_r = recirculation flowrate
Q = wastewater flowrate

The equation for the second stage filter is:

$$E_2 = \frac{100}{1 + \frac{0.0561}{1 - E}\sqrt{\frac{W'}{VF}}}$$

Where:

E_2 = efficiency of BOD removal for second stage filter at 20°C, including recirculation and settling, percent
W' = BOD loading applied to second stage filter, lb/d

Adjustments to the efficiency based on wastewater temperature are given by:

$$E_T = E(1.035)^{T-20}$$

Where:

E_T = Efficiency of BOD removal at temperature T, percent
E = efficiency of BOD removal at 20°C given by the NRC equations, percent
T = wastewater temperature, °C

Germain and Schultz Equation

$$\ln\frac{S_e}{S_i} = -k_{20}D(Q_v)^{-n}$$

Where:

S_e = BOD of settled effluent from filter, mg/L
S_i = BOD of wastewater applied to filter, mg/L
k_{20} = treatability constant corresponding to a specific filter medium of depth D at 20°C, units vary with exponent n, as shown $(FT^3/d)^{0.5}/FT$.
D = filter depth, FT
Q_v = volumetric flowrate applied per unit area of filter, $FT^3/d \cdot FT^2$
n = experimental constant, usually 0.5

Match units

$$k_T = k_{20}(1.035)^{T-20}$$

$$k_2 = k_1\left(\frac{D_1}{D_2}\right)^x$$

Where:

k_1 = treatability constant corresponding to a filter of depth, D_1.
k_2 = treatability constant corresponding to a filter of depth, D_2.
D_1 = depth of filter one, FT.
D_2 = depth of filter two, FT.
x = 0.5 for vertical plastic media or rock media filters
0.3 for cross flow plastic media filters

Digesters

Anaerobic Digestion

Methane production:

$$V_{CH_4} = (5.62)[Q(S_0 - S)(8.34) - 1.42P_x]$$

sludge daily vol

daily mass of sludge

$$V = \frac{M}{S_s \gamma_w P_s \%}$$

sp. grav, density H2O, solids decimal

Where:

V_{CH_4} = volume of methane produced, FT^3/d.
5.62 = theoretical conversion factor
Q = flowrate (MGD)
S_0 = ultimate BOD in influent, mg/L
S = ultimate BOD in effluent, mg/L
P_x = net mass of cell tissue produce per day, lb/d

Volatile solids produced in the Anaerobic Digester:

mass microorg.

$$P_x = \frac{Y[Q(S_0 - S)8.34]}{1 + k_d\theta_c}$$

lb/d

$$\frac{Y(S_0 - S)}{1 + K_d\theta_c}$$

Where:

Y = yield coefficient, lb/lb
k_d = endogenous coefficient, d^{-1}
θ_c = mean cell residence time, d

$\theta_c = \theta$ for complete mix

high rate = 15 day low = 30-60 day w/out recycle

Aerobic Digestion

Volatile solids produced in aerobic digester:

$$\frac{dm}{dt} = -k_d m$$

Where

$\frac{dm}{dt}$ = rate of change of biodegradable volatile solids per unit time.
k_d = reaction rate constant, d^{-1}.

waste utilization =

$Se = BOD_u(1 - X\%)$ 20%

↳ % of waste util.

1 MGD = $\frac{FT^3}{1.5472\ S}$ = 133,678.08 FT^3/d
1.5472

m = concentration of volatile solids remaining at time t, mg/L.

Digester Volume Required:

$$V = \frac{Q_i(X_i + YS_i)}{X(k_d P_v + 1/\theta_c)} = \theta_c Q$$

Where:
V = digester volume, FT^3
Q_i = average influent flowrate, FT^3/d
X_i = average influent suspended solids concentration, mg/L
Y = fraction of the influent BOD_5 consisting of raw primary sludge
S_i = influent BOD_5, mg/L
X = digester suspended solids, mg/L
k_d = reaction rate constant, d^{-1}
P_v = volatile fraction of digester suspended solids
θ_c = mean solids retention time, days

The term YS_i can be neglected if no primary sludge is included in the sludge load to the digester.

Disinfection

Chlorine Disinfection

$$C_c = 0.7C_0 + 0.3C_0 e^{-t} \text{ for } t < 1 \text{ min.}$$

$$C_c = 0.7C_0 e^{-0.003t} \text{ for } t > 1 \text{ min.}$$

Where:

C_c = residual chlorine concentration after t minutes, mg/L.
C_0 = chlorine dose, mg/L.
t = chlorine contact time, min.

Method 1 uses the two relationships below:

$$\frac{N_t}{N_0} = (0.175C_0 t + 0.75C_0 te^{-t})^{-2.82} \text{ for } t < 1 \text{ min.}$$

$$\frac{N_t}{N_0} = (0.175C_0 te^{-0.003t})^{-2.82} \text{ for } t > 1 \text{ min.}$$

Where:

N_0 = number of coliform organisms at t=0.
N_t = number of coliform organisms at time t.
aka microbial destruction

Method 2 is derived from the Chick-Watson equation and may be used in the following form:

$$-\ln\frac{N_t}{N_0} = 2.86 \times 10^{-3} C_0^{1.46} + 14.4C_0^{1.25} - 14.5C_0^{1.25}e^{-0.00375t}$$

Nitrification and Denitrification

Nitrification

Oxygen required for nitrification alone:

$$\frac{lb}{d} \quad O_2 = 4.57\underset{mgd}{Q}(\underset{mg/L}{N_0} - N)8.34$$

Where:

O_2 = Oxygen required for nitrification, LB/d.
Q = Flow treated, MGD.
N_0 = Influent ammonia-nitrogen concentration, mg/L.
N = Effluent ammonia-nitrogen concentration, mg/L.

Oxygen required for carbon conversion and for simultaneous nitrification:

more effect at ↑ temp
requires aerobic conditions
DO > 1.0 mg/L
pH 7.2 to 9.0
Nitrification: 1 mg/L NH_4^+-N destroys 7.1 mg/L alk. as $CaCO_3$
$\boxed{4.57 \text{ lb } O_2 \text{ for 1 lb Am-Nit.}}$
Ammonia → Nitrite → Nitrate
Denitrification:
Nitrate → N_2 gas + O_2

complete nitrification:

$$O_2 = \frac{Q(S_0 - S_e)8.34}{f} - 1.42P_x + 4.57Q(N_0 - N)8.34$$

Where:

O_2 = Oxygen required for nitrification and Carbon conversion, LB/d.
Q = Flow treated, MGD.
S_0 = Influent BOD_5 concentration, mg/L.
S_e = Effluent BOD_5 concentration, mg/L.
P_x = Waste biomass, LB/d.
N_0 = Influent ammonia-nitrogen concentration, mg/L.
N = Effluent ammonia-nitrogen concentration, mg/L.

Denitrification

$$\frac{1}{MCRT} = \mu_m - k_d$$

Where:

$MCRT$ = Mean Cell Residence Time in Denitrification Reactor, d.
μ_m = denitrification rate, d^{-1}.
k_d = endogenous decay rate, d^{-1}.

Bioaccumulation Factor $BAF = \frac{C_{org}}{C}$ — C_{org}: tox. concen. in tissue (mg/kg); C — conc. of toxicant in ambient H_2O (mg/L)

Bio concentration Factor $BCF = \frac{C_{org} \text{ — mg/kg}}{C \text{ — mg/L}}$

Exponents

$$X = Y^{-N}$$

$$X^{-1/N} = Y$$

denitrif. 1 mg/l NH_4^+ needs 7.1 mg/l of alk. as $CaCO_3$

$$NH_4^+ + \underset{\text{2 moles req'd}}{2O_2} \rightarrow NO_3^- + H_2O + 2H^+$$

$$\underset{(mg/L)}{TSS} = \frac{(A - B) \times 1000}{C}$$

A — dry mass of filter + residue (mg); B — mass filter (mg); C — volume filtered in sample (mL)

$$\underset{(mg/L)}{TDS} = \frac{(\underset{mg}{A} - \underset{mg}{B}) \times 1000}{C}$$

C — vol dish (mL)

$$\frac{CFU}{100 mL} = \frac{A}{S} = \frac{A \times 100}{S}$$

A — # colonies; S — dilution fraction; S — volume (mL)

Solubility of Oxygen in Fresh Water *Salinity = 0*				
Absolute Pressure	***mm Hg***	760	1520	3040
	psi	14.7	29.3	58.7
	bar	1	2	4
	kPa	101.1	202.2	404.3
Temperature		**Solubility**		
°C	**°F**	***mg/l***	***mg/l***	***mg/l***
0	32	14.6	29.2	58.4
5	41	12.8	25.5	51.1
10	50	11.3	22.6	45.1
15	59	10.1	20.2	40.3
20	68	9.1	18.2	36.4
25	77	8.3	16.5	33.1
30	86	7.6	15.2	30.3
35	95	7	14	27.9
40	104	6.5	12.9	25.9
45	113	6	12	24
50	122	5.6	11.3	22.7

Solubility of Oxygen in Sea Water *Salinity = 35*				
Absolute Pressure	***mm Hg***	760	1520	3040
	psi	14.7	29.3	58.7
	bar	1	2	4
	kPa	101.1	202.2	404.3
Temperature		**Solubility**		
°C	**°F**	***mg/l***	***mg/l***	***mg/l***
0	32	11.2	22.4	44.8
5	41	9.9	19.7	39
10	50	8.8	17.6	35.2
15	59	7.9	15.9	31.7
20	68	7.2	14.4	28.8
25	77	6.6	13.2	26.4
30	86	6.1	12.2	24.4
35	95	5.6	11.3	22.6
40	104	5.3	10.5	21.1
45	113	4.9	9.9	19.7
50	122	4.6	9.4	18.7

Adapted from: http://www.engineeringtoolbox.com/oxygen-solubility-water-d_841.html

Major Wastewater Constituents**

Constituent	Concentration (mg/L)		
	Strong	Medium	Weak
Total Solids	1200	700	350
Dissolved Solids (TDS)	850	500	250
Suspended Solids	350	200	100
Nitrogen (as N)	85	40	20
Phosphorus (as P)	20	10	6
Chloride	100	50	30
Alkalinity (as $CaCO_3$)	200	100	50
Grease	150	100	50
BOD_5	300	200	100

**Information provided per PE Review Notes by Michael G. Morrison, P.E. of Freese and Nichols, Inc.

ALGAE - water plants HIGHER CO_2 AT NIGHT → LOWERS pH
HIGHER O_2 AT DAY

DEAD Algae create O_2 demand to decompose
toxins from algae
MUSSELS - clog intake structures

Eutrophication - happen in decades w/ WW pointsource

OLIGOTROPHIC PHASE - LOW TURBIDITY
HIGH DO, HIGH ALKALINITY, STABLE pH, LOW TEMP
LOW NUTRIENTS
LOW PRODUCTIVITY (PLANTS)
HIGH FLOW RATES

NEW LAKE/STREAM

MESOTROPHIC PHASE - MOD. TURBIDITY - increase of algae/sediments
MOD. DO (REDUCED) - because of plant decay
MOD NUTRIENTS - SPURS PRODUCT.
MOD. PRODUCTIVITY
SEDIMENT LAYER GROWS
DECAY RELEASES AMMONIA/PHOSPHORUS
LAKES HAVE ANAEROBIC BOTTOMS/AEROBIC TOPS

PLANT DECAY/LIFE CYCLE
|Algae seasonally

EUTROPHIC PHASE - HIGH TURBIDITY
LOW DO, LOW pH
HIGH NUTRIENTS
HIGH PRODUCTIVITY
ANAEROBIC bottom layer produces methane, hydrog. sulfide
rises → odorous lake
AT NIGHT O_2 PRODUCTION from algae STOPS, BECOMES ALL ANAEROBIC

DEEP SEDIMENT
ALGAE
NO FISH (CARP & SCAVENGERS ARE FINE)

SOLIDS

mg/L $TSS = \frac{(A_{mg} - B_{mg}) \times 1000}{C_{mL}}$

mg/L $TDS = \frac{(A - B) \times 1000}{C_{mL}}$

A - mass of residue in dish (after drying for TSS)
B - tare mass
C - volume of sample

Alkalinity - introducing a strong base & weak acid
no effect if introducing a acid or strong base known as "buffering"

Hardness - multivalent metallic cations
Mg^{+2}
Ca^{+2}
interferes w/soap to form suds or lather
Forms precipitate - soap scum
Forms scale on pipes, plumbing, etc.

soft	<60	mg/L as $CaCO_3$
mod. hard	61-120	
hard	121-180	
very hard	181-350	
saline, brackish	>350	

HPC - heterotrophic plate test

$$\frac{CFU}{100\,mL} = \frac{A}{S}$$ A - # colonies, S - dilution fraction of sample

membrane filter technique - MPN/100mL → use table 2 pg. 23

CLEAN WATER ACT

1972
Title I - VI
40 CFR part 133 - secondary stds
Section 303 - states need to have regs for water quality
Section 305 - biennial inventory of waters of US
Section 309 - creates EPA to enforce
Section 319 - amended 1987 for nonpoint sources
316 - thermal (cooling towers)
312 - marine sanitation (ships)
313 - federal facilities (bases, parks)
314 - clean lakes

section 402 - NPDES national pollution discharge elimination system
405 - sewage sludge disposal
503 - benef. reuse

318 - aquaculture
320 - National Estuary Prog.
403 - ocean discharge
404 - Dredging & filling
406 - coastal recreat.

THEORETICAL:

- sanitary dual-pipe is better than combined; cheaper for treatment
- lift station pumps for 2.5-3 in solids
- solids velocity 2-10 ft/s
- installed capacity - all pumps including redundant.
- firm pumping capacity - largest pump off
  - and suction - centrifugal
  - self priming
  - submersible
  - vacuum priming

wet well design: 6 starts/hr; 10 min between starts to stabilize

↑hp ↓starts

triplex lift station: 1 pump is redundant. Design capacity for pumps for 2 pumps

PRIMARY: SCREEN, GRIT REMOVAL, SETTLING, DAF

SECONDARY: DISS/SOLUBLE CONTAMINANTS

wetland facilities don't need a solids train

screens - high head loss captures more solids

grinding - head loss is design constraint

grit - $SG = 2.6$ fine enough for #100 mesh, design #100/150 mesh screen

primary clarifiers:

HRT 1.5-2.5 HR

SLR 800-1200 gpd/ft^2

WLR 10,000-40,000 gpd/ft^2

for type I discrete particle settling

min 2:1 ratio

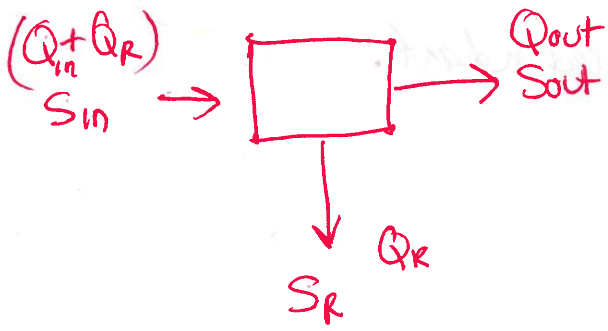
*do calcs for average and peak HRTs to see which flow controls the system

Secondary Clarifiers

MLSS = waste

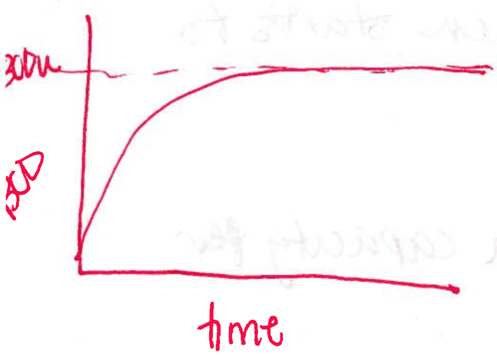
Radius < 5 depth

$Q_{in} = Q_{out} = Q_{design}$ mgd

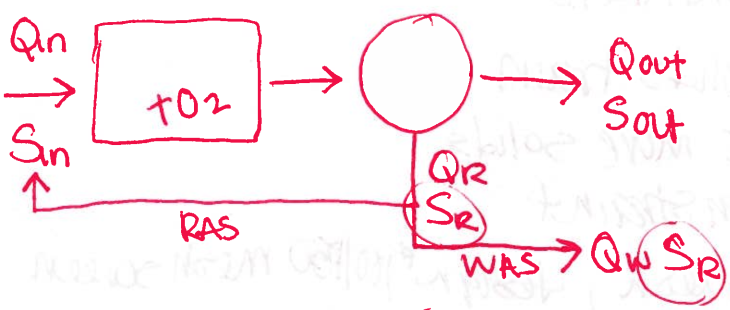


$$(Q_{in}+Q_R)S_{in} = Q_R S_R + Q_{out}S_{out}$$

BOD



Activated Sludge



$$Q_{in}S_{in} + Q_R S_R = Q_{out}S_{out} + Q_w S_R$$

Solids Treatment

Dewatering 15-20% solids

10% moisture = 90% solids

Supernatant - water which comes off

Disinfection

0.5 mg/L residual after 15-20 min → 100 CFU/100 mL

Sulfur dioxide to dechlorinate before discharge.

Chlorine

Chlorine is effected by temp, pH, contact time, ammonia c, residual Cl_2 concen

Effective at ↑ temp, ↓ pH

Chloramines are weaker

↳ Ammonia + Cl_2

WATER QUALITY

THERMAL – WARM WATER HAS LESS CAPACITY FOR DISSOL. OXYGEN
↳ SUPPORTS LESS AQUATIC LIFE

BASE FLOW – BETTER FLOW, BETTER QUALITY

SUSPENDED SOLIDS – RETAINED ON 1.5 μm FILTER

OXYGEN DEMAND – INCREASED BY ORGANIC MATERIAL

STREETER-PHELPS:

K_1 – deoxygen. rate constant
K_2 – reoxygen. rate constant

$L_u = BOD_u$ (mg/L) – ultimate BOD – aka when streams combine
∴ use mass balance at t=0 to find mixed BOD → $X_u = \frac{X_1Q_1 + X_2Q_2}{(Q_1+Q_2)}$

$DO_{min} = DO_{sat} - DO_{max}$

$DO_i = DO_{sat} - DO$

$$D_{max} = \frac{K_1}{K_2} L_u 10^{-K_1 t_c}$$

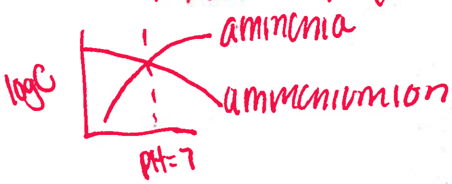
* change 10 to e if using base e not base 10

t_c – critical time for max O_2 deficit (d) – when O_2 is at max / critical time / min time DO is achieved

D_{max} – max O_2 deficit concentration (mg/L)
aka critical O_2 deficit

$$t_c = \frac{1}{K_2 - K_1} \log\left[\frac{K_2}{K_1}\left(1 - \frac{D_0(K_2-K_1)}{L_u K_1}\right)\right]$$

AMMONIA – TOXIC TO AQUATIC LIFE, MORE TOXIC AT pH LEVELS ABOVE pH=7



NOAEL – No observed acute effect level
Acute toxicity – NOAEL is 10% or more mortality
NOEC – No observable effect concentration
LOEC – Lowest observed effect concentration

CHRONIC – EPA recommends 4-day average not exceed CCC.
ACUTE – EPA recommends 1-HR. avg. exposure not exceed CMC.
Freq. not more than 1 TIME IN 3 YRS.

MINIMUM 20 psig FIRE FLOWS FOR 2-8 HR. design 10-12 HR. OF STORAGE
35 psig NORMAL FLOWS (based on MHD)

MATERIALS: DI, STEEL, COPPER, PVC, HPVC, RCP → usually prestressed

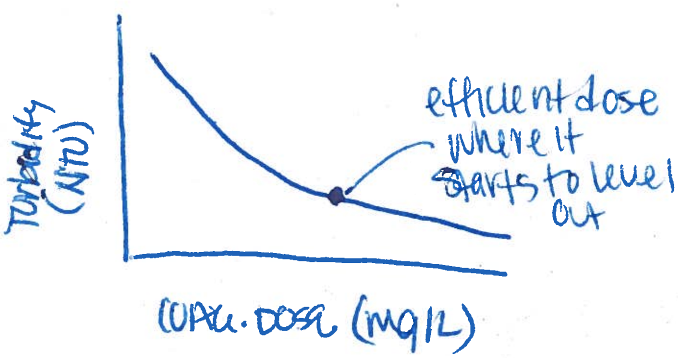
OUTDATED MATERIALS: WOOD, CI, LEAD/ASBESTOS CEMENT

VALVES: GATES <16 IN
BUTTERFLY >16 IN
BALL/PLUG FOR SERVICES & THROTTLING FLOW

THRESHOLD ODOR TREATMENT (TON) – 3 IS IDEAL, 5 CUSTOMER COMPLAINS, RIVER IS 6-24

COAGULATION – GETS RID OF REPULSIVE FORCES/BONDS; REDUCES TURBIDITY
FLOCCULATION – MAKES IT STICK TOGETHER; CAN ADD POLYMERS TO HELP
↳ SLOW MIXING

COAGULATION:
HYDROPHOBIC – ADD SALTS OF Al_2 OR Fe, RAPID MIXING



-99.9%

LOW pH
HIGH TEMP] Cl_2 is very effect.

This is the chlorine amount that is required.

Table 4 - Values of CT for 3-Log Inactivation of Giardia Lamblia

Disinfectant mg/L	pH	Temperature, °C 0.5	5	10	15	20	25
Free Chlorine ≤0.4	6	137	97	73	49	36	24
	7	195	139	104	70	52	35
	8	277	198	149	99	74	50
	9	390	279	209	140	105	70
1.0	6	148	105	79	53	39	26
	7	210	149	112	75	56	37
	8	306	216	162	108	81	56
	9	437	312	236	156	117	78
1.6	6	157	109	83	56	42	28
	7	226	155	119	79	59	40
	8	321	227	170	116	87	58
	9	466	329	236	169	126	82
2.0	6	165	116	87	58	44	29
	7	236	165	126	83	62	41
	8	346	263	182	122	91	61
	9	500	353	265	177	132	88
3.0	6	181	126	95	63	47	32
	7	261	182	137	91	68	46
	8	382	268	201	136	101	67
	9	552	389	292	195	146	97
ClO_2	6-9	63	26	23	19	15	11
Ozone	6-9	2.9	1.9	1.43	0.95	0.72	0.48
Chloramine	6-9	3800	2200	1850	1500	1100	750

Table 5 – Values of CT for the Inactivation of Viruses at pH between 6 and 9

Disinfectant	Log Inactivation	Temperature, °C ≤1	5	10	15	20	25
Free Chlorine	2	6	4	3	2	1	1
	3	9	6	4	3	2	1
	4	12	8	6	4	3	2
Chlorine Dioxide	2	8.4	5.6	4.2	2.8	2.1	1.4
	3	25.6	17.1	12.8	8.6	6.4	4.3
	4	50.1	33.4	25.1	16.7	12.5	8.4
Ozone	2	0.9	0.6	0.5	0.3	0.25	0.15
	3	1.4	0.9	0.8	0.5	0.4	0.25
	4	1.8	1.2	1.0	0.6	0.5	0.3
Chloramine	2	1243	857	643	428	321	214
	3	2063	1423	1067	712	534	356
	4	2883	1988	1491	994	746	497

The diagram below shows a typical disinfection process. The disinfectant is added before the treatment unit at some set dose. The water and disinfectant flow into the treatment unit and the residual concentration of disinfectant is measured on the flow exiting the treatment unit.

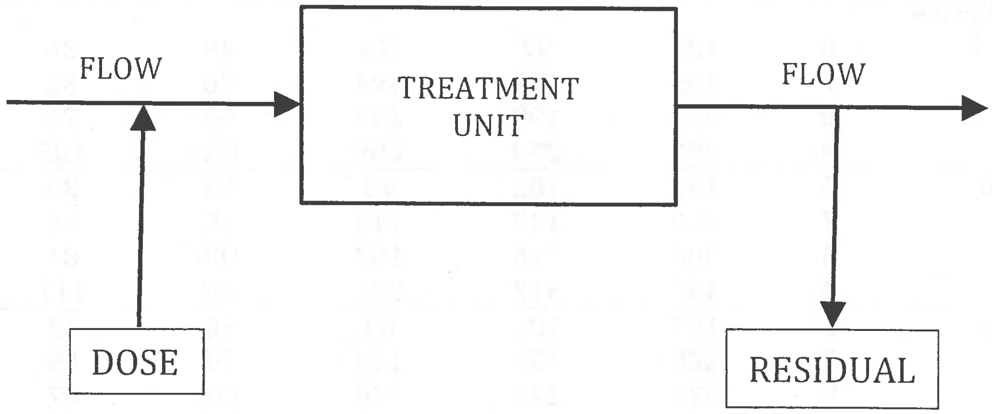


Figure 13 - Typical Disinfection Flow Sheet

The difference between the dose and the residual is called the disinfectant demand:

$$Demand = Dose - Residual$$

The disinfectant consumption can be estimated for both complete mix and plug flow regimes as follows:

$$\text{Plug Flow:} \quad C_0 = Ce^{kT_{10}}$$

$$\text{Completely Mixed:} \quad C_0 = C(1 + kT_{10})$$

Where:

C_0 = Initial concentration of disinfectant (mg/L)
C = Concentration of disinfectant (mg/L)
k = Decay Rate (min^{-1})
T_{10} = 10th percentile of the Hydraulic Retention Time as define below (min)

Use the residual concentration for the concentration, *C*, in the equation for CT. The time, *T*, in the CT equation is the 10th percentile of the hydraulic retention time in the treatment unit or T_{10}. T_{10} can be calculated using the mean hydraulic retention time, *HRT*:

$$T_{10} = HRT \times B_f = \frac{V}{Q} \times B_f$$

The baffling factor, B_f, is a function of the hydraulic characteristics of the treatment unit or contactor. Higher baffling factors result from units that come closest to achieving plug flow. Units with a high degree of hydraulic short-circuiting use a low baffling factor. Perfect plug flow uses a baffling factor of one, as shown in Table 6 with other baffling conditions.

Table 7 - Pathogen Removal and Inactivation Requirements for Various Treatment Processes

	Expected Log Removal			Required Log Inactivation		
Treatment Process	Crypto	Giardia	Viruses	Giardia	Viruses	Crypto
Conventional filtration	2.0	2.5	2.0	0.5	2.0	1.0
Direct filtration	2.0	2.0	1.0	1.0	3.0	1.0
Slow sand filtration	2.0	2.0	2.0	1.0	2.0	1.0
Diatomaceous earth filtration	2.0	2.0	1.0	1.0	3.0	1.0

Conventional filtration includes coagulation, flocculation and sedimentation prior to filtration as shown below in Figure 14.

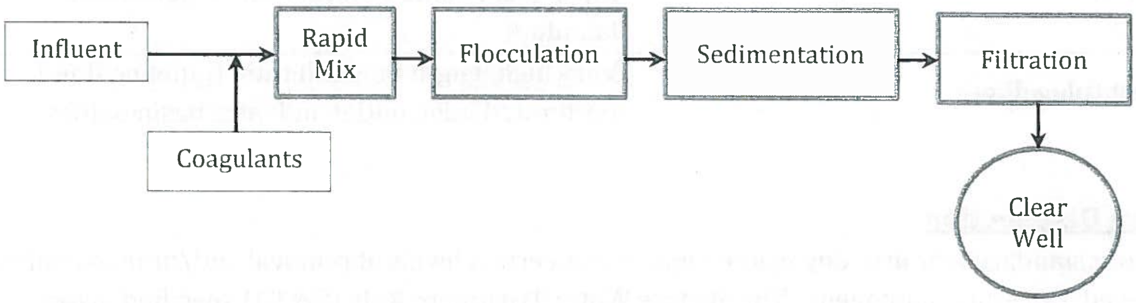


Figure 14 - Conventional Filtration Flow Sheet

Direct filtration is filtration with little or no pretreatment. The filters may be preceded by a coagulant addition and sometimes flocculation as shown below in Figure 15.

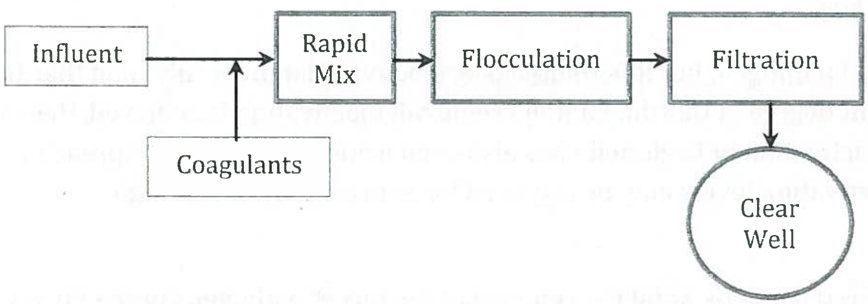


Figure 15 - Flow Sheet for Direct Filtration with Flocculation

Slow sand filtration is discussed in the filtration section of the notes. Slow sand filters typically require pretreatment such as a roughing filter or sedimentation similar to conventional filtration. Diatomaceous earth filtration is also discussed in the filtration section of the notes. DE filters should be preceded by coagulant feed of alum or polymer to enhance removal of viruses, bacteria and turbidity.

Treatment plants must determine the compliance with pathogen removal/inactivation requirements daily. Compliance is determined by calculating the actual CT, CT_{cal}, for various units in the process to determine the total CT_{cal} at the maximum flow rate for that day. Process units may

Table 6 - Baffling Factors for Calculation of Contact Time

Baffling Condition	Baffling Factor T_{10}/T	Baffling Description
Unbaffled (mixed flow)	0.1	None, agitated basin, very low length to width ratio, high inlet and outlet flow velocities
Poor	0.3	Single or multiple unbaffled inlets and outlets, no intra-basin baffles
Average	0.5	Baffled inlet or outlet with some intra-basin baffles
Superior	0.7	Perforated inlet baffle, serpentine or perforated intra-basin baffles, outlet weir or perforated launders
Perfect (plug flow)	1.0	Very high length to width ratio (pipeline flow), perforated inlet, outlet and intra basin baffles

Primary Disinfection

Regulatory standards for drinking water require that certain levels of removal and/or inactivation be achieved for certain pathogens. The Surface Water Treatment Rule (SWTR) specified target pathogens and required level of removal/inactivation include:

- Cryptosporidium: 99% or 2-Log.
- Giardia Lamblia: 99.9% or 3 Log.
- Viruses: 99.99% or 4 Log.

Legionella is another regulated pathogen; but it is removed or inactivated more easily than Giardia Lamblia, and thus if a sufficient degree of Giardia Lamblia removal/inactivation is achieved, then a sufficient level of removal/inactivation of Legionella has also been achieved to prevent spread of disease. Higher removal/inactivation levels may be required for source waters with high concentrations of pathogens.

The level of disinfection required depends upon the removal of the target pathogens in the filtration and associated treatment processes. Different processes achieve varying levels of pathogen removal. The above regulatory levels of removal/inactivation must be achieved through the combined processes of filtration and disinfection. If no filtration is used, then the inactivation achieved by disinfection must meet or exceed the same level. The level of removal from filtration is added to level of inactivation by disinfection to determine compliance with the requirements of SWTR. Table 7 summarizes the requirements for Giardia and viruses. **All the processes listed in Table 7 will provide a 2 Log removal of Cryptosporidium and thus meet those minimum requirements.**

Solids are generally fed with volumetric or gravimetric dry feeders. These feeders usually feed to a mixing basin where the solid chemicals are dissolved or mixed in a suspension or slurry before being fed to the process water. Some solids are dissolved in water in day tanks and fed to the process as liquids.

Facilities to feed chemicals are generally designed to provide a certain feed rate. Design considerations include minimum and maximum doses, daily consumption, storage quantities and the feed rate. Feed rate, R_F, is generally computed as follows:

$$R_F = \frac{DQ8.34}{P}$$

Where:

- R_F = Chemical feed rate in lb/day of active chemical.
- D = Dose of chemical required, mg/L.
- Q = Process water flow rate, MGD.
- P = Potency, or the Decimal fraction of the chemical that is active.

Equipment is selected to provide a variety of feed rates between minimum doses and minimum process flow rates to maximum doses and maximum process flow rates. Often, to achieve the wide variation in feed rate required multiple feed units are provided.

Daily consumption is used to size day tanks. Day tanks are used to provide one day of storage for the actual feeder. Feeders draw from a day tank. Every day, operators refill the day tanks. The average daily consumption also is used to determine costs for chemicals, since chemicals represent a major cost of operating a water treatment plant. The day tanks are filled from larger storage tanks. These storage facilities are designed to store sufficient chemicals to prevent running out between deliveries. Knowledge of local availability of chemicals is important when sizing storage facilities. The delivery method is also an important consideration in the design of storage facilities. Means for unloading tanker trucks, rail cars, etc. are required.

For example, many dry chemicals like lime can be unloaded from trucks or rail cars with forced air. The powdered dry chemicals can be vacuumed from the delivery vehicle into silos.

Calcium Carbonate Equivalents

Since calcium carbonate is the source of alkalinity and hardness in drinking water or because it's molecular weight is an even 100, it is often used as the base for describing quantities and concentrations of chemicals in water and used in water treatment. Table 1 provides the name, chemical formula, molecular weight, equivalent weight and the $CaCO_3$ conversion factor for the chemicals most commonly encountered in water treatment.

For Example, $CaCO_3$ has a molecular weight of 100 grams per mole and an equivalent weight of 50 grams per mole. Thus one mole, 40 grams, of Ca^{+2} with an equivalent weight of 20 grams per mole imparts a hardness equivalent to 2.5 moles, 250 grams, of $CaCO_3$ since 50/20=2.50(ratio of equivalent weights). Thus the factor for Ca^{+2} is 2.50.

To convert to equivalents as $CaCO_3$ refer to the factors in Table 1. This table presents factors to convert masses and concentrations of the substances indicated to equivalents as $CaCO_3$:

$$\mathrm{mg}/\mathrm{L}\ \text{substance} \times \text{Factor} = \mathrm{mg}/\mathrm{L}\ \text{of substance as } CaCO_3$$

For example, convert the following to equivalents as $CaCO_3$:

Ca^{+2} = 80 mg/L : $80\ \mathrm{mg}/\mathrm{L} \times 2.50 = 200\ \mathrm{mg}/\mathrm{L}$ as $CaCO_3$

Mg^{+2} = 25 mg/L: $25\ \mathrm{mg}/\mathrm{L} \times 4.10 = 102.5\ \mathrm{mg}/\mathrm{L}$ as $CaCO_3$

CO_2 = 12 mg/L: $12\ \mathrm{mg}/\mathrm{L} \times 2.27 = 27.2\ \mathrm{mg}/\mathrm{L}$ as $CaCO_3$

HCO_3^- = 207 mg/L $207\ \mathrm{mg}/\mathrm{L} \times 0.82 = 170\ \mathrm{mg}/\mathrm{L}$ as $CaCO_3$

Table 1 - Water Chemistry $CaCO_3$ Equivalents

Chemical	Formula	Molecular Weight	Equivalent Weight	Factor
Cations				
Aluminum	Al^{+3}	27.0	9.0	5.56
Ammonium	NH_4^+	18.0	18.0	2.78
Calcium	Ca^{+2}	40.1	20.0	2.50
Cupric copper	Cu^{+2}	63.6	31.8	1.57
Cuprous copper	Cu^{+3}	63.6	21.2	2.36
Ferric iron	Fe^{+3}	55.8	18.6	2.69
Ferrous iron	Fe^{+2}	55.8	27.9	1.79
Hydrogen	H^+	1.0	1.0	50.0
Manganese	Mn^{+2}	54.9	27.5	1.82
Magnesium	Mg^{+2}	24.3	12.2	4.10
Potassium	K^+	39.1	39.1	1.28
Sodium	Na^+	23.0	23.0	2.18
Anions				
Bicarbonate	HCO_3^-	61.0	61.0	0.82
Carbonate	CO_3^{-2}	60.0	30.0	1.67
Chloride	Cl^-	35.5	35.5	1.41
Fluoride	F^-	19.0	19.0	2.66
Hydroxide	OH^-	17.0	17.0	2.94
Nitrate	NO_3^-	62.0	62.0	0.81
Nitrite	NO_2^-	46.0	46.0	1.09
Phosphate (tribasic)	PO_4^{-3}	95.0	31.7	1.58
Phosphate (dibasic)	HPO_4^{-2}	96.0	48.0	1.04
Phosphate (monobasic)	$H_2PO_4^-$	97.0	97.0	0.52
Sulfate	SO_4^{-2}	96.1	48.0	1.04
Sulfite	SO_3^{-2}	80.1	40.0	1.25

Nitrogen N_2 14.0
Water H_2O 18.0

Chemical	Formula	Molecular Weight	Equivalent Weight	Factor
Compounds				
Aluminum hydroxide	$Al(OH)_3$	78.0	26.0	1.92
Aluminum Sulfate	$Al_2(SO_4)_3$	342.1	57.0	0.88
Alumina	Al_2O_3	102	17	1.94
Sodium aluminate	$Na_2Al_2O_4$	164.0	27.3	1.83
Calcium bicarbonate	$Ca(HCO_3)_2$	162.1	81.1	0.62
Calcium carbonate	$CaCO_3$	100.1	50.1	1.00
Calcium chloride	$CaCl_2$	111.0	55.5	0.90
Calcium hydroxide (pure)	$Ca(OH)_2$	74.1	37.1	1.35
Cacium hydroxide (90%)	$Ca(OH)_2$	-	41.1	1.22
Calcium oxide (Lime)	CaO	56.1	28.0	1.79
Calcium sulfate (anhydrous)	$CaSO_4$	136.2	68.1	0.74
Calcium sulfate (gypsum)	$CaSO_4 \cdot 2H_2O$	172.2	86.1	0.58
Calcium phosphate	$Ca_3(PO_4)_2$	310.3	51.7	0.97
Disodium phosphate	$Na_2HPO_4 \cdot 12H_2O$	358.2	119.4	0.42
Disodium phosphate (anhydrous)	Na_2HPO_4	142.0	47.3	1.06
Ferric chloride	$FeCl_3$	162.2	54.1	0.92
Ferric oxide	Fe_2O_3	159.6	26.6	1.88
Iron oxide (magnetic)	Fe_3O_4	321.4	-	-
Ferric sulfate	$Fe_2(SO_4)_3$	399.9	133.3	0.38
Ferrous sulfate (copperas)	$FeSO_4 \cdot 7H_2O$	278.0	139.0	0.36
Hydrogen Peroxide	H_2O_2	34.0	17.0	2.94
Magnesium oxide	MgO	40.3	20.2	2.48
Magnesium bicarbonate	$Mg(HCO_3)_2$	146.3	73.2	0.68
Magnesium carbonate	$MgCO_3$	84.3	42.2	1.19
Magnesium chloride	$MgCl_2$	95.2	47.6	1.05
Magnesium hydroxide	$Mg(OH)_2$	58.3	29.2	1.71
Magnesium phosphate	$Mg_3(PO_4)_2$	263.0	43.8	1.14
Magnesium sulfate	$MgSO_4$	120.4	60.2	0.83
Monosodium phosphate	$NaH_2PO_4 \cdot H_2O$	138.1	46.0	1.09
Monosodium phosphate (anhydrous)	NaH_2PO_4	120.1	40.0	1.25
Metaphosphate	$NaPO_3$	102.0	34.0	1.47
Potassium permanganate	$KMnO_4$	158.0	19.8	2.53
Silica	SiO_2	60.1	30.0	1.67
Sodium bicarbonate	$NaHCO_3$	84.0	84.0	0.60
Sodium carbonate (Soda Ash)	Na_2CO_3	106.0	53.0	0.94
Sodium chloride	$NaCl$	58.5	58.5	0.85
Sodium hydroxide	$NaOH$	40.0	40.0	1.25
Sodium hypochlorite	$NaOCl$	74.4	37.2	1.34
Sodium nitrate	$NaNO_3$	85.0	85.0	0.59
Sodium sulfate	Na_2SO_4	142.0	71.0	0.70
Sodium sulfite	Na_2SO_3	126.1	63.0	0.79
Tetrasodium EDTA	$(CH_2)_2N_2(CH_2OONa)_4$	380.2	95.1	0.53
Trisodium phosphate	$Na_3PO_4 \cdot 12H_2O$	380.2	126.7	0.40
Trisodium phosphate (anhydrous)	Na_3PO_4	164.0	54.7	0.91

Chemical	Formula	Molecular Weight	Equivalent Weight	Factor
Trisodium NTA	$(CH_2)_3N(COONa)_3$	257.1	85.7	0.58
Gases				
Ammonia	NH_3	17	17	2.94
Carbon dioxide	CO_2	44	22	2.27
Chlorine	Cl_2	70.9	35.5	1.41
Chlorine dioxide	ClO_2	67.5	13.5	1.48
Hydrogen	H_2	2.0	1	50.00
Hydrogen sulfide	H_2S	34	17	2.94
Oxygen	O_2	32	8	6.25
Ozone	O_3	48	8	6.25
Acids				
Carbonic	H_2CO_3	62.0	31.0	1.61
Hydrochloric	HCl	36.5	36.5	1.37
Nitric	HNO_3	63.0	63.0	0.79
Phosphoric	H_3PO_4	98.0	32.7	1.53
Sulfuric	H_2SO_4	98.1	49.1	1.02

Hydrogen Ion Concentration and pH

Water, H_2O, dissociates into hydrogen and hydroxide ions: H^+ and OH^-, at the rate of 10^{-7} mole hydrogen atoms per liter of water. We define the dissociation constant for water, K_w as:

$$K_w = [H^+]\cdot[OH^-] = 1\times 10^{-14} \text{ moles}^2/\text{liter}^2$$

If we take the logarithm of the dissociation constant, we get:

$$\log K_w = \log[H^+]\cdot[OH^-] = -14$$

Thus when water is neutral, i.e. it has equal hydrogen and hydroxide ions the logarithm of the concentration of Hydrogen ions is equal to:

$$\log[H^+] = \log[OH^-] = -7$$

Thus, we define pH as:

$$pH = \log\frac{1}{[H^+]}$$

and thus when water is neutral the pH is 7 and when Hydrogen ions dominate, the pH is less than 7 and when hydroxide ions dominate, the pH is greater than 7. We call a solution with a predominance of hydrogen ions acidic, and a solution with a predominance of hydroxide ions basic.

Compounds dissolve in water through dissociation into ions, similar to the dissociation of water into hydrogen and hydroxide ions. Addition of an acid such as hydrochloric acid, *HCl*, results in the dissociation of the acid into hydrogen ions and chlorine ions increasing the number of hydrogen

ECONOMICS

Nominal Rate

SEMI ANNUAL - DOUBLE TIME, HALF INTEREST. to get effective rate

CAPITALIZED COSTS - PRESENT WORTH W/INFINITE LIFE >100 YEARS = INFIN
$n = 100$

$$1.25 = \lambda^{10}$$

$$\frac{\ln(1.25)}{10} = \frac{10\ln(\lambda)}{10}$$

$$\frac{\ln(1.25)}{10} = \ln\lambda$$

$$\frac{0.22}{10} = \ln\lambda$$

$$e^{0.022} = \lambda$$

$$\lambda = 1.022$$

$\frac{X}{Y}$ → WANT / → KNOW

$$4^x = 16$$

$$\log 4^x = \log 16$$

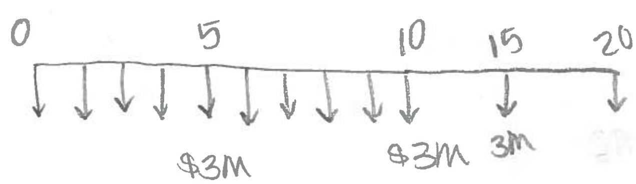
$$x \log 4 = \log 16$$

$$e^{2x} = 54$$

$$\ln e^{2x} = \ln 54$$

$$2x = \ln 54$$

UNIFORM DISTRIBUTION



$$\text{ANNUAL} = \$3M\left[\left(\frac{P}{F}, i, 5\right) + \left(\frac{P}{F}, i, 10\right) + \left(\frac{P}{F}, i, 15\right)\right] \times \left(\frac{A}{P}, i, 20\right)$$

$3M maintenance every 5 years

* NO MAINTENENCE IN LAST YEAR

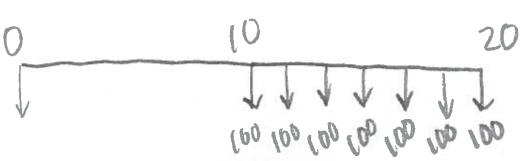
GRADIENT SERIES

G TO P $\left(\frac{P}{G}, i\%, n\right)$ G TO A $\left(\frac{A}{G}, i\%, n\right)$

MARR MINIMUM ATTRACTIVE RATE OF RETURN

P OR F USE TABLES TO FIND n

HALFWAY TIME SERIES SCENARIO



BRING ANNUAL BACK TO "PRESENT" AT $t=10$ THEN BRING FUTURE BACK TO $t=0$ PRESENT

Benefit Cost Ratio Analysis

Should not be used to rank alternatives

Benefit - positive result of project; reduced accidents, decreased emmisions, improved travel time

Cost - a cost related to the project; construction costs, Raw, utilities

Disbenefit - negative benefit; traffic noise, traffic redistribution impacts, specific land use disbursements

Inflation

keeping dollar values constant

replace effective i with a corrected value i'

$$i' = i + e + (i \times e)$$

e = inflation rate (decimal)

Depreciation

Straight line depreciation

deprec per year

$$D_i = \frac{C - SV}{n}$$

C = initial cost

SV = salvage value

n = estimated useful life

ACRS Depreciation

$$D_i = f_i \times C$$

f_i = factor from ACRS table

	Recovery Period (Years)			
Year	3	5	7	10
	Recovery Rate (%)			
1	33.3	20	14.3	10
2	44.5	32	24.5	18
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		11.5	8.9	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

Book Value = Initial cost - depeciation

ENGINEERING ECONOMICS

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n-1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\frac{(1+i)^n-1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\frac{(1+i)^n-1}{i(1+i)^n}$
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2(1+i)^n}-\frac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2}-\frac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\frac{1}{i}-\frac{n}{(1+i)^n-1}$

NOMENCLATURE AND DEFINITIONS

A.......... Uniform amount per interest period
B.......... Benefit
BV........ Book value
C.......... Cost
d.......... Combined interest rate per interest period
D_j......... Depreciation in year j
F.......... Future worth, value, or amount
f.......... General inflation rate per interest period
G.......... Uniform gradient amount per interest period
i .... Interest rate per interest period
i_e.......... Annual effective interest rate
m.......... Number of compounding periods per year
n.......... Number of compounding periods; or the expected life of an asset
P.......... Present worth, value, or amount
r.......... Nominal annual interest rate
S_n......... Expected salvage value in year n

Subscripts

j .... at time j
n.......... at time n
†.......... $F/G = (F/A - n)/i = (F/A) \times (A/G)$

NON-ANNUAL COMPOUNDING

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

BREAK-EVEN ANALYSIS

By altering the value of any one of the variables in a situation, holding all of the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical. This value is the break-even point.

Break-even analysis is used to describe the percentage of capacity of operation for a manufacturing plant at which income will just cover expenses.

The payback period is the period of time required for the profit or other benefits of an investment to equal the cost of the investment.

INFLATION

To account for inflation, the dollars are deflated by the general inflation rate per interest period f, and then they are shifted over the time scale using the interest rate per interest period i. Use a combined interest rate per interest period d for computing present worth values P and Net P.
The formula for d is $d = i + f + (i \times f)$

DEPRECIATION

Straight Line

$$D_j = \frac{C - S_n}{n}$$

Accelerated Cost Recovery System (ACRS)

D_j = (factor) C

A table of modified factors is provided below.

Sum of the Years Digits

$$D_j = \frac{n + 1 - j}{\sum_{j=1}^{n} j}(C - S_n)$$

BOOK VALUE

BV = initial cost – ΣD_j

TAXATION

Income taxes are paid at a specific rate on taxable income. Taxable income is total income less depreciation and ordinary expenses. Expenses do not include capital items, which should be depreciated.

CAPITALIZED COSTS

Capitalized costs are present worth values using an assumed perpetual period of time.

Capitalized Costs = $P = \frac{A}{i}$

BONDS

Bond Value equals the present worth of the payments the purchaser (or holder of the bond) receives during the life of the bond at some interest rate i.

Bond Yield equals the computed interest rate of the bond value when compared with the bond cost.

RATE-OF-RETURN

The minimum acceptable rate-of-return (MARR) is that interest rate that one is willing to accept, or the rate one desires to earn on investments. The rate-of-return on an investment is the interest rate that makes the benefits and costs equal.

BENEFIT-COST ANALYSIS

In a benefit-cost analysis, the benefits B of a project should exceed the estimated costs C.

$B - C \geq 0$, or $B/C \geq 1$

MODIFIED ACRS FACTORS				
	Recovery Period (Years)			
	3	5	7	10
Year	Recovery Rate (Percent)			
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		11.5	8.9	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

Factor Table - i = 0.50%

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9950	0.9950	0.0000	1.0050	1.0000	1.0050	1.0000	0.0000
2	0.9901	1.9851	0.9901	1.0100	2.0050	0.5038	0.4988	0.4988
3	0.9851	2.9702	2.9604	1.0151	3.0150	0.3367	0.3317	0.9967
4	0.9802	3.9505	5.9011	1.0202	4.0301	0.2531	0.2481	1.4938
5	0.9754	4.9259	9.8026	1.0253	5.0503	0.2030	0.1980	1.9900
6	0.9705	5.8964	14.6552	1.0304	6.0755	0.1696	0.1646	2.4855
7	0.9657	6.8621	20.4493	1.0355	7.1059	0.1457	0.1407	2.9801
8	0.9609	7.8230	27.1755	1.0407	8.1414	0.1278	0.1228	3.4738
9	0.9561	8.7791	34.8244	1.0459	9.1821	0.1139	0.1089	3.9668
10	0.9513	9.7304	43.3865	1.0511	10.2280	0.1028	0.0978	4.4589
11	0.9466	10.6770	52.8526	1.0564	11.2792	0.0937	0.0887	4.9501
12	0.9419	11.6189	63.2136	1.0617	12.3356	0.0861	0.0811	5.4406
13	0.9372	12.5562	74.4602	1.0670	13.3972	0.0796	0.0746	5.9302
14	0.9326	13.4887	86.5835	1.0723	14.4642	0.0741	0.0691	6.4190
15	0.9279	14.4166	99.5743	1.0777	15.5365	0.0694	0.0644	6.9069
16	0.9233	15.3399	113.4238	1.0831	16.6142	0.0652	0.0602	7.3940
17	0.9187	16.2586	128.1231	1.0885	17.6973	0.0615	0.0565	7.8803
18	0.9141	17.1728	143.6634	1.0939	18.7858	0.0582	0.0532	8.3658
19	0.9096	18.0824	160.0360	1.0994	19.8797	0.0553	0.0503	8.8504
20	0.9051	18.9874	177.2322	1.1049	20.9791	0.0527	0.0477	9.3342
21	0.9006	19.8880	195.2434	1.1104	22.0840	0.0503	0.0453	9.8172
22	0.8961	20.7841	214.0611	1.1160	23.1944	0.0481	0.0431	10.2993
23	0.8916	21.6757	233.6768	1.1216	24.3104	0.0461	0.0411	10.7806
24	0.8872	22.5629	254.0820	1.1272	25.4320	0.0443	0.0393	11.2611
25	0.8828	23.4456	275.2686	1.1328	26.5591	0.0427	0.0377	11.7407
30	0.8610	27.7941	392.6324	1.1614	32.2800	0.0360	0.0310	14.1265
40	0.8191	36.1722	681.3347	1.2208	44.1588	0.0276	0.0226	18.8359
50	0.7793	44.1428	1,035.6966	1.2832	56.6452	0.0227	0.0177	23.4624
60	0.7414	51.7256	1,448.6458	1.3489	69.7700	0.0193	0.0143	28.0064
100	0.6073	78.5426	3,562.7934	1.6467	129.3337	0.0127	0.0077	45.3613

Factor Table - i = 1.00%

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9901	0.9901	0.0000	1.0100	1.0000	1.0100	1.0000	0.0000
2	0.9803	1.9704	0.9803	1.0201	2.0100	0.5075	0.4975	0.4975
3	0.9706	2.9410	2.9215	1.0303	3.0301	0.3400	0.3300	0.9934
4	0.9610	3.9020	5.8044	1.0406	4.0604	0.2563	0.2463	1.4876
5	0.9515	4.8534	9.6103	1.0510	5.1010	0.2060	0.1960	1.9801
6	0.9420	5.7955	14.3205	1.0615	6.1520	0.1725	0.1625	2.4710
7	0.9327	6.7282	19.9168	1.0721	7.2135	0.1486	0.1386	2.9602
8	0.9235	7.6517	26.3812	1.0829	8.2857	0.1307	0.1207	3.4478
9	0.9143	8.5650	33.6959	1.0937	9.3685	0.1167	0.1067	3.9337
10	0.9053	9.4713	41.8435	1.1046	10.4622	0.1056	0.0956	4.4179
11	0.8963	10.3676	50.8067	1.1157	11.5668	0.0965	0.0865	4.9005
12	0.8874	11.2551	60.5687	1.1268	12.6825	0.0888	0.0788	5.3815
13	0.8787	12.1337	71.1126	1.1381	13.8093	0.0824	0.0724	5.8607
14	0.8700	13.0037	82.4221	1.1495	14.9474	0.0769	0.0669	6.3384
15	0.8613	13.8651	94.4810	1.1610	16.0969	0.0721	0.0621	6.8143
16	0.8528	14.7179	107.2734	1.1726	17.2579	0.0679	0.0579	7.2886
17	0.8444	15.5623	120.7834	1.1843	18.4304	0.0643	0.0543	7.7613
18	0.8360	16.3983	134.9957	1.1961	19.6147	0.0610	0.0510	8.2323
19	0.8277	17.2260	149.8950	1.2081	20.8109	0.0581	0.0481	8.7017
20	0.8195	18.0456	165.4664	1.2202	22.0190	0.0554	0.0454	9.1694
21	0.8114	18.8570	181.6950	1.2324	23.2392	0.0530	0.0430	9.6354
22	0.8034	19.6604	198.5663	1.2447	24.4716	0.0509	0.0409	10.0998
23	0.7954	20.4558	216.0660	1.2572	25.7163	0.0489	0.0389	10.5626
24	0.7876	21.2434	234.1800	1.2697	26.9735	0.0471	0.0371	11.0237
25	0.7798	22.0232	252.8945	1.2824	28.2432	0.0454	0.0354	11.4831
30	0.7419	25.8077	355.0021	1.3478	34.7849	0.0387	0.0277	13.7557
40	0.6717	32.8347	596.8561	1.4889	48.8864	0.0305	0.0205	18.1776
50	0.6080	39.1961	879.4176	1.6446	64.4632	0.0255	0.0155	22.4363
60	0.5504	44.9550	1,192.8061	1.8167	81.6697	0.0222	0.0122	26.5333
100	0.3697	63.0289	2,605.7758	2.7048	170.4814	0.0159	0.0059	41.3426

Factor Table - $i = 1.50\%$

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9852	0.9852	0.0000	1.0150	1.0000	1.0150	1.0000	0.0000
2	0.9707	1.9559	0.9707	1.0302	2.0150	0.5113	0.4963	0.4963
3	0.9563	2.9122	2.8833	1.0457	3.0452	0.3434	0.3284	0.9901
4	0.9422	3.8544	5.7098	1.0614	4.0909	0.2594	0.2444	1.4814
5	**0.9283**	**4.7826**	**9.4229**	**1.0773**	**5.1523**	**0.2091**	**0.1941**	**1.9702**
6	0.9145	5.6972	13.9956	1.0934	6.2296	0.1755	0.1605	2.4566
7	0.9010	6.5982	19.4018	1.1098	7.3230	0.1516	0.1366	2.9405
8	0.8877	7.4859	26.6157	1.1265	8.4328	0.1336	0.1186	3.4219
9	0.8746	8.3605	32.6125	1.1434	9.5593	0.1196	0.1046	3.9008
10	**0.8617**	**9.2222**	**40.3675**	**1.1605**	**10.7027**	**0.1084**	**0.0934**	**4.3772**
11	0.8489	10.0711	48.8568	1.1779	11.8633	0.0993	0.0843	4.8512
12	0.8364	10.9075	58.0571	1.1956	13.0412	0.0917	0.0767	5.3227
13	0.8240	11.7315	67.9454	1.2136	14.2368	0.0852	0.0702	5.7917
14	0.8118	12.5434	78.4994	1.2318	15.4504	0.0797	0.0647	6.2582
15	**0.7999**	**13.3432**	**89.6974**	**1.2502**	**16.6821**	**0.0749**	**0.0599**	**6.7223**
16	0.7880	14.1313	101.5178	1.2690	17.9324	0.0708	0.0558	7.1839
17	0.7764	14.9076	113.9400	1.2880	19.2014	0.0671	0.0521	7.6431
18	0.7649	15.6726	126.9435	1.3073	20.4894	0.0638	0.0488	8.0997
19	0.7536	16.4262	140.5084	1.3270	21.7967	0.0609	0.0459	8.5539
20	**0.7425**	**17.1686**	**154.6154**	**1.3469**	**23.1237**	**0.0582**	**0.0432**	**9.0057**
21	0.7315	17.9001	169.2453	1.3671	24.4705	0.0559	0.0409	9.4550
22	0.7207	18.6208	184.3798	1.3876	25.8376	0.0537	0.0387	9.9018
23	0.7100	19.3309	200.0006	1.4084	27.2251	0.0517	0.0367	10.3462
24	0.6995	20.0304	216.0901	1.4295	28.6335	0.0499	0.0349	10.7881
25	**0.6892**	**20.7196**	**232.6310**	**1.4509**	**30.0630**	**0.0483**	**0.0333**	**11.2276**
30	0.6398	24.0158	321.5310	1.5631	37.5387	0.0416	0.0266	13.3883
40	0.5513	29.9158	524.3568	1.8140	54.2679	0.0334	0.0184	17.5277
50	0.4750	34.9997	749.9636	2.1052	73.6828	0.0286	0.0136	21.4277
60	0.4093	39.3803	988.1674	2.4432	96.2147	0.0254	0.0104	25.0930
100	**0.2256**	**51.6247**	**1,937.4506**	**4.4320**	**228.8030**	**0.0194**	**0.0044**	**37.5295**

Factor Table - $i = 2.00\%$

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9804	0.9804	0.0000	1.0200	1.0000	1.0200	1.0000	0.0000
2	0.9612	1.9416	0.9612	1.0404	2.0200	0.5150	0.4950	0.4950
3	0.9423	2.8839	2.8458	1.0612	3.0604	0.3468	0.3268	0.9868
4	0.9238	3.8077	5.6173	1.0824	4.1216	0.2626	0.2426	1.4752
5	**0.9057**	**4.7135**	**9.2403**	**1.1041**	**5.2040**	**0.2122**	**0.1922**	**1.9604**
6	0.8880	5.6014	13.6801	1.1262	6.3081	0.1785	0.1585	2.4423
7	0.8706	6.4720	18.9035	1.1487	7.4343	0.1545	0.1345	2.9208
8	0.8535	7.3255	24.8779	1.1717	8.5830	0.1365	0.1165	3.3961
9	0.8368	8.1622	31.5720	1.1951	9.7546	0.1225	0.1025	3.8681
10	**0.8203**	**8.9826**	**38.9551**	**1.2190**	**10.9497**	**0.1113**	**0.0913**	**4.3367**
11	0.8043	9.7868	46.9977	1.2434	12.1687	0.1022	0.0822	4.8021
12	0.7885	10.5753	55.6712	1.2682	13.4121	0.0946	0.0746	5.2642
13	0.7730	11.3484	64.9475	1.2936	14.6803	0.0881	0.0681	5.7231
14	0.7579	12.1062	74.7999	1.3195	15.9739	0.0826	0.0626	6.1786
15	**0.7430**	**12.8493**	**85.2021**	**1.3459**	**17.2934**	**0.0778**	**0.0578**	**6.6309**
16	0.7284	13.5777	96.1288	1.3728	18.6393	0.0737	0.0537	7.0799
17	0.7142	14.2919	107.5554	1.4002	20.0121	0.0700	0.0500	7.5256
18	0.7002	14.9920	119.4581	1.4282	21.4123	0.0667	0.0467	7.9681
19	0.6864	15.6785	131.8139	1.4568	22.8406	0.0638	0.0438	8.4073
20	**0.6730**	**16.3514**	**144.6003**	**1.4859**	**24.2974**	**0.0612**	**0.0412**	**8.8433**
21	0.6598	17.0112	157.7959	1.5157	25.7833	0.0588	0.0388	9.2760
22	0.6468	17.6580	171.3795	1.5460	27.2990	0.0566	0.0366	9.7055
23	0.6342	18.2922	185.3309	1.5769	28.8450	0.0547	0.0347	10.1317
24	0.6217	18.9139	199.6305	1.6084	30.4219	0.0529	0.0329	10.5547
25	**0.6095**	**19.5235**	**214.2592**	**1.6406**	**32.0303**	**0.0512**	**0.0312**	**10.9745**
30	0.5521	22.3965	291.7164	1.8114	40.5681	0.0446	0.0246	13.0251
40	0.4529	27.3555	461.9931	2.2080	60.4020	0.0366	0.0166	16.8885
50	0.3715	31.4236	642.3606	2.6916	84.5794	0.0318	0.0118	20.4420
60	0.3048	34.7609	823.6975	3.2810	114.0515	0.0288	0.0088	23.6961
100	**0.1380**	**43.0984**	**1,464.7527**	**7.2446**	**312.2323**	**0.0232**	**0.0032**	**33.9863**

Factor Table - $i = 4.00\%$

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9615	0.9615	0.0000	1.0400	1.0000	1.0400	1.0000	0.0000
2	0.9246	1.8861	0.9246	1.0816	2.0400	0.5302	0.4902	0.4902
3	0.8890	2.7751	2.7025	1.1249	3.1216	0.3603	0.3203	0.9739
4	0.8548	3.6299	5.2670	1.1699	4.2465	0.2755	0.2355	1.4510
5	**0.8219**	**4.4518**	**8.5547**	**1.2167**	**5.4163**	**0.2246**	**0.1846**	**1.9216**
6	0.7903	5.2421	12.5062	1.2653	6.6330	0.1908	0.1508	2.3857
7	0.7599	6.0021	17.0657	1.3159	7.8983	0.1666	0.1266	2.8433
8	0.7307	6.7327	22.1806	1.3686	9.2142	0.1485	0.1085	3.2944
9	0.7026	7.4353	27.8013	1.4233	10.5828	0.1345	0.0945	3.7391
10	**0.6756**	**8.1109**	**33.8814**	**1.4802**	**12.0061**	**0.1233**	**0.0833**	**4.1773**
11	0.6496	8.7605	40.3772	1.5395	13.4864	0.1141	0.0741	4.6090
12	0.6246	9.3851	47.2477	1.6010	15.0258	0.1066	0.0666	5.0343
13	0.6006	9.9856	54.4546	1.6651	16.6268	0.1001	0.0601	5.4533
14	0.5775	10.5631	61.9618	1.7317	18.2919	0.0947	0.0547	5.8659
15	**0.5553**	**11.1184**	**69.7355**	**1.8009**	**20.0236**	**0.0899**	**0.0499**	**6.2721**
16	0.5339	11.6523	77.7441	1.8730	21.8245	0.0858	0.0458	6.6720
17	0.5134	12.1657	85.9581	1.9479	23.6975	0.0822	0.0422	7.0656
18	0.4936	12.6593	94.3498	2.0258	25.6454	0.0790	0.0390	7.4530
19	0.4746	13.1339	102.8933	2.1068	27.6712	0.0761	0.0361	7.8342
20	**0.4564**	**13.5903**	**111.5647**	**2.1911**	**29.7781**	**0.0736**	**0.0336**	**8.2091**
21	0.4388	14.0292	120.3414	2.2788	31.9692	0.0713	0.0313	8.5779
22	0.4220	14.4511	129.2024	2.3699	34.2480	0.0692	0.0292	8.9407
23	0.4057	14.8568	138.1284	2.4647	36.6179	0.0673	0.0273	9.2973
24	0.3901	15.2470	147.1012	2.5633	39.0826	0.0656	0.0256	9.6479
25	**0.3751**	**15.6221**	**156.1040**	**2.6658**	**41.6459**	**0.0640**	**0.0240**	**9.9925**
30	0.3083	17.2920	201.0618	3.2434	56.0849	0.0578	0.0178	11.6274
40	0.2083	19.7928	286.5303	4.8010	95.0255	0.0505	0.0105	14.4765
50	0.1407	21.4822	361.1638	7.1067	152.6671	0.0466	0.0066	16.8122
60	0.0951	22.6235	422.9966	10.5196	237.9907	0.0442	0.0042	18.6972
100	**0.0198**	**24.5050**	**563.1249**	**50.5049**	**1,237.6237**	**0.0408**	**0.0008**	**22.9800**

Factor Table - $i = 6.00\%$

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9434	0.9434	0.0000	1.0600	1.0000	1.0600	1.0000	0.0000
2	0.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854
3	0.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612
4	0.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272
5	**0.7473**	**4.2124**	**7.9345**	**1.3382**	**5.6371**	**0.2374**	**0.1774**	**1.8836**
6	0.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304
7	0.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676
8	0.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1952
9	0.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133
10	**0.5584**	**7.3601**	**29.6023**	**1.7908**	**13.1808**	**0.1359**	**0.0759**	**4.0220**
11	0.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213
12	0.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113
13	0.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920
14	0.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5635
15	**0.4173**	**9.7122**	**57.5546**	**2.3966**	**23.2760**	**0.1030**	**0.0430**	**5.9260**
16	0.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794
17	0.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240
18	0.3505	10.8276	75.3569	2.8543	30.9057	0.0924	0.0324	6.9597
19	0.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867
20	**0.3118**	**11.4699**	**87.2304**	**3.2071**	**36.7856**	**0.0872**	**0.0272**	**7.6051**
21	0.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151
22	0.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166
23	0.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099
24	0.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951
25	**0.2330**	**12.7834**	**115.9732**	**4.2919**	**54.8645**	**0.0782**	**0.0182**	**9.0722**
30	0.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422
40	0.0972	15.0463	185.9568	10.2857	154.7620	0.0665	0.0065	12.3590
50	0.0543	15.7619	217.4574	18.4202	290.3359	0.0634	0.0034	13.7964
60	0.0303	16.1614	239.0428	32.9877	533.1282	0.0619	0.0019	14.7909
100	**0.0029**	**16.6175**	**272.0471**	**339.3021**	**5,638.3681**	**0.0602**	**0.0002**	**16.3711**

Factor Table - i = 8.00%

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9259	0.9259	0.0000	1.0800	1.0000	1.0800	1.0000	0.0000
2	0.8573	1.7833	0.8573	1.1664	2.0800	0.5608	0.4808	0.4808
3	0.7938	2.5771	2.4450	1.2597	3.2464	0.3880	0.3080	0.9487
4	0.7350	3.3121	4.6501	1.3605	4.5061	0.3019	0.2219	1.4040
5	**0.6806**	**3.9927**	**7.3724**	**1.4693**	**5.8666**	**0.2505**	**0.1705**	**1.8465**
6	0.6302	4.6229	10.5233	1.5869	7.3359	0.2163	0.1363	2.2763
7	0.5835	5.2064	14.0242	1.7138	8.9228	0.1921	0.1121	2.6937
8	0.5403	5.7466	17.8061	1.8509	10.6366	0.1740	0.0940	3.0985
9	0.5002	6.2469	21.8081	1.9990	12.4876	0.1601	0.0801	3.4910
10	**0.4632**	**6.7101**	**25.9768**	**2.1589**	**14.4866**	**0.1490**	**0.0690**	**3.8713**
11	0.4289	7.1390	30.2657	2.3316	16.6455	0.1401	0.0601	4.2395
12	0.3971	7.5361	34.6339	2.5182	18.9771	0.1327	0.0527	4.5957
13	0.3677	7.9038	39.0463	2.7196	21.4953	0.1265	0.0465	4.9402
14	0.3405	8.2442	43.4723	2.9372	24.2149	0.1213	0.0413	5.2731
15	**0.3152**	**8.5595**	**47.8857**	**3.1722**	**27.1521**	**0.1168**	**0.0368**	**5.5945**
16	0.2919	8.8514	52.2640	3.4259	30.3243	0.1130	0.0330	5.9046
17	0.2703	9.1216	56.5883	3.7000	33.7502	0.1096	0.0296	6.2037
18	0.2502	9.3719	60.8426	3.9960	37.4502	0.1067	0.0267	6.4920
19	0.2317	9.6036	65.0134	4.3157	41.4463	0.1041	0.0241	6.7697
20	**0.2145**	**9.8181**	**69.0898**	**4.6610**	**45.7620**	**0.1019**	**0.0219**	**7.0369**
21	0.1987	10.0168	73.0629	5.0338	50.4229	0.0998	0.0198	7.2940
22	0.1839	10.2007	76.9257	5.4365	55.4568	0.0980	0.0180	7.5412
23	0.1703	10.3711	80.6726	5.8715	60.8933	0.0964	0.0164	7.7786
24	0.1577	10.5288	84.2997	6.3412	66.7648	0.0950	0.0150	8.0066
25	**0.1460**	**10.6748**	**87.8041**	**6.8485**	**73.1059**	**0.0937**	**0.0137**	**8.2254**
30	0.0994	11.2578	103.4558	10.0627	113.2832	0.0888	0.0088	9.1897
40	0.0460	11.9246	126.0422	21.7245	259.0565	0.0839	0.0039	10.5699
50	0.0213	12.2335	139.5928	46.9016	573.7702	0.0817	0.0017	11.4107
60	0.0099	12.3766	147.3000	101.2571	1,253.2133	0.0808	0.0008	11.9015
100	**0.0005**	**12.4943**	**155.6107**	**2,199.7613**	**27,484.5157**	**0.0800**		**12.4545**

Factor Table - i = 10.00%

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9091	0.9091	0.0000	1.1000	1.0000	1.1000	1.0000	0.0000
2	0.8264	1.7355	0.8264	1.2100	2.1000	0.5762	0.4762	0.4762
3	0.7513	2.4869	2.3291	1.3310	3.3100	0.4021	0.3021	0.9366
4	0.6830	3.1699	4.3781	1.4641	4.6410	0.3155	0.2155	1.3812
5	**0.6209**	**3.7908**	**6.8618**	**1.6105**	**6.1051**	**0.2638**	**0.1638**	**1.8101**
6	0.5645	4.3553	9.6842	1.7716	7.7156	0.2296	0.1296	2.2236
7	0.5132	4.8684	12.7631	1.9487	9.4872	0.2054	0.1054	2.6216
8	0.4665	5.3349	16.0287	2.1436	11.4359	0.1874	0.0874	3.0045
9	0.4241	5.7590	19.4215	2.3579	13.5735	0.1736	0.0736	3.3724
10	**0.3855**	**6.1446**	**22.8913**	**2.5937**	**15.9374**	**0.1627**	**0.0627**	**3.7255**
11	0.3505	6.4951	26.3962	2.8531	18.5312	0.1540	0.0540	4.0641
12	0.3186	6.8137	29.9012	3.1384	21.3843	0.1468	0.0468	4.3884
13	0.2897	7.1034	33.3772	3.4523	24.5227	0.1408	0.0408	4.6988
14	0.2633	7.3667	36.8005	3.7975	27.9750	0.1357	0.0357	4.9955
15	**0.2394**	**7.6061**	**40.1520**	**4.1772**	**31.7725**	**0.1315**	**0.0315**	**5.2789**
16	0.2176	7.8237	43.4164	4.5950	35.9497	0.1278	0.0278	5.5493
17	0.1978	8.0216	46.5819	5.0545	40.5447	0.1247	0.0247	5.8071
18	0.1799	8.2014	49.6395	5.5599	45.5992	0.1219	0.0219	6.0526
19	0.1635	8.3649	52.5827	6.1159	51.1591	0.1195	0.0195	6.2861
20	**0.1486**	**8.5136**	**55.4069**	**6.7275**	**57.2750**	**0.1175**	**0.0175**	**6.5081**
21	0.1351	8.6487	58.1095	7.4002	64.0025	0.1156	0.0156	6.7189
22	0.1228	8.7715	60.6893	8.1403	71.4027	0.1140	0.0140	6.9189
23	0.1117	8.8832	63.1462	8.9543	79.5430	0.1126	0.0126	7.1085
24	0.1015	8.9847	65.4813	9.8497	88.4973	0.1113	0.0113	7.2881
25	**0.0923**	**9.0770**	**67.6964**	**10.8347**	**98.3471**	**0.1102**	**0.0102**	**7.4580**
30	0.0573	9.4269	77.0766	17.4494	164.4940	0.1061	0.0061	8.1762
40	0.0221	9.7791	88.9525	45.2593	442.5926	0.1023	0.0023	9.0962
50	0.0085	9.9148	94.8889	117.3909	1,163.9085	0.1009	0.0009	9.5704
60	0.0033	9.9672	97.7010	304.4816	3,034.8164	0.1003	0.0003	9.8023
100	**0.0001**	**9.9993**	**99.9202**	**13,780.6123**	**137,796.1234**	**0.1000**		**9.9927**

Factor Table - $i = 12.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.8929	0.8929	0.0000	1.1200	1.0000	1.1200	1.0000	0.0000
2	0.7972	1.6901	0.7972	1.2544	2.1200	0.5917	0.4717	0.4717
3	0.7118	2.4018	2.2208	1.4049	3.3744	0.4163	0.2963	0.9246
4	0.6355	3.0373	4.1273	1.5735	4.7793	0.3292	0.2092	1.3589
5	**0.5674**	**3.6048**	**6.3970**	**1.7623**	**6.3528**	**0.2774**	**0.1574**	**1.7746**
6	0.5066	4.1114	8.9302	1.9738	8.1152	0.2432	0.1232	2.1720
7	0.4523	4.5638	11.6443	2.2107	10.0890	0.2191	0.0991	2.5515
8	0.4039	4.9676	14.4714	2.4760	12.2997	0.2013	0.0813	2.9131
9	0.3606	5.3282	17.3563	2.7731	14.7757	0.1877	0.0677	3.2574
10	**0.3220**	**5.6502**	**20.2541**	**3.1058**	**17.5487**	**0.1770**	**0.0570**	**3.5847**
11	0.2875	5.9377	23.1288	3.4785	20.6546	0.1684	0.0484	3.8953
12	0.2567	6.1944	25.9523	3.8960	24.1331	0.1614	0.0414	4.1897
13	0.2292	6.4235	28.7024	4.3635	28.0291	0.1557	0.0357	4.4683
14	0.2046	6.6282	31.3624	4.8871	32.3926	0.1509	0.0309	4.7317
15	**0.1827**	**6.8109**	**33.9202**	**5.4736**	**37.2797**	**0.1468**	**0.0268**	**4.9803**
16	0.1631	6.9740	36.3670	6.1304	42.7533	0.1434	0.0234	5.2147
17	0.1456	7.1196	38.6973	6.8660	48.8837	0.1405	0.0205	5.4353
18	0.1300	7.2497	40.9080	7.6900	55.7497	0.1379	0.0179	5.6427
19	0.1161	7.3658	42.9979	8.6128	63.4397	0.1358	0.0158	5.8375
20	**0.1037**	**7.4694**	**44.9676**	**9.6463**	**72.0524**	**0.1339**	**0.0139**	**6.0202**
21	0.0926	7.5620	46.8188	10.8038	81.6987	0.1322	0.0122	6.1913
22	0.0826	7.6446	48.5543	12.1003	92.5026	0.1308	0.0108	6.3514
23	0.0738	7.7184	50.1776	13.5523	104.6029	0.1296	0.0096	6.5010
24	0.0659	7.7843	51.6929	15.1786	118.1552	0.1285	0.0085	6.6406
25	**0.0588**	**7.8431**	**53.1046**	**17.0001**	**133.3339**	**0.1275**	**0.0075**	**6.7708**
30	0.0334	8.0552	58.7821	29.9599	241.3327	0.1241	0.0041	7.2974
40	0.0107	8.2438	65.1159	93.0510	767.0914	0.1213	0.0013	7.8988
50	0.0035	8.3045	67.7624	289.0022	2,400.0182	0.1204	0.0004	8.1597
60	0.0011	8.3240	68.8100	897.5969	7,471.6411	0.1201	0.0001	8.2664
100		**8.3332**	**69.4336**	**83,522.2657**	**696,010.5477**	**0.1200**		**8.3321**

Factor Table - $i = 18.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.8475	0.8475	0.0000	1.1800	1.0000	1.1800	1.0000	0.0000
2	0.7182	1.5656	0.7182	1.3924	2.1800	0.6387	0.4587	0.4587
3	0.6086	2.1743	1.9354	1.6430	3.5724	0.4599	0.2799	0.8902
4	0.5158	2.6901	3.4828	1.9388	5.2154	0.3717	0.1917	1.2947
5	**0.4371**	**3.1272**	**5.2312**	**2.2878**	**7.1542**	**0.3198**	**0.1398**	**1.6728**
6	0.3704	3.4976	7.0834	2.6996	9.4423	0.2859	0.1059	2.0252
7	0.3139	3.8115	8.9670	3.1855	12.1415	0.2624	0.0824	2.3526
8	0.2660	4.0776	10.8292	3.7589	15.3270	0.2452	0.0652	2.6558
9	0.2255	4.3030	12.6329	4.4355	19.0859	0.2324	0.0524	2.9358
10	**0.1911**	**4.4941**	**14.3525**	**5.2338**	**23.5213**	**0.2225**	**0.0425**	**3.1936**
11	0.1619	4.6560	15.9716	6.1759	28.7551	0.2148	0.0348	3.4303
12	0.1372	4.7932	17.4811	7.2876	34.9311	0.2086	0.0286	3.6470
13	0.1163	4.9095	18.8765	8.5994	42.2187	0.2037	0.0237	3.8449
14	0.0985	5.0081	20.1576	10.1472	50.8180	0.1997	0.0197	4.0250
15	**0.0835**	**5.0916**	**21.3269**	**11.9737**	**60.9653**	**0.1964**	**0.0164**	**4.1887**
16	0.0708	5.1624	22.3885	14.1290	72.9390	0.1937	0.0137	4.3369
17	0.0600	5.2223	23.3482	16.6722	87.0680	0.1915	0.0115	4.4708
18	0.0508	5.2732	24.2123	19.6731	103.7403	0.1896	0.0096	4.5916
19	0.0431	5.3162	24.9877	23.2144	123.4135	0.1881	0.0081	4.7003
20	**0.0365**	**5.3527**	**25.6813**	**27.3930**	**146.6280**	**0.1868**	**0.0068**	**4.7978**
21	0.0309	5.3837	26.3000	32.3238	174.0210	0.1857	0.0057	4.8851
22	0.0262	5.4099	26.8506	38.1421	206.3448	0.1848	0.0048	4.9632
23	0.0222	5.4321	27.3394	45.0076	244.4868	0.1841	0.0041	5.0329
24	0.0188	5.4509	27.7725	53.1090	289.4944	0.1835	0.0035	5.0950
25	**0.0159**	**5.4669**	**28.1555**	**62.6686**	**342.6035**	**0.1829**	**0.0029**	**5.1502**
30	0.0070	5.5168	29.4864	143.3706	790.9480	0.1813	0.0013	5.3448
40	0.0013	5.5482	30.5269	750.3783	4,163.2130	0.1802	0.0002	5.5022
50	0.0003	5.5541	30.7856	3,927.3569	21,813.0937	0.1800		5.5428
60	0.0001	5.5553	30.8465	20,555.1400	114,189.6665	0.1800		5.5526
100		**5.5556**	**30.8642**	**15,424,131.91**	**85,689,616.17**	**0.1800**		**5.5555**