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# The Inflation Tax in a Convex Model of Equilibrium Growth

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A general equilibrium monetary economy is described that exhibits equilibrium growth. The production technology is consistent with Jones and Manuelli (1990) and Rebelo (1991). The inflation tax and nominal interest rates are shown to be inversely related to the equilibrium growth rates of real variables until a critical point is reached. At this point, equilibrium growth is eliminated and the predictions of Stockman (1981) and Abel (1985) concerning the 'level effects' of distortionary monetary policies appear. For the nominal side of the model, a version of the prediction of Friedman (1969) is shown to hold for a particular balanced growth path. Following King and Rebelo (1990), a linear growth version of the model is calibrated to illustrate the potentially large growth and welfare effects of moderate inflations.

## Introduction

In a series of papers by Stockman (1981), Abel (1985), Coleman (1990) and Marquis and Reffett (1992a), it has been shown that the level of real output in the steady state is inversely related to the inflation tax when credit markets for investment are imperfect.[1] The transmission mechanism in these models for the inflation tax is the 'investment channel' as opposed to the 'labour–leisure' channel discussed in Cooley and Hansen (1989, 1991).[2] We extend this work on the investment channel to examine the relationship between the inflation tax and equilibrium economic growth. We describe a pure currency cash-in-advance economy with single-sector production that is consistent with technological primitives described in the convex models of economic growth discussed in recent papers by Jones and Manuelli (1990) and Rebelo (1991).

Extending the work of Stockman (1981) and Abel (1985), we show that for the real sector nominal interest rates and inflation taxes are generally inversely related to the rate of economic growth for all real variables along the equilibrium growth path. This relationship is shown to be monotonic up to a point where the monetary distortion eliminates growth altogether. From this latter point, additional increases in the inflation tax and nominal interest rates are then shown to generate the 'level' effects on real output discussed in Stockman (1981) and Abel (1985).[3] For the nominal side of the model, the equilibrium is shown to deliver a modified version of the prediction in Friedman (1969): (1) the optimal growth rate in the money stock is related to the rate of time preference (and other preference parameters); (2) at this growth rate the inflation tax is zero and all real variables grow at their Pareto-optimal rates; and (3) over this competitive equilibrium balanced growth path deflation in prices occurs.

In addition, we calibrate a linear growth version of the model following King and Rebelo (1990) to assess the quantitative importance of monetary policies influencing the inflation tax on long-run equilibrium growth rates and social welfare. We find that distortionary monetary policies can introduce large reductions in both economic growth and social welfare. For example, using 5% inflation as a benchmark and logarithmic preferences, we find that a 5% increase in inflation past the benchmark reduces equilibrium growth by 0·62%. This corresponds to a reduction in social welfare measured in terms of lost consumption of 4·5–4·75% depending on whether welfare is measured as in King and Rebelo (1990) or Cooley and Hansen (1989, 1991).

In the next section of the paper, we describe the economic environment, including preferences, technology and the underlying financial structure. In Section II representative household decision problems are characterized and the definition of competitive equilibrium balanced growth is formulated. In Section III we construct the dynamic equilibrium growth rates for real and nominal variables for a special case of the economy, and we derive asymptotic growth conditions that are shown to generalize Stockman (1981) and Abel (1985). Section IV calibrates a linear growth version of the model to quantify the implications of the inflation tax in a linear growth model. Section V concludes.

## I. A Description of a Monetary Economy with Convex Growth

The model is formulated in discrete time as an infinite-horizon perfect-foresight single-sector growth model. The underlying trading environment for the model follows the pure currency cash-in-advance framework described in Stockman (1981) and Abel (1985). One major difference in this model is that it incorporates a single-sector version of the production technology described in Jones and Manuelli (1990). As in Stockman (1981), liquidity constraints are applied to investment purchases as well as consumption purchases. Following the notational convention for general equilibrium economies with distortions, we distinguish the behaviour of the aggregate economy and that of individual agents.[4] Aggregate *per capita* decision variables that are taken as given by the representative agent are notated by capital letters, while the individual's decision variables are denoted by lower-case letters. Of course, in a competitive equilibrium the decision rules for all agents will be identical.

There are three distinct goods pertinent to representative agent decisions in any period $t$: labour $l_t$, capital $k_t$ and consumption goods $c_t$. All consumption goods $c_t$ and investment goods $x_t$ must be purchased with cash. The number of agents and firms that populate the economy is assumed to be large, so markets for labour, capital and consumption goods are all competitive. For simplicity, we shall assume that labour is constrained to be unity for all periods, and is supplied inelastically in each period.

The production technology available to households is summarized by the function $Y_t=f(K_t, l)$, where $Y$ is the *per capita* level of real output, $K_t$ is taken to be the aggregate *per capita* capital–labour ratio at the beginning of period $t$ that is rented by firms from representative households that begin period $t$ with capital–labour ratios $k_t$, and $L_t=1$ for all $t$. Let $K_{t+1}=(1-\delta)K_t+X_t$, with $\delta$ being the depreciation rate for capital and $X_t$ the aggregate *per capita* investment. The technology $f$ is assumed to satisfy the following assumption:

*Assumption 1.* $f(K_t)$ is a weakly concave, increasing, continuous and continuously differentiable function such that $f(K_t) = bK_t + q(K_t)$, where $b \in \mathbb{R}_{++}$ and $q(K_t)$ is a weakly concave, increasing, continuous, continuously differentiable, constant-returns-to-scale technology in capital and labour that satisfies (i) $q'(0) = \infty$, (ii) $\lim_{K \to \infty} q'(K_t) = 0$, and (iii) $q(0) = 0$, $q(K_t) > 0$ when $K_t > 0$, (iv) $\exists a \bar{K} > 0$ such that: $K < q(K) \leq \bar{K}$ for all $K \leq \bar{K}$ and $q(K) < K$ for all $K > \bar{K}$.

Assumption 1 allows for the possibility of sustained growth as in Jones and Manuelli (1990). If $b$ is not sufficiently large, the technology is simply a version of the neoclassical technology assumed in Stockman (1981) and Abel (1985). As in Jones and Manuelli, if $Y$ is the set of feasible choices of $c_t$ and $x_t$, then if $K_0 > 0$, Assumption 1 implies that there exists a sequence of constraints $B = \{B_t\}_{t=0}^{t=\infty}$ such that, for all $t$, $c_t + x_t \leq B_t$.

Given the sequence $B$, household period preferences are described by a period utility function $u$. For convenience, we make the following assumption about each household's period preferences:

*Assumption 2.* Households have period preferences summarized by a function $u: \mathbb{R}_+ \to \mathbb{R}$ where $u$ is monotone increasing in its argument, continuous, bounded below, $\lim_{c \to 0} u'(c) = \infty$, and there exists $\gamma \in \mathbb{R}_{++}$ and $\hat{U} < \infty$ with $\gamma\beta < 1$ such that $u(B_t) \leq \hat{U} + \gamma^t \forall B_t$. Each household's intertemporal preferences are then represented by

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0, 1)$.

In Jones and Manuelli (1990) a restriction on preferences and production technologies is necessary to generate convex endogenous growth. They call this restriction condition G (see Jones and Manuelli 1990, p. 1014). We will make the identical restriction on technology. For Pareto-optimal versions of our monetary economy, the following assumption will be shown to imply equilibrium endogenous growth:

*Assumption 3 (Condition G):* Preferences and technology are such that

$$\lim_{K_t \to \infty} \beta[1 - \delta + f'(K_t)] > 1.$$

Assumption 3 is essentially a restriction on the linear growth term $b$ in Assumption 1. Assumptions 1–3 describe a set of conditions sufficient to guarantee that, absent equilibrium distortions, investment in each period is strictly positive and $\lim_{t \to \infty} c_t = \infty$ (see Jones and Manuelli 1990, theorem 1, (a)). To anticipate our results, under Assumption 3 it can be shown that there is a competitive equilibrium balanced growth path such that endogenous growth is displayed if the nominal rate of interest is below a critical value.

With the production technologies specified, we can now consider the payments system. Each household is given an initial endowment of fiat money $M_0$. Fiat money is used by households to facilitate the acquisition of consumption and investment goods in their appropriate markets. Fiat money is taken to be an intrinsically worthless piece of paper that is used to communicate outside indebtedness. Let a representative household's next period holdings of money be denoted by $m^d_{t+1}$, and denote the aggregate *per capita* money stock at time $t$ by $M_t$. Let the aggregate *per capita* gross monetary growth rate at date $t$ be given by a function $h(K_t)$. The function $h$ is assumed to satisfy the following:

*Assumption 4.* $h: \mathbb{R} \to \mathbb{R}_{++}$ is a continuous function such that $h(K_t) \geq \beta$.

It is assumed that the single activity of government is to supply fiat money. The law of motion governing the aggregate *per capita* money stock is

$$(2) \qquad M_{t+1} = h(K_{t+1})M_t.$$

At the beginning of each period, households are assumed to receive a lump-sum monetary transfer $z_t = [h(K_t) - 1]M_{t-1}$.[5] Then the household begins each period with a post-transfer stock of money $m_t = m^d_t + z_t$.

*The aggregate economy*

The behaviour of the aggregate economy is taken as given by each household when solving its representative decision problems. It is assumed to evolve recursively according to a set of stationary functions. The money price of consumption goods (and investment goods), the real rental price of capital and real wage payments are given, respectively, by the following functions:

$$(3) \qquad P(K_t) = p(K_t)M_t$$

$$(4) \qquad W(K_t) = f'(K_t)$$

$$(5) \qquad W_L(K_t) = [f(K_t) - K_t f'(K_t)],$$

where $p: \mathbb{R} \to \mathbb{R}_{++}$ is the inverse of real balances. The aggregate capital stock expressed in terms of the aggregate *per capita* capital–labour ratio is assumed to evolve recursively according to the following law of motion:

$$(6) \qquad K_{t+1} = G(K_t) + (1-\delta)K_t.$$

Then, for $K_0 > 0$, the state of the aggregate economy at time $t$ is completely characterized by $K_t \in [0, B_t]$ where $B_t \in \mathbb{R}_{++}$.

## II. Household Decision Problems and Competitive Equilibrium

To describe each representative household's decision problem, we begin by constructing its feasible correspondence for any period $t$. Let $\hat{m}_t = m_t/M_t$. The state of a household at the beginning of time $t$ is denoted by $s_t = (k_t, \hat{m}_t, K_t) \in [0, B_t] \times \mathbb{R}_{++} x[0, B_t]$. Then the partial history of a household at date $T$ is $\mathbb{S}_{hT} := \Pi^T_{t=0}\,[0, B_t] \times \mathbb{R}_{++} \times [0, B_t]$. We can then write the household's budget constraint as:

$$(7) \qquad p(K_t)[c_t + k_{t+1} - (1-\delta)k_t] + m^d_{t+1}/M_t \leq \hat{m}_t + p(K_t)[f(K_t) + (k_t - K_t)f'(K_t)].$$

In addition, the cash-in-advance constraint can be written as follows:

$$(8) \qquad p(K_t)[c_t + k_{t+1} - (1-\delta)k_t] \leqslant \hat{m}_t.$$

The standard non-negativity constraints on consumption, capital and money holdings e given by

(9) $c_t, k_t, m^d_{t+1} \geq 0$ for all $t$.

Consider stationary solutions to the household's problem; i.e. let the vector of household decision rules be given by $\mathbf{q}(s_t) = [c(s_t), g(s_t), d(s_t)]$ where $c(s_t)$ is a stationary consumption decision rule, $g(s_t)$ is a stationary capital decision rule and $d(s_t)$ is a stationary function governing the ratio of individual next-period money holdings relative to the aggregate *per capita* holding $m_t/M_t$. Then let $\Phi(s_t) \subseteq \mathbb{R}^3_+$ be each household's feasible correspondence; i.e. $\Phi(s_t)$ is the set of $\mathbf{q}(s_t)$ that satisfy (7)–(9). Let $V(s)$ be the value of (1) at the optimum given functions $p(K)$, $h(K)$, $G(K)$, $W_L(K)$, $W(K)$ and $C(K)$ for any individual household entering the period in state $s$. Then $V(s)$ must satisfy the following functional equation:

$$V(s) = \sup_{\mathbf{q}(s) \in \Phi(s)} \{u(c) + \beta V(s')\},$$

where $s'$ is next period's state of the individual.[6] Carrying out the maximization, the household policy functions $c(s_t)$, $g(s_t)$ and $d(s_t)$ must satisfy the following optimality conditions:

(10) $u'(c(s_t)) = [\lambda(s_t) + \phi(s_t)]p(K_t)$

(11) $[\lambda(s_t) + \phi(s_t)]p(K_t) = \beta[\lambda(s_{t+1}) + \phi(s_{t+1})]p(K_{t+1}) \times \{1 - \delta + [\lambda(s_{t+1})/(\lambda(s_{t+1}) + \phi(s_{t+1}))]f'(K_{t+1})\}$

(12) $\lambda(s_t) = \beta\{[\lambda(s_{t+1}) + \phi(s_{t+1})]/h(K_{t+1})\}$

(13) $p(K_t)[c(s_t) + g(s_t)] = 1,$

where $\lambda$ and $\phi$ are the Lagrange multipliers associated with (7) and (8) respectively,

$$s_{t+1} = \{g(s_t), d(s_t) + [h(K_{t+1}) - 1]/h(K_{t+1}), K_{t+1}\},$$

where $K_{t+1} = (1 - \delta)K_t + G(K_t)$. The one-period nominal interest rate can be obtained by constructing an intertemporal valuation equation for monetary assets. Following Townsend (1987, Section 5), it is straightforward to show that the nominal rate of interest, $r(s)$, can be defined as follows:[7]

(14) $[(\lambda(s) + \phi(s))/\lambda(s)] = 1 + r(s).$

Then, combining (10), (11) and (14), the intertemporal valuation equation for capital is given by

(15) $[u'(c(s_t))/\beta u'(c(s_{t+1}))] = \{1 - \delta + [f'(K_{t+1})/(1 + r(s_{t+1}))]\},$

where the left side of (15) is the intertemporal marginal rate of substitution and the right side represents the distorted marginal rate of technical substitution (or, equivalently, the one period after inflation tax return on investment goods).

As in Jones and Manuelli (1990), we are interested in a particular equilibrium path. Specifically, the competitive equilibrium balanced growth path is defined as follows:

*Definition.* A *recursive competitive equilibrium* for this economy is a set of stationary functions for prices $P(K_t)$, $W(K_t)$ and $W_L(K_t)$ with $0<P(K_t)<\infty$, and an aggregate investment decision rule $G(K_t)$, an aggregate consumption decision rule $C(K_t)$, a monetary policy function $h(K_t)$, a collection of functions for household decision rules $c(s_t)$, $d(s_t)$ and $g(s_t)$, and a value function $V(s)$ such that

(i) *individual optimization:* Given the functions $G$, $W$, $W_L$, $p$ and $h$, each household maximizes (1) subject to $\mathbf{q}(s_t)\in\Phi(s_t)$; and
(ii) *market clearing:* $G(K_t)=g(s_t)$, $d(s_t)=1$ and $c(s_t)=C(K_t)$, and $c$ and $g$ satisfy

$$C(K_t)+G(K_t)=f(K_t)\forall t. \tag{16}$$

Notice that, to characterize the competitive equilibrium explicitly, we must restrict our attention to recursive competitive equilibria with constant distortions (as in Jones and Manuelli 1990 Section 4). As will be evident in the following section, to construct balanced growth paths where some or all of the variables in the model grow at the same rate along a dynamic path, we must fix the nominal rate of interest, which amounts to fixing the monetary distortion in the model. It will also be shown that, along an equilibrium path with a constant distortion, the asymptotic growth rate for all variables will be the same.

## III. Construction of the Competitive Equilibrium Growth Paths

### *Dynamic behaviour of growth rates*

To focus our results, we shall restrict our consideration to recursive competitive equilibria with constant distortions ($r(K_t)=\bar{r}$ for all $t$). Further, we will restrict our attention to dynamic paths where consumption, investment and output all grow at the same rates. For such equilibria, it can be shown that the growth rates for these real magnitudes are monotonically decreasing along a dynamic equilibrium growth path and asymptotically converge to a strictly positive constant. To formulate our results more concisely, we impose the following specialization of the economy described in Sections I and II:

*Assumption 5.* Period preferences are given by $u(c_t)=(c_t^{1-\sigma}-1)/(1-\sigma)$ for $\sigma>1$ and $\ln c_t$, for $\sigma=1$. The production function is $f(K)=bK+K^{\alpha}$, $\alpha\in[0, 1]$.

We begin by constructing the balanced growth path for output, consumption and investment given a constant distortion $\bar{r}$. Let $\theta_t$ be the gross growth rate in $C$. Then, given the distortion $\bar{r}$, (10), (11) and Assumption 4 imply that the growth rate $\theta_t$ varies over the transition path according to a stationary function $\theta(K_{t-1}, r)$, and is given by

$$\begin{aligned}\theta_{t+1}&=\theta(K_t, \bar{r})=[C(K_{t+1})/C(K_t)]\\&=(\beta[1-\delta+\{b+\alpha[(1-\delta)K_t+G(K_t)]^{\alpha-1}\}(1+\bar{r})^{-1}])^{1/\sigma}\geq 1.\end{aligned} \tag{17}$$

Then the equilibrium condition (16) implies that a balanced dynamic growth path can be constructed such that the gross growth rate in investment and output is $\theta_t$ if suitable restrictions on the growth rate in the capital stock are made.[9] To show that real balances $M/P$ are also growing at the same rate $\theta_t$ along this particular dynamic path, note that (13) implies

(18) $$\{p(K_{t+1})[C(K_{t+1})+G(K_{t+1})]/p(K_t)[C(K_t)+G(K_t)]\}=1.$$

Then (18) implies that $p(K_t)$ grows at the inverse of the rate of $C(K_t)$ and $G(K_t)$. Since $p(K_t)$ is defined as the inverse of real balances, $M_t/P_t$ must grow at the rate $\theta(K_{t-1}, \bar{r})$.

To examine the behaviour of nominal variables along this path with a constant distortion, we find a policy function $h(K_t, \bar{r})$ and a gross inflation rate $\pi_{t+1}$ (measured as the ratio of next period's price level to the current price level $P_{t+1}/P_t$) that absorb the effects, period by period, of equilibrium growth via the quantity equation. To find the function $h$, rewrite (12) using (10) and (14) as

(19) $$[p(K_{t+1})C(K_t)^{-\sigma}/p(K_t)C(K_{t+1})^{-\sigma}]=\beta(1+\bar{r})/h(K_{t+1}, \bar{r}).$$

Since $p_t$ grows at the inverse rate of $C_t$, then along this growth path (17) and (19) imply that, given $K_0$, the monetary policy that uniquely implements the distortion $\bar{r}$ is given by

(20) $$h(K_t, \bar{r})=\beta(1+\bar{r})\theta(K_t, \bar{r})^{1-\sigma}.$$

Notice that (20) implies that an optimal monetary policy (a monetary policy that generates zero nominal interest rates) sets $h=\beta\theta_t^{1-\sigma}$. When $\sigma=1$ (logarithmic preferences), or when $b$ is sufficiently small to generate no equilibrium growth (i.e. $\theta_t=1\ \forall t$), then the standard optimal policy rule applies: set $h^*=\beta$. For the more general case where $\sigma=1$ and $\theta_t>1$, optimal monetary policy is a function of both the rate of time preference and the intertemporal elasticity of substitution.

To see what is happening to the inflation rate $\pi$ along the balanced growth path, rewrite (3) as

(21) $$[p(K_{t+1})/p(K_t)]=\pi_{t+1}/h(K_{t+1}, \bar{r}).$$

Then, since the inverse of real balances $p(K_t)$ grows at rate $[\theta(K_{t-1}, \bar{r})]^{-1}$, we know the stationary function for the inflation rate $\pi$ is

(22) $$\pi_t=[h(K_t, \bar{r})/\theta(K_{t-1}, \bar{r})].$$

Since $\bar{r}$ determines $\theta$, and $\theta$ and $\bar{r}$ (along with $K$) determine $h$, $\pi$ is determined along a path for $\bar{r}$.

*The modified asymptotic growth conditions*

To discuss how monetary distortions alter long-run growth rates for real variables, we calculate the limiting expressions for (17), (20) and (22) given Assumptions 1 and 3 and the distortion $\bar{r}$. Under Assumptions 1–3 investment is strictly positive for each period $t$, $K_{t+1}$ is growing over time, and the asymptotic marginal product of capital is $[1-\delta+b(1+\bar{r})^{-1}]$. Therefore the asymptotic growth rate is independent of $K$; i.e. $\lim_{K\to\infty}\theta(K, \bar{r})=\theta(\bar{r})$. By (17), $\theta(\bar{r})=\{\beta[1-\delta+b(1+\bar{r})^{-1}]\}^{1/\sigma}$ and $h(\bar{r})$ and $\pi(\bar{r})$ are constants given by (20) and (22) respectively.

To examine the relationship between monetary distortions and asymptotic growth, we can consider the following special cases for monetary policy:

*Case 1:* $h=\beta\theta^{1-\sigma}$ (the Friedman Rule).

Equation (20) implies that $\bar{r}=0$ for such a monetary policy, and therefore (17) implies that $\theta=[\beta(1-\delta+b)]^{1/\sigma}$. Then under Assumption 3 (Condition G), $\theta>1$ and the model exhibits endogenous growth. Furthermore, (17) implies that this growth rate corresponds to the Pareto-optimal growth rate. As for nominal variables, note that $\pi=h/\theta<1$ along this path; that is, deflation is occurring along this balanced growth path where monetary policy is optimal, such that all allocations are Pareto-optimal.

*Case 2:* $h=\beta(1+\bar{r})\theta^{1-\sigma}$ for $0<\bar{r}<r^*$ with $r^*$ being the $\bar{r}$ that satisfies $\{\beta[1-\delta+b/(1+\bar{r})]\}^{1/\sigma}=1$.

In this case, Assumption 3 and (17) imply that

$$\theta(\bar{r})=\{\beta[1-\delta+b(1+\bar{r})^{-1}]\}^{1/\sigma}>1;$$

that is, asymptotic growth is still occurring along this path although it is lower than in Case 1. The monetary distortion alters capital decisions and therefore generates both asymptotic growth and level effects. Along these paths, it is not clear that prices are falling. To see this, (20) and (22) imply that

$$\pi=[\beta(1+\bar{r})\theta(\bar{r})^{-\sigma}]. \tag{23}$$

Therefore, depending on $\delta$, $b$, $\sigma$ and $\bar{r}$, the price level could be increasing, decreasing or constant along the balanced growth path.

*Case 3:* $h\geq\beta(1+r^*)\theta(r^*)^{1-\sigma}=\beta(1+r^*)$.

This monetary policy $h$ corresponds to the case where $\bar{r}>r^*$. In this case, (17) implies that $\theta(r^*)=1$; that is, the distortion eliminates asymptotic growth given that there is only one steady-state level of capital consistent with (17). Equation (21) implies that $\pi=h$ since $p_t/p_{t+1}=1$ for zero asymptotic growth. Therefore in this case the economy converges to a steady state described in Stockman (1981) and Abel (1985). For this case, we have Stockman and Abel's results on 'level' effects on equilibrium real output and capital as special cases of our economy that are consistent with asymptotic growth. That is, higher monetary growth rates lower the steady-state level of both real output and capital.

## IV. Moderate Inflations, Growth and Welfare: The Linear Technology Case

In this section, a numerical example is used to illustrate the predicted relationship between inflation and asymptotic growth. Welfare calculations are then performed for the special case of linear technology.[10] The model is calibrated using parameter settings for the postwar US economy constructed by King and Rebelo (1989, 1990). These include a net growth rate for the economy of 2%, a depreciation rate of 10% and a return on capital of 6·5%. In general, the gross rate of return on capital is a function of the distortion, and is denoted $R(\pi)$ below. The benchmark setting for the net inflation rate is taken as 5%.[11]

The gross rate of return on capital can be written as a function of the inflation factor by expressing the nominal interest rate in terms of inflation, where the mapping from $\pi$ to $\bar{r}$ can be found from (20) and (22):

(24) $$R(\pi)=\{b/[1+\bar{r}(\pi)]\}+(1-\delta)=(\beta b/\pi\theta^{\sigma})+(1-\delta).$$

Using the benchmark values for the gross rate of return on capital, $R(\pi_{bm})$, and the gross growth rate, $\theta(\pi_{bm})$, a relationship between the discount factor, $\beta$, and the intertemporal elasticity of substitution, $\sigma$, can be found by substituting (24) into (17):

(25) $$\beta=\theta_{bm}^{\sigma}/R(\pi_{bm}).$$

Given $\beta$, $\sigma$ and the benchmark values of $\theta_{bm}$ and $\pi_{bm}$, the limiting marginal product of capital, $b$, can be found from (24) and (25):

(26) $$b=[(\theta_{bm}^{\sigma}/\beta)-(1-\delta)]/[(\beta/\pi_{bm}\theta_{bm}^{\sigma})].$$

For any known distortion, $\pi$, (26) is in general a nonlinear equation in $\theta$ where $b$, $\beta$, $\delta$ and $\sigma$ are given parametrically. Sensitivity analysis is performed by recalibrating the model for alternative preferences by allowing the intertemporal elasticity of substitution to range over $1/\sigma\in[1, 1/2{\cdot}5]$. Results are reported in Table 1 for logarithmic preferences and for the case where $\sigma=2{\cdot}5$.

When preferences are logarithmic, Table 1 indicates that eliminating the benchmark monetary distortion in this economy ($\pi=1{\cdot}05$) by choosing a Pareto-optimal money rule ($h=\beta$ in Case 1) would raise the growth rate of the economy by 1·87% (from 2% to 3·87%). By reducing the (net) inflation rate from 5% to zero, growth would increase by 0·68% (from 2% to 2·68%). Similar large growth effects (examples of Case 2) are evident in the table for

TABLE 1
GROWTH EFFECTS OF MODERATE INFLATIONS
Benchmark $\pi_{\text{net}}=5\%$

| Asymptotic growth rates (%) | | | Cooley and Hansen's measure of welfare loss,[a] $\Lambda_{ch}$ (%) | |
|---|---|---|---|---|
| $\pi_{\text{net}}$ | $\sigma=1$ | $\sigma=2{\cdot}5$ | $\sigma=1$ | $\sigma=2{\cdot}5$[b] |
| −7·79[c] | 3·87 | 2·75 | −6·72 | −2·44 |
| 0 | 2·68 | 2·28 | −3·84 | −1·49 |
| 5 | 2·00 | 2·00 | 0 | 0 |
| 10 | 1·38 | 1·75 | 4·75 | 1·97 |
| 15 | 0·80 | 1·52 | 10·25 | 4·56 |
| 20 | 0·27 | 1·31 | 16·20 | 7·65 |

[a] Welfare calculations using the welfare measure in Cooley and Hansen (1989).
[b] The reduction in the distortionary effects for $\sigma=2{\cdot}5$ versus $\sigma=1$ are due primarily to the calibration procedure's implied $\beta$ lowering the welfare effects on the growth rate $\theta$ of distortionary policies. That is, the calibration procedure described in King and Rebelo (1990) for $\sigma=2{\cdot}5$ has the undesirable feature of yielding a higher implied $\beta$, in this case $\beta=0{\cdot}9866$. For $\sigma=1$, $\beta=9577$.
[c] Non-distortionary rate of inflation consistent with the Friedman rule that nominal interest rates are zero. This sets the gross inflation rate to 0·9221%, yielding a net inflation rate of −7·79%.

higher inflation rates. For a 10% increase in the inflation rate to 15% from the benchmark 5% level, growth declines by 1·20% (from 2% to 0·80%). Finally, the inflation rate that just eliminates growth (Case 3) is 22·62% (not reported in Table 1). These growth effects are less pronounced as the degree of intertemporal elasticity of substitution decreases, implying that households have a higher rate of discount (lower $\beta$). Refer to Table 1 for $\sigma = 2\cdot5$. In this case, optimal monetary policy raises growth from the benchmark 2% level to 2·75%. Since the growth rate is less sensitive to the distortion when $\sigma = 2\cdot5$, the inflation rate would now have to rise up to 64·22% to eliminate growth.

In a representative agent model, social welfare can be measured by calculating lifetime utility from (1).[12] One measure of the effect of distortions on welfare could be found by comparing the consumption path for the benchmark settings with the consumption path under an alternative distortion. Define $u(\bar{c}_t)$ as period $t$ utility for the benchmark settings, and $u(\hat{c}_t)$ as period $t$ utility under an alternative distortion. Then, following Cooley and Hansen (1989, 1991), a measure of welfare losses relative to the benchmark settings could be taken as the percentage of consumption forgone each period induced by the change in the distortion, which we denote by $\Lambda_{ch}$. For an initial $k_0$, $\Lambda_{ch} = 100\% \cdot x_{ch}$, where $x_{ch}$ solves $\Sigma\beta^t u[\hat{c}_t(1 + x_{ch})] = \Sigma\beta^t(\bar{c}_t)$.[13]

For this example economy, the welfare losses associated with even moderate inflation are seen to be large.[14] To illustrate this point, the results from column 3 of Table 1 are compared with the welfare losses induced by inflation taxes in Cooley and Hansen (1989) and in Marquis and Reffett (1992a). For example, Cooley and Hansen (1989) examine the welfare costs of moderate inflation in a standard neoclassical growth model, where the channel by which the inflation tax affects output is via a labour–leisure decision when consumption goods are liquidity-constrained. They find that a 10% inflation rate is associated with a reduction in period consumption ($\Lambda_{ch}$) of 0·38%. Under the same calibration, Marquis and Reffett (1992a) find that replacing the labour–leisure channel with the investment channel in the standard neoclassical growth model increases these costs approximately 50%; i.e. $\Lambda_{ch} \cong 0\cdot56\%$.[15] By using the numbers in Table 1, the welfare effects of a 10% inflation rate relative to optimal rate of deflation (−7·79%) in our equilibrium growth model generates over a thirty-fold reduction in period consumption (to $\Lambda_{ch} = 11\cdot47$[16]) relative to a similar comparison for the Cooley and Hansen labour–leisure channel in a standard neoclassical model, and more than twenty times the welfare loss predicted by the models of Stockman (1981) and Abel (1985). For our growth model, a moderate increase in inflation of 5% is seen to be associated with a 4·75% decrease in consumption from the benchmark.[17]

## V. Conclusion

In this paper, we demonstrate that distortionary monetary policies can have a profound impact on the growth rates of economies with financing constraints on the acquisition of investment goods. We generalize the work of Stockman (1981) and Abel (1985) to the context of neoclassical growth models that potentially exhibit asymptotic economic growth. Specifically, the inflation tax associated with positive nominal interest rates encourages households to consume more today and reduce investment. This friction in the financing of investment projects can impose binding liquidity constraints on households that drives a 'wedge of inefficiency' in the optimality conditions governing both consumption and investment decisions when inflation taxes are positive. In the context of a model with endogenous growth, the welfare loss is magnified since lower investment implies lower transitional and asymptotic growth rates. This lower level of investment generates potentially large reductions in social welfare associated with moderate inflations.

The 'level' effects of anticipated inflation discussed in Stockman (1981) and Abel (1985) are shown to be special cases of a general result in a monetary economy endowed with production technologies similar to those of Jones and Manuelli (1990) and Rebelo (1991). In a convex model of endogenous growth, positive nominal rates produce the 'level' effects as described in Stockman (1981) and Abel (1985) when the distortions are sufficiently large to eliminate the potential for endogenous growth. Finally, for the special case of linear growth, we show that this inflation rate could be as low as 20%.

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## NOTES

1. Empirical work documenting the negative correlation between economic growth and inflation includes Easterly *et al.* (1991) and Degregorio (1991, 1992).
2. Additional recent research into Arrow–Debreu economies with equilibrium-financing constraints on investment goods include the liquidity effects models of Fuerst (1992) and Christiano (1991), the endogenous growth models of Marquis and Reffett (1991, 1992b), Easterly *et al.* (1991), Easterly (1991), and the asset-pricing models of Brock and LeBaron (1989) and Refett (1992).
3. There are unique alternative channels by which inflation could negatively affect long-run economic growth. One important channel not discussed here is outlined in a series of papers by Bencivenga and Smith (1990, 1991). In these papers, monetized deficits and financial repression via reserve requirements can generate many of the relationships between nominal and real magnitudes discussed here. They show that there are similarities between the financial repression in their model and the stylized facts discussed in McKinnon (1973).
4. See e.g. Stokey *et al.* (1989), Cooley and Hansen (1989, 1991), and Coleman (1990, 1991).
5. As is common in the literature, we are using a standard device in public finance of a lump-sum transfer to make monetary transfers independent of individual decision rules on $k$; that is, household decisions concerning $k$ do not change the aggregate $K$.
6. Using Assumptions 1–3 and Theorem 5 from Blackwell (1965), it is straightforward to prove that there exists a function $V(s)$ that satisfies the above functional equation; that this $V(s)$ is strictly increasing and strictly concave in $k$ and $m$; and that $V(s)$ is continuously differentiable in both $k$ and $m$.
7. Following Coleman (1990), we could define the inflation tax as $\tau(S) = [\phi(S)/(\lambda(S) + \phi(S))]$. Then (10), (12) and (14) would imply $1 - \tau(S) = [1/(1 + r(S))]$. Therefore inflation taxes and nominal rates are related measures of the monetary distortion.
8. Notice that $(1 - \alpha)$ is the labour share for this constant-returns technology.
9. This is for the dynamic path where consumption, investment and output grow at identical rates. For this path, (16) can be shown to imply that $[(bK_t + K_t^\alpha)/(bK_{t-1} + K_{t-1}^\alpha)] = \theta(K_t, r)$ where $\theta$ is given by (17). Solving this equation implicitly for $K_t$ as a function of $K_{t-1}$, we can obtain the implied growth rate for the capital stock along this dynamic path, which is not in general equal to $\theta_t$ (except asymptotically).

10. Jones and Manuelli (1990) have argued that changes in the asymptotic growth rate arising from distortions may not have significant welfare effects. This would be true if the initial capital stock from which lifetime utility calculations were made was relatively 'low' so that the linear component of the production technology contributed very little to the period marginal product of capital in the early periods of the calculation when period utility levels were not heavily discounted. However, if this initial capital stock were relatively 'high', then the marginal product of capital would be dominated by the linear or asymptotic, term and these distortionary effects on lifetime utility could be very pronounced. King and Rebelo (1989) have argued that transitional dynamics cannot play an all-important role in determining long-run growth, since this would imply counterfactual evidence, such as the extraordinary returns to capital that would have been required at some time in the past to account for observed growth rates. They opt instead for models that rely on purely linear technology. In King and Rebelo (1990), they then examine welfare losses associated with marginal income tax distortions in these models and find them to be extremely large. Changes in welfare arising from changes in the marginal income tax distortions are independent of the initial state of the economy (initial capital stock) in linear models. This is also true for the monetary distortions examined in this paper.
11. This is approximately equal to the average annualized, monthly inflation rate of 4·78% for the US economy over the period 1959(1)–1991(6).
12. This approach to calculating social welfare is limited in the sense that it fixes a functional form for $u$, and then calculates welfare losses measured as lost consumption based on the particular $u$. A monotonic transformation of $u$ might alter the numerical results.
13. A variant of this measure is employed by King and Rebelo (1990), where welfare losses are measured as the percentage consumption that would have to be given up each period to render the benchmark lifetime utility equivalent to the lifetime utility that would be attained by the household with the economy under the alternative distortion. This is given by the value $\Lambda_{kr}=100\%\cdot x_{kr}$ where $x_{kr}$ solves

$$\Sigma\, \beta^t u(\hat{c}_t)=\Sigma\, \beta^t[\bar{c}_t(1-x_{kr})].$$

It is noteworthy that there is both a quantitative and a qualitative difference between $\Lambda_{ch}$ and $\Lambda_{kr}$ induced by the curvature of the utility function. Both $\Lambda_{ch}$ and $\Lambda_{kr}$ exhibit nonlinear relationships with the inflation rate, such that an incremental increase in inflation induces a larger reduction in 'welfare' at high inflation rates than in low inflation rates. However, for log utility, the Cooley–Hansen measure suggests larger welfare losses than the King–Rebelo measure associated with inflation rate increases above the benchmark rate, whereas the reverse is true of welfare gains for inflation rate reductions below the benchmark rate. For example, in comparison with the Cooley–Hansen measure reported in Table 1, for $\sigma=1$ and $\pi=FR$, $\Lambda_{kr}=-7\cdot16$; whereas for $\sigma=1$ and $\pi=20\%$, $\Lambda_{kr}=13\cdot94\%$. The more curvature to the utility function, i.e. the higher is $\sigma$, the greater the disparity between these two measures. For $\sigma=2\cdot5$, the cost of eliminating growth yields values for $\Lambda_{kr}=38\cdot95\%$ and $\Lambda_{ch}=64\cdot22\%$.
14. This numerical example is intended only to highlight the potential importance of the inflation tax via this 'investment channel' on welfare. There are many ways that this effect can be mitigated, but not completely eliminated. For example, in Marquis and Reffett (1992b), private financial arrangements that facilitate investment goods transactions are allowed to develop endogenously in response to the inflation tax.
15. Marquis and Reffett (1992a) also find that these two channels yield roughly the same welfare losses when approximately 65% of the investment goods purchases are liquidity-constrained.
16. Since Cooley and Hansen (1989) benchmark welfare losses with respect to optimal monetary rules, these welfare losses would be calculated with respect of the Friedman rule of a deflation of −7·79% in Table 1; i.e. 11·47% = 6·72% + 4·75%.
17. The welfare effects of inflation taxes in this model are comparable with the welfare effects of income taxes in King and Rebelo (1990). For example, with an increase of inflation from the benchmark value of 5% to 22·62%, the distortion is sufficient to eliminate growth. Using King and Rebelo's welfare measure, $\Lambda_{kr}$, this is seen to be associated with a perpetual loss of period consumption of 16·37%. This roughly coincides with the 16·3% reported in King and Rebelo (1990, p. S147) for a similar calibration ($\beta=0\cdot9576$) that resulted from an increase in the income tax from their benchmark value of 0% to 10%.

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