MECS 570-1: Organizational Economics I

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Incentives in Organizations, Part I: Risk–Incentives

Incentives in Organizations (Updated: Jan 5 2017)

In order to move away from the Neoclassical view of a firm as a single individual pursuing a single objective, different strands of the literature have proposed different approaches. The first is what is now known as "team theory" (going back to the 1972 work of Marschak and Radner). Team-theoretic models focus on issues that arise when all members of an organization have the same preferences—these models typically impose constraints on information transmission between individuals and information processing by individuals and look at questions of task and attention allocation.

The alternative approach, which we will focus on in the majority of the course, asserts that different individuals within the organization have different preferences (that is, "People (i.e., individuals) have goals; collectivities of people do not." (Cyert and March, 1963: 30)) and explores the implications that these conflicts of interest have for firm behavior. In turn, this approach examines how limits to formal contracting restrict a firm's ability to resolve these conflicts of interest and how unresolved conflicts of interest determine how decisions are made. We will talk about several different sources of limits to formal contracts and the trade-offs they entail.

We will then think about how to motivate individuals in environments where formal contracts are either unavailable or they are so incomplete that they are of little use. Individuals can be motivated out of a desire to convince "the market" that they are intrinsically productive in the hopes that doing so will attract favorable outside offers in the future—that is, they are motivated by their own career concerns. Additionally, individuals may form long-term attachments with an organization. In such long-term relationships, goodwill can arise as an equilibrium phenomenon, and fear of shattering this goodwill can motivate individuals to perform well and to reward good performance.

Formal Incentive Contracts (Updated: Jan 5 2017)

We will look at several different sources of frictions that prevent individuals from writing contracts with each other that induce the same patterns of behavior they would choose if they were all acting as a single individual receiving all the payoffs. The first will be familiar from core microeconomics—individual actions chosen by an agent are not observed but determine the distribution of a verifiable performance measure. The agent is risk-averse, so writing a high-powered contract on that noisy performance measure subjects him to costly risk. As a result, there is a trade-off between incentive provision (and therefore the agent's effort choice) and inefficient risk allocation. This is the famous **risk–incentives trade-off**.

The second contracting friction that might arise is that an agent is either liquidity-constrained or is subject to a limited-liability constraint. As a result, the principal is unable to extract all the surplus the agent generates and must therefore provide the agent with **incentive rents** in order to motivate him. That is, offering the agent a higher-powered contract induces him to exert more effort and therefore increases the total size of the pie, but it also leaves the agent with a larger share of that pie. The principal then, in choosing a contract, chooses one that trades off the creation of surplus with her ability to extract that surplus. This is the **motivation–rent extraction trade-off.**

The third contracting friction that might arise is that the principal's objective simply cannot be written into a formal contract. Instead, the principal has to rely on imperfectly aligned performance measures. Increasing the strength of a formal contract that is based on imperfectly aligned performance measures may increase the agent's efforts toward the principal's objectives, but it may also motivate the agent to exert costly effort towards objectives that either hurt the principal or at least do not help the principal. Since the principal ultimately has to compensate the agent for whatever effort costs he incurs in order to get him to sign a contract to begin with, even the latter proves costly for the principal. Failure to account for the effects of using distorted performance measures is sometimes referred to as **the folly of rewarding A while hoping for B** (Kerr, 1975) or the **multi-task problem** (Holmstrom and Milgrom, 1991).

All three of these sources of contractual frictions lead to similar results—under the optimal contract, the agent chooses an action that is not jointly optimal from his and the principal's perspective. But in different applied settings, different assumptions regarding what is contractible and what is not are more or less plausible. As a result, it is useful to master at least elementary versions of models capturing these three sources of frictions, so that you are well-equipped to use them as building blocks.

In the elementary versions of models of these three contracting frictions that we will look at, the effort level that the Principal would induce if there were no contractual frictions would solve:

$$\max_e pe - \frac{c}{2}e^2,$$

so that $e^{FB} = p/c$. All three of these models yield *equilibrium* effort levels $e^* < e^{FB}$.

Risk-Incentives Trade-off (Updated: Jan 5 2017)

The exposition of an economic model usually begins with a rough (but accurate and mostly complete) description of the players, their preferences, and what they do in the course of the game. The exposition should also include a precise treatment of the timing, which includes spelling out who does what and when and on the basis of what information, and a description of the solution concept that will be used to derive predictions. Given the description of the economic environment, it is then useful to specify the program(s) that players are solving.

I will begin with a pretty general description of the standard principal-agent model, but I will shortly afterwards specialize the model quite a bit in order to focus on a single point—the risk–incentives trade-off.

Description There is a risk-neutral Principal (P) and a risk-averse Agent (A). The Agent chooses an effort level $e \in \mathbb{R}_+$ at a private cost of $c(e)$, with $c'', c' > 0$, and this effort level affects the distribution over output $y \in Y$, with y distributed according to cdf $F(\cdot|\, e)$. This output can be sold on the product market at price p. The Principal can write a contract $w \in W \subset \{w : Y \to \mathbb{R}\}$ that determines a transfer $w(y)$ that she is compelled to pay the Agent if output y is realized. The Agent has an outside option that provides utility $\bar{u}$ to the Agent and $\bar{\pi}$ to the Principal. If the outside option is not exercised, the Principal's and Agent's preferences are, respectively,

$$\begin{aligned} \Pi(w,e) &= \int_{y\in Y} (py - w(y))\, dF(y|\, e) = E_y[py - w|\, e] \\ U(w,e) &= \int_{y\in Y} u(w(y) - c(e))\, dF(y|\, e) = E_y[u(w - c(e))|\, e]. \end{aligned}$$

Timing The timing of the game is:

1. P offers A a contract $w(y)$, which is commonly observed.
2. A accepts the contract ($d = 1$) or rejects it ($d = 0$) and receives $\bar{u}$, and the game ends. This decision is commonly observed.
3. If A accepts the contract, A chooses effort level e and incurs cost $c(e)$. e is only observed by A.
4. Output y is drawn from distribution with cdf $F(\cdot|\, e)$. y is commonly observed.
5. P pays A an amount $w(y)$. This payment is commonly observed.

A couple remarks are in order at this point. First, behind the scenes, there is an implicit assumption that there is a third-party contract enforcer (a judge or arbitrator) who can costlessly detect when agreements have been broken and costlessly exact harsh punishments on the offender. Second, it is not necessarily important that e is unobserved by the Principal—given that the Principal takes no actions after the contract has been offered, as long as the contract cannot be conditioned directly on effort, the outcome of the game will be the same whether or not the Principal observes e. Put differently, one way of viewing the underlying source of moral-hazard problems is that contracts cannot be conditioned on relevant variables, not that the relevant variables are unobserved by the Principal. We will return to this issue when we discuss the Property Rights Theory of firm boundaries.

Solution Concept A **pure-strategy subgame-perfect equilibrium** is a contract $w^* \in W$, an acceptance decision $d^* : W \to \{0,1\}$, and an effort choice $e^* : W \times \{0,1\} \to \mathbb{R}_+$ such that, given the contract w^*, the Agent optimally chooses d^* and e^*, and given d^* and e^*, the Principal optimally offers contract w^*. We will say that the optimal contract **induces** effort e^*.

The Program The principal offers a contract $w \in W$ and proposes an effort level e in order to solve

$$\max_{w \in W, e \in \mathbb{R}_+} \int_{y \in Y} (py - w(y))\, dF(y|\, e)$$

subject to two constraints. The first constraint is that the agent actually prefers to choose effort level e rather than any other effort level $\hat{e}$. This is the standard **incentive-compatibility constraint**:

$$e \in \underset{\hat{e} \in \mathbb{R}_+}{\operatorname{argmax}} \int_{y \in Y} u(w(y) - c(\hat{e}))\, dF(y|\, \hat{e}).$$

The second constraint is that, given that the agent knows he will choose e if he accepts the contract, he prefers to accept the contract rather than to reject it and receive his outside utility $\bar{u}$. This is the standard **individual-rationality constraint** or **participation constraint**:

$$\int_{y \in Y} u(w(y) - c(e))\, dF(y|\, e) \geq \bar{u}.$$

CARA-Normal Case with Affine Contracts In order to establish a straightforward version of the risk-incentives trade-off, we will make a number of simplifying assumptions.

Assumption 1. The Agent has CARA preferences over wealth and effort costs, which are quadratic:

$$u(w(y) - c(e)) = -\exp\left\{-r\left(w(y) - \frac{c}{2}e^2\right)\right\},$$

and his outside option yields utility $-\exp\{-r\bar{u}\}$.

Assumption 2. Effort shifts the mean of a normally distributed random variable. That is, $y \sim N(e, \sigma^2)$.

Assumption 3. $W = \{w : Y \to \mathbb{R}, w(y) = s + by\}$. That is, the contract space permits only affine contracts.

Discussion. In principle, there should be no exogenous restrictions on the functional form of $w(y)$. Applications, however, often restrict attention to affine contracts: $w(y) = s + by$. In many environments, an optimal contract does not exist if the contracting space is sufficiently rich, and situations in which the agent chooses the first-best level of effort, and the principal receives all the surplus can be arbitrarily approximated with a sequence of sufficiently perverse contracts (Mirrlees, 1974; Moroni and Swinkels, 2014). In contrast, the optimal affine contract often results in an effort choice that is lower than the first-best effort level, and the principal receives a lower payoff.

There are then at least three ways to view the exercise of solving for the optimal affine contract.

1. From an applied perspective, many pay-for-performance contracts in the world are affine in the relevant performance measure—franchisees pay a franchise fee and receive a constant fraction of the revenues their store generates, windshield installers receive a base wage and a constant piece rate, fruit pickers are paid per kilogram of fruit they pick. And so given that many practitioners seem to restrict attention to this class of contracts, why don't we just make sure they are doing what they do optimally? Put differently, we can brush aside global optimality on purely pragmatic grounds.

2. Many pay-for-performance contracts in the world are affine in the relevant performance

measure. Our models are either too rich or not rich enough in a certain sense and therefore generate optimal contracts that are inconsistent with those we see in the world. Maybe the aspects that, in the world, lead practitioners to use affine contracts are orthogonal to the considerations we are focusing on, so that by restricting attention to the optimal affine contract, we can still say something about how real-world contracts ought to vary with changes in the underlying environment. This view presumes a more positive (as opposed to normative) role for the modeler and hopes that the theoretical equivalent of the omitted variables bias is not too severe.

3. Who cares about second-best when first-best can be attained? If our models are pushing us toward complicated, non-linear contracts, then maybe our models are wrong. Instead, we should focus on writing down models that generate affine contracts as the optimal contract, and therefore we should think harder about what gives rise to them. (And indeed, steps have been made in this direction—see Holmstrom and Milgrom (1987), Diamond (1998) and, more recently, Carroll (2013) and Barron, Georgiadis, and Swinkels (2017)) This perspective will come back later in the course when we discuss the Property Rights Theory of firm boundaries.

Given the assumptions, for any contract $w(y) = s + by$, the income stream the agent receives is normally distributed with mean $s + be$ and variance $b^2\sigma^2$. His expected utility over monetary compensation is therefore a moment-generating function for a normally distributed random variable, (recall that if $X \sim N(\mu, \sigma^2)$, then $E[\exp\{tX\}] = \exp\left\{\mu t + \frac{1}{2}\sigma^2 t^2\right\}$), so his preferences can be written as

$$E\left[-\exp\left\{-r\left(w\left(y\right)-c\left(e\right)\right)\right\}\right] = -\exp\left\{-r\left(s+be\right)+\frac{r^2}{2}b^2\sigma^2+r\frac{c}{2}e^2\right\}.$$

We can take a monotonic transformation of his utility function ($-\frac{1}{r}\log(-x)$) and represent his preferences as:

$$\begin{aligned} U(e,w) &= E[w(y)] - \frac{r}{2}Var(w(y)) - \frac{c}{2}e^2 \\ &= s + be - \frac{r}{2}b^2\sigma^2 - \frac{c}{2}e^2. \end{aligned}$$

The Principal's program is then

$$\max_{s,b,e} pe - (s + be)$$

subject to incentive-compatibility

$$e \in \operatorname*{argmax}_{\hat{e}} b\hat{e} - \frac{c}{2}\hat{e}^2$$

and individual-rationality

$$s + be - \frac{r}{2}b^2\sigma^2 - \frac{c}{2}e^2 \geq \bar{u}.$$

Solving this problem is then relatively straightforward. Given an affine contract $s + be$, the agent will choose an effort level $e(b)$ that satisfies his first-order conditions

$$e(b) = \frac{b}{c},$$

and the Principal will choose the value s to ensure that the agent's individual-rationality constraint holds with equality (for if it did not hold with equality, the Principal could reduce s, making herself better off without affecting the Agent's incentive-compatibility constraint, while still respecting the Agent's individual-rationality constraint). That is,

$$s + be(b) = \frac{c}{2}e(b)^2 + \frac{r}{2}b^2\sigma^2 + \bar{u}.$$

In other words, the Principal has to ensure that the Agent's total expected monetary compensation, $s + be(b)$, fully compensates him for his effort costs, the risk costs he has to bear if he accepts this contract, and his opportunity cost. Indirectly, then, the Principal bears these costs when designing an optimal contract.

The Principal's remaining problem is to choose the incentive slope b to solve

$$\max_b pe(b) - \frac{c}{2}e(b)^2 - \frac{r}{2}b^2\sigma^2 - \bar{u}.$$

This is now an unconstrained problem with proper convexity assumptions, so the Principal's optimal choice of incentive slope solves her first-order condition

$$\begin{aligned} 0 &= pe'(b^*) - ce^*(b^*)e'(b^*) - rb^*\sigma^2 \\ &= \frac{p}{c} - c\frac{b^*}{c}\frac{1}{c} - rb^*\sigma^2 \end{aligned}$$

and therefore

$$b^* = \frac{p}{1 + rc\sigma^2}.$$

Also, given b^* and the individual-rationality constraint, we can back out s^*.

$$s^* = \bar{u} + \frac{1}{2}\left(rc\sigma^2 - 1\right)\frac{(b^*)^2}{c}.$$

Depending on the parameters, it may be the case that $s^* < 0$. That is, the Agent would have to pay the Principal if he accepts the job and does not produce anything.

In this setting, if the Principal could contract directly on effort, she would choose a contract that ensures that the Agent's individual-rationality constraint binds and therefore would solve

$$\max_e pe - \frac{c}{2}e^2,$$

so that

$$e^{FB} = \frac{p}{c}.$$

If the Principal wanted to implement this same level of effort using a contract on output, y, she would choose $b = p$ (since the Agent would choose $\frac{b}{c} = \frac{p}{c}$).

Why, in this setting, does the Principal not choose such a contract? Let us go back to the Principal's problem of choosing the incentive slope b.

$$\max_{b} pe(b) - \frac{c}{2}e(b)^2 - \frac{r}{2}b^2\sigma^2 - \bar{u}$$

Many fundamental points in models in the Organizational Economics literature can be seen as a comparison of first-order losses or gains against second-order gains or losses. Suppose the Principal chooses $b = p$, and consider a marginal reduction in b away from this value. The change in the Principal's profits would be

$$\begin{aligned} & \frac{d}{db}\left(pe(b) - \frac{c}{2}e(b)^2 - \frac{r}{2}b^2\sigma^2\right)\Big|_{b=p} \\ = & \underbrace{\frac{d}{db}\left(pe(b) - \frac{c}{2}e(b)^2\right)\Big|_{b=p}}_{=0} - rp\sigma^2 < 0 \end{aligned}$$

This first term is zero, because $b = p$ in fact maximizes $pe(b) - \frac{c}{2}e(b)^2$ (since it induces the first-best level of effort). The second term is strictly negative. That is, relative to the contract that induces first-best effort, a reduction in the slope of the incentive contract yields a first-order gain resulting from a decrease in the risk costs the Agent bears, while it yields a second-order loss in terms of profits resulting from moving away from the effort level that maximizes revenues minus effort costs. The optimal contract balances the incentive benefits of higher-powered incentives with these risk costs.

This trade-off seems first-order in some settings (e.g., insurance contracts in health care markets, some types of sales contracts in industries in which individual sales are infrequent, large, and unpredictable) and for certain types of output. There are many other environments in which contracts provide less-than-first-best incentives, but the first-order reasons for these low-powered contracts seem completely different, and we will turn to these environments shortly.

Before doing so, it is worth pointing out that many models in this course will involve trade-offs that determine the optimal way of organizing a firm. In many of the settings these models examine, results that take the form of "X is organized according to Y, because player A is more risk-averse than player B" often seem intuitively unappealing. For example, suppose a model of hierarchies generated the result that less risk-averse individuals should be at the top of an organization, and more risk-averse individuals should be at the bottom. This sounds somewhat sensible—maybe richer individuals are better able to diversify their wealth, and they can therefore behave as if they are less risk averse with respect to the income stream they derive from a particular organization. But it sounds less appealing as a general rule for who should be assigned to what role in an organization—a model that predicts that more knowledgeable or more experienced workers should be assigned to higher positions seems more consistent with experience.

Finally, connecting this analysis back to the Neoclassical view of the firm, what does the risk-incentives trade-off imply about the firm's production set? Let Y^f denote the firm's **technological possibilities set**, in which the firm's input is labor costs C, and its expected output is y. This is the set of input-output vectors that would be feasible if there were no contracting frictions.

We can write

$$C = c(e) = \frac{c}{2}e^2,$$

and since expected output is just equal to the Agent's effort choice, we have that

$$y(C) = e = \left(\frac{2C}{c}\right)^{1/2}.$$

The technological possibilities set is therefore

$$Y^f = \{(y, -C) : y \leq y(C)\}.$$

We will now augment the technological possibilities set with the contractual considerations we have just derived. Because the Principal can increase s without bound, the **contract-augmented possibilities set** can be characterized by its frontier, which is the highest level of expected output the firm can produce for a given level of costs. If the Principal puts in place an optimal contract with incentive slope b (in which case the Agent's effort choice will be $e^*(b) = \frac{b}{c}$) and an s that pins the Agent to his individual-rationality constraint, the firm's profits are

$$p\frac{b}{c} - \frac{c}{2}\left(\frac{b}{c}\right)^2 - \frac{r}{2}\sigma^2 b^2 = p\frac{b}{c} - \frac{1}{2}\frac{1+rc\sigma^2}{c}b^2.$$

Therefore, producing expected output $\frac{b}{c}$ costs the firm

$$C = \frac{1}{2}\frac{1+rc\sigma^2}{c}b^2.$$

Finally, we can rearrange this equation to solve for the b such that the total costs to the Principal are C:

$$b = \left(\frac{2Cc}{1+rc\sigma^2}\right)^{1/2}.$$

In this case, the firm produces expected output $y = \tilde{y}(C)$, which is given by

$$\tilde{y}(C) = \left(\frac{2C}{c}\right)^{1/2}\left(\frac{1}{1+rc\sigma^2}\right)^{1/2} = \left(\frac{1}{1+rc\sigma^2}\right)^{1/2} y(C).$$

The contract-augmented possibilities set is therefore

$$\tilde{Y}^f = \{(y, -C) : y \leq \tilde{y}(C)\}.$$

Because of contractual frictions, we have that $\tilde{Y}^f \subset Y^f$, and any change in the parameters of the model for which the divergence between e^{FB} and e^* grows (such as an increase in the Agent's risk aversion or an increase in output uncertainty) will tend to increase the difference between $y(C)$ and $\tilde{y}(C)$ and therefore will shrink the contract-augmented possibilities set.

In this elementary version of this model, the contract-augmented possibilities set is a convex set. More generally, given a production-possibilities set that is convex, it need not be the case that the contract-augmented possibilities set is convex.

Further Reading Many papers restrict attention to linear contracts, even in environments in which the optimal contract (if it exists) is not linear. Holmstrom and Milgrom (1987) examines an environment in which the principal and the agent have CARA preferences and the agent controls the drift of a Brownian motion for a finite time interval. An optimal contract conditions payments only on the value of the Brownian motion at the end of the time interval. Diamond (1998) considers an environment in which the agent can choose the mean of the output distribution as well as the entire distribution itself and shows (essentially by a convexification argument) that linear contracts are optimal. Carroll (2015) shows that linear contracts can be max-min optimal when the Principal is sufficiently uncertain about the class of actions the Agent can take.

A key comparative static of the risk–incentives moral-hazard model is that incentives are optimally weaker when there is more uncertainty in the mapping between effort and contractible output, but this comparative static is inconsistent with a body of empirical work suggesting that in more uncertain environments, agency contracts tend to involve higher-powered incentives. Prendergast (2002) resolves this discrepancy by arguing that in more uncertain environments, it is optimal to assign greater responsibility to the agent and to complement this greater responsibility with higher-powered incentives. Holding responsibilities fixed, the standard risk–incentives tradeoff would arise, but the empirical studies that fail to find this relationship do not control for workers' responsibilities. Raith (2003) argues that these empirical studies examine the relationship between the risk the firm faces and the strength of the agent's incentives, while the theory is about the relationship between the risk the *agent* faces and his incentives. For an examination of several channels through which uncertainty can impact an agent's incentives, see Rantakari (2008).