Productivity and Credibility in Industry Equilibrium*

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Abstract

I analyze a model of production in a competitive environment with heterogeneous firms. Efficient production requires individuals within the organization to take noncontractible actions for which rewards must be informally promised rather than contractually assured. The credibility of such promises originates from a firm's future competitive rents. In equilibrium, heterogeneous firms are heterogeneously constrained, and competitive rents are inefficiently concentrated at the top. I explore several policy and empirical implications of this result.

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1 Introduction

To make a firm more productive, managers have to figure out how to get a given set of people with a given set of resources to work together more effectively. This can be done through harder work—asking for more personal sacrifice on the part of the workers. It can be done through smarter work—putting in place more effective management practices. And it can be done through improving internal resource allocation—ensuring that the right people have the right resources for the job at hand. Getting people to make sacrifices, getting them to cooperate with new managerial initiatives, and getting them to use the firm's resources appropriately requires that they be rewarded for doing so. However, many of these objectives and whether they have been met are not easily describable to third-party enforcers. Instead, firms have to rely, at least in part, on informal promises of rewards.[1] A firm's ability to improve its productivity is therefore constrained by its ability to make credible promises. In this paper, I explore the question of why some firms are able to put in place effective practices and others are not by studying how credibility originates in a model of competition among heterogeneous firms.

I model credibility as self enforcement in a repeated game (Bull (1987), MacLeod and Malcomson (1989), Levin (2003)) between a firm's owner and a team of managers. The owner allocates some resources to each manager. She would like each manager to utilize those resources appropriately, but formal contracts are unavailable. She can promise to pay a pre-specified reward if the manager appropriately utilizes the resources he has been allocated. The owner lacks commitment, so in a one-shot game, after the manager's utilization choice has been made, the owner would always prefer not to pay the reward and will therefore not do so; forward-looking managers working for such a "fly-by-night" firm will squander their firm's resources and therefore will not be given any to begin with. A long-lived firm, however, can make credible promises of future rewards, since failure to uphold promises may put the future of the firm at stake: the future competitive rents the firm generates can be used as collateral in the firm's promises.

Explicitly modeling the source of competitive rents is therefore important for understanding the set of opportunities individual firms possess. Output generated by the owner-manager problem is sold into a competitive product market. The market consists of many firm owners of heterogeneous ability, and production exhibits decreasing returns to scale. As in Lucas (1978), this implies that firms of different total factor productivity levels will coexist in equilibrium. The novel ele-

[1] See Malcomson (2013) for a survey on the importance of informal agreements for motivating effort; Gibbons and Henderson (2013) on how productivity-enhancing managerial practices rely on informal agreements; and Bloom, Sadun, and Van Reenen (2012) on how lack of trust constrains decentralization and therefore productivity.

ment of this model is that firms of different *marginal* productivities will coexist in equilibrium, even though all firms face the same factor prices: heterogeneous firms will be heterogeneously constrained, and therefore there will be misallocation of production. The credibility necessary to sustain decentralization is determined by each firm's potential future competitive rents. Competitive rents, credibility, firms' decentralization levels, and therefore firms' productivity levels are jointly determined in industry equilibrium. The model offers two sets of results.

First, because competitive rents serve as collateral, their allocation matters for efficiency. Initial advantage begets further advantage: in equilibrium, high-ability owners achieve high levels of rents and hence collateral, which in turn gives rise to even greater rents. This "Matthew Effect" (Merton (1968)) is limited by decreasing returns to scale, but it nevertheless results in aggregate inefficiencies: competitive rents are allocated too progressively. High-ability firms overproduce, imposing first-order pecuniary externality losses on low-ability firms. The competitive equilibrium is therefore constrained-inefficient, and policies that redistribute profits away from the most profitable firms, such as a progressive corporate income tax, may increase overall welfare.

Second, by augmenting a standard Neoclassical production function with non-contractible managerial decisions, the model provides a framework for thinking about the Bloom, Sadun, and Van Reenen (2014) view of good managerial practices as a technology. This approach highlights the scarcity of credibility as a barrier to the spread of such practices and illustrates how the distribution of good managerial practices depend on the underlying institutional environment in which firms operate. An improvement in the strength of formal contracting institutions reduces the importance of credibility, disproportionately benefiting smaller, constrained firms. The model is therefore consistent with the widely documented facts that the manufacturing sector in less-developed countries is characterized by much greater productivity dispersion than in developed countries (Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013)) and that this dispersion is largely driven by the presence of poorly managed, unproductive firms (Bloom and Van Reenen (2007)).

Related Literature This paper is related to the literature on the large and persistent differences in productivity levels across producers (for a survey, see Syverson (2011)), and it is methodologically closest to Board and Meyer-ter-Vehn (Forthcoming), who augment Shapiro and Stiglitz (1984)'s model of efficiency wages with on-the-job search and show that wage, and hence productivity, dispersion emerges in a stationary industry equilibrium, even with ex-ante identical firms. In their model, credible incentives are derived from endogenous quasi-rents: workers are motivated by the prospect of obtaining or losing high-paying jobs. In my model, credibility is derived from competitive rents: a firm upholds its promises out of fear of losing future profits. Both quasi-rents and competitive rents are important in determining the strength of ongoing relationships, and since their determinants differ, these approaches are complementary.

Also closely related are Chassang (2010) and Gibbons and Henderson (2013), who argue that firm-level productivity differences are due to differences in (ex ante identical) firms' success in developing efficient relational contracts. I assume that all firms succeed in implementing optimal relational contracts. Small differences in the ability of firm owners translate into differences in continuation values and potentially large differences in decentralization and productivity. Relational incentive contracts can therefore amplify existing differences. The analysis in this paper does not address firm dynamics, unlike Chassang (2010) and Ellison and Holden (Forthcoming). It provides a theory of steady-state misallocation, not a theory of the process that leads to it.

Further, this paper contributes to the literatures on firm governance in industry equilibrium (Grossman and Helpman (2002), Legros and Newman (2013), and Gibbons, Holden, and Powell (2012)) and on the aggregate implications of contractual incompleteness (Caballero and Hammour (1998), Francois and Roberts (2003), Martimort and Verdier (2004), Cooley, Marimon, and Quadrini (2004), and Acemoglu, Antras, and Helpman (2007)). My analysis is most similar to Acemoglu, Antras, and Helpman (2007), who examine the role of incomplete contracts and unresolved hold-up on technology adoption. In contrast, I explore how the success of attempts to resolve contractual incompleteness using relational contracts varies with underlying firm characteristics and with the competitive environment in which the firm operates.

Finally, this paper is related to the recent literature on misallocation and economic growth (Banerjee and Duflo (2005), Jeong and Townsend (2007), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009)), which has argued that cross-country differences in the efficiency of resource allocation across firms can explain a substantial portion of the differences in per-capita GDP. How to improve resource allocation across firms depends, of course, on why resources were not allocated efficiently to begin with. Several recent papers in the macro tradition (Banerjee and Moll (2010), Buera, Kaboski, and Shin (2011), Moll (2014), and Midrigan and Xu (2014)) focus on the role of underdeveloped financial markets. Other explanations include heterogeneous markups that distort relative output prices (Peters (2013)), adjustment costs (Collard-Wexler, Asker, and De Loecker (2014)), and size-dependent public policies (Guner, Ventura, and Xu (2008) and Garicano, Lelarge and Van Reenen (2013)). This paper provides an alternative and complementary mechanism that generates persistent misallocation in a perfectly competitive environment with no adjustment costs or credit rationing.

2 The Model

There is a unit mass of firms, indexed by $i \in [0,1]$, each run by a risk-neutral owner (she) who is the residual claimant. Output requires capital and managers (he), who must be given resources in order to be productive. How the managers utilize these resources cannot be directly contracted upon. Managers are homogeneous and risk-neutral, and they are on the long side of the market, so that in equilibrium, they will receive no rents. It will be notationally convenient, but not consequential, to assume that each firm draws potential managers from its own firm-specific pool, so I make this assumption.

Play is infinitely repeated, and I denote by $t = 1, 2, 3, \ldots$ the period. All players share a common discount factor, which I express in terms of a discount rate $\frac{1}{1+r}$ with $r < 1$. Output is homogeneous across firms and sold into a competitive product market. Aggregate demand is stationary, $D_t(p_t) = D(p_t)$, where p_t is the output price in period t, downward-sloping ($D' < 0$), it satisfies $\lim_{p\to 0} D(p) = \infty$ and $\lim_{p\to\infty} D(p) = 0$, and it is generated by consumers who have quasilinear preferences.

Stage Game At the beginning of the stage game, owner i decides whether or not to pay a fixed cost of production, F. If she does, then she decides how much capital K_{it} to rent at exogenous rental rate R and the mass M_{it} of managers to whom she makes an offer.

She then offers each manager $m \in [0, M_{it}]$ a triple $(\delta_{itm}, s_{itm}(\rho_{itm}), b_{itm})$, where δ_{itm} is a level of discretionary resources she allocates to manager m, $s_{itm}(\rho_{itm})$ is a payment that may depend on a contractible measure ρ_{itm} of the level of resources manager m utilizes in production, and b_{itm} is a reward that the owner intends to pay manager m if and only if he utilizes all the resources he has been allocated.[2] Perfectly enforceable contracts can be written on ρ_{itm}, but no contracts can be written directly on the manager's utilization choice.

Each manager m then decides whether to accept this proposed contract or reject it in favor of an outside opportunity that yields exogenous utility $W > 0$. If manager m accepts the contract, the owner transfers resources δ_{itm} to manager m who then chooses a resource utilization level $\hat{\delta}_{itm} \geq 0$ and keeps the remaining resources, $\delta_{itm} - \hat{\delta}_{itm}$, which he values dollar-for-dollar. This utilization level is commonly observed, and the owner subsequently decides whether or not to pay manager m a reward of b_{itm}. Output for firm i is then realized and sold into the market at price p_t.

[2]The model is qualitatively similar to one in which the owner asks each manager to exert observable effort at a private cost.

Technology and Profits Owners have heterogeneous ability, which I denote by φ_i. Assume the realized distribution of ability is given by the distribution function Φ, which is absolutely continuous. Given capital K_{it} and a mass M_{it} of managers who choose utilization levels $\hat{\delta}_{it} \equiv \left\{\hat{\delta}_{itm}\right\}_{m \in M_{it}}$, firm i's production in period t is given by

$$y_i\left(\hat{\delta}_{it}, K_{it}, M_{it}\right) = \varphi_i K_{it}^{\alpha} \left(\int_0^{M_{it}} \hat{\delta}_{itm}^{\frac{\theta}{1-\alpha-\theta}} dm\right)^{1-\alpha-\theta}.$$

Utilization levels across managers are substitutes. Further, I assume that $2\theta < 1-\alpha$, which ensures that the unconstrained problem has a solution, and this solution can be characterized by the firm's first-order conditions. In period t, if owner i pays all rewards, her profits are

$$\pi_i\left(\hat{\delta}_{it}, K_{it}, M_{it}, p_t\right) = p_t y_i\left(\hat{\delta}_{it}, K_{it}, M_{it}\right) - RK_{it} - \int_0^{M_{it}} \left(\delta_{itm} + s_{itm}\left(\rho_{itm}\right) + b_{itm}\right) dm - F.$$

As a benchmark, Section 3 analyzes the case where $\rho_{itm}\left(\hat{\delta}_{itm}\right) = \hat{\delta}_{itm}$, so that formal contracts can be written directly on utilization levels, eliminating the need to use relational incentives. Section 4 examines the pure relational incentives case, where $\rho_{itm}\left(\hat{\delta}_{itm}\right)$ is constant, as well as intermediate cases.

Equilibrium Industry equilibrium must specify (a) the complete plan for the relationship between firm i and its managers and (b) how these plans within each firm aggregate up to determine industry-wide variables.

To describe the former, I define a **relational contract** for firm i as a complete contingent plan for its relationships with its workers, which specifies capital and management choices $\{K_{it}, M_{it}\}_t$ and offers $\{\delta_{itm}, s_{itm}\left(\rho_{itm}\right), b_{itm}\}_{tm}$ as a function of the history of past play within the firm as well as the history of output prices up to, and including, date t. A relational contract is **self-enforcing** if it describes a subgame-perfect equilibrium of the game between owner i and her managers. Note that I am implicitly assuming that firm i's actions can depend on firm j's actions only inasmuch as the latter affect output prices p_t. An **optimal relational contract** for firm i is a self-enforcing relational contract that yields higher expected profits for firm i than any other self-enforcing relational contract.

The notion of industry equilibrium that I will use is one in which all firms conjecture the same price sequence and choose optimal relational contracts, and this conjectured price sequence in fact clears the market in each period. Formally, a **rational-expectations equilibrium (REE)** is a set of sequences of prices $\{p_t\}_t$, capital and management $\{K_{it}, M_{it}\}_{it}$, offers $\{\delta_{itm}, s_{itm}, b_{itm}\}_{itm}$, and utilization choices $\left\{\hat{\delta}_{itm}\right\}_{itm}$ such that at each time t :

1. Given promised reward b_{itm} and resources δ_{itm}, manager m for firm i optimally chooses full utilization $\hat{\delta}_{itm} = \delta_{itm}$.

2. Given the conjectured price sequence $\{p_t\}_{t=1}^{\infty}$, owner i optimally chooses capital and management levels $\{K_{it}, M_{it}\}_t$ and makes offers $\{\delta_{tm}, s_{tm}, b_{tm}\}_{tm}$.

3. p_t clears the output market in period t.

A rational-expectations equilibrium is a **stationary REE** if prices are constant: $p_t = p$.

Throughout, I assume that the rental rate of capital is exogenous and constant at R. Additionally, I will maintain the assumption of perfect competition in the product market, though an equivalent monopolistic competition model can be written in which φ_i is a function of the size of the market for the variety that firm i produces. Finally, the mass of firms in the economy is fixed at 1, though the mass of firms that produce in equilibrium will be endogenous. The model can be extended to allow for endogenous firm entry in which a firm can incur a sunk cost to draw an ability $\varphi_i \sim \Phi$, as in Hopenhayn (1992), and the resulting mass of entrants is determined by an indifference condition. Allowing for entry in this manner does not qualitatively change the results.

3 Complete-Contracts Benchmark

Production described in the previous section differs from standard Neoclassical production in two ways. First, managers are not passive after accepting employment, as they must also make resource-utilization decisions. Second, these resource-utilization decisions are not contractible. To isolate the role of the first assumption, I derive the firm's optimal production decisions when resource-utilization decisions are perfectly contractible.

Assume that $\rho_{itm}\left(\hat{\delta}_{itm}\right) = \hat{\delta}_{itm}$, so that the owner can use the contractible portion of the payment, s_{itm}, both to pin each manager to his (IR) constraint and to directly choose his utilization level, say, by setting $s_{itm}\left(\hat{\delta}_{itm} \neq \delta_{itm}\right) = -\infty$. Because there are no intertemporal linkages in the problem, each firm solves its profit-maximization problem period-by-period. Given a price level p_t, owner i wants to choose K_{it}, M_{it}, and $\{\delta_{itm}\}_m$ to solve the following problem.

$$\max_{K_{it}, M_{it}, \{\delta_{itm}, s_{itm}\}_m} p_t \varphi_i K_{it}^{\alpha} \left(\int_0^{M_{it}} \delta_{itm}^{\frac{\theta}{1-\alpha-\theta}} dm \right)^{1-\alpha-\theta} - RK_{it} - \int_0^{M_{it}} \left(\delta_{itm} + s_{itm}\right) dm - F \tag{1}$$

subject to each manager's individual rationality constraint.

Managers are on the long side of the market, so their individual-rationality constraints will hold with equality: $s_{itm}\left(\delta_{itm}\right) = W$. Additionally, since $2\theta < 1 - \alpha$, the firm's problem is concave in

$\{\delta_{itm}\}_m$. Managers are symmetric, so any optimal solution must satisfy $\delta_{itm} = \delta_{it}$ for all m. The production function therefore collapses into a constant returns-to-scale Cobb-Douglas production function, but the costs are not linear in $(K_{it}, M_{it}, \delta_{it})$, since they depend on the total amount of resources allocated to managers, $\delta_{it} \cdot M_{it}$. The firm's problem becomes

$$\max_{K_{it}, M_{it}, \delta_{it}} p_t \varphi_i \delta_{it}^{\theta} K_{it}^{\alpha} M_{it}^{1-\alpha-\theta} - RK_{it} - (W + \delta_{it}) M_{it} - F.$$

There will be some shutdown value of ability, φ_S, for which $\varphi_i < \varphi_S$ implies that a firm with ability φ_i should optimally not produce. Define the following value, which is increasing in both the firm's ability and prices:

$$H(\varphi_i, p_t) = (p_t \varphi_i)^{1/\theta} (\alpha/R)^{\alpha/\theta} ((1-\alpha-2\theta)/W)^{(1-\alpha-\theta)/\theta}$$

The solution to the firm's problem is summarized in the following proposition.

Proposition 1. There exists a φ_S such that if $\varphi_i < \varphi_S$, firm i optimally does not produce. If $\varphi_i \geq \varphi_S$, firm i optimally chooses

$$\begin{aligned} \delta^{FB}(\varphi_i, p_t) &= \theta \frac{W}{1-\alpha-2\theta}; \\ M^{FB}(\varphi_i, p_t) &= \theta H(\varphi_i, p_t); \\ K^{FB}(\varphi_i, p_t) &= \frac{\alpha}{R} H(\varphi_i, p_t) \delta^{FB}. \end{aligned}$$

Equilibrium total factor productivity for a firm with ability φ_i is given by

$$A_i^{FB}(\varphi_i, p_t) \equiv \frac{y_i}{K_i^{\alpha} M_i^{1-\alpha-\theta}} = \varphi_i \left(\delta^{FB}\right)^{\theta}.$$

First, observe that δ^{FB} does not depend on φ_i or on p_t. When resource utilization is contractible, higher-ability firms or firms that face higher output prices produce more by hiring more managers and renting more capital rather than by allocating more resources to each manager. Next, the solution to the period-t problem does not depend on variables from any other period, and demand is stationary, so output prices will be constant, $p_t = p$ for all t. A stationary REE is then a price level p and a vector of firm-level choices $\{K_i, M_i, \delta_i\}_i$ such that these choices are optimal in each period given the price level, and the price level clears the market in each period. It is straightforward to verify that a stationary REE exists, is unique, and it is Pareto-efficient.

It is also worth noting that firm i's equilibrium total factor productivity depends only on the firm's ability, φ_i, and the first-best level of resource utilization, δ^{FB}. It does not depend on the interest rate, r, or the equilibrium prices, p. This will stand in contrast to the results of the following section, where managers' utilization choices are not directly contractible.

4 Relational Incentive Contracts

I now turn to the heart of the model and assume that resource utilization is not contractible. That is, $\rho_{itm} = \emptyset$ for all $\hat{\delta}_{itm}$, and therefore s_{itm} is constant. The owner would like to provide incentives for her managers to utilize resources, but she can only do so by making a promise that she will pay a pre-specified reward if the manager chooses a particular utilization level. The owner cannot commit to doing so, so in a one-shot game, after the manager's utilization choice has been sunk, the owner would always prefer not to pay the reward. A forward-looking manager will therefore not choose a positive utilization level. Consequently, the owner will not allocate any resources to the manager. However, the owner may use future competitive rents as a partial commitment device.

Her ability to do so depends on the clarity with which her failure to pay promised rewards is communicated to her current and potential future managers. Throughout, I make the following assumption of perfect observability.

Assumption 1. A firm's potential future managers commonly observe allocated resources and utilization choices of individual managers and whether they were paid their promised rewards.

It is worth pausing to comment on the starkness of this assumption. In an organization with internal labor markets, next period's managers come from the ranks of this period's workers, so this assumption does not require strong external monitoring. Additionally, perfect observability can be relaxed to all-or-nothing public monitoring, which I show in the Appendix. All-or-nothing public monitoring makes the goal of dynamic enforcement more difficult to achieve, which corresponds to an increase in the discount rate r in the present model.

In addition, I assume that a manager's outside option is independent of his employment history, preventing the firm from leveraging quasi-rents from labor-market frictions to aid in dynamic enforcement. I similarly assume that capital is not firm-specific.

Assumption 2. Managers' outside options are independent of their employment histories. Capital is not firm-specific.

The upshot of Assumption 1 is that the totality of a firm's future competitive rents can be used as collateral in its promises. Assumption 2 eliminates market frictions and ensures that only a firm's future competitive rents, rather than quasi-rents, can be used in this manner.

4.1 Dynamic Enforcement

This section derives conditions under which utilization levels $\{\delta_{itm}\}_{t,m\in M_{it}}$ are sustainable as part of a relational contract. The first observation is that any equilibrium in which manager m does not fully utilize the resources he has been allocated is payoff-equivalent to an equilibrium in which he does, because the manager can be compensated for the difference between δ_{itm} and $\hat{\delta}_{itm}$ ex-ante through an increased s_{itm} equal to this difference. Therefore, without loss of generality, I focus on relational contracts in which managers fully utilize the resources they have been allocated. Under what conditions does an owner have the credibility necessary to promise rewards sufficient to ensure that managers will choose to utilize resources $\{\delta_{itm}\}_{t,m\in M_{it}}$?

Utilization levels $\{\delta_{itm}\}_{t,m\in M_{it}}$ are sustainable as part of a relational contract if and only if they are sustainable as part of a relational contract that involves grim-trigger punishment off the equilibrium path. I therefore look for an equilibrium in which a firm's managers begin by fully utilizing the resources they have been allocated, and owners reward managers as promised. In any given period, if in the past, the owner failed to pay any number of managers their promised reward following full utilization, players revert to the unique SPNE of the stage game: each manager utilizes zero resources, the owner does not allocate any resources to any manager, managers reject all offers, and the owner does not pay the fixed cost of production. This relational contract is, of course, not renegotiation-proof, but as in Levin (2003), since monetary transfers are possible, on the equilibrium path, it is payoff-equivalent to a renegotiation-proof relational contract in which all competitive rents are transferred to the managers following a deviation by the owner.

Suppose manager m believes the owner will pay reward b_{itm} if and only if he chooses utilization level $\hat{\delta}_{itm} = \delta_{itm}$. Then he will choose $\hat{\delta}_{itm} = \delta_{itm}$ rather than his maximal reneging temptation of $\hat{\delta}_{itm} = 0$, in which case he walks away with δ_{itm}, if

$$b_{itm} + \frac{1}{1+r}\left(U_{i,t+1,m} - \tilde{U}_{i,t+1,m}\right) \geq \delta_{itm}, \tag{2}$$

where $U_{i,t+1,m}$ is the continuation utility manager m receives from $t+1$ on if the relationship is not terminated, and $\tilde{U}_{i,t+1,m}$ is the continuation utility he receives if separation occurs. He will therefore choose full utilization if and only if the sum of the reward and the change in the continuation value exceeds the value of the resources.

If the manager chooses any utilization level other than full utilization, the owner has no incentive to pay the reward and therefore will not. If the manager fully utilizes resources δ_{itm}, the owner will pay the promised reward b_{itm} if

$$\frac{1}{1+r}\left(\Pi_{i,t+1,m} - \tilde{\Pi}_{i,t+1,m}\right) \geq b_{itm}, \tag{3}$$

where $\Pi_{i,t+1,m}$ and $\tilde{\Pi}_{i,t+1,m}$ are, respectively, owner i's continuation value if he pays the promised reward and if she does not pay the promised reward. Thus, the change in continuation value of the firm must exceed the size of the promised bonus.

MacLeod and Malcomson (1989) and Levin (2003) show that (2) and (3) can be pooled together to provide necessary and sufficient conditions for the manager to choose full utilization and the principal to pay the promised reward. Let $S_{i,t+1.m} = U_{i,t+1,m} + \Pi_{i,t+1,m}$ and $\tilde{S}_{i,t+1,m} = \tilde{U}_{i,t+1,m} + \tilde{\Pi}_{i,t+1,m}$. δ_{itm} is sustainable in a relational contract if

$$\frac{1}{1+r}\left(S_{i,t+1,m} - \tilde{S}_{i,t+1,m}\right) \geq \delta_{itm} \tag{4}$$

is satisfied.

In principle, $\tilde{S}_{i,t+1,m}$ is not a straightforward object. Following a deviation, other relationships within the firm may be altered, and the owner may choose to renege on multiple managers simultaneously. However, the candidate equilibrium described above involves multilateral punishment: an owner's choice to renege on a single manager leads all current and potential future managers to stop cooperating. The owner's maximal reneging temptation is therefore to pay no bonuses to any manager. Thus, as in Levin (2002), a necessary and sufficient condition for $\{\delta_{itm}\}_{t,m}$ to be sustainable as part of a relational contract is that the following aggregate dynamic enforcement constraint is satisfied:

$$\frac{1}{1+r}\left(S_{it+1} - \tilde{S}_{it+1}\right) \geq \int_0^{M_{it}} \delta_{itm} dm, \tag{5}$$

where $S_{i,t+1} - \tilde{S}_{i,t+1}$ represents the total profits generated by the owner and the managers she hires net of their outside opportunities.

Finally, note that $S_{i,t+1}$ depends on the whole future stream of prices and future promises. Given a conjecture $\{p_\tau\}_{\tau=t}^{\infty}$ that is shared by the owner and the managers, S_{it+1} is given by

$$\sum_{\tau=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{\tau-t-1}\left[p_\tau \varphi_i K_{i\tau}^{\alpha}\left(\int_0^{M_{it}} \delta_{itm}^{\frac{\theta}{1-\alpha-\theta}} dm\right)^{1-\alpha-\theta} - RK_{i\tau} - \int_0^{M_{it}}\left(W + \delta_{i\tau m}\right) dm - F\right]. \tag{6}$$

$\{p_\tau\}_{\tau=t}^{\infty}$ are determined jointly by the demand conditions as well as the production capabilities and relational contracts of all the firms in the economy.

4.2 Rational-Expectations Equilibrium

Throughout, I will focus on stationary REEs with constant prices $p_t = p$. The following theorem establishes existence and uniqueness of a stationary REE. The proof is constructive and forms the basis of the analysis in the next section.

Theorem 1. There exists a unique stationary REE.

Proof. Suppose all firms conjecture price sequence $p_t = p$ for all t. I will show that aggregate supply is well-defined and stationary. Fix a firm i and assume all other firms choose stationary offers $\{\delta_{jtm}, s_{jtm}, b_{jtm}\} = \{\delta_{jm}, s_{jm}, b_{jm}\}$ and constant capital and management levels $\{K_{jt}, M_{jt}\} = \{K_j, M_j\}$. Further, suppose firm i chooses constant capital and management levels $\{K_{it}, M_{it}\} = \{K_i, M_i\}$. From firm i's perspective, the environment is stationary. By Levin (2003), firm i can replicate any optimal relational contract with a stationary relational contract. Thus, $\{\delta_{itm}, s_{itm}, b_{itm}\} = \{\delta_{im}, s_{im}, b_{im}\}$, which in turn rationalizes the firm's choice of a constant capital and management sequence. This implies a constant aggregate production sequence, which yields aggregate supply $S(p)$.

The remaining task is to find the constant price sequence consistent with supply and demand in each period. Aggregate supply is upward-sloping, since future competitive rents, and hence today's output, are increasing in p for all firms. Further, it is smooth, since Φ is absolutely continuous. Since aggregate demand has an infinite choke price and is decreasing, smooth, and asymptotes to 0, existence and uniqueness of such a price p follows.■

I pause to comment briefly on uniqueness of equilibrium. First, given constant prices, the game played within a particular firm is a repeated game and there may therefore exist many self-enforcing relational contracts. However, choice of a sub-optimal relational contract within a firm is ruled out by the definition of REE. It is also worth noting that, as in Levin (2003), unrestricted transfers ensures that the maximal surplus generated within a firm is independent of the distribution of that surplus–in principle, the surplus could be allocated to the managers, but this would constitute a suboptimal relational contract from the firm's perspective. Further, because transfers are allowed at the beginning and the end of each stage of the stage game, the constrained-optimal resource allocation could also be implemented in a payoff-equivalent way through efficiency wages (i.e., high contingent wages paid at the beginning of next period) rather than through bonuses.

Even if all firms choose optimal relational contracts, there may exist a nonstationary REE with price cycles. The basic intuition is the following. Suppose all firms believe that output prices will cycle between a given pair of low and high values. Then from the perspective of a high-price period, the future looks relatively grim, as prices will be low in the future. This constrains the resource-utilization level firms can sustain as part of an optimal relational contract today, which leads to a restriction in quantity and therefore is consistent with today's high prices. A similar argument in low-price periods establishes that this two-point alternating price sequence is consistent with equilibrium. Throughout, I will focus on stationary REEs, since they are the direct analogue of the unique stationary REE in the complete-contracts case considered in Section 3.

4.3 Equilibrium Optimal Relational Contracts

The remaining sections characterize optimal relational contracts in the stationary REE and examine the aggregate implications of dynamic enforcement constraints. The proof of Theorem 1 charts a road map for constructing the stationary REE: (1) fix output prices $p_t = p$ and solve for each firm's optimal stationary relational contract, (2) aggregate up the production of individual firms to generate the industry supply curve $S(p)$, and (3) solve for the equilibrium price p^* that satisfies $S(p^*) = D(p^*)$. In this section, I will drop the i subscript and work directly firm ability φ.

Since $2\theta < 1 - \alpha$, production is concave in individual utilization levels. Since managers are symmetric, any optimal relational contract will involve $\delta_m = \delta$ for all m. At the steady state, per-period profits for a firm with ability φ are given by

$$\pi(K, M, \delta) = p\varphi\delta^\theta K^\alpha M^{1-\alpha-\theta} - RK - (W + \delta) M - F.$$

In an optimal relational contract, firms maximize their per-period profits subject to their pooled dynamic-enforcement constraint. That is, each firm takes p as given and solves

$$\max_{K,M,\delta} \pi(K, M, \delta) \tag{7}$$

subject to

$$\pi(K, M, \delta) \geq rM\delta. \tag{8}$$

In the formulation of the production function, if all managers choose the same utilization levels, production exhibits decreasing returns to scale in K and M. This is a standard assumption in models in which firms of different productivities coexist in equilibrium (e.g., Lucas (1978)). Proposition 6 in the appendix shows that if production exhibits constant returns in K and M, there does not exist an REE. With constant returns to scale, equilibrium prices will be such that firms that produce make zero profits, which in turn precludes such firms from producing at all.

I view the interest rate the firm faces as an effective interest rate that combines firm turnover (i.e., an exogenous probability of firm destruction), pure time preferences, monitoring technology on the part of the firm (i.e., can the firm see whether or not a manager has chosen the correct utilization level?), social connections on the part of the population of managers (i.e., can future managers see if the owner has paid the promised rewards?). Proposition 7 in the appendix shows that if with probability q_O and q_M, deviations by the owner and by each manager, respectively, are publicly detected, and if with probability $1 - q_X$, the firm exogenously exits the industry, then the effective interest rate is $\tilde{r} = r/(q_O q_M q_X)$. In other words, think of r as fairly large.

Define cutoffs $\varphi_L(p) \equiv (1+r)^{\theta} \varphi_S(p)$ and $\varphi_H(p) = (1-r)^{-\theta} \varphi_S(p)$.The next proposition characterizes the solution to the constrained problem (7) subject to (8).

Proposition 2. There exists a weakly increasing function $\mu^*(\varphi, p)$ satisfying $\mu^*(\varphi, p) = 0$ for all $\varphi < \varphi_L(p)$ and $\mu^*(\varphi, p) = 1$ for all $\varphi \geq \varphi_H(p)$ such that the solution to the constrained problem satisfies

$$\delta^*(\varphi, p) / \delta^{FB} = M^*(\varphi, p) / M^{FB}(\varphi, p) = K^*(\varphi, p) / K^{FB}(\varphi, p) = \mu^*(\varphi, p),$$

Equilibrium total factor productivity is given by $A(\varphi, p) = \mu^*(\varphi, p)^{\theta} A^{FB}(\varphi, p)$.

Each of the firm's three choice variables at the constrained optimum is proportional to its first-best value with the same constant of proportionality, given by $\mu^*(\varphi, p)$. The firm's solution can therefore be characterized entirely by the function $\mu^*(\varphi, p)$ and its first-best solution. The proof of Proposition 2 and the exact expression for $\mu^*(\varphi, p)$ are in the appendix, and Figure 1 below characterizes $\mu^*(\varphi, p)$ as a function of φ for a fixed price level p. In the complete-contracts benchmark of Section 3, $\delta^*(\varphi, p)$ equals zero if φ is not large enough for the firm to cover its fixed costs and $\delta^*(\varphi, p) = \delta^{FB}$ otherwise.

When formal contracts are unavailable, however, future competitive rents are a determinant of the firm's current productivity. Higher-ability firms have higher future competitive rents and therefore are less constrained in equilibrium. These considerations introduce three additional regions relative to the complete-contracts benchmark. For $\varphi_S \leq \varphi < \varphi_L$, the firm should produce but is unable to. Such firms are constrained on the extensive margin. For $\varphi_L \leq \varphi < \varphi_H$, the dynamic enforcement constraint is binding, and the firm is unable to produce efficiently. Such firms are constrained on the intensive margin. For $\varphi \geq \varphi_H$, the firm is unconstrained and therefore produces according to first-best.

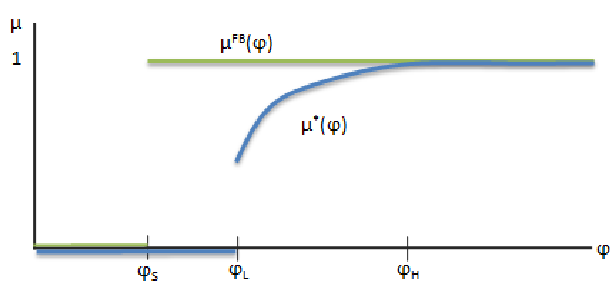

Figure 1

Figure 1 plots the equilibrium (top) and first-best (bottom) solutions for given price p

Consequently, equilibrium total factor productivity for a firm with ability φ is proportional to its first-best total factor productivity. That is, $A(\varphi, p) = \mu^*(\varphi, p)^\theta A^{FB}(\varphi, p)$. A firm's total factor productivity depends on the effective discount rate it faces and is therefore decreasing in firm turnover and increasing in the clarity with which deviations are communicated. The quality of communication technology and the strength of social connections may therefore play a role in determining a firm's total factor productivity. In addition, total factor productivity is jointly determined with the equilibrium price—a firm's production possibilities set is endogenous to market conditions, unlike in the standard Neoclassical model of the firm.

In the complete-contracts competitive equilibrium, firms of heterogeneous total factor productivity coexist in equilibrium. However, because firms are unconstrained and face identical factor prices, firms' *marginal* productivities are equalized. Relative to this benchmark, in the absence of formal contracts, firms of heterogeneous marginal productivity coexist in equilibrium: high-ability firms are less constrained, implying a lower marginal productivity. Further, high-ability firms are able to sustain higher levels of decentralization and therefore have higher total factor productivity. This implies a negative relationship between total factor productivity and marginal productivity in equilibrium.

The analysis in this section so far has held output prices constant and derived firm-level production. Given price p, a firm of ability φ produces $y^*(\varphi, p)$. If all firms expect the same constant price p, then aggregate supply is given by

$$S(p) = \int_{\varphi_L(p)}^{\infty} y^*(\varphi, p)\, d\Phi(\varphi),$$

where $\varphi_L(p)$ is the cutoff value of ability such that $\varphi < \varphi_L(p)$ implies that a firm of ability φ will not have enough credibility to sustain any positive level of decentralization if prices are constant at p. It is worth noting that $\varphi_L(p)$ is continuous and decreasing in p: if prices are higher, then future competitive rents are higher, and therefore firms with lower ability will be able to sustain positive levels of decentralization. Further, $y^*(\varphi, p)$ is increasing in p: unconstrained firms choose to produce more if prices are higher, and constrained firms are able to produce more, because their future competitive rents are greater. Therefore, $S(p)$ is strictly increasing in p. Equilibrium prices, p^* therefore solve $D(p^*) = S(p^*)$ in each period.

4.4 Imperfect Formal Contracts

Until now, I have considered two opposite extremes: the perfect formal-contracts benchmark model in Section 3 and the no formal-contracts model. In this section, I will consider intermediate cases in which imperfect formal contracts are available. That is, contracts can be written directly on manager m's output, but they are imperfect. Contracts such as equity stakes that are written directly on overall firm profits are ineffective at providing motivation in this setting, because there are a continuum of managers, each of whom has an infinitesimal effect on firm profits.

Suppose a third-party enforcer observes δ_{itm} and $\hat{\delta}_{itm}$. However, the third-party enforcer will only enforce deviations that are at least $(1-\omega)$-egregious, for $\omega \in [0,1]$, which implies that the formal portion of the contract can be contingent on whether or not $\rho_{itm}\left(\hat{\delta}_{itm}, \delta_{itm}\right) = 1\left\{\hat{\delta}_{itm} \geq \omega\delta_{itm}\right\}$, and hence $s_{itm}\left(\rho_{itm} = 0\right)$ can be set to $-\infty$. Enforcement is otherwise costless. I refer to ω as the **quality of formal contracting institutions**.

I restrict attention to **full-utilization relational contracts**, in which any choice $\hat{\delta}_{itm} < \delta_{itm}$ is viewed as a deviation, which results in punishment. The definition of REE must also be modified to account for this restriction. In contrast to the no formal-contracts model, restricting to full-utilization relational contracts is consequential: relaxing this assumption enables each firm to achieve first-best utilization levels by setting $\delta_i = \delta^{FB}/\omega$, allowing each manager to choose $\hat{\delta}_i = \omega\delta_i = \delta^{FB}$ and keep the remaining $(1-\omega)\,\delta^{FB}/\omega$. The salary component of the contract then extracts this ex post "reward." This equilibrium restriction can be sidestepped if I allow instead $\rho_{itm} = 1\left\{\hat{\delta}_{itm} \leq \omega\delta^{FB}\right\}$, which would deliver qualitatively similar results, but its solution can only be computed numerically.

I also assume that a management team operating to the letter of a formal contract yields no more profits than the firm could realize if it simply shuts down. That is, no matter how strong a formal contract is, certain non-contractible actions must be taken for any production to take place. A firm is therefore unlikely to survive if its management team works "to rule."

Assumption 3. The firm's outside option is independent of the strength of formal contracting institutions.

I now examine the effect of an increase in the quality of formal contracting institutions on the level of resource allocation within firms in a stationary REE. Suppose a manager has been allocated resources δ. He can "get away" with utilizing a resource level as low as $\omega\delta$, but if he utilizes any less than this level, he will face harsh third-party punishment. As a result, his maximal reneging temptation is to walk away with resources $(1-\omega)\,\delta$, so his bonus and continuation-value differential must only exceed $(1-\omega)\,\delta$ rather than δ as in the no formal-contracts model. The remaining point to note is that since the firm's continuation value if it fails to pay any promised bonuses is again zero, the pooled dynamic enforcement constraint that is required for resource allocation $\{\delta_i\}$ to be sustainable as part of a full-utilization relational contract is simply

$$\pi_i \geq (1-\omega)\, r M_i \delta_i = \tilde{r} M_i \delta_i.$$

In other words, the strength of the formal contracting institutions enters the dynamic-enforcement constraint as an effective decrease in the interest rate.

5 Efficiency of Rational-Expectations Equilibrium

When formal contracting institutions are weak, a firm's future profits are an input into its current production. Importantly, and in contrast to the standard Neoclassical model, they are an input that is determined not only by the firm itself but also by the environment in which the firm operates. As a result, the distribution of profits across heterogeneous firms affects the overall efficiency of production, and in this section, I show that profits are not distributed efficiently.

To build intuition for the nature and cause of the inefficient profit distribution, let

$$\pi^*(\varphi, p^*, F) = \max_{K,M,\delta} \left\{ \pi(K, M, \delta; \varphi, p^*, F) : \pi(K, M, \delta; \varphi, p^*, F) \geq r M \delta \right\}$$

denote the optimal per-period profits of a firm with ability φ when equilibrium prices are p^*, and let $\lambda^*(\varphi, p^*, F)$ denote the shadow cost of the dynamic enforcement constraint at the optimum. By the envelope theorem,

$$\frac{d\pi^*}{d(-F)} = 1 + \lambda^*(\varphi, p^*, F).$$

In addition to the static effect on per-period profits, a reduction in fixed costs increases future profits—this in turn increases the firm's credibility and allows it to increase decentralization. The dynamic effect is greater the more constrained the firm is. From the previous section, $\mu(\varphi, p^*)$ is increasing in φ. In the appendix, I show that $\lambda^*(\varphi, p^*, F)$ is decreasing in φ and in fact is equal to ∞ at $\varphi_L(p^*)$, and it is equal to 0 for all $\varphi \geq \varphi_H(p^*)$. Higher-ability firms are less constrained in equilibrium and therefore benefit less from an increase in future profits.

In principle, a social planner could improve upon the competitive-equilibrium allocation. To see this, suppose the support of Φ is unbounded from above, so that for any price level p, there will be a positive mass of firms with $\varphi_i > \varphi_H(p) + \zeta$ for some small but positive ζ, so they are unconstrained. Consider a persistent proportional output tax τ on such firms. Let $T(\tau)$ be the tax revenues generated by this tax scheme, and define p^τ to solve $D(p^\tau) = S(p^\tau; \tau)$. Total per-period welfare is given by

$$\mathcal{W}(\tau) = \underbrace{\int_{\varphi_L(p^\tau)}^{\varphi_H(p^\tau)+\zeta} \pi^*(p^\tau, \varphi; 0)\, d\Phi(\varphi)}_{\text{Producer surplus of untaxed firms}} + \underbrace{\int_{\varphi_H(p^\tau)+\zeta}^{\infty} \pi^*(p^\tau, \varphi; \tau)\, d\Phi(\varphi)}_{\text{Producer surplus of taxed firms}} + \underbrace{\int_{p^\tau}^{\infty} D(p)\, dp}_{\text{Consumer surplus}} + \underbrace{T(\tau)}_{\text{Revenues}}, \tag{9}$$

where $\pi^*(p, \varphi; \tau)$ is the equilibrium per-period profits a firm with ability φ receives if prices are given by p and it faces a tax τ, so that the effective prices it faces are $(1-\tau)\,p$.

Theorem 2. $\mathcal{W}'(0) > 0$.

The basic idea of the proof, which is in the appendix, is that a small increase in taxes on unconstrained firms leads to an output-price increase, which induces a transfer from consumers to the constrained firms. Statically, this is merely a transfer. However, the dynamic effects of this transfer in each period result in a relaxation of the dynamic enforcement constraint and hence an increase in efficiency of the constrained firms. Theorem 2 highlights the source of the market inefficiency: high-ability firms overproduce, inducing a first-order negative pecuniary externality on low-ability firms. The result of this is that competitive rents are allocated too progressively.

From a policy perspective, Theorem 2 suggests that when formal contracting institutions are weak, a small progressive corporate-revenue tax may improve aggregate welfare by boosting the profits of low-ability firms. It is interesting to note that what seems like a similar policy, a limited small-business tax credit funded by a non-distortionary head tax, does not lead to unambiguous gains. This is because, while a persistent output subsidy for small, constrained firms increases their profits by more than the monetary cost of the subsidy, such a policy increases aggregate output, driving down output prices. This in turn reduces the profits of those firms that do not receive the subsidy, which may lead to further losses if such firms are constrained in equilibrium.

A full treatment of optimal corporate taxation in the presence of credibility constraints is beyond the scope of this paper, but it is interesting to note that, in contrast to classical results on optimal-tax theory (Diamond and Mirrlees (1971)), taxing the output of a subset of firms may lead to an increase in total surplus. This is because, in the Neoclassical model of production that Diamond and Mirrlees (and the ensuing literature) study, absent any distortionary taxes on production, aggregate production is carried out efficiently. That is, there is no misallocation of productive resources across independent production units.

Policies that increase the concentration of profits can potentially have harmful effects. Consider two countries opening up to trade. In a Melitz (2003) model of heterogeneous firms, high-ability—and hence high profitability—firms will export. In so doing, they will drive up domestic factor prices, reducing the profitability of those firms that do not export (as well as the marginal exporting firms). Trade liberalization therefore leads to a further concentration of profits at the top. If the countries involved have poor formal contracting institutions, this concentration of profits could result in a reduction in aggregate productivity among the smaller firms, and these losses may exceed the Melitz (2003) reallocation benefits. Trade liberalization, therefore, may harm aggregate productivity in countries with weak formal contracting institutions.

6 Empirical Implications

The model in this paper augments the Neoclassical model of the firm with non-contractible managerial decisions and supposes that incentives are provided to these managers primarily through promises of future compensation. This additional feature, which is widely studied in the organizational-economics literature (see Malcomson (2013) for a survey), generates a mechanism through which future profits can determine current productivity, resulting in firm-level income effects with efficiency consequences. These income effects are decreasing—the marginal returns to a dollar-a-day increase in profits is higher for less productive firms—and therefore, profits are inefficiently concentrated at the top in a competitive equilibrium. Consequently, non-standard economic policy aimed at manipulating the distribution of profits among heterogeneous producers can improve welfare.

A natural question to ask is whether the model also generates implications that are consistent with the data. This section explores empirical implications of this model. First, I examine the model's within-country implications for firm-level productivity changes in response to persistent changes in aggregate demand. Second, I consider its implications for cross-country differences in the distribution of firm-level productivity. Both sets of implications build upon the idea that low-ability firms are more sensitive than high-ability firms to changes in future competitive rents. To formalize this result, recall that, in equilibrium, the TFP for a firm with ability φ is

$$A\left(\varphi, p^{*}, F\right)=\varphi \mu^{*}\left(\varphi, p^{*}, F\right)^{\theta}\left(\delta^{FB}\right)^{\theta}.$$

The following proposition shows that the sensitivity of total factor productivity to future competitive rents is greater for low-ability firms than for high-ability firms.

Proposition 3. $\log A^{*}\left(\varphi, p, F\right)$ is increasing and exhibits decreasing differences in $\left(\varphi, -F, p\right)$.

In response to an unexpected, persistent increase (decrease) in aggregate demand, this proposition suggests that more constrained firms will see a proportionally larger increase (decrease) in their productivity than less constrained firms. This is consistent with the micro data from Baily, Bartelsman, and Haltiwanger (2001) on firm-level responses to the business cycle from 1979 to 1989. Section 6.1 explores this implication.

In Section 6.2, I compare productivity distributions across countries that differ in the strength of formal contracting institutions, ω. An increase in the strength of formal contracting institutions reduces the importance of credibility in sustaining decentralization, disproportionately benefiting low-ability firms that are more constrained in equilibrium. Proposition 4 captures this result.

Proposition 4. $\log y^*(\varphi, p, F, \omega)$ and $\log A^*(\varphi, p, F, \omega)$ are increasing in ω and exhibit decreasing differences in (φ, ω).

Proposition 4 implies that holding prices constant, an increase in ω leads to an increase in total factor productivity and output. Further, low-ability firms disproportionately benefit from this increase in ω. This leads to a convergence in the productivity distribution among existing firms. However, it will also potential lead to the entry of low-ability firms. The new entrants and all existing firms produce more the greater is ω, so supply increases and therefore prices must fall. Let p^ω solve $D(p^\omega) = S(p^\omega)$. Then p^ω is decreasing in ω. This decrease in prices leads to a net reduction in production of unconstrained firms, since the increase in ω does not allow them to produce more. Provided that the price effects are not too large, the effects identified in Proposition 4 hold even after allowing for price adjustments, as Proposition 5 shows.

Proposition 5. Suppose that either (a) φ has a log-convex distribution and $|\varepsilon_{D,p}| > 1$ or (b) φ has a log-concave distribution and $|\varepsilon_{D,p}| < 1$. Then $Var\left(A^*(\varphi, p^\omega, \omega) \middle| \varphi \geq \varphi_L^\omega\right)$ is greater for $\omega = 0$ than for $\omega = 1$. Additionally, there exists some $\hat{\varphi}^\omega$ such that $y^*(\varphi, p^\omega, \omega)$ is increasing in ω for $\varphi < \hat{\varphi}^\omega$ and decreasing in ω for $\varphi > \hat{\varphi}^\omega$.

Finally, if all firms produce more at a given price, equilibrium prices must fall. This reduction in prices in equilibrium reduces the output of high ability, unconstrained firms, which leads to a compression in the distribution of output across firms.

6.1 Responses to Sustained Changes in Aggregate Demand

Macroeconomic evidence, dating back to at least Hultgren (1960), strongly suggests that aggregate productivity is pro-cyclical, at least until the most recent recessions (See Gali and van Rens (2014)). Bartelsman and Doms (2000) decompose the changes in aggregate productivity into between- and within-firm productivity changes over the period of 1977-1987 and find that this procyclicality was driven by within-firm productivity declines during the slump that occurred between 1977 and 1982 and within-firm productivity increases during the boom that occurred between 1982 and 1987. Within-firm productivity changes were therefore found to be procyclical.

Baily, Bartelsman, and Haltiwanger (2001) decompose these productivity changes further. They examine firm-level changes in productivity over the period between 1979 and 1988 and find that the productivity of firms with bright long-run prospects (as predicted by variables that are observable in 1979) was procyclical, but not very much so. Firms with poor long-run predicted prospects, on the other hand, exhibited much greater degrees of procyclicality. This result is also consistent with recent work by Kehrig (2015), who shows that productivity dispersion is greater during recessions than during booms, and the change in productivity dispersion is driven primarily by the left tail of the productivity distribution: during recessions, there are more unproductive firms, and during booms, there are fewer.

In summary, two key facts regarding productivity dynamics over the business cycle are: (1) within-firm productivity changes are pro-cyclical, and (2) these changes are primarily concentrated in the left tail of the distribution. Fact 1 is inconsistent with a Neoclassical model in which firm-level productivity is exogenous and aggregate productivity changes are driven only by selection (i.e., the "cleansing" effect of recessions). It is further inconsistent with a standard efficiency-wage story: downward-rigid wages and increased unemployment during recessions should enable firms to implement higher levels of effort, and therefore we would expect to see within-firm productivity increase during a recession.

Fact 2 is inconsistent with the standard labor-hoarding story in which labor adjustment costs cause successful firms to ride out a recession by holding on to their existing workers and asking less of them, as such an explanation would generate a compressed right tail during recessions. Finally, it is also inconsistent with a model in which aggregate fluctuations are driven by independent and exogenous firm-level productivity shocks. As Gabaix (2011) points out, only when the firm-size distribution is sufficiently right-skewed do independent firm-level shocks aggregate up to economy-wide shocks. A boom occurs when the productivity of the largest, most successful firms increases, and a recession occurs when the productivity of such firms decreases. Such an explanation would imply procyclical, rather than countercyclical, productivity dispersion.

These facts are, however, consistent with Proposition 3. I will compare the stationary REEs in two economies: a low-demand economy in which aggregate demand is given by $D_L(p)$ and a high-demand economy in which $D_H(p) > D_L(p)$ for all p. Denote the stationary REE price levels in the low- and high-demand economies by p_L^* and p_H^*. It will necessarily be the case that $p_H^* > p_L^*$, because aggregate demand is strictly decreasing, and aggregate supply is strictly increasing. For all

φ such that firms with ability φ operate in both economies, Proposition 3 shows that we will have $A(\varphi, p_H^*) \geq A(\varphi, p_L^*)$. That is, all firms will have weakly higher TFP in the high-demand economy than in the low-demand economy. Further, by Proposition 3, $A(\varphi, p_H^*) - A(\varphi, p_L^*)$ is larger for firms with lower levels of φ. If we interpret this exercise of unexpectedly (and permanently) changing aggregate demand as the dawning of a boom or a bust, the model predicts pro-cyclical within-firm productivity changes. Additionally, these productivity changes will be primarily centered around low-ability firms. Proposition 3 is therefore consistent with both Facts 1 and 2.

These patterns are driven by the fact that the productivity of the firms in the left tail of the productivity distribution is more sensitive to changes in future profits. The comparison of steady states can be viewed as the limiting case of a model with an aggregate demand state that follows a finite-state Markov process with persistence. Introducing such persistent demand fluctuations into this model is straightforward but involved. Optimal relational contracts in an environment with a Markovian public state variable are sequentially optimal (see Proposition 1 in Barron and Powell (2014)) and can therefore be replicated by a Markovian relational contract. The extension of the industry-equilibrium concept would be a Markovian REE in which all firms conjecture a Markov process for aggregate prices and choose optimal Markovian relational contracts, and output markets clear in each period. Under regularity conditions, the Markovian REE would be unique. In that equilibrium, productivity of low-ability firms would change more in response to a change in the demand state (because this change in demand would be associated with a directionally equivalent change in expectations of future profits), which would deliver a result analogous to Proposition 3.

6.2 Differences in Formal Contracting Institutions

Bartelsman, Haltiwanger, and Scarpetta (2013) and Hsieh and Klenow (2009) document both substantial dispersion in within-country productivity, controlling for industry composition, and heterogeneity in productivity dispersion across countries. Loosely speaking, there is more productivity dispersion in less-developed countries. Other authors have similarly documented "... huge variation among countries in the speed and quality of courts." (Djankov, La Porta, Lopez-de-Silanes, and Shleifer (2003)). The objective of this section is to connect these two sets of facts.

Proposition 4 shows that stronger formal contracts complement relational contracts, leading to improvements in firm-level productivity. This result is in line with Johnson, McMillan, and Woodruff's (2002) finding that "... entrepreneurs who say the courts are effective have measurably more trust in their trading partners..." and Laeven and Woodruff's (2007) finding that firms operating in Mexican states with stronger legal environments are more productive than those operating in states with weaker legal environments. With stronger formal contracting institutions, credibility becomes relatively less important for sustaining decentralization, consistent with the positive correlation between Kaufmann, Kraay, and Mastruzzi's (2006) country-level measure of "rule of law" and Bloom, Sadun, and Van Reenen's (2012) measure of decentralization in organizations. Since in equilibrium, low-ability firms are more constrained by lack of credibility, stronger formal contracting institutions disproportionately benefit such firms by allowing them to decentralize more.

In addition, Propositions 4 and 5 predict that in high-ω countries, relative to low-ω countries, (1) productivity dispersion will be lower, (2) the distribution of productivity will have a thinner left tail, and (3) output (firm-size) dispersion will be lower.

In order to examine (1), I gathered country-level measures of (a) labor productivity dispersion from Bartelsman, Haltiwanger, and Scarpetta (2013),[3] and (b) the quality of formal contracting institutions ("Rule of Law") from Kaufmann, Kraay, and Mastruzzi (2006).[4] Figure 2 plots labor productivity dispersion against the measure of formal contracting institutions and confirms that countries with higher measures of formal contracting institutions tend to have less productivity dispersion. The results are qualitatively similar using either of Djankov, La Porta, Lopez-de-Silanes, and Shleifer's (2003) measures of the quality of court enforcement.

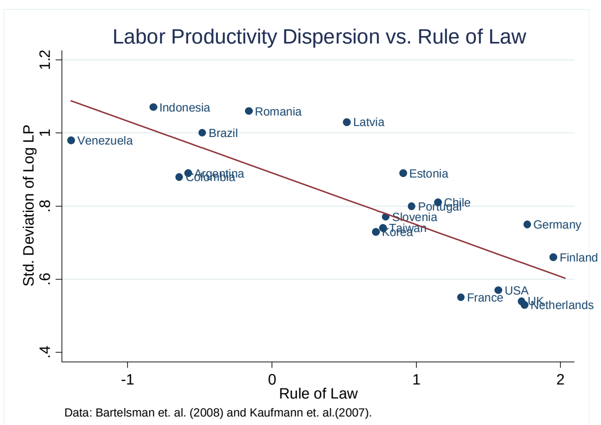

Figure 2

Figure 2 plots Bartelsman et. al.'s labor-productivity dispersion measure against Kaufmann et. al.'s Rule of Law measure

[3] Bartelsman, Haltiwanger, and Scarpetta (2013) construct a harmonized database (standardizing definitions for meaningful cross-country comparisons) that covers 24 industrial and emerging economies from the 1990s.

[4] This commonly used measure in the international trade literature is an aggregate survey indicator "measuring the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement... ." The values I use are from 2005.

Of course, "Rule of Law" is not the only factor that varies across countries. If, as we would expect, "Rule of Law" is highly correlated with the quality of capital markets, which in turn largely determine the level of productivity dispersion in a country, then Figure 2 may simply be capturing this relationship. I explore this possibility below.

For the second prediction, Figure 3 shows Hsieh and Klenow (2009)'s plots of the $\log(TFP)$ distributions in India and the U.S., controlling for industry composition. A striking feature is the thickness of the left tail of productivity in India relative to the U.S..

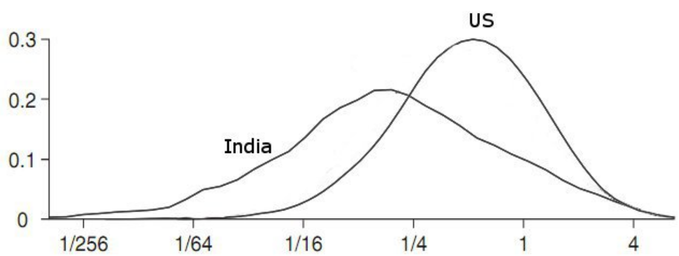

Figure 3

Figure 3 overlays the log(TFP) distribution for India and the US from Hsieh and Klenow

For prediction (3), the evidence is limited. Alfaro, Charlton, and Kanczuk (2008) show that *establishment* size is less variable in countries with higher GDP per capita (which is correlated with rule of law). Of course, it may be the case that, in countries with higher GDP per capita, firms expand by adding establishments rather than by expanding existing establishments. Others (Tybout (2000), Ayyagari, Demirguc-Kunt, and Beck (2007)) describe the phenomenon of the "missing middle" in developing countries: in high-income countries, medium-sized firms are responsible for a much larger share of GDP ($\approx 50\%$) than in low-income countries ($\approx 17\%$). Low-income countries tend to be dominated by firms that are either very small, often informal, or very large.

Though this model does not literally generate a "missing middle," it is straightforward to extend the model to allow for this possibility. In such an extension, firms can potentially choose between two technologies: one is a low productivity (traditional) technology that does not require the owner to decentralize decision-making to managers, and the other (modern) technology is given by the current model. For sufficiently weak contracting institutions, both types of firms could coexist in equilibrium–low-ability owners will choose the traditional technologies, and owners with sufficiently high ability to sustain decentralization will choose the modern technology. Improved formal contracting will cause some marginal traditional producers to switch to modern technologies.

This will lead to increased output and hence decreased prices, which could in turn drive out some of the less productive traditional producers.

There are, of course, other potential explanations for these cross-country facts, including differences in the quality of capital markets, differences in product-market competition, and differences in the underlying ability distribution across countries. To explore whether the relationship between productivity dispersion and Rule of Law is driven entirely by correlation between the quality of formal contracting institutions and capital markets, I collected Manova (2013)'s measure of "Private Credit," which proxies for the quality of capital markets.[5] I show in Figure 4 that the relationship between "Rule of Law" and productivity dispersion is robust to controlling for this measure. This result does not suggest that firm-level financial constraints and differences in capital markets are not important factors driving productivity dispersion differences but merely that other factors also appear to be important. Further, there may be important interactions between a firm's inability to make credible promises to its workers and its inability to access capital markets.

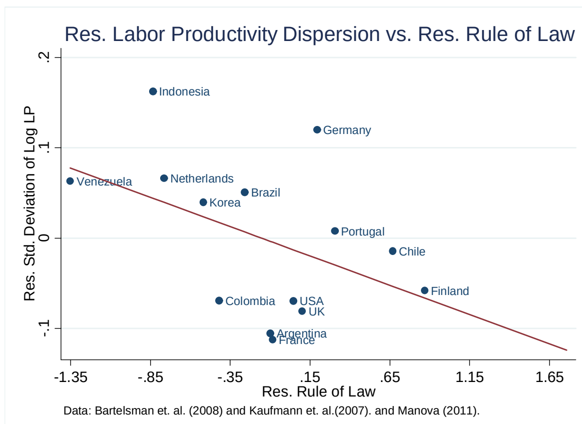

Figure 4

Figure 4 plots residuals from a regression of Bartelsman et. al.'s labor-productivity dispersion on Manova's measure of "private credit" against residuals from a regression of Kaufmann et. al.'s Rule of Law measure on "private credit."

It is also worth observing that, in countries with weak formal contracting institutions, an entrepreneur who is constrained by the credibility of her promises may pursue alternative strategies for relaxing this constraint. For example, she may purchase capital that is specific to the firm and therefore loses value if the firm is dissolved. In a multiproduct firm, she may leverage the profits

[5] "Private credit" is the amount of credit by banks and other financial intermediaries to the private sector as a share of GDP during the years 1985-1995.

earned in one product line as collateral for promises made to managers responsible for other product lines. Finally, she may hire managers with whom she interacts more frequently (perhaps relatives). The upshot is that conglomerates and family firms are likely to be more prevalent in countries with poor formal contracting institutions, consistent with evidence from La Porta, Lopez-de-Silanes, and Shleifer (1999).

Though privately (and potentially socially) beneficial, these alternative firm-level policies do not eliminate the inefficiency of the competitive equilibrium, however. A firm investing in capital that is otherwise suboptimally firm-specific will have inefficiently low capital productivity. A conglomerate pursuing breadth for the sake of leveraging its profits may crowd out more efficient (but narrow) producers of other goods. Firms may overemploy trustworthy family members, even if they are not a good fit for the job; further, skilled entrepreneurs may lack the familial connections necessary to profitably expand her enterprise to its optimal size. To the extent that a firm's size and scope is determined by factors orthogonal to its marginal profitability, the allocation of profits will be inefficient: some firms will be too small and others will be too large.

Identifying the nature of organizational decreasing returns to scale is an unresolved question and is beyond the scope of the current paper. Such decreasing returns to scale are, however, partly offset by the credibility channel, and this might lead to inefficiently large (or in a richer model with multiproduct firms, inefficiently broad) firms.

7 Conclusion

In order for a large firm to produce efficiently, the owner of the firm must decentralize daily operating decisions to individuals further down in the organization. Absent perfect formal contracts, decentralization requires trust: the owner must trust that the managers will not make reckless decisions for their own private gains, and the managers must trust that the owner will reward them appropriately for judiciously using the firm's resources. This paper views trust as credibility in a relational contract–the credibility of the owner's promises is derived from the value of the owner's reputation in the labor market. This value is, in turn, limited by the firm's potential future competitive rents. Competitive rents, credibility, and therefore firms' decentralization levels and hence productivity are jointly determined in industry equilibrium.

The theory of relational contracts generates a mechanism through which future profits can be an important determinant of current productivity, resulting in firm-level income effects that have efficiency consequences. These firm-level income effects are decreasing–the marginal returns to a dollar-a-day increase in profits is higher for less productive firms–and therefore, profits are inefficiently concentrated at the top in a competitive equilibrium. On the normative side, this view suggests that in weak formal-contracting environments, a progressive corporate tax may improve aggregate productivity by distorting production away from high average- but low marginal-productivity firms to low average- but high marginal-productivity firms.

On the positive side, low-ability firms face tighter credibility constraints, making their productivity more sensitive to the environment in which they operate. This effect forms the basis for two sets of empirical implications: (1) within-country, over time, and (2) across-country. Low-ability firms are uniformly more responsive to persistent changes in aggregate demand, which is consistent with micro evidence on firm-level productivity responses to business cycles (Baily, Bartelsman, and Haltiwanger (2001) and Kehrig (2015)). Improvements in the strength of formal contracting institutions reduce the importance of credibility in sustaining decentralization and therefore disproportionately improve the productivity of low-ability firms, leading to a reduction in the dispersion of productivity. Cross-country evidence supports the predictions of an upward compression of the left tail of the productivity distribution in high rule-of-law countries and a negative relationship between the strength of formal contracting institutions and productivity dispersion.

These patterns, while potentially of independent interest, are only indirect tests of the theory. The underlying causal mechanisms involved are (1) an increase in expected future profits increases current productivity and (2) the effect of an increase in future profits on current productivity is decreasing in future profits. An important future direction for the results in this paper is establishing direct evidence of these mechanisms.

This paper has focused on the distortions that arise in the steady state of an economy. Taking a more dynamic view, if we think of firm growth as being made possible only by non-contractible investments by a firm's managers, then the rate at which a firm grows may be limited by its medium-run profitability. Small, but productive, firms may be unable to grow, and as a result, there may be inefficiently slow industrial churn in countries with weak formal contracting institutions. Such a model may be able to generate results consistent with the recent Hsieh and Klenow (2014) facts on firm growth.

Appendix

Solution to the Model

Proposition 6. If production exhibits constant returns to scale in labor and management, there does not exist a stationary REE.

Proof of Proposition 6. Suppose production is $y_{it}(\delta_{it}, K_{it}, M_{it}) = \varphi_i \delta_{it}^{\theta} K_{it}^{\alpha} M_{it}^{1-\alpha}$. Then, in period t, the firm with the highest value of φ_i will continue to produce as long as $p_t y_{it} - RK_{it} - (W + \delta_{it}) M_{it} \geq 0$. Market clearing with finite demand thus implies that $p_t y_{it} - RK_{it} - (W + \delta_{it}) M_{it} = 0$ for all t. This in turn implies that the left-hand side of the dynamic enforcement constraint is equal to $-F$, which implies that no production can be sustained.■

Proposition 7. Suppose with probability q_O, deviations by the owner are publicly detected, and with probability q_M, deviations by a manager are publicly detected. Finally, suppose with probability $1 - q_X$, the firm exogenously is forced to exit the industry. Then the effective interest rate in (8) is $\tilde{r} = \frac{r}{q_O q_M q_X}$.

Proof of Proposition 7. If we rewrite (2) and (3) recognizing that (a) the owner will choose s_t to pin each manager to his (IR) constraint and that (b) the optimal relational contract will be stationary, and we introduce $q_O, q_M, q_X > 0$, these become

$$b_{im} \geq \frac{1}{q_M}\delta_{im} \text{ and } q_O q_X \frac{\pi_{im}}{r} \geq b_{im}.$$

If we pool these across agents, this becomes

$$q_O q_X \frac{\pi_{im}}{r} \geq \int b_{im} \geq \frac{1}{q_M} M_i \delta_i,$$

and therefore a reward scheme supporting δ_i exists if and only if $\pi_i \geq \frac{r}{q_O q_M q_X} M_i \delta_i \equiv \tilde{r} M_i \delta_i$, which is the desired result.■

Proposition 2. In this model, the solution to the constrained problem satisfies

$$\frac{\delta^*(\varphi)}{\delta^{FB}} = \frac{M^*(\varphi)}{M^{FB}(\varphi)} = \frac{K^*(\varphi)}{K^{FB}(\varphi)} = \mu^*(\varphi),$$

where $0 \leq \mu^*(\varphi) \leq 1$ is (weakly) increasing in p and (weakly) decreasing in R, W, and r. Further,

$$\mu^*(\varphi) = \begin{cases} 1 & \varphi \geq \varphi_H \\ \frac{1}{1+r}\left(1 + \left(1 - (\varphi_L/\varphi)^{1/\theta}\right)^{1/2}\right) & \varphi_L \leq \varphi < \varphi_H \\ 0 & \varphi < \varphi_L, \end{cases}$$

where

$$\begin{aligned} \varphi_L &= \frac{F^\theta}{p}\left((1+r)\frac{1}{\theta^2}\right)^\theta \left(\frac{R}{\alpha}\right)^\alpha \left(\frac{W}{1-\alpha-2\theta}\right)^{1-\alpha-2\theta} \\ \varphi_H &= \frac{F^\theta}{p}\left(\frac{1}{1-r}\frac{1}{\theta^2}\right)^\theta \left(\frac{R}{\alpha}\right)^\alpha \left(\frac{W}{1-\alpha-2\theta}\right)^{1-\alpha-2\theta}. \end{aligned}$$

Proof of Proposition 2. Throughout this proof, I drop the i subscript for the firm. Proposition 3 allows us to focus on the stationary problem. Manager symmetry and decreasing returns to utilization imply that $\delta_m = \delta$ for all $m \in [0, M]$. The firm's problem is then

$$\max_{K,M,\delta} p\varphi\delta^{\theta} K^{\alpha} M^{1-\alpha-\theta} - RK - (W+\delta) M - F$$

subject to

$$p\varphi\delta^{\theta} K^{\alpha} M^{1-\alpha-\theta} - RK - (W+\delta) M - F \geq rM\delta.$$

Since an increase in K increases the objective function as well as the left-hand side of the constraint, capital will be chosen efficiently, given M and δ. Define

$$\pi\left(K^*\left(M,\delta\right), M, \delta\right) = py\left(K^*\left(M,\delta\right), M, \delta\right) - RK^*\left(M,\delta\right) - \left(W+\delta\right)M - F.$$

The firm's problem is then to $\max \pi\left(K^*\left(M,\delta\right), M, \delta\right)$ subject to $\pi\left(K^*\left(M,\delta\right), M, \delta\right) \geq rM\delta$. Suppose the firm is constrained at the optimum. Define $M\left(\delta\right)$ such that the constraint holds with equality. The unconstrained problem is then

$$\max_{\delta} rM\left(\delta\right)\delta.$$

Taking first-order conditions, the firm chooses δ such that $\frac{M'(\delta)}{M(\delta)}\delta = -1$. Implicitly differentiating the constraint with respect to δ and substituting this into the first-order condition yields

$$\frac{py\left(K^*, M^*, \delta^*\right) - RK^*}{M^*} = \frac{1-\alpha}{1-\alpha-2\theta} W \tag{10}$$

and we know from the constraint that

$$\frac{py\left(K^*, M^*, \delta^*\right) - RK^*}{M^*} = \left(W + \left(1+r\right)\delta^*\right) + \frac{F}{M^*}. \tag{11}$$

(10) implies

$$M^*\left(\delta^*\right) = \left(\frac{1-\alpha-2\theta}{W}\right)^{\frac{1-\alpha}{\theta}} \left(pA\right)^{\frac{1}{\theta}} \left(\frac{\alpha}{R}\right)^{\frac{\alpha}{\theta}} \delta^*,$$

and substituting this into (11), we have that δ^* solves a quadratic equation. The linearity of $M^*\left(\delta^*\right)$ results from the assumption that production is constant returns to scale in $\left(K, M, \delta\right)$. Without this assumption, δ^* would be the solution to a nonlinear equation. If we define φ_L as in the statement of the proposition, the solution to this quadratic equation is

$$\delta^* = \frac{1}{1+r}\left(1 + \left(1 - \left(\varphi_L/\varphi\right)^{1/\theta}\right)^{1/2}\right).$$

Optimal capital and management are linear in δ^*. It is then easy to show that the constraint is binding for $\varphi < \varphi_H$. For $\varphi \geq \varphi_H$, the solution to the constrained problem is the same as the solution to the unconstrained problem.■

Structure of Inefficiencies

Theorem 2. Let $\mathcal{W}(\tau)$ be as defined in (9). Then $\mathcal{W}'(0) > 0$.

Proof of Theorem 2. At $\tau = 0$ and p^0, a marginal increase in τ leads to a reduction in production. In order for markets to clear, prices must increase. Thus, $\left.\frac{dp^\tau}{d\tau}\right|_{\tau=0,p^0} > 0$. We proceed by examining the effects of a marginal increase in τ from $\tau = 0$ on the four expressions in $\mathcal{W}(\tau)$. Since consumers have quasilinear preferences, the effect of a change in taxes on consumers is straightforward:

$$\left.\frac{d}{d\tau}\int_{p^\tau}^{\infty} D(p)\,dp\right|_{\tau=0,p^0} = -D\left(p^0\right)\left.\frac{dp^\tau}{d\tau}\right|_{\tau=0,p^0}.$$

Let $T(\varphi;\tau) = \pi^*(p^\tau,\varphi;0) - \pi^*(p^\tau,\varphi;\tau) - O\left(\tau^2\right)$ denote the revenues that the tax scheme generates from a firm of ability φ, so

$$\int_{\varphi_L(p^\tau)}^{\varphi_H(p^\tau)+\zeta} \pi^*(p^\tau,\varphi;0)\,d\Phi(\varphi) + \int_{\varphi_H(p^\tau)+\zeta}^{\infty} \pi^*(p^\tau,\varphi;\tau)\,d\Phi(\varphi) + T(\tau) = \int_{\varphi_L(p^\tau)}^{\infty} \pi^*(p^\tau,\varphi;0)\,d\Phi(\varphi).$$

Next, using Leibniz's rule,

$$\left.\frac{d}{d\tau}\int_{\varphi_L(p^\tau)}^{\infty} \pi^*(p^\tau,\varphi;0)\,d\Phi(\varphi)\right|_{\tau=0,p^0} = \left(S\left(p^0\right) + \Delta + E\left[\chi\left(p^0,\varphi;0\right)\middle|\,\varphi \geq \varphi_L\left(p^0\right)\right]\right)\left.\frac{dp^\tau}{d\tau}\right|_{\tau=0,p^0}$$

where $\Delta > 0$ and $\chi\left(p^0,\varphi;0\right) > 0$ and is decreasing in μ. Finally, since $S\left(p^0\right) = D\left(p^0\right)$,

$$\mathcal{W}'(0) = \left(\Delta + E\left[\chi\left(p^0,\varphi;0\right)\middle|\,\varphi \geq \varphi_L\left(p^0\right)\right]\right)\left.\frac{dp^\tau}{d\tau}\right|_{\tau=0,p^0} > 0,$$

which establishes the claim.■

Partial-Equilibrium Comparative Statics

Notation. Let $\underline{\mu}(\omega) = \frac{1}{1+(1-\omega)r}$. For applications with $\omega = 0$, $\underline{\mu} = \frac{1}{1+r}$. Let $\xi(\varphi,p,F,\omega) = \frac{\underline{\mu}(\omega)-\frac{1}{2}\mu(\varphi,p,\omega)}{\mu(\varphi,p,\omega)-\underline{\mu}(\omega)}$. For applications with $\omega = 0$, denote $\xi(\varphi,p,F) = \xi(\varphi,p,F,0)$. Finally, the following definitions will be useful in what follows

$$\begin{aligned}
\mu(\varphi,p,F,\omega) &= \frac{1+\left(1-\left(\frac{F^\theta}{p\varphi}\left(\frac{1+(1-\omega)r}{\theta^2}\right)^\theta\left(\frac{R}{\alpha}\right)^\alpha\left(\frac{W}{1-\alpha-2\theta}\right)^{1-\alpha-2\theta}\right)^{\frac{1}{\theta}}\right)^{\frac{1}{2}}}{1+(1-\omega)r} \\
y^{FB}(\varphi,p) &= \frac{1}{p}\delta^{FB}\left[p\varphi\left(\frac{\alpha}{R}\right)^\alpha\left(\frac{1-\alpha-2\theta}{W}\right)^{1-\alpha-\theta}\right]^{\frac{1}{\theta}} \\
Z_\varphi &= \theta\varphi;\ Z_p = \theta p;\ Z_{-F} = F;\ Z_\omega = \left(r\underline{\mu}\right)^{-1} \\
\Xi &= \xi(1+\xi)(1+2\xi).
\end{aligned}$$

Remark. For $X \in \{\varphi, p, -F\}$,

$$\frac{\partial \mu}{\partial X} = \frac{\mu \xi}{Z_X}; \ \frac{\partial \mu}{\partial \omega} = \frac{\mu \xi}{Z_\omega} \frac{1+\xi}{\xi},$$

and for $X \in \{\varphi, p, -F, \omega\}$, $\frac{\partial \xi}{\partial X} = -\frac{\Xi}{Z_X}$. Finally, note that

$$\frac{\partial y^{FB}}{\partial \varphi} = \frac{y^{FB}}{Z_\varphi}; \ \frac{\partial y^{FB}}{\partial p} = (1-\theta)\frac{y^{FB}}{Z_p}.$$

Lemma 1. $\log \mu (\varphi, p, F, \omega)$ is increasing in and exhibits decreasing differences in $(\varphi, p, -F, \omega)$.
Proof of Lemma 1. That $\log \mu$ is increasing in $(\varphi, p, -F, \omega)$ follows from their characterizations in the remark above. To examine decreasing differences, we simply must check the cross-partials. For $X \in \{\varphi, p, -F\}$, the first derivatives are

$$\frac{\partial \log \mu}{\partial X} = \frac{1}{\mu}\frac{\partial \mu}{\partial X}.$$

With some effort, it can be shown that for $X, Y \in \{\varphi, p, -F, \omega\}$, $X \neq Y$,

$$\frac{\partial^2 \log \mu}{\partial X \partial Y} = -\frac{\Xi}{Z_X Z_Y}.$$

Since $Z_X > 0$ for all $X \in \{\varphi, p, -F, \omega\}$ and $\Xi > 0$, $\frac{\partial^2 \log \mu}{\partial X \partial Y} < 0$ for all $X \neq Y$.■
Proposition 3. $\log A^* (\varphi, p, F)$ is increasing and exhibits decreasing differences in $(\varphi, -F, p)$.
Proof of Proposition 3. Since $\log A^* = \log \varphi + \theta \log \mu + \theta \log \delta^{FB}$, $\log A^*$ is increasing in $(\varphi, -F, p)$ since $\log \varphi$ is increasing in φ and $\log \mu$ is increasing in $(\varphi, -F, p)$ from the previous lemma. Since μ is the only term that contains interactions, $\log A^*$ exhibits decreasing differences in $(\varphi, -F, p)$ if $\log \mu$ exhibits decreasing differences in $(\varphi, -F, p)$, which it does by the previous lemma.■
Proposition 4. $\log y^* (\varphi, p, F, \omega)$ and $\log A^* (\varphi, p, F, \omega)$ are increasing in ω and exhibit decreasing differences in (φ, ω).
Proof of Proposition 4. Note that

$$\begin{aligned} \log A^* &= \log \varphi + \theta \log \mu + \theta \log \delta^{FB} \\ \log y^* &= \log y^{FB} + \log \mu. \end{aligned}$$

y^{FB} does not depend directly on ω. Since μ is increasing in ω, $\log A^*$ and $\log y^*$ are increasing in ω. The only terms in $\log A^*$ and $\log y^*$ that depend both on φ and ω are the $\log \mu$ term. We know from the lemma that $\log \mu$ exhibits decreasing differences in (φ, ω). The proposition then follows.■

Industry-Equilibrium Comparative Statics

This section provides a proof of proposition 5. It proceeds first by establishing three lemmas. The first lemma connects the equilibrium price response to properties of the industry supply and demand curves. The second lemma shows that when price effects are small (large), the equilibrium ability cutoff is decreasing (increasing) in the strength of formal contracts. The third lemma shows that the lowest observed productivity level will be lower when formal contracts are weaker. Lemma 4 shows that the slope of TFP with respect to ability is higher when formal contracts are weaker. These lemmas are used in the proof of proposition 5.

Lemma 2. Let p^{ω} solve $D(p^{\omega}) = S(p^{\omega}, \omega)$. Then

$$\left.\frac{dp}{d\omega}\frac{\omega}{p}\right|_{p^{\omega}} = -r\omega\theta\underline{\mu}\frac{\frac{\partial S}{\partial p}p + D}{\frac{\partial S}{\partial p}p + |\varepsilon_{D,p}|\, D}.$$

Proof of Lemma 2. If we totally differentiate the market-clearing condition and rearrange, we get the following

$$\frac{dp^{\omega}}{d\omega}\frac{\omega}{p} = -\frac{\frac{\partial S}{\partial \omega}\omega}{\frac{\partial S}{\partial p}p + \left|\frac{dD}{dp}\right| p}$$

We now seek to derive a relationship between $\frac{\partial S}{\partial \omega}\omega$ and $\frac{\partial S}{\partial p}p$. Supply is

$$S(p,\omega) = \int_{\varphi_L(p,\omega)}^{\infty} y^*(\varphi, p, \omega)\, g(\varphi)\, d\varphi$$

and therefore, using Leibniz's rule,

$$\begin{aligned}
\frac{\partial S}{\partial p} &= -\frac{\partial \varphi_L}{\partial p} y^*(\varphi_L, p, \omega)\, g(\varphi_L) + \int_{\varphi_L}^{\infty} \frac{\partial}{\partial p}\left(\mu y^{FB}\right) g(\varphi)\, d\varphi \\
\frac{\partial S}{\partial \omega} &= -\frac{\partial \varphi_L}{\partial \omega} y^*(\varphi_L, p, \omega)\, g(\varphi_L) + \int_{\varphi_L}^{\infty} \frac{\partial}{\partial \omega}\left(\mu y^{FB}\right) g(\varphi)\, d\varphi
\end{aligned}$$

Recall that $\varphi_L(p,\omega) = (1 + (1-\omega) r)^{\theta}\, \varphi_S(p)$, so that

$$\begin{aligned}
\frac{\partial \varphi_L}{\partial p}\frac{p}{\varphi_L} &= -1; \ \frac{\partial \varphi_L}{\partial \omega}\frac{\omega}{\varphi_L} = -\theta\omega r\underline{\mu} \\
\frac{\partial}{\partial p}\left(\mu y^{FB}\right) &= \frac{y^*}{p}\left[\frac{1+\xi}{\theta} - 1\right],
\end{aligned}$$

we get

$$\begin{aligned}
\frac{\partial S}{\partial p}p &= \varphi_L y^*(\varphi_L, p, \omega)\, g(\varphi_L) + \int_{\varphi_L}^{\infty}\left[\frac{1+\xi}{\theta} - 1\right] y^* g(\varphi)\, d\varphi \\
\frac{\partial S}{\partial \omega}\frac{\omega}{r\omega\theta\underline{\mu}} &= \varphi_L y^*(\varphi_L, p, \omega)\, g(\varphi_L) + \int_{\varphi_L}^{\infty}\frac{1+\xi}{\theta} y^* g(\varphi)\, d\varphi
\end{aligned}$$

Finally, note that

$$\frac{\partial S}{\partial \omega}\frac{\omega}{r\omega\theta\underline{\mu}} - \frac{\partial S}{\partial p}p = S(p,\omega).$$

At $p = p^{\omega}$, $S(p^{\omega}, \omega) = D(p^{\omega})$, so the result follows.■

Lemma 3. If $|\varepsilon_{D,p}| > 1$, then $\varphi_L^0 > \varphi_S^1$. If $|\varepsilon_{D,p}| < 1$, then $\varphi_L^0 < \varphi_S^1$.

Proof of Lemma 3. We know that $\varphi_L(p^\omega,\omega) = (1+(1-\omega)r)^\theta \varphi_S(p^\omega)$ and

$$\frac{d\varphi_S(p^\omega)}{d\omega} = -\frac{\varphi_S(p^\omega)}{p^\omega}\frac{dp^\omega}{d\omega}.$$

Therefore,

$$\begin{aligned}\frac{d\varphi_L}{d\omega} &= -r\theta(1+(1-\omega)r)^{\theta-1}\varphi_S(p^\omega) + (1+(1-\omega)r)^\theta \frac{d\varphi_S}{d\omega} \\ &= -\frac{\varphi_L(p^\omega,\omega)}{\omega}\left(r\omega\theta\underline{\mu} + \frac{\omega}{p^\omega}\frac{dp^\omega}{d\omega}\right) = \frac{\varphi_L(p^\omega,\omega)}{\omega} r\omega\theta\underline{\mu}\left(\frac{D(1-|\varepsilon_{D,p}|)}{\frac{\partial S}{\partial p}p + |\varepsilon_{D,p}|D}\right).\end{aligned}$$

By definition, $\varphi_S^1 = \varphi_L^1$, and by the fundamental theorem of calculus,

$$\varphi_L^1 = \varphi_L^0 + \int_0^1 \frac{d\varphi_L}{d\omega}d\omega.$$

This is less than φ_L^0 if $|\varepsilon_{D,p}| > 1$, so that $\frac{d\varphi_L}{d\omega} < 0$ for all ω, and it is greater than φ_L^0 if $|\varepsilon_{D,p}| < 1$, so that $\frac{d\varphi_L}{d\omega} > 0$ for all ω.■

Lemma 4. $\varphi_L^0\mu(\varphi_L^0)^\theta(\delta^{FB})^\theta < \varphi_S^1(\delta_{FB})^\theta$.

Proof of Lemma 4. We know that $\mu(\varphi_L^0) = \frac{1}{1+r}$ and $\varphi_L = (1+r)^\theta\varphi_S$. This implies that

$$\varphi_L^0\mu(\varphi_L^0)^\theta(\delta^{FB})^\theta = \varphi_S^0(\delta^{FB})^\theta < \varphi_S^1(\delta^{FB})^\theta,$$

where in the last inequality I used the facts that $p^1 < p^0$ and therefore $\varphi_S^1 > \varphi_S^0$.■

Lemma 5. $\varphi\mu(\varphi,p,F,0)$ increases faster than φ for $\varphi \geq \varphi_L^0$.

Proof of Lemma 5. We know that

$$\frac{d}{d\varphi}(\varphi - \varphi\mu) = 1 - \mu^\theta\left[1 + \theta\frac{d\mu}{d\varphi}\frac{\varphi}{\mu}\right] = 1 - \mu^\theta(1+\xi)$$

If this expression is negative, then $\frac{d}{d\varphi}\varphi < \frac{d}{d\varphi}\varphi\mu(\varphi,p,F,0)$. Note that

$$\frac{\partial}{\partial\varphi}\mu^\theta(1+\xi) = -\mu^{\theta-1}(1-\theta+2\xi)(1+\xi)\frac{\partial\mu}{\partial\varphi} < 0,$$

so $1-\mu^\theta(1+\xi)$ is minimized at φ_H (and all $\varphi > \varphi_H$), where it equals $\frac{\frac{1}{2}-\underline{\mu}}{1-\underline{\mu}}$, which is positive, since $\underline{\mu} < \frac{1}{2}$ since $r < 1$.■

Proposition 5. Suppose that either (a) φ has a log-convex distribution and $|\varepsilon_{D,p}| > 1$ or (b) φ has a log-concave distribution and $|\varepsilon_{D,p}| < 1$. Then $Var(A^*(\varphi,p^\omega,\omega)|\,\varphi \geq \varphi_L^\omega)$ is greater for $\omega = 0$ than for $\omega = 1$. Additionally, there exists some $\hat{\varphi}^\omega$ such that $y^*(\varphi,p^\omega,\omega)$ is increasing in ω for $\varphi < \hat{\varphi}^\omega$ and decreasing in ω for $\varphi > \hat{\varphi}^\omega$.

Proof of Proposition 5. From the previous lemma, we know that $\left|\frac{dA^0}{\partial\varphi}\right| < \left|\frac{dA^1}{\partial\varphi}\right|$ for all $\varphi \geq \varphi_L^0$. Tang and See (2009) show that if f and g are functions of a random variable and $|f| < |g|$ almost everywhere, then $Var(f) < Var(g)$. This implies that

$$Var\left(A^0(\varphi)\middle|\,\varphi \geq \varphi_L^0\right) > Var\left(A^1(\varphi)\middle|\,\varphi \geq \varphi_L^0\right) = \left(\delta^{FB}\right)^{2\theta} Var\left(\varphi|\,\varphi \geq \varphi_L^0\right).$$

If $|\varepsilon_{D,p}| > 1$ and φ is log-convex, then we have that $Var(\varphi|\,\varphi \geq k)$ is increasing in k (see Burdett 1996), and the result follows, since $\varphi_L^0 > \varphi_S^1$. If $|\varepsilon_{D,p}| < 1$ and φ is log-concave, then $Var(\varphi|\,\varphi \geq k)$ is decreasing in k and the result follows since $\varphi_L^0 < \varphi_S^1$.

For the second result, let $y(\varphi, p^\omega, \omega) = \mu(\varphi, p^\omega, \omega)\, y^{FB}(\varphi, p^\omega)$ for $\varphi < \varphi_H(p^\omega, \omega)$ and $y(\varphi, p^\omega, \omega) = y^{FB}(\varphi, p^\omega)$ for $\varphi > \varphi_H(p^\omega, \omega)$. For $\varphi < \varphi_H$,

$$\begin{aligned}\frac{dy}{d\omega} &= \underbrace{\frac{\partial\mu}{\partial\omega}y^{FB}}_{>0} + \underbrace{\left(\frac{\partial\mu}{\partial p}y^{FB} + \mu\frac{\partial y^{FB}}{\partial p}\right)\frac{dp}{d\omega}}_{<0} \\ &= r\underline{\mu}\left(\frac{\left(|\varepsilon_{D,p}| - 1\right)D}{\frac{\partial S}{\partial p}p + |\varepsilon_{D,p}|\,D}(1+\xi) + \frac{\frac{\partial S}{\partial p}p + D}{\frac{\partial S}{\partial p}p + |\varepsilon_{D,p}|\,D}\theta\right)\mu y^{FB}\end{aligned}$$

and for $\varphi \geq \hat{\varphi}^\omega = \varphi_H$, $\frac{dy}{d\omega} = \left(\frac{\partial\mu}{\partial p}y^{FB} + \mu\frac{\partial y^{FB}}{\partial p}\right)\frac{dp}{d\omega}$. When $|\varepsilon_{D,p}| > 1$, all firms with $\varphi < \hat{\varphi}^\omega$ expand production, and all firms with $\varphi > \hat{\varphi}^\omega$ reduce production. When $|\varepsilon_{D,p}| < 1$, there is a cutoff value $\hat{\varphi}^\omega < \varphi_H$ such that all firms with $\varphi < \hat{\varphi}^\omega$ expand production and all firms with $\varphi > \hat{\varphi}^\omega$ reduce production.$\blacksquare$

Remark. In fact the Burdett 1996 result shows that a sufficient condition for $Var(\varphi|\,\varphi \geq k)$ to be increasing in k is that the triple cumulative integration of φ is log-convex, which is a significantly weaker condition.

Remark. For the $|\varepsilon_{D,p}| > 1$ case, it can be seen that this result will hold for distributions that are not "too log-concave" in the sense that all that is required is that

$$\left[\left.\frac{d\left(\varphi\mu\left(\varphi, p^0\right)^\theta\right)}{d\varphi}\right|_{\varphi^*}\right]^2 Var\left(\varphi|\,\varphi \geq \varphi_L^0\right) > Var\left(\varphi|\,\varphi \geq \varphi_S^1\right),$$

where φ^* is the approximation point for a variance approximation. Similarly, if $|\varepsilon_{D,p}| < 1$, then this result will hold for distributions that are not "too log-convex."

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