

A Guide to Casino Mathematics

Robert C. Hannum
University of Denver

This guide contains a brief, non-technical discussion of the basic mathematics governing casino games and shows how casinos make money from these games. The article addresses a variety of topics, including house advantage, confusion about win rates, game volatility, player value and comp policies, casino pricing mistakes, and regulatory issues. Statistical advantages associated with the major games are also provided.

Introduction

At its core the business of casino gaming is pretty simple. Casinos make money on their games because of the mathematics behind the games. As Nico Zographos, dealer-extraordinaire for the 'Greek Syndicate' in Deauville, Cannes, and Monte Carlo in the 1920s observed about casino gaming: "There is no such thing as luck. It is all mathematics."

With a few notable exceptions, the house always wins – in the long run – because of the mathematical advantage the casino enjoys over the player. That is what Mario Puzo was referring to in his famous novel *Fools Die* when his fictional casino boss character, Gronevelt, commented: "Percentages never lie. We built all these hotels on percentages. We stay rich on the percentage. You can lose faith in everything, religion and God, women and love, good and evil, war and peace. You name it. But the percentage will always stand fast."

Puzo is, of course, right on the money about casino gaming. Without the "edge," casinos would not exist. With this edge, and because of a famous mathematical result called the law of large numbers, a casino is guaranteed to win in the long run.

Why is Mathematics Important?

Critics of the gaming industry have long accused it of creating the name "gaming" and using this as more politically correct than calling itself the "gambling industry." The term "gaming," however, has been around for centuries and more accurately describes the operators' view of the industry because most often casino operators are not gambling. Instead, they rely on mathematical principles to assure that their establishment generates positive gross gaming revenues. The operator, however, must assure the gaming revenues are sufficient to cover deductions like bad debts, expenses, employees, taxes and interest.

Despite the obvious, many casino professionals limit their advancements by failing to understand the basic mathematics of the games and their relationships to casino profitability. One casino owner would often test his pit bosses by asking how a casino could make money on blackjack if the outcome is determined simply by whether the player or the dealer came closest to 21. The answer, typically, was because the casino maintained "a house advantage." This was fair enough, but many could not identify the amount of that advantage or what aspect of the game created the advantage. Given that products offered by casinos are games, managers must understand why the games provide the expected revenues. In the gaming industry, nothing plays a more important role than mathematics.

Mathematics should also overcome the dangers of superstitions. An owner of a major Las Vegas strip casino once experienced a streak of losing substantial amounts of money to a few “high rollers.” He did not attribute this losing streak to normal volatility in the games, but to bad luck. His solution was simple. He spent the evening spreading salt throughout the casino to ward off the bad spirits. Before attributing this example to the idiosyncrasies of one owner, his are atypical only in their extreme. Superstition has long been a part of gambling – from both sides of the table. Superstitions can lead to irrational decisions that may hurt casino profits. For example, believing that a particular dealer is unlucky against a particular (winning) player may lead to a decision to change dealers. As many, if not most, players are superstitious. At best, he may resent that the casino is trying to change his luck. At worst, the player may feel the new dealer is skilled in methods to “cool” the game. Perhaps he is even familiar with stories of old where casinos employed dealers to cheat “lucky” players.

Understanding the mathematics of a game also is important for the casino operator to ensure that the reasonable expectations of the players are met. For most persons, gambling is entertainment. It provides an outlet for adult play. As such, persons have the opportunity for a pleasant diversion from ordinary life and from societal and personal pressures. As an entertainment alternative, however, players may consider the value of the gambling experience. For example, some people may have the option of either spending a hundred dollars during an evening by going to a professional basketball game or at a licensed casino. If the house advantage is too strong and the person loses his money too quickly, he may not value that casino entertainment experience. On the other hand, if a casino can entertain him for an evening, and he enjoys a “complimentary” meal or drinks, he may want to repeat the experience, even over a professional basketball game. Likewise, new casino games themselves may succeed or fail based on player expectations. In recent years, casinos have debuted a variety of new games that attempt to garner player interest and keep their attention. Regardless of whether a game is fun or interesting to play, most often a player will not want to play games where his money is lost too quickly or where he has an exceptionally remote chance of returning home with winnings.

Mathematics also plays an important part in meeting players’ expectations as to the possible consequences of his gambling activities. If gambling involves rational decision-making, it would appear irrational to wager money where your opponent has a better chance of winning than you do. Adam Smith suggested that all gambling, where the operator has an advantage, is irrational. He wrote “There is not, however, a more certain proposition in mathematics than that the more tickets [in a lottery] you advertise upon, the more likely you are a loser. Adventure upon all the tickets in the lottery, and you lose for certain; and the greater the number of your tickets, the nearer you approach to this certainty.”

Even where the house has an advantage, however, a gambler may be justified if the amount lost means little to him, but the potential gain would elevate him to a higher standing of living. For example, a person with an annual income of \$30,000 may have \$5 in disposable weekly income. He could save or gamble this money. By saving it, at the end of a year, he would have \$260. Even if he did this for years, the savings would not elevate his economic status to another level. As an alternative, he could use the \$5 to gamble for the chance to win \$1 million. While the odds of winning are remote, it may provide the only opportunity to move to a higher economic class.

Since the casino industry is heavily regulated and some of the standards set forth by regulatory bodies involve mathematically related issues, casino managers also should understand the mathematical aspects relating to gaming regulation. Gaming regulation is principally dedicated

to assuring that the games offered in the casino are fair, honest, and that players get paid if they win. Fairness is often expressed in the regulations as either requiring a minimum payback to the player or, in more extreme cases, as dictating the actual rules of the games offered. Casino executives should understand the impact that rules changes have on the payback to players to assure they meet regulatory standards. Equally important, casino executives should understand how government mandated rules would impact their gaming revenues.

The House Edge

The player's chances of winning in a casino game and the rate at which he wins or loses money depends on the game, the rules in effect for that game, and for some games his level of skill. The amount of money the player can expect to win or lose in the long run – if the bet is made over and over again – is called the player's wager *expected value* (EV), or *expectation*. When the player's wager expectation is negative, he will lose money in the long run. For a \$5 bet on the color red in roulette, for example, the expectation is $-\$0.263$. On the average the player will lose just over a quarter for each \$5 bet on red.

When the wager expectation is viewed from the casino's perspective (i.e., the negative of the player's expectation) and expressed as a percentage, you have the house advantage. For the roulette example, the house advantage is 5.26% ($\$0.263$ divided by $\$5$). The formal calculation is as follows:

$$EV = (+5)(18/38) + (-5)(20/38) = -0.263$$

(House Advantage = $0.263/5 = 5.26\%$)

When this EV calculation is performed for a 1-unit amount, the negative of the resulting value is the house edge. Here are the calculations for bets on a single-number in double-zero and single-zero roulette.

Double-zero roulette (single number bet):

$$EV = (+35)(1/38) + (-1)(37/38) = -0.053$$

(House Advantage = 5.3%)

Single-zero roulette (single number bet):

$$EV = (+35)(1/37) + (-1)(36/37) = -0.027$$

(House Advantage = 2.7%)

The house advantage represents the long run percentage of the wagered money that will be retained by the casino. It is also called the house edge, the "odds" (i.e., avoid games with bad odds), or just the "percentage" (as in Mario Puzo's *Fools Die*). Although the house edge can be computed easily for some games – for example, roulette and craps – for others it requires more sophisticated mathematical analysis and/or computer simulations. Regardless of the method used to compute it, the house advantage represents the price to the player of playing the game.

Because this positive house edge exists for virtually all bets in a casino (ignoring the poker room and sports book where a few professionals can make a living), gamblers are faced with an uphill and, in the long run, losing battle. There are some exceptions. The odds bet in craps has zero house edge (although this bet cannot be made without making another negative expectation wager) and there are a few video poker machines that return greater than 100% if played with perfect strategy. Occasionally the casino will even offer a promotion that gives the astute player a positive expectation. These promotions are usually mistakes – sometimes casinos don't check

the math – and are terminated once the casino realizes the player has the edge. But by and large the player will lose money in the long run, and the house edge is a measure of how fast the money will be lost. A player betting in a game with a 4% house advantage will tend to lose his money twice as fast as a player making bets with a 2% house edge. The trick to intelligent casino gambling – at least from the mathematical expectation point of view – is to avoid the games and bets with the large house advantages.

Some casino games are pure chance – no amount of skill or strategy can alter the odds. These games include roulette, craps, baccarat, keno, the big-six wheel of fortune, and slot machines. Of these, baccarat and craps offer the best odds, with house advantages of 1.2% and less than 1% (assuming only pass/come with full odds), respectively. Roulette and slots cost the player more – house advantages of 5.3% for double-zero roulette and 5% to 10% for slots – while the wheel of fortune feeds the casino near 20% of the wagers, and keno is a veritable casino cash cow with average house advantage close to 30%.

Games where an element of skill can affect the house advantage include blackjack, video poker, and the four popular poker-based table games: Caribbean Stud poker, Let It Ride, Three Card poker, and Pai Gow poker. For the poker games, optimal strategy results in a house edge in the 3% to 5% range (CSP has the largest house edge, PGP the lowest, with LIR and TCP in between). For video poker the statistical advantage varies depending on the particular machine, but generally this game can be very player friendly – house edge less than 3% is not uncommon and some are less than 1% – if played with expert strategy.

Blackjack, the most popular of all table games, offers the skilled player some of the best odds in the casino. The house advantage varies slightly depending on the rules and number of decks, but a player using *basic strategy* faces little or no disadvantage in a single-deck game and only a 0.5% house edge in the common six-deck game. Despite these numbers, the average player ends up giving the casino a 2% edge due to mistakes and deviations from basic strategy. Complete basic strategy tables can be found in many books and many casino-hotel gift shops sell color-coded credit card size versions. Rule variations favorable to the player include fewer decks, dealer stands on soft seventeen (worth 0.2%), doubling after splitting (0.14%), late surrender (worth 0.06%), and early surrender (uncommon, but worth 0.24%). If the dealer hits soft seventeen it will cost you, as will any restrictions on when you can double down.

Probability versus Odds

Probability represents the long run ratio of (# of times an outcome occurs) to (# of times experiment is conducted). Odds represent the long run ratio of (# of times an outcome does not occur) to (# of times an outcome occurs). If a card is randomly selected from a standard deck of 52 playing cards, the *probability* it is a spade is $1/4$; the *odds* (against spade) are 3 to 1. The true odds of an event represent the payoff that would make the bet on that event fair. For example, a bet on a single number in double-zero roulette has probability of $1/38$, so to break even in the long run a player would have to be paid 37 to 1 (the actual payoff is 35 to 1).

Confusion about Win Rate

There are all kinds of percentages in the world of gaming. Win percentage, theoretical win percentage, hold percentage, and house advantage come to mind. Sometimes casino bosses use these percentages interchangeably, as if they are just different names for the same thing. Admittedly, in some cases this is correct. House advantage is just another name for theoretical

win percentage, and for slot machines, hold percentage is (in principle) equivalent to win percentage. But there are fundamental differences among these win rate measurements.

The house advantage – the all-important percentage that explains how casinos make money – is also called the house edge, the theoretical win percentage, and expected win percentage. In double-zero roulette, this figure is 5.3%. In the long run the house will retain 5.3% of the money wagered. In the short term, of course, the actual win percentage will differ from the theoretical win percentage (the magnitude of this deviation can be predicted from statistical theory). The actual win percentage is just the (actual) win divided by the handle. Because of the law of large numbers – or as some prefer to call it, the law of averages – as the number of trials gets larger, the actual win percentage should get closer to the theoretical win percentage.

Because handle can be difficult to measure for table games, performance is often measured by hold percentage (and sometimes erroneously called win percentage). Hold percentage is equal to win divided by drop. In Nevada, this figure is about 24% for roulette. The drop and hold percentage are affected by many factors; we won't delve into these nor the associated management issues. Suffice it to say that the casino will *not* in the long term keep 24% of the money bet on the spins of roulette wheel – well, an honest casino won't.

To summarize: House advantage and theoretical win percentage are the same thing, hold percentage is win over drop, win percentage is win over handle, win percentage approaches the house advantage as the number of plays increases, and hold percentage is equivalent to win percentage for slots but not table games.

- Hold % = Win/Drop
- Win % (actual) = Win/Handle
- H.A. = Theoretical Win % = Limit(Actual Win %) = Limit(Win/Handle)
- Hold Percentage \neq House Edge

Furthermore, the house advantage is itself subject to varying interpretations. In Let It Ride, for example, the casino advantage is either 3.51% or 2.86% depending on whether you express the advantage with respect to the base bet or the average bet. Those familiar with the game know that the player begins with three equal base bets, but may withdraw one or two of these initial units. The final amount put at risk, then, can be one (84.6% of the time assuming proper strategy), two (8.5%), or three units (6.9%), making the average bet size 1.224 units. In the long run, the casino will win 3.51% of the hands, which equates to 2.86% of the money wagered. So what's the house edge for Let It Ride? Some prefer to say 3.51% per hand, others 2.86% per unit wagered. No matter. Either way, the bottom line is the same either way: assuming three \$1 base bets, the casino can expect to earn 3.5¢ per hand (note that $1.224 \times 0.0286 = 0.035$).

The question of whether to use the base bet or average bet size also arises in Caribbean Stud Poker (5.22% vs. 2.56%), Three Card Poker (3.37% vs. 2.01%), Casino War (2.88% vs. 2.68%), and Red Dog (2.80% vs. 2.37%).

For still other games, the house edge can be stated including or excluding ties. The prime examples here are the player (1.24% vs. 1.37%) and banker (1.06% vs. 1.17%) bets in baccarat, and the don't pass bet (1.36% vs. 1.40%) in craps. Again, these are different views on the casino edge, but the expected revenue will not change.

That the house advantage can appear in different disguises might be unsettling. When properly computed and interpreted, however, regardless of which representation is chosen, the same truth (read: money) emerges: expected win is the same.

Volatility and Risk

Statistical theory can be used to predict the magnitude of the difference between the actual win percentage and the theoretical win percentage for a given number of wagers. When observing the actual win percentage a player (or casino) may experience, how much variation from theoretical win can be expected? What is a normal fluctuation? The basis for the analysis of such volatility questions is a statistical measure called the *standard deviation* (essentially the average deviation of all possible outcomes from the expected). Together with the central limit theorem (a form of the law of large numbers), the standard deviation (SD) can be used to determine confidence limits with the following volatility guidelines:

Volatility Analysis Guidelines

- Only 5% of the time will outcomes will be more than 2 SD's from expected outcome
- Almost never (0.3%) will outcomes be more than 3 SD's from expected outcome

Obviously a key to using these guidelines is the value of the SD. Computing the SD value is beyond the scope of this article, but to get an idea behind confidence limits, consider a series of 1,000 pass line wagers in craps. Since each wager has a 1.4% house advantage, on average the player will be behind by 14 units. It can be shown (calculations omitted) that the wager standard deviation is for a single pass line bet is 1.0, and for 1,000 wagers the SD is 31.6. Applying the volatility guidelines, we can say that there is a 95% chance the player's actual win will be between 49 units ahead and 77 units behind, and almost certainly between 81 units ahead and 109 units behind.

A similar analysis for 1,000 single-number wagers on double-zero roulette (on average the player will be behind 53 units, wager SD = 5.8, 1,000 wager SD = 182.2) will yield 95% confidence limits on the player win of 311 units ahead and 417 units behind, with win almost certainly between 494 units ahead and 600 units behind.

Note that if the volatility analysis is done in terms of the *percentage* win (rather than the number of units or amount won), the confidence limits will converge to the house advantage as the number of wagers increases. This is the result of the law of large numbers – as the number of trials gets larger, the actual win percentage should get closer to the theoretical win percentage. Risk in the gaming business depends on the house advantage, standard deviation, bet size, and length of play.

Player Value and Complimentaries

Using the house advantage, bet size, duration of play, and pace of the game, a casino can determine how much it expects to win from a certain player. This player earning potential (also called player value, player worth, or theoretical win) can be calculated by the formula:

Earning Potential = Average Bet × Hours Played × Decisions per Hour × House Advantage

For example, suppose a baccarat player bets \$500 per hand for 12 hours at 60 hands per hour. Using a house advantage of 1.2%, this player's worth to the casino is \$4,320 ($500 \times 12 \times 60 \times$

.012). A player who bets \$500 per spin for 12 hours in double-zero roulette at 60 spins per hour would be worth about \$19,000 ($500 \times 12 \times 60 \times .053$).

Many casinos set comp (complimentary) policies by giving the player back a set percentage of their earning potential. Although comp and rebate policies based on theoretical loss are the most popular, rebates on actual losses and dead chip programs are also used in some casinos. Some programs involve a mix of systems. The mathematics associated with these programs will not be addressed in this article.

Casino Pricing Mistakes

In an effort to entice players and increase business, casinos occasionally offer novel wagers, side bets, increased payoffs, or rule variations. These promotions have the effect of lowering the house advantage and the effective price of the game for the player. This is sound reasoning from a marketing standpoint, but can be disastrous for the casino if care is not taken to ensure the math behind the promotion is sound. One casino offered a baccarat commission on winning banker bets of only 2% instead of the usual 5%, resulting in a 0.32% player advantage. This is easy to see (using the well-known probabilities of winning and losing the banker bet):

$$EV = (+0.98)(.4462) + (-1)(.4586) = 0.0032$$

(House Advantage = -0.32%)

A casino in Biloxi, Mississippi gave players a 12.5% edge on Sic Bo bets of 4 and 17 when they offered 80 to 1 payoffs instead of the usual 60 to 1. Again, this is an easy calculation. Using the fact that the probability of rolling a total of 4 (same calculation applies for a total of 17) with three dice is $1/72$ ($1/6 \times 1/6 \times 1/6 \times 3$), here are the expected values for both the usual and the promotional payoffs:

$$\text{Usual 60 to 1 payoff: } EV = (+60)(1/72) + (-1)(71/72) = -0.153$$

(House Advantage = 15.3%)

$$\text{Promotional 80 to 1 payoff: } EV = (+80)(1/72) + (-1)(71/72) = +0.125$$

(House Advantage = -12.5%)

In other promotional gaffes, an Illinois riverboat casino lost a reported \$200,000 in one day with their “2 to 1 Tuesdays” that paid players 2 to 1 (the usual payoff is 3 to 2) on blackjack naturals, a scheme that gave players a 2% advantage. Not to be outdone, an Indian casino in California paid 3 to 1 on naturals during their “happy hour,” offered three times a day, two days a week for over two weeks. This promotion gave the player a whopping 6% edge. A small Las Vegas casino offered a blackjack rule variation called the “Free Ride” in which players were given a free right-to-surrender token every time they received a natural. Proper use of the token led to a player edge of 1.3%, and the casino lost an estimated \$17,000 in eight hours. Another major Las Vegas casino offered a “50/50 Split” blackjack side bet that allowed the player to stand on an initial holding of 12-16, and begin a new hand for equal stakes against the same dealer up card. Although the game marketers claimed the variation was to the advantage of the casino, it turned out that players who exercised the 50/50 Split only against dealer 26 had a 2% advantage. According to one pit boss, the casino suffered a \$230,000 loss in three and a half days.

In the gaming business, it's all about “bad math” or “good math.” Honest games based on good math with positive house advantage minimize the short-term risk and ensure the casino will

make money in the long run. Players will get “lucky” in the short term, but that is all part of the grand design. Fluctuations in both directions will occur. We call these fluctuations good luck or bad luck depending on the direction of the fluctuation. *There is no such thing as luck. It is all mathematics.*

Gaming Regulation and Mathematics

Casino gaming is one of the most regulated industries in the world. Most gaming regulatory systems share common objectives: keep the games fair and honest and assure that players are paid if they win. Fairness and honesty are different concepts. A casino can be honest but not fair. Honesty refers to whether the casino offers games whose chance elements are random. Fairness refers to the game advantage – how much of each dollar wagered should the casino be able to keep? A slot machine that holds, on average, 90% of every dollar bet is certainly not fair, but could very well be honest (if the outcomes of each play are not predetermined in the casino’s favor). Two major regulatory issues relating to fairness and honesty – ensuring random outcomes and controlling the house advantage – are inextricably tied to mathematics and most regulatory bodies require some type of mathematical analysis to demonstrate game advantage and/or confirm that games outcomes are random. Such evidence can range from straightforward probability analyses to computer simulations and complex statistical studies. Requirements vary across jurisdictions, but it is not uncommon to see technical language in gaming regulations concerning specific statistical tests that must be performed, confidence limits that must be met, and other mathematical specifications and standards relating to game outcomes.

Summary Tables for House Advantage

The two tables below show the house advantages for many of the popular casino games. The first table is a summary of the popular games and the second gives a more detailed breakdown.

House Advantages for Popular Casino Games	
<i>Game</i>	<i>House Advantage</i>
Roulette (double-zero)	5.3%
Craps (pass/come)	1.4%
Craps (pass/come with double odds)	0.6%
Blackjack – average player	2.0%
Blackjack – 6 decks, basic strategy*	0.5%
Blackjack – single deck, basic strategy*	0.0%
Baccarat (no tie bets)	1.2%
Caribbean Stud*	5.2%
Let It Ride*	3.5%
Three Card Poker*	3.4%
Pai Gow Poker (ante/play)*	2.5%
Slots	5% - 10%
Video Poker*	0.5% - 3%
Keno (average)	27.0%
*optimal strategy	

HOUSE ADVANTAGES FOR MAJOR CASINO WAGERS		
Game	Bet	HA*
Baccarat	Banker (5% commission)	1.06%
Baccarat	Player	1.24%
Big Six Wheel	Average	19.84%
Blackjack	Card-Counting	-1.00%
Blackjack	Basic Strategy	0.50%
Blackjack	Average player	2.00%
Blackjack	Poor Player	4.00%
Caribbean Stud	Ante	5.22%
Casino War	Basic Bet	2.88%
Craps	Any Craps	11.11%
Craps	Any Seven	16.67%
Craps	Big 6, Big 8	9.09%
Craps	Buy (any)	4.76%
Craps	C&E	11.11%
Craps	don't pass/Don't Come	1.36%
Craps	don't pass/Don't Come w/1X Odds	0.68%
Craps	don't pass/Don't Come w/2X Odds	0.45%
Craps	don't pass/Don't Come w/3X Odds	0.34%
Craps	don't pass/Don't Come w/5X Odds	0.23%
Craps	don't pass/Don't Come w/10X Odds	0.12%
Craps	Don't Place 4 or 10	3.03%
Craps	Don't Place 5 or 9	2.50%
Craps	Don't Place 6 or 8	1.82%
Craps	Field (2 and 12 pay double)	5.56%
Craps	Field (2 or 12 pays triple)	2.78%
Craps	Hard 4, Hard 10	11.11%
Craps	Hard 6, Hard 8	9.09%
Craps	Hop Bet – easy (14-1)	16.67%
Craps	Hop Bet – easy (15-1)	11.11%
Craps	Hop Bet – hard (29-1)	16.67%
Craps	Hop Bet – hard (30-1)	13.89%
Craps	Horn Bet (30-1 & 15-1)	12.50%
Craps	Horn High – any (29-1 & 14-1)	16.67%
Craps	Horn High 2, Horn High 12 (30-1 & 15-1)	12.78%
Craps	Horn High 3, Horn High 11 (30-1 & 15-1)	12.22%
Craps	Lay 4 or 10	2.44%
Craps	Lay 5 or 9	3.23%
Craps	Lay 6 or 8	4.00%
Craps	Pass/Come	1.41%
Craps	Pass/Come w/1X Odds	0.85%
Craps	Pass/Come w/2X Odds	0.61%
Craps	Pass/Come w/3X Odds	0.47%
Craps	Pass/Come w/5X Odds	0.33%
Craps	Pass/Come w/10X Odds	0.18%
Craps	Place 4 or 10	6.67%
Craps	Place 5 or 9	4.00%
Craps	Place 6 or 8	1.52%
Craps	Three, Eleven (14-1)	16.67%
Craps	Three, Eleven (15-1)	11.11%

Craps	Two, Twelve (29-1)	16.67%
Craps	Two, Twelve (30-1)	13.89%
Keno	Typical	27.00%
Let It Ride	Base bet	3.51%
Pai Gow Poker	Skilled player (non-banker)	2.54%
Pai Gow Poker	Average player (non-banker)	2.84%
Red Dog	Basic bet (six decks)	2.80%
Roulette	Single-zero	2.70%
Roulette	Double-zero (except five-number)	5.26%
Roulette	Double-zero, five-number bet	7.89%
Sic Bo	Big/Small	2.78%
Sic Bo	One of a Kind	7.87%
Sic Bo	7, 14	9.72%
Sic Bo	8, 13	12.50%
Sic Bo	10, 11	12.50%
Sic Bo	Any three of a kind	13.89%
Sic Bo	5, 16	13.89%
Sic Bo	4, 17	15.28%
Sic Bo	Three of a kind	16.20%
Sic Bo	Two-dice combination	16.67%
Sic Bo	6, 15	16.67%
Sic Bo	Two of a kind	18.52%
Sic Bo	9, 12	18.98%
Slots	Dollar Slots (good)	4.00%
Slots	Quarter Slots (good)	5.00%
Slots	Dollar Slots (average)	6.00%
Slots	Quarter Slots (average)	8.00%
Sports Betting	Bet \$11 to Win \$10	4.55%
Three Card Poker	Pair Plus	2.32%
Three Card Poker	Ante	3.37%
Video Poker	Selected Machines	-0.50%
*House Advantages under typical conditions, expressed "per hand" and including ties, where appropriate. Optimal strategy assumed unless otherwise noted.		

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- Weaver, Warren (1982). *Lady Luck: The Theory of Probability*, Dover Publications, New York, NY.
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Selected Internet Resources

Urbino – a casino and gaming management site.
www.urbino.net

Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno.
www.unr.edu/gaming

The Wizard of Odds – an excellent site for general game mathematics and odds.

www.thewizardofodds.com

Blackjack (mainly) sites.

www.bjmath.com

www.bj21.com

www.advantageplayer.com

About the Author

Bob Hannum is an Associate Professor of Statistics at the University of Denver where he teaches courses in probability, statistics and the theory of gambling. His publications include *Practical Casino Math* (co-authored with Anthony N. Cabot) and numerous articles in scholarly and gaming industry journals. Hannum regularly speaks on casino mathematics to audiences around the globe. (Some of this guide has been excerpted from *Practical Casino Math*.)