MECS 570-1: Organizational Economics I

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Decision Making in Organizations: Delegation

Decision Making in Organizations (Updated: Feb 16 2016)

In the first couple weeks of the class, we considered environments in which one party (the Agent) chose actions that affected the payoffs of another party (the Principal), and we asked how the Principal could motivate the Agent to choose actions that were more favorable for the Principal. We considered the use of formal incentive contracts, and we also looked at environments in which such contracts were unavailable. Throughout, however, we took as given that only the agent was able to make those decisions. In other words, **decision rights** were **inalienable**. This week, we will allow for the possibility that control can be transferred from one party to the other, and we will ask when one party or the other should possess these decision rights. Parts of this discussion will echo parts of the discussion on the boundaries of the firm, where asset allocation was tantamount to decision-rights allocation, but the trade-offs we will focus on here will be different.

If in principle, important decisions could be made by the Principal, why would the Principal ever want to delegate such decisions to an Agent? In his book on the design of bureaucracies, James Q. Wilson concludes that "In general, authority [decision rights] should be placed at the lowest level at which all essential elements of information are available." A Principal may therefore want to delegate to a better-informed Agent who knows more about what decisions are available or what their payoff consequences are. But delegation itself may be costly as well, because the Principal and the Agent may disagree about the ideal decision to be made. This conflict is resolved in different ways in different papers in the literature.

First, if the Principal can commit to a decision rule as a function of an announcement by the Agent, then the formal allocation of control is irrelvant. This mechanism-design approach to delegation (Holmstrom, 1984; Alonso and Matouschek, 2008; Amador and Bagwell, 2013;

Frankel, 2016) focuses on the idea that while control is irrelevant, implementable decision rules can be implemented via constrained delegation: the Principal delegates to the Agent, but the Agent is restricted to making decisions from a restricted "delegation set." The interesting results of these papers is their characterization of optimal delegation sets.

If the Principal cannot commit to a decision rule, then the allocation of control matters. The optimal allocation of control is determined by one of several trade-offs identified in the literature. The most direct trade-off that a Principal faces is the trade-off between a loss of control under delegation (since the Agent may not necessarily make decisions in the Principal's best interest) and a loss of information under centralization (since the Principal may not be able to act upon the Agent's information). This trade-off occurs even if the Agent is able to communicate his information to the Principal in a cheap-talk manner (Dessein, 2002). Next, if the Agent has to exert non-contractible effort in order to become informed, then his incentives to do so are greater if he is able to act upon that information: delegation improves incentives for information acquisition. There is therefore a trade-off between loss of control under delegation and loss of initiative under centralization (Aghion and Tirole, 1997).

The previous two trade-offs are only relevant if the preferences of the Principal and the Agent are at least somewhat well-aligned. Even if they are not, however, delegation can serve a role. It may be beneficial to promise the Agent future control as a reward for good decision making today in order to get the Agent to use his private information in a way that is beneficial for the Principal. There is therefore a dynamic trade-off between loss of information today and loss of control in the future (Li, Matouschek, and Powell, 2017; Lipnowski and Ramos, 2017).

Mechanism-Design Approach to Delegation

Description There is a Principal (P) and an Agent (A) and a single decision $d \in \mathbb{R}$ to be made. Both P and A would like the decision to be tailored to the state of the world,

$s \in S$, which is privately observed only by A. The Principal selects (and commits to) a control-rights allocation $g \in \{P, A\}$, a mechanism (M, d), which consists of a message space M and a deterministic decision rule $d : M \to \mathbb{R}$, which selects a decision $d(m)$ as a function of a message $m \in M$ sent by the Agent, and a delegation set $D \subset \mathbb{R}$. If $g = P$, then P makes decisions according to $d(\cdot)$. If $g = A$, then A makes decision $d_A \in D \subset \mathbb{R}$. Players' preferences are given by

$$\begin{aligned} u_P(d, s) &= -(d - s)^2 \\ u_A(d, s) &= -(d - y_A(s))^2, \end{aligned}$$

where $y_A(\cdot)$ is strictly increasing in s. Given state of the world s, P would like the decision to be $d = s$, and A would like the decision to be $d = y_A(s)$. There are no transfers.

Timing The timing of the game is:

1. P chooses control-rights allocation $g \in \{P, A\}$, mechanism (M, d), and delegation set D. g, M, d, and D are commonly observed.

2. A privately observes s.

3. A sends message $m \in M$ and chooses $d_A \in D$, which are commonly observed.

4. If $g = P$, the resulting decision is $d = d(m)$. If $g = A$, the resulting decision is $d = d_A$.

Equilibrium A **pure-strategy subgame-perfect equilibrium** is a control-rights allocation g^*, a mechanism (M^*, d^*), a delegation set D^*, an announcement function $m^* : S \to M^*$, and a decision rule $d_A^* : S \to D^*$ such that given g^* and (M^*, d^*), the Agent optimally announces $m^*(s)$ and chooses $d_A^*(s)$ when the state of the world is s, and the Principal optimally chooses control-rights allocation g^*, mechanism (M^*, d^*), and delegation set D^*.

The Program The Principal chooses (g, M, d, D) to solve

$$\max_{g,M,d,D} \int_s \left[u_P \left(d \left(m^* \left(s \right) \right), s \right) 1_{g=P} + u_P \left(d_A^* \left(s \right), s \right) 1_{g=A} \right] dF \left(s \right)$$

subject to

$$m^* \left(s \right) \in \underset{m \in M}{\operatorname{argmax}} \int_s u_A \left(d \left(m \right), s \right) dF \left(s \right)$$

and

$$d_A^* \left(s \right) \in \underset{d \in D}{\operatorname{argmax}} \int_s u_A \left(d, s \right) dF \left(s \right).$$

Functional-Form Assumptions We will assume that $s \sim U \left[-1, 1 \right]$ and $y_A \left(s \right) = \beta s$, where $\beta > 1/2$.

Outline of the Analysis I will begin by separating out the problem of choosing a mechanism (M, d) from the problem of choosing a delegation set D. Define

$$V^P = \max_{M,d} \int_s u_P \left(d \left(m^* \left(s \right) \right), s \right) dF \left(s \right)$$

subject to

$$m^* \left(s \right) \in \underset{m \in M}{\operatorname{argmax}} \int_s u_A \left(d \left(m \right), s \right) dF \left(s \right)$$

and define

$$V^A = \max_{D} \int_s u_P \left(d_A^* \left(s \right), s \right) dF \left(s \right)$$

subject to

$$d_A^* \left(s \right) \in \underset{d \in D}{\operatorname{argmax}} \int_s u_A \left(d, s \right) dF \left(s \right).$$

The Coasian program can then be written as

$$\max_{g} V^g.$$

I will now proceed in several steps, for the most part following the analysis of Alonso and Matouschek (2008).

1. First, I will show that under $g = P$, there is an analog of the revelation principle that simplifies the search for an optimal mechanism: it is without loss of generality to set $M = S$ and focus on incentive-compatible decision rules $d(s)$ that satisfy

$$u_A(d(s), s) \geq u_A(d(s'), s) \text{ for all } s', s \in S.$$

2. I will then show that all incentive-compatible decision rules have some nice properties.

3. Further, each incentive-compatible decision rule $d(s)$ is associated with a range $\tilde{D} = \{d(s) : s \in S\}$, and the incentive-compatibility condition is equivalent to

$$u_A(d(s), s) \geq u_A(d', s) \text{ for all } d' \in \tilde{D}.$$

   This result implies that **the allocation of control is irrelevant**. For any incentive-compatible direct mechanism (Θ, d), there is a delegation set D such that under either control-rights allocation g, the decision rule is the same: $d(s) = d_A(s)$, which implies that $V^A = V^P$. It is therefore without loss of generality to solve for the optimal delegation set D.

4. I will restrict attention to **interval delegation sets** $D = [d_L, d_H]$, which under the specific functional-form assumptions I have made, is indeed without loss of generality. The Principal's problem will then be to

$$\max_{d_L, d_H} \int_s u_P(d_A^*(s), s)\, dF(s)$$

   subject to

$$u_A(d_A^*(s), s) \geq u_A(d, s) \text{ for all } d_L \leq d \leq d_H.$$

Step 1: Revelation Principle Given $g = P$, any choice (M, d) by the Principal implements some distribution over outcomes $\sigma(s)$, which may be a nontrivial distribution, since the Agent might be indifferent between sending two different messages that induce two different decisions. Since $y_A(s)$ is strictly increasing in s, it follows that $\sigma(s)$ must be increasing in s in the sense that if $d \in \operatorname{supp} \sigma(s)$ and $d' \in \operatorname{supp} \sigma(s')$ for $s > s'$, then $d > d'$. This distribution determines some expected payoffs (given state s) for the Principal:

$$\pi(s) = E_{\sigma(s)}\left[u_P(d(m), s)\right],$$

where the expectation is taken over the distribution over messages that induces $\sigma(s)$. For each s, take $\hat{d}(s) \in \operatorname{supp} \sigma(s)$ such that

$$u_P\left(\hat{d}(s), s\right) \geq \pi(s).$$

The associated direct mechanism (S, d) is well-defined, incentive-compatible, and weakly better for the Principal, so it is without loss of generality to focus on direct mechanisms.

Step 2: Properties of Incentive-Compatible Mechanisms The set of incentive-compatible direct mechanisms $d : S \to \mathbb{R}$ satisfies

$$u_A(d(s), s) \geq u_A(d(s'), s) \text{ for all } s, s' \in S.$$

or

$$|d(s) - y_A(s)| \leq |d(s') - y_A(s)| \text{ for all } s, s'.$$

This condition implies a couple properties of $d(\cdot)$, but the proofs establishing these properties are fairly involved (which correspond to Proposition 1 in Melumad and Shibano (1991)), so I omit them here. First, $d(\cdot)$ must be weakly increasing, since $y_A(\cdot)$ is increasing. Next, if it is strictly increasing and continuous on an open interval (s_1, s_2), it must be the case that

$d(s) = y_A(s)$ for all $s \in (s_1, s_2)$. Finally, if d is not continuous at s', then there must be a jump discontinuity such that

$$\lim_{s \uparrow s'} u_A(d(s), s') = \lim_{s \downarrow s'} u_A(d(s), s'),$$

and $d(s)$ will be flat in an interval to the left and to the right of s'.

Step 3: Control-Rights Allocation is Irrelevant For any direct mechanism d, we can define the range of the mechanism to be $\tilde{D} = \{d(s) : s \in S\}$. The incentive-compatibility condition is then equivalent to

$$u_A(d(s), s) \geq u_A(d', s) \text{ for all } d' \in \tilde{D}.$$

That is, given a state s, the Agent has to prefer decision $d(s)$ to any other decision that he could induce by any other announcement s'. Under $g = P$, choosing a decision rule $d(s)$ therefore amounts to choosing its range $\tilde{D}$ and allowing the Agent to choose his ideal decision $d \in \tilde{D}$. The Principal's problem is therefore identical under $g = P$ as under $g = A$, so that $V^P = V^A$. Therefore, the allocation of control rights is irrelevant when the Principal has commitment either to a decision rule or to formal constraints on the delegation set. It is therefore without loss of generality to solve for the optimal delegation set D, so the Principal's problem becomes

$$\max_D \int u_P(d_A^*(s), s)\, dF(s)$$

subject to

$$u_A(d_A^*(s), s) \geq u_A(d', s) \text{ for all } s \text{ and for all } d' \in D.$$

Step 4: Optimal Interval Delegation Under the specific functional-form assumptions I have made, it is without loss of generality to focus on interval delegation sets of the form

$D = [d_L, d_H]$, where $d_L \leq d_H$ and d_L can be $-\infty$ and d_H can be $+\infty$ (this result is nontrivial and follows from Proposition 3 in Alonso and Matouschek (2008)). Any interval $[d_L, d_H]$ will be associated with an interval of states $[s_L, s_H] = [d_L/\beta, d_H/\beta]$ such that

$$d_A^*(s) = \begin{cases} d_L & s \leq s_L \\ \beta s & s_L < s < s_H \\ d_H & s \geq s_H. \end{cases}$$

The Principal's problem will then be to

$$\max_{d_L, d_H} \int_{-1}^{s_L} u_P(d_L, s)\, dF(s) + \int_{s_L}^{s_H} u_P(\beta s, s)\, dF(s) + \int_{s_H}^{1} u_P(d_H, s)\, dF(s)$$

or since $dF(s) = 1/2ds$, $s_L = d_L/\beta$ and $s_H = d_H/\beta$,

$$\max_{d_L, d_H} -\frac{1}{2}\left[\int_{-1}^{d_L/\beta} (d_L - s)^2\, ds + \int_{d_L/\beta}^{d_H/\beta} (\beta s - s)^2\, ds + \int_{d_H/\beta}^{1} (d_H - s)^2\, ds\right].$$

Applying the Kuhn-Tucker conditions (using Leibniz's rule), with some effort, we get

$$d_L^* = \max\left\{-\frac{\beta}{2\beta - 1}, -1\right\}, d_H^* = \min\left\{\frac{\beta}{2\beta - 1}, 1\right\},$$

if interior.

It is worth noting that if $\beta = 1$, so that P and A are perfectly aligned, then $d_L^* = -1$ and $d_H^* = 1$. That is, the Principal does not constrain the Agent's choices if their ideal decisions coincide. If $\beta > 1$, $d_L^* > -1$ and $d_H^* < 1$. In this case, the Agent's ideal decision is more responsive to the state of the world than the Principal would like, and the only instrument the Principal has to reduce the sensitivity of the Agent's decision rule is to constrain his decision set.

Finally, if $\beta < 1$, then again $d_L^* = -1$ and $d_H^* = 1$. In this case, the Agent's ideal decision is not as responsive to the state of the world as the Principal would like, but the Principal cannot use interval delegation to make the Agent's decision rule more responsive to the state of the world. Alonso and Matouschek (2008) provide conditions under which the Principal may like to remove points from the Agent's delegation set precisely in order to make the Agent's decision rule more sensitive to the state of the world.

Exercise If in addition to a message-contingent decision rule, the Principal is able to commit to a set of message-contingent transfers, it will still be the case that the allocation of control is irrelevant. Show that this is the case. In doing so, assume that the Agent has an outside option that yields utility $\bar{u}$ and that the Principal makes a take-it-or-leave-it offer of a mechanism (M, d, t), where $d : M \to \mathbb{R}$ is a decision rule and $t : M \to \mathbb{R}$ is a set of transfers from the Principal to the Agent.