Policies in Relational Contracts

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Managers Motivate...

Managers motivate agents in long-term relationships

Managers Motivate... and Manage

Managers motivate agents in long-term relationships

Managers implement policies

These policies are sometimes biased towards some agents over others

- Firms delay hiring when demand increases (Ariely, et al., 2014)
- Promotions fall prey to the "Peter Principle"
- Supplier allocations are skewed (Asanuma, 1989)
- Capital allocations are biased (Graham, et al., 2013)

Biased Policies in Relational Contracts

How should policies be set in a relational contract?

- With multiple agents, future decisions determine future surplus produced by each
- Relational contract: agent's future surplus make current incentives credible
- Trade-off: decisions tomorrow affect surplus tomorrow **and** credibility today

Favoring one agent in future might lead to lower total surplus

- Biased policies → lower total surplus, stronger incentives credible to favored agent
- Can be optimal even if principal can pay bonuses to / demand fines from agents

Private monitoring: agents cannot coordinate to punish principal

- "Bilateral relationships:" principal can betray one agent without alerting others

Outline of Results

General model of policies in relational contracts

- Principal chooses a decision (production function) in each period
- This decision determines how agent efforts map to agent outputs

Result: biased decisions optimal in class of games

- Look at recursive equilibrium refinement
- Agents who performed well in the past produce "too much" surplus relative to continuation play that maximizes total surplus
- Agents who perform poorly produce "too little"

Applications and Extensions:

- Show equilibrium refinement is not driving result
- "Jobless recoveries": hiring lags increase in demand
- "Distorted investment": irreversibly invest in less productive agent

Agenda

- Illustrative Example
- The Model
- General Results
- Applications
- Extensions

Example Illustrating Mechanism

One principal, two agents – risk-neutral, discount δ

- Agent i observes only own wage, output, bonus
- Principal observes everything except efforts

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In first period:

1. Principal and each agent exchange wage payments $w_{i,t} \in \mathbb{R}$
2. Agent i privately chooses effort $e_{i,t} \in \{0,1\}$ at cost $ce_{i,t}$
3. From i, principal earns output $y_{i,t} \in \{0, H_i\}$, $\Pr[H_i] = pe_{i,t} < 1$
4. Principal and each agent exchange bonus payments $\tau_{i,t} \in \mathbb{R}$

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At beginning of second period...

- Principal chooses one of the agents (agent i with probability q_i)
- Plays game repeatedly with chosen agent

Reward Schemes Provide Incentives

Payoffs:

$$\pi = (1-\delta)\sum_{i=1}^{2}(y_{i,t} - w_{i,t} - \tau_{i,t})$$

$$\mathrm{u_i} = (1-\delta)(w_{i,t} + \tau_{i,t} - ce_{i,t})$$

Assume chosen agent can be motivated from second period onwards

What motivates agent i in first period? Following output y,

$$B_i(y) = (1-\delta)\tau_{i,t} + \delta U_{i,t}$$

What Can the Principal Do?

Agent i works hard if:

$$E[B_i(H_i, y_{-i,t})] - E[B_i(0, y_{-i,t})] \geq (1-\delta)\frac{c}{p}$$

Lack of commitment constrains $B_i(y)$:

$$0 \leq B_i(y) \leq q_i \delta [pH_i - c]$$

- Agent i can walk away by choosing $e_i = 0$: $B_i \geq 0$
- Principal can renege on each agent separately: $B_i(y) \leq q_i \delta [pH_i - c]$

Why are These Constraints Sufficient?

$$0 \leq B_i(y) \leq q_i \delta [pH_i - c]$$

In second period onwards...

- If agent i chosen, total surplus is $pH_i - c$
- Transfers split this surplus between principal and chosen agent

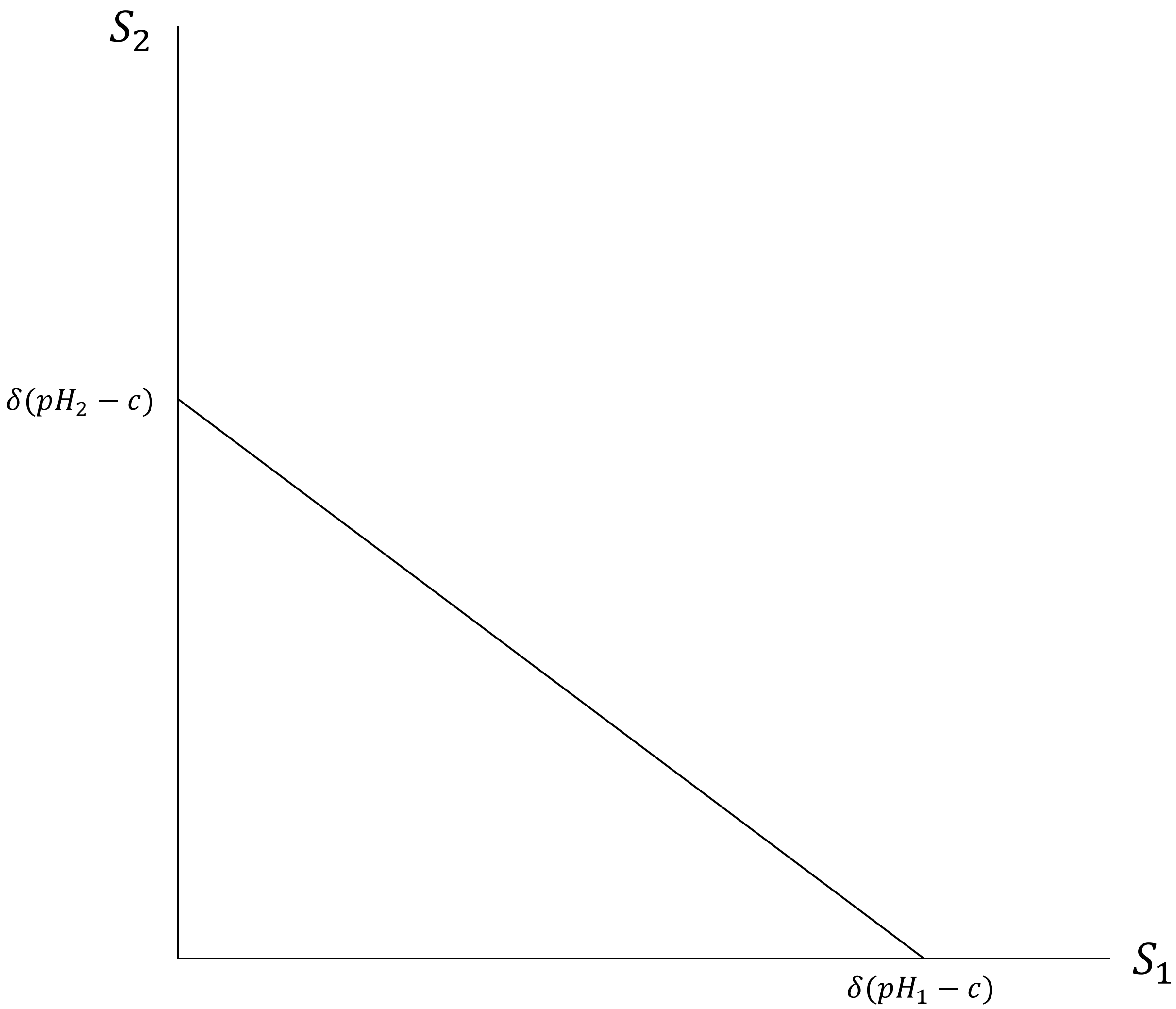
Split surplus so principal earns 0, chosen agent earns $pH_i - c$

- Principal willing to choose either player

Then in first period, $\tau_{i,t}$ used to punish agent

- Punish: $\tau_{i,t} = -\frac{\delta}{1-\delta} q_i [pH_i - c]$ yields $B_i = 0$
- Reward: $\tau_{i,t} = 0$ yields $B_i = q_i \delta [pH_i - c]$

Dynamic Enforcement Constraint



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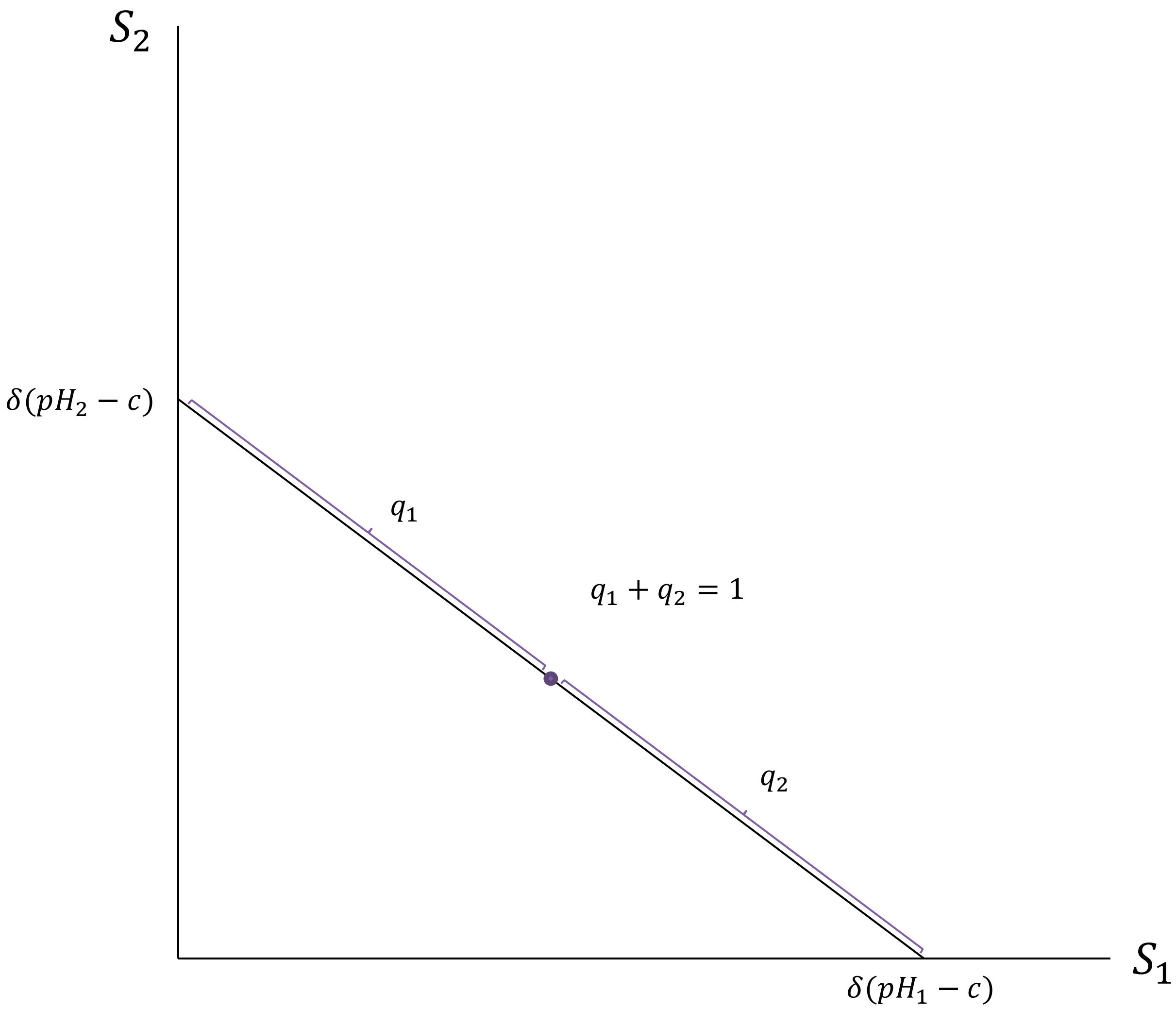

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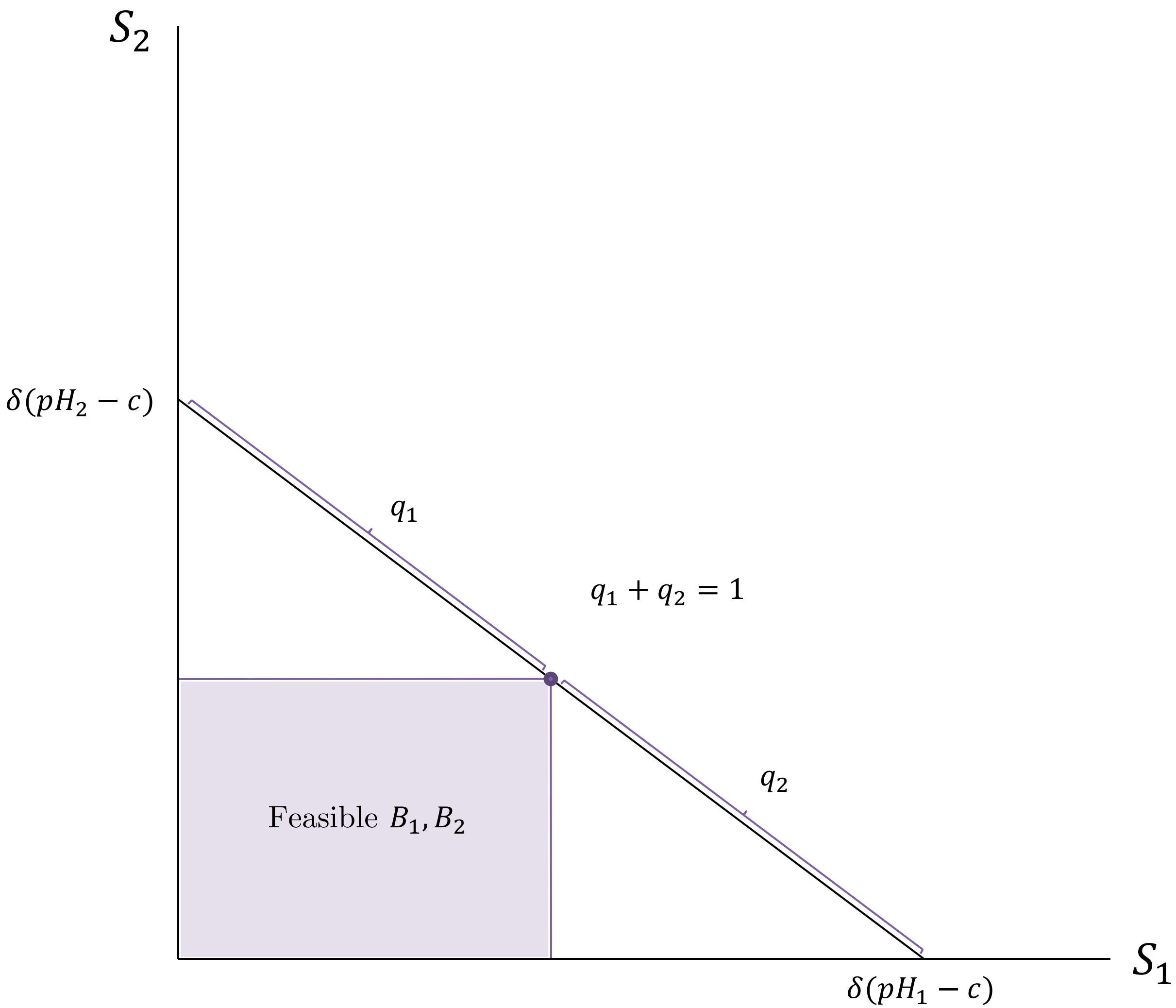
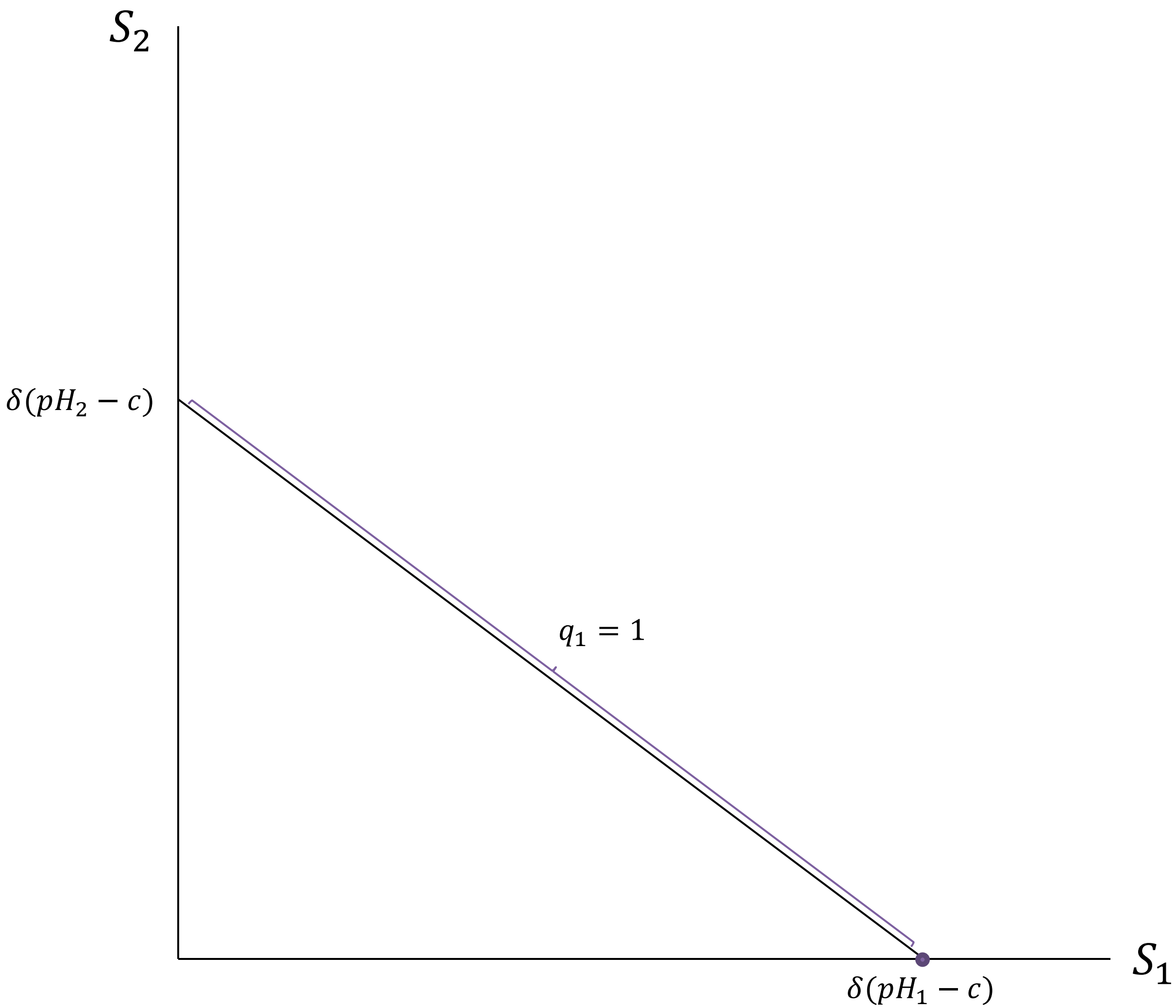


$$0 \leq B_1(y) \leq \delta(pH_1 - c)q_1$$

$$0 \leq B_2(y) \leq \delta(pH_2 - c)q_2$$

Option 1: Ex-post Efficiency



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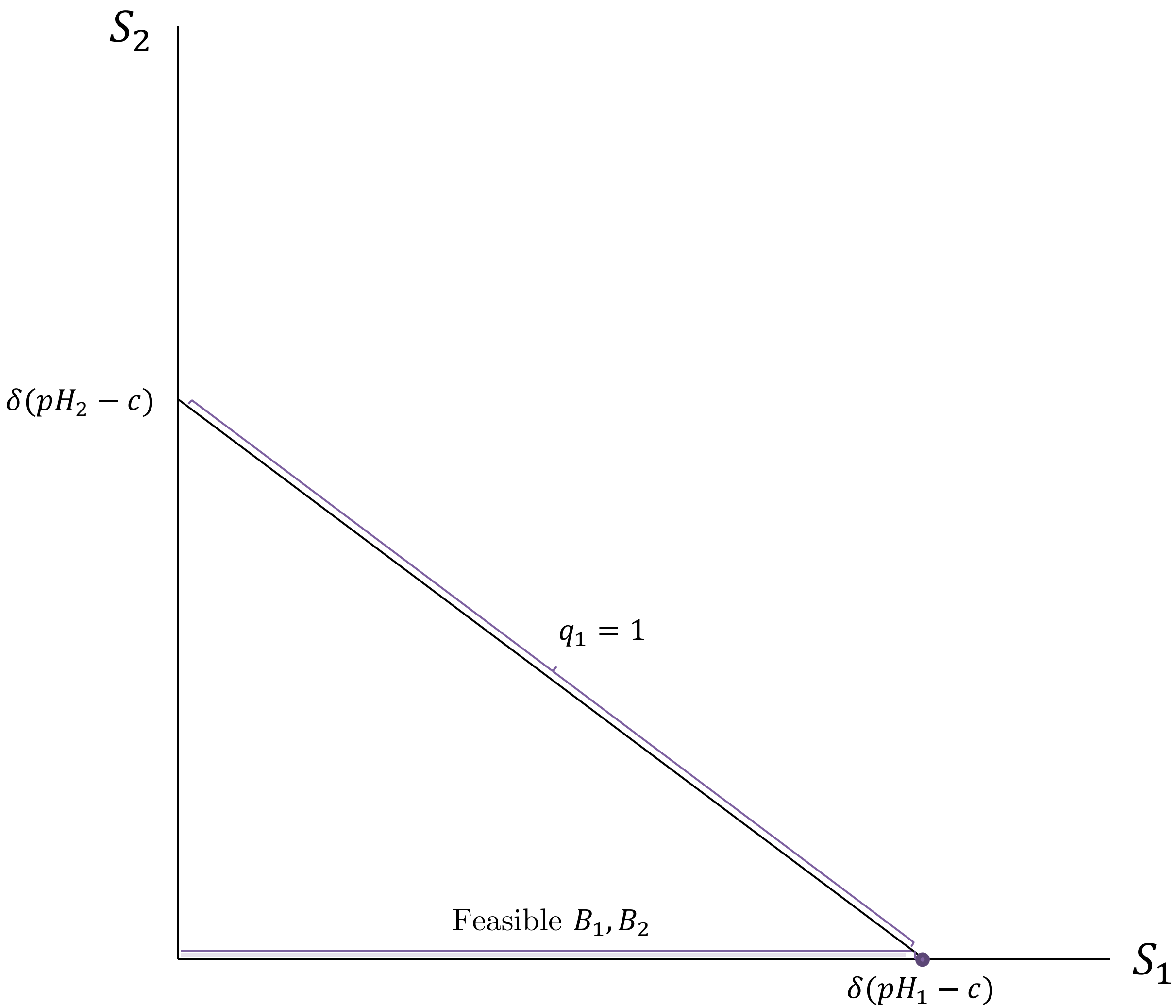

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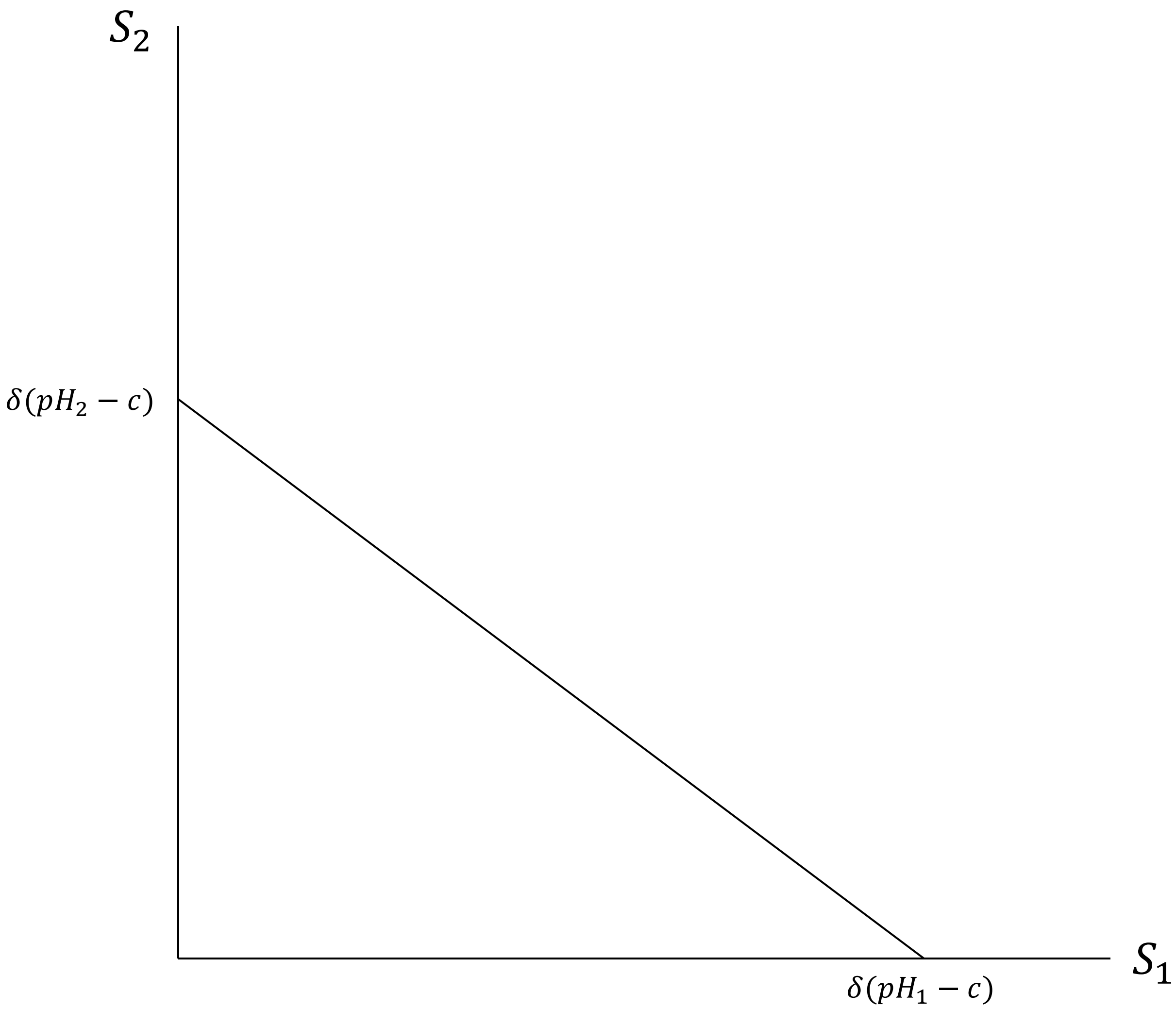

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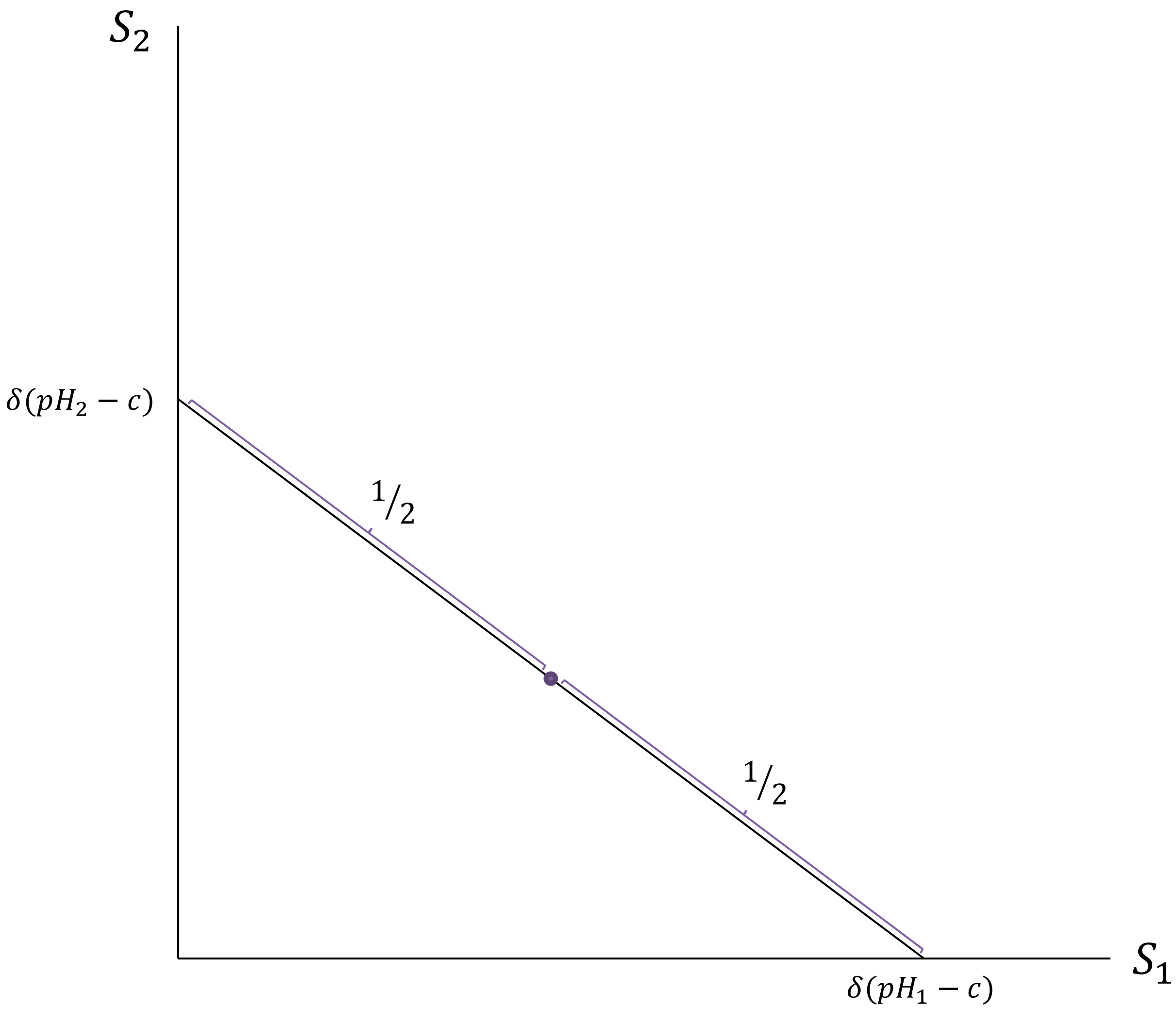

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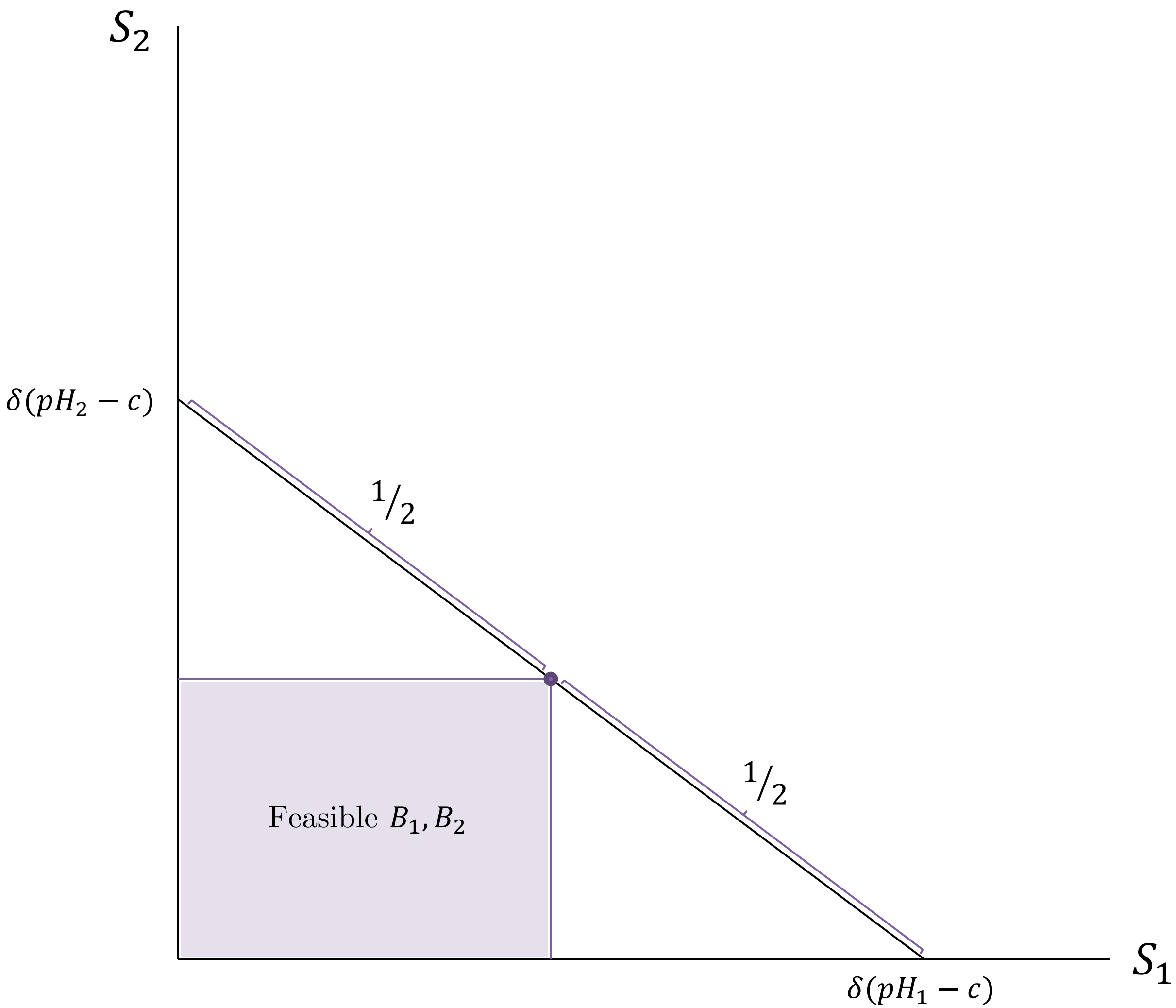

$$0 \leq B_1(y) \leq \delta(pH_1 - c)1/2$$

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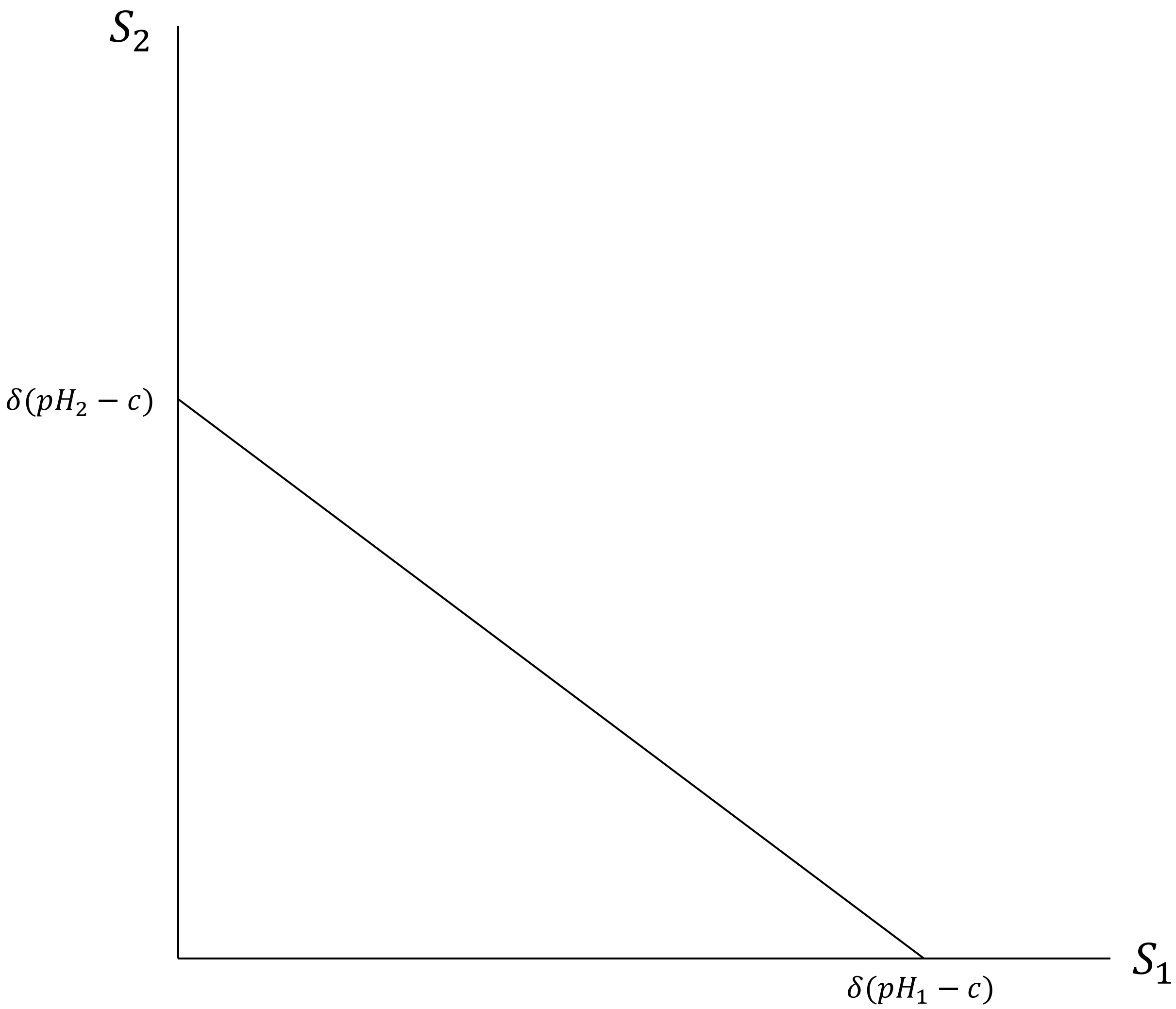
Option 3: History-Dependent Inefficiencies



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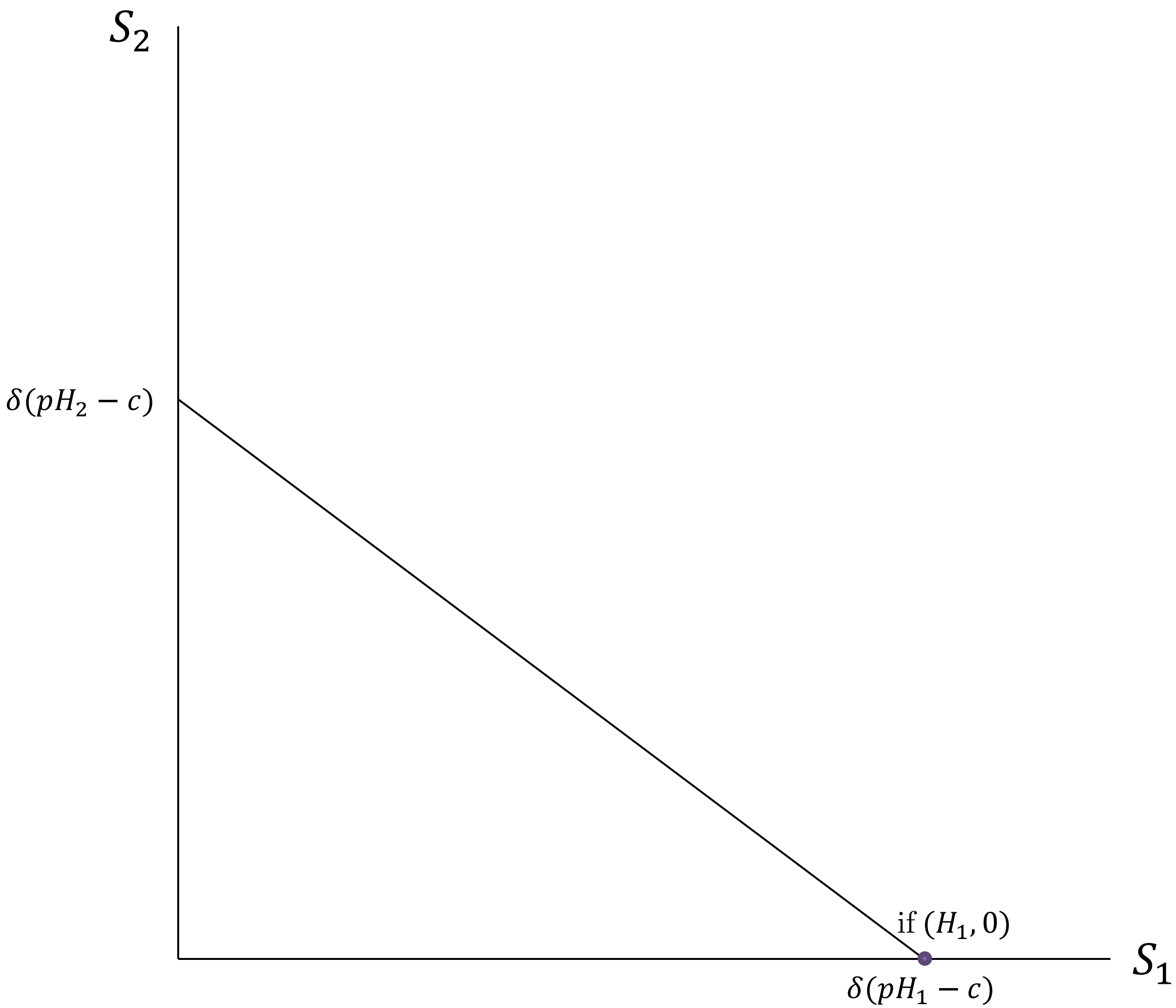

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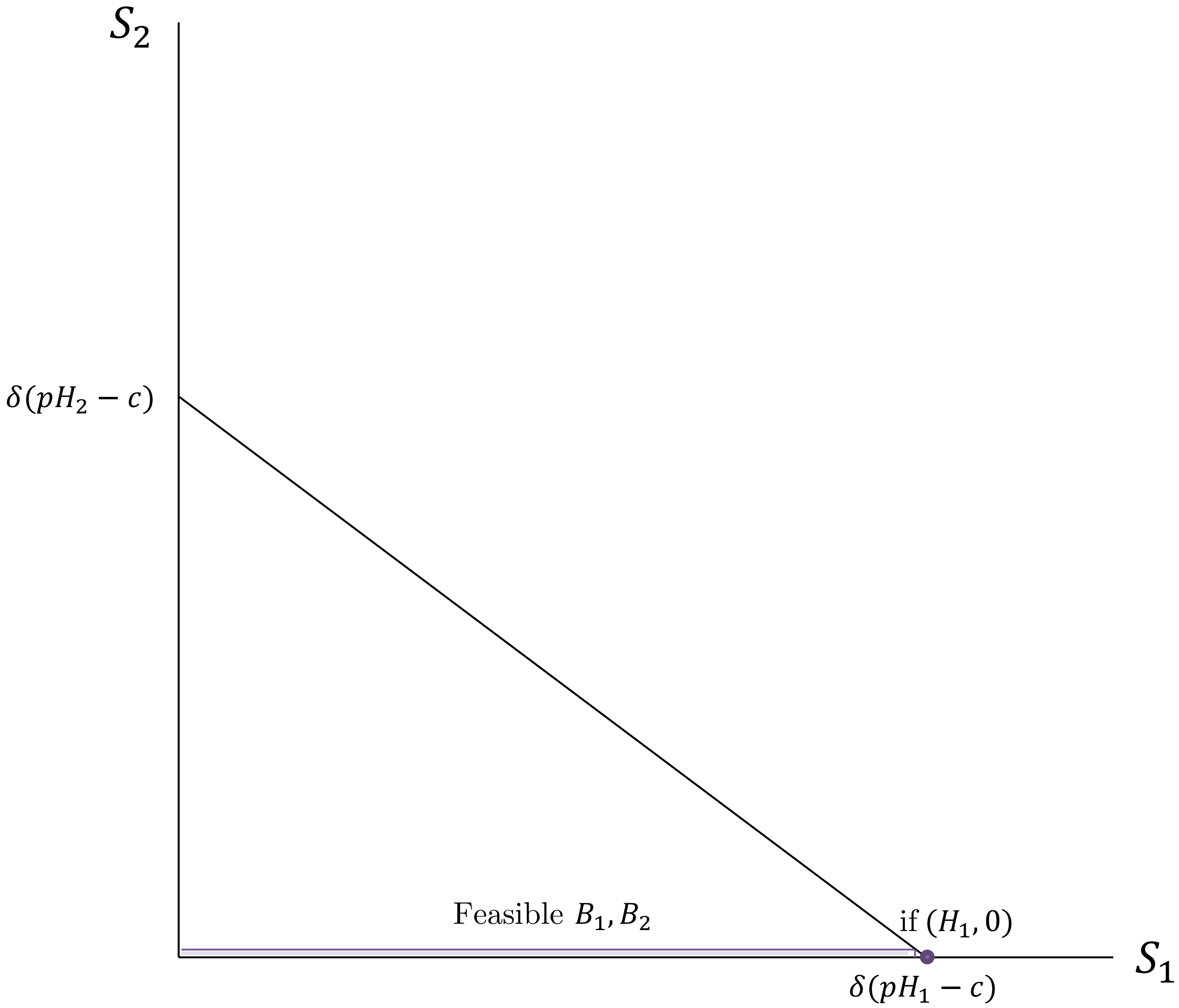

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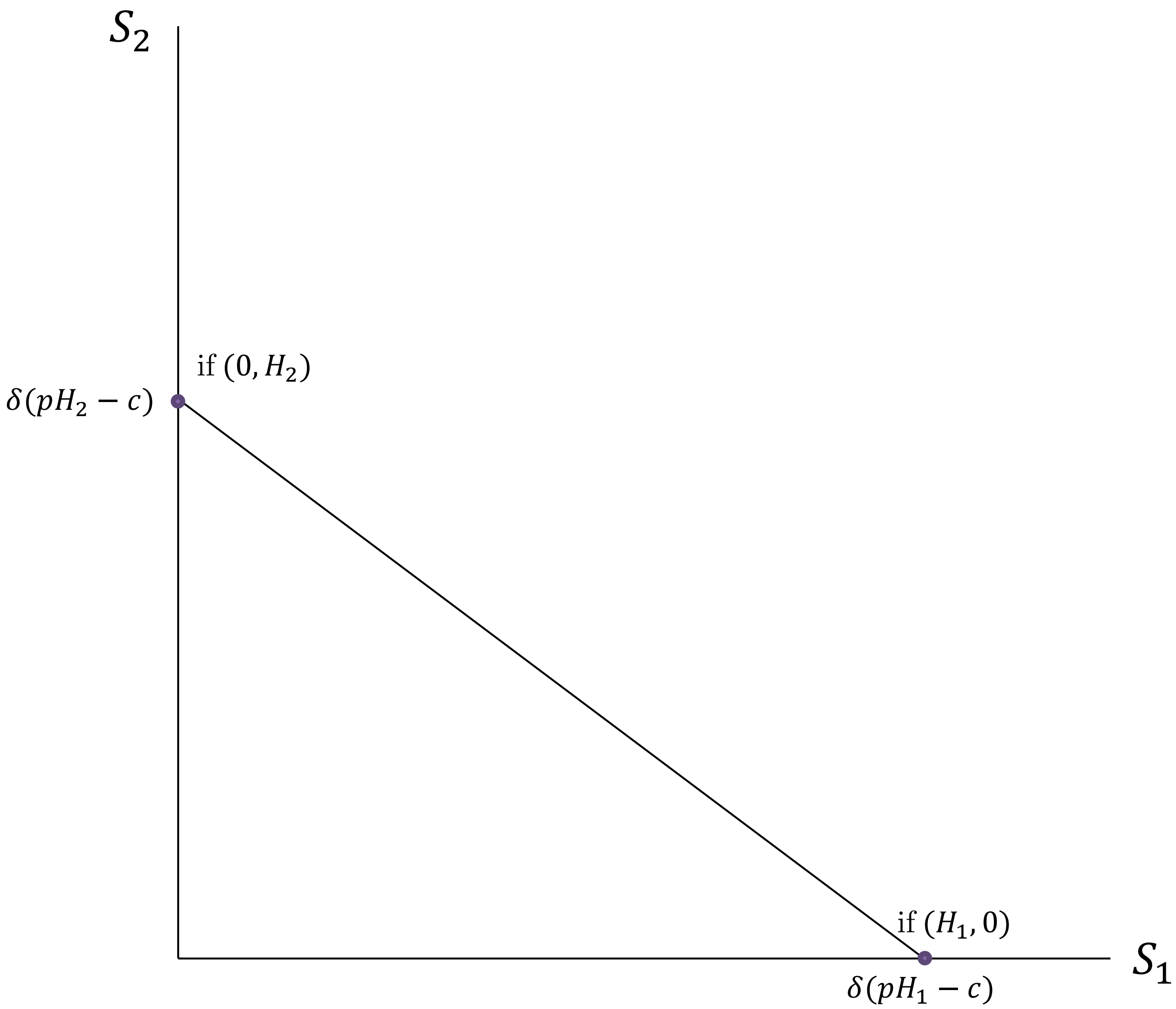

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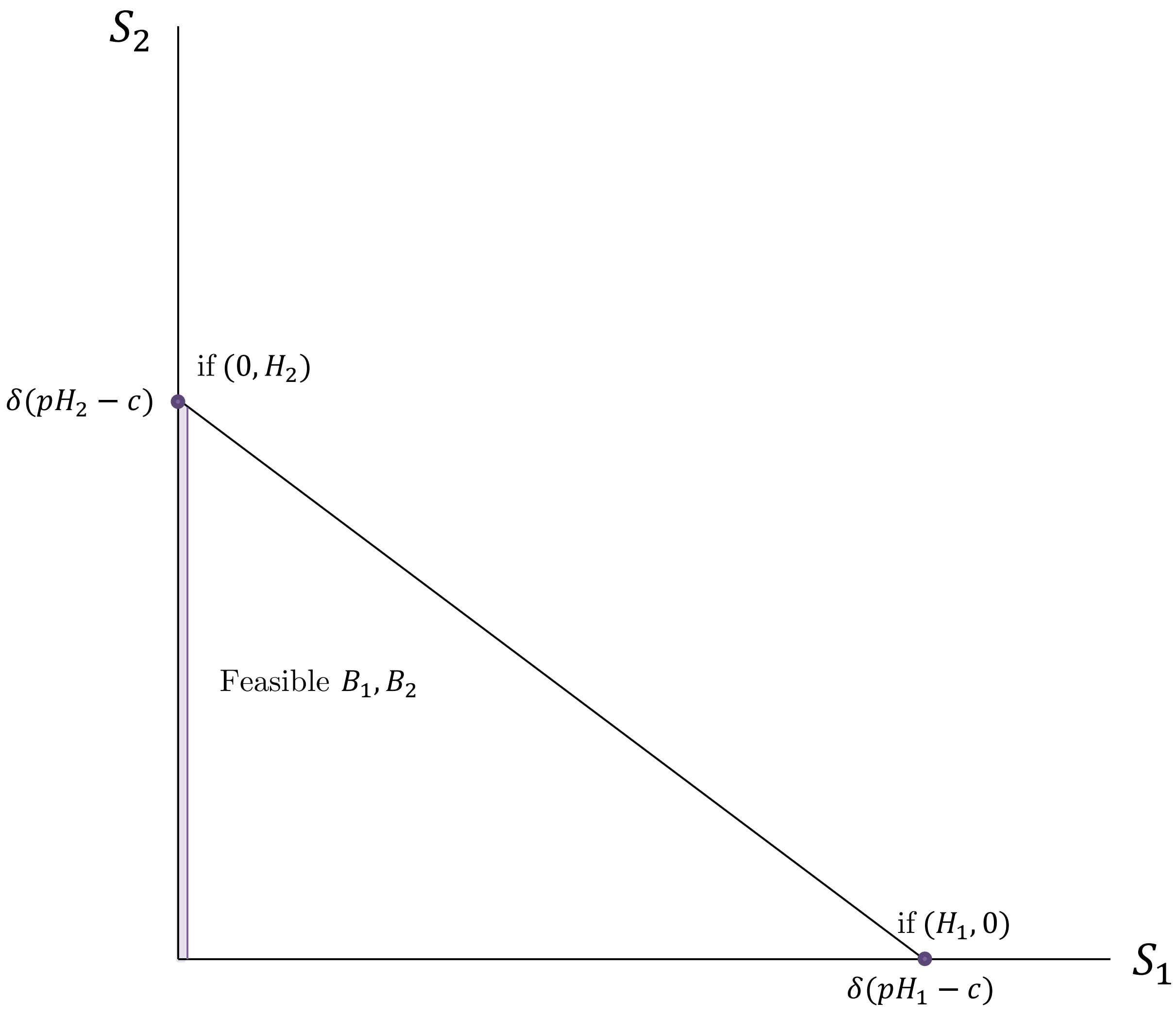

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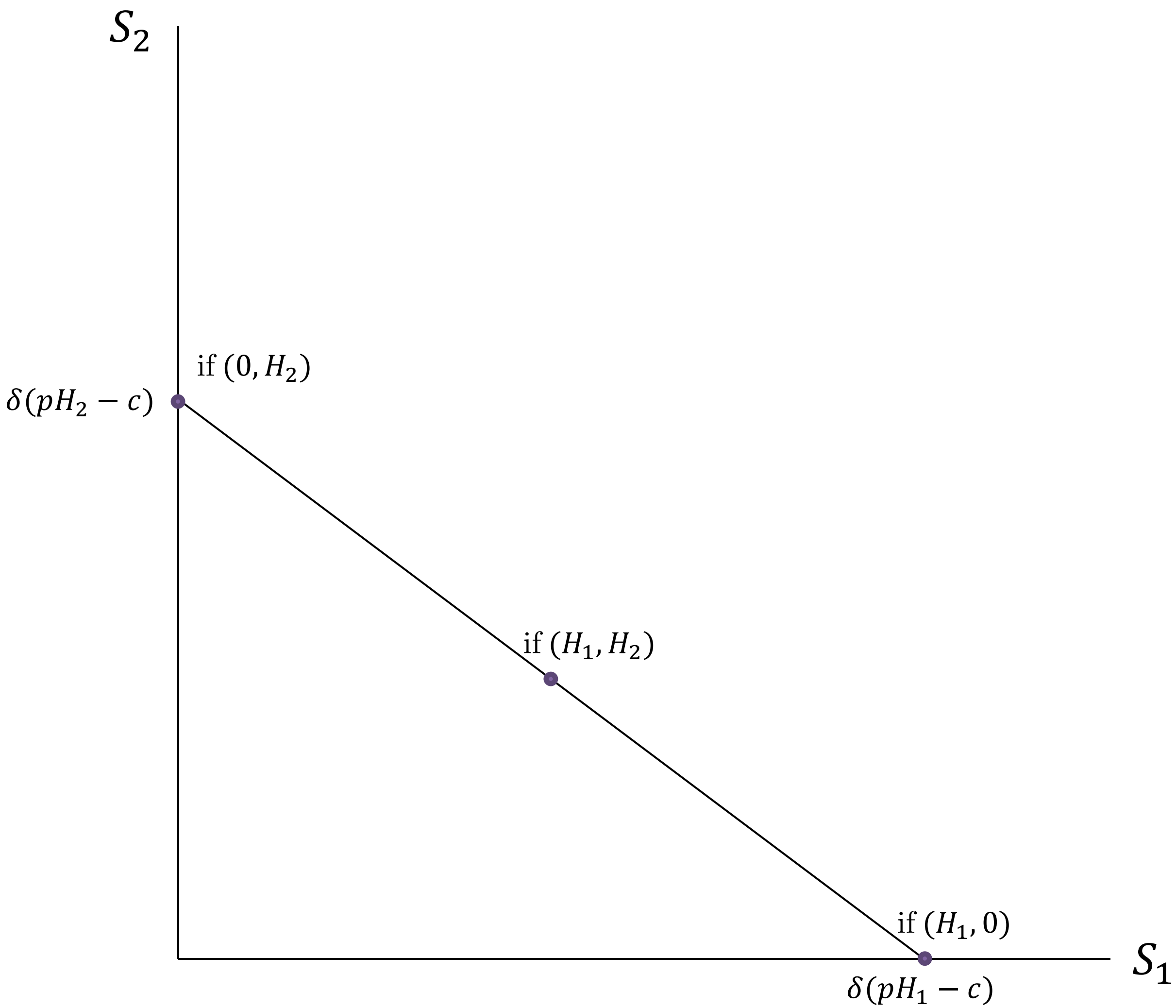

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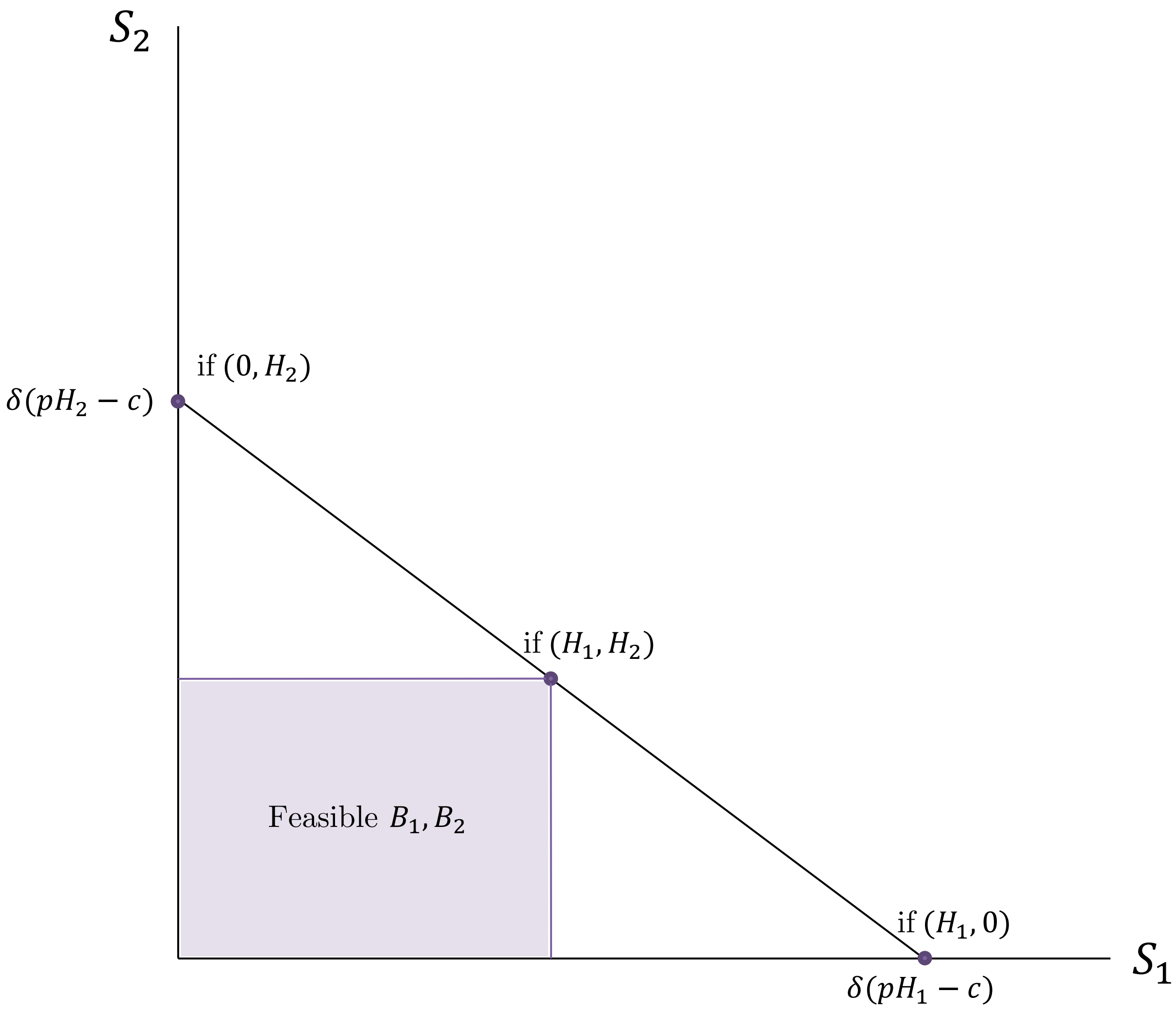

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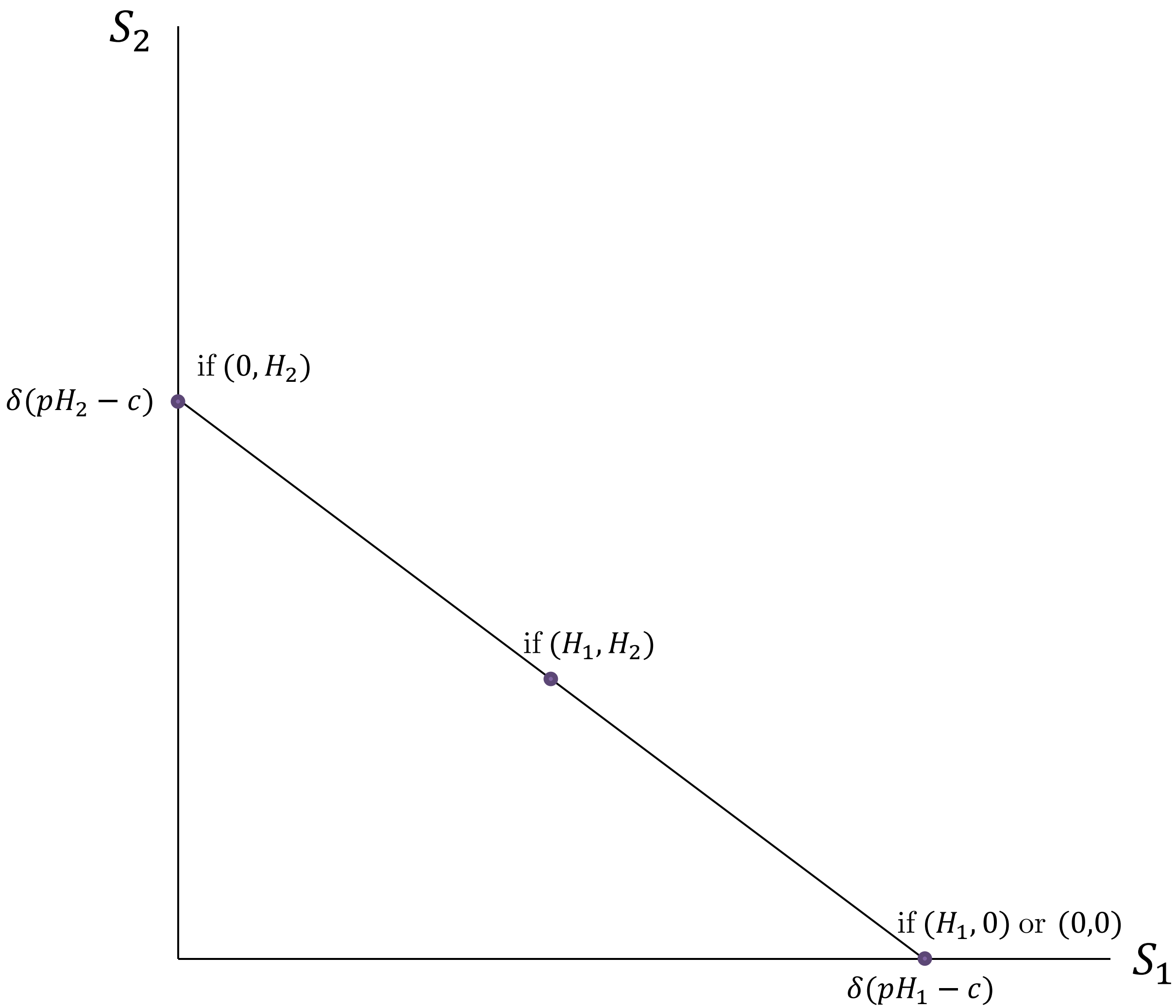

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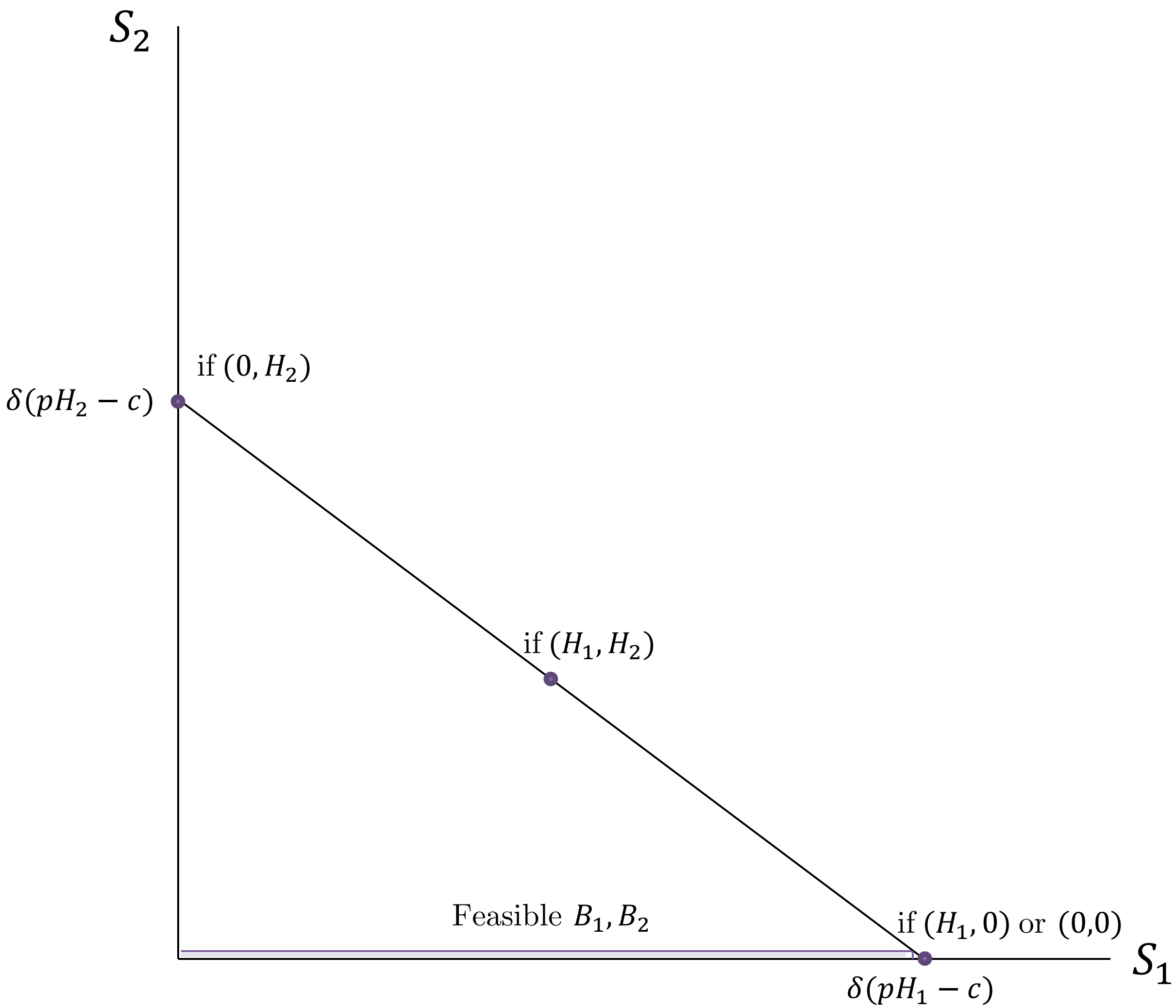

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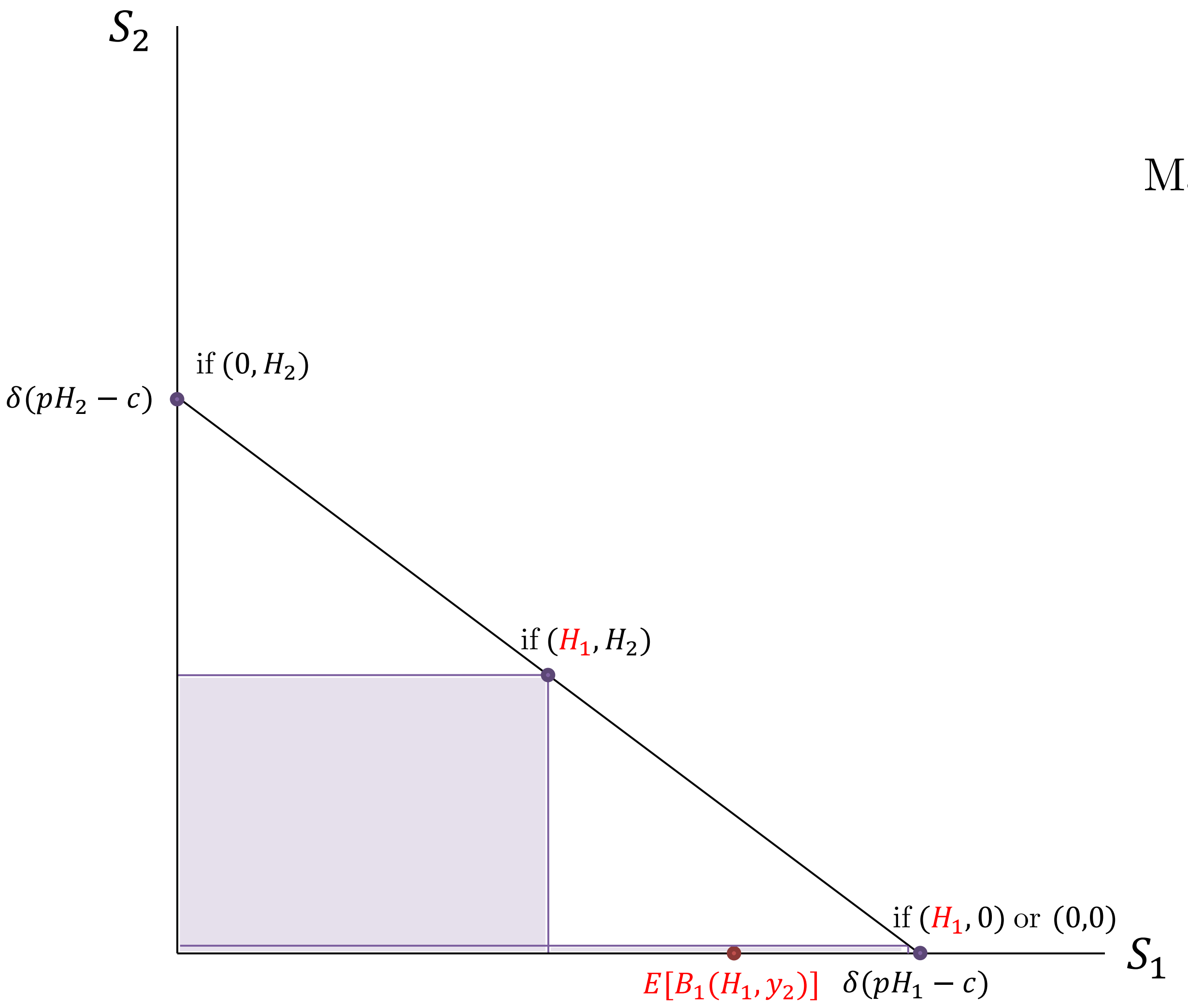

Maximal incentives for Agent 1:

$E[B_1(H_1, y_2)] - 0$

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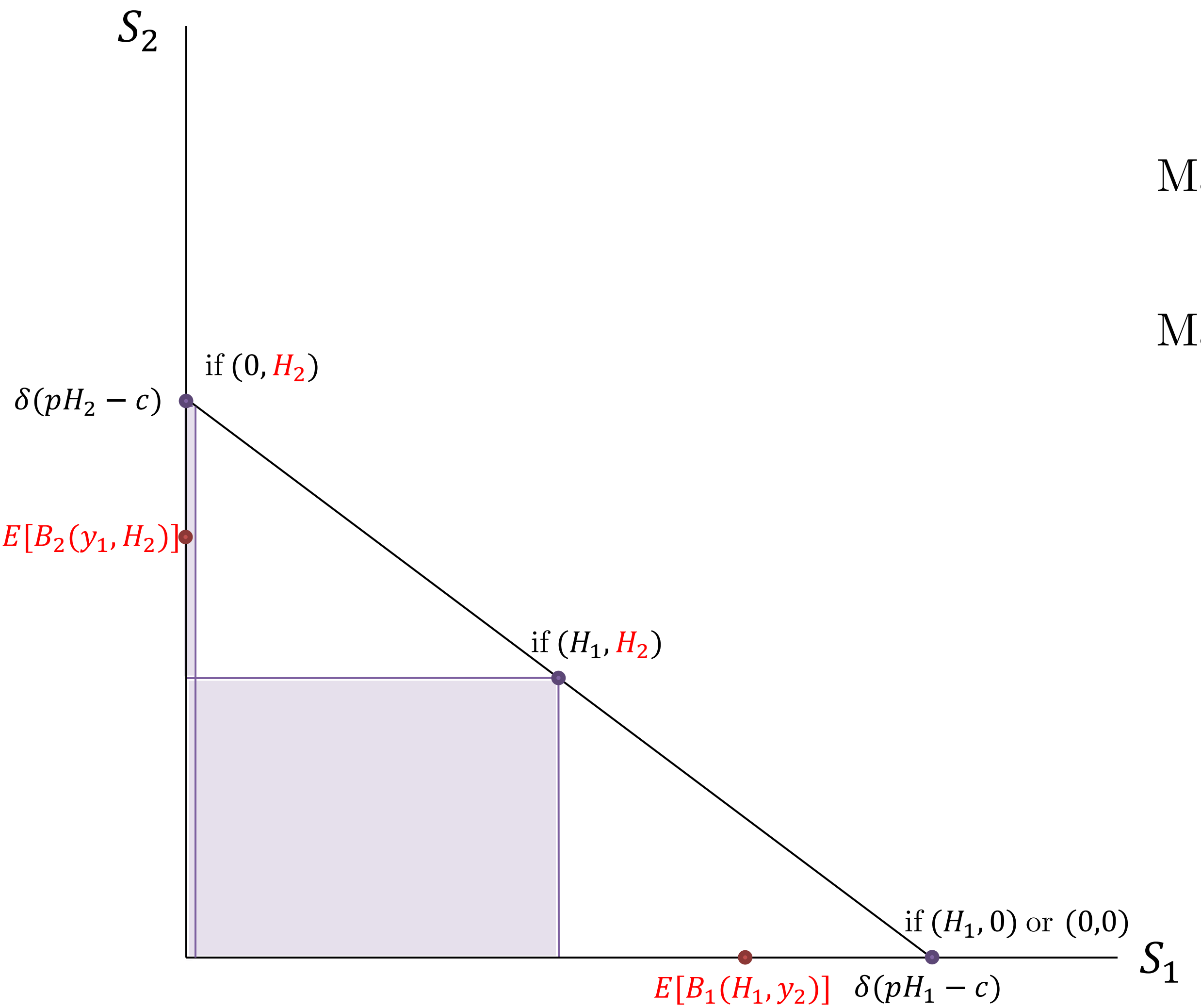


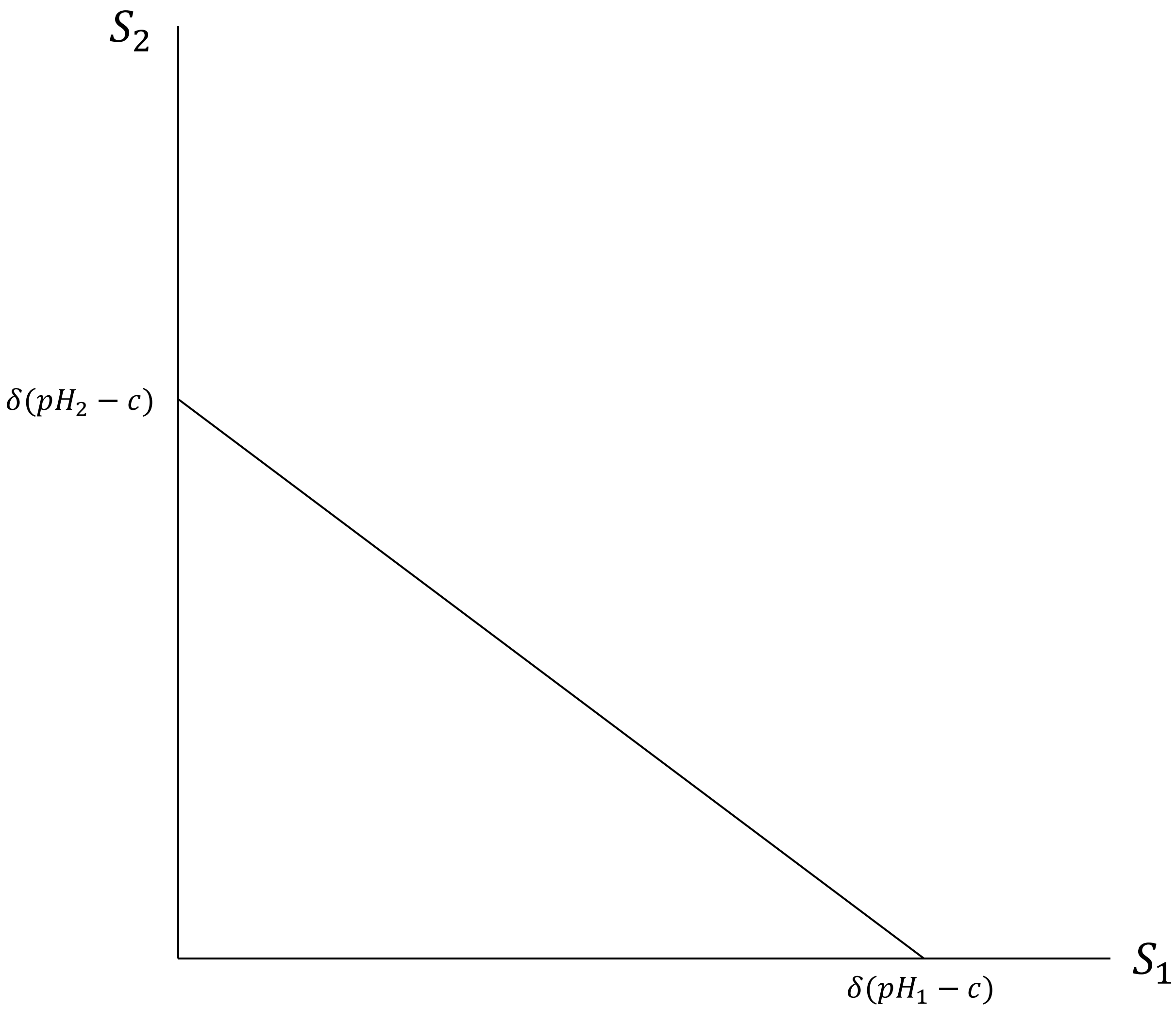
Maximal incentives for Agent 1:

$E[B_1(H_1, y_2)] - 0$

Maximal incentives for Agent 2:

$E[B_2(y_1, H_2)] - 0$

What if Everything (but e) is Public?



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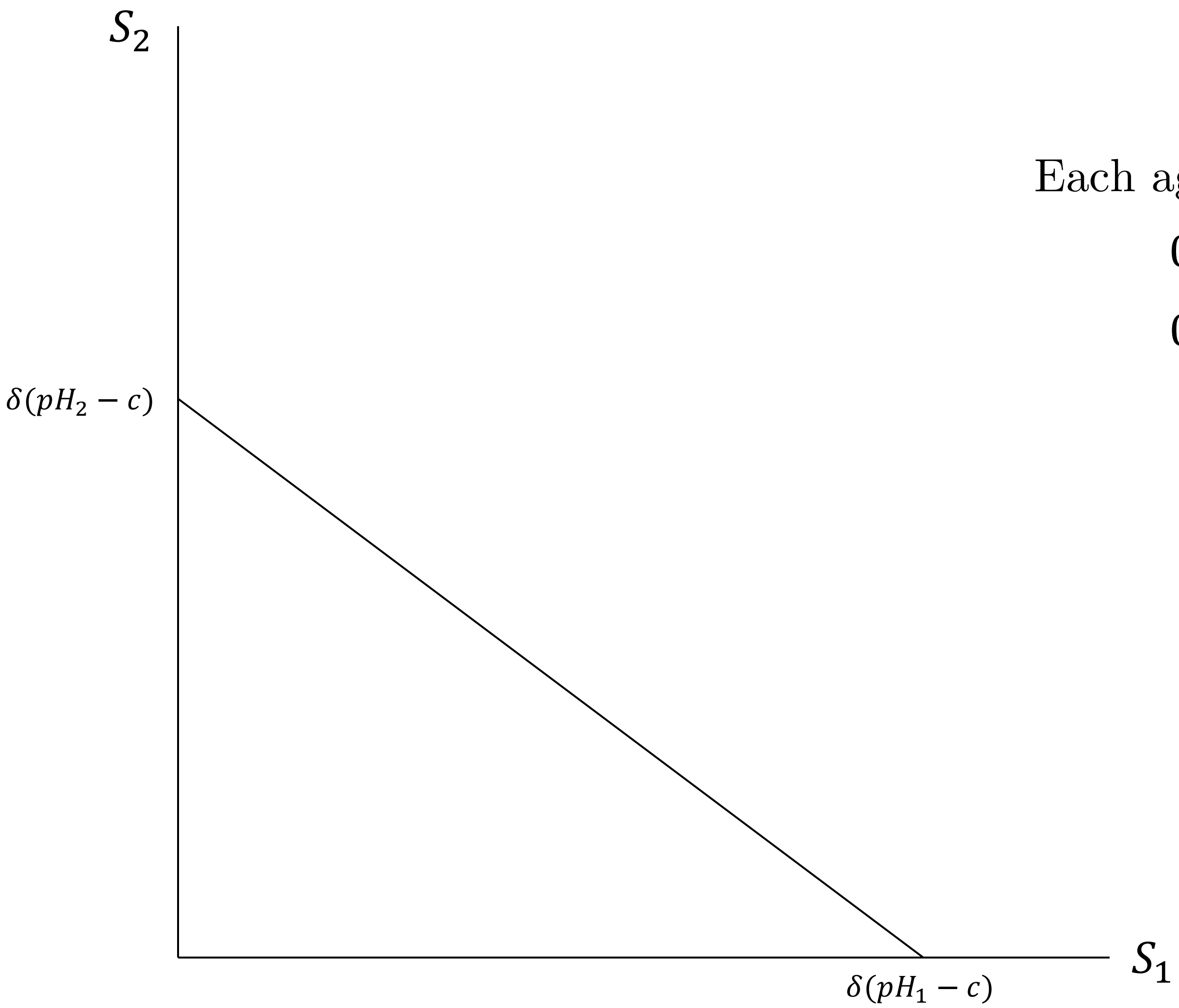
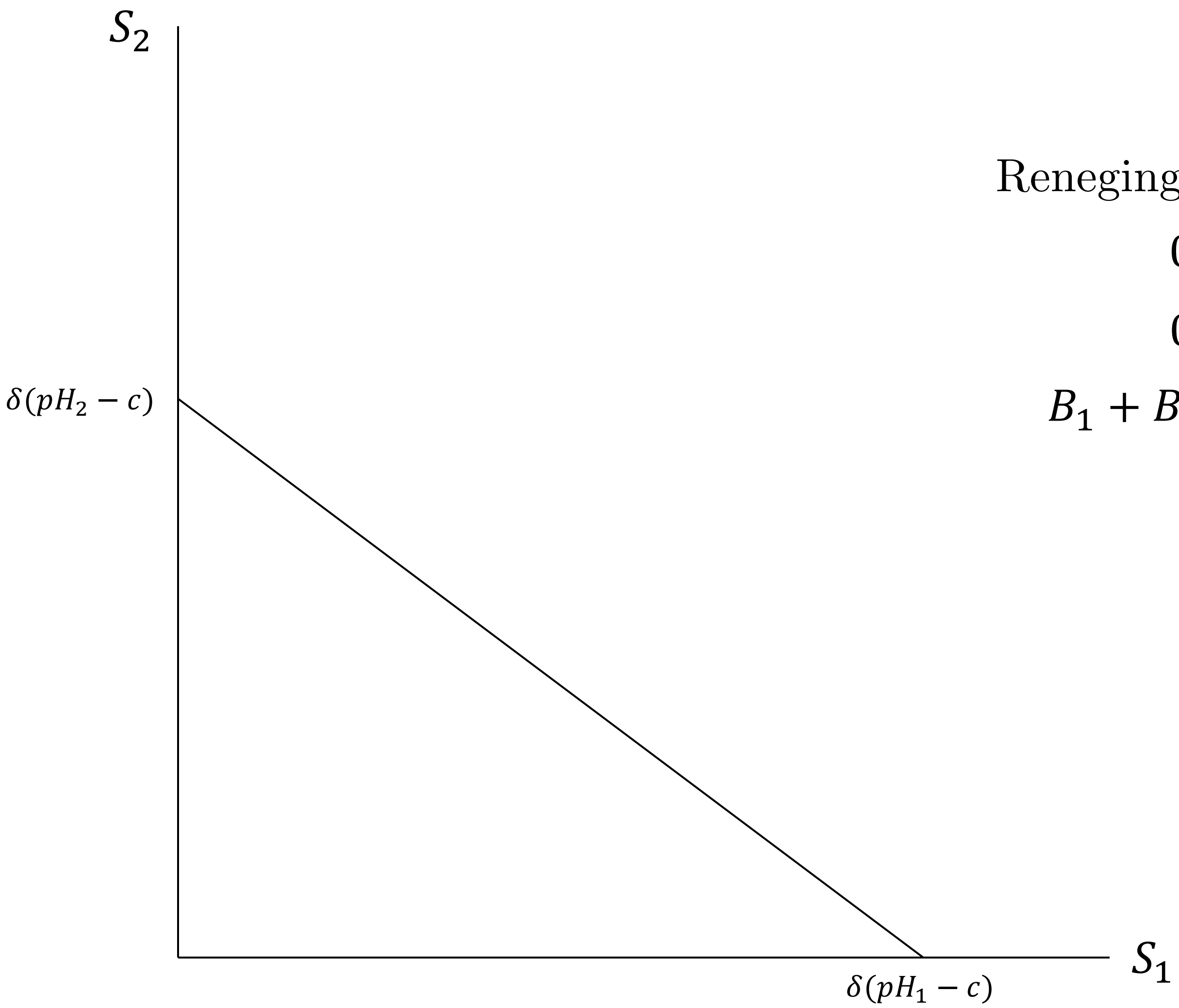


Each agent can walk away:

$0 \leq B_1$

$0 \leq B_2$

WHAT IF EVERYTHING (BUT e) IS PUBLIC?



Reneging can be jointly punished:

$$0 \leq B_1$$

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$$B_1 + B_2 \leq \delta(p(q_1 H_1 + q_2 H_2) - c)$$

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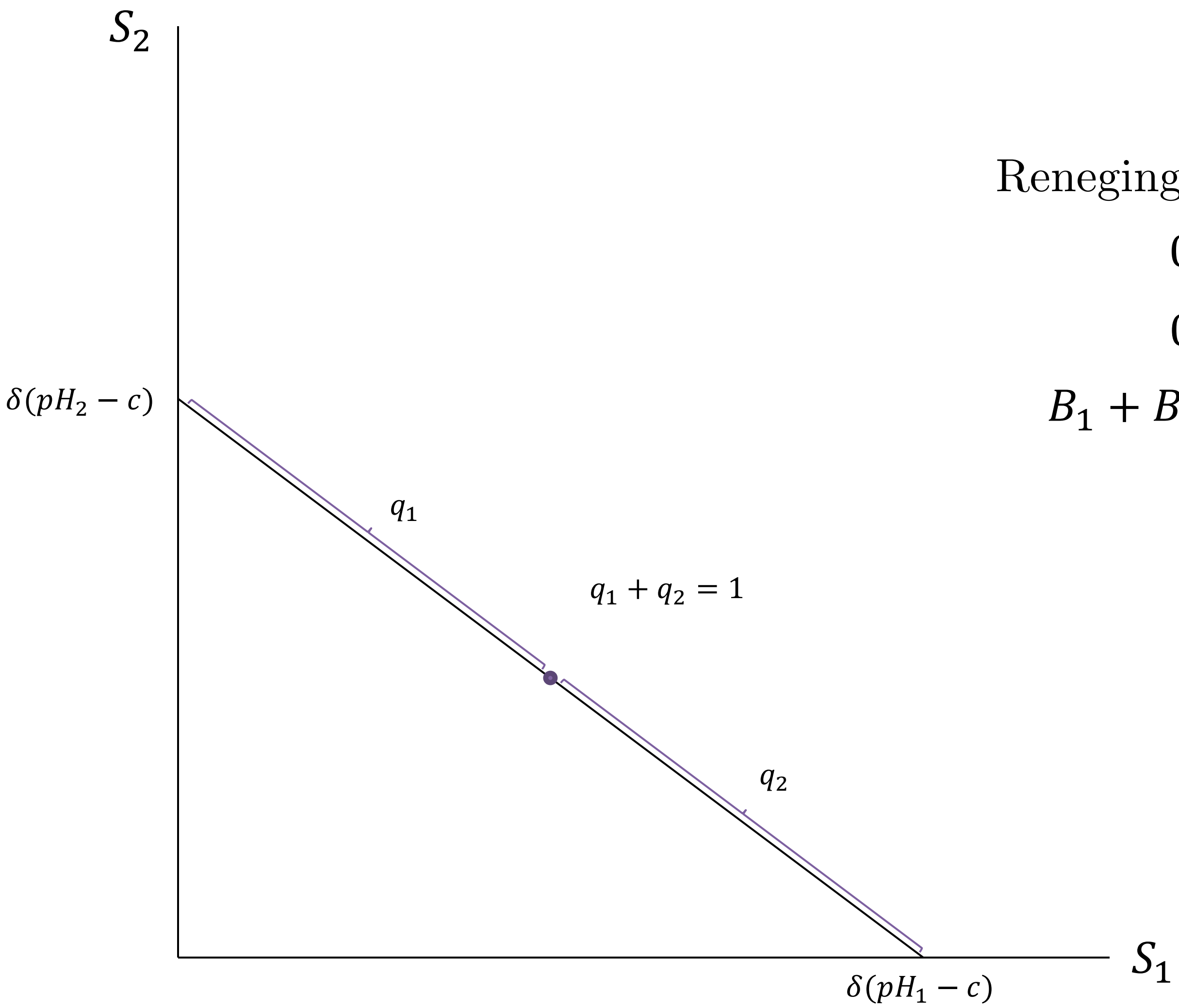

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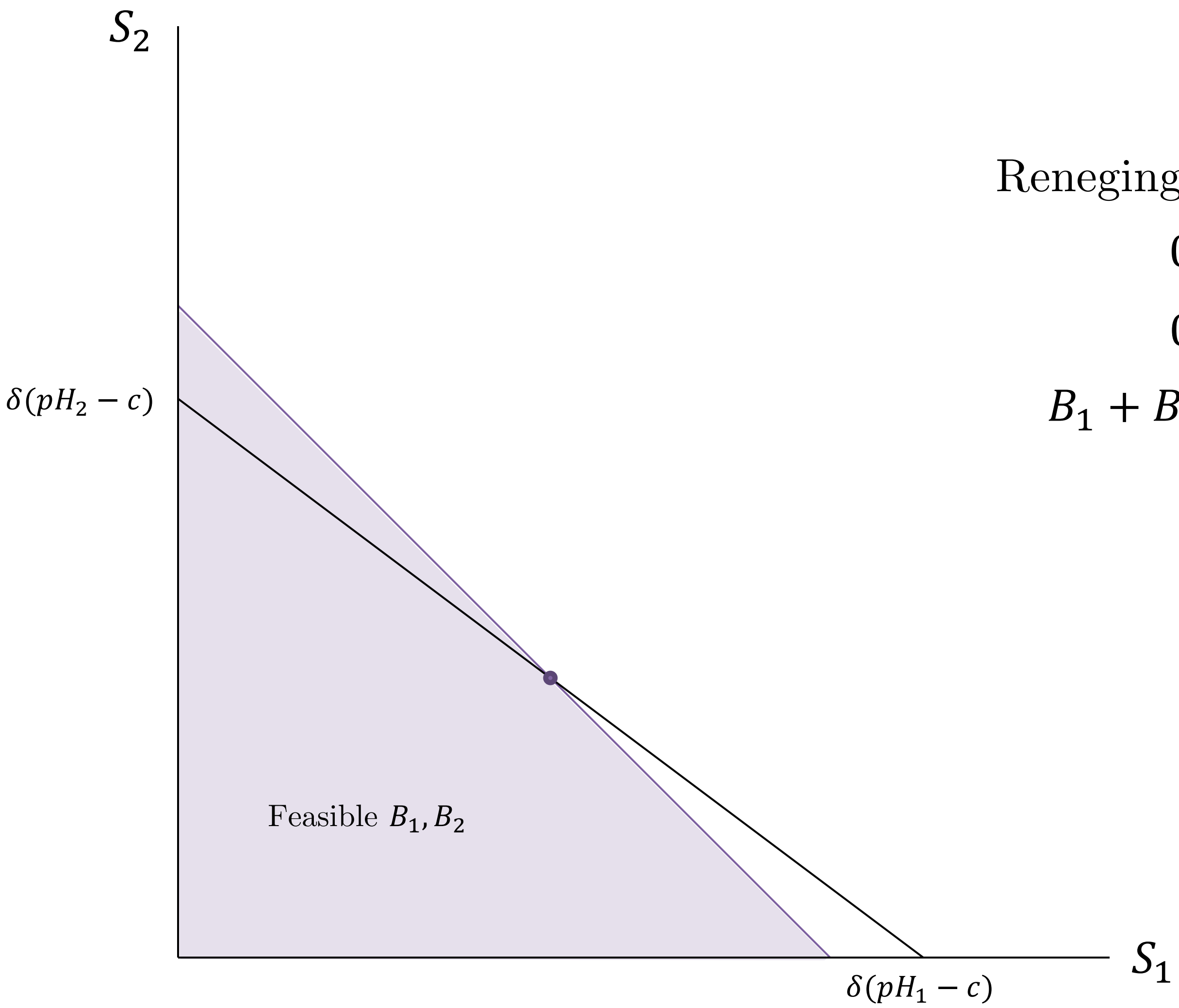

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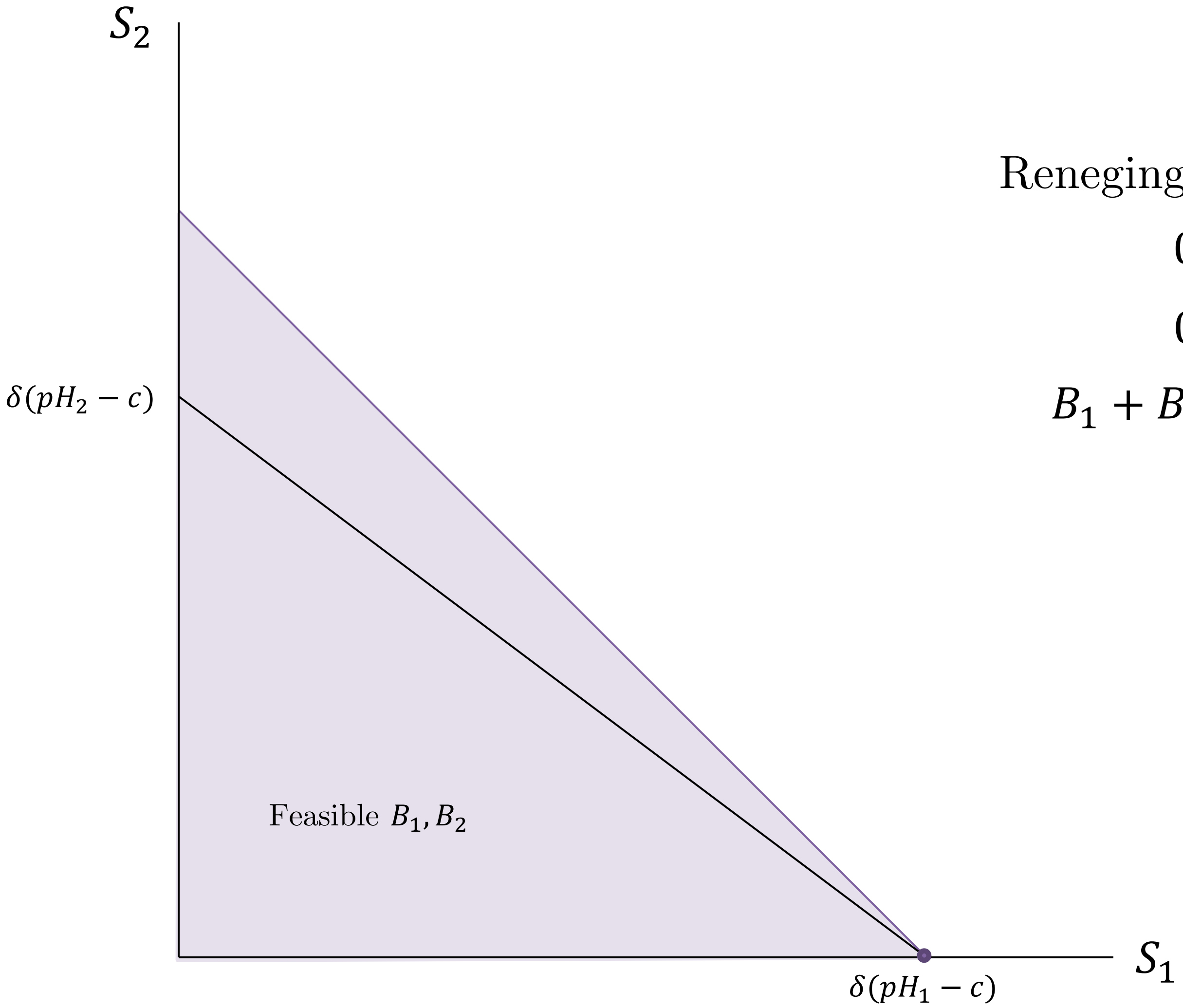
No Biases if Monitoring is Public



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Agenda

- Illustrative Example
- The Model
- General Results
- Applications
- Extensions

What is a Policy?

- Principal makes a **decision** each period

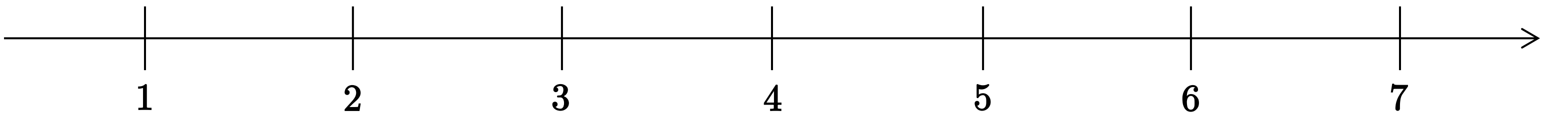
- Decision determines how agent efforts map to outputs

$$P_i(y_i | e_i, \theta, d)$$

effort state decision

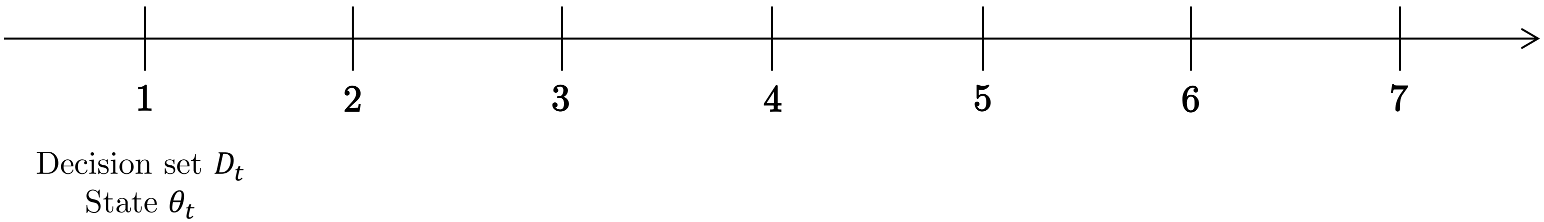
- A **policy** is a history-contingent decision plan

Stage Game



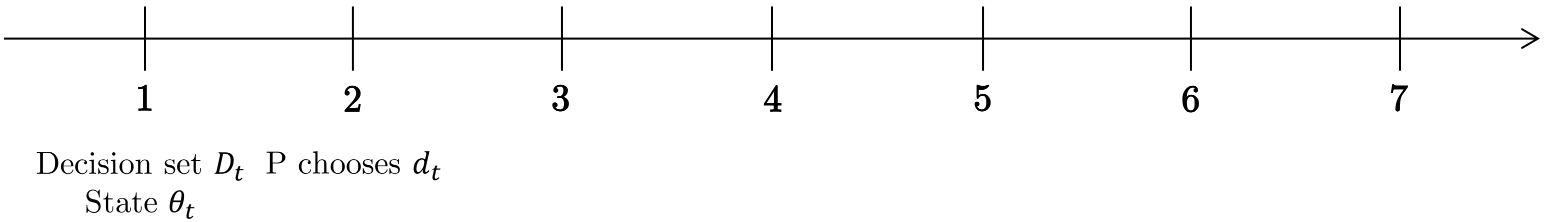
One principal, N agents, common discount factor $\delta < 1$

Stage Game



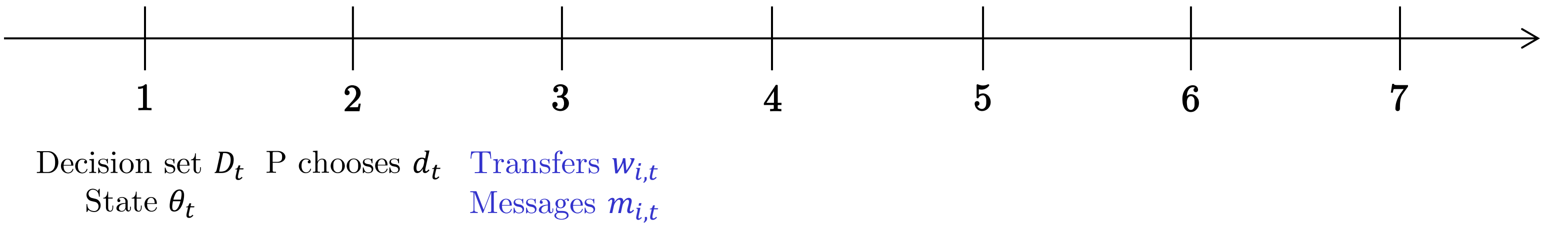
1: Decision set D_t and state θ_t drawn from $F(\cdot \,|\{\theta_{t'}, D_{t'}, d_{t'}\}_{t'=0}^{t-1})$.
Publicly observed.

Stage Game



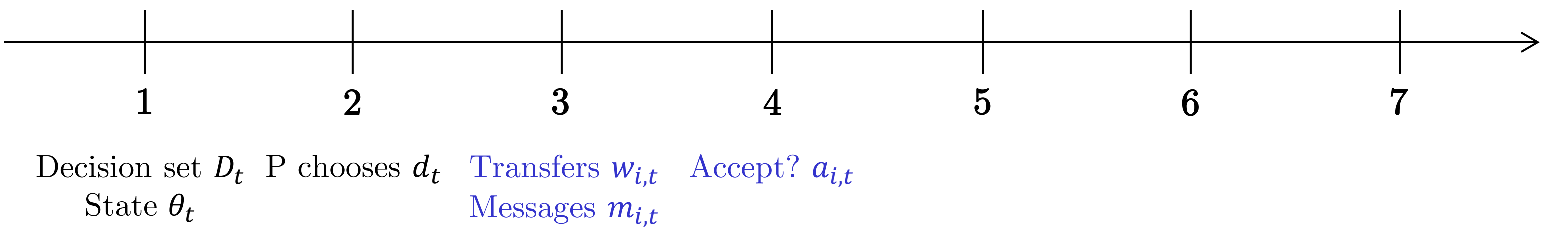
2: Principal chooses decision $d_t \in D_t$. Publicly observed.

Stage Game



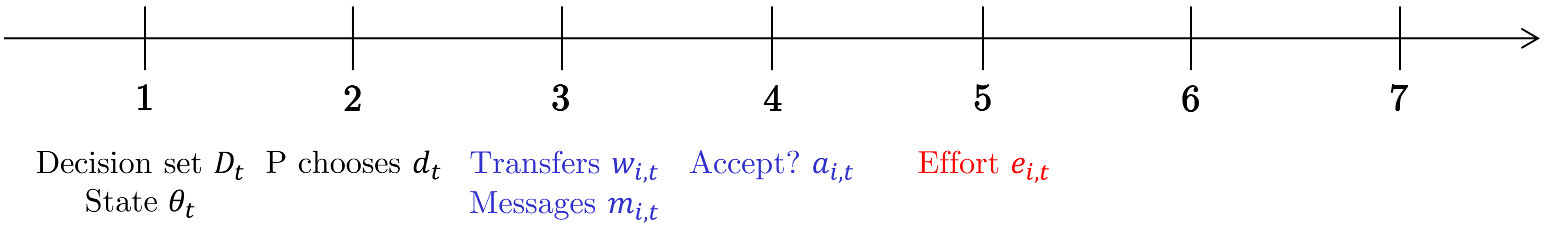
3: Principal and each agent pay each other $w_{i,t} \in \mathbb{R}$. Principal sends messages $\{m_{i,t}\}_{i=1}^{N}$ to each agent. Bilaterally observed (by the principal and agent i).

Stage Game



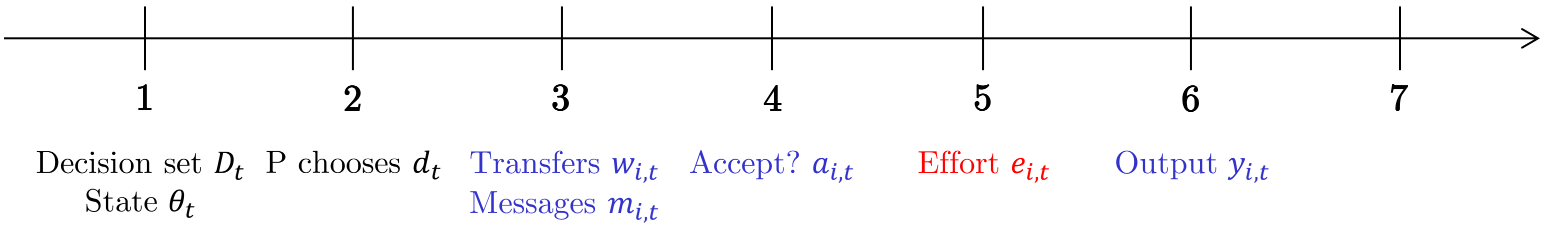
4: Each agent i accepts or rejects, $a_{i,t} \in \{0,1\}$. Outside option $\bar{u}_i(d_t, \theta_t) \geq 0$ results in $y_{i,t} = 0$. Bilaterally observed.

Stage Game



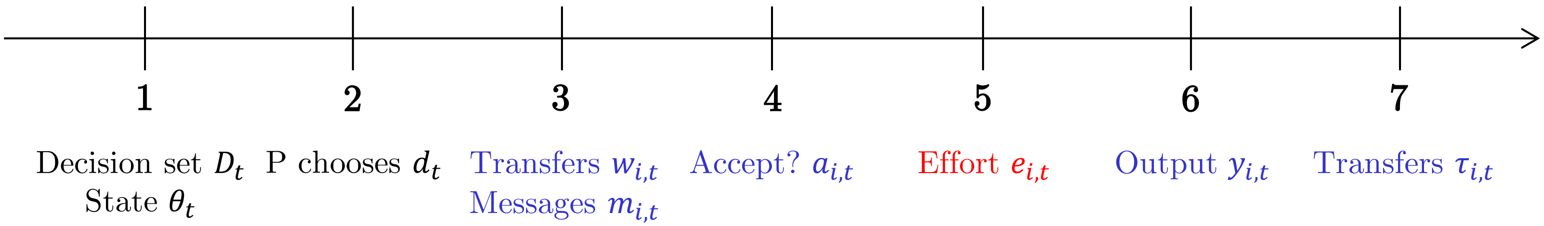
5: If i accepts, chooses effort $e_{i,t} \in \mathbb{R}_+$ at cost $c(\cdot)$. Privately observed.

Stage Game



6: Output $y_{i,t} \in \mathbb{R}_+$ realized according to $P_i(\cdot\,|e_{i,t}, \theta_t, d_t)$.
Bilaterally observed.

Stage Game



7: Principal and agent i exchange (net) transfers $\tau_{i,t} \in \mathbb{R}$.
Bilaterally observed.

Payoffs and Information

Define $C_{i,t} = a_{i,t}c(e_{i,t}) - (1 - a_{i,t})\bar{u}_i$

Payoffs:

$$\pi_t = (1 - \delta) \sum_{i \leq N} (y_{i,t} - w_{i,t} - \tau_{i,t})$$

$$u_{i,t} = (1 - \delta)(w_{i,t} + \tau_{i,t} - C_{i,t})$$

Dyad-surplus: $S_{i,t} = \sum_{t' \geq t} \delta^{t'-t}(1 - \delta)(y_{i,t'} - C_{i,t'})$

Histories: h_0^t at start of period, h_x^t after variable x, agent i sees $\phi_i(h^t)$

Recursive Equilibrium

A Perfect Bayesian Equilibrium σ^* is a **recursive equilibrium** if, for each h_0^t on eq'm path, $\sigma^*|h_0^t$ is a PBE

- Private monitoring: PBE not recursive
- Recursive eq'm tractable alternative ($\approx$ BFE for extensive form)

Intuition for this refinement:

- Agent i's effort IC constraint conditions on h_0^t , not $\phi_i(h_0^t)$
- When paying $\tau_{i,t}$, agent i has Bayesian expectations over $y_{-i,t}$
- Off-path, agents don't observe if principal deviates in other relationships

Surplus-Maximizing Relational Contracts

A recursive equilibrium σ^* is

- **Surplus-maximizing** = maximizes ex ante total surplus (among recursive equilibria)
- **Sequentially surplus-maximizing** = $\sigma^*|h_0^t$ is surplus-maximizing for every on-path h_0^t

A **biased** decision is not sequentially surplus-maximizing

A **backward-looking** policy involves (on-path) biased decisions

Examples of Decisions

Hiring / Firing: D = agents available; θ = demand; d = agents hired

- Possible bias: low-productivity or wrong # of agents hired

Promotion: D = set of agents up for promotion; d = agent promoted

- Possible bias: low-productivity agent promoted

Irreversible investment: D = set of agents (if no investment yet), chosen agent otherwise; d = agent chosen for investment

- Possible bias: delay investment or invest in low-productivity agent

Sourcing decision: D = set of available suppliers; θ = each supplier's productivity; d = supplier chosen

- Possible bias: pick low productivity supplier

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General Results

Develop necessary and sufficient conditions for relational contract

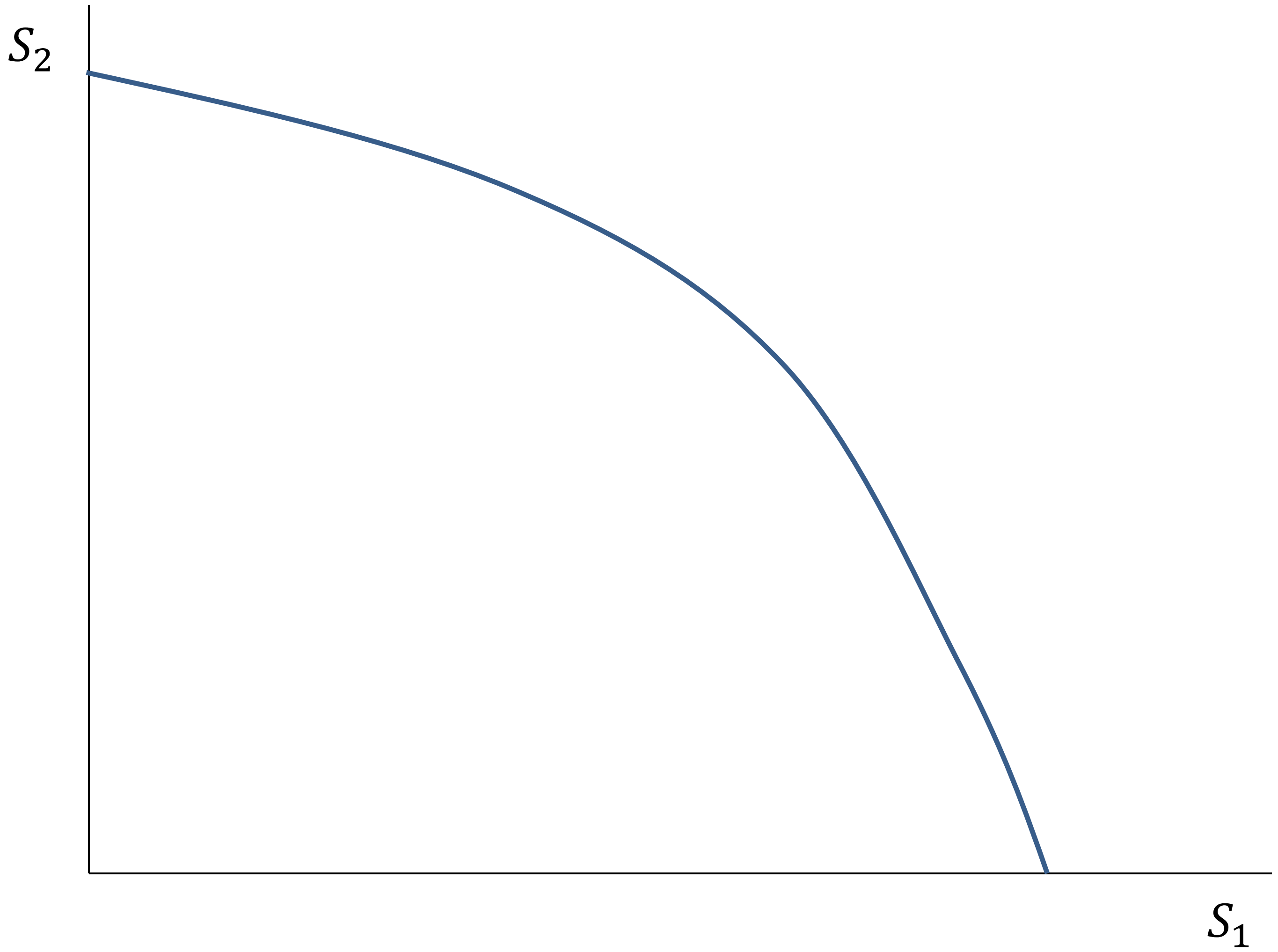
- Rewards must **motivate** agents and satisfy **dynamic enforcement constraints**
- **Dynamic enforcement** depends on future policies

Biased decisions are surplus-maximizing for broad class of games

- Biased towards those who performed well in past, against those who did not
- Resembles a **tournament** for future biased decisions

More detail on Main Results

Intuition: Why are Decisions Biased?



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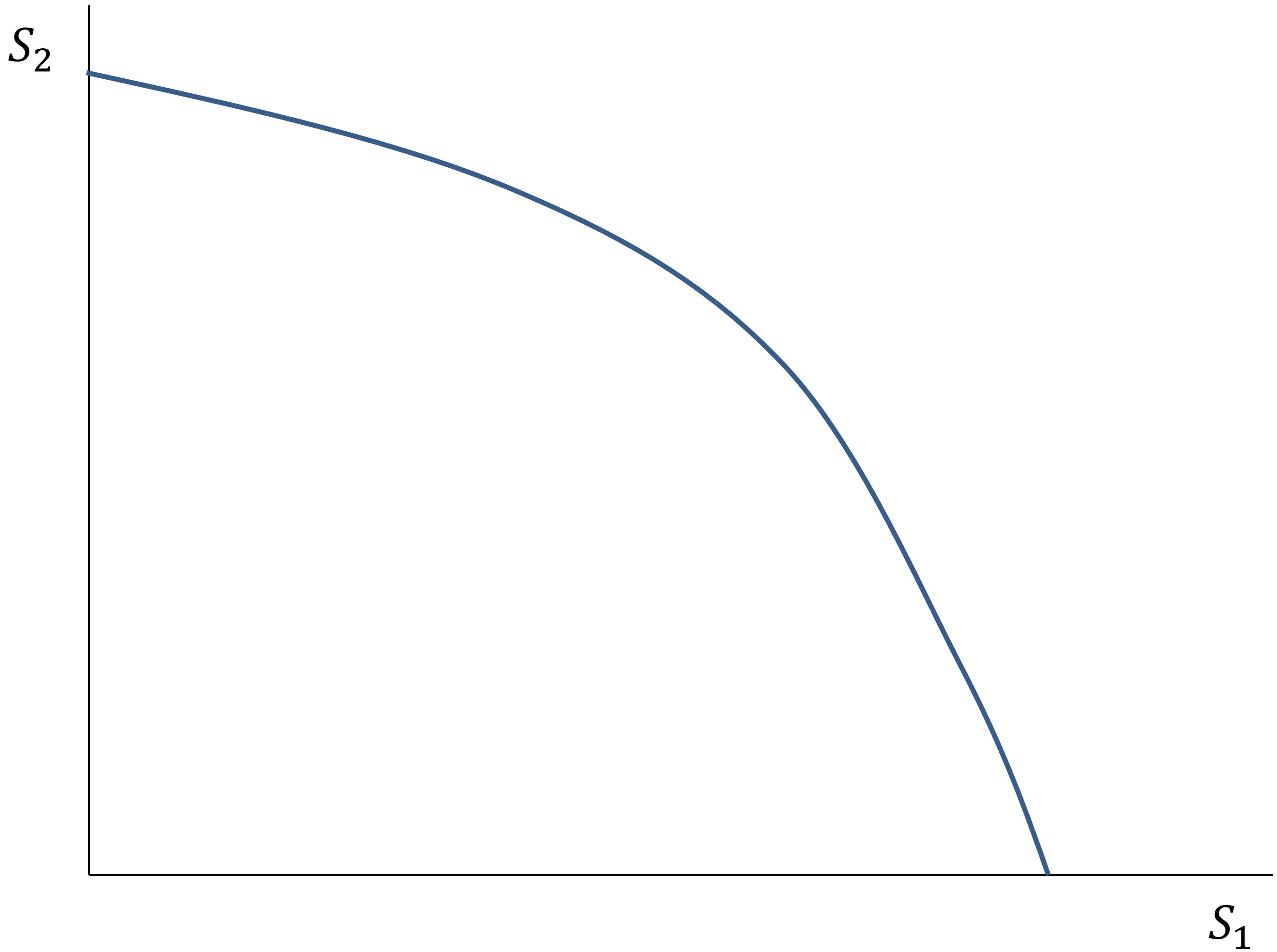
If frontier is smooth...



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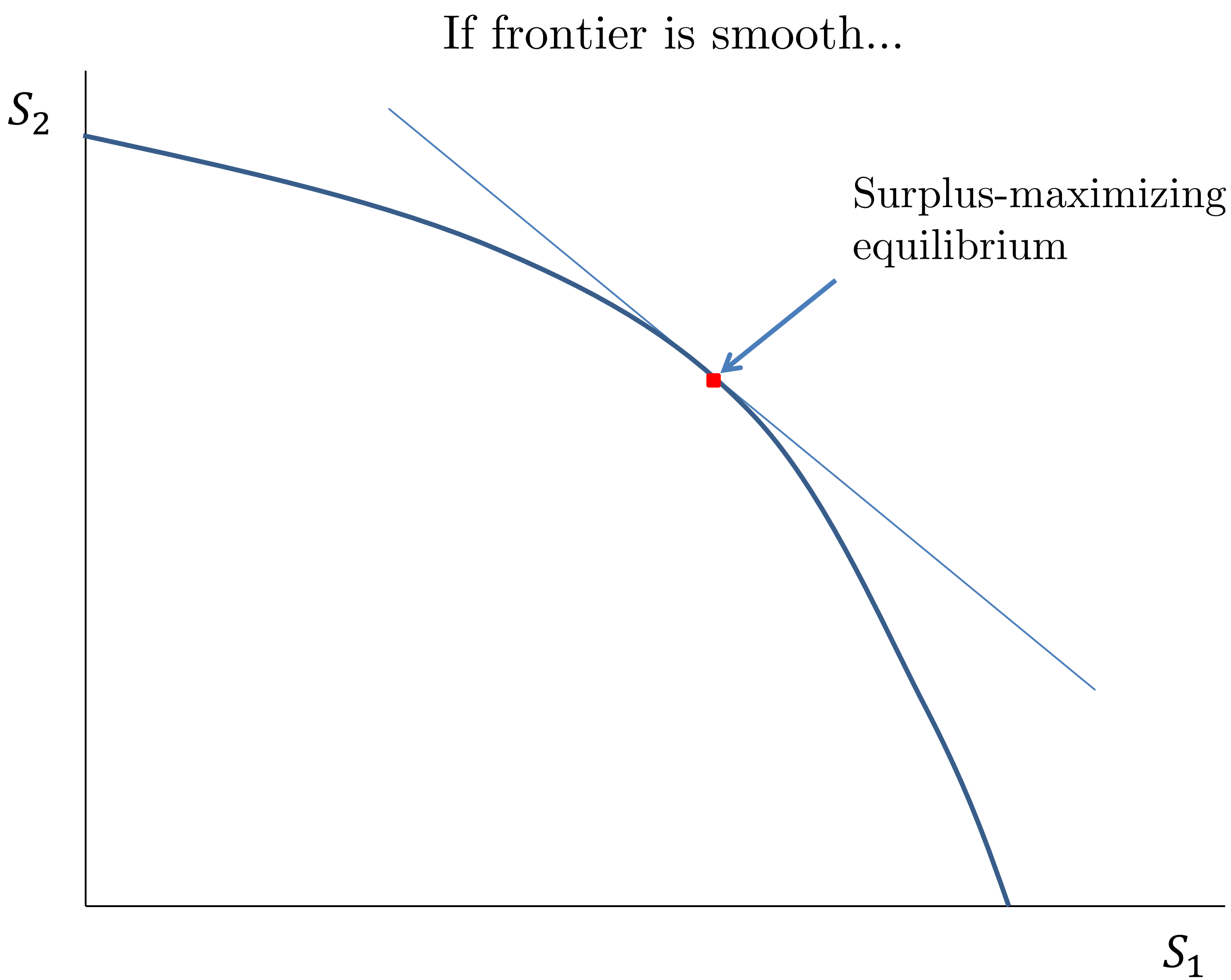

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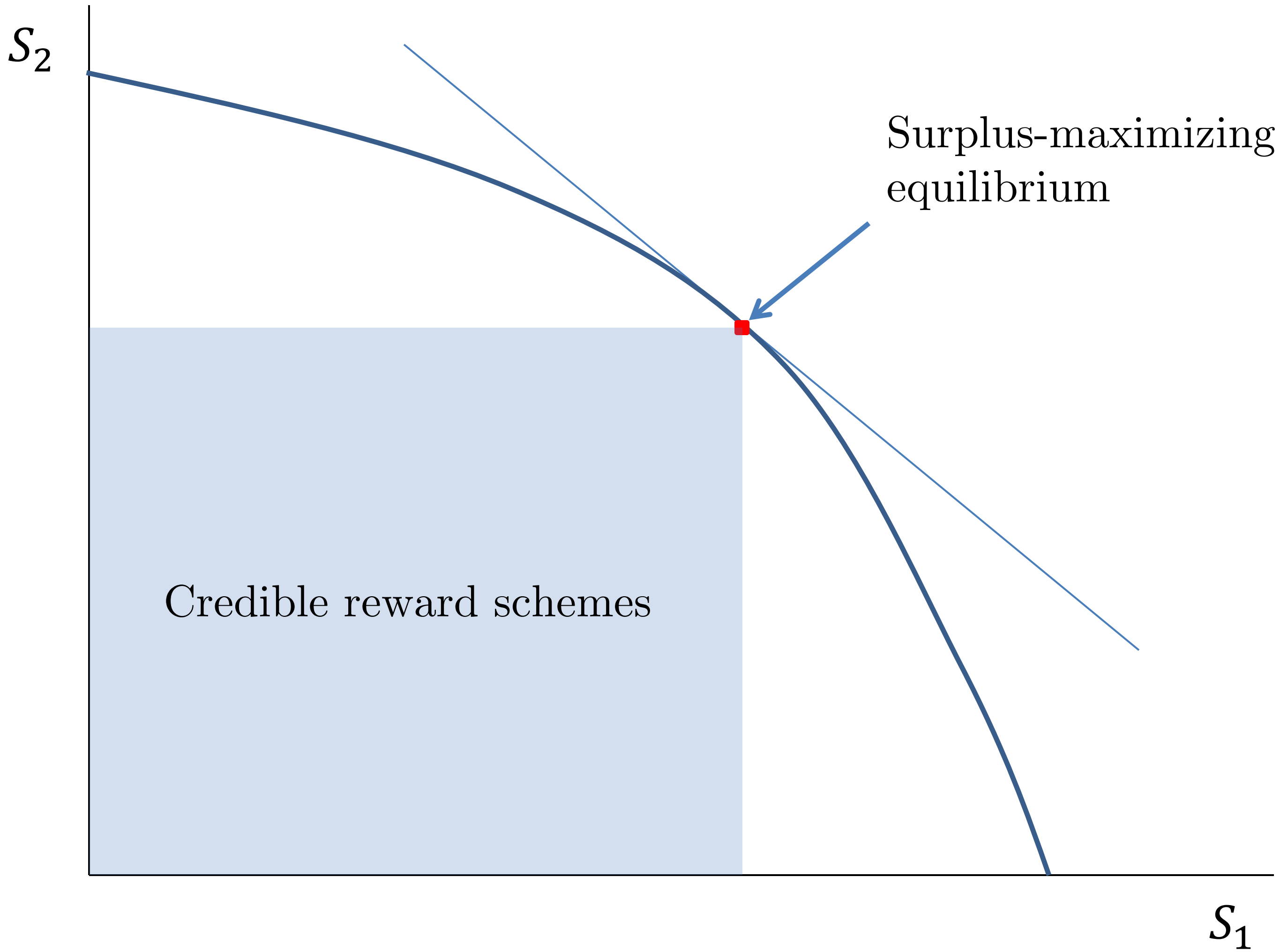

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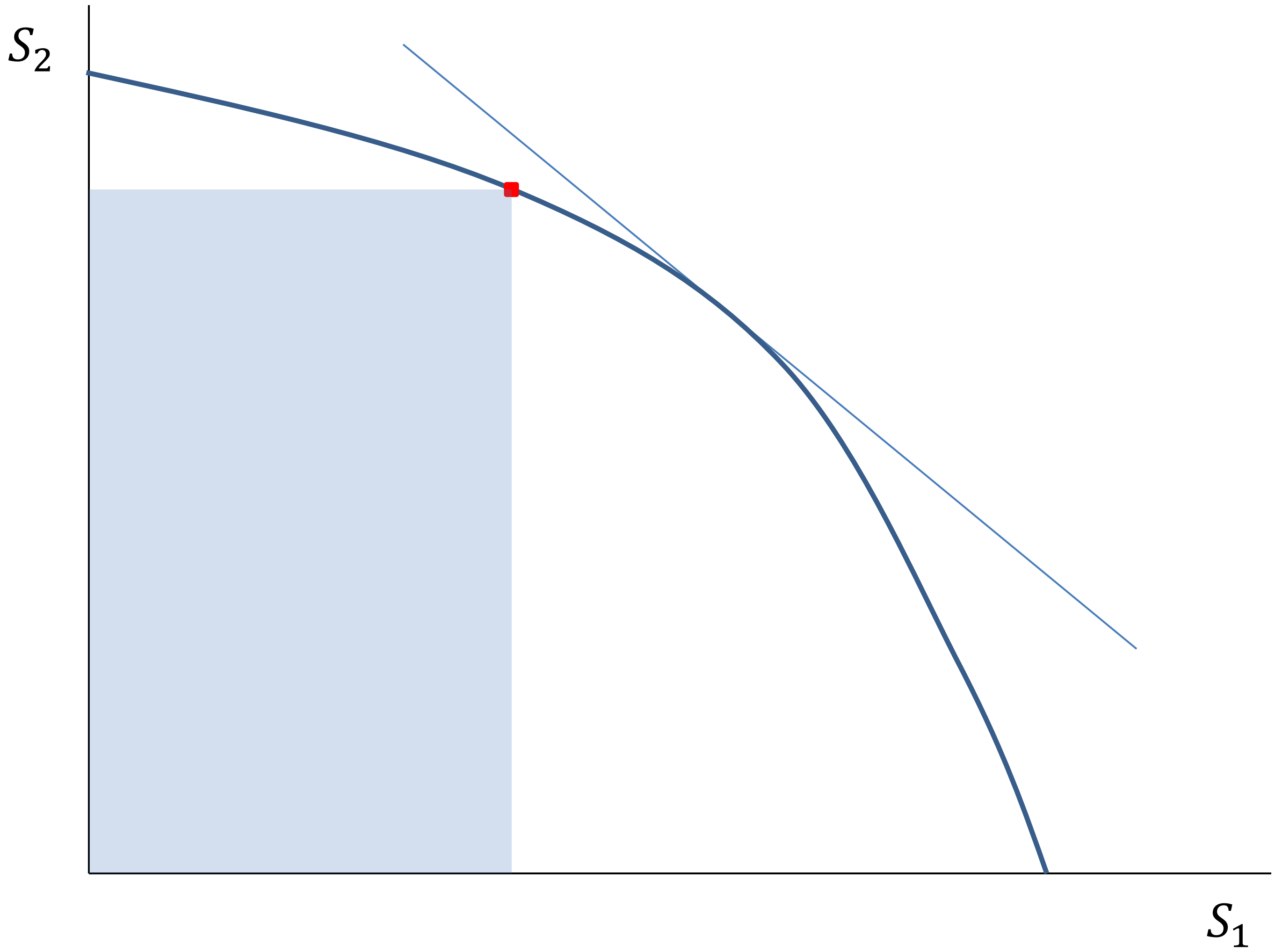

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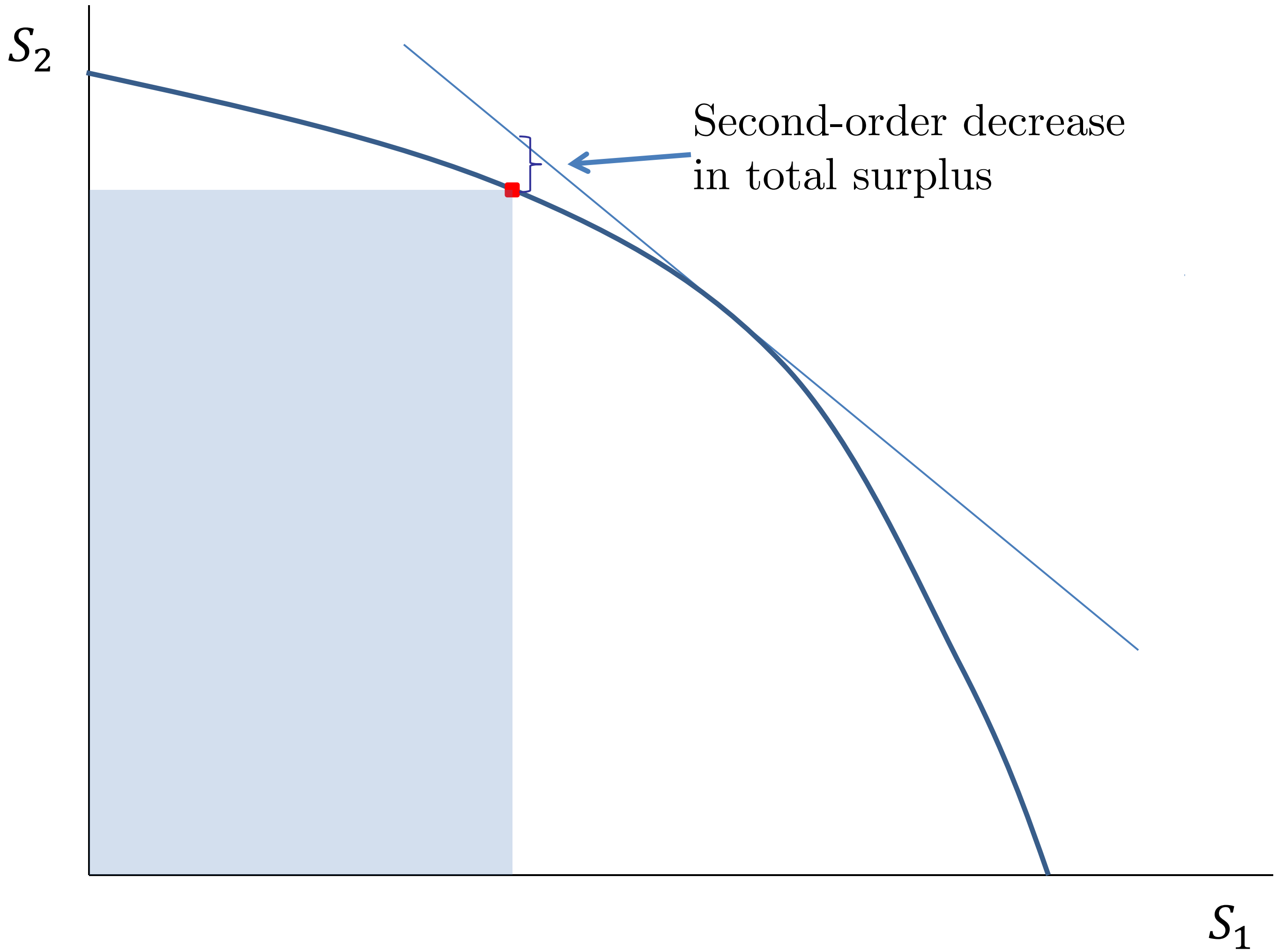

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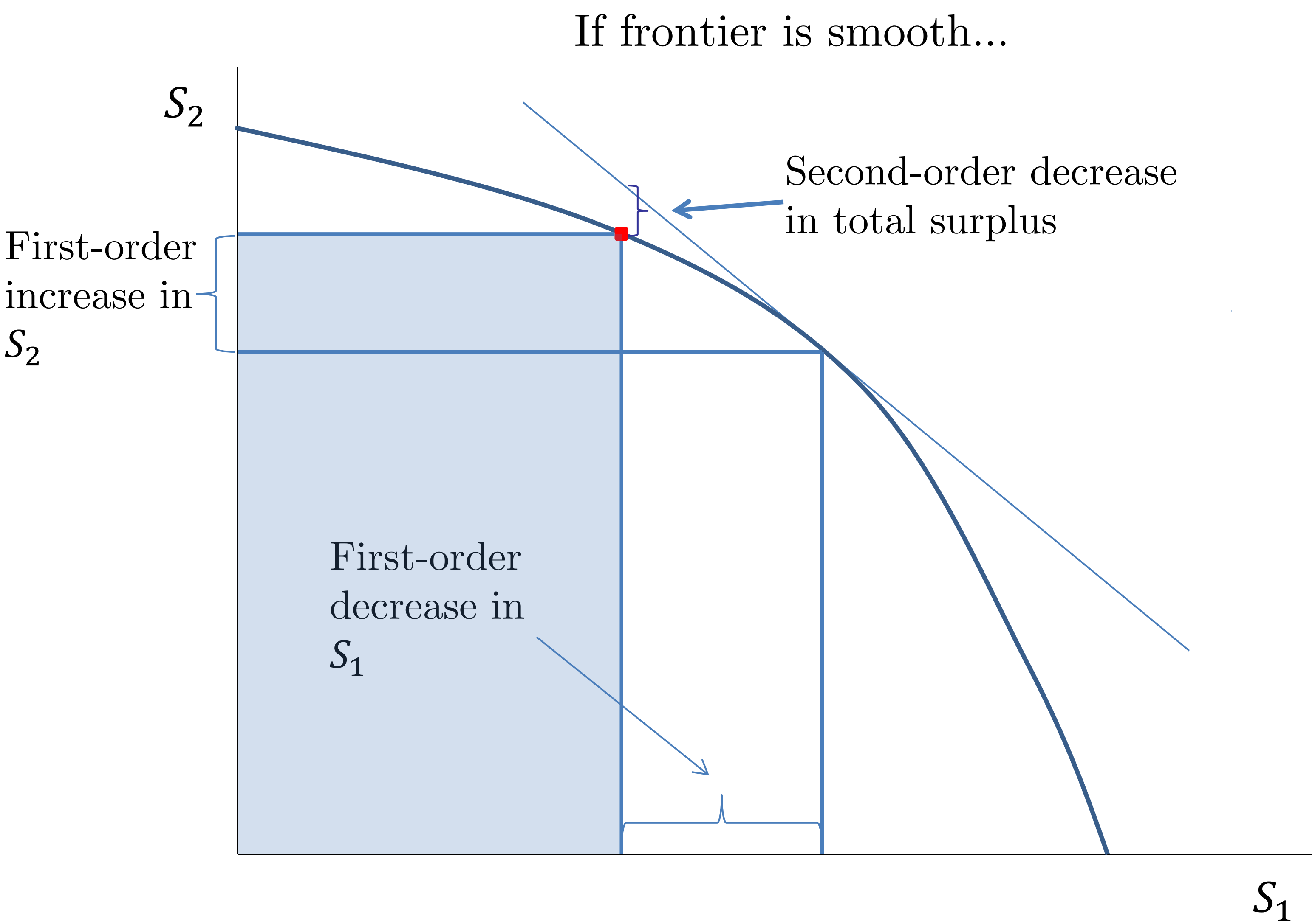
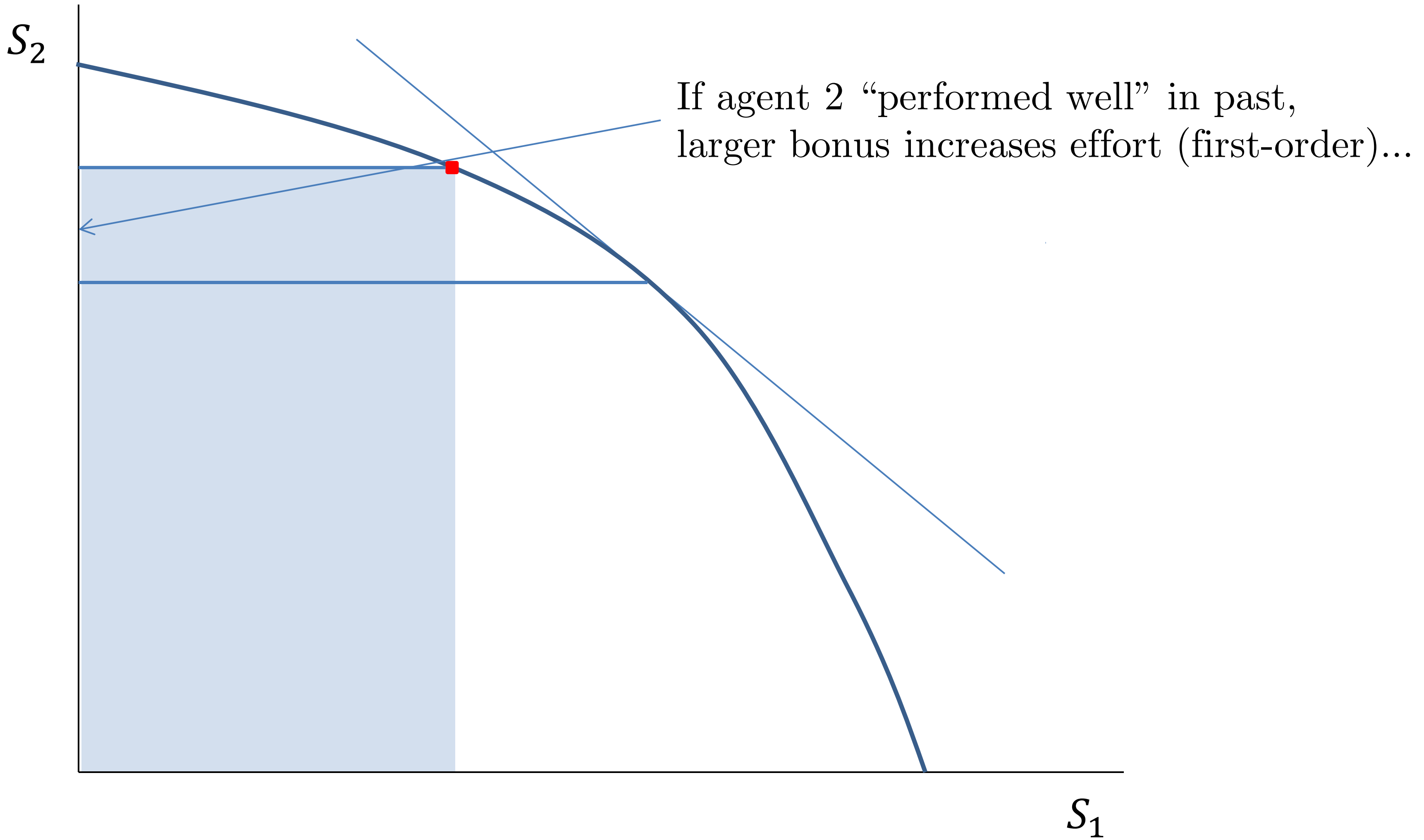


Intuition: Why are Decisions Biased?

If frontier is smooth...



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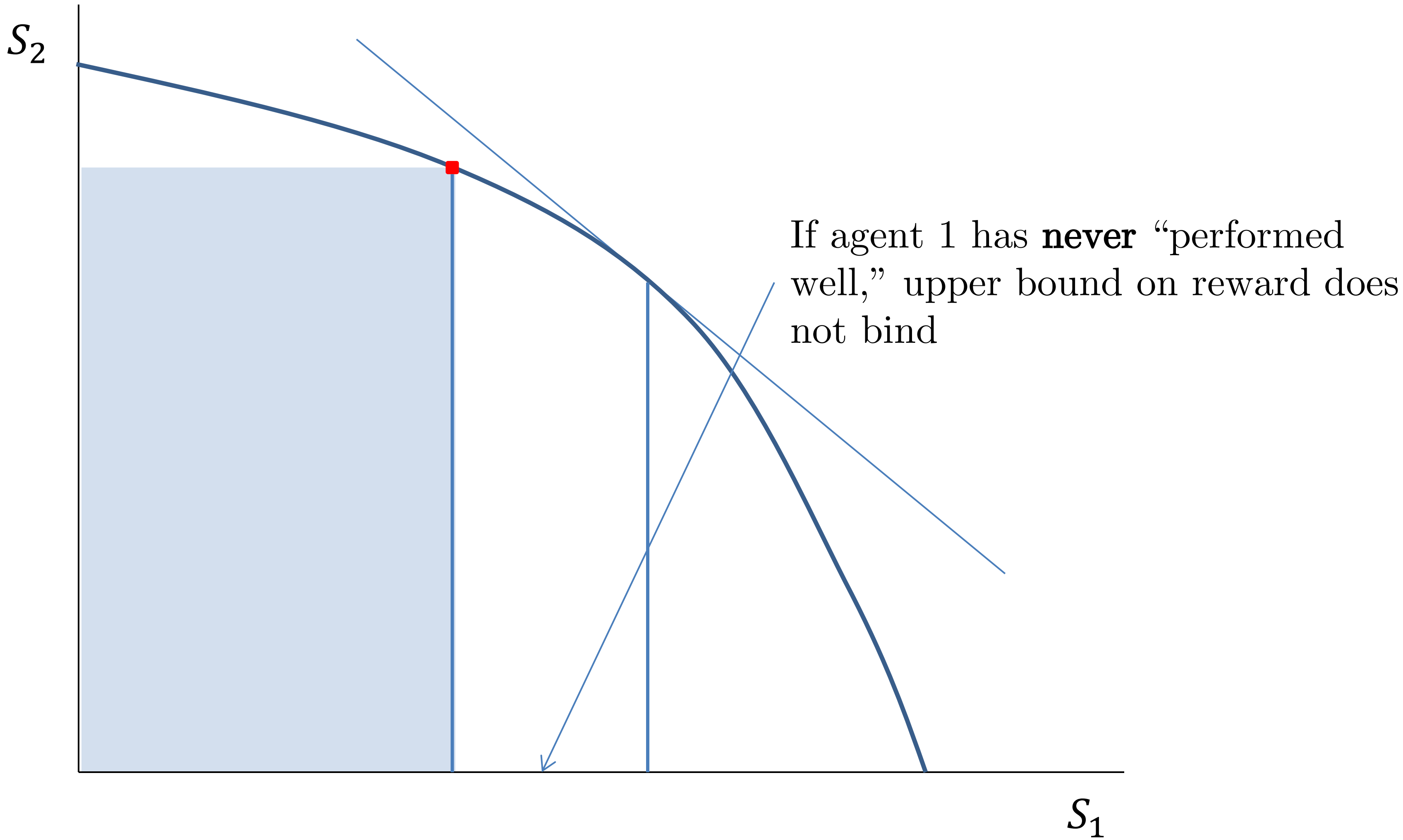

Intuition: Why are Decisions Biased?

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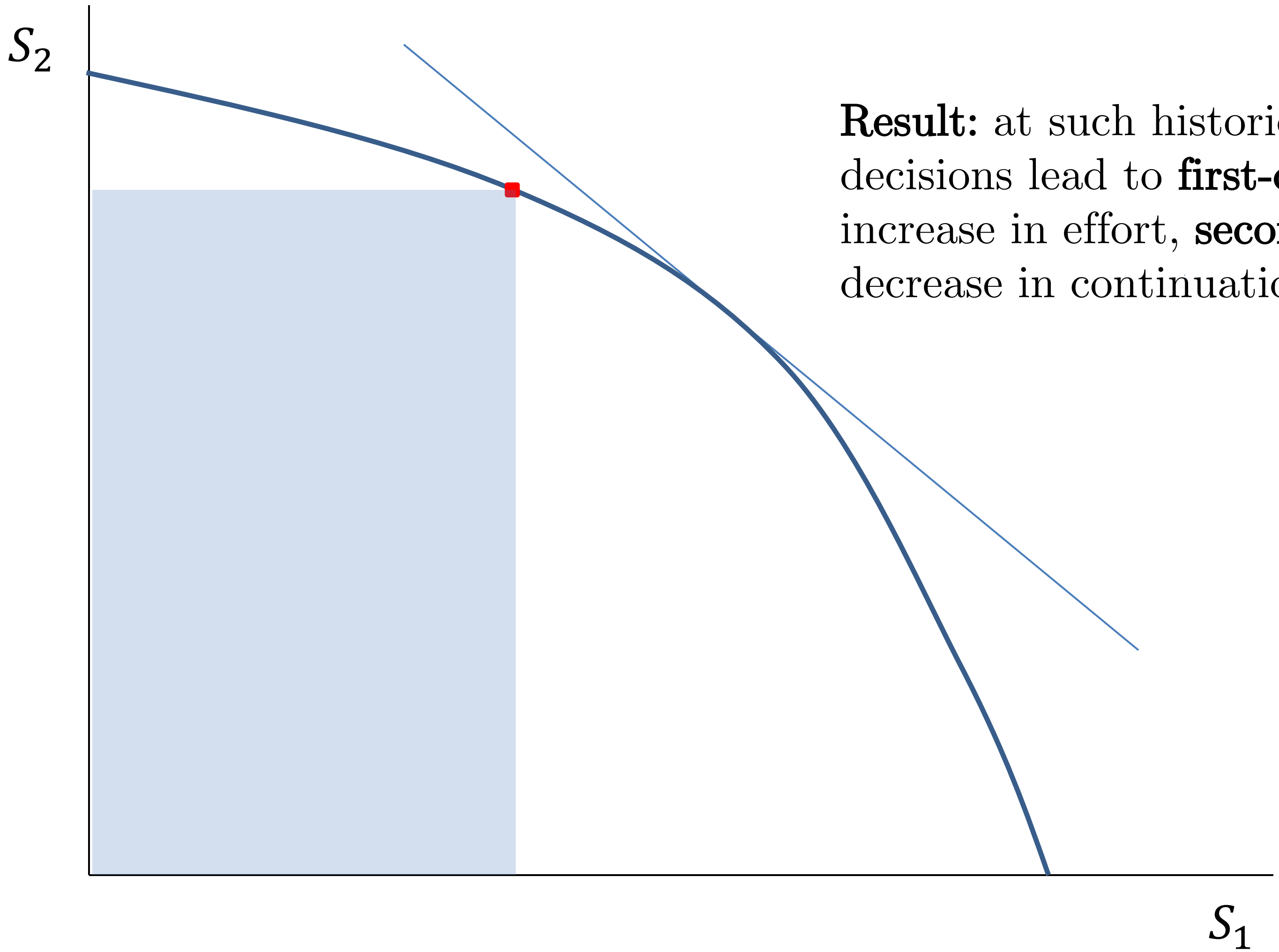

Result: at such histories, biased decisions lead to **first-order** increase in effort, **second-order** decrease in continuation surplus

When is Dyad-Surplus Frontier Smooth?

1. S_i frontier is downward-sloping

- **Decisions** are weights $d_{i,t} \geq 0$ assigned to each agent $\left(\sum_i d_{i,t} \leq 1\right)$
- **Higher $\boldsymbol{d_i}$** means: higher expected y_i (strictly concave) that is (weakly) more informative of effort (effort-independent garbling). No effect on y_{-i}.

2. S_i frontier is smooth

- **States of the world $\boldsymbol{\theta}$** are independent of past decisions
- **Outside options** $\bar{u}_i$ depend only on states of the world
- **Effort** costs $c(\cdot)$ are smooth, strictly increasing, and strictly convex

3. Changing one agent's effort can affect others' incentives

- **Output distributions $\boldsymbol{P_i}$** are smooth and satisfy Mirrlees-Rogerson conditions

Formal statement of assumptions

Statement of Main Result

Define $e_i^{FB}(\mathrm{d_i}, \theta) = arg \max_{e_i} E[y_i | d_i, \theta, e_i] - c(e_i)$

In a smooth game, let σ^* be a surplus-maximizing recursive equilibrium

For agents i, j, consider a history h_0^{t+1} such that:

1. Agent i chooses positive effort less than e_i^{FB} in t
2. Agent i's output had strictly positive likelihood ratio in t
3. Agent j's output had weakly negative likelihood ratio for all $t' \leq t$
4. Both i and j have positive weight $(d_{i,t}, d_{j,t} > 0)$

For almost all such h_0^{t+1}, $\sigma^* | h_0^{t+1}$ is **not** surplus-maximizing

Why is Dyad-Surplus Frontier Smooth?

Formal statement

Agenda

- Illustrative Example
- The Model
- General Results
- Applications
- Extensions

Hiring

Decision = how many workers to hire in each period

- State of the world = demand, persistent and increasing over time
- Per-worker productivity **falls** in number hired
- If agents work hard, hire more as demand increases

Result: hiring growth lags demand increases

- Hiring more workers makes current workers less productive
- If firm hires rapidly in future, then current workers produce less future surplus
- So firm optimally delays hiring after increase in demand

Permanent Investment

Decision = one-time, permanent investment in one agent

- Investment increases agent output for fixed effort
- Agents have differing returns from investment
- Moral hazard: output is stochastic

Result: award investment in a tournament

- Distort investment: if low-return agent performs well, gets investment
- Agent with investment produces more in future, so can be promised larger reward

Agenda

- Illustrative Example
- The Model
- General Results
- Applications
- Extensions

Outline of Extensions

Public monitoring:

If all variables (except effort) are publicly observed, then all surplus-maximizing equilibria are sequentially surplus-maximizing

Extension 1: Public Monitoring

Perfect Bayesian Equilibrium:

In a simple class of games, our main result extends to the full set of Perfect Bayesian Equilibria

Extension 2: Biased Decisions in PBE

Conclusion

Flexible framework of backward-looking policies in relational contracts

- Decisions make past promises credible, rather than maximizing future surplus

Biases important for broad class of games

- If (and only if) agents cannot coordinate punishments
- Relational contracts evolve in history-dependent ways

Biases manifest in realistic ways

- Lagged hiring, delayed investment

Literature: Dynamic Inefficiencies

We have a sense for what contracting frictions lead to "history-dependent" **formal** contracts...

- Fudenberg, Holmstrom, and Milgrom (1990, "FHM")

We have a sense for how **similar** contracting frictions affect relational contracts...

- **Limited liability**: Board (2011), Fong and Li (2015)...
- **Asymmetric information about future preferences:** Halac (2012), Malcomson (2014)...
- **Asymmetric information about past outputs:** MacLeod (2003), Fuchs (2007)...
- Absent these, **stationary** relational contracts: Levin (2002, 2003), Kranz (2014).,.

We know less about how lack of principal commitment leads to **new** sources of history-dependence

- Related to literature on **private monitoring:** Kandori (2002), etc.
- Builds on the intuition of Andrews and Barron (2015)

Return to Main Talk

Definition: Smooth Games (Formal)

When is dyad-surplus frontier smooth?

A game is **smooth** if…

- For every t, $D_t = \{(d_1, \ldots, d_N) | d_i \geq 0, \sum_i d_i \leq 1\}$ and θ_t is iid
- Outside options depend only on θ_t
- Effort costs $c(\cdot)$ are smooth, strictly increasing, and strictly convex
- P_i depends only on d_i, θ, e_i; is smooth in all arguments with density p_i; has full support; is strictly MLRP-increasing in e_i; and satisfies CDFC
- Expected output $E[y_i | d_i, \theta, e_i]$ is strictly increasing and strictly concave in $\{d_i, e_i\}$
- Higher decisions are more informative: if $d_i \geq \tilde{d}_i$, then there exists an effort-independent garbling $R(x_i | y_i)$ with density r_i such that

$$\int_{y_i \leq \overline{y_i}} p_i(y_i | \theta, \tilde{d}_i, e_i) dy_i = \int_{y_i \leq \bar{y}_i} r_i(x | y_i) p_i(y_i | \theta, d_i, e_i) dy_i$$

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Statement of Main Result (Formal)

Define $e_i^{FB}(\mathrm{d_i}, \theta) = arg \max_{e_i} E[y_i | d_i, \theta, e_i] - c(e_i)$

In a smooth game, let σ^* be a surplus-maximizing recursive equilibrium

For agents i, j, let E_t be a set of histories h_0^{t+1} such that:

1. $e_{i,t} > 0$ but $e_{i,t} < e_i^{FB}(d_i, \theta)$
2. $\frac{\partial p_i / \partial e_i}{p_i}\left(y_{i,t} \middle| d_{i,t}, \theta_t, e_{i,t}\right) > 0$
3. $\frac{\partial p_j / \partial e_j}{p_j}\left(y_{j,t'} \middle| d_{j,t'}, \theta_{t'}, e_{j,t'}\right) \leq 0$ for all $t' \leq t$
4. $d_{i,t+1} < 1$ and $d_{j,t+1} > 0$ with positive probability

For almost every $\mathrm{h}_0^{t+1} \in E_t$, $\sigma^* | h_0^{t+1}$ is **not** surplus-maximizing

Why is Dyad-Surplus Frontier Smooth?

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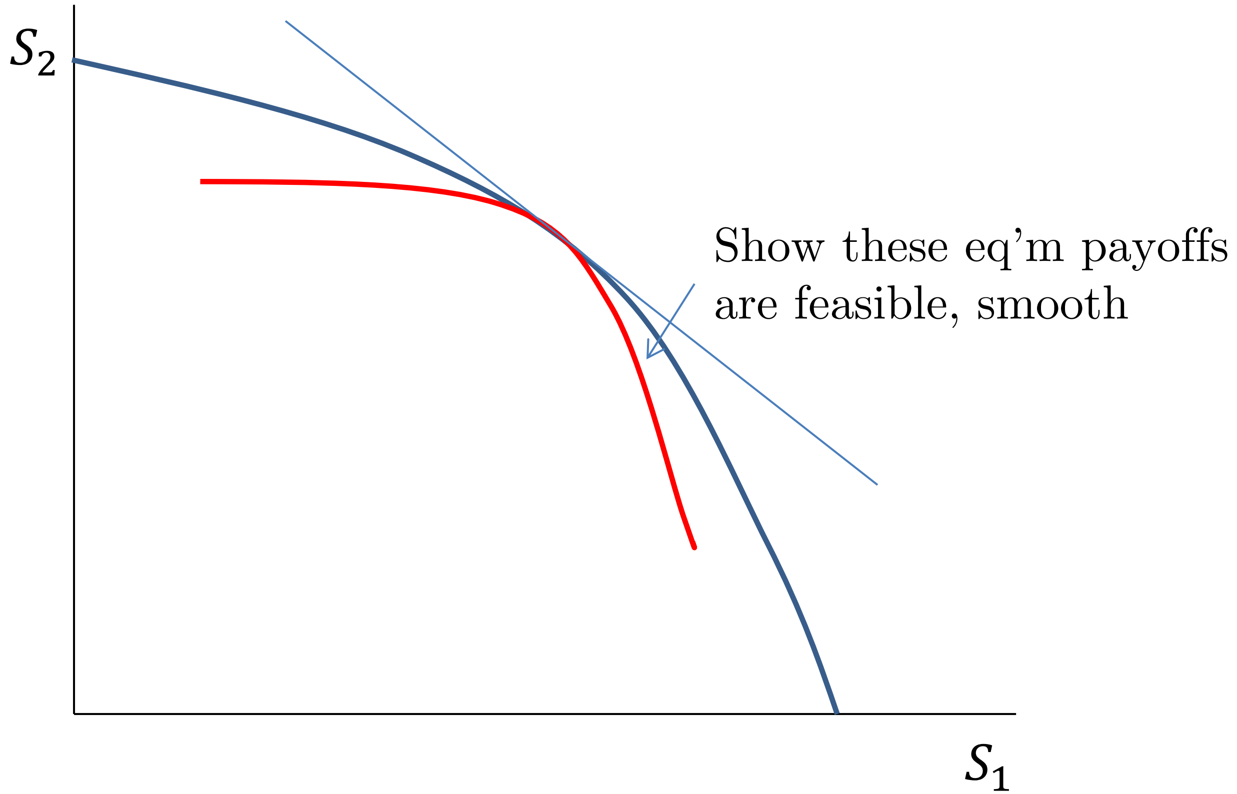
Why is Dyad-Surplus Frontier Smooth?

Goal: given eq'm σ^*, construct **feasible and IC** perturbations that smoothly change dyad-surplus

- Can use these perturbations to show biased decisions have second-order cost



Why is Dyad-Surplus Frontier Smooth?

Given P_i satisfies strict MLRP, exists unique $y_i^*(d_i, \theta, e_i)$ such that

$$\frac{\partial p_i / \partial e_i}{p_i}(y_i^* | d_i, \theta, e_i) = 0$$

Effort-maximizing reward scheme:

$$B_i(h_y^t) = \begin{cases} \overline{U}_i & if\ y_{i,t} < y_i^*(d_{i,t}, \theta_t, e_{i,t}) \\ S_i & otherwise \end{cases}$$

Since $y_{j,t'} < y_j^*$ for all $t' \leq t$, agent j's dyad-surplus irrelevant for $e_{j,t'}$

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Why is Dyad-Surplus Frontier Smooth?

Goal of construction:

- Increase $d_{i,t+1}$, decrease $d_{j,t+1}$
- Show that this change allows larger $e_{i,t'}$ and same $e_{j,t'}$ for all $t' \leq t$
- Apply first-order gain / second-order loss argument

Challenge:

- Changing $d_{i,t+1}$ or $e_{i,t}$ affects the distribution over agent i's output
- Changing i's output distribution changes distribution over continuation
- Incentives for all agents determined by distribution over continuation

Solution:

- Hold distribution over continuation play **constant** as $d_{i,t+1}$ or $e_{i,t}$ changes
- Must show that larger $e_{i,t'}$ incentive-compatible, holding continuation distribution fixed

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Why is Dyad-Surplus Frontier Smooth?

As $d_{i,t+1}$ increases...

- Output becomes more informative of effort
- Apply garbling to output to determine continuation play → same $e_{i,t+1}$ still IC, leading to same distribution over continuation play

As $e_{i,t}$ increases...

- Treat $y_{i,t}$ as output from same quantile of distribution induced by old effort
- Can show that $e_{i,t}$ increases smoothly in $d_{i,t+1}$

As $d_{j,t+1}$ decreases...

- Treat $y_{j,t+1}$ as output from same quantile of distribution induced by old effort
- Can show that $e_{j,t+1}$ decreases smoothly as $d_{j,t+1}$ decreases
- $e_{j,t'}$ unaffected because $y_{j,t'} < y_j^*$

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Hiring: Model ($N = 2$)

Demand is $\Theta = \{W, R\}$ with $0 < W < R$.

- If $\theta_t = R$, then $\theta_{t+1} = R$
- If $\theta_t = W$, then $\theta_{t+1} = R$ with probability $\rho \in (0,1)$

In each period, $D_t \in \{1,2\}$

- The principal hires d_t agents
- WLOG: if $d_t = 1$, then Agent 1 is hired

Effort is binary: $e_{i,t} \in \{0,1\}$ at cost $ce_{i,t}$

- Outside option is $\bar{u}_i = 0$

Output depends on effort and demand for hired workers

- If agent i is not hired, then $y_{i,t} = 0$
- If agent i is hired, then

$$y_{i,t} = \begin{cases} \theta_t e_{i,t} & if\ d_{i,t} = 1 \\ \theta_t \alpha e_{i,t} & if\ d_{i,t} = 2 \end{cases}$$

with $\alpha \in (0,1)$

Hiring: Result

Assume:

1. In first-best, employment increases with demand: $R > \frac{c}{2\alpha-1} > W$
2. Dyad-surplus larger with robust demand: $\alpha R > W$

There exist $\delta_L < \delta_H$ such that for $\delta \in (\delta_L, \delta_H)$, any surplus-maximizing equilibrium satisfies:

1. If $\theta_0 = R$, then $d_t = 2$ in every period t
2. If $\theta_0 = W$, then $d_t = 1$ whenever $\theta_t = W$. Moreover, there exists $t' > 0$ such that $\Pr\{d_{t'} = 1, \theta_{t'} = G\} > 0$

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Permanent Investment: Model ($N = 2$)

No state of the world: $|\Theta| = 1$

Principal makes one-time, irreversible investment in second period:

- $D_0 = \{0\}$, $D_1 = \{1,2\}$, $D_t = \{d_1\}$ for $t \geq 2$
- $d_1 = i$ interpreted as investment in agent i

Effort is binary: $e_{i,t} \in \{0,1\}$ at cost $ce_{i,t}$

- Outside option is $\bar{u}_i = 0$

Output continuous, depends on effort and investment

- $P_i(y_i|d_i = 0, e_i) = P_i(y_i|d_i = -i, e_i) = P_i(\alpha_i y_i|d_i = i, e_i)$ for $\alpha_1 \geq \alpha_2 > 1$,
- $e_i = 1$ efficient for any d
- $P_1 = P_2$, P_i is smooth and satisfies MLRP, with density p_i

Permanent Investment: Main Result

Define

$$L(y) = \frac{p_i(y_i | e_i = 1)}{p_i(y_i | e_i = 0)}$$

There exist $0 \leq \delta_L < \delta_H < 1$ and $\bar{\Delta} > 0$ such that if $\delta \in (\delta_L, \delta_H)$ and $\alpha_1 - \alpha_2 < \bar{\Delta}$, then any surplus-maximizing contract σ^* satisfies:

1. Both agents work in first period: $e_{1,0} = e_{2,0} = 1$
2. The principal sometimes invests in Agent 2: $d_1 = 2$ if $L\left(y_{2,0}\right) > 1$ and
$$\frac{1}{L(y_{2,0})} < \gamma + \beta\left(\frac{1}{L(y_{1,0})}\right)$$
for some $\gamma \in \mathbb{R}$ and $\beta \geq 0$

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Extension 1: Public Monitoring

Suppose all variables (except effort) publicly observed

- Agents can coordinate to punish the principal
- Following deviation, **total** continuation surplus destroyed as punishment

Biased decisions...

- ...decrease total continuation surplus...
- ...which decreases amount of surplus destroyed following punishment...
- ...which constrains set of credible incentives more tightly

Result: if monitoring is imperfect but public, then any surplus-maximizing relational contract is sequentially surplus-maximizing

- Small caveat: proof requires a public randomization device after each action

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Extension 2: Perfect Bayesian Equilibrium

Main result restricts to recursive equilibrium

- Want to show main intuition not dependent on this restriction
- To do so, consider the full set of Perfect Bayesian Equilibrium

Suppose (θ_t, d_t) are iid in each period

Let

$$\bar{V} = \max_{PBE\ \sigma^*, t \geq 0} E_{\sigma^*}\left[\sum_{t' \geq t} \delta^{t'-t}(1-\delta)\left(\pi_t + \sum_{i \leq N} u_{i,t}\right)\right]$$

- Max ex ante total expected continuation surplus, for any PBE and any period t

Lemma: there exists a PBE with ex ante total surplus $\bar{V}$

- A **surplus-maximizing** PBE has ex ante total surplus $\bar{V}$
- A **sequentially surplus-maximizing** PBE has ex ante continuation surplus $\bar{V}$ in each period t

Extension 2: PBE - Biased Decisions

For a smooth game with iid (θ_t, D_t), suppose

$$y_i = x_i + \gamma_i(\theta, d_i)$$

for some random variable x_i that depends only on (θ, e_i).

If no surplus-maximizing recursive equilibrium is sequentially surplus-maximizing, then no surplus-maximizing PBE is sequentially surplus-maximizing

Takeaway: In this class of games, if recursive equilibria entail biased decisions, so do PBE!

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