Economics 2010b Problem Set 1

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Due: Friday, 2/2 by 11:59pm

Exercise 1 (Adapted from MWG 15.B.2). Consider an Edgeworth box economy in which the consumers have Cobb-Douglas utility functions $u_1(x_{1,1}, x_{2,1}) = x_{1,1}^{\alpha} x_{2,1}^{1-\alpha}$ and $u_2(x_{1,2}, x_{2,2}) = x_{1,2}^{\beta} x_{2,2}^{1-\beta}$, where $\alpha, \beta \in (0,1)$. Consumer i's endowments are $(\omega_{1,i}, \omega_{2,i}) >> 0$ for $i = 1, 2$. Solve for the Walrasian equilibrium price ratio and allocation. How do these change as you increase $\omega_{1,1}$? Note: Feel free to avoid writing expressions out as much as possible. For example, if you solve for price, feel free to leave the solutions for demand in terms of the price variable instead of plugging in. For comparative statics, if you can find the sign without having to write it out, that's fine.

Exercise 2 (Adapted from MWG 15.B.6). Compute the Walrasian equilibria for the following Edgeworth box economy (there is more than one Walrasian equilibrium):

$$
\begin{aligned}
u_1(x_{1,1}, x_{2,1}) &= \left(x_{1,1}^{-2} + \left(\frac{12}{37}\right)^3 x_{2,1}^{-2} \right)^{-1/2}, \qquad \omega_1 = (1, 0), \\
u_2(x_{1,2}, x_{2,2}) &= \left(\left(\frac{12}{37}\right)^3 x_{1,2}^{-2} + x_{2,2}^{-2} \right)^{-1/2}, \qquad \omega_2 = (0, 1).
\end{aligned}
$$

Exercise 3 (Adapted from MWG 15.B.9). Suppose that in a pure exchange economy, we have two consumers, Alphanse and Betatrix, and two commodities, Perrier and Brie. Alphanse and Betatrix have the utility functions:

$$
u_\alpha(x_{p,\alpha}, x_{b,\alpha}) = \min\{x_{p,\alpha}, x_{b,\alpha}\} \text{ and } u_\beta(x_{p,\beta}, x_{b,\beta}) = \min\left\{x_{p,\beta}, (x_{b,\beta})^{1/2}\right\},
$$

(where $x_{p,\alpha}$ is Alphanse's consumption of Perrier, and so on). Alphanse starts with an endowment of 30 units of Perrier (and none of Brie); Betatrix starts with 20 units of Brie (and none of Perrier). Neither can consume negative amounts of a commodity. If the two consumers behave as price takers, what is the equilibrium? [Hint: consider the market-clearing condition in the cases when both prices are positive, when only the price of Perrier is positive, and when only the price of Brie is positive.]

Exercise 4. Consider an exchange economy with two consumers. The utility functions and endowments are given by

$$
\begin{aligned}
u_1(x_{1,1}, x_{2,1}) &= x_{1,1} - \frac{x_{2,1}^{-3}}{3}, \qquad \omega_1 = (K, r) \\
u_2(x_{1,2}, x_{2,2}) &= x_{2,2} - \frac{x_{1,2}^{-3}}{3}, \qquad \omega_2 = (r, K).
\end{aligned}
$$

Assume that K is sufficiently large so that each consumer can achieve an interior solution to her optimal consumption problem. Note that $p^* = (1, 1)$ is an equilibrium price vector.

(a) For what values of r will there be multiple Walrasian equilibria in this economy? [Hint: first solve for $q = p_y/p_x$ by showing that $rt^4 - t^3 + t - r = 0$, where $t = q^{1/4}$. Note this expression factors as $(t+1)(t-1)(rt^2 - t + r) = 0$.]

(b) For what value of r will $p^* = (1, 3)$ be an equilibrium price vector?

(c) [Optional: algebra intensive] Assume that $K = 10$ and that r takes the value identified in part (b). Find all equilibrium prices and allocations.

(d) [Optional: algebra intensive] Rank the outcomes identified in part (d) in terms of most preferred to least preferred for each consumer.

Exercise 5 (Adapted from MWG 15.B.1). Consider an Edgeworth box economy in which the two consumers' preferences satisfy local nonsatiation. Let $x_{l,i}(p, p \cdot \omega_i)$ be consumer i's demand for commodity l at prices $p = (p_1, p_2)$.

(a) Show that $p_1 \sum_{i \in \mathcal{I}} (x_{1,i} - \omega_{1,i}) + p_2 \sum_{i \in \mathcal{I}} (x_{2,i} - \omega_{2,i}) = 0$ for all prices $p \neq 0$.

(b) Argue that if the market for commodity 1 clears at prices $p^* >> 0$, then so does the market for commodity 2; hence p^* is a Walrasian equilibrium price vector.