Markets for Rhetorical Services

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May 8, 2018

Abstract

This paper studies markets for "rhetorical services" such as advertising. An agency prices a service that can be purchased by a client and that alters the distribution of a signal observed by a audience. We show that the audience's beliefs about purchase behavior influence the client's benefit from this service. The feedback from (audience) beliefs to (client) benefit can lead to upward-sloping equilibrium demand and other unusual market features, and it implies that these markets have subtle effects on both client and audience welfare. If the agency can freely design its rhetorical service, then we give conditions under which communication completely breaks down, to the potential detriment of both client and audience. To serve as a foundation for this analysis, we develop a parsimonious model of client-audience communication in which the client has private information about her rhetorical ability.

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> "If we should enquire into the principle in the human mind on which this disposition of [trade] is founded, it is clearly the natural inclination every one has to persuade....every one is practicing oratory on others thro the whole of his life."
>
> - Adam Smith, *Lectures on Jurisprudence*

1 Introduction

Many economic transactions involve the buying and selling of *arguments* rather than physical products. Firms hire advertising companies to craft compelling claims and spur demand. Politicians hire consultants to present their positions in ways that will sway voters. Plaintiffs and defendants hire lawyers to devise logical (or logical-seeming) arguments for arbitration or court. Media organizations hire pundits to put the news into a (potentially biased) context. These *rhetorical services* are different from other products and services because their value depends on how the arguments they enable are interpreted by their intended audience.

The history of the advertising industry is replete with examples of spillovers from audience beliefs to demand for advertising services. As told by Wu (2016), such spillovers marked the very first mass-market advertising campaign in the United States, which consisted of mailed pamphlets for a patent medicine named Dr. Shoop's Restorative. These pamphlets included many outright lies but nevertheless initially encouraged demand for Dr. Shoop's tonics. However, this initial success was met by a strong backlash, as potential customers learned that any firm could make fantastic claims regardless of the underlying quality of their products. As advertising flooded the market, it was met by concomitant increases in customer skepticism, until the advertising market essentially collapsed in the United States. Demand for advertising agencies recovered after the success of World War I propoganda, but cycles of extraordinary claims, customer skepticism, and eventual collapse recurred with the introduction of new advertising media, arguably including the recent concerns about fake news on social media.

This paper proposes a framework for analyzing markets for rhetorical services, which we model as changing the distribution over a signal observed by an audience. We show that the value of such services depends crucially on the audience's beliefs about the veracity of purchased arguments, which depend in turn on their beliefs about whether or not the sender is likely to purchase rhetorical services. We give conditions under which audiences are unlikely to be swayed by rhetorical services if they believe that those services are commonly employed, which means that quantity sold has a negative equilibrium spillover on demand. In contrast, other services "create their own demand:" the sender's willingness-to-pay for such services is increasing in the audience's beliefs about quantity sold, giving rise to unusual market features like upward-sloping equilibrium demand. We argue that sellers of rhetorical services do not internalize either these belief spillovers or the effects of these services on the ultimate audience's utility, raising the possibility of welfare-improving regulation. To give a strategic foundation for how we model rhetorical services, we develop a parsimonious sender-receiver model of communication that is of potential independent interest.

Section 2 presents a stylized model of a market for rhetorical services. A **sender** ("she;" for example, a company) with private information about a binary state would like to convince a **receiver** ("he;" a customer) that that state is high. To do so, the sender chooses whether or not to purchase a rhetorical service at a price determined by a monopolistic **agency** ("it"), which could be an advertising agency, political consultancy, law firm, or any other entity that can craft an argument on behalf of the sender. The receiver, who does not see either the agency's price or the sender's purchase decision, then observes a signal whose distribution is determined by both the state and the sender's purchase decision. Signals are ordered—higher signals are "stronger" in the sense that they statistically suggest that the state of the world is high—but the receiver's interpretation of a signal depends on her beliefs about the sender's purchase decision.

While we assume that the rhetorical service always makes higher signals more likely, its value depends crucially on *how* it does so. Suppose the rhetorical service **complements quality,** in the sense that it disproportionately improves the distribution over signals when the state is high. Such a service is most attractive to "high-quality" senders who believe the state is likely to be high. If the receiver believes that the sender is likely to purchase this service, then she infers that weak signals strongly indicate that the state is low, which means that the sender benefits more from having access to stronger signals. We show that this feedback effect can lead to upward-sloping equilibrium demand, in the sense that a higher equilibrium price can lead to a larger equilibrium quantity sold. We also show that such services lead to more informative signals in equilibrium and so increase the receiver's expected utility.

In contrast, services that **substitute quality** disproportionately improve the signal distribution in the low state and have exactly the opposite effects. In particular, such services appeal most to "low-quality" senders, since having access to them allows signals in the low state to better pool with signals sent if the state is high. The sender's value from such a service is decreasing in the receiver's belief about the likelihood of purchase, since a receiver does not put much weight on strong signals if she believes those signals are likely even if the state is low. Consequently, an increase in equilibrium quantity sold has a negative spillover on sender willingness-to-pay. This intuition suggests that rhetorical services that substitute quality decrease the information conveyed in equilibrium and so harm the receiver.

Rhetorical services are valuable because they influence the mapping between the state and the distribution over signals observed by the receiver. In Section 4, we interpret this signal in the context of a model of strategic communication. This model provides a parsimonious way to think about how a sender's rhetorical ability affects her persuasiveness and is therefore of potential independent interest.

We model the sender's **rhetorical ability** as a two-dimensional private type that determines the set of feasible arguments that the sender can make to the receiver. An argument consists of both a message about the state and a strength. Rhetorical ability is a two-dimensional type: the first dimension gives the maximum feasible strength of an argument if the sender is truthful—her message equals the true state—while the second gives the maximum feasible strength if she lies. Subject to an intuitive equilibrium refinement, we show that the sender essentially always argues that the state of the world is high, but that the distribution over the strength of these arguments depends on the state of the world. This distribution over strengths therefore serves as the signal distribution in our model of rhetorical services. In this interpretation, the agency sells access to a better distribution over rhetorical abilities, where the service complements or substitutes quality exactly when it disproportionately improves the marginal distribution over the sender's truth-telling or lying ability, respectively. This model of rhetorical ability therefore provides a foundation for our analysis of rhetorical services.

Our market model assumes that the rhetorical service has an exogenous effect on the distribution over signals. Section 5 turns to optimal product design: if the agency could privately choose to sell *any* mapping from state to signal distribution, what is its profit-maximizing choice? This question is related to models of selling experiments (Horner and Skrzypacz (2016); Bergemann et al. (2018)), with the crucial difference that the sender purchases an experiment with the intent of using it to persuade a third party rather than to inform her own decision-making. Our stark result is that if senders are homogeneous, then giving the agency the freedom to design a profit-maximizing rhetorical service leads to completely uninformative equilibrium communication. As the patent medicine industry learned first-hand, the unrestricted ability to design rhetorical services ensures that such services enable cheap talk, which ultimately undermines their social value. Consequently, truth-in-advertising laws or other regulatory interventions might restore the functioning of these markets to the ultimate benefit of all participants.

Related Literature

To the best of our knowledge, our paper is the first to analyze markets for rhetorical services using a rational model of communication. McCloskey and Klamer (1995) argues that a substantial fraction of all economic transactions in the United States occur in such markets.

The existing literature tends to focus on the "downstream" effects of rhetorical services, for instance by studying how advertising expenditures affect customer demand. See Bagwell (2007) for a comprehensive survey of the literature on advertising and DellaVigna and Gentzkow (2010) for an overview of persuasion in politics, media organizations, and other settings. Classic theories of advertising focus on its role in reducing search frictions (Stigler (1961); Butters (1977)), improving match quality (Grossman and Shapiro (1984)), or serving as a costly signal of quality (Nelson (1974); Milgrom and Roberts (1986a)). More recently, Mullainathan et al. (2008) argues that persuasive advertising may lead even uninformative messages to be persuasive, with empirical evidence from Bertrand et al. (2010). The recent theoretical literature has also focused on welfare and regulation using complementary approaches to our own. For example, Dellarocas (2006) studies firms that can manipulate online recommendations and derives conditions under which that manipulation leads to either more or less information in equilibrium, while Rhodes and Wilson (2017) consider optimal penalties for false advertising.

Other literatures study the effects of rhetorical services in other contexts. Mullainathan and Shleifer (2005) argue that competition does not lead media organizations to accurately cover the news; Besley and Prat (2006) develop a model of government capture of the media; Prat (2017) develops measures that capture the media's power over its audience; and Murphy and Shleifer (2004) presents a simple model to study networks of political persuasion. We differ from these papers in two ways. First, we focus on the *upstream market* for these rhetorical services. Second, rhetorical services are valuable in our setting because they alter the distribution over the signals observed by the (rational) audience, rather than those services signaling quality directly, or reducing market frictions, or "tricking" behavioral agents.

Rhetorical services serve a "signal jamming" role in our model (Fudenberg and Tirole (1986); Dewatripont et al. (1999); Holmstrom (1999)), in the sense that our sender secures a favorable signal distribution by purchasing a rhetorical service, but the receiver correctly predicts the likelihood of purchase in equilibrium. We model these services in an abstract way that nests many of the commonly studied signal jamming technologies from other applications. This approach allows us to emphasize types of signal jamming that are not widely studied, with a particular focus on services that *disproportionately* change the signal distribution following some state realizations relative to others. As we show, this differential state-by-state impact determines which senders find the rhetorical service most attractive, as well as the types of belief spillovers that arise in equilibirum and the welfare implications of the market.

Markets for rhetorical services transact informative signals and so are related to markets for advice (Bergemann et al. (2018); Horner and Skrzypacz (2016)), with the difference that the sender in our setting purchases rhetorical services to persuade a third party rather than for her own consumption. We argue that markets for rhetorical services are also closely connected to markets with network effects (Farrell and Klemperer (2007)), with the key difference that the sender's value for such services depend on the *receiver's* beliefs about purchase behavior and how that behavior affects the signal distribution. These belief spillovers can lead to upward-sloping demand, which is closely connected to Akerlof et al. (2018)'s analysis of demand with network externalities.

We microfound our model of rhetorical services using a sender-receiver model of rhetorical ability. This model builds on papers that study communication and persuasion, as surveyed in Sobel (2013). Within this literature, our model nests both cheap talk (Crawford and Sobel (1982); Lipnowski and Ravid (2017)) and verifiable disclosure (Milgrom (1981); Milgrom and Roberts (1986b)) as special cases, though we make simplifying assumptions about both the state of the world (binary) and the sender's preferences (monotonic in the receiver's action).

Our paper is closely related to Kartik (2009) and especially Frankel and Kartik (2017), both of which study settings in which lying is costly and thereby similarly nest cheap talk and verifiable communication. Frankel and Kartik (2017) is the more closely related of the two, and indeed, one can interpret our model of rhetorical ability as a particularly tractable limiting case of their framework. Our focus on markets for rhetorical services complements these papers, since a typical equilibrium in the communication literature maps states to signals, and our model takes such a mapping as a primitive.

As in the literature on type-dependent message spaces, our communication model assumes that the sender's set of *feasible* communications depends on her private information (Hagenbach et al. (2014); Blume and Board (2013)). We differ from these models by focusing on games with misaligned preferences and partially informative communication. Dziuda (2011) shows why a sender might make arguments that are counter to her preferred action in order to mimic a truthful behavioral type. In contrast, we have no behavioral types, and we focus on equilibria in which the sender only makes arguments that support her preferred action. Dewan and Myatt (2008) argue that clear communication—modeled as the precision of a signal as a function of a sender's message—can help coordinate listeners on a course of action and so allow the sender to be an effective leader.

2 Model

Our model of a market for rhetorical services has three players: a monopolistic **agency** ("it") that prices the rhetorical service, a **sender** ("she") who chooses whether or not to purchase it, and a **receiver** ("he") who uses the resulting signal to inform a decision. We assume that the receiver's optimal decision is strictly increasing in his posterior belief that the state is high and the sender's payoff is strictly increasing in the receiver's decision.

The timing of the game is:

1. Quality $q \sim H(\cdot)$ is realized and privately observed by the sender, with $E[q] = \bar{q}$.

2. The agency chooses a price $p \in \mathbb{R}_+$, which is observed by the sender but not the receiver.

3. The sender chooses whether or not to purchase, $x \in \{0, 1\}$, which is observed by the agency but not the receiver.

4. A binary state of the world is realized, $\omega \in \{0, 1\}$, with $\Pr\{\omega = 1\} = q$.

5. The receiver observes a signal $k \in [0, 1]$, where $k \sim xG_\omega(\cdot) + (1 - x)F_\omega(\cdot)$.

6. The receiver makes a decision $d \in \mathbb{R}$.

The agency's, sender's, and receiver's payoffs are respectively $\pi = px$, $v_S = u_S(d) - px$, and $v_R = u_R(d, \omega)$. We assume that both $u_S(\cdot)$ and $u_R(\cdot, 1) - u_R(\cdot, 0)$ is strictly increasing, so the sender's payoff is strictly increasing in the receiver's decision, and the receiver's decision is strictly increasing in his posterior belief that $\omega = 1$. Our solution concept is (weak) Perfect Bayesian Equilibrium, where we denote the receiver's equilibrium posterior belief that $\omega = 1$ after observing signal k by $\mu^*(k)$.

The agency is a monopolist who sells a service that changes the distribution over signals from $F_\omega(\cdot)$ to $G_\omega(\cdot)$. Depending on setting, the agency might be an advertising firm, pundit, political consultancy, or law firm. We model the resulting advertisements, speeches, or other communication between sender and receiver by the exogenous signal distributions $F_\omega(\cdot)$ and $G_\omega(\cdot)$, which collectively determine the receiver's posterior belief about ω. These distributions are designed to nest a variety of ways in which the sender and receiver can communicate, since the equilibrium outcome of a typical communication game is a mapping between states of the world and messages. Section 4 develops our preferred interpretation of $F_\omega(\cdot)$ and $G_\omega(\cdot)$ as the outcome of a model in which the sender has private information about her rhetorical ability. The agency can then be interpreted as improving the sender's rhetorical ability.

We require that $F_\omega(\cdot)$ and $G_\omega(\cdot)$ have differentiable densities.

Assumption 1 *The game is* ***smooth:*** *$H(\cdot)$, $G_\omega(\cdot)$, and $F_\omega(\cdot)$ have full support and are twice continuously differentiable with respective densities $h(\cdot)$, $g_\omega(\cdot)$, and $f_\omega(\cdot)$ that are strictly positive on their domains.*

We assume that $F_\omega(\cdot)$ and $G_\omega(\cdot)$ satisfy a set of MLRP orderings to ensure that higher k are "stronger" indicators of $\omega = 1$ in a statistical sense. We also assume a single-crossing condition on $g_\omega(\cdot) - f_\omega(\cdot)$, which implies that purchasing the rhetorical service shifts the distribution towards higher signal realizations.

Assumption 2 *Higher signals indicate higher states: $\frac{f_1(\cdot)}{f_0(\cdot)}$, $\frac{g_1(\cdot)}{g_0(\cdot)}$, $\frac{f_1(\cdot)}{g_0(\cdot)}$, and $\frac{g_1(\cdot)}{f_0(\cdot)}$ are all increasing, with $\frac{f_1(\cdot)}{f_0(\cdot)}$ and $\frac{g_1(\cdot)}{g_0(\cdot)}$ strictly so. Buying the rhetorical service leads to higher signals: $G_\omega \neq F_\omega$ for at least one $\omega \in \{0, 1\}$, and $g_\omega(\cdot) - f_\omega(\cdot)$ single-crosses 0 from below for each $\omega \in \{0, 1\}$.*

Assumptions 1 and 2 are satisfied if, for instance, $F_1(\cdot)$, $F_0(\cdot)$, $G_1(\cdot)$, and $G_0(\cdot)$ are exponential distributions with respective parameters λ_1^F, λ_0^F, λ_1^G, and λ_0^G that satisfy $\lambda_0^F \leq \lambda_0^G \leq \lambda_1^F \leq \lambda_1^G$ with $\lambda_0^F < \lambda_1^F$ and $\lambda_0^G < \lambda_1^G$.

We assume that the sender does *not* know the state of the world when she makes her purchase decision. In advertising, for instance, ω is the value of the product to an end customer and q is the probability that a product is a "home run" when an advertising agency is chosen. Similarly, if the sender is a defendant in a court case, then q represents her beliefs about her guilt when she decides which law firm should represent her, while ω represents the severity of her liability and is revealed only after she has retained counsel. In equilibrium, the distribution over q generates a demand curve for rhetorical services.

We also assume that the receiver can observe neither the price nor the sender's purchase decision. As we will show, the receiver's beliefs in equilibrium depend on both expected prices and expected purchase decisions. The agency might benefit from committing to make prices or purchase decisions public in order to manipulate these beliefs. However, committing to public prices or purchases might be difficult in practice, since the agency has the incentive to then negotiate private sales or secret price changes. Assuming that the receiver observes neither p nor x also clarifies the "signal-jamming" role of the rhetorical service. We discuss equilibria with public prices or purchase decisions at the end of Section 3.

As stated, our model does not allow the agency to offer menus of contracts. In principle, such menus could allow the agency to induce the sender to reveal information about her *ex ante* quality, q. However, one can show that this restriction is without loss in our setting: the agency cannot benefit from offering a menu because both the probability of sale and q enter linearly in the sender's utility (see, e.g., Myerson (1981)).

3 The Market for Rhetorical Services

This section shows that markets for rhetorical services differ from other markets because demand depends on the receiver's beliefs. We illustrate how this feedback influences pricing, welfare, and equilibrium persuasion.

Suppose the sender purchases the service if and only if $q \in \mathcal{Q}$ for some measurable set $\mathcal{Q} \subseteq [0,1]$. Define $Q_H \equiv \int_{q\in\mathcal{Q}} qh(q)dq$, $Q_L \equiv \int_{q\in\mathcal{Q}} (1-q)h(q)dq$, and the receiver's posterior belief that $\omega = 1$ after seeing a signal k as

$$\mu^*(k|\mathcal{Q}) = \frac{Q_H g_1(k) + (\bar{q} - Q_H) f_1(k)}{Q_H g_1(k) + (\bar{q} - Q_H) f_1(k) + Q_L g_0(k) + ((1-\bar{q}) - Q_L)\, f_0(k)}.$$

Note that $\mu^*(\cdot|\mathcal{Q})$ is a strictly increasing function of the score,

$$s(\cdot|\mathcal{Q}) \equiv \frac{Q_H g_1(\cdot) + (\bar{q} - Q_H) f_1(\cdot)}{Q_L g_0(\cdot) + ((1-\bar{q}) - Q_L)\, f_0(\cdot)}. \tag{1}$$

Therefore, the receiver's optimal decision $d^*(s)$ is a strictly increasing function of the score, which implies that the sender's payoff $u_S^*(s)$ is similarly strictly increasing in the score. That is, the sender wants to induce the receiver to believe the state is high with the highest possible probability, which corresponds to the highest possible score. Define

$$\Delta(q|\mathcal{Q}) \equiv \int u_S^*(s(k|\mathcal{Q}))\,[q(g_1(k) - f_1(k)) + (1-q)(g_0(k) - f_0(k))]\,dk \tag{2}$$

as the sender's willingness-to-pay for the service if her quality equals q and the receiver believes that the sender purchases if and only if $q \in \mathcal{Q}$.

We begin by identifying a natural source of horizontal differentiation in the market for rhetorical services. We say the service **complements quality** or **substitutes quality** if $(g_1 - g_0) - (f_1 - f_0)$ single-crosses 0 from below or above, respectively. A rhetorical service that complements quality disproportionately impacts the signal distribution following $\omega = 1$ and is therefore particularly attractive if q is high, while a service that substitutes quality disproportionately impacts the signal following $\omega = 0$ and so is attractive if q is low.

Lemma 1 *Suppose Assumptions 1 and 2 hold. In any equilibrium,* $\Delta(q|\mathcal{Q}) > 0$ *for any* $q \in (0,1)$ *and* $\mathcal{Q} \subseteq [0,1]$.

1. *If the service complements quality, then* $\frac{\partial \Delta}{\partial q}(q|\mathcal{Q}) > 0$ *and so* $\mathcal{Q} = [q^*, 1]$ *in equilibrium. Moreover,* $\frac{\partial \Delta}{\partial q^*}(1|[q^*,1])|_{q^*=1} < 0$*, and if* $f_0 = g_0$*, then* $\frac{\partial \Delta}{\partial q^*}(q|[q^*,1]) < 0$ *for all* $q, q^* > 0$.

2. *If the service substitutes quality, then* $\frac{\partial \Delta}{\partial q}(q|\mathcal{Q}) < 0$ *and so* $\mathcal{Q} = [0, q^*]$ *in equilibrium. Moreover,* $\frac{\partial \Delta}{\partial q^*}(0|[0,q^*])|_{q^*=0} < 0$*, and if* $f_1 = g_1$*, then* $\frac{\partial \Delta}{\partial q^*}(q|[0,q^*]) < 0$ *for all* $q, q^* < 1$.

Proof: See Appendix A.

We discuss the proof of Lemma 1 for the case in which the service complements quality (the proof of the other case is similar). To show that willingness-to-pay is strictly increasing in q, we apply Beesack's Inequality[1] to the derivative of (2) with respect to q. Since all senders pay the same price p, $\mathcal{Q} = [q^*, 1]$ in any equilibrium. Increasing the receiver's beliefs about the likelihood of a purchase—as parameterized by q^*—increases the receiver's

[1] The relevant version of Beesack's Inequality states that if a function $\gamma(\cdot)$ single-crosses 0 from below and satisfies $\int \gamma(x)dx = 0$, then for any increasing function $\lambda(\cdot)$, $\int \gamma(x)\lambda(x)dx \geq 0$, and strictly so if $\lambda(\cdot)$ is strictly increasing and $\gamma(\cdot)$ is not everywhere 0. See Beesack (1957)

posterior beliefs following some signal realizations while decreasing it following others, with corresponding increases and decreases in the sender's value for that realization. The sender's value from the service depends on a weighted average of these effects, where the weights depend on both $g_\omega(\cdot) - f_\omega(\cdot)$ and expected quality q. We can unambiguously sign the change in this weighted average with respect to q^* if either $q^* = q = 1$ or $f_0 = g_0$.

As Lemma 1 demonstrates, the difference between rhetorical services and "typical" products is that $\Delta(q|\mathcal{Q})$ depends on the receiver's beliefs about purchase behavior, as represented by $\mathcal{Q}$. Under certain conditions, the sender's willingness-to-pay is increasing (decreasing) in the receiver's beliefs about quantity if the service complements (substitutes) quality. Our next goal is to characterize demand in markets for rhetorical services given these spillovers.

If the service complements quality, define **equilibrium demand** as

$$\Delta_T(q^*) \equiv \Delta(q^*|[q^*, 1]).$$

Then $\Delta_T(q^*)$ is the willingness-to-pay of a sender with quality q^*, given that the receiver believes this sender to be the marginal purchaser of the service. Define

$$\Delta_L(q^*) = \Delta(q^*|[0, q^*])$$

as the analogous equilibrium demand if the service substitutes quality.

Holding beliefs fixed, the marginal sender's willingness-to-pay is decreasing in quantity sold. If the service substitutes quality and $f_1 = g_1$, then Lemma 1 says that willingness-to-pay is also decreasing in the receiver's beliefs about quantity. Equilibrium demand is always downward-sloping for such services. In contrast, if the service complements quality and $f_0 = g_0$, then willingness-to-pay is increasing in the receiver's belief about quantity. We identify conditions under which this belief spillover dominates, leading to an upward-sloping equilibrium demand curve.

Proposition 1 *Suppose Assumptions 1 and 2 hold. If the service substitutes quality and*

$g_1 = f_1$, then $\frac{d\Delta_L}{dq^*} < 0$ for all $q^* \in [0,1)$. If the service complements quality and $g_0 = f_0$, then $\frac{d\Delta_T}{dq^*} < 0$ if and only if

$$\frac{\Delta_T(q^*)}{(q^*)^3 h(q^*)} < \int \frac{\partial u_S^*(s(k|[q^*,1])}{\partial s} \frac{(g_1(k) - f_1(k))^2}{(1-\bar{q}) f_0(k)} dk. \tag{3}$$

Proof: See Appendix A.

If the service substitutes quality and $g_1 = f_1$, then $\Delta_L(\cdot)$ is decreasing because $\Delta(q|[0,q^*])$ is decreasing in both q and q^*. If the service complements quality and $g_0 = f_0$, then (3) follows from the expression for $\frac{\partial \Delta_T}{\partial q^*}$. While this inequality is somewhat complicated, it is likely to hold if $h(q^*)$ is large for a fixed marginal quality $q^* \in (0,1)$ and average quality $\bar{q} \in (0,1)$. Intuitively, if $h(q^*)$ is large, then a small decrease in q^* has a large impact on the likelihood that the sender purchases the service and hence on the receiver's equilibrium beliefs about the probability of purchase. Since sender willingness-to-pay is increasing in the receiver's belief about this probability, a large enough $h(q^*)$ ensures that this positive spillover dominates the negative slope of demand for fixed beliefs, in which case equilibrium demand is upward-sloping. Indeed, in the limit where $H(\cdot)$ is degenerate (and hence $h(\cdot)$ is a Dirac function), we can show that equilibrium demand is always upward-sloping. See Figure 1 for an illustration.

Since equilibrium purchase probabilities affect willingness-to-pay, markets for rhetorical services resemble those for *network goods* (Farrell and Klemperer (2007)). Unlike network goods, our equilibrium spillovers do not serve as a barrier to entry and operate through the beliefs of a third party (the receiver) rather than the beliefs of the purchaser. As with network goods, however, a single price might correspond to multiple purchase probabilities, raising the possibility of multiple equilibria.

Akerlof et al. (2018) resolves this equilibrium multiplicity with a refinement that selects an equilibrium based on the history of purchases in the market. We can similarly apply this refinement to markets for rhetorical services. We focus on optimal pricing if the agency

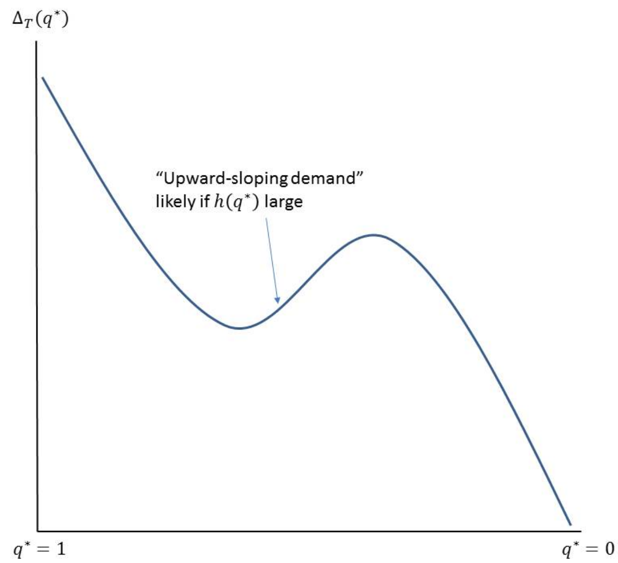

Figure 1: Upward-sloping equilibrium demand is possible if the service complements quality. Note that the horizontal axis orders sender qualities from highest ($q = 1$) to lowest ($q = 0$) willingness-to-pay.

can select her preferred equilibrium, which corresponds to the equilibrium for the "inside firm" in Akerlof et al. (2018). Formally, an earnest equilibrium is **agency-optimal** if given any conjectured price p, the receiver's beliefs maximize the agency's expected payoff at that price.

In a agency-optimal equilibrium, we can treat the agency as choosing a marginal sender quality q^*. The equilibrium price is then $\Delta_T(q^*)$ or $\Delta_L(q^*)$ if the service complements or substitutes quality, respectively. Our next result characterizes the profit-maximizing q^*.

Proposition 2 *Suppose Assumptions 1 and 2 hold, and let q^* be the marginal customer in an agency-optimal earnest equilibrium. Define*

$$V = \frac{\int u_S^*(s(k|\mathcal{Q}))\,(g_0(k) - f_0(k))\,dk}{\int u_S^*(s(k|\mathcal{Q}))\,(g_1(k) - f_1(k) - (g_0(k) - f_0(k)))\,dk}.$$

If the service complements quality, then

$$\frac{1 - H(q^*)}{h(q^*)} \leq q^* + V,$$

with equality if $q^* > 0$. *If the service substitutes quality, then*

$$\frac{H(q^*)}{h(q^*)} \leq q^* + V,$$

with equality if $q^* < 1$.

Proof: See Appendix A.

These conditions follow immediately from the first-order conditions of the agency's profit-maximization problem, assuming that the agency can choose a marginal customer quality q^* rather than a price. The details of $F_\omega(\cdot)$ and $G_\omega(\cdot)$ influence this profit-maximization problem in two ways. First, they influence q^* via the term V, which is constant in q^*. Second, these distributions influence $\Delta(q|\mathcal{Q})$ and hence the profit-maximizing price for a fixed q^*.

Next, we characterize how the existence of a market for rhetorical services affects welfare. The receiver prefers more informative communication to better tailor his decision to the state. We show that the existence of a rhetorical service that complements quality improves the receiver's information and welfare, while rhetorical services that substitute quality have the opposite effects.

Proposition 3 *Suppose Assumptions 1 and 2 hold. If the service complements (substitutes) quality and* $g_0 = f_0$ $(g_1 = f_1)$*, then the receiver's expected utility is higher (lower) in any equilibrium of the market game relative to a setting in which* $x = 0$ *with probability 1.*

Proof: See Appendix A.

Proposition 3 is a straightforward implication of Lehmann (1988), Theorem 5.1. Since the receiver's decision problem is monotone, his preferences over information structures can be ranked using the Lehmann rather than the more restrictive Blackwell ordering (Lehmann (1988); Athey and Levin (2018)). Intuitively, a service that complements quality is valuable to the sender precisely because it "disentangles" truthful and lying arguments, so that the resulting mapping from state to argument is more informative. The opposite intuition holds if the service substitutes quality.

In addition to this externality on the receiver, the market for rhetorical services also imposes an equilibrium externality on the sender's utility. Prices and purchase decisions are both private, so the profit-maximizing price does not account for the spillover from receiver beliefs to sender willingness-to-pay. To make this point, we compare agency-optimal equilibria to a **game with public prices**, which is identical to Section 2except that p is publicly observed. We show that making prices public leads to higher prices if the service substitutes quality and lower prices if the service complements quality, as the agency uses prices to influence the receiver's beliefs about purchase behavior and hence sender willingness-to-pay.

Proposition 4 *Let Assumptions 1 and 2 hold in the game with public prices. Denote the equilibrium marginal customer in an agency-optimal equilibrium by q_{PUB}, and let q_{PVT} be the corresponding profit-maximizing marginal customer from Proposition 2. If the service complements (substitutes) quality with $f_0 = g_0$ ($f_1 = g_1$), and if $\Delta_T(\cdot)(1-H(\cdot))$ ($\Delta_L(\cdot)H(\cdot)$) is concave, then $q_{PUB} < q_{PVT}$.*

Proof: See Appendix A.

If the service complements quality, then the agency chooses the marginal customer in the game with public prices to solve

$$q_{PUB} \in \arg\max_{q} \Delta_T(q)(1 - H(q)), \tag{4}$$

since $\Delta_T(q)$ is the maximum equilibrium price that results in all senders with quality above q purchasing the service. Unlike the profit-maximization problem in the market game, the receiver's belief about the sender's purchase behavior changes with q. If this problem is concave, then $q_{PUB} < q_{PVT}$ because $\frac{\partial \Delta_T(q^*)}{\partial q^*} < \frac{\partial \Delta(q|[0,q^*))}{\partial q}|_{q=q^*}$ by Lemma 1. A similar logic yields the result for a service that substitutes quality.

Eliminating the market for rhetorical services entirely can lead to a Pareto welfare improvement. For example, if the service substitutes quality and $f_1 = g_1$, then Proposition 3 says that the receiver would be better off without it. If $G_\omega(\cdot)$ is also less informative than $F_\omega(\cdot)$ in the stronger Blackwell sense, and if the sender's payoff is strictly convex in the receiver's posterior belief, then eliminating the market would also increase the joint payoff of the sender and agency. In that case, the sender and receiver would benefit from paying the agency to credibly and permanently leave the market.

The agency might also internalize its equilibrium effects on sender utility if it could commit to make the sender's purchase decision publicly observable. For example, a defendant in a legal case (the sender) typically reveals her counsel (the agency) to the judge or jury (the receiver) during court proceedings. Even in such settings, the sender might privately purchase additional rhetorical services, for instance by retaining experts or lawyers that give informal advice but never appear before the jury. For example, in some legal settings, the barristers who present a case in court are distinct from the solicitors who do behind-the-scenes work crafting the arguments for that case.

If purchase decisions are public, then purchasing the service changes the continuation game from one in which the signal is *commonly known* to be distributed according to $F_\omega(\cdot)$, to one in which it is *commonly known* to be distributed according to $G_\omega(\cdot)$. The sender's willingness-to-pay for this change depends on two factors. First, how does the sender value the distribution over posteriors induced by these two games? For instance, a sender whose payoff is concave in the receiver's posterior would not benefit at all from publicly purchasing a (Blackwell) more informative signal distribution. Second, what does the purchase decision itself signal about q? Making purchase decisions public essentially transforms the game from one of signal-jamming to one of signaling, which introduces the possibility of multiple pooling and separating equilibria.

4 Rhetorical Ability in a Sender-Receiver Game

This section introduces a model of strategic communication in order to provide an interpretation of $F_\omega(\cdot)$ and $G_\omega(\cdot)$. We assume that the sender has private information about her *rhetorical abilities*, which determines the feasible communications in each state of the world and thereby induces an equilibrium mapping between $\omega \in \{0,1\}$ and a signal distribution.

4.1 A Model of Rhetorical Ability

Timing and Payoffs

Consider the following **communication game** between the sender and receiver. After learning $\omega \in \{0,1\}$, the sender privately observes her rhetorical ability θ. She then chooses an argument a from a type- and state-dependent set $\mathcal{A}(\theta,\omega)$. This argument is observed by the receiver, who then makes a decision.

The game has the following timing:

1. The sender privately learns her **rhetorical ability** $\theta \equiv (\theta_T, \theta_L) \in \Theta \subseteq \mathbb{R}^2_+$, with $\theta \sim F(\cdot)$, and the **state** $\omega \in \{0,1\}$, with $\Pr\{\omega = 1\} = q$.

2. The sender makes an **argument** $a \equiv (m,k) \in \mathcal{A}(\theta,\omega)$, where $\mathcal{A}(\theta,\omega) \equiv \mathcal{A}^T(\theta_T,\omega) \cup \mathcal{A}^L(\theta_L)$ and

$$\mathcal{A}^T(\theta_T,\omega) = \{(\omega,k) | k \in [0,\theta_T]\},$$
$$\mathcal{A}^L(\theta_L) = \{(m,k) | m \in \{0,1\}, k \in [0,\theta_L]\}.$$

3. The receiver observes a and makes a **decision** $d \in \mathbb{R}$.

Payoffs are as in Section 2 with $p = 0$.

We call $m \in \{0,1\}$ the **message** and $k \in \mathbb{R}$ the **strength** of argument $a = (m,k)$. A sender's rhetorical ability restricts the strength of her argument: she can either lie ($m \neq \omega$), in which case the argument's strength cannot exceed θ_L, or she can tell the truth ($m = \omega$), in which case the strength cannot exceed $\max\{\theta_L, \theta_T\}$. Without loss, we assume $\theta_T \geq \theta_L$

for all θ in the support of $F(\cdot)$. Let $F_1(\cdot)$ and $F_0(\cdot)$, with densities $f_1(\cdot)$ and $f_0(\cdot)$, be the marginal distributions over θ_T and θ_L, respectively.

Earnest Equilibrium

Our results focus on a subset of PBE that we call **earnest**. In an earnest equilibrium, the sender chooses $a = (1, \theta_T)$ with probability 1 whenever $\omega = 1$. In other words, we require a sender who observes her preferred state to make the strongest feasible argument in favor of that state. This refinement rules out two phenomena in equilibrium: a strong argument might never be made if it is assigned a low (off-path) posterior, or senders might "reverse" the meaning of the message by choosing (with positive probability) $m = 0$ when $\omega = 1$ and $m = 1$ when $\omega = 0$.

We show that earnest equilibria are essentially unique under Assumption 2 and in many other settings as well. We justify this refinement in Appendix B by showing that under Assumption 2, earnest equilibria are the most informative PBE in the Lehmann sense and so are receiver-optimal. This appendix also explores non-earnest equilibria in a simple example.

Discussion

This model nests both cheap talk and verifiable disclosure as special cases. Cheap talk obtains if $\theta_T = \theta_L$ for all θ in the support of $F(\cdot)$, so that it is common knowledge that the set of feasible arguments is independent of the state. Verifiable disclosure obtains if, for example, $\theta_T = 1 > \theta_L$ for all θ in the support of $F(\cdot)$, so that the argument $(1, 1)$ can be sent if and only if $\omega = 1$.

More generally, one can interpret our model as capturing a "middle ground" between verifiable disclosure, where an argument incontrovertible establishes the truth of a claim, and cheap talk, which is a message without any supporting argument. Under that interpretation, our model reflects settings in law, politics, and advertising in which persuasiveness depends on the judgment and skill of the sender and the beliefs of the receiver (Aldisert et al. (2007)).

	Logical	Clever
Advertising	"This TV is 1080p"	"This TV is high definition" [meaning either 720p or 1080p]
	"Use this code to save 30% off your first order!"	"It's toasted!" [Lucky Strike cigarettes]
	"This car has the best highway gas mileage of cars in its class."	"9 out of 10 dentists can't be wrong!" [for toothpaste]
Law	"The Judicial Department determines the law; the Supreme Court is the Judicial Department; therefore, the Supreme Court's duty is to say what the law is." [Marbury v. Madison, adapted from Aldisert, Clowney, and Peterson (2007)]	"If [the glove] doesn't fit, you must acquit." [OJ Simpson trial]
	"Unequal educational facilities are not permitted under the Constitution; a separate facility for black children is inherently unequal; therefore, a separate facility is not permitted under the Constitution." [Brown v. Board of Education, again adapted from Aldisert, Clowney, and Peterson (2007)	
Politics	"It may be true that the law cannot make a man love me but it can keep him from lynching me and I think that is pretty important also." [Martin Luther King, Jr, in a speech at Western Michigan University, responding to the claim that morality cannot be legislated.]	

Figure 2: Examples of logical and clever arguments.

This observation motivates our focus on the sender's private information about her rhetorical ability.

One way to think about θ is as a measure of types of arguments available to the sender. For example, suppose that regardless of the true state, the sender can devise θ_L "clever" arguments in favor of $\omega = 1$. If the state is in fact $\omega = 1$, then the receiver can also devise $\theta_T - \theta_L$ "logical" arguments in favor of $\omega = 1$. The receiver observes the arguments that the sender chooses to make, but he cannot tell whether those arguments are logical or clever.

Figure 2 gives illustrative examples of logical and clever arguments in different contexts. Our categorization is potentially controversial; the reader might believe that some logical arguments are instead clever, or vice versa.[2] This ambiguity actually strengthens our central claim, which is that it is difficult for a listener to cleanly separate logical and clever arguments, so that a clever argument might persuade even if it is not actually evidence of the underlying state. Note that clever arguments are made to support *both* true and false messages in our framework, so categorizing an argument as "clever" does not imply that the underlying claim is false.

[2]Indeed, the authors themselves do not totally agree on this categorization.

4.2 Earnest Equilibrium Persuasion

This section shows that under earnest equilibrium communication leads to the signal distributions from Section 2 so long as Assumption 2 holds. It also considers earnest equilibria if this assumption is not satisfied.

Define

$$\mu(k) \equiv \frac{qf_1(k)}{qf_1(k) + (1-q)f_0(k)}. \tag{5}$$

Intuitively, $\mu(k)$ is the receiver's posterior probability that $\omega = 1$ if he sees argument $(1, k)$ and believes that the sender makes the strongest possible argument that $\omega = 1$, regardless of the true state. Note that $\mu(\cdot)$ is increasing in the likelihood ratio $\frac{f_1(\cdot)}{f_0(\cdot)}$ and coincides with $\mu^*(\cdot|\emptyset)$ if $q = \bar{q}$.

Suppose $f_1(\cdot)/f_0(\cdot)$ is increasing and consider the strategy profile used to define $\mu(\cdot)$: if $\omega = 1$, then $a = (1, \theta_T)$, and if $\omega = 0$, then $a = (1, \theta_L)$. Since $\mu(\cdot)$ is increasing, the sender has no profitable deviation to weaker arguments $(1, k)$, and it is easy to find off-path beliefs such that the sender does not want to deviate to $(0, k)$ for any k. This argument implies that there exists an earnest equilibrium in which the sender always makes the strongest possible argument that the state is high, so long as $\mu(\cdot)$ is increasing. We show that this is the essentially unique earnest equilibrium in these settings.

Proposition 5 *Let Assumptions 1 and 2 hold. Then there exists an earnest equilibrium, and in any such equilibrium, the receiver's on-path posterior belief equals $\mu(\theta_T)$ if $\omega = 1$ and $\mu(\theta_L)$ if $\omega = 0$.*

Proof: See Appendix A.

We have already argued that at least one such earnest equilibrium exists. To show that any earnest equilibrium entails these beliefs, first note that any on-path $(0, k)$ must induce posterior 0 in an earnest equilibrium, since $m = 1$ if $\omega = 1$ in an earnest equilibrium. The mapping from state and rhetorical ability to posterior is therefore "as if" the sender makes an argument $a = (1, k)$ for some k. Fix an earnest equilibrium, and let $\mu^*(k)$ be the posterior following $a = (1, k)$. Then $\mu^*(\cdot)$ is weakly increasing because the sender can always weaken her argument.

If $\mu^*(\cdot) \neq \mu(\cdot)$, then there must exist some θ such that the sender chooses $k < \theta_L$ when $\omega = 0$. This choice is incentive compatible only if $\mu^*(k) = \mu^*(\theta_L)$, so $\mu^*(\cdot)$ must be constant on some interval $\mathcal{I}$ with $[k, \theta_L] \subseteq \mathcal{I}$. But $\mu^*(\cdot) \leq \mu(\cdot)$ near $\underline{k} \equiv \inf \mathcal{I}$, since a sender with θ_L just above $\underline{k}$ cannot make a stronger argument than $(1, \theta_L)$ and is unwilling to make an argument weaker than $(1, \underline{k})$. Since $\mu(\cdot)$ is increasing, we infer that $\mu^*(\cdot) \leq \mu(\cdot)$ on the entire interval $\mathcal{I}$ and strictly so on part of this interval. But that cannot hold, since if $\omega = 0$, then any sender with $\theta_L \in \mathcal{I}$ must make an argument with $k \in \mathcal{I}$ and hence $\mu^*(k)$ must equal the *average* of $\mu(\cdot)$ on $\mathcal{I}$. We conclude that $\mu^*(\cdot) = \mu(\cdot)$ in any earnest equilibrium.

Proposition 5 says that the receiver essentially observes θ_T if $\omega = 1$ and θ_L if $\omega = 0$ in any earnest equilibrium. Since $\theta_T \sim F_1(\cdot)$ and $\theta_0 \sim F_0(\cdot)$, earnest equilibria in this game generates the signal distributions assumed in Section 2. Indeed, we can easily combine this sender-receiver game with our model of the market and show that the results from Section 3 hold in any earnest equilibrium. Under this interpretation, the agency devises clever slogans, improving θ_L; or it identifies clear and catchy logical arguments, improving θ_T; or it performs some combination of these roles.

Assumption 2 implies that $\mu(\cdot)$ is increasing, but we can also analyze equilibrium persuasion if $\mu(\cdot)$ is non-monotonic. The receiver's posterior beliefs cannot coincide with $\mu(\cdot)$ in such settings because stronger arguments must induce higher posteriors. We show that the receiver's equilibrium posterior "irons" $\mu(\cdot)$ so that it is increasing.

Proposition 6 *Let Assumption 1 hold, and suppose there exist $0 \leq k_L < k_H \leq 1$ such that $\mu(\cdot)$ is strictly increasing on $[0, k_L]$ and $[k_H, 1]$ and strictly decreasing on $[k_L, k_H]$. Then an earnest equilibrium exists. There exists an increasing and continuous function $\mu^* : \mathbb{R}_+ \to [0, 1]$ such that in any earnest equilibrium, the receiver's on-path posterior belief equals $\mu^*(\theta_T)$ if $\omega = 1$ and $\mu^*(\theta_L)$ if $\omega = 0$. There exists an interval $\mathcal{I}$ with $[k_L, k_H] \subseteq \mathcal{I}$ such that $\mu^*(\cdot)$ is*

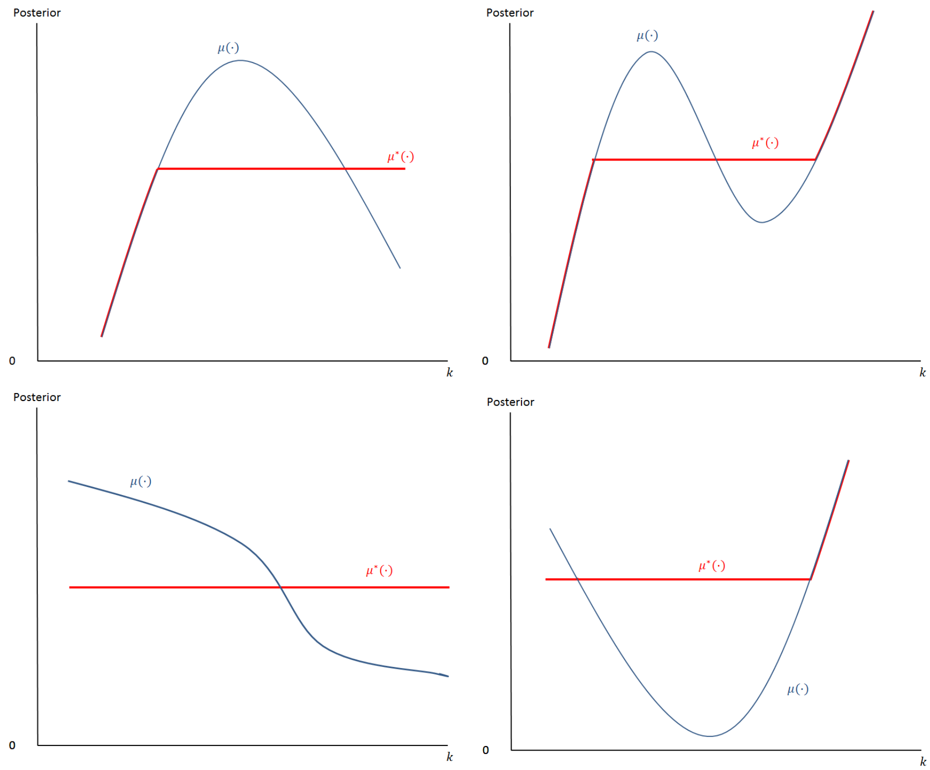

Figure 3: Illustrative pictures of non-monotonic $\mu(\cdot)$ and the corresponding unique equilibrium posteriors $\mu^*(\cdot)$.

constant on $\mathcal{I}$ and $\mu^*(k) = \mu(k)$ for $k \notin \mathcal{I}$.

Proof: See Appendix A

The proof of this result builds on the intuition from Proposition 6. The receiver's posterior must be constant over the entire interval $[k_L, k_H]$, and must be continuous and coincide with $\mu(\cdot)$ at the endpoints of this constant interval, or else we can show that the sender would have a profitable deviation. Outside of this interval, $\mu(\cdot)$ is strictly increasing and so $\mu^*(\cdot) = \mu(\cdot)$ by an argument identical to that of Proposition 5. Note that within the flat interval $\mathcal{I}$, $\mu^*(\cdot)$ must equal $\mu(\cdot)$ *in expectation,* which is the sense in which $\mu^*(\cdot)$ is an ironed version of $\mu(\cdot)$.

On $\mathcal{I}$, some sender types must choose $k < \theta_L$ when $\omega = 0$. That is, the equilibrium must exhibit "gaps" in persuasion, over which the sender would gain nothing from a marginal improvement to her rhetorical ability. It would be straightforward (if somewhat cumbersome) to extend these ironing techniques to more complicated $\mu(\cdot)$ functions. In those cases, several different ironed $\mu^*(\cdot)$ might be consistent with different earnest equilibria.

5 Optimal Product Design

This section allows the agency to design the signal distribution that it offers. If the agency can privately choose any $G_\omega(\cdot)$, then we give conditions under which informative communication (though not necessarily the agency's profit) completely collapses in equilibrium, which is consistent with the collapse of advertising for patent medicines discussed in the Introduction.

We consider a **product design game** that is identical to the model in Section 2 except that the agency costlessly chooses a distribution $G_\omega(\cdot)$ when it sets its price p. The choice of $G_\omega(\cdot)$ is observed by the sender but not the receiver, so the agency optimally offers a distribution that puts weight on only those signals that induce the highest posteriors. But then those signals are not very informative of ω, which limits the price that the agency can charge for its services. Allowing an unrestricted choice of $G_\omega(\cdot)$ essentially turns the game into one of cheap talk.

If the sender's quality q is commonly known, then we prove that communication is completely uninformative in any equilibrium of the product design game.

Proposition 7 *Suppose $H(\cdot)$ is degenerate at $\bar{q} \in (0,1)$. An equilibrium of the product design game exists. In any equilibrium, $\mu_{\sigma^*}(k) = \bar{q}$ with probability 1 under the equilibrium signal distribution, and at least one of the following two conditions must hold: (i) $p = 0$, or (ii) the sender purchases the service with probability 1.*

Proof: See Appendix A.

Suppose the sender is willing to pay a strictly positive amount for the service. The sender's willingness-to-pay is commonly known because $H(\cdot)$ is degenerate. If $p > 0$ and the sender does not always purchase, then either price exceeds this willingness-to-pay, or the sender is indifferent between purchasing and not. In either case, the agency can profitably decrease its price, so either $p = 0$ or the sender purchases with probability 1 in any equilibrium.

Willingness-to-pay is maximized if $G_\omega(\cdot)$ puts weight only on those signals that induce the highest posterior beliefs. If those signals induce a posterior that is strictly higher than $\bar{q}$, then the sender is willing to pay a strictly positive price for the service, which by the previous argument implies that the sender much purchase with probability 1. But then on-path signals are uninformative because the posterior induced by $G_\omega(\cdot)$ is independent of ω. If $p = 0$, then it must be no signal induces a posterior higher than $\bar{q}$ and so again on-path signals are uninformative.

Note that Proposition 7 does *not* imply that the agency earns no profit. To see why the agency might be profitable, suppose that $G_\omega(\cdot)$ has support on a subset of the signal space and that the sender always purchases the service. Every k not in the support of $G_\omega(\cdot)$ is off-path, so we can assign those signals the posterior $\mu^*(k) = 0$. But $F_\omega(\cdot)$ assigns positive probability to these off-path signals, which means the sender is willing to pay $p > 0$ for the service and so the agency earns strictly positive profit.

Proposition 7 suggests a role for "truth in advertising" laws or other regulations that limit the kinds of claims that can be made in markets for rhetorical services. If effective, such regulations would unambiguously benefit the receiver by facilitating informative communication. Under some conditions, these regulations would also benefit both the sender (if she prefers informative communication *ex ante*) and the agency (if absent regulation the equilibrium would entail $p = 0$). In other words, regulations that force the agency to internalize the spillovers that its actions have on equilibrium beliefs can (but do not always) generate Pareto improvements in welfare.

6 Conclusion

We view this paper as a first step towards analyzing markets for rhetorical services. This section informally discusses competition among agencies or senders and outlines two other ways in which we could substantially enrich the analysis.

Competition among agencies or senders: Competition among agencies leads to lower prices, which encourages widespread adoption of those agencies' rhetorical services. Consequently, even very simple models of competition—such as undifferentiated Bertrand competition—can have complicated effects of both receiver and sender utility. As suggested by Proposition 3, widespread adoption of services that complement quality leads to more informative communication and improves receiver welfare. In contrast, competition between agencies whose services substitute quality would lead to less informative communication and so would *decrease* receiver welfare. Competition among agencies also changes equilibrium communication in ways that might either increase or decrease the sender's expected equilibrium utility. Therefore, encouraging entry and competition in markets for rhetorical services is not an unalloyed social good. Moving beyond the simplest models of agency competition, Lemma 1 suggests a natural form of horizontal differentiation in these markets: agencies can focus on services that either complement or substitute quality to mitigate price competition.

The sender in our advertising application is a firm, and therefore her incentives to purchase rhetorical services presumably depend on the strategies of other firms in her industry. It is straightforward to derive conditions under which rhetorical service purchases are strategic complements or substitutes for competing senders. One might also imagine that a firm can purchase advertising in order to deter rivals from entering a market, and indeed, one can show that rhetorical services deter entry *for fixed receiver beliefs*. Since receiver beliefs are correct in equilibrium, it is not necessarily the case that there exists an equilibrium in which an incumbent purchases rhetorical services in order to deter entry. That is, the value of a rhetorical service as a barrier to entry depends on the receiver's beliefs about its purchase, which reinforces the idea that rhetorical services are different from other kinds of investments.

Dynamics: Our introductory example of patent medicine advertising and the subsequent collapse of the advertising industry suggests that markets for rhetorical services can exhibit interesting dynamics. The advent of a new communication technology—whether it be mass mailing campaigns, radio, television, or the Internet—can be accompanied by uncertainty about the extent to which that technology can be manipulated. Recent concerns over "fake news" and viral propaganda on social media provide examples of this uncertainty. As these technologies are more widely used (and misused), their audiences better learn whether to trust communication on them. In a future version of this paper, we plan to model this learning process by assuming that the receiver is initially uncertain about $G_\omega(\cdot)$ and updates his beliefs about it based on his experiences.

Other applications of the communication model: The model of rhetorical ability in Section 4 can be applied to other settings that prominently features rhetoric and persuasion in organizations, capital markets, or politics. As a new product diffuses across society, how does an early adopter's rhetorical ability affect the extent of its eventual success? How does the presence of a charismatic founder or early investor in a start-up affect follow-on investment and the eventual success of that firm? How do employees wield their rhetorical talents to manipulate firm decisions, and how should management structure its incentives and hierarchy to take advantage of its employees' talents? Our hope is that our communication game provides a simple building block to address these (and other) questions.

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A Omitted Proofs

A.1 Proof of Lemma 1

The derivative of $s(\cdot|\mathcal{Q})$ is equal *in sign* to

$$(Q_L g_0 + (1-\bar{q}-Q_L) f_0)\left(Q_H g_0' + (\bar{q}-Q_H) f_1'\right) - \left(Q_L g_0' - (1-\bar{q}-Q_L) f_0'\right)(Q_H g_1 + (\bar{q}-Q_H) f_1)$$

Rearranging terms in this expression, $s(\cdot|\mathcal{Q})$ is strictly increasing if

$$\left(\begin{array}{c} Q_L Q_H \left(g_0 g_1' - g_0' g_1\right) + \\ Q_L(\bar{q}-Q_H)\left(g_0 f_1' - g_0' f_1\right) + \\ (1-\bar{q}-Q_L) Q_H \left(f_0 g_1' - f_0' g_1\right) + \\ (1-\bar{q}-Q_L)(\bar{q}-Q_H)\left(f_0 f_1' - f_0' f_1\right) \end{array}\right) > 0.$$

Assumption 2 implies that each of these terms is weakly positive, with the first and last terms strictly so because either $Q_L Q_H > 0$ or $(1-\bar{q}-Q_L)(\bar{q}-Q_H) > 0$.

Next, we argue that $\Delta(q|\mathcal{Q}) > 0$ for any $q \in (0,1)$ and $\mathcal{Q} \subseteq [0,1]$. Indeed,

$$\Delta(q|\mathcal{Q}) = \int u_S^*(s(k|\mathcal{Q}))\,[q(g_1(k) - f_1(k)) + (1-q)(g_0(k) - f_0(k))]\,dk.$$

By Assumption 2, $g_1(k) - f_0(k)$ and $g_1(k) - f_0(k)$ single-cross 0 from below, and at least one of these is not identically 0. Furthermore, $\int (g_1(k) - f_1(k))\,dk = \int (g_0(k) - f_0(k))\,dk = 0$ and $u_S^*(s(\cdot|\mathcal{Q}))$ is strictly increasing, so Beesack's Inequality implies that $\Delta(q|\mathcal{Q}) > 0$.

We can also argue that

$$\frac{\partial \Delta}{\partial q}(q|\mathcal{Q}) = \int u_S^*(s(k|\mathcal{Q}))\,[g_1(k) - f_1(k) - (g_0(k) - f_0(k))]\,dk \qquad (6)$$

is strictly increasing (decreasing) if the service complements (substitutes) quality. We have already shown that $s(\cdot|\mathcal{Q})$ is strictly increasing. Since $u_S^*(\cdot)$ is also strictly increasing,

$u^*_S(s(\cdot|\mathcal{Q}))$ is strictly increasing. Moreover,

$$\int (g_1(k) - g_0(k) - f_1(k) + f_0(k))\, dk = 0.$$

If $g_T(k) - g_L(k) - f_T(k) + f_L(k)$ single-crosses 0 from below and is not identically 0, Beesack's Inequality implies that (6) is strictly positive. If $g_T(k) - g_L(k) - f_T(k) + f_L(k)$ single-crosses 0 from below, then exactly the same argument implies that (6) is strictly negative.

If the service complements quality, then $\frac{\partial \Delta}{\partial q}(q|\mathcal{Q}) > 0$ implies that $\mathcal{Q} = [q^*, 1]$ in any equilibrium. Then $\frac{\partial Q_H}{\partial q^*} = -q^* h(q^*)$ and $\frac{\partial Q_L}{\partial q^*} = -(1-q^*)h(q^*)$, so

$$\frac{\partial s(q|q^*, 1])}{\partial q^*}|_{q^*=1} = -\frac{h(1)(g_1(k) - f_1(k))}{(1-\bar{q})f_0(k)}.$$

Taking the derivative of (2) with respect to q^* and setting $q = 1$ yields

$$\begin{aligned} \frac{\partial \Delta}{\partial q^*}(1|[q^*, 1])|_{q^*=1} &= \int \frac{du^*_S}{ds} \frac{\partial s(q|[q^*,1])}{\partial q^*}|_{q^*=1}(g_1(k) - f_1(k))dk \\ &= -\frac{h(1)}{(1-\bar{q})} \int \frac{du^*_S}{ds} \frac{(g_1(k)-f_1(k))^2}{f_0(k)} dk \\ &< 0, \end{aligned}$$

where the inequality follows because $\frac{du^*_S}{ds} > 0$ and $h(1) > 0$ by Assumption 1. If $g_0 = f_0$ as well, then

$$\frac{\partial s(q|[q^*, 1])}{\partial q^*} = -q^* h(q^*) \frac{g_1(k) - f_1(k)}{(1-\bar{q})f_0(k)}$$

and so

$$\frac{\partial \Delta}{\partial q^*}(q|[q^*, 1]) = -q^* h(q^*) \frac{q}{1-q} \int \frac{\partial \tilde{u}_S}{\partial s} \frac{(g_1(k) - f_1(k))^2}{f_0(k)} dk < 0$$

for any $q^* > 0$.

If the service substitutes quality, then $\frac{\partial \Delta}{\partial q}(q|\mathcal{Q}) < 0$ implies that $\mathcal{Q} = [0, q^*]$ in equilibrium. Following the same derivations as in the previous paragraph yields

$$\frac{\partial \Delta}{\partial q^*}(0|[0, q^*])|_{q^*=0} =_s -\bar{q}h(0) \int \frac{du^*_s}{ds} f_1(k)(g_0(k) - f_0(k))^2 dk < 0,$$

where "$=_s$" denotes equality of sign. If $g_1 = f_1$ as well, then

$$\frac{\partial \Delta}{\partial q^*}(q|[0,q^*]) = -(1-q)(1-q^*)h(q^*)\int \frac{\partial \tilde{u}_S}{\partial s} f_1(k) \frac{(g_0(k)-f_0(k))^2}{(Q_L g_0(k) + ((1-q)-Q_L)f_0(k))^2} dk < 0.$$

These calculations collectively prove the result. ■

A.2 Proof of Proposition 1

Suppose the service substitutes quality and $g_1 = f_1$. Then

$$\Delta_L(q^*) = \int u_S^*(s(k|[0,q^*]))(1-q^*)(g_0(k)-f_0(k))dk$$

and so

$$\frac{d\Delta_L}{dq^*}(q^*) = -\int u_S^*(s(k|[0,q^*])(g_0(k)-f_0(k))dk + (1-q)\int \frac{\partial u_S^*}{\partial \mu}\frac{\partial \mu}{\partial q^*}(g_0(k)-f_0(k))dk.$$

Since $\int(g_0(k)-f_0(k))dk = 0$, $g_0(\cdot)$ single-crosses $f_0(\cdot)$ from below, and $u_S^*(\cdot)$ and $s(\cdot|[0,q^*])$ are strictly increasing, Beesack's Inequality implies that the first term in this expression is strictly negative. The second term equals $\frac{\partial \Delta}{\partial q^*}(q|[0,q^*])$ and so is strictly negative by Lemma 1. So $\frac{d\Delta_L}{dq^*} < 0$ as desired.

Suppose instead that the service complements quality. Then

$$\Delta_T(q^*) = q^* \int u_S^*(s(k|[q^*,1]))(g_1(k)-f_1(k))dk$$

and

$$\frac{d\Delta_T}{dq^*} = \frac{\Delta_T(q^*)}{q^*} + q^* \int \frac{\partial u_S^*}{\partial s}\frac{\partial s}{\partial q^*}(k|[q^*,1])(g_1(k)-f_1(k))dk.$$

Now,

$$\frac{\partial s}{\partial q^*} = -q^* h(q^*)\frac{g_1(\cdot)-f_1(\cdot)}{(1-\bar{q})f_0(\cdot)}$$

and so

$$\frac{d\Delta_T}{dq^*} = \frac{\Delta_T(q^*)}{q^*} - (q^*)^2 h(q^*) \int \frac{\partial u_S^*}{\partial s} \frac{(g_1(k) - f_1(k))^2}{(1-\bar{q})f_0(k)} dk.$$

The first term in this expression is strictly positive, while the second term is strictly negative.

If $q^* \in (0, 1)$, then

$$\frac{d\Delta_T}{dq^*} =_s \Delta_T(q^*) - (q^*)^3 h(q^*) \int \frac{\partial u_S^*}{\partial s} \frac{(g_1(k) - f_1(k))^2}{(1-\bar{q})f_0(k)} dk.$$

Therefore, $\frac{d\Delta_T}{dq^*} < 0$ (and so the marginal sender's willingness-to-pay is increasing in likelihood of purchase) if and only if

$$\frac{\Delta_T(q^*)}{(q^*)^3 h(q^*)} < \int \frac{\partial u_S^*}{\partial s}(s(k|[q^*, 1])) \frac{(g_1(k) - f_1(k))^2}{(1-\bar{q})f_0(k)} dk,$$

as desired. ■

A.3 Proof of Proposition 2

Suppose the service complements quality. Given Lemma 1, the agency's price equilibrium satisfies $p = \Delta(q^*|\mathcal{Q})$ and the sender purchases if and only if $q \in [q^*, 1]$. If the agency can choose q^*, it will never choose $q^* = 1$ because that yields 0 profit. The optimal q^* therefore solves

$$\max_q \Delta(q|\mathcal{Q})(1 - H(q)),$$

which has first-order condition

$$\frac{1 - H(q^*)}{h(q^*)} \leq \frac{\Delta(q^*|\mathcal{Q})}{\frac{\partial \Delta}{\partial q}(q^*|\mathcal{Q})}$$

with equality if $q^* > 0$. Writing out this expression yields

$$\frac{1 - H(q^*)}{h(q^*)} \leq q^* + \frac{\int u_S^*(s(k|\mathcal{Q}))\,(g_0(k) - f_0(k))\,dk}{\int u_S^*(s(k|\mathcal{Q}))\,(g_1(k) - f_1(k) - (g_0(k) - f_0(k)))\,dk},$$

as desired.

If the service substitutes quality, then q^* maximizes $\Delta(q|\mathcal{Q})H(q)$. Clearly, $q^* > 0$, and so we can follow the same procedure as above (noting that $\frac{\partial \Delta}{\partial q} < 0$) to yields the first-order condition

$$\frac{H(q^*)}{h(q^*)} \leq q^* + \frac{\int u_S^*(s(k|\mathcal{Q}))\,(g_0(k) - f_0(k))\,dk}{\int u_S^*(s(k|\mathcal{Q}))\,(g_1(k) - f_1(k) - (g_0(k) - f_0(k)))\,dk},$$

with equality if $q^* < 1$, as desired. ■

A.4 Proof of Proposition 3

Since $d^*(\cdot)$ is increasing and $\mu(\cdot|\mathcal{Q})$ is increasing for any $\mathcal{Q}$, the receiver faces a monotone decision problem.

For fixed purchase decisions $\mathcal{Q}$, denote

$$R_1(\cdot) = \frac{Q_H}{\bar{q}} G_1(\cdot) + \frac{\bar{q} - Q_H}{\bar{q}} F_1(\cdot)$$

as the distribution over the signal if $\omega = 1$, and similarly

$$R_0(\cdot) = \frac{Q_L}{1 - \bar{q}} G_0(\cdot) + \frac{1 - \bar{q} - Q_L}{1 - \bar{q}} F_0(\cdot)$$

if $\omega = 0$. These distributions have respective densities $r_1(\cdot)$ and $r_0(\cdot)$.

If the service complements quality and $f_0 = g_0$, then Assumption 2 implies that

$$\frac{r_1(\cdot)}{r_0(\cdot)} = \frac{Q_H}{\bar{q}} \frac{g_1(\cdot)}{f_0(\cdot)} + \frac{\bar{q} - Q_H}{\bar{q}} \frac{f_1(\cdot)}{f_0(\cdot)}$$

is strictly increasing. By Lehmann (1988), our result follows so long as for any $x \in [0, 1]$,

$$R_1^{-1}(F_1(x)) \geq R_0^{-1}(F_0(x)).$$

But $R_1(\cdot)$ is strictly increasing and $R_0(\cdot) = F_0(\cdot)$, so this expression is equivalent to $F_1(x) \geq R_1(x)$; that is, $R_1(\cdot)$ strictly dominates $F_1(\cdot)$ in the sense of first-order stochastic dominance. This ordering holds because

$$r_1(\cdot) - f_1(\cdot) = \frac{Q_H}{\bar{q}}(g_1(\cdot) - f_1(\cdot))$$

single-crosses 0 from below by Assumption 2. So such a service strictly improves receiver welfare in equilibrium.

If the service instead substitutes quality and $f_1 = g_1$, then as above,

$$\frac{r_1(\cdot)}{r_0(\cdot)} = \frac{f_1(\cdot)}{\frac{Q_L}{1-\bar{q}}g_0(\cdot) + \frac{1-\bar{q}-Q_L}{1-\bar{q}}f_0(\cdot)} = \frac{1}{\frac{Q_L}{1-\bar{q}}\frac{g_0(\cdot)}{f_1(\cdot)} + \frac{1-\bar{q}-Q_L}{1-\bar{q}}\frac{f_1(\cdot)}{f_1(\cdot)}}$$

is strictly increasing by Assumption 2. Therefore, it suffices to show that for all $x \in [0, 1]$,

$$F_1^{-1}(R_1(x)) \geq F_0^{-1}(R_0(x))$$

or $F_0(x) \geq R_0(x)$. But this relationship again holds because

$$f_0(\cdot) - r_0(\cdot) = -\frac{Q_L}{1-\bar{q}}\left(g_0(\cdot) - f_0(\cdot)\right),$$

which again single-crosses 0 from below by Assumption 2. ■

A.5 Proof of Proposition 4

In the agency-optimal equilibrium of the market game with public prices, the receiver's beliefs maximize the probability of sale conditional on each price p, among the set of beliefs that are consistent with equilibrium continuation play.

Suppose the service substitutes quality and $g_1 = f_1$. The principal's profit-maximization problem is

$$\max_{p,q^*|\Delta_L(q^*)=p} pH(q^*).$$

If there are two marginal qualities, $q_L < q_H$, such that $\Delta_L(q_L) = \Delta_L(q_H) = p$, then $pH(q_L) < pH(q_H)$. Therefore, any solution to this problem is also a solution to

$$\max_{q^*} \Delta_L(q^*)H(q^*).$$

Since $\Delta_L(1) = H(0) = 0$, any solution to this problem is interior and so satisfies the first-order condition

$$\frac{\partial \Delta_L(q^*)}{\partial q^*} H(q^*) + h(q^*)\Delta_L(q^*) = 0.$$

By assumption, $\Delta_L(\cdot)H(\cdot)$ is strictly concave, and so the left-hand side of this expression is strictly decreasing in q^*.

Consider the first-order condition of the profit-maximization problem in the market game:

$$\left(\frac{\partial \Delta_L}{\partial q}(q|[0,q^*])|_{q=q^*}\right) H(q^*) + h(q^*)\Delta_L(q^*) = 0.$$

Let q_{PVT} be any solution to this problem. It is straightforward to show that

$$0 > \frac{\partial \Delta}{\partial q}(q|[0,q^*])|_{q=q^*} > \frac{\partial \Delta_L(q^*)}{\partial q^*}.$$

Therefore, $\frac{\partial \Delta_L(q_{PVT})}{\partial q^*} H(q_{PVT}) + h(q_{PVT})\Delta_L(q_{PVT}) < 0$, and hence $q_{PUB} < q_{PVT}$ as desired.

If the service complements quality, then the profit-maximization problem has first-order condition

$$\frac{\partial \Delta_T(q^*)}{\partial q^*}(1 - H(q^*)) - h(q^*)\Delta_T(q^*) = 0. \tag{7}$$

Lemma 1 implies that $\frac{\partial \Delta_T(q^*)}{\partial q^*} < \frac{\partial \Delta}{\partial q}(q|[q^*,1])$, and so for any profit-maximizing q_{PVT} in the market game, the left-hand side of (7) is strictly negative. Hence, if $\Delta_T(\cdot)(1 - H(\cdot))$ is strictly concave, then the unique profit-maximizing q_{PUB} of the game with public prices satisfies $q_{PUB} < q_{PVT}$. ■

A.6 Proof of Proposition 5

The first step in this proof is to state a definition and prove a lemma, both of which will also be useful in the proof of Proposition 6.

Definition 1 *Let Assumption 1 hold. An increasing function* $\mu^* : [0,1] \to [0,1]$ *is a* ***candidate posterior*** *if for any* $k \in [0,1]$*, either* $\mu^*(k) = \mu(k)$ *or* $\mu^*(\cdot)$ *is constant on a closed interval that contains* k*. Let* $I \subseteq [0,1]$ *be an interval such that* $\mu^*(\cdot)$ *is constant on* I *but not on any other interval in* $[0,1]$ *that contains* I*. Letting* $\underline{k} = \inf I$*, for any* $k \in I$*,*

$$\mu^*(k) \equiv \mu^* \leq \int_{\underline{k}}^{k} \mu(x) z(x|[\underline{k}, k]) dx, \tag{8}$$

where (8) holds with equality as $k \uparrow \sup I$*.*

Lemma 2 *In any earnest equilibrium of a smooth game, there exists a candidate posterior* $\mu^*(\cdot)$ *such that for any realization of* $\theta \in \Theta$*, the receiver's equilibrium posterior belief that* $\omega = 1$ *equals* $\mu^*(\theta_T)$ *if* $\omega = 1$ *and* $\mu^*(\theta_L)$ *if* $\omega = 0$*.*

Proof of Lemma 2

Let σ^* be an earnest equilibrium of a smooth game. Define $\tilde{\mu}(a)$ as the receiver's posterior belief that $\omega = 1$ following argument a. Because F_1 has full support on $[0,1]$, for any $k \in [0,1]$, $(1,k)$ is sent with positive probability. But then $\tilde{\mu}(1,k)$ must be weakly increasing in k: if $\tilde{\mu}(1,k') < \tilde{\mu}(1,k)$ for $k' > k$, then a sender with $\theta_T = k'$ and $\omega = 1$ would have a feasible and profitable deviation to $a = (1,k)$.

By definition, if $\omega = 1$, then $m = 1$. Therefore, any $a = (0,k)$ that is sent on the equilibrium path satisfies $\tilde{\mu}(a) = 0$ and so either $m = 0$ is never sent on the equilibrium path, or there exists some k such that $\tilde{\mu}(1,k) = 0$. In the former case, we can equivalently define the receiver's posterior $\mu^*(k) \equiv \tilde{\mu}(1,k)$. In the latter case, $\tilde{\mu}(1,0) = 0$ because $\tilde{\mu}((1,\cdot))$ is weakly increasing, so we can interpret any argument $a = (0,k)$ as the argument $a = (1,0)$, and thereby similarly define the receiver's posterior as a function of the argument's strength alone.

Now, $\mu^*(k)$ must be increasing because $\tilde{\mu}(1,\cdot)$ is increasing. So in equilibrium, a sender with type θ must induce posterior $\mu^*(\theta_T)$ if $\omega = 1$ and $\mu^*(\theta_L)$ if $\omega = 0$, since otherwise she would have a profitable deviation to a stronger feasible argument. Hence, it remains to show that $\mu^*(\cdot)$ is a candidate posterior.

We have already shown that $\mu^*(\cdot)$ is increasing. Suppose there exists some $k \in (0,1)$ such that $\mu^*(k) \neq \mu(k)$. If $\omega = 1$, then $a = (1,k)$ if and only if $\theta_T = k$. If $\omega = 0$ leads to $a = (1,k)$ if and only if $\theta_L = k$, then $\mu^*(k) = \mu(k)$ by definition of $\mu(\cdot)$. Therefore, either (i) the sender does not choose $a = (1,k)$ with probability 1 if $\omega = 0$ and $\theta_L = k$, or (ii) a sender with $\theta_L \neq k$ makes argument $a = (1,k)$ with positive probability if $\omega = 0$ (or both).

In case (i), the sender must instead make another feasible argument: either $(1,k')$ for some $k' < k = \theta_L$, or $(0,k')$ for some $k' \leq \theta_T$. If the former, then the sender has the incentive to do so only if $\mu^*(k') = \mu^*(k)$ because $\mu^*(\cdot)$ is increasing. If the latter, then $\mu^*(k) = 0$ by the argument above, which implies that $\mu^*(k') = 0$ for all $k' \in [0,k]$. So $\mu^*(\cdot)$ is constant on some non-empty interval $[k',k]$.

In case (ii), the sender could have instead made argument $(1,\theta_L)$. For $(1,k)$ to be a feasible argument for the sender, $\theta_L > k$, and so it must be that $\mu^*(\cdot)$ is constant on the non-empty interval $[k,\theta_L]$. So $\mu^*(\cdot)$ is flat on a non-empty interval interval about k whenever it does not coincide with $\mu(\cdot)$.

Finally, let $(\underline{k},\bar{k})$ be such that $\mu^*(\cdot)$ is constant on this interval but not on any open interval in $[0,1]$ that contains $(\underline{k},\bar{k})$. Every argument $(1,\tilde{k})$ with $k \in (\underline{k},\bar{k})$ induces the same posterior, which we denote by μ^*. Therefore, for any $k \in (\underline{k},\bar{k})$,

$$\mu^* = \frac{q\Pr_{\sigma^*}\{a = (1,x) \text{ for } x \in (\underline{k},k)|\omega = 1\}}{q\Pr_{\sigma^*}\{a = (1,x) \text{ for } x \in (\underline{k},k)|\omega = 1\} + (1-q)\Pr_{\sigma^*}\{a = (1,x) \text{ for } x \in (\underline{k},k)|\omega = 0\}}.$$

In any earnest equilibrium,

$$\Pr_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{k}, k) | \omega = 1\} = F_1(k) - F_1(\underline{k}).$$

Furthermore, any sender with $\theta_L \in (\underline{k}, k)$ cannot make an argument stronger than k and is unwilling to make an argument that is weaker than $\underline{k}$, since such an argument would induce a strictly lower posterior by definition of $\underline{k}$. Therefore,

$$\Pr_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{k}, k) | \omega = 0\} \geq F_0(k) - F_0(\underline{k}). \tag{9}$$

Consequently,

$$\begin{aligned} \mu^* &\leq \frac{q(F_1(k) - F_1(\underline{k}))}{q(F_1(k) - F_1(\underline{k})) + (1-q)(F_0(k) - F_0(\underline{k}))} \\ &= \int_{\underline{k}}^{k} \mu(x) z(x | [\underline{k}, k]) dx, \end{aligned}$$

where the equality follows from the definitions of $\mu(x)$ and $z(x|[\underline{k}, k])$. So (8) must hold for every $k \in (\underline{k}, \bar{k})$. For $k = \bar{k}$, (9) must hold with equality because no sender with $\theta_L > \bar{k}$ is willing to make an argument with strength in $[\underline{k}, \bar{k}]$. Therefore, the steps given above imply that (8) holds with equality for $k = \bar{k}$. ■

Completing the proof of Proposition 5

Suppose that $\mu(\cdot)$ is increasing. We first show that an earnest equilibrium exists. Consider the strategy profile used to construct $\mu(\cdot)$: for every $\theta \in \Theta$, a sender with ability θ makes argument $(1, \theta_T)$ if $\omega = 1$ and $(1, \theta_L)$ if $\omega = 0$. A belief system consistent with this strategy profile is: the receiver's posterior equals $\mu(k)$ if he observes $(1, k)$ and equals 0 if he observes $(0, k)$ for any $k \in [0, 1]$. We claim the sender has no profitable deviation from this strategy profile.

Any deviation to $a = (0, k)$ induces posterior belief 0 and so cannot be profitable. To be feasible, a deviation to $(1, k)$ must satisfy $k < \theta_T$ if $\omega = 1$ or $k < \theta_L$ if $\omega = 0$. But these deviations are not profitable because $\mu(\cdot)$ is increasing. So the specified strategy profile is an earnest equilibrium.

Next, uniqueness. Lemma 2 implies that it suffices to show that $\mu(\cdot)$ is the only candidate posterior. Consider some candidate posterior $\mu^*(\cdot)$ such that there exists a $k \in [0,1]$ with $\mu^*(k) \neq \mu(k)$. By Definition 1, there exists some open interval $(\underline{k}, \bar{k})$ such that $\mu^*(k) \equiv \mu^*$ is constant for all $k \in (\underline{k}, \bar{k})$ but not on any larger open interval.

Suppose that $\mu^* > \mu(\underline{k})$. Assumption 1 ensures that $\mu(\cdot)$ is continuous, so there exists $\bar{\epsilon} > 0$ such that for any $\epsilon \in [0, \bar{\epsilon}]$, $\mu^* > \mu(\underline{k} + \epsilon)$. But this inequality contradicts (8), which requires μ^* to be smaller than the conditional probability of $\mu(k)$ on the interval $(\underline{k}, \underline{k} + \epsilon)$.

Suppose that $\mu^* \leq \mu(\underline{k})$. Since $\mu(\cdot)$ is increasing, $\mu^* \leq \mu(k')$ for all $k' \in (\underline{k}, \bar{k})$ as well. But $\mu^* < \mu(k)$ by assumption, so $\mu^* < \mu(k')$ for a positive measure of $k' \in (\underline{k}, \bar{k})$ because $\mu(\cdot)$ is continuous. But then (8) cannot hold with equality on the interval $(\underline{k}, \bar{k})$. Contradiction: $\mu^*(\cdot)$ cannot be a candidate posterior. Therefore, Lemma 2 implies that in any earnest equilibrium, the equilibrium posterior must equal $\mu(\theta_T)$ if $\omega = 1$ and $\mu(\theta_L)$ if $\omega = 0$. ■

A.7 Proof of Proposition 6

By Lemma 2, it suffices to show that there exists a unique candidate posterior. Let $\tilde{\mu}(\cdot)$ be a candidate posterior. We claim that there exists an interval $I \subseteq [0,1]$ such that $[k_L, k_H] \subseteq I$ and $\tilde{\mu}(\cdot)$ is constant on I.

Suppose not. Then there exist points $\underline{k} < \bar{k}$ such that $\underline{k}, \bar{k} \in [k_L, k_H]$ and $\tilde{\mu}(\underline{k}) < \tilde{\mu}(\bar{k})$. Because $\mu(\cdot)$ is decreasing on $[k_L, k_H]$, we can choose $\underline{k}$ and $\bar{k}$ to be part of intervals $\underline{I}$ and $\bar{I}$, respectively, where $\tilde{\mu}(\cdot)$ is constant on both $\underline{I}$ and $\bar{I}$ and $\sup \underline{I} \leq \inf \bar{I}$. Now, $\tilde{\mu}(\bar{k}) \leq \mu(\inf \bar{I})$, since otherwise (8) would be violated on a small enough sub-interval of $\bar{I}$ because $\mu(\cdot)$ is continuous. But $\mu(\inf \bar{I}) \leq \mu(\sup \underline{I})$ because $\mu(\cdot)$ is decreasing on $[k_L, k_H]$, so $\tilde{\mu}(\underline{k}) < \mu(\sup \underline{I})$. If $\tilde{\mu}(\underline{k}) \leq \mu(\inf \underline{I})$, then $\tilde{\mu}(\cdot)$ lies everywhere below $\mu(\cdot)$ on $\underline{I}$ because $\mu(\cdot)$ is increasing and then decreasing on $\underline{I}$, and so (8) cannot hold with equality on $\underline{I}$. If $\tilde{\mu}(\underline{k}) > \mu(\inf \underline{I})$, then (8) is violated for a sufficiently small sub-interval of $\underline{I}$. So $\tilde{\mu}(\cdot)$ must be constant on some interval $I \supseteq [k_L, k_H]$.

Define $\bar{k} = \sup I$ and $\underline{k} = \inf I$, and suppose the candidate posterior equals $\tilde{\mu}$ on I. We claim that $\tilde{\mu}(\bar{k}) \geq \mu(\bar{k})$, with equality unless $\bar{k} = 1$. Suppose $\tilde{\mu}(\bar{k}) < \mu(\bar{k})$. The candidate posterior is increasing, so $\tilde{\mu} < \mu(\bar{k})$. But (8) holds with equality on $[\underline{k}, \bar{k}]$ and $\mu(\cdot)$ is continuous, so (8) must be violated on $[\underline{k}, \bar{k} - \epsilon]$ for $\epsilon > 0$ sufficiently small. Hence, $\tilde{\mu} \geq \mu(\bar{k})$. If $\bar{k} < 1$, then $\tilde{\mu}(\bar{k} + \epsilon) > \mu(\bar{k} + \epsilon)$ for any sufficiently small $\epsilon > 0$ because $\tilde{\mu}(\cdot)$ is increasing and $\mu(\cdot)$ is continuous. But then $\tilde{\mu}(\cdot)$ must be constant on some interval about $\bar{k} + \epsilon$, I_ϵ, where $\inf I_\epsilon \geq \bar{k}$ and so $\tilde{\mu}(\inf I_\epsilon) > \mu(\inf I_\epsilon)$. Hence, $\tilde{\mu}(\cdot)$ violates (8) on a small enough sub-interval of I_ϵ. So $\tilde{\mu} = \mu(\bar{k})$ if $\bar{k} < 1$.

Next, we claim that $\tilde{\mu} \leq \mu(\underline{k})$, with equality unless $\underline{k} = 0$. If $\tilde{\mu} > \mu(\underline{k})$, then (8) is violated on $[\underline{k}, \underline{k} + \epsilon]$ for sufficiently small $\epsilon > 0$. If $\tilde{\mu} < \mu(\underline{k})$ and $\underline{k} > 0$, then $\tilde{\mu}(\underline{k} - \epsilon) < \mu(\underline{k} - \epsilon)$ for any $\epsilon > 0$ sufficiently small. Therefore, there exists an interval I_ϵ such that $\underline{k} - \epsilon \in I_\epsilon$ and $\tilde{\mu}(\cdot)$ is constant on I_ϵ. But $\sup I_\epsilon \leq \underline{k}$ and so $\mu(\cdot)$ is strictly increasing on I_ϵ. Therefore, either $\tilde{\mu}(k) < \mu(k)$ for almost all $k \in I_\epsilon$, in which case (8) cannot hold with equality on I_ϵ, or $\tilde{\mu}(\underline{k} - \epsilon) > \mu(\inf I_\epsilon)$, in which case (8) is violated on a sufficiently small sub-segment of I_ϵ. So $\tilde{\mu} \leq \mu(\underline{k})$, with equality unless $\underline{k} = 0$.

$\mu(\cdot)$ is strictly increasing on $[0,1]\backslash I$. Therefore, $\tilde{\mu}(\cdot)$ cannot be constant on any interval $\tilde{I} \in [0,1]\backslash I$ for reasons similar to those given in the proof of Proposition 5. Consequently, $\tilde{\mu}(\cdot)$ must be continuous and coincide with $\mu(\cdot)$ except on a single interval at which it is constant. To satisfy (8), this constant region must cross $\mu(\cdot)$. Therefore, for any candidate posterior, there exists some $\tilde{k} \in [k_L, k_H]$ such that the candidate posterior equals

$$\tilde{\mu}(k|\tilde{k}) \equiv \begin{cases} \min\left\{\mu(k), \mu(\tilde{k})\right\} & k < k_L \\ \mu(\tilde{k}) & k \in [k_L, k_H] \\ \max\left\{\mu(k), \mu(\tilde{k})\right\} & k > k_H \end{cases}.$$

Finally, we argue that $\tilde{\mu}(\cdot|\tilde{k})$ is a candidate posterior for exactly one $\tilde{k} \in [k_L, k_H]$. Letting

$[\underline{k}(\tilde{k}), \bar{k}(\tilde{k})]$ be the interval on which $\tilde{\mu}(\cdot|\tilde{k})$ is constant, consider

$$\mu(\tilde{k}) - \int_{\underline{k}(\tilde{k})}^{\bar{k}(\tilde{k})} \mu(x) z(x|[\underline{k}(\tilde{k}), \bar{k}(\tilde{k})]) dx.$$

If $\tilde{\mu}(\cdot|\tilde{k})$ is a candidate posterior, then (8) implies that this expression equals 0. Since $\tilde{\mu}(\cdot|\tilde{k}) = \mu(\cdot)$ everywhere outside the interval $[\underline{k}(\tilde{k}), \bar{k}(\tilde{k})]$, this expression has the same sign as

$$\int_0^1 \left(\tilde{\mu}(x|\tilde{k}) - \mu(x)\right) z(x|[0,1]) dx. \tag{10}$$

Now, $\mu(\cdot)$ is decreasing and continuous on $[k_L, k_H]$, so $\tilde{\mu}(k|\cdot)$ is likewise decreasing and continuous for each $k \in [0,1]$. Further, $\tilde{\mu}(\cdot|k_L) \geq \mu(\cdot)$ and $\tilde{\mu}(\cdot|k_H) \leq \mu(\cdot)$. Therefore, (10) is strictly positive at $\tilde{k} = k_L$, strictly negative at $\tilde{k} = k_H$, and strictly decreasing in $\tilde{k}$. So (10) equals 0 for exactly one $\tilde{k}$. But then the unique candidate posterior in this game equals $\tilde{\mu}(\cdot|\tilde{k})$.

It remains to argue that there exists an earnest equilibrium of the game. Such an equilibrium must have equilibrium posterior $\mu^*(\cdot) \equiv \tilde{\mu}(\cdot|\tilde{k})$ for the unique appropriate $\tilde{k}$. The definition of an earnest equilibrium pins down the sender's strategy if $\omega = 1$. Consider the following strategy if $\omega = 0$. If θ_L is such that $\mu^*(\theta_L) \leq \mu(\theta_L)$, then $a = (1, \theta_L)$. If θ_L is such that $\mu^*(\theta_L) > \mu(\theta_L)$, then $a = (1, \theta_L)$ with probability $\gamma(\theta_L) < 1$ defined by

$$\frac{qf_1(\theta_L)}{qf_1(\theta_L) + (1-q)\gamma(\theta_L) f_0(\theta_L)} = \mu^*(\theta_L)$$

With the complementary probability, the sender chooses an argument with a strength in $\mathcal{K} = \{k \in [\underline{k}(\tilde{k}), \bar{k}(\tilde{k})] | \mu^*(k) < \mu(k)\}$. Every argument in this set is feasible because $\mu(\cdot)$ reverses direction only once, and any k with $\mu^*(k) > \mu(k)$ is strictly larger than every k with $\mu^*(k) < \mu(k)$. The distribution of arguments over $\mathcal{K}$ is chosen (independent of the sender's type) so that the posterior equals $\mu^*(k)$ at every $k \in \mathcal{K}$. Since (10) holds with equality on $[\underline{k}(\tilde{k}), \bar{k}(\tilde{k})]$, such a distribution exists. No sender has a profitable deviation from this strategy by construction, so it is an earnest equilibrium. ■

A.8 Proof of Proposition 7

Claim 1: Given an equilibrium σ^* of the product design game, let $\mu_{\sigma^*}(k)$ be the receiver's posterior belief that the state is high conditional on signal k. Define

$$\mathcal{K}_M = \arg\max_k \mu_{\sigma^*}(k).$$

Then $\mathcal{K}_M$ is non-empty and $G_1(\mathcal{K}_M) = G_0(\mathcal{K}_M) = 1$.

Proof of Claim 1: Suppose that $\mathcal{K}_M$ is non-empty, but that $G_\omega(\mathcal{K}_M) < 1$ for some $\omega \in \{0, 1\}$. Let $\bar{\mu} = \max_k \mu_{\sigma^*}(k)$, and consider the alternative product that sets $\tilde{G}_\omega(\mathcal{K}_M) = 1$ and is otherwise identical to G_ω. This perturbed product increases the sender's willingness-to-pay by

$$\int_0^1 u_S^*(\mu_{\sigma^*}(k)) \left[q \left(d\tilde{G}_1 - dG_1 \right) + (1-q) \left(d\tilde{G}_0 - dG_0 \right) \right] =$$

$$(1 - qG_1(\mathcal{K}_M) - (1-q)G_0(\mathcal{K}_M))u_S^*(\bar{\mu}) - \int_{k \notin \mathcal{K}_M} u_S^*(\mu_{\sigma^*}(k)) \left[qdG_1 + (1-q)dG_0 \right] > 0$$

where the inequality follows because $\mu_{\sigma^*}(k) < \bar{\mu}$ for all $k \notin \mathcal{K}_M$, $q \in (0, 1)$, and $G_\omega(\mathcal{K}_M) < 1$ for at least one ω. The agency can increase its price by this amount without affecting quantity sold, so such a deviation is profitable.

It remains to show that $\mathcal{K}_M$ is non-empty in any equilibrium. If $\mathcal{K}_M$ is empty, then for any $G_\omega(\cdot)$, there exists an alternative $\tilde{G}_\omega(\cdot)$ that induces weakly higher posteriors everywhere and strictly higher posteriors with positive probability. Therefore, the agency always has a profitable deviation in such settings, which are therefore inconsistent with equilibrium. ■

Claim 2: An equilibrium of the product design game exists. In any equilibrium, $\mu_{\sigma^*}(k) = q$ on the support of the equilibrium signal distribution, and at least one of the following two conditions must hold: (i) $p = 0$, or (ii) the sender purchases the service with probability 1.

Proof of Claim 2: To prove existence, consider the following strategy profile: for $\omega \in \{0,1\}$, $G_\omega(\cdot)$ is a uniform distribution over $[0,1]$, $p=0$, the sender purchases this service with probability 1, and $\mu_{\sigma^*}(k)=q$ for all $k \in [0,1]$. Given $G_\omega(\cdot)$, $\mu_{\sigma^*}(\cdot)$ follows from Bayes' Rule. The sender is indifferent between purchasing or not and so is willing to purchase this service with probability 1. Any $G_\omega(\cdot)$ results in the same (degenerate) posterior distribution, so the sender is willing to pay 0 regardless of $G_\omega(\cdot)$ and the agency has no profitable deviation. Therefore, this strategy profile is an equilibrium of the product design game.

Consider any equilibrium σ^*. If $p>0$ in this equilibrium, then the sender must purchase the service with probability 1; otherwise, the agency could deviate to $p-\epsilon$ with $\epsilon>0$ and generate a discrete increase in quantity sold, which would therefore be profitable for $\epsilon>0$ sufficiently close to 0. But Claim 1 implies that $G_\omega(\cdot)$ induces the same posterior regardless of the true state in any equilibrium; since the sender purchases with probability 1, $\mu_{\sigma^*}(k)=q$ must hold on the support of $G_\omega(\cdot)$.

Suppose instead that $p=0$ in this equilibrium. By claim 1, $G_\omega(\mathcal{K}_M)=1$. Since $F_\omega(\cdot)$ has full support on $[0,1]$, sender willingness-to-pay is strictly positive unless $\mu_{\sigma^*}(k)=q$ for all $k \in [0,1]$, which proves the claim. ■

B Exploring Non-Earnest Equilibria

B.1 Earnest Equilibrium as Most Informative PBE

In the text, we justified our focus on earnest equilibria by arguing that they result in a natural mapping between the message and strength of an arguments and its persuasiveness. The appendix provides a second justification that relies on the information conveyed in these equilibria. If $\frac{f_1(\cdot)}{f_0(\cdot)}$ and hence $\mu(\cdot)$ are increasing, then communication in an earnest equilibrium is weakly more informative (in the Lehmann sense) than in any other PBE. Consequently, earnest equilibria maximize the receiver's expected payoff among all PBE in this setting. Note that Assumption 2, which is maintained throughout Section 3, requires

$\frac{f_1(\cdot)}{f_0(\cdot)}$ to be increasing.

Proposition 8 *Suppose* $\frac{f_1(\cdot)}{f_0(\cdot)}$ *is increasing. Among PBE, the unique earnest equilibrium induces the most Lehmann-informative equilibrium mapping from* ω *to* a.

Proof of Proposition 8

Fix a PBE, and let $\mu(a)$ be the receiver's posterior belief that the state is high after observing a. Without loss, assume that $\mu(a) = 0$ whenever a is not sent on the equilibrium path. Define

$$\mu_0^*(k) = \sup_{k' \leq k} \mu(0, k)$$

and

$$\mu_1^*(k) = \sup_{k' \leq k} \mu(1, k).$$

Note that $\mu_0^*(\cdot)$ and $\mu_1^*(\cdot)$ are both increasing. In equilibrium, the sender induces posterior $\mu_1^*(\theta_T)$ if $m = 1$ and $\mu_0^*(\theta_L)$ if $m = 0$.

We first argue that if $(1, k)$ is sent on-path, then $\mu_1^*(k) \geq \mu_0^*(k)$. Moreover, if $(0, k')$ with $k' \leq k$ is on-path as well, then $\mu_1^*(k) = \mu_0^*(k)$ without loss. If $(1, k)$ is on-path, suppose towards contradiction that $\mu_1^*(k) < \mu_0^*(k)$. If $\omega = 0$, then any sender who can send $(1, k')$ with $k' \leq k$ can also send $(0, k')$ with $k' \leq k$. Therefore, no sender chooses $a = (1, k)$ if $\omega = 0$. But $(1, k)$ is sent on the equilibrium path, so it must be sent only if $\omega = 1$ and so $\mu(1, k) = 1 \geq \mu_0^*(k)$; contradiction.

Suppose there exists $k' \leq k$ such that $(0, k')$ is on-path as well. If $\mu_1^*(k) > \mu_0^*(k)$, then no sender chooses $(0, k')$ with $k' \leq k$ if $\omega = 1$. So any on-path $(0, k')$ satisfies $\mu(0, k') = 0$, and moreover, it must be the case that $\mu_1^*(0) = 0$ in order for $(0, k)$ to be on-path. But then we can perturb the equilibrium so that all senders who make argument $(0, k')$ with $k' \leq k$ instead choose $(1, 0)$ without affecting equilibrium persuasion. In this perturbed equilibrium, $\mu_1^*(k) = \mu_0^*(k)$ whenever $(1, k)$ is on-path and there exists a $k' \leq k$ such that $(0, k')$ is on-path.

Now, define

$$\mathcal{K}_1(k) \equiv \{k' | \mu_1^*(k') = \mu_1^*(k)\}$$

and

$$\mathcal{K}_0(k) \equiv \{k' | \mu_0^*(k') = \mu_0^*(k)\}.$$

Then let

$$\mathcal{K}(k) \equiv \mathcal{K}_1(k) \cap \mathcal{K}_0(k),$$

and note that $k \in \mathcal{K}(k)$ and so $\mathcal{K}(\cdot)$ is nonempty on its domain. Moreover, for each $\mathcal{K}(k)$, there exists some $k' \in \mathcal{K}(k)$ such that either $(1, k)$ or $(0, k)$ is on the equilibrium path.

Fixing the sender's strategy, suppose that rather than observing $a = (m, k)$, the receiver instead observes $\mathcal{K}(k)$. Clearly, this alternative information structure results in the receiver having weakly less information. We claim that it results in the receiver having *exactly the same* amount of information. Indeed, for each $\mathcal{K}(k)$, one of three possibilities obtains.

First, it might be that for all $k' \in \mathcal{K}(k)$, $a = (0, k')$ is off-path and so the receiver infers posterior $\mu_1^*(k')$ when observing $\mathcal{K}(k)$. Second, it might be that for all $k' \in \mathcal{K}(k)$, $a = (1, k')$ is off-path and so the receiver infers posterior $\mu_0^*(k')$ when observing $\mathcal{K}(k)$. Finally, there might exist $k', k'' \in \mathcal{K}(k)$ such that $(1, k')$ and $(0, k'')$ are both on-path. If $k'' \leq k'$, then $\mu_1^*(k) = \mu_0^*(k)$ by the argument above and so the receiver infers posterior $\mu^*(1, k)$ when observing $\mathcal{K}(k)$. If $k'' > k'$, then $\mathcal{K}(k) \subsetneq \mathcal{K}_0(k)$ because $\mu_0^*(k)$ can change only if $(0, k)$ is sent on-path. But then either $\mu_0^*(k) = 0$ for all $k \in \mathcal{K}(k)$, in which case the receiver can infer $\mu^*(1, k)$ when observing $\mathcal{K}(k)$, or $\mu_0^*(k) > 0$ for all $k \in \mathcal{K}(k)$, in which case there exists some $\hat{k} < k'$ with $\hat{k} \in \mathcal{K}_0(k) \backslash \mathcal{K}(k)$ such that $(0, \hat{k})$ is sent on-path. But then $\mu_0^*(\hat{k}) = \mu_1^*(k)$, implying $\mu_1^*(k) = \mu_0^*(k)$ because $\mu_0^*(\cdot)$ is constant on $\mathcal{K}_0(k)$ and $\hat{k} \in \mathcal{K}_0(k)$.

Therefore, we have established that we can treat the sender as communicating $\mathcal{K}(k)$ rather than the argument (m, k) in equilibrium. By the argument above, the posterior induced by $\mathcal{K}(k)$ equals $\mu_1^*(k)$ if there exists a $k' \in \mathcal{K}(k)$ such that $(1, k')$ is on-path and equals $\mu_0^*(k)$ otherwise. Next, we claim that the posteriors induced by $\mathcal{K}(k)$ are increasing in k. Suppose not. Note that $\mathcal{K}(\cdot)$ partitions $[0,1]$, and suppose $k < k'$ are such that $\mathcal{K}(k')$ induces a strictly lower posterior than $\mathcal{K}(k)$. This cannot be the case if $m = 1$ is on-path in both $\mathcal{K}(k)$ and $\mathcal{K}(k')$, since $\mu_1^*(k)$ is increasing in k. Similarly if $m = 1$ is *not* on-path in both $\mathcal{K}(k)$ and $\mathcal{K}(k')$.

Suppose $m = 1$ is on-path in $\mathcal{K}(k')$ but not in $\mathcal{K}(k)$, and $\mu_0^*(k) > \mu_1^*(k')$. But then $\mathcal{K}(k')$ is sent only if $\omega = 1$, and so $\mu_1^*(k') = 1$; contradiction. Suppose instead that $m = 1$ in on-path in $\mathcal{K}(k)$ but not in $\mathcal{K}(k')$, and $\mu_1^*(k) > \mu_0^*(k')$. Then $\mathcal{K}(k')$ is sent only if $\omega = 0$, and so $\mu_1^*(k') = 0$. But then without loss, $\mathcal{K}(k')$ is never sent when $\omega = 0$ either; contradiction. So beliefs are increasing in $\mathcal{K}(k)$. One implication of this result is that we can construct an information structure that is equivalent to $\mathcal{K}(\cdot)$ with a density that satisfies MLRP.[3]

If $\omega = 1$, then $\sup \mathcal{K}(k) \leq \theta_T$ because only $k \leq \theta_T$ are feasible. If $\omega = 0$, then $\sup \mathcal{K}(k) \geq \theta_L$; otherwise, the sender could induce a strictly higher posterior by making some argument with $k \in \mathcal{K}(\theta_L)$. Now, consider the alternative information structure in which the sender chooses $\mathcal{K}(\theta_T)$ if $\omega = 1$ and $\mathcal{K}(\theta_L)$ if $\omega = 0$. This alternative information structure has the property that posteriors are increasing in $\mathcal{K}(k)$ because $\frac{f_1(\cdot)}{f_0(\cdot)}$ is increasing. Moreover, if $\omega = 1$, then the sender chooses a weakly higher $\mathcal{K}(\cdot)$; if $\omega = 0$, then the sender chooses a weakly lower $\mathcal{K}(\cdot)$. Hence, this alternative information structure entails in a FOSD shift upwards of the marginal distribution over $\mathcal{K}(\cdot)$ if $\omega = 1$, and a FOSD shift downwards of the marginal distribution over $\mathcal{K}(\cdot)$ if $\omega = 0$. Consequently, it is Lehmann more informative.

The garbling $k \to \mathcal{K}(k)$ transforms the information conveyed in the earnest equilibrium to that of this alternative information structure. Therefore, the earnest equilibrium is Blackwell more informative than the alternative information structure. Hence, the earnest equilibrium is Lehmann more informative than an arbitrary PBE. ■

[3] For example, consider the signal structure in which, whenever the receiver would observe $\mathcal{K}(k)$, they instead observe some $k' \in \mathcal{K}(k)$ drawn uniformly at random from $\mathcal{K}(k)$. This equivalent signal structure has a distribution that is differentiable almost everywhere and satisfies MLRP.

B.2 PBE in a Simple Example

This appendix characterizes the full set of Perfect Bayesian Equilibrium payoffs in a simple example of the communication model from Section 4. This analysis shows how the refinement to earnest equilibrium constrains the set of equilibrium payoffs in an example that does not satisfy Assumption 1.

Suppose that sender's rhetorical ability is one of three types, $\Theta = \{(1,0),(1,1),(2,1)\}$, where $F(\cdot)$ has full support on Θ. We refer to a sender with ability $(1,0)$, $(1,1)$, and $(2,1)$ as a **normal**, a **BSer**, and an **orator**, respectively. Let $\rho(\theta)$ be the probability that $\theta \in \Theta$ is realized, with marginals $\rho^T(\cdot)$ over θ_T and $\rho^L(\cdot)$ over θ_L. In this example, $2 > \max_{\theta\in\Theta}\theta_L$, so an argument with $k = 2$ can be made only if $\omega = 1$ and so reveals that the state is high. More generally, the stronger an argument, the more likely it is to be the maximum feasible argument under truth-telling in this example. Therefore, by an argument analogous to that of Proposition 5, there exists an essentially unique earnest equilibrium in this setting.

Proposition 9 *In this example, define*

$$\mu^* = \frac{q\rho^T(1)}{q\rho^T(1) + (1-q)\rho^L(1)} \in (0,1).$$

In any earnest equilibrium, high-state orators choose $a = (1,2)$ and induce posterior $\mu = 1$, low-state orators, low-state BSers, and high-state normals choose $a = (1,1)$ and induce posterior $\mu = \mu^*$, and low-state normals induce posterior $\mu = 0$.

Proof: Suppose $\omega = 0$. If $\theta = (1,0)$, then the sender's feasible arguments are $\{(1,0)\} \cup \{(0,k)|k \leq 1\}$, none of which are sent if $\omega = 1$. So a normal must induce posterior $\mu = 0$ in any earnest equilibrium. If $\rho^T(1) > 0$, then $a = (1,1)$ induces a strictly positive posterior in equilibrium, while any $a \in \{(1,k)|k < 1\} \cup \{(0,k)|k \leq 2\}$ induces posterior 0. Therefore, the orator and BSer must both make argument $a = (1,1)$ when $\omega = 0$, which induces posterior μ^* in equilibrium.

It is straightforward to show that no sender has a profitable deviation from this strategy. So every earnest equilibrium must satisfy the desired properties. ■

The essentially unique earnest equilibrium in this setting conforms closely to the earnest equilibria in smooth games with increasing $\mu(\cdot)$. Next, we show that there exists an essentially unique PBE that is **not** earnest in this setting. To do so, we impose the (mild) condition that if $\theta_L < k$ for all θ in the support of $F(\cdot)$, then the receiver's posterior if he observes $a = (m, k)$ equals $\Pr\{\omega = m|a\} = 1$.

Proposition 10 *In this example, define*

$$\bar{\mu} = \frac{(1 - \rho(2,1))q}{1 - \rho(2,1)q} \in (0, q).$$

In any PBE satisfying $Pr\{\omega = 1|(1,2)\} = 1$, either (i) the equilibrium mapping from (θ, ω) to the receiver's posterior belief is identical to an earnest equilibrium, or (ii) the receiver's posterior belief equals $\bar{\mu}$unless $\theta = (2,1)$ and $\omega = 1$, in which case $a = (1,2)$ and $\mu = 1$.

Proof: Any $a = (m, k)$ with $k \in (1, 2]$ can be sent only if $\theta = (2, 1)$ and $\omega = m$, and similarly any $a = (m, k)$ with $k \in (0, 1]$ can be sent only if $\theta \in \{(1, 1), (2, 1)\}$ or $\theta = (1, 0)$ and $\omega = m$. Therefore, for the purposes of identifying equilibrium beliefs, we can restrict attention to equilibria in which any on-path arguments satisfy $k \in \{0, 1, 2\}$.

By our definition of PBE, $a = (1, 2)$ must induce posterior $\mu = 1$ in any equilibrium. Let $\{\mu^1, ..., \mu^L\}$ be the set of posteriors induced on the equilibrium path, where $\mu^1 < ... < \mu^L$. If $\omega = 1$ and $\theta = (2, 1)$, then $a = (1, 2)$ is feasible and induces posterior $\mu = 1$. So $\mu^L = 1$ and hence $L > 1$. Any other argument is feasible when $\omega = 0$ for at least one ability type, so no other argument can induce posterior $\mu = 1$.

If $\omega = 1$ but $\theta \neq (2, 1)$, then $a = (1, 2)$ and hence $\mu^{L-1} > 0$. If $\omega = 0$ and $\theta = (2, 1)$, then the sender can make the argument that induces μ^{L-1}, so she never chooses $a = (0, 2)$ because doing so would induce posterior 0. But then a sender with $\theta = (1, 1)$ can also induce posterior μ^{L-1}, regardless of ω.

Suppose $L = 2$. Then $\mu^{L-1} = \Pr\{\omega = 1|(\theta, \omega) \neq ((2,1),1)\} \equiv \bar{\mu}$. Such an equilibrium always exists. For example, consider the following strategy profile. If $\theta = (1,0)$, then $a = (1,0)$ for any ω. If $\theta = (2,1)$ and $\omega = 0$, or $\theta = (1,1)$, then the sender mixes over $a = (1,0)$ and $a = (1,1)$ with probability such that both arguments induce posterior $\bar{\mu}$. Note that the average posterior induced by $a = (1,0)$ and $a = (1,1)$ must equal $\bar{\mu}$. Therefore, such a mixture is always possible by the Intermediate Value Theorem, since $a = (1,1)$ induces posterior q if $\theta = (1,1)$ always makes argument $a = (1,1)$ and $\theta = (2,1)$ never does, and equals 0 if $\theta = (1,1)$ always makes argument $a = (1,0)$.

Now, suppose that $L \geq 3$. We claim that $\mu^{L-2} = \mu^1 = 0$, so this equilibrium induces the same distribution over posteriors as an earnest equilibrium. First, $a = (m,0)$ cannot induce μ^{L-1} for any $m \in \{0,1\}$, since then no sender would choose an a that induces μ^{L-2}. So μ^{L-1} must be induced by $a = (1,1)$, $a = (0,1)$, or both.

If $\mu^{L-2} > 0$, then any a that induces μ^{L-2} must be sent with positive probability if $\theta = (1,0)$ and $\omega = 1$. So $a = (1,1)$ cannot induce μ^{L-2}, since a sender with $\theta = (1,0)$ and $\omega = 0$ cannot choose $a = (1,1)$, no other θ induces posterior μ^{L-2}, and $\mu^{L-2} < 1$. So $a = (0,1)$ must be the only argument that induces μ^{L-2}. Since $a = (0,2)$ is never sent on the equilibrium path, some $a = (m,1)$ must induce μ^{L-1}. Then a sender with type $\theta = (1,0)$ induces posterior μ^{L-1} when either $\omega = 1$, in which case the mapping from (ω, θ) to posterior beliefs is identical to an earnest equilibrium, or $\omega = 0$, in which case μ^{L-2} is induced only if $\omega = 1$ and hence $\mu^{L-2} = 1$, a contradiction. This argument proves the claim. ■