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Exogeneity and forward rate unbiasedness

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Recent studies have rejected the forward rate efficiency hypothesis because of a failure to find support for the forward rate unbiasedness condition (FRUC). This condition states that spot and forward rates should be cointegrated with a unit cointegrating vector. These studies have focused on inferences drawn from partial system error-correction approaches that make critical assumptions concerning the exogeneity of forward rates. Taking a full systems approach to estimation without any *a priori* weak exogeneity assumptions, we find support for FRUC, and that spot rates, not forward rates, are weakly exogenous in the sense of Engle, Hendry, and Richard (1983). The latter finding implies that the rejection of the FRUC in prior research may be due to the incorrect implied exogeneity assumptions. (JEL F31, F40, C32). Copyright © 1996 Elsevier Science Ltd

With the recent emphasis in the time series literature on unit root and cointegration techniques, the possibility that the potential nonstationarity of both spot and forward exchange rates could be generated by a common stochastic trend has recently led researchers to reevaluate the previous findings of biasedness and inefficiency in the forward market for exchange rates. Simply stated, the forward rate efficiency hypothesis (FREH) requires that three conditions be met: (i) the future spot rate and the forward rate for a given country must be cointegrated; (ii) the coefficient on the cointegrating relationship for the forward rate must be unity; (iii) the residual in the cointegrating regressions must be white noise. We shall refer to the first two conditions

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jointly as the forward rate unbiasedness condition (FRUC).[1] Previously, the results in work such as Hansen and Hodrick (1980) and Bilson (1981) had brought into question the validity of the FRUC. But since many of these papers exploited statistical tests in first-differences, it has been conjectured that how one handled issues relating to the nonstationary in these two rates might be critical in reconciling the failure of these authors to find support for FRUC.

Unfortunately, tests of this conjecture have met with mixed results. Many papers have failed to find support for the FRUC, and this work has therefore rejected the FREH. In fact, many papers have actually both supported and rejected the hypothesis implying that the power of different test procedures may be important. In a recent paper by Hakkio and Rush (1989), they find some evidence for the existence of a long-run equilibrium relationship between the spot and forward rates using Engle and Granger (1987) cointegration tests. However, they conclude that based on the results of the estimation of a single equation error correction model that forward market cointegration, and thus unbiasedness, can be rejected for the British pound and German mark (DM). In echoing these results, Barnhart and Szakmary (1991) reject the hypothesis of cointegration using monthly data for the pound, DM, yen, and Canadian dollar on the basis of the insignificant explanatory power of the disequilibrium error variable from their single equation error correction model.

In contrast, several authors have found support for the FRUC. On the empirical side of the issue, Baillie and Bollerslev (1989) provide supporting empirical evidence for FRUC using 'levels' tests estimated using OLS as advocated by Engle and Granger (1987). Further, Corbae, Lim, and Ouliaris (1992) find support for FRUC using a canonical cointegrating regression approach in bivariate specifications. In this paper, we reexamine the empirical support for the FRUC. Using generalizations of the error correction methods discussed in Engle and Granger (1987), we attempt to unify some of the findings in the existing literature. Using exchange rate data from Japan, Germany, the United Kingdom, Switzerland, and Canada, we find support for a unit cointegrating vector between spot rates and forward rates. Furthermore, we explicitly test for exogeneity in the vector error correction model (VECM). In each of these five systems, we find strong support that spot rates are exogenous. Therefore, the disequilibrium error is important in explaining the dynamics of *only* forward rates for all five countries; not spot rates. We also show how our findings are consistent with the results reported from single equation error-correction models such as Hakkio and Rush (1989) and Barnhart and Szakmary (1991), and we demonstrate that the implicit assumption that forward rates are exogenous in these models led to the erroneous rejection of the FRUC.

I. Exogeneity in reduced form error correction models

Consider estimating a levels VAR for a bivariate system consisting of for country i a future spot rate, S_{it+1} and current forward rate F_{it}. Let $X_{it} = [S_{it}, F_{it-1}]$. If X_{it} is an I(1) vector, then FRUC implies that S_{it} and F_{it-1} are cointegrated.[2] Exploiting this conjecture, we can consider the following VECM of order K for ΔX_{it}:

$$\langle 1a \rangle \qquad \Delta S_{it} = \mu_{is} + \mu_{is}(S_{i,t-1} - \beta F_{i,t-2}) + \sum_{j}^{K-1} \Gamma_{isj} \Delta X_{i,t-1} + \varepsilon_{i1t},$$

and

$$\langle 1b \rangle \qquad \Delta F_{it-1} = \mu_{if} + \alpha_{if}(S_{i,t-1} - \beta F_{i,t-2}) + \sum_{j}^{K-1} \Gamma_{ifj} \Delta X_{i,t-1} + \varepsilon_{i2t},$$

where the errors for country i, $\varepsilon_i = [\varepsilon_{i1}, \varepsilon_{i2}]$, are i.i.d Gaussian, and the vector $\alpha = [\alpha_{is}, \alpha_{if}]$ are the adjustment coefficients for the VECM for the disequilibrium error terms $\beta X'_{it} = [1, -\beta][S_{it}, F_{it-1}]'$. The FRUC is more than a simple test for the presence of cointegration in ⟨1⟩. The hypothesis also imposes a unit elasticity of $\beta = 1$ on the forward rates if forward rates are to be unbiased estimators for spot rates. Therefore we would find support for the FRUC if the following are true (i) we find support for the presence of cointegration in the system ΔX_{it} (*i.e.* in the error correction model a test of the restriction that one or both coefficients α_{is} and α_{if} are non-zero[3]); and (ii) we find support that $\beta = 1$.

The FRUC is silent on the issue of statistical exogeneity in the system X_{it}. Therefore, ⟨1⟩ should be estimated as a fully specified system unless we can statistically find evidence of exogeneity. Estimating ⟨1⟩, we can test for the presence of any form of exogeneity by estimating the system ⟨1⟩ and simply testing linear restriction on the parameters of ⟨1⟩. If either component of α is zero, then we can identify the structure of the FRUC. If $\alpha_{is} = 0$, spot rates would be weakly exogenous for forward rates, and the parameters in the forward rate equation could be estimated efficiently by the maximum likelihood estimate obtained from estimating ⟨1⟩ as a single equation. Similarly if $\alpha_{if} = 0$, forward rates would be weakly exogenous. Now consider the case when $\alpha_{is} = 0$ and $\alpha_{if} \neq 0$. If the underlying VAR is of order 1, weak exogeneity implies strong exogeneity and *all* of the adjustment in the dynamics between spot rates and forward rates would be captured by the error-correction mechanism in the forward rate equation. In this case, the dynamic adjustment of the system would completely correspond to the adjustments in ΔF_{it} to deviations from the long-run 'static' equilibrium in which $S_{it} - F_{it-1} = 0$. If $\alpha_{if} = 0$ and $\alpha_{is} \neq 0$ and the VECM was still of order 1, spot rates would be adjusting while forward rates would be exogenous to the system.[4]

We can also relate the model in ⟨1⟩ to many of the important tests of FRUC in the recent literature. For example, Hakkio and Rush (1989, Table 3) report the results of the estimation of a partial system similar to ⟨1a⟩ under the implicit assumption of weak exogeneity of forward rates. These results were critical in their rejection of the FREH. Reproducing their specification, they estimate the following single equation error correction model:

⟨2⟩

$$\Delta S_{it} = \mu_{is} + \alpha_{is}(S_{it-1} - \beta F_{it-2}) + \gamma_{is1} \Delta F_{it-1} + \gamma_{is2} \Delta S_{it-1} + \gamma_{is3} \Delta F_{it-2} + \varepsilon_{i1t}.$$

The estimation of ⟨2⟩ as a partial system is fully efficient only if forward rates are weakly exogenous. Note that if the true exogeneity is the reverse, *i.e.* spot rates are weakly or strongly exogenous, then α_{is} would be by definition zero. Therefore any estimation of a partial system such as ⟨2⟩ may lead one to erroneously reject any hypothesis that requires α_{is} to be non-zero. Similarly, we can view the tests in Barnhart and Szakmary (1991) as partial system tests. If $\alpha_{is} = \gamma_{is1}$ and $\beta = 1$, then we can rewrite ⟨2⟩ as:

$$\langle 3 \rangle \qquad \Delta S_{it} = \mu_{is} + \alpha_{is}(S_{it-1} - F_{it-1}) + \gamma_{is2}\Delta S_{it-1} + \gamma_{is3}\Delta F_{it-1} + \varepsilon_{i1t},$$

which is the specification used in Barnhart and Szakmary (1991). Therefore, they estimate a *constrained* version of Hakkio and Rush's partial system, and are equally sensitive to the potentially erroneous exogeneity assumption.

II. Forward rate efficiency results

Before we discuss the results we briefly discuss the data. The forward rates are end of period 3-month forward rates from the IFS database and OECD Main Economic Indicators. The spot rates are all end of period rates from the same sources.[5] The sample frequency is quarterly and the data are from 1973:1–1992:4. The sample frequency was chosen to avoid potential problems caused by a finer sampling frequency than the forecast period. All data show evidence of being first order integrated processes.[6] Before we can estimate a system such as ⟨1⟩, we need to determine the order of the underlying VAR. We use the Schwarz BIC criterion to determine the VAR lag length. The results show that all of the currencies are first order VARs.

For our tests using the theoretical cointegrating vector, we assume that the cointegrating vector is (1, − 1), and we estimate a system such as ⟨1⟩ which, given the underlying VAR is order one, simplifies to:

$$(4a) \qquad \Delta S_{it} = \mu_{is} + \alpha_{is}(S_{it-1} - \beta F_{it-2}) + \varepsilon_{i1t},$$

and

$$(4b) \qquad \Delta F_{i,t-1} = \mu_{if} + \alpha_{if}(S_{it-1} - \beta F_{it-2}) + \varepsilon_{i2t},$$

where β is set to its theoretical value of unity. The results are reported in Table 1. What is interesting is that error correction equation for the forward rate for each of the five countries exhibits a highly significant response to deviations from the equilibrium relationship. All the coefficients on both the error correction variables and the lagged spot rates are significantly different from zero at a significance level of 5 percent.[7] All the coefficients are very close to unity. Note that this coefficient is not the β cointegrating vector, but the speed of adjustment to return the forward rate to the long-run equilibrium relationship. The coefficients range from 0.942 to 0.994 implying that the forward rate almost completely adjusts within one quarter when a deviation occurs between the spot and past forward rates. In the second column of Table 1, we test if this adjustment could be equal to unity. We cannot reject the unit error correction coefficient for any of the currencies. Thus, a complete adjustment by the forward rate within the quarter cannot be ruled out by the data.

TABLE 1. Vector error correction models

	$(S_{t-1} - F_{t-2})^{a}$	Test of[b] unit error correction coefficient	Lagrange[c] multiplier test for serial correlation	R^2
Spot rates				
$\Delta \text{Spot}_{\text{DM}, t}$	0.054	9.361**	8.497	0.01
	(0.113)		(0.075)	
$\Delta \text{Spot}_{\text{Can\$}, t}$	0.095	9.919**	4.531	0.01
	(0.110)		(0.339)	
$\Delta \text{Spot}_{\text{Pound}, t}$	0.177	10.436**	10.269*	0.03
	(0.119)		(0.036)	
$\Delta \text{Spot}_{\text{SFranc}, t}$	0.028	9.858**	5.237	0.01
	(0.115)		(0.264)	
$\Delta \text{Spot}_{\text{Yen}, t}$	0.137	8.959**	2.266	0.02
	(0.109)		(0.687)	
Forward rates				
$\Delta \text{Forward}_{\text{DM}, t-1}$	0.964**	2.853	53.755**	0.99
	(0.013)		(0.001)	
$\Delta \text{Forward}_{\text{Can\$}, t-1}$	0.942**	2.862	41.328**	0.97
	(0.020)		(0.001)	
$\Delta \text{Forward}_{\text{Pound}, t-1}$	0.994**	1.220	48.165**	0.96
	(0.024)		(0.001)	
$\Delta \text{Forward}_{\text{SFranc}, t-1}$	0.956**	0.232	50.892**	0.98
	(0.014)		(0.001)	
$\Delta \text{Forward}_{\text{Yen}, t-1}$	0.966**	3.084	32.304**	0.94
	(0.027)		(0.001)	

Standard errors in parentheses.
[a] MacKinnon (1991) critical value is 4.042 at 1 percent significance and 3.412 at 5 percent significance.
[b] T-statistic for test of $\alpha = 1$ in equation 4. The critical values are from MacKinnon (1991) and are 4.042 at 1 percent significance and 3.412 at 5 percent significance.
[c] $\chi^2(4)$ test with p-value in parentheses.
* Indicates significance at 5 percent.
** Indicates significance at 1 percent.

In contrast, the spot rate in Table 1 has no significant predictive effects from the error correction component. Thus the exchange rate can be thought of as a strongly exogeneous process to the system according to our earlier definition in Section I. This finding is not surprising as we would not expect information in time $t - 1$ to affect the spot rate at time t. The dynamic process of the system ⟨4⟩ can then be described as exogenous 'news' affecting the spot rate which causes a temporary disequilibrium between the spot and forward rate that is eliminated by adjustments to only the forward rate. This table thus has two important findings. First it cannot reject the FRUC hypothesis. Further, it

TABLE 2. Single equation ECM, assuming forward exogeneity[a]

	$(S_{t-1}-F_{t-2})$	ΔF_{t-1}	ΔS_{t-1}	ΔF_{t-2}	R^2
$\Delta \text{Spot}_{\text{DM},t}$	0.891	1.462	−2.271	−0.290*	0.09
	(1.065)	(1.641)	(1.706)	(0.119)	
$\Delta \text{Spot}_{\text{Can\$},t}$	0.025	−1.310	1.376	−0.001	0.04
	(0.666)	(0.919)	(0.919)	(0.122)	
$\Delta \text{Spot}_{\text{Pound},t}$	−0.602	1.476	−0.647	−0.276*	0.12
	(0.619)	(0.897)	(0.886)	(0.120)	
$\Delta \text{Spot}_{\text{SFranc},t}$	0.764	0.158	−0.897	−0.213	0.04
	(0.994)	(1.598)	(1.589)	(0.129)	
$\Delta \text{Spot}_{\text{Yen},t}$	0.141	−0.344	0.357	−0.023	0.03
	(0.527)	(0.577)	(0.597)	(0.115)	

[a]Coefficients with standard errors in parentheses.
*Signifies a rejection at the 5 percent level.

points out that examining the error correction equation of the spot rates is going to lead the researcher to mistakenly reject the FRUC.[8]

To elaborate on this last point, we present the estimates of equation ⟨2⟩ in Table 2 that incorrectly assumes that the forward rate is strongly exogenous. This equation is identical to the single equation error correction model estimated in Hakkio and Rush (1989). Clearly none of the error correction coefficients are significant, but this does not mean that the FRUC can be rejected.[9] Instead, a single equation error correction model with the forward rate as a dependent variable and an assumed exogenous spot rate would result in a conclusion that supports forward rate unbiasedness, as shown in Table 1.[10]

III. Conclusion

In this paper, we present evidence supporting the hypothesis of unbiased long-run forward rates. We also test the unit cointegrating vector implied by the FRUC hypothesis in a VECM framework. We find strong support for a unit cointegrating vector. We also find support for a complete adjustment of forward rates to any short-run disequilibria between forward rates and spot rates. However, we find no significant adjustment in spot rates. This has two important implications. First, the forward rate adjustment is very quick and complete which is consistent with an efficient market. Secondly, the spot rate exogeneity implies that only forward rate adjustments appear to preserve the long-run equilibrium in the FRUC. This latter point may seem obvious, but we show in the paper how testing a single equation error correction model with the incorrect causality assumption will lead to an incorrect inference of no cointegration between the forward and spot rates. Therefore, it is always important to test the causality assumption in the VECM system prior to estimating single equation versions of the FRUC.

Notes

1. Actually, this characterization of the FREH is somewhat strong. In a recent paper, Brenner and Kroner (1992) show that stationary interest rate differentials in the underlying uncovered interest rate parity condition may lead to the presence of common features in the residuals of the cointegrating regression. Therefore, tests of FREH and FRUC in their work coincide. In the present paper, we maintain the distinction between the FREH and FRUC.
2. For the technical details of this discussion concerning error-correction models and statistical exogeneity, see Engle, Hendry, and Richard (1983) and Johansen (1992).
3. Notice that we only need to find one of these coefficients to be significantly different from zero, or that both are jointly different from zero, for the FRUC to hold. We do not need both to differ from zero. See Norrbin and Reffett (1994) for a formal treatment.
4. See Norrbin and Reffett (1994) for cases where the underlying VAR exceeds one.
5. Germany and Canada are from the IFS database and Japan, UK, and Switzerland are from the OECD database.
6. See the working draft of this paper (Norrbin and Reffett, 1994) for details.
7. Note that we use the MacKinnon (1991) distribution to test for the significance of the error correction term, as suggested by Kremers, Ericsson, and Dolado (1992) in the cases where the 'signal-to-noise' ratio is unknown. This is a conservative critical value to allow for the fact that the two variables are nonstationary in the null hypothesis and that the adjustment coefficients are zero.
8. For the sake of robustness, we estimated the VECM as a seemingly unrelated regression (SUR). The parameter estimates change only slightly, and the null of unitary adjustments is rejected only in two cases due to the increase in precision of the estimates. One explanation for these differences could be that the true coefficient indicates less than complete adjustment. This would imply a very small forward rate stickiness for two countries. On the other hand, the MacKinnon (1991) critical values may be too low for a system procedure such as SUR. In either case, we can conclude that forward rates almost completely adjust and that a unit cointegrating vector appears consistent with the data for all five countries.
9. It is surprising, however, that the past forward rate is marginally significant for both the German mark and the pound. This would imply a forecastable spot rate.
10. The analogous single equation error correction model with assumed spot rate exogeneity would involve a spot rate lead as a regressor. As this violates the information requirements, the single equation error correction model becomes identical to ⟨4b⟩.

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