Exploring All Bipedal Gaits with a United Simplistic Model

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Introduction

In nature, bipedal animals switch gaits at different speeds to minimize their energy expenditure [1]. The most common bipedal gaits are walking and running. Geyer et al. explored the parameter space of the bipedal SLIP (Spring Loaded Inverted Pendulum) model, showing that it is able to reveal the underlying dynamics of both walking and running with one mechanical system. Rummel et al. investigated regions of *walking* patterns with 2, 3, and 4 humps in the vertical GRFs (Ground Reaction Forces), *level walking*, *running*, and *grounded running*.

However, in the previous SLIP models, the dynamics of the swing leg is replaced by one contact angle. This means, at touch-down events, both limbs have the same contact angle. This limitation confines these models to producing only symmetrical gaits such as *walking* and *running*. Moreover, due to the lack of articulated knee joints in the model, the leg length is set to its uncompressed spring length during swing, some of the realistic solutions at low speeds were ignored.

Besides *walking* and *running*, terrestrial animals in nature use many other bipedal gaits such as *hopping*, *skipping*, and *galloping*. For instance, *skipping* is often used by some primates such as lemurs [4], and *hopping* is often observed in many birds and mammals including crows and kangaroos [5, 6]. Bipedal *galloping* is also adopted by humans in special situations such as fast turning, stairs descending, or in low gravity environments [7, 8].

In order to reveal the relationships among all these gaits, there is still a need to develop a single unified model that can reproduce the motions of all these gaits. For example, as shown in the previous research [3], a smooth transition from walking to running was identified. We are wondering if similar relationships can be extended to other bipedal gaits.

Therefore, in this project, we propose a bipedal model with two compliant legs that is similar to the Geyer SLIP model. Unlike the previous models, we assign different contact angles (angles of attack) for different legs, so that *walking*, *running*, *hopping*, and *skipping/galloping* gaits can be found.

With respect to the model's dynamics, the touch-down and lift-off events are triggered by additional timers whose exact values are found by the nonlinear solver. By conducting a detailed continuation in the selected parameter space (Energy and angles of attack), regions of all the bipedal gaits are revealed.

Methods

The method used in this study is similar to the implementation of the BVP (Boundary Value Problem) [9]. The generalized position and velocity in the model are $\mathbf{q} = [x, y]^T$ and $\dot{\mathbf{q}} = [\dot{x}, \dot{y}]^T$. We select the apex transition as our Poincaré section. Five timing variables $\mathbf{e} = [t_{Ltd}, t_{Llo}, t_{Rtd}, t_{Rlo}, t_{Apx}]^T$ are used to determine the touch-down and lift-off events. To find solutions with arbitrary footfall patterns, the order of these events are not specified, but their values are restricted within the time interval of $[0, t_{Apx}]$. The active parameters in the system $\mathbf{p}$ include the total energy E_t, and two angles of attack (α_l, α_r). Therefore, the dynamics of the model are governed by a set of differential equations (EOM):

$$\ddot{\mathbf{q}} = f\left(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{e}, \mathbf{p}\right). \tag{1}$$

To identify all regions of periodic solutions from this model, we have conducted a multidimensional continuation in the parameter space. That is, we uniformly mesh the whole parameter space spanned by E_t, α_l, and α_r. For each periodic solution from this model, we conduct Floquet Analysis on the Poincaré map, and create candidate solutions in all nearby grids. The initial guesses of the candidates are based on the given periodic solution as well as the prediction of the corresponding eigenvectors. Then we run the root search to see if the candidate solution is periodic.

When the borders of regions that connects different gaits are reached, we run separate instances of continuation algorithm to track the bifurcations that are characterized by having two "+1" Floquet multipliers that correspond to the two continuous states: $\dot{x}$ and y. The model and the searching algorithm are implemented in MATLAB. The root search is solved numerically with an accuracy of 10^{-9}.

Results & Discussion

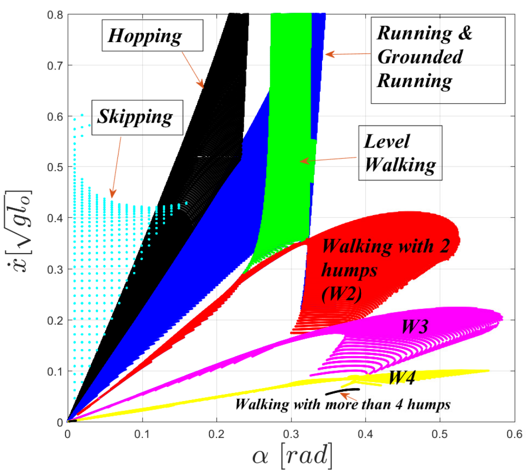

Figure 1: All the periodic solutions of the proposed SLIP model are shown in this figure. They include multiple walking and running gaits, and several new gaits like hopping and skipping/galloping.

From the current results, we have successfully reproduced all solutions found in previous research in addition to several new bipedal gaits as shown in Fig. 1. The manifold of bipedal hopping has very similar shape of the running solution. Skipping gait is found as a pitch-fork bifurcation from the hopping manifold. In this gait, one leg strikes the ground first as the trailing leg and the other one touches the ground in the front as the leading leg. The grounded running and the level walking are essentially the same gait that reside in the different regions of the same surface. One side of this surface joins to the running through saddle-node bifurcations and the other side of this surface is directly connected to walking with 2 humps in vertical GRFs.

Different from the solutions reported in the previous literature, our model suggests that the walking regions with 2, 3, and 4 maxima (W2, W3, and W4) in the vertical GRFs can be extended to lower speed regions. Eventually, all these solutions join together at zero speed and become the hopping in-place gaits.

We found that all these bipedal gaits are just different oscillation modes of the same mechanical system. These modes of solutions live on multiple manifolds on the same Poincaré section, and they are connected at multiple landmark solutions where we see smooth transitions from one mode to another. These results may help understanding the relationships among bipedal gaits in nature, and designing controllers for efficient gait transitions in legged robots. Hence, this model has the potential to serve as the general template for the study of bipedal locomotion.

References

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