

SHSAT Math Reference Sheet



Factor Completely:

- 1) Greatest Common Factor: Ex. $(7x^3 - 14x^2 - 49x)$ GCF is $7x$. A common factor to all terms of a polynomial can be factored out. All 3 terms contain a factor of $7x$. Pulling out the GCF creates a new equivalent expression $7x(x^2 - 2x - 7)$
- 2) Difference of Two Perfect Squares: Anything in the form $(a^2 - b^2) = (a+b)(a-b)$
- 3) Factoring Trinomials: To factor a polynomial in the form $y = ax^2 + bx + c$ you need to find two binomials that would multiply together to create the polynomial. Consider factors of c that add to the b term. Ex. $x^2 + 7x + 12$, consider factor pairs of the c term "12" that will add to the b term "7". The only way to get a leading term of x^2 is when you multiply x by x . The factor pair of 12 that adds to 7 would be 3 and 4. $(x + 3)(x + 4)$

Percent Increase or Decrease

$$\left(\frac{\text{Amount of change}}{\text{Original}} \right) \times 100$$

Arithmetic Sequences:

To **find any term**
of an **arithmetic sequence**:

$$a_n = a_1 + (n - 1)d$$

where a_1 is the first term of the sequence,
 d is the common difference, n is the number
of the term to find.

Radicals:

Simplifying Square Roots

Sometimes it is a good idea to **simplify a square root** without approximating. This allows us to see patterns develop and create short-cuts that we would not have known existed.

Instead of $\sqrt{50} \approx 7.1$

We rather split $\sqrt{50}$ into two factors, one being a **perfect square**.

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

Answer
 $5\sqrt{2}$

Multiplying Radicals: Multiply the coefficients of the radicals, multiply the radicands or the expressions under the radical sign and simplify.

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

Dividing by a radical: When you divide by a radical, you must rationalize the denominator, by multiplying the expression by radical 7 over radical 7.

$$\frac{2}{\sqrt{7}}$$

$$\frac{2 \text{ times radical seven} = 2 \text{ radical seven}}{\text{radical seven times radical seven} = 7}$$

$$\frac{2 \sqrt{7}}{7}$$

Add or Subtracting Radicals, can only be done when the radicand or the number under the radical is the same number.

Triangle Inequality Theorem:

$$a + b > c > b - a$$

where a = smallest side, b = medium side, c = largest side of a triangle

A **tangent to a circle** is a line in the plane of the circle that intersects the circle in exactly one point.

*If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of tangency (forms a 90 degree <)

A **measure of an exterior angle of a triangle** is equal to the sum of the measures of the two non-adjacent interior angles.



Motion Problems:

distance = rate x time

Ex. A passenger train and a freight train leave at 10:30 AM from stations that are 405 miles apart and travel toward each other. The rate of the passenger train is 45 miles per hour faster than that of the freight train. If they pass each other at 1:30 PM, how fast was the passenger train travelling? Set up this chart:

Rate	Time	Distance
$x + 45$	3	$3x + 135$
x	3	$3x$

$3x + 135 + 3x = 405$ $x = 45$, passenger train travels at 90mph.

Consecutive Integer Problems:

Let the $x = 1^{\text{st}}$ consecutive integer

$x + 1 = 2^{\text{nd}}$ consecutive integer

$x + 2 = 3^{\text{rd}}$ consecutive integer

If it says consecutive odd/even integers

Set up your legend as: $x, x + 2, x + 4$ etc

Combined Rate Problems:

Angelica can mow one lawn in 30 minutes. Lawrence can mow the same lawn in 40 minutes. If they mow the lawn together, how long will it take them to complete the task?

Angelica's rate is $\frac{1 \text{ lawn}}{30 \text{ min}}$

Lawrence's rate is $\frac{1 \text{ lawn}}{40 \text{ min}}$

Since they are working together, add their rates and then create a proportion.

$$\frac{1 \text{ lawn}}{30 \text{ min}} + \frac{1 \text{ lawn}}{40 \text{ min}} = \frac{1 \text{ lawn}}{x \text{ minutes}}$$

Solve by getting a common denominator and then cross multiply.

Investment Problems: Principle x Rate = Interest Income

Solve Coin Problems: with the Substitution Method of Simultaneous Equations. A postal clerk sold 40 stamps for \$5.40. Some were ten cent stamps and others were fifteen cent stamps. Set up your equations. Let $x =$ the # of 15 cent stamps and Let $y =$ # of 10 cent stamps, therefore $x + y = 40$ and the second equation attaches value; $15x + 10y = 540$ (use whole numbers)

Solving Simultaneous Equations: (Systems of Equations)

- 1) Addition or Subtraction (Elimination): Add or Subtract both equations in order to cancel out a variable. Solve for the variable that's left and then substitute to find the other.
- 2) Substitution Method: Solve one of the equations for one variable, and substitute it into the other equation.

Permutation and Combination

$$C_{(n,r)} = \frac{n!}{r! (n-r)!}$$

$$P_{(n,r)} = \frac{n!}{(n-r)!}$$

$n =$ set size: the total number of items in the sample
 $r =$ subset size: the number of items to be selected from the sample






A **permutation** is an ordered arrangement of Distinct objects in a sequence.

A **combination** is an unordered collection of unique elements. 2 lists with the same elements in different orders in the same set.



Reference Sheet

Area		KEY
Parallelogram	$A = bh$	b = base h = height w = width d = diameter r = radius ℓ = slant height a = apothem*
Triangle	$A = \frac{1}{2}bh$	A = area B = area of base C = circumference V = volume P = perimeter of base $S.A.$ = surface area
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$	
Circle	$A = \pi r^2$	
Regular Polygon	$A = \frac{1}{2}aP$	
		*apothem: line segment from the center of a regular polygon to the midpoint of one of its sides Use 3.14 or $\frac{22}{7}$ for π . Circumference $C = \pi d$ or $C = 2\pi r$

Volume/Capacity			Total Surface Area
	Rectangular Prism	$V = bwh$ or $V = Bh$	$S.A. = 2bh + 2bw + 2hw$ or $S.A. = Ph + 2B$
	Right Circular Cylinder	$V = \pi r^2 h$ or $V = Bh$	$S.A. = 2\pi rh + 2\pi r^2$ or $S.A. = 2\pi rh + 2B$
	Right Square Pyramid	$V = \frac{1}{3}Bh$	$S.A. = \frac{1}{2}P\ell + B$
	Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$ or $V = \frac{1}{3}Bh$	$S.A. = \frac{1}{2}(2\pi r)\ell + B$
	Sphere	$V = \frac{4}{3}\pi r^3$	$S.A. = 4\pi r^2$

Sum of the measures of the interior angles of a polygon = $180(n-2)$

Measure of an interior angle of a regular polygon = $\frac{180(n-2)}{n}$

where:

n represents the number of sides



Reference Sheet

Slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where m = slope and (x_1, y_1) and (x_2, y_2) are points on the line

Slope-intercept form of a linear equation

$$y = mx + b$$

where m = slope and b = y -intercept

Point-slope form of a linear equation

$$y - y_1 = m(x - x_1)$$

where m = slope and (x_1, y_1) is a point on the line

Distance between two points

$$P_1(x_1, y_1) \text{ and } P_2(x_2, y_2)$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points

$$P_1(x_1, y_1) \text{ and } P_2(x_2, y_2)$$

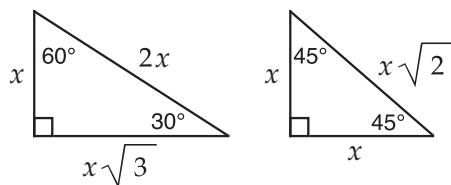
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Quadratic formula

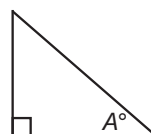
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a , b , and c are coefficients in an equation of the form $ax^2 + bx + c = 0$

Special Right Triangles



Trigonometric Ratios



$$\sin A^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

Conversions

1 yard = 3 feet

1 mile = 1,760 yards = 5,280 feet

1 acre = 43,560 square feet

1 hour = 60 minutes

1 minute = 60 seconds

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

1 pound = 16 ounces

1 ton = 2,000 pounds

1 meter = 100 centimeters = 1000 millimeters

1 kilometer = 1000 meters

1 liter = 1000 milliliters = 1000 cubic centimeters

1 gram = 1000 milligrams

1 kilogram = 1000 grams