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Using the PIAAC Numeracy Framework to Guide Instruction: An Introduction for Adult Educators

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Using the PIAAC Numeracy Framework to Guide Instruction:

An Introduction for Adult Educators

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Donna Curry

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I. Introduction

This guide is designed for practitioners, including teachers, lead instructors, and professional developers, who are ready to take up the challenge to change how numeracy is taught in adult education. If instructors shift their teaching to focus more on the use of numeracy skills, they might find that their students not only score better on formal assessments but that they are more effective in using numeracy in their daily lives.

Situations involving numeracy are embedded in our daily lives. From the minute we wake up we make multiple decisions each day based on numeracy skills such as measurement, data, patterns, and number – decisions like figuring whether it is cheaper to bring lunch or eat out to deciding what to cook for dinner all involve numeracy. Considering the importance of numeracy in our daily lives, it would seem that adults should be fairly proficient at numeracy-related tasks. However, that does not appear to be the case.

The numeracy results from the Program for International Assessment of Adult Competencies (PIAAC), an international assessment of adult competencies conducted in the U. S. in 2011-12, (and additional National Supplement was conducted in 2013 – 14 which included adults 66 – 74 and incarcerated adults) confirm what many of us in adult education already know: too many adults in the U.S. do not have the numeracy skills they need to keep up with the demands of twenty-first century life. The ongoing PIAAC study, which defines numeracy as "*the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations"* is being conducted under the auspices of the Organization for Economic Cooperation and Development in the U.S. and (to date) 32 other developed countries. Key findings of the study include the following:

- The U. S. average numeracy score (257) was lower than the international average (269). Incarcerated adults in the U. S. scored even lower – an average of 220.
- U.S. adults at every education level are below the international average in numeracy.
- U. S. employed adults had lower average numeracy scores than their peers internationally.
- U. S. adults in every age group scored below the international average for their age group in numeracy.
- U. S. adults in every income level scored below the international average for their income group in numeracy.

While global competition for jobs may be enough to raise concerns about our performance relative to adults in other countries, it is not just that U.S. adults of every age between 16 and 74 and income and education level scored below their peers in other countries. It is that their absolute performance – their ability to access, use, interpret and communicate mathematical information and ideas – is not sufficient to enable them to carry out their everyday responsibilities – at home, in the community and in the workplace.

According to PIAAC results, 9% of adults in the U.S. can only carry out very simple, concrete numeracy tasks like counting or one-step arithmetic operations with whole numbers. An additional 20% are limited to tasks that are only a little more complicated such as understanding a simple percent (e.g., 50%). And more than 36% of adults can only perform tasks that involve simple measurement, estimation, and interpretation of simple graphs and tables. In a world where we are required to solve challenging problems, make decisions and predictions based on patterns, think proportionately, and understand statistical information, we desperately need to strengthen and enhance the numeracy skills of this 65% of our adult population.

What Can We Do to Address this Need?

PIAAC not only helps us understand the scope of this problem, it provides us with tools that may enable us to address the challenge of fixing it. In developing a conceptual framework that would enable countries around the world to accurately measure how well adults could engage in numerate behaviors, the PIAAC Numeracy Expert Group focused on defining the skills and competencies required by adults to cope with tasks that are likely to appear in the adult world and that contain mathematical or quantitative information, or that require the activation of mathematical or statistical skills and knowledge.

These skills and competencies align with the adult education content standards that are being used across the country. The *College and Career-Readiness Standards for Adult Education* (CCRS) have been adopted in an effort to prepare adult learners in the U. S. for the skill demands of the 21st century. The standards articulate the mathematics skills and knowledge required to succeed in three broad areas: 1) entry-level positions of promising careers, 2) academic college courses and workforce training programs, and 3) activities required of active citizens in a demanding democracy.

In 2014, Congress passed the Workforce Innovation and Opportunity Act (WIOA) as the primary legislation directing adult education activity nationally. Together, WIOA and the CCRS provide guidance for the adult basic education field to ensure adult learners develop skills to help them succeed in college and careers. The PIAAC framework provides a set of useful tools that practitioners can use to move the WIOA and CCRS agenda forward.

The PIAAC Conceptual Framework for numeracy is based on two key concepts: 1) numeracy as being use-oriented, and 2) proficiency as a continuum. As stated above, PIAAC defines numeracy as *the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.* In other words, in PIAAC adults are expected to do something with the mathematical information they interact with. And they are expected to interact with mathematical information and ideas at all levels. The PIAAC framework includes cognitive elements (such as math skills and knowledge), and non-cognitive elements (such as attitudes, beliefs, and dispositions) that help describe what is required of an adult to use math. It includes enabling factors and also includes a set of task characteristics that enable adult educators to construct concrete learning activities that move adults along a continuum of expertise from very concrete to the abstract, from simple counting of items in a picture to abstracting and making decisions from a complex graph. Adult educators can use these concepts to develop approaches to teaching numeracy that focus on application and help adults to handle numerate activities by providing progressively challenging tasks.

The PIAAC numeracy assessment framework can be used to guide how to approach numeracy teaching so that achieving higher results - whether from a high-stakes test, a college placement test, a workforce assessment, or an international test– will be possible. More important, the results from this approach to teaching include making sure our students have the skills they need to use numeracy to carry out important tasks in their daily lives.

This guide will first examine each of the PIAAC concepts separately, focusing specifically on the definition of numeracy and the elements that help adults use mathematical information. Then we will integrate these ideas to consider how to create numeracy tasks so that our students can actually use the skills that they are learning. Before we get to the details of how these elements can impact classroom instruction, we will take a brief look at what PIAAC is.

II. Background of PIAAC

PIAAC is a policy-driven initiative meant to provide policy makers and key stakeholders at the national and international levels with information that can inform policy and planning of social interventions and programs. PIAAC has two predominant goals: to identify and measure differences within and across countries in literacy, numeracy, and digital problem-solving competencies for the information age, and to assess the relationship of these adult competencies to economic and social outcomes believed to underlie both personal and societal success.

One of the key features of the PIAAC framework is its use-oriented conception of competency. The PIAAC approach is focused not just on building skills but on *using* skills. In order to provide a richer context for understanding the level and distribution of skills among adults in the U.S., the PIAAC Background Questionnaire collected information from each respondent in five main areas:

- basic demographics of respondents;
- educational attainment and participation;
- labor-force status and employment;
- social outcomes; and
- literacy and numeracy practices and the use of other 21st century skills.

To help us better understand respondents' numeracy practices, the Background Questionnaire gathered information from respondents on their numeracy-related activities at work and in everyday life. In addition, respondents were asked whether their skills and qualifications match their work requirements and whether they have

What is PIAAC?

In 2013 The Organization for Economic Cooperation and Development (OECD) released the first results of a multi-cycle program of assessment of adult skills – the Program for International Assessment of Adult Competencies (PIAAC). Twenty-four countries – including the U.S. and most other developed countries in the world – participated in the first round of this assessment in 2011-12, which was designed to give countries critical information on how well-prepared their adult residents were to participate fully in the civic, cultural and economic life of their countries in the 21st century. In addition to assessing three key information-processing skills -- literacy, numeracy, and problem solving in technology-rich environments – PIAAC included a skills use module, which collected information from each participant on additional skills used in the workplace, including communication, interpersonal, problem-solving and learning skills, as part of an extensive background questionnaire, which collected information on education and work history, in addition to demographic data, that would help each country understand the range and distribution of skills among its adult population so that it could use the assessment data to make important policy decisions about the best ways to improve adult skills. Taken together, these features of PIAAC make it the most comprehensive assessment of adult skills undertaken to date.

autonomy over key aspects of their work. This type of information enables us to understand more about the relationship between skill level and skill use. In fact, the data from the Background Questionnaire informed the new WIOA at the policy level. This focus on how adults use their mathematical skills is something that would be very useful for every math teacher to integrate into their teaching.

The PIAAC Background Questionnaire (BQ) posed questions about contexts in which adults used numerate behavior. Questions included those such as:
In your current or past job (similar questions are repeated for the home context), how often (never, less than once a month, less than once a week, everyday) do you usually…

- *Calculate prices, costs, or budgets?*
- *Use or calculate fractions, decimals, or percentages?*
- *Use a calculator – either hand-held or computer based?*
- *Prepare charts, graphs, or tables?*
- *Use simple algebra or formulas? [For clarification, the BQ included this definition: By simple algebra or formula, we mean a mathematical rule that enables us to find an unknown number or quantity, for example a rule for finding an area when knowing length and width, or for working out how much more time is needed to travel a certain distance if speed is reduced.]*
- *Use more advanced math or statistics such as calculus, complex algebra, trigonometry, or use of regression techniques?*

There is no reason that adult education instructors could not ask similar questions of their own students in order to determine the types of numerate behaviors they already engage in. (These same questions, by the way, can apply to different contexts – society and community, education and training, and students' personal lives). Or, if students are not yet in the workplace, instructors might think about how to create tasks so students have opportunities to become numerate in workplace situations.

Teachers might want to consider discussions about where their students apply numerate behavior as part of their opening activities each semester. Perhaps if students realized that they do engage in numerate tasks on a daily basis, they might be more convinced about the need to become more numerate. The teacher could use a variety of strategies to accomplish this. For example, students could interview each other about where they use math in their lives or how they addressed a situation where math was needed. When they realize how integral math is to their daily lives, students themselves may start to nudge their teachers to teach for application.

Now let's take a closer look at the way in which the PIAAC Numeracy Expert Group defined numeracy and numerate behavior. In section IV we will look at how the PIAAC Numeracy Expert Group defined and used complexity criteria to make tasks easier or more challenging.

III. PIAAC's Definition of Numerate Behavior

In this section, we will first look at PIAAC's definition of numeracy. Since numeracy requires the use of numerate behaviors, we will examine these closely, focusing on the four facets of numerate behavior along with the enabling factors and processes that help an individual successfully manage situations involving numerate behavior.

Defining Numeracy

The PIAAC Numeracy Framework was developed by an international group of experts in numeracy tasked with defining the skills and competencies required by adults to cope with tasks that are likely to appear in the adult world and that contain mathematical or quantitative information. Their numeracy definition aligns with the Adult Literacy and Lifeskills (ALL) survey and is similar to that used in the Program for International Student Assessment (PISA). It provides the basis for examining competencies in the information age. In PIAAC, numeracy is evidenced in situations that have mathematical components or involve quantitative information. Numeracy, therefore, requires cognitive elements (such as math skills and knowledge) and non-cognitive elements (such as attitudes, beliefs, and dispositions) (*Conceptual Framework*, p. 10).

PIAAC defines numeracy as follows:

> *Numeracy is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.*

A key verb in this definition is *engage*. For one to engage with mathematics suggests that the individual has a disposition that allows him or her to address a mathematical situation. This is a much broader definition of numeracy than a set of procedures to memorize or skills that are taught in isolation in order to pass high stakes assessments. As a result, for some, this definition of numeracy will require that we think quite differently about what we typically teach in adult education classes.

Defining Numerate Behavior

PIAAC's definition of numerate behavior will resonate with those practitioners who are familiar with the National Council of Teachers of Mathematics (NCTM) Process and Content Standards and the *College and Career Readiness Standards for Adult Education*, especially the Standards for Mathematical Practice (p. 48).

In order to better understand numeracy, we need to consider the actions that suggest someone is behaving in a numerate manner. Just as literacy requires literate behaviors, so does numeracy require numerate behaviors. According to PIACC, numerate behavior involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways (Conceptual Framework, p. 21).

Table 1: Numerate behavior – key facets and their components (from Conceptual Framework, p. 21)

Numerate behavior involves managing a situation or solving a problem… 1. in a real context: – everyday life – work – society – further learning 2. by responding: – identify, locate or access – act upon and use: order, count, estimate, compute, measure, model – interpret – evaluate/analyze – communicate 3. to mathematical content/information/ideas: – quantity and number – dimension and shape – pattern, relationships, change – data and chance 4. represented in multiple ways: – objects and pictures – numbers and mathematical symbols – formulae – diagrams and maps, graphs, tables – texts – technology-based displays
5. Numerate behavior is founded on the activation of several enabling factors and processes: – mathematical knowledge and conceptual understanding – adaptive reasoning and mathematical problem-solving skills – literacy skills – beliefs and attitudes – numeracy-related practices and experience – context/world knowledge

Key to the definition of numeracy is the phrase "managing a situation or solving a problem." Clearly, in order to do so, an individual must use math; he must apply what he knows about math concepts and skills in order to deal with an issue.

These facets of numerate behavior listed above were key to the development of the PIAAC assessment framework and the development of PIAAC assessment tasks. They also should be considered key to numeracy instruction in the adult education classroom. The four facets - contexts, responses, mathematic content/ information/ ideas, and representations – should be fundamental in guiding teachers in using (or creating) instructional activities that move students toward increased numerate behavior that they can apply in multiple contexts.

Let's take a look at each of these a little more closely.

- **Facet 1: Contexts.** In our everyday life we encounter many numeracy tasks, some simple and some more complicated. The tasks may come from work, society and community, education and training, or from students' personal/family lives. An effective teacher is someone who can point to many instances of these tasks to show her students the prominence of numeracy tasks in everyday life -- or to ask her students to point out numeracy tasks and their importance in their own lives. The teacher needs to be a good listener at times to tune in to contexts important in students' lives. This is critical since students' needs and concerns may be different from those of their teachers. For example, students may be struggling to cope from weekly paycheck to weekly paycheck, or to cope with stressful family situations, or a history of incarceration.

In an adult education classroom, the contexts might overlap for students with different levels of skills. Childcare, for example could be a context that applies to all levels of learners; it is part of adults' personal lives. If students are working – or preparing for a particular kind of work – the context can readily come from situations that they could encounter on the job. Students ready to take a high-stakes assessment may need to also have contexts that they will encounter in those test situations as they prepare for further learning in college or training.

- **Facet 2: Responses.** The ways in which adults react to mathematical tasks or information are shaped by their purposes for using those data or information. PIAAC classified these purposes into three groups of cognitive strategies: those required to identify, locate, or access mathematical information; those required to act upon or use mathematical information; and those required to interpret, evaluate, analyze, or communicate mathematical information.

  In almost any mathematical task, adults need to identify, locate, or access information in order to address the situation. This could be as simple as finding the price tag on a piece of clothing to determine its cost, or checking the estimated area that a can of paint will cover.

  However, locating information isn't always as easy as reading a label. Information can be more challenging to access or locate, such as scouring through a set of data to look for patterns or relationships or to determine what is needed to make a point. (In section IV of this document you will see that the ease of locating or accessing information influences the difficulty of the task.)

  Often, the response of identifying, locating, or accessing information is performed in tandem with one of the other two sets of cognitive strategies, particularly when the purpose of the task is not simply finding or locating information. Once an adult has located information, he may use that information in different ways such as ordering, counting, estimating, computing, measuring, or modeling. For example, once someone finds the price tag on an article of clothing, he will need to decide whether the price is within his budget or may want to add that price to other items to estimate the total cost. Or, once an individual finds how much area a can of paint will cover, he will need to determine the area of the room to be painted and then will have to figure out just how many cans of paint he should buy. In fact, in order to determine the area to be painted, the individual might need to measure the room, then use a formula to determine the area of each wall.

The third group of responses involves interpreting, evaluating/analyzing, and communicating. This group also usually is used in connection with at least the first type of response – locate, identify, and access – since clearly an individual has to have information to interpret or communicate. When a situation does not require any direct action or manipulation of information, an individual may only have to interpret that information. The interpretation may include judging or giving opinions. For example, in a simpler task, an individual might need to locate the scale on a map, then estimate the length of time to get from one place to another. Tasks requiring the interpretation of data from graphs vary in difficulty depending on the type of graph (simple bar graph to very complex graphs illustrating lots of data) and question to be answered (such as how the categories compare vs. what patterns or trends the graph suggests).

The response of evaluate and analyze requires an interpretation but is based on some criteria or demands. A task requiring evaluation and analysis would be a high level situation where an individual might review a huge set of data to determine whether the data are valid, or needed for the situation, or include gaps. (Evaluate/analyze is lumped together for the PIAAC assessment analysis.)

This third group of responses also includes communication. Communication could include a description of how one arrived at a conclusion, or a justification of the reasoning used in an analysis or interpretation. Communication could be oral or written; written communication could be through a drawing or graph or map, or even a computer-generated display such as a spreadsheet. A simple task might require someone to draw a picture to represent the total number of bottles stocked on a shelf. A higher-level task might require an individual to give an oral presentation justifying why one situation is better than another.

These three types of responses are usually used in various combinations, depending on the numeracy task at hand. Over time, the teacher needs to pay attention to the types of responses she is asking her students to provide so that they have practice not just with locating or acting upon mathematical information but also with evaluating the information and communicating about it.

- **Facet 3: Mathematical Content/information/ideas.** Mathematical information has been classified in different ways. The College and Career Readiness Standards for Adult Education (CCRS) use domains to describe mathematical content. The National Council of Teachers of Mathematics (NCTM) uses five strands: numbers and operations, functions, relations, and algebra, data, geometry, and measurement. In PIAAC, mathematical content and ideas are classified as quantity and number, dimension and shape, data and chance, and patterns, relationships and change. In the table below, you see how these different approaches align.

Table 2: Mathematical Content

Mathematical Content/Information/Ideas Classifications		
(NCTM)	(CCRS) (domains for K – 8 only)	PIAAC
Numbers and operations	Number and operations in base ten (K – 5) The number system (6 – 8) Number and operations – fractions (3 – 5) Ratios and proportional relationships (6 – 7)	Quantity and number
Data	Statistics and probability (6 - 8)	Data and chance
Measurement	Measurement and data (K – 5)	Dimensions and shape
Geometry	Geometry (K – 8)	
Functions, relations, and algebra	Operations and algebraic thinking (K – 5) Expressions and equations (6 – 8) Functions (8)	Patterns, relationships and change

Table 3 below further details the content specific to the PIAAC framework.

Table 3: Mathematical content/information/ideas (adapted from *Conceptual Framework*, pp. 27 – 28)

Mathematical content/ information/ ideas	Description	Sample types of tasks
Quantity and number	*Quantity* is using attributes that allow individual to quantify the world (such as cost, temperature, growth rate). *Number* is fundamental to quantification: whole numbers; fractions, decimals, percents; positive and negative numbers. In addition to quantification, numbers are used to put things in order and as identifiers (e.g., telephone numbers or zip codes).	A numeracy task with less cognitive demand might be figuring out the cost of one can of soup, given the cost of 4 for $2.00. A task with a higher cognitive demand could involve figuring out the cost when buying 0.28 pound of cheese at $4.95 per pound.
Dimension and shape	*Dimension* includes 'big ideas' for 1-, 2-, and 3-dimensional items. *Shape* describes real 2- and 3- dimensional images such as a house or sign.	A task with lower cognitive demand could be shape identification. A higher level could involve a description of the change in capacity of an object when one of its dimensions is changed.
Pattern, relationships, and change	*Pattern* relates to those patterns seen around us (including music, nature, etc.) *Relationship* and *change* refers to how things in the world are related or develop.	A task with lower cognitive demand may ask someone to describe the simple pattern in how bottles are stocked on shelves. A task with a much higher cognitive demand would require the use of spreadsheets to compare rates.
Data and chance	*Data* involves ideas such as variability, sampling, data collection, etc. *Chance* relates to probability and relevant statistical methods.	A task with lower cognitive demand might be the interpretation of a simple pie chart. A task with a higher cognitive demand could involve determining the likelihood of an event occurring, based on past information.

In the approach to these content areas, teachers need to be aware of how students might need to use this content. It's not enough to teach rote procedures in a particular content area; students need to know how they are going to use those procedures in order to accomplish a task. As noted above, at all levels of tasks, individuals are expected to interact with the content. At the most basic level a student learns about quantity and number so he can address a question or situation that has a mathematical component to it. It could be as simple as someone trying to figure out if he has enough money to buy several items. The situation requires the use of quantity and number – a clear purpose within a real-life situation.

- **Facet 4: Representations of mathematical information.** Mathematical information can be represented in various formats. We often think of mathematical information being presented as numbers and symbols, including formulae. For example, the idea of five can be represented using the symbol 5, or the Roman numeral V or even four tick marks with a slash mark through them (~~IIII~~). Information can be represented by simply using objects to be counted. A simple picture could also be used by an individual to count or address a question. Mathematical information can be represented using visual displays such as charts, graphs, tables, and maps, including technology-based displays. And mathematical information can be represented by text. For example, five is the text version that corresponds to 5 and V.

  It is important for teachers to be aware of different representations so they make sure they include them all as they work with students. They can differentiate tasks in their classroom by varying the type or representation required of students, or that students use in addressing a situation with a mathematical component. For example, teachers might present data in chart form, or in a table where students have to extrapolate the necessary information in order to address a task. And, teachers might ask their students to represents the results of their task in a chart or graph or even an equation. Students need to be exposed to these various representations and explicitly taught how to identify and get to the math sitting behind the real world stimulus that has math embedded in it.

How the Facets of Numerate Behavior Work

The four facets of numerate behavior are all represented in each task identified in the BQ and discussed earlier (e.g. calculate prices, costs, or budgets, use or calculate fractions, decimals, or percentages, use a calculator, prepare charts, graphs, or tables). And, these four facets also can be part of learning about how to use math for adults at all levels. Let's look at some examples of how these four facets could play out in the classroom.

Scenario 1: Example A – An intermediate level class

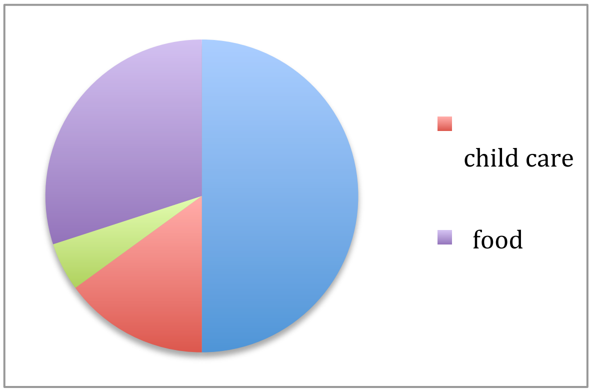
<table>
<tr><td colspan="2">Childcare: A Look at the Four Facets of Numeracy
Example A
Unit Goal: By the end of this unit, students will be able to apply their understanding of percentages and circle graphs in order to create two different budgets represented as circle graphs and explain how they are different.</td></tr>
<tr><td>Val, the adult education teacher, has been hearing bits and pieces from her students about their childcare issues. And, she has noted that several times students have missed class because of issues related to childcare.</td><td>Context: Childcare, an issue for students</td></tr>
<tr><td>Val asks her students to bring to class any information that they can about the various childcare offerings in the area. She also collects some information herself so that students have a variety of options to consider as they explore the issue in detail in class.</td><td>Representations: The teacher expects students to use a variety of information from different sources – textual or graphical representation embedded in various brochures and other childcare information provided through a phone conversation or from a web search. She also encourages students to make connections between fractions and percentages, which also is a way to reinforce the fact that mathematical information takes many forms, and that the same information can be represented in different ways.</td></tr>
<tr><td>Val's students have been working on percentages, so she uses the topic of childcare to focus on a big idea about percentages and pie charts – that when the whole changes, it impacts the individual 'slices' or parts of the whole. Val gives students time to explore some lessons on percentages and graphing.

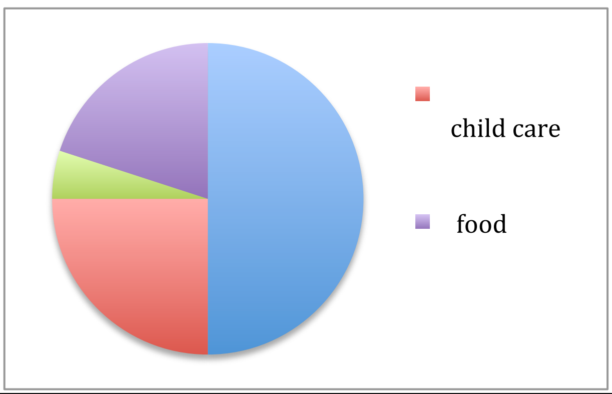
Val then asks students to create a pie chart of their present estimated monthly budget. She asks students to estimate their monthly budget (she also provides some sample realistic budgets for those students who do not have a sense of their monthly income and outgo, or who do not want to use their own budget estimates). Depending on the students' comfort level with graphing, she has them limit the number of categories (rent, childcare, and 'other' for those less comfortable with graphing).</td><td>Mathematical content: The teacher knows that her students have had some exposure to percentages and to graphing and she wants to continue to focus on these ideas. She wants students to understand that when the whole (100%) changes it impacts the parts. She is also linking percentages to graphs.</td></tr>
<tr><td>Students then choose one of the childcare options, review the details of the option they have chosen, and then estimate the monthly cost of the service. Val asks them to first predict how the new childcare option will impact their monthly budget. She asks them, "What do you think your new pie chart will look like when you change your present monthly childcare estimate with the new one?" She also asks, "If some categories of expenses (such as rent) cannot change, what might the new pie chart look like? Why?"

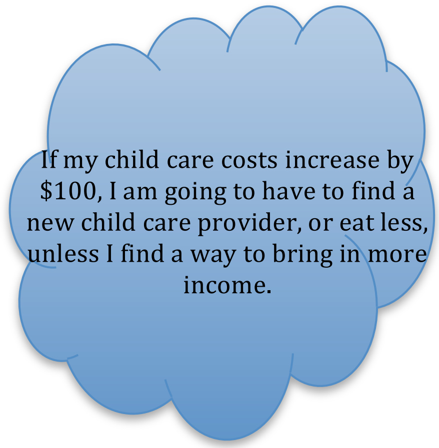
Val takes time to teach her students how to use Excel to create their pie charts after having them sketch one by hand. Students create pie charts based on their estimated childcare costs in relation to their other monthly expenses. Each student explains, based on the data alone, which</td><td>Response: Students must draw on all three types of responses: accessing childcare information, using childcare information to create a pie chart, analyzing and evaluating the information in order to make a comparison based on monthly budgets, and communicating their reasoning to others, explaining which childcare option seems to be the best fit,</td></tr>
</table>

childcare situation seems to be best suited based on their monthly budget estimates. Then they share what they discovered about how changes to one expense impact what money is left for other expenses, especially when the total monthly income does not change.

Here's a sample of how a student might begin to respond at this level:







In creating an activity that focuses on the four facets above, the teacher is doing more than helping her students gain skills in mathematical content areas (such as those related to percentages and graphs) she is helping them build the knowledge and skills that enable them to apply those skills and knowledge in a variety of situations, including but not limited to high-stakes tests. She is giving students opportunities to use a variety of mathematical representations; she is having them locate information, then interact with that information and evaluate the results. By using computer applications to address a mathematical situation she is also preparing them for a more active role in the technology age.

Teachers adopting this use-oriented approach to numeracy instruction still have to teach individual skills, strategies, math concepts, and specific content knowledge (including vocabulary), but the goal of their instruction will be that students can actually apply that new learning. Teachers will assess more than just whether a student can follow a particular procedure or strategy; they need to assess whether a student knows which strategy might be most appropriate in a given situation, and if a student knows *when* to use a specific process or operation. Rather than checking understanding by asking students to compute decontextualized problems (often already with the appropriate operation to perform), teachers need to give students the opportunity to figure out which of those operations to use in a given situation. Students do not get this experience if they are only focused on procedures without context. To make sure that students are prepared to succeed on such an applied assessment teachers should make sure students have opportunities to apply their learning and to help them develop numerate behaviors so that they actually understand what they are doing and why.

In the scenario above we saw that Val developed a unit goal: *By the end of this unit, students will be able to apply their understanding of percentages and circle graphs in order to create two different budgets represented as circle graphs and explain how they are different.* Her goal statement makes explicit how students will be expected to demonstrate that they can use the math content being taught.

Enabling Processes

Along with the four facets of numerate behavior, the PIAAC Numeracy Expert Group identified several enabling processes that adults need to activate (and teachers need to help develop in their students) in order to engage in numerate behaviors. These enabling factors and processes influence the successful completion of numeracy tasks. Without these processes, adults would have difficulty addressing situations that have a mathematical aspect to them. These processes include context/world knowledge, mathematical knowledge and conceptual understanding, adaptive reasoning and problem-solving skills, beliefs and attitudes, numeracy-related practices and experiences, and literacy skills. While described individually below, these enabling processes work together as an individual tackles a numeracy task.

- **Context/world knowledge.** In order to interpret mathematical information, individuals must have a sense for the context by accessing their world knowledge and personal experiences. World knowledge is critical to sense-making of mathematical information adults are interacting with. Being able to think critically about statistical claims about data requires that individual to have the background knowledge and experience to make comparisons. For example, there are many statistical claims made during election years. Students need to understand what seems reasonable, even making the effort to learn more in order to judge the veracity of the claims.

  In the classroom scenario presented earlier, Val chose the issue of childcare because she had heard her students discussing it and knows that those students who have personal experience and knowledge of approximate cost and quality of daycare center are in a better position to evaluate their options than those who do not have this background.

- **Mathematical knowledge and conceptual understanding.** PIAAC refers to conceptual understanding as an integrated and functional grasp of mathematical ideas (*Conceptual Framework*, p. 29). Terms such as *meaning making* and *relationships* are synonymous with conceptual understanding. Without conceptual understanding, adult learners will continue to have to memorize procedures over and over again. Conceptual understanding can help individuals make reasonable estimates and rely more on sense making rather than rules and procedures. Unfortunately, there is no quick way to teach for understanding. Students will need many opportunities to explore and to make connections among math ideas.

In the previous scenario, Val understood that in order for her students to realize that changing one part of the whole influences other parts, they will need to understand the concept of percents and how a pie chart graphically represents that concept. This might require Val to back up and ensure that her students understand what a percent means, not just the process for finding the part, whole, or percent. As part of building her students' conceptual understanding, she will want to make sure that they connect the concept of percents to proportional reasoning.

- **Adaptive reasoning and problem-solving skills.** Adults have a variety of strategies for solving problems - some learned through formal schooling and other developed informally or intuitively over time. Problem-solving strategies may include, e.g., extracting relevant information from the task; rewriting or restating the task; drawing pictures, diagrams or sketches; guessing and checking; making a table; or generating a concrete model or representation (*Conceptual Framework*, p. 29). The more connections students are able to make among big math ideas, the more they will be able to adapt their reasoning; they will be able to use different strategies to work through situations.

  In the scenario with Val, the teacher wants to have students explicitly discuss the strategies they use in order to continue to build reasoning and problem-solving skills. For example, they may have learned one way to calculate a percent in school but use another strategy when they are shopping. Val will want to have students explicitly share their various strategies and then ensure that students see how all the various strategies are connected. Students may need to explore, for example, that there are many ways to figure out 50% of an amount. Students need to reason about these different ways so that they can reason adaptively, i.e., consider which strategy might make the most sense in a given situation and decide whether the calculation makes sense. For example, a student may discover that it is efficient to divide by 2 when figuring out 50% of 480, yet use a calculator to determine 27% of 479. But, with conceptual understanding of benchmark percents, students will be able to reason whether the answer they arrived at using the calculator is correct. (Students may reason that 25% of 480 is 120, so 129.33 is a reasonable answer while 1293.3 is not.)

- **Beliefs and attitudes.** How an individual responds to a numeracy task depends on attitudes, beliefs, habits of mind, and prior experiences, not just knowledge and skill. The belief that one is not good with numbers, for example, affects how someone reacts to a task involving math. Such negative attitudes and beliefs about numeracy tasks are often exemplified in the adult education classroom. Productive disposition, the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (National Research Council, 2001, p. 116), is key to being able to stick it out long enough to work through a situation involving numerate behavior. In real-life situations, adults with negative attitudes and beliefs about math often seek the help of others, thereby avoiding engagement in mathematical tasks.

Addressing adult learners' attitudes about math to help them build a productive disposition is possibly the greatest challenge for teachers.

Val may need to work with students to help them experience success – to believe that they can 'do math' as they begin to understand the math that they are learning. This may require a great deal of discussion over time as students slowly overcome any past negative experiences with math.

- **Numeracy-related practices and experiences.** Mathematical knowledge and skills develop through formal schooling and informally throughout life. The frequency with which individuals engage in mathematical tasks also influences numerate behavior. The more often adults engage in numeracy tasks and the more varied the tasks, the more likely their attitudes and beliefs will be affected. These processes are closely intertwined.

  This notion of past practices and experiences can also be another challenging factors to address in adult education. Many adult learners have only been exposed to procedures, and that is what they believe math is: "fractions, decimals, percentages, algebra." Mathematical knowledge develops both in and out of school. If students are only exposed to procedures without context, chances are that they will not develop a strong mathematical understanding regarding when and where and how to use math. Adult students may need to have classroom experiences different from what they have had in the past; teachers may need to help students link 'school math' with the math they use in everyday life.

- **Literacy skills.** One form of mathematical information is text - words rather than symbols or numerals; for example, "Seventy-five percent of the voters…" But, more than just words written in place of numerals, words within a mathematical context can be quite challenging. For example, fifty divided *by* vs. fifty divided *into* can stump students, especially if they have not had a great deal of experience with division to draw upon. If students have consistently been exposed to decontextualized division problems that have already been set up for them to compute, students' exposure to the words that express the 'set-up' of the problem may be limited.

  Let's again think about Val's class. Most numbers used in life are embedded in some type of literacy material – perhaps a paragraph in an article, or data from signs, a table, or other information that will require students to read closely. For example, as students read about child care options, they will need to determine whether there are extra charges for late pick-ups, charges for cancellations, better prices monthly vs. yearly, price discounts, etc. Similar to strategies used in a reading class, the teacher needs to ensure that students have the requisite vocabulary for the context as well as the math content. This could entail vocabulary lessons to build the literacy skills students need.

Adjusting Numerate Behaviors for Different Levels of Learners

These enabling processes, along with the four facets of numerate behavior, are needed for all adults engaging in numeracy tasks. The enabling processes are integrated; numeracy tasks require mathematical knowledge and conceptual understanding which also elicit particular attitudes, beliefs, prior experiences, etc. So, the four facets of numeracy interact with the enabling processes to help or hinder a person's ability to successfully address a numeracy task.

Since the four facets of numerate behavior show up at all levels, it is important for teachers to be able to create activities, or tasks, that are at an appropriate level for their students. And, as students develop numerate behaviors, the teachers need to continue to adjust the tasks so that students are progressing in their ability to tackle more and more challenging situations that require numerate behaviors.

Let's look at how our teacher, Val, creates an activity for both lower and higher level students, still ensuring that students have to engage in all aspects of numerate behavior, and still using the same context - childcare.

Scenario 2: Example B – A lower level class

Childcare: A Lower Level Class Example B Unit Goal: By the end of this unit, students will be able to apply their understanding of whole number operations and in/out tables in order to determine which of two childcare options is less costly.	
Val has yet another class with similar childcare issues. Again, she asks her students to bring to class any information that they can about the various childcare offerings in the area. She also collects some information herself so that students have a variety of options to consider as they explore the issue in detail in class.	**Context:** Especially at the lowest level, students need to connect what they are learning to real-life contexts; this provides them an opportunity to see the value of what they are learning. In doing so, Val hopes to change their *numeracy-related practices and experiences* and to affect their *disposition* toward numeracy tasks.
These students are in Val's ABE (Adult Basic Education) class and are focusing on operations with whole numbers. They have been working on multiplying and dividing whole numbers so she reviews all of the childcare situations to see which ones lend themselves to use with her students. She tweaks the options to make sure that they are written in various ways – some give a monthly cost, others a weekly cost, and others a yearly cost.	**Representations:** Val reviews the information that they bring in, as she is sensitive to the *literacy* needs of her students. She simplifies the information that students use, including less information and numbers that are less complex. At this level, their *mathematical knowledge* is more limited and she knows she will have to focus on ensuring that students gain *conceptual understanding* of the math that they are going to be learning as they work toward the final product.
Val works with the students to round the weekly and monthly options so that the numbers are friendlier (for example, they might round $188 to $200 for a weekly cost of one child; then they can practice using whole numbers with zeros to discover a pattern.) She makes sure that one of the options is given in cost per week and another by month or year so that students have to manipulate the costs to make a comparison.	**Content:** Even when dealing with basic concepts such as whole number operations, the teacher wants to show how concepts are applied; she also is anticipating algebra by asking students to look for patterns over time by looking at in/out tables. In doing so, she is slowing helping them change their *attitude* regarding numeracy, helping them build their confidence in being able to 'do math'.

She then has students choose two childcare options from the very limited set. She gives them this task: *Determine the total yearly cost of each option in order to figure out which is the better deal (based on cost only) by creating two in/out tables*.

Students each give oral presentations, explaining their reasoning for their choice for childcare.

Here's a sample of how a student might begin to respond at this level:

Weeks	Cost		Weeks	Cost
1	200		4	800
2	400		8	1600
3	600		48	9600
4	800		52	10,400

Response: At this level, students are asked not just to react to the information, but also make decisions based on their simple analysis; they also orally communicate their decision based on the information they analyzed. Adult learners who have struggled with numeracy often do not have effective *numeracy-related practices*, so Val needs to ensure that students begin to experience success and to see math as more than simply doing decontextualized calculations.

If 4 weeks costs $800, then 8 weeks costs $1600. I can either continue to add $800 each month or I can do some multiplying and adding.

As a teacher designs activities or tasks for her students, she needs to ensure that the task requires students to engage in all facets of numerate behavior. She also needs to ask herself questions about the enabling processes students will need to use in order to work through tasks. She can use questions such as the following to guide the development of her task.

Table 4: Facets of numerate behavior

Facets of numerate behavior	**Questions a teacher might ask herself**
Context	Is the context meaningful for students? Are students familiar with the context? Can the actual context be transformed into a more familiar context if working at lower levels?
Responses	What are students going to do with the mathematical information? What actions do they need to undertake? What strategies will they use? What cognitive process(es) is required to address the situation? Is there an opportunity to ask students to communicate something about the information?
Mathematical content/ information/ ideas	What is the math that students are learning? What do students already know? How have students shown what they know and understand? How can students show that they can apply the math that they are learning?
Representations of mathematical information	In what format (chart, graph, picture, etc.) is the information presented and needed for the task? How difficult is it to locate and identify the math embedded in the materials?

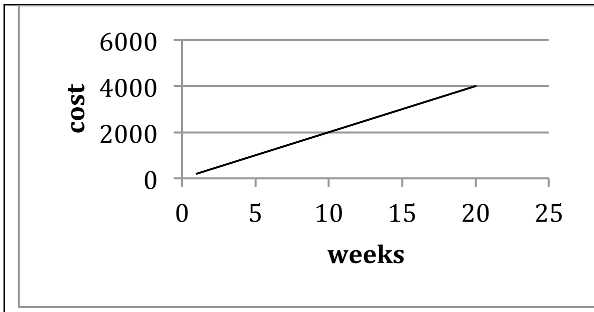
As she asks herself these questions, the teacher adjusts the task to fit the needs of her particular level of students.

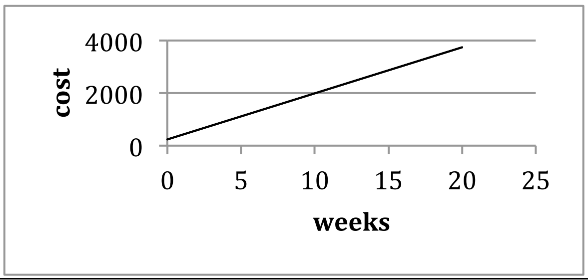
Scenario 3: Example C – A higher-level class

<table>
<tr><td colspan="2" align="center">Childcare: A Higher Level Class
Example C</td></tr>
<tr><td colspan="2">Unit Goal: By the end of this unit, students will be able to apply their understanding of algebraic reasoning and functions in order to show with graphs and equations which childcare option is the best deal (based on cost only).</td></tr>
<tr><td>Val, the adult education teacher, has another class with similar childcare issues.</td><td>Context: Even if her student are working toward a high school equivalency diploma, they will need to use math in their lives after the test so it is still appropriate to begin with real-life situations. Val has chosen a topic that allows her students to engage their world knowledge to the situation.</td></tr>
<tr><td>At this level, Val asks students to do a web search to find information about childcare centers in their area. If costs are not available, she encourages them to make phone contact in order to collect data on local childcare costs.</td><td>Representations: By asking students to do a web search, Val anticipates that the information will come in a variety of representations, with information not always obvious, so will require some critical reasoning by the students. Because the teacher is doing less monitoring of the information, students will have to activate their literacy skills to a greater degree.</td></tr>
<tr><td>These students are in Val's ASE (Adult Secondary Education) or College-and-Career Readiness class. They have been working on linear functions so she reviews all of the childcare information they have collected to see which ones lend themselves to developing linear functions. She analyzes each to see whether the costs follow a predictable pattern, such as weekly, monthly, or yearly costs.</td><td>Mathematical content: The teacher uses childcare situations to model real-life uses of algebra in order to make predictions over time. Val will conduct many lessons with her students before they tackle this final task so she can ensure that students are gaining conceptual understanding of graphing and of simple linear functions. In using real-life scenarios to illustrate algebraic concepts, Val hopes to change their attitudes about algebra (often thought of as symbolic manipulation without meaning or real-life use according to our adult education students).</td></tr>
<tr><td>Students choose two or three childcare options from which they have to decide which offers the best deal for the price (although they all know that criteria other than cost are involved in making decisions about childcare). They first make in-out tables to organize their data. They then graph their options and derive an equation based on the pattern that they find.

Students each give oral presentations, sharing their graphs and explaining their equation and their choice for childcare.

Here's a sample of how a student might begin to respond at this level:</td><td>Response: She asks students to go beyond just acting upon the information. They create a visual representation of the situation and an equation, then explain their reasoning, which requires much deeper thinking. They may have to use adaptive reasoning, especially if their calculations or graph do not align with their personal knowledge of daycare situations.</td></tr>
</table>





In the first graph, y = 200x, I would pay $200 per week. In the second graph, y = 175x + 250, I have to pay a non-refundable $250 no matter what. But then it costs only $175 per week. At 10 weeks, the cost is the same, but after that the second option is better.

Notice that in both of the examples above Val creates a unit goal that is appropriate to the level of her students and that focuses on practical application of the skills learned. Even at the most basic level, adults should be expected to use what they are learning. As the teacher develops the unit goal, which includes a statement of how she will assess performance, she also considers what concepts she is going to teach, what cognitive skills are going to be required, and what strategies or approaches might be appropriate.

In setting her learning goals for the unit, the teacher not only has to ask questions about the four facets of numerate behavior, but she also needs to consider the processes that enable students to engage in that numerate behavior.

Table 5: Enabling processes of numerate behavior

Enabling processes of numerate behavior	Questions a teacher might ask herself
Mathematical knowledge and conceptual understanding	Do students have the requisite understandings on which to build new understanding? Do students really understand the math or do they simply try to follow procedures?
Numeracy-related practices and experiences	What kinds of prior experiences have students had with the content? How much number sense do students have regarding the content?
Adaptive reasoning and problem-solving skills	Do students know how to self-monitor to see if they are on the right track? Do students know how to begin to tackle the situation?
Literacy skills	How much reading is required for the situation? What is the level of text complicity? How difficult is it to locate and identify the math embedded in the materials? How embedded is the math? Is there writing involved? If not, is there a way that it might be included?

Context/world knowledge	What experiences and prior knowledge do students have related to the context? Is there something that we need to do in class to familiarize students with the context?
Beliefs and attitudes	How comfortable are students with the math content to be addressed? How comfortable are the students with the context?

The three childcare scenarios presented above all focus on the same context – childcare – but it is obvious that students in each of the three classes will be focusing on different skills. As the students move from basic to intermediate to advanced classes, the teacher needs to design increasingly challenging tasks that will engage students in skills appropriate to their level. Ultimately, over time, the teacher wants to move students along the continuum suggested by the PIAAC assessment framework.

IV. Adding Complexity Factors to the Mix

In examining the facets of numerate behavior and the related enabling factors, it makes sense that the difficulty of a numeracy task is influenced by changes in both the enabling factors and the facets. Understanding how these influence the difficulty can help teachers figure out how to adjust numeracy tasks based on the needs and abilities of their students. One of the key features of the PIAAC framework is that it is based on the belief that numeracy proficiency runs along a continuum. The continuum, as used in the PIAAC tasks, is based on several criteria, described in PIAAC as **factors that affect complexity**:

- type of match/problem transparency
- plausibility of distractors
- complexity of mathematical information/data
- type of operation/skill
- expected number of operations

These five complexity factors are intended to take into account all facets of numerate behavior – the variety of contexts, mathematical ideas/content, responses, and representations. The first two aspects of complexity of a task relate to text – how the information is presented. The last three aspects relate specifically to the math operations or activities that an adult has to engage with to accomplish the task; however, the five factors are interconnected. While these complexity factors were used in developing PIAAC assessment tasks, they can also be useful in developing meaningful tasks in the classroom.

Let's look at each of these complexity factors in more detail and then we will examine how teachers can use these in the classroom.

- **Type of match/problem transparency**. This factor combines two factors related to how straightforward the problem is. Transparency describes the problem itself. It has to do with the explicitness of the information needed to solve a problem. At a lower level, there may be little or no text, and it is obvious what must be done with the information. At higher levels, there could be very dense text from which an adult must extract relevant mathematical information. The concept of type of match has to do with what an individual has to do with the text in order to find the information necessary to solve the problem. For example, a highly transparent problem might require a person to simply locate information in a text. A less transparent problem might require the person to search several pieces of information in order to extract the information necessary to address the task. A simple set of examples to illustrate this involves the task of comparing data.

> *Ex. 1:Based on the GED® test data in 2001, the median score for test takers in math was 475. In 2003, it was 470 and in 2010 it was 469. What is the difference between the highest and lowest median score for those three years?*
> *Ex. 2:Review the GED® test data from 1999 – 2010 to determine which year test takers had the highest median score in math.*

In the first example, the data are provided, but in the second example, no data are provided. An individual would have to search through several years of reports to find the data needed for the situation.

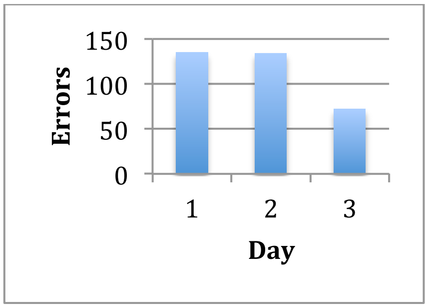
- **Plausibility of distractors.** Easy tasks would involve using text that has just the right amount of information. Many typical word problems that give just enough information would be considered simple. Moving toward the complex end of the spectrum would be tasks that include a lot of information that has to be read to find the relevant information and sometimes may not even include relevant information such as a formula needed to address the problem.

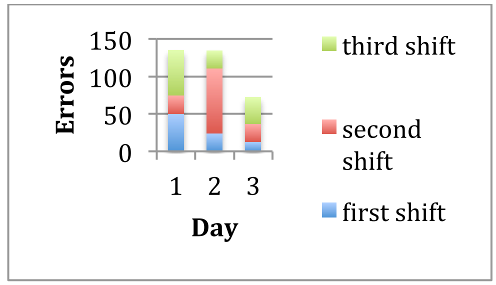
  Consider these two simple examples to illustrate the complexity factor related to distractors:

  > *Ex 1:According to the bus schedule, John can catch it at 6:33 p.m. for a 7:45 p.m. arrival downtown. How long will his ride be?*
  > *Ex 2: Use the bus schedule to determine which bus to take to be sure to arrive downtown before 8:00 pm.*

  In the first example, the information is straightforward; the beginning and ending times are given so the student does not have to look for the information. In the second example, the information is imbedded in the schedule. The times are not provided so the student has to use the bus schedule to locate the necessary information.

- **Complexity of mathematical information/data.** This complexity factor looks at the range from concrete to more abstract mathematical information, such as counting the number of items in each box vs. analyzing a complex graph or applying a detailed formula. Complexity also ranges from the familiar to the unfamiliar. The more familiar the context in which the mathematical information is presented, the easier the task. Tasks that are situated in an unfamiliar context can ratchet up the complexity level, even if the numbers themselves are not too challenging. Imagine two different bar graphs, both depicting the number of errors made per day. In the first graph, there are three simple bars, each representing only the total number of errors for the day. In the second graph, the number of errors is broken down by shift in a series of stacked bars.





Students who have only been exposed to the typical bar graphs where each bar represents only one category will find this 2nd graph challenging. The bars are more challenging to read and the graph itself may be unfamiliar.

- **Type of operation/skill**. Clearly a mathematical situation involving addition and subtraction of whole numbers is much simpler than one requiring dividing fractions. Likewise, it is easier to give facts about simple bar graphs and more complex to make predictions from a line graph. An example of varying complexity based on type of operation or skill is determining simple vs. compound interest. The compound interest formula [$A = P(1 + r/n)^{nt}$] requires an understanding of exponents and the graph of the situation is exponential.

- **Expected number of operations**. Tasks that involve one step are considered less complex than those involving more than one step. We all have seen students who stop working on a problem after they have completed the first of a series of steps. It seems that the more steps there are, the more likely students are to get lost in the process. Here are two simple examples to show the difference in complexity. In the first example, an individual only needs to calculate the amount saved. In the second example, there could be a second step required – to figure out the new price once the amount saved based on sale is determined. Or, an individual could first determine the percent of the regular price (100% - 25%), then determine the new price. But, in either case, the second example requires two steps.

  Ex. 1:A coat that regularly costs $456 is on sale for 25% off. How much would you save by buying it at the sale price?

  Ex. 2: A coat that regularly costs $456 is on sale for 25% off. What is the sales price?

Let's look again at one of the tasks from Val's classroom to see how she used the above criteria in developing the activity for her students. For the most part, unless we can actually see the childcare information used by the students, we cannot know how simple or complex the text is. We don't know whether the mathematical information is obvious or embedded, nor do we know whether there are distractors or not. However, we can look at the mathematical aspects of the complexity of the task Val asked of her students. We can also look at the type and number of operations required for the task.

Scenario 4: Example A revisited

Val has been hearing bits and pieces from her students about their childcare issues. She asks her students to bring to class any information that they can about the various childcare offerings in the area. She also collects some information herself so that students have a variety of options to consider as they explore the issue in class. Val's students have been working on percentages so she uses the topic of childcare to focus on a big idea about percentages and pie charts. Val gives students time to explore some lessons on percentages and graphing. Val then asks students to estimate their monthly budget. Depending on the students' comfort level with graphing, she has them limit the number of categories. She asks them to create a pie chart of their present estimated monthly budget. Students then choose one of the childcare options, review the details of the childcare option, and then estimate the monthly cost of the service. Val asks them to first predict how the new childcare option will impact their monthly budget. She asks them, "What do you think your new pie chart will look like when you change your present monthly childcare estimate with the new one?" She also asks, "If some categories of expenses (such as rent) cannot change, what might the new pie chart look like? Why?" Students create pie charts based on their estimated childcare costs in relation to their other monthly expenses. Each student explains, based on the data alone, which childcare situation seems to be best suited based on their monthly budget estimates. Then they share what they discovered about how changes to one expense impact what money is left for other expenses, especially when the total monthly income does not change.	Complexity of mathematical information: Students have familiarity with the topic and are tasked with find the monthly cost in the text. Type of operation/skill: Simple percentages and pie charts of a monthly budget Expected number of operations: If the information provided was not by month, this will be something students will have to determine. Because students will have to consider what changes in a pie chart when a category changes, this increases the complexity level.

Just as Val did in the example above, teachers need to consider complexity factors as they design activities or tasks. Teachers need to ask themselves several questions related to these factors.

Table 6: Complexity factors

Complexity factors	Questions a teacher might ask herself
Type of match/ problem transparency	Is the needed information explicit or hidden? Will the student need to search through the information several times in order to access everything that is needed to address the situation?
Plausibility of distractors	Are there any distractors? If so, are there numerous ones that get in the way of easily deciding what information is needed?
Complexity of mathematical information/data	How is the information presented – is it concrete or visual so that the information can be readily counted, or is it very abstract such as a decontextualized equation? What are the quantities involved – whole numbers vs. fractions vs. decimals, for example?
Type of operation/ skill	Are students already familiar with the operation? Do they know the relationship between and among operations so that they can add up for a subtraction operation, for example? How friendly are the numbers (for example, does the task require a comparison of two ratios such as 2/3 = 6/□ vs. 2/3 = 5/□ ?)
Expected number of operations	How many operations will be required for the situation?

Connecting Numerate Behavior and Complexity Factors
So far, we have discussed several key elements of the PIAAC assessment framework, including facets of numerate behavior, enabling processes that support such behavior, and complexity factors that determine the difficulty of the task requiring numerate behavior. While we have discussed these in isolation, they are all integrated and interdependent.

Let's take a look at how they can influence each other with a couple of specific examples.

Table 7: Decontextualized problems vs. real-life tasks

A typical decontextualized fraction problem: 1/2 x 1/3 = *x* **A real-life task involving fractions**	**Facets of Numerate Behavior**	**Enabling Processes to Support Numerate Behavior**	**Complexity Factors that Determine the Difficulty of the Task for Adults**
 *You work on the late shift at Getz Bakery and need to prepare a report of what is sold at the end of each day. On Monday 1/3 of the slices of cake were sold. On Tuesday, ½ of what was left was sold. What portion of the original cake would you report was sold on Tuesday?*	Context: could be work or adults' personal lives Responses: act upon Mathematical content/ information/ ideas: number sense - fractions Representations of mathematical information: includes text and a picture; both are needed for the task	Mathematical knowledge and conceptual understanding: requires an understanding of what happens when a fraction of a fraction is taken Adaptive reasoning and problem-solving: students might choose a variety of strategies as they work through the situation. Many may try different operations (adding, subtracting, multiplying, dividing) but should be able to reason whether the result makes sense; drawing a picture to visual this situation could be beneficial to make sense of the situation Literacy skills: some skill needed, although fractions are in symbols and numbers Context/world knowledge: most students probably know that cakes are sliced into equal parts Beliefs and attitudes: for adult learners to succeed with this task, they need to believe that they can work with fractions Numeracy-related practices and experiences: learners will tend to perform some operation, possibly depending on their memory of fraction procedures	Problem transparency: mathematical information is obvious Plausibility of distractors: none Complexity of mathematical information/ data: somewhat abstract but object is familiar Type of operation/skill: challenging for most adult learners because it involves multiplying fractions Number of operations: requires more than one operation, but also involves counting

A typical decontextualized algebra problem: $x + 20 = 4x$ **A real-life task that involves algebraic reasoning**	**Facets of Numerate Behavior**	**Enabling Processes to Support Numerate Behavior**	**Complexity Factors that Determine the Difficulty of the Task for Adults**
Lavonne's granddaughter wants to go to the community fair. There are two prices for tickets: either an individual pays $20 entrance fee and then pays only $1 per ride, or the individual pays no entrance fee but has to pay $4 per ride. How can Lavone explain in a graph and an equation the options based on the number of rides her granddaughter thinks she might want to go on?	Context: adults' personal lives Responses: interpret, evaluate, analyze, communicate Mathematical content/ information/ ideas: Patterns, relationships and change Representations of mathematical information: needed information is presented in text; but individual will represent the solution graphically and symbolically	Mathematical knowledge and conceptual understanding: requires an understanding of variables and operations Adaptive reasoning and problem-solving: Students could use concrete manipulatives to support their reasoning Literacy skills: some skill needed, especially understanding terms such as 'per' Context/world knowledge: understanding that there are situations in which you pay up front to enter an environment even if you don't participate in activities Beliefs and attitudes: because this situation is grounded in a real-life experience, students are more likely to try to work through the problem than they would if they only had the original problem; many students fear algebra Numeracy-related practices and experiences: depending on adult learners' experiences, they may initially tackle this situation by creating two tables to compare the two options as the number of rides increases	Problem transparency: mathematical information is obvious but it needs to be separated into two different options Plausibility of distractors: none Complexity of mathematical information/ data: money is familiar to students and amounts are in whole numbers Type of operation/skill: what makes this task more challenging is that students are asked to create equations; if they only had to use a table of other strategy, this would have been considered an easier task Number of operations: more than one operation and a comparison of two situations

For a teacher using the PIAAC framework to guide instruction, what might go through her mind as she tries to think of numeracy tasks - tasks that require students to engage in numerate behavior? She will consider the four facets of numerate behavior, but she will also think about the enabling process and complexity factors that influence task design.

Let's say she knows that the content is ratio and proportions. She knows she will teach lessons to ensure that students continue to build on their conceptual understanding of ratios and proportions and their ability to think proportionately.

She has an idea about the **content**, but she still needs to consider several factors:

- She suspects that most of her students have been exposed to the 'cross-product' method of finding a missing amount in a proportion, but do they have any conceptual understanding about what a ratio is? A proportion?
- Have they made the connection between equivalent fractions and equal ratios? (If not, she may have to add some lessons to help students make those connections.)
- Have they had opportunities to visually explore equal ratios (long before being introduced to the cross-product method)?

As the teacher thinks about **context**, she realizes that proportional reasoning is used often – at work, at home, and in the community (including such prevalent comparisons as unit price, miles per gallon, miles per hour, etc.). So, she thinks about her students:

- Are students aware of how they use proportional reasoning already in their lives?
- What are some contexts in which students have already applied their understanding of proportions? For example, have they had to mix cleaning chemicals, adjust recipes, buy items based on unit prices or even sale price?)
- What are some new contexts that they might explore? (Have they ever considered what happens when pictures are not in proportion? What in a recipe does not remain in proportion? How can they create a smaller scale version of a larger picture?)
- What kinds of materials might I use? Should these materials embed information that the students need, or should the information be fairly obvious? Will the materials used require a lot of reading? If so, will I need to address vocabulary for the particular context that we use?

Speaking of materials to be used, she will need to look closely not only at whether the text is challenging for students, but she will also need to examine the mathematical **representations** used.

- Will students have to interpret graphs in order to address the task?
- Will they be using scale models, such as maps?
- Will they use information from different websites?
- And, how will students be expected to present the results of their tasks?

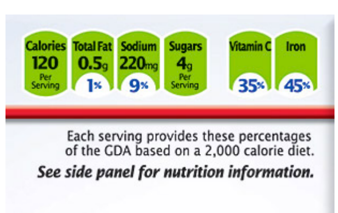
As she designs the task for her students, she needs to consider what she wants students to do with the information that they find. She needs to think about how they will **respond** to the material.

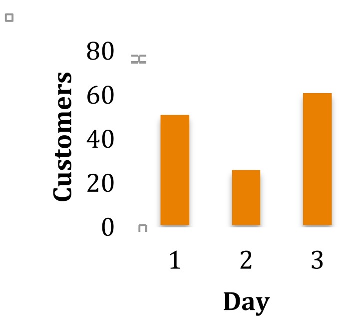
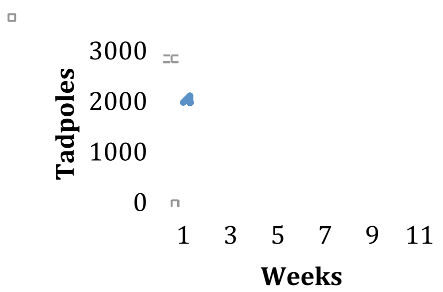
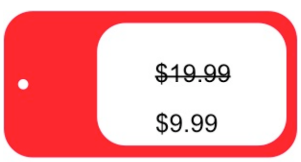
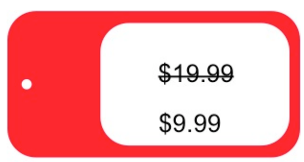
- Will they readily find information from a text in order to make a simple comparison?
- Will they have to use data from a table to make a simple comparison or will there be some analysis needed?
- Will they have to use a scale in order to explain distance?
- Should students be asked to communicate their results? If so, how?

While this seems like a lot of thinking that has to go into designing lessons, it is this type of thinking that will help students develop into numerate adults.

Let's look at some simple examples of how a teacher uses this continuum to think through the development of tasks.

Table 8: Complexity factors affecting tasks

Old task	New task	What changed?
How many bottles of water does the shelf hold? 	The bottles are stacked 4 deep and there are 6 rows of bottles. How many water bottles are there altogether? 	In the first example, the information is obvious. There are no hidden bottles that need to be considered. The new task has been ratcheted up with a new picture and more information. Not all the bottles can be readily counted by just looking at the picture.
Jeff needs to get to his mother's house before noon. He knows his mother lives about 130 miles away from his own home. If he plans on following the speed limit, by what time does he need to leave home? 	Jeff needs to get to his mother's house before noon. He knows his mother lives about 130 miles away but he also needs to stop by his office which is 20 miles in the opposite direction. If he plans on following the speed limit, by what time does he need to leave? 	The new task has been ratcheted up with the added information. It requires another step to the process of figuring out the total amount of driving time needed and then working backwards to determine what time Jeff needs to leave his own home.
Amy can only eat 10 g. of sugar per day. If she eats one serving of this cereal, how many more grams of sugar can she have during the rest of the day? 	Amy can only eat 10 g. of sugar per day. If she eats one serving of this cereal, how many more grams of sugar can she have during the rest of the day? 	The new task has been ratcheted up by adding extraneous information.

<table>
<tr>
<td>Tamara’s son has set up his own little business and has been keeping track of his customer calls. On which day did he receive the most calls?
</td>
<td>The conservation committee, with the help of volunteers, has been counting the number of tadpoles in the pond at one of their nature walks. Jot down some questions that you would want to answer in order to better understand why this graph looks the way it does.
</td>
<td>The new task is ratcheted up quite a bit. The line graph is harder to read and interpret than the bar graph. There is more information in the line graph, many more years and much larger numbers. The task involves interpreting the graph, not just getting obvious information from it.</td>
</tr>
<tr>
<td>What is the new price?
</td>
<td>How much of a discount is the new price?
</td>
<td>The representation in both examples is the same. The second task is at a much higher level because of the mathematical content that needs to be applied.</td>
</tr>
</table>

Once teachers begin to get comfortable with the notion of numeracy (rather than math as a set of discrete skills) and the complexity factors, they should be able to readily tweak tasks to challenge their students without overwhelming them. Teachers want to ensure that tasks they design are at an appropriate level for their students. Tweaking these tasks allows them to readily do so without creating new tasks for different students. As students develop skills, teachers should consider the complexity factors as they create more challenging tasks to move students developmentally forward along the continuum of numerate behavior.

V. Conclusion

While there are several components to the PIAAC assessment framework, in this Guide we have focused mainly on the critical component of numeracy as use-oriented. The definitions of numeracy and numerate behavior illustrate the concept of ‘use-oriented’. Teachers adopting PIAAC’s use-oriented definition of numeracy are more likely to realize that teaching discrete, decontextualized skills is not benefiting their students. Students need to be able to apply their skills in increasingly challenging situations in order to effectively manage their lives, whether it is at work, home, in further schooling, or in the community.

We also examined the PIAAC concept of numeracy as a continuum, and how different factors influence whether a task is simple or more complex.

Simple Tasks
- explicit information
- no distractors
- concrete information (actual or visual)
- whole numbers and benchmark fractions, decimals, percents
- one operation (+, -, x, ÷)

→

Moving along the Continuum
- information becomes less obvious
- distractors begin to appear in the information
- mathematical ideas and content becomes more challenging
- multi-steps are needed to address the situation

→

Complex Tasks
- information hidden, challenging to sort through
- lots of distractors
- abstract information or presentation of material
- complex mathematical information
- multiple operations

We explored how these complexity factors can help teachers create tasks that are at an appropriate level for their students. At the same time, teachers can use these complexity factors to develop tasks that push students to move beyond what they can do now, so that they can attempt more complex tasks. (While focusing on the complexity factors used for assessment tasks in the PIAAC, in this guide we have intentionally not labeled them at a particular level. Although we have talked in general about "levels" of student performance, care must be taken not to assume that PIAAC levels correspond with the National Reporting System levels used in the adult basic education field. Correspondences between those two schemas have not been attempted. Instead, we tried to focus on movement along a continuum of numeracy development.)

Adopting PIAAC's use-oriented approach to teaching numeracy will lead to literacy instruction that more effectively prepares adults for real life numeracy tasks. Practitioners who are interested in adopting this use-oriented approach to teaching numeracy should focus on is the definition of numeracy, the enabling factors that influence numerate behavior, and the complexity factors that affect task difficulty. (For a quick guide to using PIAAC, see page 32.) These aspects of PIAAC, if incorporated into classroom settings, can help create an environment where students develop numeracy skills that can be applied throughout their lives. Practitioners who embrace the idea that adults need to become numerate rather than 'learn math' should be able to create tasks that require all aspects of numerate behavior – context, content, responses, and representations. Practitioners will be able to think about what supporting processes enable a student to be able to tackle a situation that requires numerate behavior. And, these same practitioners should be able to create new tasks and tweak present ones using the complexity criteria so that all students can continue to become more numerate adults.

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A Quick Review: How to Use the PIAAC Elements in Your Classroom

As we mentioned from the onset of this paper, too many adults in the U. S. are not able to successfully tackle situations involving math. In this paper we have discussed how PIAAC can help teachers change the adult education math classroom. How do we do this?

1. Start with PIAAC's definition of numeracy. Teachers need to embrace the PIAAC definition of numeracy: *the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.* Math is not a set of isolated skills that typically show up in the table of contents of adult education workbooks – topics such as Whole numbers, Fractions, Decimals, Ratio and Proportion, Algebra, Data, and Geometry that are taught procedurally. This definition should drive how teachers (and students) think about 'math' and what needs to happen in the adult education classroom.
(See Section III above, pp. 5-11.)

2. Ask students how they use math in their lives. A teacher who uses PIAAC's definition of numeracy is more likely to ask her learners how they use math in their lives, providing a starting place for math instruction. For ideas on types of questions to ask, she might want to revisit the Background Questionnaire of PIAAC. (See Section II above, p. 3.)

3. Design learning activities based in real-life contexts. Once a teacher has an idea of the kinds of math situations their students typically engage in, she needs to design learning activities that enable them to strengthen their enabling processes and develop and refine the skills they need to be comfortable using math in these situations — and many more.

With her students providing real-life contexts as a starting point, she needs to consider what knowledge and understanding they already bring to the situations. Then she needs to nudge students so that they develop new knowledge and understanding.

Based on the contexts from students, a teacher can focus on the specific content that she will have to teach. She will need to determine what her students already know about the content.

- Have students previously been exposed to the content?
- If so, was it simply procedurally? Do students have only one strategy when addressing a particular topic?
- What are students' attitudes about the content to be addressed?

The teacher will still need to teach specific skills, but she should be mindful that she is teaching for understanding, not just procedurally, so that students can develop strategies and problem-solving skills, not just memorization techniques.

As the teacher begins to develop lessons and gather materials for class, she will need to consider how the math information is presented. Is it in graphic form? Are there many pieces of information that make it challenging for a student to figure out what is needed? Is the math information imbedded in text? If so, what are the literacy levels of the students? What might she need to do in order to make the material more accessible to students, or what teaching might she need to do in order for the students to be able to effectively use the textual math information?

The teacher also will need to consider how students will be expected to interact with the math information. Will they simply find information? Or, will they have to evaluate information, or communicate something about the information? Obviously, this facet of numeracy (response) requires that the teacher expose her students to much more than computation problems from a workbook. In such situations, students only have to superficially engage in the math information. The teacher needs to provide rich sources of math information in the classroom, similar to what students have to interact with in real-life. (See Enabling Processes, pp. 13 – 16, Table 7, above, pp. 25 – 26.)

4. Build in performance assessments. Teachers need to build in assessments that help them see how well the students can use the skills they are learning and what else they need to do in class to help them develop their skills further. If the teacher has considered context early on, she will have in mind a unit goal long before she gets to the end of the unit. This unit goal provides the opportunity for students to apply the content that they have been learning, using the skills they have been learning, in a real-life context. The teacher can tweak this unit goal so that all students are nudged to push themselves further along the continuum of math proficiency. (See Examples from Val's classes above, pp. 11-12, 17, and18-19.)