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The time trade-off: A note on the effect of lifetime reallocation of consumption and discounting

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Abstract

This paper considers the extent to which responses to time trade-off (TTO) questions can provide unbiased estimates of ratios of individual marginal rates of substitution (MRS) of wealth for risk of various health state impairments relative to the corresponding MRS for risk of death. It is shown that if there is reallocation of lifetime consumption and/or discounting of future utilities, then a TTO response that is not adjusted for these effects will unambiguously overestimate the ratios of individual MRS. While the effect of reallocation is likely to be very small, discounting can lead to significant overestimation, the magnitude of which depends in part upon the severity of the health state impairment. © 1997 Elsevier Science B.V.

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1. Background

The willingness-to-pay (WTP) approach has been widely used by economists to establish direct preference-based values of health and safety (see Jones-Lee,

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1989a). However, the approach has recently become the subject of a heated controversy in the literature (particularly following the Exxon Valdez oil spillage; see Hausman, 1993) and there are now serious doubts as to the reliability of monetary values of health and safety established on the basis of responses to WTP questions. In view of all this, many researchers have used alternative methods of measuring preferences over health and safety, such as the risk–risk (RR) and standard gamble (SG) approaches. [1] Responses to both RR and SG questions can then be used to provide indirect estimates of the ratios of individual marginal rates of substitution (MRS) of wealth for risk of various severities of health impairment *relative to* the corresponding (WTP-based) MRS for risk of death.

The WTP, RR and SG approaches to benefit valuation are all built upon the same expected utility theory (EUT) foundations and thus *in principle* should provide the same estimates of the ratios of individual MRS. However, there now exists a very considerable amount of experimental evidence (see Camerer, 1993) to suggest that *in practice* the restrictions on behaviour imposed by EUT may be too severe for many people. Thus, responses to WTP, RR and SG questions are unlikely to provide unbiased estimates of the ratios of individual MRS. However, the direction and, to some extent, the magnitude of these biases are now at least partially understood (see Jones-Lee et al., 1993, for details).

Another method which is increasingly being used by health economists, and one which is also capable of being used to generate indirect estimates of the ratios of individual MRS, is the time trade-off (TTO) method, as originally developed by Torrance (1976). There has been considerable debate in the literature regarding the relative merits of this technique, particularly vis-à-vis the standard gamble (see Gafni et al., 1993; Richardson, 1994; Dolan et al., 1996). It is not the intention of the present paper to contribute directly to this already fruitful debate but rather to consider the extent to which responses to TTO questions can provide unbiased estimates of ratios of individual MRS of wealth for risk of various health state severities relative to the corresponding MRS for risk of death.

Although a number of authors have discussed the *sources* of potential bias in TTO responses (see, for example, Loomes and McKenzie, 1989), little attention has been directed towards explicitly setting out the likely magnitude of these biases. This is the aim of the present paper. In considering the impact of time preference, this paper could be properly regarded as a logical tidying up exercise. However, another potentially important source of bias considered in this paper, namely, the reallocation of lifetime consumption, has to our knowledge not been discussed elsewhere.

[1] The RR method has been applied to the valuation of life and safety in the US (see Viscusi, 1994) and to the valuation of non-fatal road accidents in the UK (see Dolan et al., 1995), whilst the SG has been most widely used by health economists (see Gafni, 1994).

2. Introduction

Consider first a Von Neumann Morgenstern (NM) expected utility maximiser facing current-year probabilities p and q of death and a permanently disabling illness/injury respectively. For simplicity, death and disability will be treated as mutually exclusive events, so that assuming strong separability on the time dimension (see Broome, 1993), and ignoring the possibility of injury or premature death in other than the current year, the individual's lifetime expected utility is given by

$$EU = \left[(1-p-q)\left(\sum_{t=0}^{T-1} \rho^t L(C) + \rho^T D\right)\right] + \left[q\left(\sum_{t=0}^{T-1} \rho^t I(\hat{C}) + \rho^T D\right)\right] + pD \tag{1}$$

where $L(.)$ and $I(.)$ denote the NM annual utility of consumption functions for full health and injury/illness respectively, C denotes constant consumption per annum conditional on full health, $\hat{C}$ denotes constant consumption per annum conditional on injury/illness (where $\hat{C}$ may differ from C), ρ is a discount factor reflecting the individual's rate of time preference, T is the individual's maximum life expectancy and D denotes the NM utility associated with the prospect of death.

Note that for simplicity it has been assumed that consumption is time-invariant. Indeed, it is probably impossible to interpret the response to a TTO question in relative utility terms without imposing a time-invariant condition on consumption. Of course, given that the only uncertainty in our model is associated with the determination of the lifetime health state at the beginning of the first period, time-invariant consumption would in fact be optimal provided that the time preference and interest rates were equal. [2]

In the model developed in this paper, it will be assumed for simplicity that ρ is independent of the health state and that D is constant, so that without loss of generality, $L(.)$, $I(.)$ and D can be scaled such that $D = 0$. In addition, it will be assumed that

$$L(C) > I(\hat{C}) \tag{2}$$

$$L'(.), I'(.) > 0 \tag{3}$$

[2] This can be illustrated by considering a simple two-period model. Suppose that the initial uncertainty is resolved such that an individual experiences full health. This individual's optimal life-cycle consumption decision is to maximise $L(C_1) + \rho L(C_2)$ subject to $C_1 + \mu C_2 = W$, where ρ and μ are discount factors reflecting the time preference and interest rates respectively, and W is initial wealth, including first and discounted second period income. It is then straghtforward to show that the solution to this constrained maximisation problem is such that if $\rho = \mu$, then $C_1 = C_2$. Precisely the same argument applies if the initial uncertainty results in ill health. Here the optimal lifetime consumption decision is to maximise $I(\hat{C}_1) + \rho I(\hat{C}_2)$ subject to $\hat{C}_1 + \mu \hat{C}_2 = \hat{W}$, where $\hat{W}$ may differ from W to the extent that ill health (presumably adversely) affects income.

and

$$0 < \rho \leq 1 \tag{4}$$

Denoting the individual's MRS of wealth for risk of death and wealth for risk of illness/injury by M_D and M_I respectively, with $D = 0$ it is then straightforward to show that [3]

$$\frac{M_I}{M_D} = \frac{\sum_{t=0}^{T-1} \rho^t L(C) - \sum_{t=0}^{T-1} \rho^t I(\hat{C})}{\sum_{t=0}^{T-1} \rho^t L(\bar{C})} \tag{5}$$

$$= \frac{L(C) - I(\hat{C})}{L(C)} \tag{5'}$$

$$= 1 - \frac{I(\hat{C})}{L(C)} \tag{5''}$$

Whilst this result has been derived on the admittedly somewhat restrictive assumption of expected utility maximisation, it can be shown that with appropriate reinterpretation of $L(.)$ and $I(.)$, a similar result follows in the case of a wide range of non-expected utility maximisation theories provided that the latter satisfy the betweenness axiom [4] (see for example Jones-Lee, 1989b, pp. 34–36, or Jones-Lee et al., 1993, appendix).

Clearly then, in assessing the extent to which the response to a TTO question provides the basis for estimating the ratio M_I/M_D, the principal focus will be upon whether or not the response to such a question yields an unbiased estimate of $I(\hat{C})/L(C)$. Suppose that in response to a TTO question, the individual, whose lifetime expected utility is as specified in Eq. (1), indicates that she would be indifferent between the certainty of spending ten years in the state of injury/illness referred to above, followed by death on the one hand, and the certainty of spending τ years in full health, followed by death on the other.

Clearly, from Eq. (2) we shall have $\tau < 10$. In addition, in what follows, we will focus upon the case in which the injury/illness concerned is not judged to be

[3] While the argument is developed for the single-period case, precisely the same result follows for the multi-period case with lifetime expected utility expressed as in Eq. (1), given that D has been set equal to zero (see Jones-Lee, 1989a).

[4] Betweenness entails that an individual who is indifferent between X and Y will also be indifferent between those alternatives and every probability mixture $pX + (1-p)Y$, $0 < p < 1$. This ensures that indifference curves in the Marschak–Machina triangle are linear.

as bad as or worse than death, so that $L(C) > I(\hat{C}) > 0$ and hence $\tau > 0$. With $D = 0$, it follows that

$$\sum_{t=0}^{\tau-1} \rho^t L(\tilde{C}) = \sum_{t=0}^{9} \rho^t I(\hat{C}) \tag{6}$$

where $\tilde{C}(\geq C)$ is the individual's planned annual consumption given that she expects to live for only $\tau(<10)$ years. From Eq. (6), it is immediate that

$$0 < \rho < 1 \Rightarrow \frac{I(\hat{C})}{L(\tilde{C})} > \frac{\tau}{10} \tag{7}$$

and

$$\rho = 1 \Rightarrow \frac{I(\hat{C})}{L(\tilde{C})} = \frac{\tau}{10} \tag{8}$$

Furthermore, given $L'(.) > 0$

$$\tilde{C} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} C \Rightarrow \frac{I(\hat{C})}{L(C)} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \frac{I(\hat{C})}{L(\tilde{C})} \tag{9}$$

Since it is highly unlikely that the individual would consume less per annum if she lived for $\tau(<10)$ years rather than the full ten years, it seems reasonable to impose the restriction $\tilde{C} \geq C$. Given this restriction, and given that the individual does not have a negative rate of time preference over life years (i.e. $0 < \rho \leq 1$), it follows from Eqs. (7)–(9) that

$$\frac{I(\hat{C})}{L(C)} = \frac{\tau}{10} \text{ iff } \tilde{C} = C \text{ and } \rho = 1 \tag{10}$$

That is, in order for the response to the TTO question to provide a direct and unbiased estimate of the ratio M_I/M_D, it is necessary that there should be *no* reallocation of lifetime consumption and *no* discounting of future utilities. If, by contrast, there is either reallocation (i.e. $\tilde{C} > C$) and/or discounting (i.e. $0 < \rho < 1$), then $\tau/10$ will unambiguously *under*estimate $I(\hat{C})/L(C)$, so that $1-(\tau/10)$ will *over*estimate M_I/M_D.

3. Lifetime reallocation of consumption

If the individual has the opportunity to reallocate lifetime consumption then it seems likely that $\tilde{C}$ will exceed C. For example, an individual who is to some extent consuming out of accumulated wealth would, in the model specified above, plan to consume at a greater rate if she knew for certain that her life expectancy were to be reduced. Since the TTO is based on the comparison of two alternatives for which the respective life expectancies *are* known for certain, lifetime reallocation of consumption is clearly a source of potential bias in TTO responses.

However, it transpires that there are grounds for believing that the impact of such reallocation will be negligible. By applying a variant of the argument developed in Jones-Lee (1989a, pp. 115–116) to Eq. (1), with $D=0$ and assuming that $L(.)$ and $I(.)$ are bounded above, [5] it is fairly straightforward to show that

$$\frac{L^*}{L(C)} < \frac{1-p-q}{1-p-q-\Delta p^*} \tag{11}$$

where L^* denotes $\sup L(.)$ and Δp^* is the individual's 'maximum acceptable increase in p' i.e. the increase in p for which the compensating variation in terms of an increase in C becomes unbounded.

Now, for most people it seems reasonable to assume that: (i) their risk of death in the current year is less than 1 in 100 (i.e. $p < 10^{-2}$); (ii) the risk of suffering other than the most minor injury/illness is less than 1 in 10 (i.e. $q < 10^{-1}$); and (iii) the maximum increase in the risk of death they would be prepared to accept is also likely to be less than 1 in 10 (i.e. $\Delta p^* < 10^{-1}$). It follows from Eq. (11) that even an unbounded increase in C will cause $L(.)$ to increase by, at most, about 13%. Indeed, with $p=10^{-3}$, $q=10^{-2}$ and $\Delta p^* = 10^{-2}$, which are not entirely implausible orders of magnitude, the increase would only be about 1%. It therefore seems clear that for the (relatively modest) increase from C to $\tilde{C}$ that might be expected from a lifetime reallocation of consumption in the context of a TTO question, $L(\tilde{C})$ would exceed $L(C)$ only by a very small percentage. Of course, this ignores the fact that in practice most people will be consuming out of current income (not accumulated wealth) and will therefore have little scope for reallocating consumption in any case.

4. Discounting

If $\rho = 1$, then each year of life in constant quality (L or I) yields the same utility. However, in responding to a TTO question, the individual may be prepared to sacrifice more years of life in the future relative to years of life now, in which case they have a positive rate of time preference and will be discounting the future; hence $0 < \rho < 1$. To get some feel for the extent to which $1-(\tau/10)$ might overestimate M_I/M_D in the presence of positive time preference, consider integer values for τ between 1 and 9 and, in addition to $\rho = 1$, three further values

[5] It is necessary for $L(.)$ and $I(.)$ to be bounded above if the individual is to be immune to versions of the St. Petersburg paradox.

Table 1
Values of M_I/M_D for various values of ρ and τ

τ	$\rho = 1$	$\rho = 0.95$	$\rho = 0.91$	$\rho = 0.87$
1	0.9	0.88	0.85	0.83
2	0.8	0.76	0.72	0.68
3	0.7	0.65	0.59	0.54
4	0.6	0.54	0.48	0.43
5	0.5	0.44	0.38	0.33
6	0.4	0.34	0.29	0.25
7	0.3	0.25	0.21	0.17
8	0.2	0.16	0.13	0.11
9	0.1	0.08	0.06	0.05

for ρ of 0.95, 0.91 and 0.87 (corresponding, approximately, to annual time preference rates of 5%, 10% and 15% respectively). In view of Eqs. (5″) and (6), Table 1 shows the values of M_I/M_D that would result.

Although the effect of a positive rate of time preference is to reduce the ratio M_I/M_D for any given value of τ, the effect is not uniform across all values of this ratio. The absolute difference between undiscounted and discounted values of M_I/M_D is smallest for high and low values (i.e. for values of M_I/M_D close to 1 and 0) and largest for values around 0.5. Clearly then, even setting aside the possible impact of lifetime reallocation of consumption, assuming that $\rho = 1$ (which is an assumption made in almost all previous studies using the TTO method), will overestimate M_I/M_D: the extent of the overestimation clearly depends upon how far ρ deviates from 1 but also on the severity of the permanently disabling illness/injury.

However, valuations generated by the TTO method do not have to be predicated on the assumption of no discounting. All respondents indicate in answering a TTO question is the number of years in L that is regarded as equivalent to a longer period of time in I. The value that is attached to $I(\hat{C})/L(C)$ (even assuming that $\tilde{C} = C$) is a separate issue. Table 1 shows how *different* values for $I(C)/L(C)$ (and hence M_I/M_D) can be generated from the *same* point of indifference established in a TTO question, depending on the value of ρ used.

5. Discussion

The willingness-to-pay, risk–risk and standard gamble procedures have been widely used to establish preference-based values for health and safety and, perhaps as a result, the extent to which they provide unbiased representations of underlying preferences is now at least partially understood. Another method often used by health economists is the time trade-off (TTO) but, although it was developed more than 20 years ago, surprisingly little attention has been directed at the extent to which TTO valuations will under- or overestimate the required ratios of MRS.

This paper has shown that in order for a response to a TTO question to provide a direct and unbiased estimate of the ratio M_I/M_D it is necessary that: (i) there is no reallocation of lifetime consumption, and (ii) there is no discounting of future utilities. If there is either reallocation and/or discounting, then it is shown that a TTO response that is not adjusted for these effects will unambiguously overestimate M_I/M_D.

In the case of reallocation of lifetime consumption, the extent to which M_I/M_D is overestimated is likely to be very small. Since most people consume out of current income, the extent to which they will be able to consume at a greater rate if their life expectancy were to be reduced is therefore highly constrained. Even for those people who are to some extent consuming out of accumulated wealth, it transpires that by making entirely plausible assumptions about the risk of death and the maximum increase in that risk that an individual would be prepared to accept, the impact of lifetime reallocation of consumption is almost certainly trivial. Therefore, it seems reasonable to assume that $\tilde{C} = C$.

The effect of discounting, however, is *non*-trivial. Assuming an annual time preference rate of 5%, an undiscounted TTO response will overestimate M_I/M_D by as much as 0.06 (in relation to a true value of 0.44). The issue of time preference has been discussed elsewhere in the health economics literature (see Fuchs, 1982) and a number of authors have attempted to measure its extent (see Redelmeier and Heller, 1993; Dolan and Gudex, 1995). There is unquestionably the need for further research in this area. Unless a reliable method of exploring the effect of time preference on benefit valuation generally can be constructed, then choices between alternative uses of resources that have different benefit streams are unlikely to fully represent individual or social preferences. Of course, this is an issue beyond the scope of this paper, but we hope that we have identified the extent to which undiscounted TTO valuations may overestimate M_I/M_D.

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