Smart-dating in speed-dating: How a simple Search model can explain matching decisions

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ABSTRACT

How do people in a romantic matching situation choose a potential partner? We study this question in a new model of matching under search frictions, which we estimate using data from an existing speed dating experiment. We find that attraction is mostly in the eye of the beholder and that the attraction between two potential partners has a tendency to be mutual. These results are supported by a direct measure of subjective attraction. We also simulate the estimated model, and it predicts rejection patterns, matching rates, and sorting outcomes that fit the data very well. Our results are consistent with the hypothesis that people in a dating environment act strategically and have at least an implicit understanding of the nature of the frictions and of the strategic equilibrium.

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1. Introduction

How do people choose a romantic partner? In addition to personal preferences, this decision will also be shaped by the availability of potential partners. To analyze this process we develop a new search-and-matching model that extends previous work to incorporate a richer set of preferences. We then test our model using data from the “Northwestern Speed Dating Study” (NSDS henceforth; Eastwick and Finkel, 2008; Finkel et al., 2007); the speed dating environment has the valuable property that we are able to observe preferences, decisions, and matching outcomes at the same time.

Our model is an extension of Burdett and Coles (1997) and Burdett and Wright (1998), both of which are search-and-matching models with random meetings and non-transferable utility. In the former, attractiveness is a person’s permanent type that everyone knows and agrees upon, and in the latter, attraction is a meeting-specific random variable (entirely in the eye of the beholder). Our model combines these insights: attraction is largely subjective but not purely so. Following much of the marriage matching literature, we call the consensus component “vertical” (because it can be ranked), and the subjective component “horizontal” (because it is orthogonal to “vertical”).

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In the frictionless matching literature, a vertical component of preferences is often called a “common value”, and a horizontal component an “idiosyncratic shock”.

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There are two possible ways to quantify our model using data from the NSDS. First, we estimate the model using maximum likelihood applied to participants' matching decisions, and second, we analyze their directly measured partner preferences. Using the first method, we find that about one third of the variation in overall attraction is vertical and the remaining two thirds are horizontal. Using the second method, the vertical share of the variation is one quarter, which confirms that the consensus component of romantic preferences is strong but not overwhelming. In addition, both methods confirm that the horizontal component of attraction is correlated among partners in a meeting – subjective attraction has a tendency to be mutual. The two methods do not quite agree on how mutual: the direct measurement of preferences yields a correlation of 0.09, but the participants' matching decisions are best explained if the true correlation is 0.30. Both numbers are statistically distinct from zero, however, and this is important because many theoretical results on the asymptotic stability and efficiency of matching outcomes require independent preferences (as discussed by Che and Tercieux, 2014).

Using the vertical component of attraction, we can show that the decision by the NSDS participants to say 'yes' to exchanging contact information with one another is not random or independent, but consistent with strategic thinking. Participants who are judged to be one standard deviation above average in their vertical component are much more likely (23 percentage points) to receive such a 'yes' and significantly (4 percentage points) less likely to say 'yes' to their partner, compared to an average 'yes' rate of 46%. Overall, this makes them 9 percentage points more likely to be 'matched' with a given partner (both saying 'yes' to each other), compared to an average matching rate of 22%. The average matching rate is close to the square of the 'yes' rate which may make it appear to be the result of random decisions, but the fact that decisions and outcomes vary so strongly with the vertical component of preferences shows that this cannot be the case. Vertical preferences alone would imply a negative correlation between decisions and a much lower matching rate. Instead, the high matching rate which we actually observe reveals that horizontal attraction tends to be mutual. All of these numbers are very similar in a simulation of our estimated model, and the simulation also suggests that the noise in the data is consistent with sampling variation.

Our results have some implications for the study of decision making under uncertainty. There are two sources of uncertainty in our framework: the randomness of who one will meet and how attractive one might find them, but also strategic uncertainty about the decisions of strangers. We find that the decisions and matching outcomes in the NSDS are consistent with the notion that the participants had developed a sophisticated understanding of both kinds of uncertainty in romantic matching. As the fit of our model is very good and the implied level of frictions is large, we conclude that the search framework has a role to play in explaining real world matching decisions.

We next analyze matched partners with respect to certain characteristics. We would expect that attractive participants are more likely to match with attractive partners, or that participants match with others similar to them on some other dimension. This phenomenon is called “assortative matching” in the literature; empirically, this simply means that there is a correlation between observables among partners in a couple or in a marriage, such as income, education, or ethnicity. But in terms of interpretation, we would argue that there is more than one kind of sorting, corresponding to our distinction between vertical and horizontal preferences. Along the vertical dimension, high-rank singles match with other high-rank singles not because they prefer high-rank partners, but because everyone prefers high-rank partners and other high-rank singles happen to be the ones who end up matching with them. In the study, we measure a positive correlation of matching value between matched participants of 0.15; our simulated model agrees that such a small correlation is natural given the search frictions in our environment combined with the substantial disagreement on who is attractive. Along horizontal dimensions, in contrast, partners would match because of a preference for compatibility rather than a preference for status. Indeed, we find that matched partners tend to be particularly attracted to each other – and it is not trivial that this must be
the case for initial attraction even if we routinely observe it in long-term couples. While Eastwick and Hunt (2014) demonstrate that attraction between opposite-sex friends and acquaintances grows increasingly idiosyncratic as two individuals get to know each other over time, our finding provides evidence that the roots of this divergence are already present at the initial attraction stage.

We think that our distinction between two sources of sorting is meaningful in general contexts. For example, in an analysis of commercial speed-dating data, Kurzban and Weeden (2005) argue that vertical sorting is more prevalent than horizontal sorting. But they come to this conclusion by classifying some observable characteristics as vertical (e.g., looks) and others as horizontal (e.g., race), and then find stronger evidence for sorting on the former than on the latter. Our results do not directly contradict this finding but suggest caution in its interpretation. For one, few real-world characteristics are purely vertical, and physical attractiveness is not one of them (as we show). For another, many horizontal characteristics such as common interests are hard to observe, even though they may be very important in determining attraction.

In our paper, we model matching according to the random search paradigm. Burdett and Coles (1997, 1999) and Bloch and Ryder (2000) studied random search models where there is only a single dimension of attractiveness that everyone observes, and agrees on. Subsequent work by Sundaram (2001) and Smith (2006) extended the framework allowing for (limited) heterogeneity in preferences, but more work is required as Smith (2011) acknowledges. In contrast, Burdett and Wright (1998) assumed completely idiosyncratic preferences; our paper is the first one to combine the models of Burdett–Coles and Burdett–Wright, and also the first paper to quantify the extent to which each channel shapes decisions in the real world. A large empirical literature has applied random matching models to the determinants of marriage and divorce, explaining existing patterns and why they may change (Booth and Coles, 2010; Fernández et al., 2005; Greenwood et al., 2003). Our goal is to connect to this literature and extend it in two ways: first, we develop a richer model of subjective attraction to a potential partner, and second, we use the NSDS data to directly estimate the new model and to analyze its implications for decision making and sorting.

The random search approach we take is different from the frictionless matching approach which has been popular in the marriage matching literature since Gale and Shapley (1962). The main benefit of that approach is that one can analyze matching outcomes with respect to stability or efficiency (e.g., Hirsch et al., 2010). Instead, we model how individuals are attracted to potential partners in general, and then describe how they make strategic choices in an environment with frictions. We can do this ambitious modeling exercise because in the NSDS data, we can observe every participant’s evaluation of several potential partners, as well as their decisions and their matching outcomes. It is worth noting that the two approaches are connected. Recent efforts to describe an optimal tradeoff between efficiency and stability in large matching markets introduce features in the environment that end up looking like frictions (Bidner, 2010; Che and Tercieux, 2014). In the other direction, Adachi (2003) shows that outcomes implied by random matching approach can under some conditions converge to a stable matching as the meeting rate approaches infinity, though Wu (2015) cautions that inefficiencies due to delay can remain substantial even for very small search frictions.

The rest of the paper is organized as follows. In Section 2, we describe the modeling framework which underpins our theory. In Section 3, we describe and analyze the empirical results from the NSDS. In Section 4, we estimate the model from Section 2 and show that it fits our empirical results very well. Section 5 concludes. Supplementary information is provided in an appendix.

2. The model

As in the model of Burdett and Coles (1997), time is continuous and indexed by $t \in [0, \infty)$. There is a fixed population of female and male agents; they discount the future at rate $\rho > 0$, and they meet randomly at the fixed rate $\mu > 0$, in pairs of one woman and one man.

In a meeting, each agent decides whether they would like to be matched with the other agent or whether they would prefer to keep searching. We call the former a ‘yes’, and the latter a ‘no’ or ‘rejection’. There is no recall of rejected meeting partners. A ‘yes’ by both agents results in a ‘match’: the agents are withdrawn from the pool of searchers and receive a utility flow while the match endures. Utility is not transferable. Existing matches break up randomly at rate $\delta > 0$, and only unmatched agents can search.

Unmatched agents receive a utility flow of $\rho b > 0$ (so that $b$ would be the present discounted value of being single forever), and matched agents receive a utility flow of $\rho u$ (ditto), which is a random variable corresponding to an agent’s attraction to their match. How can we interpret these parameters? Since $b$ is the value of being single, an agent will reject every potential partner that would deliver match utility $u < b$. Thus, $\mathbb{P}(u < b)$ is the lowest possible rejection rate; however, 

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8 Fisman et al. (2006) also used a flexible specification of preferences, but in a model without search frictions, to explain speed dating outcomes.

9 As the model is sex-symmetric, it could just as well apply to same-sex attraction. However, our data includes opposite-sex dating only. It would be very interesting to extend this.

10 Many models of search and matching assume that utility is transferable. We think that this would be more appropriate in the context of marriages than dating and initial attraction, where the assumption of non-transferable utility is more evidently justified. However, non-transferable utility settings are very general; even in the labor market, where transfers in the form of wages are possible in principle, such transfers are often restricted by law or by custom.
due to the option value of waiting for an excellent match rather than a marginally satisfactory one, most agents will be more picky than this lower bound.

We parametrize the distribution of \( u \) among the pool of available singles as follows.\(^{11}\) Each agent is described by a permanent type \( a_v \) ("vertical component of attractiveness"). The distribution of \( a_v \) among singles is \( N(0, 1-s) \), and the variance \( (1-s) \in (0,1) \) is the same for each sex. When agent \( a_v \) meets with potential partner \( a'_v \), they also draw random "horizontal" values \((a'_h, a_H)\) from the distribution:

\[
\begin{bmatrix}
  a'_H \\
a_H \\
\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} s & c \ 
  c & s \end{bmatrix}\right),
\]

where \( s \in (0, 1) \) is the variance of both \( a'_H \) and \( a_H \), and \( c \in [-1,1] \) is the correlation between them. We use \( H(a'_h, a_H) \) to denote the joint CDF. If a match is formed, agents receive the following utilities:

\[
u = \exp(a'_v + a'_h) \quad \text{and} \quad u' = \exp(a_v + a_H).
\]

Type values \( a_v \) and \( a'_v \) are the vertical components of attractiveness, which everybody can see and agree on. Value \( a'_h \) is the horizontal component of agent \((a_v)'s\) attractiveness to agent \((a'_h)'\), independent of anyone else’s evaluation of that agent, but possibly correlated with \((a'_h)'s\) reciprocal attraction to \((a_v)'\); if \( c > 0 \), then attraction tends to be mutual. The parameter \( s \) represents the share of overall attraction that is horizontal, or the amount of disagreement between agents when evaluating others. Because its components are independently distributed with means zero and variances summing to one, the distribution of \( \log(u) \) over all possible meetings is standard normal.\(^{12}\)

Each meeting is therefore characterized by four random variables: \( a_v \) and \( a'_v \), the types of the two agents in the meeting, and \( a'_h \) and \( a_H \), the meeting-specific random variables describing how the agents perceive each other. Singles thus face three dimensions of uncertainty: who they meet, how strongly they are attracted to that person, and how strongly that person is attracted to them.

Let \( V(a_v) \) be the value function of searching for an agent with type \( a_v \), and \( W(a_v, u) \) be this agent’s value function of being matched with utility \( u \). They satisfy the following Bellman equations in steady state:

\[
\rho V(a_v) = \rho b + \mu \cdot \max_{\text{yes/no}} \mathbb{E}[W(a_v, u) − V(a_v) | \text{match}] \mathbb{P}[\text{match}]
\]

\[
\rho W(a_v, u) = \rho u + \delta[V(a_v) − W(a_v, u)]
\]

We substitute \( W(a_v, u) \) in the first equation using the second equation, and rearrange some terms to obtain the following Bellman equation of an unmatched agent:

\[
(\rho + \delta) V(a_v) = (\rho + \delta) b + \mu \cdot \max_{\text{yes/no}} \mathbb{E}\left[ u - V(a_v) \mid \text{match} \right] \mathbb{P}[\text{match}]
\]

\[
\Rightarrow \quad r V(a_v) = r b + \max_{\text{yes/no}} \mathbb{E}\left[ u - V(a_v) \mid \text{match} \right] \mathbb{P}[\text{match}],
\]

where \( r \equiv (\rho + \delta)/\mu \) is a reduced-form parameter that summarizes the size of “frictions”. Frictions are large either if \( \rho \) is large (agents are impatient), or if \( \delta \) is large relative to \( \mu \) (matches are short, yet take a long time to find). Since we only observe the NSDS participants at a point in time, we will not be able to identify either their long-run meeting rate \( \mu \) or breakup rate \( \delta \). But we can infer the friction statistic, \( r \), from their decisions.

2.1. Optimal decisions

As shown by Burdett and Coles (1997), the optimal decision satisfies a reservation utility property: a single of type \( a_v \) says ‘yes’ to a potential partner whenever their attraction to the partner exceeds the threshold \( \log V(a_v) \), and rejects them otherwise. Suppose we consider the decision problem of a woman of type \( a_v \), who anticipates that men have the value function \( V^M(a'_v) \).

Then, random search implies that the value (and the corresponding rejection threshold) of this woman, \( V^F(a_v) \), satisfies the following Bellman equation, which is easiest to state in stages. We define the “horizontal attraction set”, the set of horizontal attraction terms high enough for both agents to say ‘yes’, conditional on their vertical types:

\[
H(a_v, a'_v) \equiv \{a'_h, a_H : a'_H \geq \log[V^F(a_v)] - a'_v \text{ and } a_H \geq \log[V^M(a'_v)] - a_v \}
\]

And the match surplus for woman \( a_v \), in case of a match with man \( a'_v \):

\[
S^F(a'_v, a'_h, a_H) = \max \left\{ \exp(a'_v + a'_h) - V^F(a_v), 0 \right\}
\]

\(^{11}\) Theoretical models of matching are often interested in the endogenous distribution of types among singles as an equilibrium outcome. Clearly, if some agents have more desirable traits than others, these desirable traits will be overrepresented among matched agents and underrepresented among singles. However, in our dataset, we observe a sample of singles; hence, we parametrize the type distribution among singles directly. Multiple equilibria are not an issue: while the literature has shown that an exogenous population distribution can lead to multiple endogenous type distributions among singles, here we only require that a known type distribution among singles leads to a unique matching equilibrium.

\(^{12}\) We present the log-normal utility case here, but we also solved the model with normal utility and re-did a subset of our estimates from Section 4. The results were almost identical.
Finally, let \( \Phi \) denote the CDF of the standard normal distribution. Then the woman solves:

\[
rv(f,a_v) = rb + \int_{-\infty}^{\infty} \left[ \int_{\mathcal{H}(a_v,a_{\hat{H}}) \cup \mathcal{L}(a_v,a_{\hat{H}})} S^f(a_{\hat{H}},a_{\hat{H}}) \, dh(a_{\hat{H}},a_{\hat{H}}) \right] \, d\Phi \left( \frac{a_{\hat{H}}}{\sqrt{1-s}} \right),
\]

where of course \( \mathcal{H} \) and \( S^f \) depend on the value functions, too, as defined explicitly above.

The outer integral answers the question: who will the woman meet? The inner integrals answer the question: will they like each other enough to agree to a match? Finally, the surplus term \( S^f \) answers the question: how happy will this match make her?

**Lemma 1.** Assume that \( V^M : \mathbb{R} \to \mathbb{R}_+ \) is a continuous function. Then there exists a unique function \( V^f : \mathbb{R} \to \mathbb{R}_+ \) which satisfies the Bellman Eq. (1). The function \( V^f \) is continuous, strictly monotonically increasing, and approaches the bounds \( V \in \mathbb{R}_+ \) as \( a_v \to -\infty \) and \( V \in \mathbb{R}_+ \) as \( a_v \to +\infty \). The lower bound is \( V = b \), and the upper bound solves:

\[
rV = rb + e^{0.5} \Phi(1 - \log V) - \nabla \Phi(-\log V)
\]

**Proof.** See Appendix A.1. \( \square \)

The Lemma formalizes the intuitive notion that more attractive agents are pickier in who they agree to match with (keeping in mind that this pickiness applies to a subjective evaluation of potential partners). The fact that the monotonicity is strict is in contrast to the famous result (Burdett and Coles, 1997; McNamara and Collins, 1990) where people sort into distinct classes by attractiveness (see also Chade, 2001). In those models, attractiveness is entirely vertical (everyone agrees on it), whereas in our model there is subjective disagreement; as a consequence, their thresholds form a step function and ours are smooth.

The existence of lower and upper bounds is also intuitive. The least attractive agents will only receive a ‘yes’ with infinitesimal probability, therefore their reservation utility is the value of being single. The most attractive agents will only be rejected with infinitesimal probability, therefore their only concern is to find the right tradeoff between waiting and getting matched now. As \( r > 0 \), they are not willing to wait forever and have a finite reservation utility. Importantly, the upper bound does not depend on the parameters \( s \) and \( c \), because the distribution of the sum \( a'_v + a'_{\hat{H}} \) is standard normal; hence, both bounds depend on the parameters \( r \) and \( b \) alone. They do not even depend on a particular expectation of the other agents’ rejection functions, because an agent of very low type is almost always rejected and an agent of very high type is almost never rejected.

### 2.2. Equilibrium of the matching game

If we flip the labels \( F \) and \( M \) in Eq. (1), we obtain the decision problem of a man of type \( a_v \), who anticipates that women have the value function \( V^M(a_v) \); the problem is symmetric. We are now ready to define an equilibrium.

**Definition 1.** An equilibrium of the matching game consists of functions \( V^F(a_v) \) and \( V^M(a_v) \) such that:

1. For a woman of type \( a_v \), \( V^F(a_v) \) solves Eq. (1).
2. For a man of type \( a_v \), \( V^M(a_v) \) solves Eq. (1) with labels \( F \) and \( M \) switched.

The uniqueness result in Lemma 1 rules out equilibria in which different agents of the same gender and type apply different thresholds, but it does not rule out the possibility that women and men have different threshold functions in equilibrium. If all women uniformly became more selective \( (V^F) \), all men would have an incentive to become less selective \( (V^M) \), which could in principle support multiple equilibria (Burdett and Wright, 1998). However, for our particular model, the equilibrium is unique and symmetric:

**Proposition 1.** There exists an equilibrium of the matching game. Furthermore, if the parameters of the model satisfy the condition:

\[
r \sqrt{(1-c)s} > G(b),
\]

where \( G(b) \) is defined in the appendix, then there exists a unique equilibrium of functions \( V^F \) and \( V^M \), and these functions coincide:

\[
V^F(x) = V^M(x) \quad \text{for all } x \in \mathbb{R}.
\]

**Proof.** See Appendix A.1 for the proof. We conjecture that uniqueness holds more generally, but this condition allows for a straightforward proof. The parameters we estimate in the empirical application satisfy the uniqueness condition. \( \square \)

There are no closed form solutions for the value functions. Note that both functions \( V^F(\cdot) \) and \( V^M(\cdot) \) appear inside of the expectations operator in Eq. (1), and the expectation is over the argument of one of them. Consequently, the rest of the analysis will be numerical. We use a recursive root-finding procedure: using the result that the thresholds for men and women coincide, we interpolate \( V^F(\cdot) \) over a reasonable range given a previous result for \( V^M(\cdot) = V^F(\cdot) \), and repeat until the answers converge. Examples of equilibrium rejection rates are shown in Fig. 1, and we will discuss aggregate matching and sorting in Section 4 after estimating the model.
but our relevant men they to decisions, from 2.3.

Fig. 1. Rejection rates in a symmetric equilibrium ($\Phi(log V)$) as functions of attractiveness rank ($\Phi(a_i/\sqrt{1-s})$). Upper and lower bounds are indicated with dashed lines; they only depend on the parameters $r$ and $b$, but are independent of $s$ and $c$.

c) With more disagreement ($s \uparrow$), attractive singles reject less often, and unattractive singles reject more often

d) With strong mutual attraction ($c \uparrow$), rejections are more frequent (but so are successful matches)

2.3. Richer models

As we mentioned earlier, the assumption that the distribution of singles is exogenous is not the most satisfying one from the point of view of theory. We know that when the distribution of singles is endogenously determined by matching decisions, the population distribution can be consistent with multiple equilibria, each with a different distribution of singles (Baughman, 2014; Burdett and Coles, 1997; 1999). Compared to the theoretical literature, however, our situation is special in that we are able to observe decisions among singles. And in our estimated model, we confirm that this distribution leads to a unique matching equilibrium (Proposition 1) – hence, to a unique corresponding population distribution.

In an earlier version of our model, we also considered the possibility that men and women might differ in how much they disagree in evaluating the attractiveness of potential partners; in plain terms, that the parameter $s$ might differ between men and women. However, in our dataset there was no evidence that men and women differ on any important dimension relevant to decisions and matching outcomes, including disagreement about the attractiveness of potential partners. Hence, our analysis will focus on the symmetric case.13

13 The lack of gender differences in the NSDS is in contrast with the literature on personal ads and online dating contexts (e.g. Hitsch et al., 2010), but it is consistent with large meta-analyses examining sex differences in the implications of attractiveness and earning prospects for face-to-face interactions and established relationships [Eastwick et al., 2014]. Using the NSDS data, Eastwick and Finkel (2008) and Eastwick et al. (2011) argue that the stated preferences of participants may not always match their revealed preferences and therefore their matching outcomes. Even the common finding that women are more selective than men may not be robust to features of the meeting environment, such as who happens to physically approach whom
Finally, it is worth noting that in our model there is no explicit cost of saying ‘yes’ to a potential partner (though the opportunity cost of waiting for a better one prevents saying ‘yes’ to everyone from being a dominant strategy). Among other things, an explicit cost of contact would imply that a low or medium ranked agent might not say ‘yes’ to a top ranked one, because the probability of rejection would be too high to justify the cost. Then, there might be upper threshold for rejection in addition to the lower threshold \( \log \mathcal{V}(-) \). This could cause matching rates to be lower for the most attractive people than for a more approachable ‘middle class’. We analyze this possibility at the end of the next section and find no evidence of it in our data. Neither did Hitcsh et al. (2010) in their analysis of online dating; this supports our case because we would expect that the cost of saying ‘yes’ (whether an explicit cost or the emotional cost of rejection) should if anything be higher in the online dating environment, where contact requires an active decision and effort to compose a message, rather than an anonymous yes/no decision.

2.4. Model environment vs the speed-dating environment

While we think our model captures the essentials of the decision problem in speed dating, there are two obvious differences between the model environment and a typical speed dating session. One: in speed dating, participants make a simultaneous decision about the partners they have met. Two: they can ‘match’ with more than one partner. In the model, on the other hand, agents meet in continuous time (that is to say, in sequence) and they cannot meet other singles while matched. Does this mismatch invalidate our approach?

We think not, for two reasons. First, in our model, being “matched” with a person represents dating rather than marriage. Since dating takes time, people have a finite capacity for the number of dating partners. If this capacity is on the order of 10 and overall utility is the sum of the utilities derived from current partners (or from free time if not dating), then our model, running in 10 parallel slots, would exactly describe the problem the participants in the study were facing. However, this stretches the interpretation of the model more than we want to, and at any rate a dating capacity of 10 seems high. What would be a more realistic model? For most people, the capacity for serious relationships is probably 1, the capacity for continued dating is probably 1–3, and the capacity for first dates to see whether more is possible might go up to 5. Thus a realistic description of matching could involve being open minded at first, followed by gradual winnowing if the outcome is more than a few ‘candidate matches’ (e.g. see Das and Kamenica, 2005, for a formal treatment). This kind of winnowing in a strategic environment is challenging to model. In our model, we make the simplifying assumption that break-up is random, independent of partners’ types or their initial attraction.

Second, even though the participants made their ’yes’ decisions simultaneously at the end of the study, they were not really faced with a one-shot decision problem: if unmatched during the study, they were surely (we hope!) able to find romantic partners in some other way. In other words, if we modeled a speed-dating event as a static game between a double-digit number of players, the question would be how to back out a continuation value of leaving the event unmatched. It seems reasonable to think that participants’ ability to find a partner outside of the study, and therefore their continuation value within the study, is related to the vertical component of matchability in a similar way outside of the study as within it. If we think of the study as a mere short-term spike in meeting opportunities, followed by a long continuation game, then our model would be a valid description of it. It seems realistic, at least as much as any other equally simple alternative.

On the other hand, it was proposed that we use an even simpler model, merely concluding theoretically that \( \mathcal{V}(\cdot) \) is an increasing function and using \( \log \mathcal{V}(x) = \alpha + \beta x \) for the estimation. This would not be satisfying, either. Theory strongly suggests that the rejection rate \( \log \mathcal{V}(\cdot) \) cannot be linear: as long as the value of being single is not lower than the utility from any potential partner, everybody will reject some potential partners, thus \( \log \mathcal{V}(\cdot) \) must have a lower bound. And as long as there are any kind of frictions at all, the most picky person will still accept some potential partners, thus \( \log \mathcal{V}(\cdot) \) must also have an upper bound. Now, an increasing function with an upper and lower bound has at minimum four free parameters: upper and lower bounds, location of midpoint, slope at midpoint. This is exactly the number of parameters that our model has, too; however, only one of our four parameters is reduced-form and the other three are structural, and two of them have direct counterparts in our measurement of individual preferences. For this reason, we think there is little to be gained from moving to a purely reduced-form approach.

3. Empirical results

3.1. Data source and descriptive statistics

We use the Northwestern Speed-Dating Study (NSDS) to test our model. The NSDS was a study conducted by Eastwick and Finkel (2008). The speed-dating experiments were executed in two rounds. In total, the dataset includes a total of 15 speed-dating events, and there are 350 participants and 2050 interactions between them.

(Finkel and Eastwick, 2009). Finally, Abramitzky et al. (2011) show that gender differences in outcomes can be related to the sex ratio in the marriage-able population. This is consistent with our model of strategic decision making under uncertainty, and it suggests that many commonly observed gender differences are less due to preferences than the environment that matching takes place in.

Shimer and Smith (2000) analyze such regions of matching in a model of transferable utility.
Undergraduate students from Northwestern University were recruited for the study via flyers posted in the campus and informational e-mails. No deception was used in this study and the NSDS was designed to model the commercial speed-dating experience as closely as possible. Although similar commercial events normally charge a fee, participants were paid $5 for their participation. The study consisted of three phases: intake, speed-dating, and follow-up. In the intake phase, participants completed a 30-min online survey about themselves (e.g., sex, ethnicity, religion, and personality measures) and their preferences for personal characteristics of the interaction partners. They then attended the speed-dating event 6–13 days later.

In the speed-dating phase, each speed-dating event included 9–13 female and male Northwestern students. Speed-dating events were hosted in an art gallery on campus. Tables and chairs were arranged such that each pair of speed-daters could communicate easily with each other without being distracted by other pairs. Upon arrival, participants received a set of interaction-record questionnaires. The experimenter then took photos of the participants, so that participants could easily remember their interaction partners from the photos in the follow-up phase. Once all the participants had arrived, the experimenter explained the procedures of the speed-dating event.

Each speed date lasted for 4 minutes. Participants freely chatted during this time. After each speed date, the experimenter blew a whistle to signal the participants to rotate to the next position before completing a 2-min. interaction record about the partner they had just met. In each interaction record, they scored their impression of some personal characteristics of the partner, such as physical attractiveness. In addition, participants also rated the overall matching value of each partner (“I am likely to say ‘yes’ to my interaction partner”), on a scale from 1 (strongly disagree) to 9 (strongly agree). After 2 minutes passed, the experimenter signaled the participants to begin their next 4-min date. After all possible dates had been completed, the experimenter explained to the participants how to complete the matching process and remaining questionnaires.

In the follow-up phase (i.e., immediately after the speed-dating event), participants visited a website with photographs of their meeting partners, and they indicated with a ‘yes’/’no’ button who they wanted to be in contact with. If both partners said ‘yes’, we say that their meeting resulted in a ‘match’, and they could then use the website to exchange online messages and/or personal information if they wished to. We recorded the message exchanges between participants. The procedure yielded a total of 444 matching pairs. The follow-up surveys revealed that 70% of the participants who were matched with at least one partner used the messaging system after the speed-dating event, and many ‘matches’ led to subsequent interaction (e.g., coffee dates; Finkel et al., 2007).

We provide demographic statistics in Table 1. The gender distribution of participants is balanced in the study, while the majority of the participants are Caucasian and religious. Table 2 presents a summary of meeting expectations, ratings, and decisions for all participants. In particular, we use the variable that is the agreement of a rater to the statement “I am likely to say ‘yes’ to my interaction partner” to measure the subjective match value of the partner to the participant. For each participant, we construct a ‘matchability’ index as the average “likelihood to receive yes” as judged by all of their meeting partners, and we use this ‘matchability’ index as a general matching value that we interpret as the vertical component of attraction.

Formally, let $e^{ij}$ be participant $i$’s answer to the “likely to say ‘yes’” question regarding participant $j$, measured on an integer scale from 1 to 9. Let $N \in \{9, \ldots, 13\}$ be the number of participants of each gender in the event. Then $j$’s empirical ‘matchability’ index is:

$$m_j = \frac{1}{N} \sum_{i=1}^{N} e^{ij}$$
And correspondingly, our empirical measure of i's horizontal attraction to j is:

\[ m_{ij}^H = \epsilon_{ij} - m_{ij}^V = \left(1 - \frac{1}{N}\right) \epsilon_{ij} - \frac{1}{N} \sum_{k \neq i} \epsilon_{ij} \]

When we estimate the model, we use versions of \( m_{ij}^H \) and \( m_{ij}^V \) which were standardized among the entire set of participants (i.e. not standardized by group) as the empirical counterparts of \( a_{ij}^H \) and \( a_{ij}^V \) from the model.

It is evident that estimating averages from such a small sample involves measurement error, which will cause some estimates to be exaggerated (e.g., the share of the variance in \( \epsilon_{ij} \) that can be attributed to \( m_{ij}^H \)) and others to be attenuated (e.g., \( corr(m_{ij}^H, m_{ij}^V) \)). However, since we are interested in population statistics and since we have 15 event groups, we can use a bias-correction procedure to deflate or inflate estimates properly. Details are provided in Appendix A.2. In the rest of the paper, estimates refer to the raw versions of the variables unless bias correction is specifically mentioned. Also, it is important to keep in mind that the small-sample bias only applies to higher-order statistics such as variance or correlation, and not to averages: therefore, for each individual their ‘matchability’ index is indeed our best estimate of their vertical type.

Table 2 also shows the average number of ‘yeses’ sent and received for male and female participants and the average number of successful matches. On average, men and women are almost equally likely to say and receive ‘yes’, at a rate around 45%. The average matching rate for each gender is identical, at 21.7%, because each match involved one male and one female participant. Notice that if the probability of one participant saying ‘yes’ to their partner was independent from the probability that the partner would say ‘yes’ in return, then we would predict an aggregate matching rate of 20.7%. Of course, we have good reasons to believe that these decisions are not independent, because participants’ preferences are not independent but exhibit clear patterns. We explore these patterns in the next two subsections.

It is important to recognize that there are other ways to measure ‘attraction’ or ‘attractiveness’. One way is to collect data on multiple characteristics and measure how important they are in determining match outcomes. This is the preferred approach in most of the marriage matching literature, but that is because the vast majority of these works do not have access to a direct measurement of preferences. Therefore, our preferred measure of overall attraction is the ‘matchability’ rating introduced in the previous section, because it directly answers the question of whether a person is “worth matching with”. However, we will also include other measurable traits, whether objective (height, weight) or subjective (physical attractiveness as rated by other participants) in the analysis.\(^{15}\)

3.2. Vertical component of attraction

In the previous section, we introduced our preferred measure of ‘matchability’ \( m_{ij}^V \), general matching value, as the average agreement of raters with the statement “I am likely to say ‘yes’ to my interaction partner” (\( \epsilon \)). We want to know how much people agree on who is matchable in our speed-dating setting, and we measure the degree of agreement as the ratio of between-targets variance (i.e. variance of \( m_{ij}^V \)) out of total sample variance for \( \epsilon_{ij} \):

\[ \text{agreement} = \frac{\text{Var}_i(\epsilon_j)}{\text{Var}_i(\epsilon_{ij})} = \frac{\text{Var}_i(\epsilon_j)}{\text{Var}_i(m_{ij}^V) + m_{ij}^V} = \frac{\text{Var}_i(\epsilon_j)}{\text{Var}_i(m_{ij}^V) + \text{Var}_i(m_{ij}^V)}. \]

\(^{15}\) The per-individual and per-meeting statistics differ slightly because group size varied from 9 to 13. Some groups were also unbalanced, though not by more than one person.

\(^{16}\) For most participants, we also have self-reported math and verbal SAT scores, including them did not change any of our results.
Table 3

*Matchability predicts matches,* at the observation level of participants, and thus the term “matchability” is justified. “Covariates” are physical attractiveness (average evaluation by all partners), plus height, weight, and (gender × group size) fixed effects. Variables are not standardized.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Mean ‘Yeses’ received</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Matchability (m_V)</td>
<td>0.171*** (0.0046)</td>
</tr>
<tr>
<td>Covariates</td>
<td>X</td>
</tr>
<tr>
<td>Observations</td>
<td>350</td>
</tr>
<tr>
<td>R²</td>
<td>0.788</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses. One participant did not report height/weight. *** indicates p < 0.01.

Table 4

*Matchability is normally distributed among participants.*

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Matchability (m_V) (350 observations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W</td>
</tr>
<tr>
<td>Shapiro–Wilk W test</td>
<td>0.993</td>
</tr>
<tr>
<td>Shapiro–Francia W test</td>
<td>0.995</td>
</tr>
<tr>
<td>Skewness/Kurtosis tests</td>
<td>Pr(Skewness)</td>
</tr>
<tr>
<td></td>
<td>0.428</td>
</tr>
</tbody>
</table>

where the independence used in the last equality is by construction of the m-variables.

We find that the agreement ratio is 32%. To put this number into context, the corresponding ratio for judgments of physical attractiveness is 45%. (After correcting for small-sample bias, both numbers fall, to 25% and 40%. On the other hand, noise in the reported answers would cause a bias in the opposite direction.) The disagreement on who is physically attractive is substantial in the study, and as we would expect, the participants disagreed even more on who was worth matching with in general.

Our findings are consistent with (Eastwick and Hunt, 2014, Table 4, Column 2), though they decompose ‘mate value’ (what we call ‘matchability’) into three variance terms: the variance in the target’s average score received, the rater’s average score given, and the meeting-specific residual. To focus on the distinction between the vertical and horizontal components of attraction, we only split the total variance in matchability into the sum of the variance in a target’s average score and a meeting-specific residual term. Our residual term therefore does not control for a rater’s personal rating standard. However, we do not think this is a problem for our analysis, because whether the participants were meeting with strict or lenient raters was still randomly assigned.

As it should, a participant’s matchability strongly predicts how many partners did say ‘yes’ to this participant after all the meetings were concluded (Table 3). But it is important to note that the predictive power of matchability stays very strong when we include additional controls: subjective evaluations of physical attractiveness, height and weight, and the number of partners that the participant was meeting. Furthermore, the predictive power (R²) of matchability alone is stronger than the joint predictive power of the covariates.

As part of our structural analysis, we will identify the empirical matchability value m_V with the vertical component of attraction, a_V, from the theoretical model. In the model, we had assumed that a_V is normally distributed; hence, for our analysis to be valid, m_V should be normally distributed, too. We test this and report the results in Table 4. They show that the distribution of m_V cannot be distinguished from normal.

Next, we want to understand how matchability is related to specific personal characteristics. We regress each participant’s matchability on these characteristics (where we measure the physical attractiveness as the average of subjective ratings), and not surprisingly, we find that matchability is strongly related overall to these variables with an R² of 77% (Column 3 of Table 11, in Appendix A.4). This result suggests that although people can disagree on how physically attractive they find a given person, or the importance that they attach to this trait, it is still a very strong predictor of general matching value (and indeed of matching success, as reported in Column (1) of Table 11). And including the reported judgments of physical attractiveness is important: the R² of a regression of matching success on the anthropometric variables weight and height is only 7%.

Now that we understand the vertical component of attraction, the next question is whether the vertical type of partners in a matched couple is correlated. We propose the term "rank-based sorting", one type of assortative matching, to refer to such a correlation: high-rank singles match with other high-rank singles not because they prefer to match with them, but because everyone prefers to match with them. Among the 444 successful matches in the NSDS data, we find that the correlation between the vertical components of the partners is 0.15 with a standard error of 0.05, indicating that matching is assortative but not overwhelmingly so. This positive but low value is consistent with our theory, which formalizes two
reasons for why the correlation cannot be perfect: one is the substantial disagreement between participants on who they find attractive, and the other one is the fact that matching takes place in a frictional environment.17

Finally, we can use the vertical component of attraction to address the concern whether a search model of decision making can be valid in the speed-dating context. The key assumption that makes our search framework valid is that an individual’s decision to say ‘yes’ to a potential romantic partner is not context specific. For example, this would require that our participants used the same yes-no threshold in everyday face-to-face interactions as they used in the study. We cannot directly test this, but we can test whether our participants’ thresholds varied depending on the characteristics of an event and of the people participating in it. We therefore regress the mean number of ‘yeses’ a participant received on their own matchability and on the average matchability of same-gender participants in the same event. If all subjects indeed apply the same decision rule in speed-dating as in daily life, then the group average matchability – in plain terms, the quality of one’s romantic competition – should not affect how many ‘yeses’ the participant receives, whether unconditionally or conditional on their own matchability.

We show the regression results in Table 5, and they support our hypothesis. Whether or not we control for own matchability or for covariates including own physical attractiveness, the matchability of potential romantic rivals does not affect a participant’s matching success. This suggests that our participants apply a general decision rule for all their romantic encounters rather than creating specific rules for the speed-dating event. We think this supports our approach of modeling the speed-dating decision as depending only on subjective attraction to a potential partner, on the option set summarized by the decision maker’s vertical score, and on the search friction.

### 3.3. Horizontal component of attraction

How much do people disagree in evaluating others? One contribution of our paper is to quantify the weight of subjective preferences in determining the search decision. Our definition of disagreement is the deviation of personal rating from group average rating. The deviation of one rater’s judgment of a participant’s match value from the group average corresponds to the horizontal component of attraction in the model of Section 2.

If the horizontal component was purely noise, one participant’s horizontal component would be independent from their partner’s. If the horizontal component instead represents subjective preferences that affect the match quality, then the horizontal components of participants’ attraction to each other can be correlated. For example, say that a participant has dancing as a hobby. The additional attractiveness of this participant to others may be randomly perceived, but a partner who ranks them higher in matching value probably likes dancing more, too. This correlation causes sorting in match in a different way than the “rank-based” sorting analyzed above, which is the correlation in vertical components. In contrast to rank-based sorting, we propose the term “compatibility-based sorting” to refer to sorting along horizontal dimensions: for example, people may have a preference for partners of a particular ethnicity, but disagree on what the ethnicity of the ideal partner is.18

Table 6 shows the correlation of horizontal attraction between partners, split by matching outcomes. The whole-sample correlation is significantly positive, implying that attraction tends to be mutual.19 It is also significantly larger in meetings that resulted in a match than in those that did not, which indicates the presence of compatibility-based sorting.

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17 Because in our model utility is not transferable, our results on vertical sorting come purely from the strategic choices of the agents, not from the supermodularity of a joint payoff function as in Shimer and Smith (2000). In fact, the agents in our model do not care about a potential partner’s utility, except to the extent that it determines their agreement to form the match.

18 See, for example, Eastwick et al. (2009) on the link between interracial dating and political preferences.

19 Our result of 0.08 is smaller than the 0.14 reported in Eastwick et al. (2007) for a slightly different measure of attraction, which included sexual attraction. It is plausible that sexual attraction may be more mutual than other forms of attraction.
Table 6

Mutual attraction implies compatibility-based sorting. The horizontal components of attraction between meeting partners are correlated, and we observe sorting: the correlation is higher if the meeting resulted in a match, and zero if it did not. The correlations are raw; in order to correct for small-sample attenuation, they would have to be inflated by 9.3.

<table>
<thead>
<tr>
<th></th>
<th>(1) Total</th>
<th>(2) Matched</th>
<th>(3) Unmatched</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation ((m_H^1, m_H^2))</td>
<td>0.083***</td>
<td>0.098***</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.033)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Observations</td>
<td>4,076</td>
<td>884</td>
<td>3,192</td>
</tr>
</tbody>
</table>

Standard errors clustered at individual level. *** indicates \(p < 0.01\).

Table 7

Sharing a background predicts liking and matching. Ethnicity and religion categories are listed in Table 1.

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>‘Likely-Yes’ (1)</th>
<th>‘Said-Yes’ (2)</th>
<th>‘Match’ (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner in same ethnicity category</td>
<td>0.231**</td>
<td>0.059***</td>
<td>0.066***</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.019)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Partner in same religion category</td>
<td>0.106</td>
<td>0.032*</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(0.088)</td>
<td>(0.017)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Observations</td>
<td>4,088</td>
<td>4,088</td>
<td>4,088</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.002</td>
<td>0.002</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Standard errors clustered at the target level. *** \(p < 0.01\), ** \(p < 0.05\), * \(p < 0.1\).

Next, we want to see how well this correlation could be attributed to some form of participant similarity; in other words, whether “birds of a feather flock together”. But there is a vast number of ways in which two people could be similar, and we only observe a sliver of them. Hence, we approach this problem in two ways. First, we look at background variables that we have in our dataset: namely, ethnicity and religion, as listed in Table 1. Second, we look at perceived similarity and check whether it correlates with actual outcomes.

We present the matching patterns across ethnicity and religion in Table 7. Indeed, sharing an ethnic background significantly increases the likelihood of giving a partner a higher matchability score, saying ‘yes’ to a partner, and matching with a partner. Sharing a religious background points in the same direction, but the differences are not significant. (The reason for this could be that our religion categories are fairly coarse, but also the fact that religious affiliation is hard to observe unless it comes up in conversation. Ethnicity is more visible.) For readers interested in the details, we also provide the cross-tabs of matching rates by ethnic or religious group; see Tables 13–14 in Appendix A.4.

Ethnicity and religion are important ways in which two people can be similar, but hardly exhaustive. To approach our question from a different perspective, we analyze participants’ stated perceptions of similarity with a potential partner. Specifically, we compute correlations between the horizontal component of attraction and various measures of perceived similarity; see Tables 15 and 16 in Appendix A.4 for details. We see that the horizontal component of the attraction of participant A to participant B is strongly correlated (\(corr = .54\)) with whether A thinks the two have a lot in common, which is not a surprising result. More important is the second result: the attraction of A to B is also significantly correlated (\(corr = .06\)) with whether B thinks (s)he has a lot in common with A. Together, these results indicate that not only does attraction tend to be mutual, but that participants are aware of this effect and attribute it to a shared compatibility value of a potential match.

There has been some doubt in the recent literature whether mutual liking can be attributed to similarity. For example, Luo and Zhang (2009) do not find evidence that people match with those of similar personality attributes. And according to Watson et al. (2004), spousal similarity is not related to marital satisfaction.\(^{20}\) And in a recent paper, Tidwell et al. (2013) suggest that perceived similarity is much more important for attraction than actual (or rather, measurable) similarity.

Since our argument in the preceding paragraphs is that perceptions do predict real outcomes (in this case, compatibility-based sorting), these results need to be reconciled. First, we note that true similarity is hard to measure precisely, since researchers have access to much less information than the participants do, even in the very best study designs. And things that predict similarity could vary from dyad to dyad. Participant A likes partner B because he and she share music preferences, but A likes partner C because he and she are both from the same home town. If the components that produce similarity vary randomly like this, they would be very hard to capture a priori. Our second response is empirical: the partner’s perception of similarity is significantly correlated with a rater’s attraction (Table 16), which makes it unlikely that the

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\(^{20}\) On the other hand, Luo and Klohnen (2005) find that similarity predicts marital satisfaction even after controlling for stated attraction. We think a likely explanation is that couple formation is endogenous (of course). Similarity among strangers could predict whether they like each other a lot or not at all, and therefore whether a match is formed, but the effect of similarity on whether spouses like each other a little bit more or less should be much weaker and therefore less robust, because surviving marriages are conditional on a minimum of satisfaction.
similarity-attraction link was due to perception alone. Participants tended to agree with each other on whether they had things in common or not. In general, however, it is fair to worry (and we cannot test) whether perceptions of similarity might be a consequence as much as a cause of attraction.

3.4. Meeting outcomes

What is the importance of the vertical component of attraction in predicting matching outcomes? Since we interpret this variable as measuring general matchability, and think of it as a person’s permanent type, it is a very important question whether people are aware of their standing in this variable and use it to inform their decisions. We therefore regress meeting outcomes on the participants’ matchability using a linear probability model:

\[ Y_{ij} = \beta_0 + \beta_1 V S_j + \beta_2 V S_j^2 + \beta_3 V S_i + \beta_4 V S_i^2 + \beta_5 V S_j V S_i + \epsilon_{ij}, \]  

where \( i \) is the decider, or rater, and \( j \) is the target of a decision, and where the dependent variable  \( Y_{ij} \) stands for either ‘Yes’ (whether \( i \) said ‘yes’ to \( j \)) or ‘Match’ (whether \( i \) and \( j \) said ‘yes’ to each other). In the latter case, the regression is symmetric with respect to the identity of \( i \) and \( j \), and their coefficients are identical by construction. VS is the standardized version of \( m_v \), short for “vertical score” of a participant:

\[ VS_j = \frac{m_v^j - \sum_{k=1}^{350} m_v^k / 350}{\sqrt{\text{Var}_k(m_v^k)}} \]

Lastly, \( \epsilon_{ij} \) is the error term. We report our results in Table 8. They are robust with respect to Logit or Probit specifications.

There are four important messages. First, the vertical score of a participant positively predicts whether they receive a ‘yes’ from their partner, and it does so very strongly. Second, the vertical score of a participant negatively predicts them saying ‘yes’ to their partner; it follows that participants with a high matching value are more selective, which supports our hypothesis that participants are aware of their own ranking and make decisions strategically. Third, as we would expect, more matchable participants were involved in more successful matches. Fourth, we have strongly positive rater-target interaction terms. The one in Column (2) is a subtle echo of the class formation result of Burdett and Coles (1997), and confirms a nontrivial prediction of the model: for a participant with vertical score two standard deviations above the mean, the marginal effect of the rater’s score is zero. Everyone is equally likely to say ‘yes’ to such an attractive participant, independently of their own rank. The positive interaction term in Column (4) indicates assortive matching; highly ranked men are especially likely to match with highly ranked women and vice versa.

Finally, we find that 23% of the variance in yes/no outcomes in individual meetings can be explained by the average matchability of the meeting partners, as measured by the \( R^2 \) of the regression. This number supports our result that attraction is predominantly subjective, but still admits clear objective patterns to be discerned.

We included second-order terms in the regressions to account for possible non-linear effects (reported in columns (2)–(4) of Table 8). Some coefficients are statistically significant, but they do not meaningfully change the estimated slope coefficients, nor the fit of the selectivity regression (Columns (1) and (2)). The interaction term in Column (4), on the other hand, is significant both in statistical and scientific terms. We also ran the regressions separately for each gender, and could not reject the null hypothesis of no gender difference.

Our results suggest a possible resolution to a debate in the literature on “general reciprocity”, the question of whether people who are (or appear) more willing to match with others are in turn considered more likable or matchable by others,
summarized by Luo and Zhang (2009). First, we should expect general reciprocity to be negative overall due to strategic differences in selectivity: people with high matching value will receive more romantic interest and signal less. Second, however, one might ask how people’s willingness to match with others affects their own matchability conditional on other things that determine matching value, such as physical attractiveness, high earnings potential, or a sense of humor. We think it is an open question whether such “conditional general reciprocity” is still negative, or becomes zero or even positive, assuming one is able to properly control for all other relevant traits, and an important topic for future research.

However, a potential weakness of our analysis is that we would like to interpret a participant’s ‘likelihood to say yes’ as a subjective but unbiased judgment of a partner’s general matching value, and that only the ‘yes’ decision is strategic but the ‘likelihood to say yes’ is not. This is not the case: Eastwick et al. (2007) demonstrated that ‘liking’ exhibits negative general reciprocity, which implies that it might be already subject to the same selectivity pattern that we find for the actual ‘yes’. In other words, ‘likelihood to say yes’ is halfway between a ‘like’ and a ‘yes’.

We have reason to believe this concern is not too severe. The ‘likely to say yes’ question was answered right after every interaction. After a single interaction, a participant had not finished meeting their other partners and there was no direct reminder of previous interactions, which may have made it less likely for a participant to assess their attraction to other participants strategically. The actual ‘yes’ decision was made after the event, after the participants had returned home. They logged into the NSDS website and checked either ‘yes’ or ‘no’ next to the photograph of each partner to indicate whether they would be interested in seeing that person again, a decision process which is more obviously strategic.

Our second response to this concern is that the goal of our analysis is not merely qualitative (are people considered attractive more selective?), but also quantitative (how much more so?), and to compare the quantities from our empirical analysis with the model prediction. The only question is therefore whether subjectively stated likelihood to receive ‘yeses’ is a valid measure of overall attractiveness. We think it is, especially because we have reason to believe that observed matching success would also be a consistent measure of overall attractiveness. The advantage of the “likelihood to say ‘yes’” variable is that it is more direct – collected right after each meeting – and also more granular, being measured on a 1–9 scale rather than as a dichotomous outcome.21

Finally, one might be concerned that the assumption of a one-sided rejection rule (which Fisman et al., 2006, call “straightforward behavior”) is not valid. In particular, the participants may want to avoid saying ‘yes’ to someone who does not say ‘yes’ in return; there is no physical cost of saying ‘yes’ in the study, but there could be an emotional cost of being rejected. We test this hypothesis by augmenting the main regression from Column (1) of Table 8 with the subjective evaluation of the target’s matchability by the rater. Ideally, this variable should summarize everything we need to know about the rater’s decision, bringing the coefficients on the rater’s and target’s vertical to zero. However, one way to avoid saying ‘yes’ to targets who will not reciprocate is to reject targets who are highly rated by other participants – in other words, those who have a high vertical score. If this is the case, we expect to see a positive coefficient on the target’s subjective evaluation of the rater, but a negative coefficient on the target’s vertical score.

We show the result in Table 12 in Appendix A.4. As expected, we obtain a strongly positive coefficient on the subjective evaluation, the $R^2$ doubles, and the coefficient on the target’s vertical score falls to less than a third of its earlier value. However, it does not become negative, but rather stays positive and significant.22 This result is inconsistent with a strong fear of rejection, and we infer that participants probably did not reject people whom they liked for fear of being out of their league.

4. Quantitative analysis of the model

4.1. Maximum likelihood estimation

Our model contains four parameters: the reduced-form discount rate $r > 0$ (which also captures search frictions), the utility $b > 0$ of being single forever, the parameter $s \in (0, 1)$ which represents the amount of disagreement between members of one gender in rating the attractiveness of members of the other gender, and the parameter $c \in [-1, 1]$ which measures the degree to which horizontal attraction is mutual.

We estimate the model by choosing the parameters ($r$, $b$, $s$, $c$) to maximize the log-likelihood function:

\[
L = \sum_{n=1}^{2050} Y^F_n Y^M_n \log \left( \int_{\mathbb{R}^2} \Phi[a^F_{vn} + a^M_{vn} - R(a^M_{vn})] \Phi[a^M_{vn} + a^F_{vn} - R(a^F_{vn})] dH \right) \\
+ (1 - Y^F_n) Y^M_n \log \left( \int_{\mathbb{R}^2} \Phi[a^F_{vn} + a^M_{vn} - R(a^M_{vn})] \Phi[-a^M_{vn} - a^F_{vn} + R(a^F_{vn})] dH \right)
\]

21 This is analogous to a well-known result from sports analytics: if one wants to predict the future wins of a team, one should use the team’s current scoring differential rather than the team’s current win-loss record. Why? Because scoring is more granular than wins and therefore less affected by noise.

22 So, why is the coefficient positive rather than zero? We think it has to do with the fact that as discussed earlier in this section, participants’ strategic thinking already influences the “likelihood to say yes” evaluation. Highly matchable participants are more likely to give their partners scores that are “too low”, and vice versa. Another possible explanation is that the “likelihood to say yes” evaluation is subject to noise that the actual ‘yes’ decision is not, which appears less plausible but we cannot rule it out.
\[ Y_n^f (1 - Y_n^M) \log \left( \int_{\mathbb{R}^2} \Phi[-a_{vn} - a_{vn}^{MF} + R(a_{vn})] \Phi[a_{vn} + a_{vn}^{MF} - R(a_{vn})] dH \right) + (1 - Y_n^f) (1 - Y_n^M) \log \left( \int_{\mathbb{R}^2} \Phi[-a_{vn}^{c} - a_{vn}^{MF} + R(a_{vn})] \Phi[-a_{vn} + a_{vn}^{MF} + R(a_{vn})] dH \right) \]

where \( n \) counts every meeting once. \( Y_n^f \) equals 1 if the woman said ‘yes’ and 0 if not, and \( Y_n^M \) equals 1 if the man said ‘yes’ and 0 if not. The terms \( a_{vn}^{c} \) and \( a_{vn}^{MF} \) are the vertical types of the woman and the man in meeting \( n \), respectively, \( a_{vn}^{MF} \) and \( a_{vn}^{c} \) are their (correlated) subjective attraction terms, and \( R(\cdot) = \log V(\cdot) \) is the estimated rejection threshold function, equal to the natural logarithm of the value function. The term \( dH \) is short for \( dH(a_{vn}^{MF}, a_{vn}^{c}) \), the joint density of the subjective attraction terms (defined on page 2; its parameters are \( s \) and \( c \)); the integrals are computed numerically.

We obtain the following estimate of the model parameters (and in parentheses the standard errors estimated from the Hessian of \( L \)):

<table>
<thead>
<tr>
<th>( r )</th>
<th>( b )</th>
<th>( s )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.66</td>
<td>0.85</td>
<td>0.67</td>
<td>0.30</td>
</tr>
<tr>
<td>(0.378)</td>
<td>(0.059)</td>
<td>(0.015)</td>
<td>(0.039)</td>
</tr>
</tbody>
</table>

### 4.2. Analysis of the estimated model

As we can see, the parameters \( b, s \) and \( c \) which summarize preferences over potential partners, are quite precisely identified by the observed matching decisions. However, the parameters \( s \) and \( c \) have direct counterparts in the stated preferences, and those differ a bit from the decision-based (i.e. revealed preference) estimates. As explained in Section 3, we use the variable “likely to say yes” to a meeting partner to measure overall attraction, and we measure that 32% of the variance of this variable can be attributed to the variance of the per-person averages (i.e. the vertical component). This would suggest that \( s = 0.68 \), a perfect match for the decision-based estimate. However, on the one hand, calculating the per-person average from only 9–13 observations is noisy, and we expect the vertical share of the variance to be biased upward for that reason. In Appendix A.2, we describe a bias correction procedure which yields \( s = 0.745 \). On the other hand, if some of the variation in subjective ratings reflects measurement error rather than subjective preferences, then the vertical share of the variance would be biased downward. We cannot quantify this second bias, but taking the outcome-based and the stated-preference estimates of \( s \) as bounds, we conclude that between two thirds and three quarters of of overall attraction is “in the eye of the beholder”.

The fact that preferences are neither completely common nor completely idiosyncratic is important because a substantial part of existing matching theory has focused on one of these two corners. Here, whether we use stated or revealed preferences, the null hypotheses of \( s = 0 \) or \( s = 1 \) (which correspond to the models of Burdett and Coles, 1997, and Burdett and Wright, 1998) are statistically rejected; furthermore, as we show in Appendix A.3 using counterfactual simulations, the cases \( s = 0 \) or \( s = 1 \) produce decisions and matching outcomes that are at odds with those we see in the data. Consequently, future models of matching (whether search based or frictionless) should incorporate mixed preferences.

In Table 6, we computed that the correlation of horizontal attraction between partners in a meeting is 0.081, much smaller than the 0.3 needed to rationalize the matching decisions. But unlike for \( s \), measurement error and the small-sample bias both point in the same direction: our direct measure of \( c \) is too low. The small sample bias occurs because the averaging tends to attribute too much of an individual’s attraction to the vertical type of the partner, and therefore the horizontal correlation will be biased down. However, the correction only gets us to \( c = 0.087 \), which according to the maximum likelihood estimate is still too small to explain decisions. This suggests that measurement error in the individual attraction rating is considerable. (Though the measurement error of a participant’s vertical type should be much smaller after averaging between raters, which is important for the validity both of our results in Table 6 and our maximum likelihood estimation.)

Again, however, it is important for future research that the null hypothesis of \( c = 0 \) is rejected both by stated and revealed preferences. For example, many asymptotic results in the frictionless matching literature about the stability and efficiency of a matching rely on independent preference shocks between potential partners, and our results indicate that this assumption is not satisfied in the context of romantic attraction.

Our estimate of the parameter \( r \), which in reduced form combines time preference with the strength of the search friction, is more noisy, with an approximate confidence interval of [0.9, 2.4]. According to the structural model, we can interpret \( r \) as the ratio of time preference plus the breakup rate, divided by the meeting rate. If time preference is negligible compared to the other two rates, then \( r \) is simply the breakup rate divided by the meeting rate; or, equivalently, it is the average length of time between meeting potential partners divided by the average duration of a relationship. And using our estimated confidence interval of \( r \), we conclude that these two durations have a comparable order of magnitude, unless the participants were exceptionally impatient. Certainly, the “relationships” that the NSDS was able to accomplish varied a lot in both length and depth. An average length on the order of weeks or months for the length of a speed-dating initiated
relationship, and 1–2 times as many weeks/months for how long it would take our participants to meet another potential partner, is as reasonable a guess as any. We conclude that the fairly high value of $r$ is evidence that search frictions are substantial.

The parameter $b$ is the value of being single forever, and $\Phi(\log(b)) = 44\%$ is the rejection rate of an agent who expects to never match and therefore has no option value of waiting. This suggests an interesting counterfactual: what if there was no strategic value of saying "no", say because two of our agents are stranded on a deserted island where they can either match with the other person or nobody else? Our estimates imply that each would say ‘yes’ with probability 56%, and they would match with a probability of 35% (which is slightly higher than the square of 56% due to mutual attraction). This does not depend on the vertical type of the stranded agents; how one might be evaluated by an outside option is meaningless if there is no outside option. The maximum-likelihood matching rate is only 22%; therefore, in other words, strategic delay reduces the matching probability by thirteen percentage points.

Alternatively, what if $s = 1$, so there is no vertical component of preferences and preferences are purely subjective (but still mutual with $c = 0.3$)? We can compute that in this counterfactual world, people will say ‘yes’ to matching with probability 44%, and a meeting would become a ‘match’ with probability 24%. This is a smaller ‘yes’ rate but a larger matching probability than we observe, which suggests that strategic rejection – pithily encapsulated by Groucho Marx’s remark that he would refuse to join any club that would have him as a member – is important but not overwhelmingly so in the dating environment.

We can compare these numbers to those from a third counterfactual: what if subjective attraction was not mutual, so that strategic rejection was the only operative force? We can compute this by setting $c$ to zero and leaving the other parameters unchanged. In this counterfactual world, we would see the average matching rate fall to 17%, five percentage points lower than the maximum-likelihood matching rate. We conclude that both forces, strategic rejection and correlated attraction, are empirically relevant in producing the matching outcomes we observe.

4.2.1. Decisions and matching outcomes

More attractive people are clearly pickier – but not overwhelmingly so, because so much of people’s attraction to each other is horizontal. The fit of the model prediction compared to the smoothed average of participants’ decisions is very good (Fig. 2). One might think that the substantial dispersion around the fit lines is an argument against our model, but this is not the case. First, compare the slight downward slope with the coefficient on the rater’s vertical type in Table 8, which is $-0.036$ and tightly estimated. It is small but meaningful, as it confirms that the NSDS participants were making their ‘yes’ decisions strategically. Furthermore, due to the fact that all participants were meeting a sample of 9–13 partners, we would expect there to be sampling variation in the pattern of decisions, and as we show in Section 4.3 below, a simulation of the estimated model predicts a very similar amount of sampling variation as we actually see in Table 8 and Figs. 2 and 3.

One of the most interesting predictions of our model concerns the matching rates for agents of a given gender and attractiveness. In Fig. 3, we show the matching rates for agents of a given vertical component of attractiveness. The first observation is that the matching rate is monotonically increasing, although the model does not require this: more
attractive people might in principle trade off higher match quality for lower matching rates. (Unlike the yes rate, which by Lemma 1 must be monotonically decreasing.) However, our result is intuitive. More attractive people are more selective and still have higher matching rates on average.

The second observation is, again, the substantial amount of variation around this average. Some very highly ranked people left the study without a match, whereas others matched with 8 or 10 potential dates. What we really want to know is whether this variation is explained by the strongly subjective nature of preferences, i.e. sampling variation in who the participants were meeting, or factors that our model does not account for. Answering this question is the main purpose of the simulation in Section 4.3.

4.2.2. Sorting

In our estimated model, as in the data, matching tends to be assortative in the rank-based sense, but only weakly so. There are two possible approaches to measuring this. First, we can use the estimated model to compute what the correlation between the vertical types of matched partners would be in an infinite sample. We obtain +0.24, which is higher than the +0.15 we obtain in the data, but this is not a fair comparison because the samples in the data are small. Therefore, in Section 4.3 below, we simulate the small sample nature of our data, and the simulation shows that the range of sorting correlations consistent with our model is wide but it well overlaps with the range of estimates implied by the data.

This weak sorting is partly due to the search friction. With lower values of \( r \), equivalent to more patience, longer relationship duration, or a higher meeting rate, the sorting correlation would get stronger as agents would wait more patiently for the ‘perfect’ match. But it would not approach 1, because the subjective component of preferences is so strong that even if everyone was matched with their ‘perfect’ partner, different people would not agree on who that person is.

It is important to keep in mind that we measured rank-based sorting based on the vertical component of overall attraction, not any particular feature of attractiveness such as wealth, looks, or shared interests. Strictly following our theory, we expect that sorting on any such a feature will be even weaker, because no real-world characteristic is purely vertical, with income or wealth probably coming the closest. However, two things could lead to stronger sorting empirically. One is the fact that it is easier to meet people with whom one already has some things in common, for example education or ethnicity. Second, if utility is partially transferable (which is more likely in long-term relationships than in short ones), then sorting on any particular variable becomes delinked from sorting on overall match value, because partners can compensate each other.

Just as we find in the data, the estimated model predicts that matched couples should have a higher mutual attraction to each other than people in general, and even more strongly so compared to people who did not agree to form a match (Table 9). In the estimated model, the correlations are larger than in the data, but the spreads go in the same direction.

4.3. Simulation of the study using the estimated model

We simulate the model by replicating the NSDS set-up closely. For each ‘study’, we simulate one event of size 9 × 9, five events of size 11 × 11, seven events of size 12 × 12, and two events of size 13 × 13, for a total of fifteen events with
346 participants and 2,032 interactions. (The totals differ slightly because a few events were unbalanced in the NSDS.) For each ‘event’, we model the respective number of male and female agents, and randomly draw their vertical types $a_V$ and horizontal preferences $a_H$. Then, we compare agents’ attraction to others with their rejection thresholds $\log V(a_V)$ and compute ‘yes’ decisions and ‘match’ outcomes. Finally, we estimate the equivalent of Eq. (2) for the entire ‘study’, collect coefficients of interest, and also compute the correlation coefficient between the vertical types of matched partners (which measures rank-based sorting).

We can measure ‘matchability’, the vertical component of attraction, in two ways in the simulation. First, we know what the “true” value of $a_V$ is within the simulation, so we can use that directly in the regression. However, in the data, we do not know this value but have to estimate it from a small sample of subjective ratings. In order to keep the simulation as analogous to the data as possible, we therefore estimate matchability within each simulated ‘study’ as the mean of reported match value scores $a_V + a_H$.

We simulate 999 such ‘studies’ and report the results in Table 10. We focus on the slope coefficients of the rater’s and target’s standardized matchability (corresponding to $\beta_1$ and $\beta_3$ in Eq. (2)), as well as the $R^2$ from the regression, and the sorting correlation. The confidence intervals of the regression coefficients are taken from the regression results (clustering on the target). The confidence interval of the estimated sorting correlation is constructed using the Fisher transformation, and the confidence interval of the estimated $R^2$ is constructed using a Fisher transformation of the square root of the $R^2$, which is a correlation coefficient. The confidence intervals of the simulation are the 2.5th and 97.5th percentile of the simulated results.

Our results indicate that the model performs well in matching the data. All four confidence intervals well overlap, and the estimates are also very close in practical terms. The fact that the estimated $R^2$ from the regression is so close to its simulated counterpart suggests that the difference between observed matching outcomes and the central prediction of the model (Fig. 2) is consistent with sampling error.

### 5. Conclusion

We have proposed a simple model of decision making in an environment with heterogeneous preferences and search frictions, and we have used data from the Northwestern Speed Dating Study to (a) gather direct evidence on the structure of partner preferences, and (b) estimate the model. Concerning preferences, we conclude that people’s preferences over romantic partners are predominantly subjective but correlated both within and across gender, and that matching is assortative on both vertical and horizontal characteristics (and neither type of sorting clearly dominates).

The model environment, taken literally, is clearly not the same as the speed dating environment. In reality, participants made decisions simultaneously rather than sequentially, and were able to match with more than one partner; we have discussed these issues in Section 2.4. On the other hand, our model is parsimonious, containing only four parameters, and three of the parameters are structural. The model captures the ingredients of the decision making process which we think are essential: there is limited capacity, there is therefore an option value of waiting, and this option value (and therefore the decision) depends on the structure of preferences and on everybody else’s decisions. We conclude theoretically that the decision to accept or reject should depend on how much people agree on their evaluations of others, including who agrees with whom, and on how high one’s own status is in the vertical dimension.

And even with its limitations, our simple search model of matching under frictions fits the data exceptionally well. We conclude empirically that people may have a sophisticated understanding of the structure of preferences, of the search frictions in the environment, and of their own option set when making the decision of whether to pair up or to wait.
Acknowledgments

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Appendix A

A.1. Proofs

Proof of Lemma 1. Note that the Bellman Equation (Eq. (1)) is degenerate as an agent’s type never changes. So for each \( a_v \), we need to show that \( V^\ell(a_v) \) is a unique number. The left-hand side of the Bellman equation is strictly increasing as a function of \( V^\ell \in \mathbb{R} \), and spans \((0, \infty)\). The integrals on the right-hand-side exist given that \( V^M \) is assumed to be a continuous function. The surplus \( S^f \) is clearly non-increasing in \( V^F \) and bounded below by 0; hence, the entire right-hand side is non-increasing and bounded below by \( rb \). As a result, there exists a unique intersection that determines \( V^\ell(a_v) \).

Next, consider that the agent’s own type, \( a_v \), only appears once on the right-hand-side, in the boundary of the integration set \( \mathbb{H} \). As the integral operator is continuity preserving, the right-hand side changes continuously in \( a_v \). Both sides are clearly continuous in the number \( V^\ell(a_v) \), which establishes that \( V^F \) is a continuous function of \( a_v \).

The fact that \( V^F \) is a nondecreasing function is also straightforward, because an increase in \( a_v \) must make the condition \( a_v + a_H \geq \log V^M(a_v') \) (weakly) easier to satisfy for every \( a_v' \). To see that the monotonicity is strict, consider first that for all finite values of \( \log V^F(a_v) \), the probability is non-zero that \( a_v' + a_H' \) is large enough to deliver positive match surplus. Second, as \( V^M(a_v') \) is finite for any \( a_v' \), and as \( a_H' \) is normally distributed, the probability that \( a_v + a_H \geq \log V^M(a_v') \) holds is always strictly between 0 and 1. So as \( a_v \) increases, the right-hand side of the Bellman equation increases and so does the solution \( V^F(a_v) \).

Finally, assume that \( a_v \) approaches negative infinity; in that case, the probability of \( a_v + a_H \geq \log V^M(a_v') \) converges to zero because, again, \( \log V^M(a_v') \) is finite and \( a_H' \) is normally distributed. Consequently, the entire partial expectation converges to zero and \( V^F(a_v) \) converges to \( b \). Conversely, suppose that \( a_v \) approaches positive infinity. In that case, the probability of \( a_v + a_H \geq \log V^M(a_v') \) converges to one, and \( V^F(a_v) \) therefore converges to \( V \) which solves:

\[
\begin{align*}
rb &+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max \left\{ e^{a_v + a_H - \bar{V}}, 0 \right\} \sigma d\Phi \left( \frac{a_H}{\sqrt{s}} \right) d\Phi \left( \frac{a_v}{\sqrt{1 - s}} \right).
\end{align*}
\]

Because \( a_v' \) and \( a_H' \) are normal, independent, and their variances sum to 1, we can apply the text:formula for the partial expectation of a standard log-normal random variable to obtain the equation in the Lemma.

Proof of Proposition 1. First, note that since \( a_H' \) and \( a_H \) are jointly normal with correlation \( c \) and each having variance \( s \), we can orthogonalize them as three independent normal random variables:

\[
\begin{align*}
a_z &\sim \mathcal{N}(0, (1-c)s), \quad a_5 \sim \mathcal{N}(0, (1-c)s), \quad \text{and} \quad a_7 \sim \mathcal{N}(0, cs), \quad \text{such that} \quad a_H' = a_z' + a_j \quad \text{and} \quad a_H = a_5 + a_j
\end{align*}
\]

(The letters S and J stand for “separate” and “joint”.)

This will make the rest easier; even as we are now using an extra random variable, all remaining random variables are independent of each other. To begin with, consider Eq. (1) for female agents (F) who are meeting female agents (M). Assuming that the men’s value \( V^M \) is a continuous function, we can write the women’s Bellman equation as:

\[
\begin{align*}
rV^F(a_v) &= rb + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S^F d\Phi \left( \frac{a_z'}{\sqrt{(1-c)s}} \right) d\Phi \left( \frac{a_5}{\sqrt{cs}} \right) d\Phi \left( \frac{a_j}{\sqrt{1-s}} \right)
\end{align*}
\]

\[
S^F = \max \left\{ \exp(a_v' + a_z' + a_j) - V^F(a_v'), 0 \right\}
\]

\[
H = \{(a_z', a_5, a_j) : a_z' + a_j \geq \log[V^F(a_v)] - a_v' \text{ and } a_5 + a_j \geq \log[V^M(a_v') - a_v] \}
\]

where the integration is now over four random variables: \( a_v', a_z', a_5, \text{ and } a_j \), all of which are independent. As \( a_z \) appears only once in the match constraint but not in the surplus – we can easily integrate it out, and re-order terms to obtain:

\[
\begin{align*}
rV^F(a_v) &= rb + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max \left\{ e^{a_v + a_j - \log V^M(a_v')} - a_v', 0 \right\} \sigma d\Phi \left( \frac{a_z'}{\sqrt{(1-c)s}} \right) d\Phi \left( \frac{a_j}{\sqrt{cs}} \right) d\Phi \left( \frac{a_v'}{\sqrt{1-s}} \right)
\end{align*}
\]
Or, we can write it more concisely using the expectations operator, where the expectation is over variables $a'_v, a'_s$, and $a_I$. The expectation can be written as unconstrained: the lower bound on $a'_s$ is redundant because the match surplus is zero when $a'_s$ is too low.

$$rV^F(a_v) = rb + \mathbb{E} \left\{ \max \left\{ e^{a'_v + a'_s + a_I} - V^F(a_v), 0 \right\} \cdot \Phi \left( \frac{a_v + a_I - \log V^M(a'_v)}{\sqrt{(1 - c)s}} \right) \right\}$$

(3)

Because by Lemma 1, Eq. (3) defines a unique bounded and continuous function $V^F$ for every continuous function $V^M$, the equation describes a continuous operator on the space of continuous and bounded functions. We call this functional operator $T^F$.

A symmetric equation exists to define a function $V^M$ for a given $V^F$, therefore that equation describes a functional operator $T^M$ analogous to $T^F$. We claim that the concatenation of the two operators, the operator $T = T^M \circ T^F$ which takes a particular value $V^M$ of the men, then has the women best respond, then has the men best respond in turn to yield a new $V^M$, is a contraction mapping on the space of bounded and continuous functions.

To prove existence of an equilibrium, we need to show that $T$ has a fixed point. We have already shown that $T$ defines a continuous mapping on a compact and convex subset of a vector space (the space of bounded continuous functions). We can also show that $T$ satisfies the monotonicity property: assume that there are two bounded continuous functions $V^M$ and $V^M_2$, and that $V^M_1 \geq V^M_2$. Clearly the right-hand side of Eq. (3) is lower (or equal) using $V^M_1$ compared to $V^M_2$ for every value $a_v$, therefore, $T^F(V^M_1) \leq T^F(V^M_2)$. (In intuitive terms: if all men are at least as picky or become pickier, all women will become (weakly) less picky.) The same is true in reverse, therefore $T^M[T^F(V^M)] \geq T^M[T^F(V^M_2)]$, which establishes monotonicity. Consequently, by Tarski’s fixed point theorem, $T$ has at least one fixed point. Furthermore, iterating $T$ on the constant function $V_0(x) = V$ results in convergence to the lowest fixed point, and iterating $T$ on the constant function $V_1(x) = V$ results in convergence to the highest fixed point.

To prove uniqueness, we show that if the parameters of the model satisfy a certain condition, $T$ is a contraction mapping. Of Blackwell’s sufficient conditions we have already shown monotonicity, so only discounting remains. Fix a constant $C \in (0, \infty)$, and replace $V^M(a'_v)$ in Eq. (3) with $V^M(a'_v) + C$. Suppressing the argument $a_v$ of $V^F$, we get:

$$rV^F = rb + \mathbb{E} \left\{ \max \left\{ e^{a'_v + a'_s + a_I} - V^F, 0 \right\} \cdot \Phi \left( \frac{a_v + a_I - \log \left[ V^M(a'_v) + C \right]}{\sqrt{(1 - c)s}} \right) \right\}$$

Next, for every $a_v$, take the implicit derivative of $V^F$ with respect to $C$. The derivative of the left-hand side with respect to $V^F$ is simply $r$, and the derivative of the right-hand side with respect to $V^F$ is:

$$\frac{d \text{RHS}}{dV^F} = -\mathbb{E} \left\{ \max \left\{ e^{a'_v + a'_s + a_I} - V^F, 0 \right\} \cdot \Phi \left( \frac{a_v + a_I - \log \left[ V^M(a'_v) + C \right]}{\sqrt{(1 - c)s}} \right) \right\} \in (-1, 0),$$

which is between $-1$ and 0, because the expectation is just the matching probability for this agent (the probability that he likes a woman who also likes him back). And the derivative of the right-hand side with respect to C is:

$$\frac{d \text{RHS}}{dC} = \frac{-1}{\sqrt{V^M(a'_v) + C}} \cdot \mathbb{E} \left\{ \max \left\{ 0, e^{a'_v + a'_s + a_I} - V^F \right\} \cdot \Phi \left( \frac{a_v + a_I - \log \left[ V^M(a'_v) + C \right]}{\sqrt{(1 - c)s}} \right) \right\}$$

The $\Phi(\cdot)$-term is the density of a standard normal distribution; it is positive and bounded above by its value at the peak, which is $1/\sqrt{\pi} \approx 0.4$ (where $\tau = 2\pi = 6.28 \ldots$ is the circle constant). Therefore, the term inside the expectation is less than:

$$\mathbb{E} \left\{ \max \left\{ 0, e^{a'_v + a'_s + a_I} - V^F \right\} \right\} \frac{1}{\sqrt{\tau}},$$

which is in turn less than $\mathbb{E} \left\{ \max \left\{ 0, \exp(a'_v + a'_s + a_I) - b \right\} \right\}/\sqrt{\tau}$ because $b$ is the reservation value of the least attractive woman. Using the formula for the partial expectation of a standard log-normal random variable, we can compute that this latter expectation equals:

$$\sqrt{\tau} \Phi[1 - \log(b)] - b \Phi[-\log(b)].$$

Summing up: the implicit derivative of Eq. (3) tells us that

$$\frac{dV^F}{dC} = \frac{d \text{RHS}}{dC} = \frac{d \text{RHS}}{dV^F} \cdot \frac{dV^F}{dC},$$

and by applying all the results and inequalities derived earlier, we can sign:

$$0 > \frac{dV^F}{dC} > -\frac{\sqrt{\tau} \Phi[1 - \log(b)] - b \Phi[-\log(b)]}{(r + \mathbb{P} \{\text{match}\})(V^M(a'_v) + C)\sqrt{(1 - c)s \tau}}$$
and further, using that the match probability is positive and $V^M + C > V^M > b$, we obtain:

$$0 > \frac{dV^F}{dC} > -\frac{\sqrt{C} \Phi[1 - \log(b)] - b \Phi[-\log(b)]}{r b \sqrt{(1 - C) \tau}}$$

Hence, a sufficient condition for uniqueness is that $G(b) < r \sqrt{(1 - C) s}$, where we define:

$$G(b) = \frac{1}{\sqrt{\tau}} \left( \frac{\sqrt{C} \Phi[1 - \log(b)] - \Phi[-\log(b)]}{b} \right)$$

If the sufficient condition is satisfied, as we assumed in the proposition, then the derivative $dV^F/dC$ is strictly between -1 and 0. Hence, we conclude that the values of the function $T^F (V^M + C) - V^F$ are negative and bounded away from $-C$, which establishes a ‘reflected’ form of the discounting condition.

Because the women’s problem is exactly symmetric to the men’s, the same result holds there, and we can conclude that the values of the function $T^M [T^F (V^M + C) - V^M]$ are positive and bounded by $G(b)^2/[r^2 (1 - c) \tau] < C$, which establishes Blackwell’s discounting condition. $T = T^M \circ T^F$ is therefore a contraction mapping with a unique fixed point in the space of bounded continuous functions. Applying the operator $T$ repeatedly to any continuous function will result in convergence, and we exploit this fact in our numerical algorithm. At the maximum likelihood estimate of the model, the worst-case discounting factor equals $G(b)/[r \sqrt{(1 - C) s}] = 0.40$, which is sufficient although the (very rough) inequalities above suggest that the actual rate of convergence may be even faster.

Finally, having established a unique solution, we need to show that this solution is symmetric. Assume that we have a solution which is not symmetric, i.e. an equilibrium where $V^M \neq V^F$ for at least some values; by continuity, this means that $V^M \neq V^F$ on some open set. However, because the men’s and women’s problems are symmetric, we could switch the best responses, assigning $V^M$ to the women and $V^F$ to the men. This must then also be an equilibrium, which contradicts uniqueness.

A.2. The small-sample correction

Because we measure the vertical component of attraction as the average of individual ratings of meeting partners, we will systematically overestimate the share of attraction that is vertical. For example, if we had only a single evaluation of a person, we would have to take it as the best estimate of the target’s vertical type, leaving no variance at all to horizontal factors even if we know they must be present. In our set-up, if participants are evaluated by $n$ meeting partners, we can expect that the true amount of disagreement (s in the model) will be underestimated by a factor of $(n - 1)/n$. Similarly, because our measurement of the horizontal component of attraction will be ‘contaminated’ by the target’s true vertical score, the correlation between the horizontal ratings of partners in a meeting (c in the model) will be underestimated by the same factor $(n - 1)/n$.

We therefore inflate our raw measurements for s (0.68) and c (0.081) by the factor $\hat{n}/(\hat{n} - 1) = 1.0933$, where $\hat{n} = 4100/350 \approx 11.7$ is the average group size in the NSDS. The correction suggests estimates of $c = 0.089$ and $s = 0.745$ (thereby attributing 25.5% of the variance to the vertical component of overall attraction or matching value). This correction is comparable to using an adjusted $R^2$ to evaluate the fit of a regression instead of the raw one, because the raw one is maximized within the regression and will therefore tend to overestimate the true fit of a statistical model.

A.3. Counterfactual analyses

Counterfactual 1: purely vertical preferences

As a counterfactual to the main analysis, we set $s = 0$; this corresponds to the model by Burdett and Coles (1997), where all agents can be ranked by their attractiveness type (“pizzazz”). We keep parameters $r = 1.66$ and $b = 0.85$ unchanged from the benchmark estimate, and as there are no subjective preferences, the parameter $c$ becomes irrelevant. Burdett and Coles showed that in this model, the rejection function is a step function as people endogenously sort into classes. When computing the equilibrium in this model, we find that there are three classes: 48.4% in the top class, who accept everyone in the top class, reject everyone else, and therefore only match with each other; 44% in the bottom class who reject others in the bottom class and who are therefore always rejected themselves; and 7.6% in the middle class who accept everyone in the middle and top classes but are accepted only by themselves and the bottom class. This equilibrium implies an average ‘yes’ rate of 52% and an average matching rate of 24%, not too far from our data. However, when simulating this model, the fact that it is a very bad fit becomes apparent:

<table>
<thead>
<tr>
<th></th>
<th>NSDS estimate</th>
<th>CI of estimate</th>
<th>Simulation mean</th>
<th>CI of simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target’s coefficient</td>
<td>.233</td>
<td>[.220, .246]</td>
<td>.396</td>
<td>[.384, .407]</td>
</tr>
<tr>
<td>Rater’s coefficient</td>
<td>-.036</td>
<td>[-.057, -.014]</td>
<td>-.032</td>
<td>[-.047, -.019]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.229</td>
<td>[.207, .252]</td>
<td>.635</td>
<td>[.600, .688]</td>
</tr>
<tr>
<td>Sorting correlation</td>
<td>.15</td>
<td>[.06, .24]</td>
<td>.05</td>
<td>[.05, .15]</td>
</tr>
</tbody>
</table>
The rater’s coefficients do not fit too badly but the target’s coefficient and the $R^2$ are far too high. Due to the flat rejection threshold in the top half of agents, the implied sorting correlation is too low even with the sorting into distinct classes.

**Counterfactual 2: purely idiosyncratic preferences**

As our second counterfactual, we set $s = 1$ and $c = 0$; this corresponds to the model by Burdett and Wright (1998), where preferences are purely random. Again, we keep parameters $r = 1.66$ and $b = 0.85$ unchanged from the benchmark estimate. In this scenario, the average ‘yes’ rate is 47% and an average matching rate of $(47\%)^2 = 22\%$. Again, these averages are close to our data but simulation reveals a very poor fit of the counterfactual model:

<table>
<thead>
<tr>
<th></th>
<th>NSDS estimate</th>
<th>CI of estimate</th>
<th>Simulation mean</th>
<th>CI of simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target’s coefficient</td>
<td>.233</td>
<td>[.220, .246]</td>
<td>.116</td>
<td>[.104, .128]</td>
</tr>
<tr>
<td>Rater’s coefficient</td>
<td>-.036</td>
<td>[−.057, −.014]</td>
<td>0</td>
<td>[−.015, .015]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.229</td>
<td>[.207, .252]</td>
<td>.055</td>
<td>[.044, .067]</td>
</tr>
<tr>
<td>Sorting correlation</td>
<td>.15</td>
<td>[.06, .24]</td>
<td>0</td>
<td>[−.30, .10]</td>
</tr>
</tbody>
</table>

The rater’s coefficient and the sorting correlation are simulated to be exactly zero on average. The target’s coefficient and the $R^2$ are positive due to bias; a person who gets many ‘yeses’ is also likely to receive high subjective ratings, so we can expect a spurious positive coefficient on average. As we see, however, both the coefficient and the $R^2$ are much smaller than the counterparts in our data, so we have no concern that those are spurious.

**Counterfactual 3: attraction is purely horizontal but mutual**

As our third counterfactual, we set $s = 1$ and $c = .3$. the latter as in our benchmark estimate; this is a variation of the model by Burdett and Wright (1998) where we let preferences be purely subjective but positively correlated within a meeting. Again, we keep parameters $r = 1.66$ and $b = 0.85$ unchanged from the benchmark estimate. In this scenario, the average ‘yes’ rate is 44% and the average matching rate is 24%. Still, in terms of decisions and matching outcomes, the third counterfactual fits no better than the others:

<table>
<thead>
<tr>
<th></th>
<th>NSDS estimate</th>
<th>CI of estimate</th>
<th>Simulation mean</th>
<th>CI of simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target’s coefficient</td>
<td>.233</td>
<td>[.220, .246]</td>
<td>.114</td>
<td>[.102, .127]</td>
</tr>
<tr>
<td>Rater’s coefficient</td>
<td>-.036</td>
<td>[−.057, −.014]</td>
<td>.032</td>
<td>[.018, .048]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.229</td>
<td>[.207, .252]</td>
<td>.058</td>
<td>[.047, .072]</td>
</tr>
<tr>
<td>Sorting correlation</td>
<td>.15</td>
<td>[.06, .24]</td>
<td>0</td>
<td>[−.09, .10]</td>
</tr>
</tbody>
</table>

The results are barely changed from those in Counterfactual 2, except that the rater’s coefficient has now turned positive. From our analysis of these counterfactuals, we conclude that our model of vertical and horizontal preferences has bite. Simpler versions of the model which restrict preferences to be purely vertical or purely horizontal are unable to reproduce the decision and sorting patterns we observe in the data.

**Supplementary material**

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.euroecorev.2018.04.001.

**References**


