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I(2) representations of US money demand

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Abstract

Researchers have found success in estimating I(2) representations of money demand for other countries. Using a general equilibrium cash–credit model, an I(2) model for the United States is tested. No bias is found from treating the variables as I(1).

Keywords: Cointegration; Money demand; CCR

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1. Introduction

Since the recent work by Lucas (1988), there has been a resurgence in interest in the long-run behavior of the money demand function in the United States. Much of the research has focused on identifying cointegrating relationships between variables presumed to be I(1), such as real balances, income and interest rates for the post-war period in the United States, using the traditional money demand specifications.[1] The results have been mixed. Support for stable money demand functions have generally only been forthcoming for very broad monetary aggregates such as M2 for the post-war period, whereas the long-run stability for narrower monetary aggregates such as the monetary base (MB) and M1 has generally not been found.[2] In an attempt to find long-run stable money demand relationships for narrow monetary aggregates, some researchers have modified traditional money demand models by either transforming the variables in the original specification and/or adding additional

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[1] Recent studies using this approach are numerous and include Friedman and Kuttner (1992), Norrbin and Reffett (1993), and Stock and Watson (1993).

[2] One important exception is the finding in Stock and Watson (1993) that M1 appears to be stable only when using the Lucas data set, and is not stable for the post-war period. However, they also find that when restricting the data to cover only the post-war period, they find no support for cointegration.

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variables to reflect financial and institutional innovation in an effort to capture secular movements in the long-run behavior of velocity.[3]

Recently, a new approach to resolving this problem has been advocated in a series of papers by Johansen (1992a, b, 1993), Juselius (1993a, b), and Johansen and Juselius (1993). Johansen (1992a) shows that treating a system that includes I(2) variables that are cointegrated as an I(1) system leads to a missing variable bias, i.e. the common stochastic trend generating the higher-order cointegration between the I(2) variables is missing in the I(1) specification. He further shows how the I(2) specification for money demand can be treated as a relationship between four I(1) variables: real balances, interest rates, real income, and a latent nominal common I(1) stochastic trend. Using post-war economic data from the United Kingdom, Australia, and Denmark, these papers have reported success in estimating long-run money demand relationships when treating the statistical model for money demand as an I(2) model as opposed to the I(1) model used in previous work.[4]

We follow the methodological suggestion in these papers by constructing an I(2) model of post-war US money demand to see if the possible I(2) properties in money and prices are important in the estimation of US money demand. The next section of the paper presents a simple general equilibrium cash–credit model that theoretically supports the I(2) specification of the money demand model that we test. We also show that such model implies an equivalence between using monetary growth and inflation to capture the missing stochastic trend generating the higher-order cointegration between money and prices. In Section 3 we present our results, while in Section 4 we conclude.

2. Cash-in-advance models and I(2) representations of money demand

Consider a simple version of the theoretical framework described in Lucas and Stokey (1987) and Lucas (1988). The model is formulated in discrete time as an infinite horizon stochastic pure exchange economy. Households receive a random endowment of a composite perishable output good Y_t at the beginning of each period. Each household has period preferences over purchases of subsets of a composite consumption good: a cash good and a credit denoted by c_{1t} and c_{2t}. Lifetime preferences are assumed to be given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_{1t} + \ln c_{2t}] , \tag{2.1}$$

where $\beta \in (0, 1)$, E_0 indicates the mathematical expectation taken at date 0 with the integral

[3] Important papers along these lines for the United States and the United Kingdom include Baba et al. (1992), and Hendry and Ericsson (1991).

[4] These papers have found evidence in I(2) models that the traditional long-run money demand can be stable without necessarily resorting to ad hoc variable additions and (in some cases) variable transformations. Johansen (1992a), using the Hendry and Ericsson (1991) data for the United Kingdom, finds support for I(2) representation of money demand. Juselius (1993a, b) provides strong evidence for I(2) representations in the case of Denmark, while Juselius and Hargreaves (1992) and Johansen and Juselius (1993) provide support for Australian money demand stability.

in (2.1) defined across the state space for sequences of household histories s_t, and period utility function u is assumed to be logarithmic in consumption.

Let each household be given an identical initial stock of fiat money M_0 at date 0. Denote a household's demand for next-period money holdings at date t by m^{d}_{t+1}. The single activity of the government is to issue fiat money. Let M_t denote the aggregate per capita stock of money at the beginning of period t and let this aggregate evolve according to the following rule:

$$M_t = \mu \theta_t M_{t-1} \, ; \quad \mu > \beta \, , \tag{2.2}$$

where μ is the constant growth factor and θ_t is interpreted as a random shock to the monetary growth rate. We assume the process governing this random endowment and the monetary growth rates is as follows:

$$\ln Y_t = \phi \ln Y_{t-1} + \varepsilon_{1t} \, ; \qquad \varepsilon_{1t} \sim \mathrm{N}(0, \sigma_1^2) \, , \qquad \phi \in [0, 1] \, , \tag{2.3}$$

$$\ln \theta_t = \rho \ln \theta_{t-1} + \varepsilon_{2t} \, ; \qquad \varepsilon_{2t} \sim \mathrm{N}(0, \sigma_2^2) \, , \qquad \rho \in [0, 1] \, , \tag{2.4}$$

where $\rho \in [0, 1]$ allows for a potential unit root in output and monetary growth rates, and $S = [Y, \theta] \in S$ is the aggregate state space for the economy. Each agent is assumed to receive an identical lump-sum monetary transfer of $J(S_t) = [\mu\theta - 1]M_{t-1}$ at the beginning of each period. Transfers cannot offset current period financing constraints. Households assume that the aggregate money price of consumption goods is $P(S_t) = p(S_t)M_t$ where $p: S \rightarrow \mathbb{R}_+$ is continuous and defined to be the inverse of real money balances M/P, and the aggregate per capita consumption is given by $C_1(S)$ and $C_2(S)$.

Let $\hat{m} = m/M$. Define the household's feasible correspondence $\Phi(s) \subseteq \mathbb{R}^3_+$ to be the set of $z(s) = [c_1(s), c_2(s), d(s)]$ that satisfy the following equations:

$$p(S_t)[c_{1t} + c_{2t}] + m^{\mathrm{d}}_{t+1}/M_t \leq \hat{m}_t + p(S_t)Y_t \, , \tag{2.5}$$

$$p(S_t)c_{2t} \leq \hat{m}_t \, , \tag{2.6}$$

$$c_{1t}(s_t), c_{2t}(s_t), m^{\mathrm{d}}_{t+1}(s_t) \geq 0 \, , \quad \forall S_t \in S_h \, , \tag{2.7}$$

where (2.5) is the household's budget constraint, (2.6) is the cash-in-advance constraint, (2.7) a non-negativity constraint, $c_1(s), c_2(s)$, and $d(s)$ are stationary decision rules for the individual's two consumption decisions and her next-period money holdings m_t relative to M_t, and s_t is the state vector for the individual household's decision problem where $s = (\hat{m}, S) \in S_h, S_h := \mathbb{R}_{++} \times S)$.

In a recursive monetary equilibrium, households choose a vector of stationary decision rules $z(s) = [c_1(s), c_2(s), d(s)] \in \Phi(s)$ to maximize (2.1) given the behavior of the aggregate economy summarized by the number μ and functions $p(S), C(S)$ and $\chi(S, dS')$, where χ is the stationary transition function for the exogenous processes $S = [Y, \theta]$. If $V(s)$ be the value of

(2.1) at the optimum for an individual in state s, a Bellman equation for studying the monetary equilibrium is a $V(s)$ that satisfies the following functional equation:

$$V(s) = \sup_{z \in \Phi(s)} \left[u(c(s)) + \beta \int_{\theta} V(s')\chi(\theta, \mathrm{d}\theta') \right], \tag{2.8}$$

where s' the next-period state of an individual household, and $s' = [d(s) + [\mu\theta - 1]/h(S'), S]$.

A recursive competitive equilibrium for this economy is defined in Lucas and Stokey (1987). For logarithmic preferences, it can be shown that the envelope and Euler condition for (2.8) and equilibrium conditions imply that

$$[C_1(S)/C_2(S)] = 1 + r(S) = [\mu\theta^{\rho} \exp(.5(\sigma_2^2)/\beta]\,. \tag{2.9}$$

Further, Norrbin and Reffett (1993) have shown that in a recursive competitive equilibrium, the equilibrium decision rules C_1 and C_2 can be written as $C_1(S) = \phi[r(S)]Y$; $C_2(S) = \phi[r(S)]Y$ where $\phi_1'[r] < 0$ and $\phi_2'[r] > 0$. Therefore, the cash-in-advance constraint implies under the money rule (2.2) that

$$\ln M_t - \ln P_t = \ln \phi_1(r_t) - \ln y_t\,, \tag{2.10}$$

which is the money demand equation described in Lucas (1988).

Now consider the case where the growth rate in money $\ln(\mu\theta)$ is I(1), i.e. $\rho = 1$ in (2.4). Then the log of the equilibrium nominal rate is I(1) by (2.9) and the money stock is I(2) by (2.2). If $\phi = 1$ in (2.3), then output is also I(1). With $\mu > \beta$, cash-in-advance binds, and (2.6) implies (i) the price level must be I(2), and (ii) $\ln M$ and $\ln P$ must be cointegrated with cointegrating vector $\alpha = [1, -1]$. Note that this implies that if the cash-in-advance specification is correct, monetary growth rates and inflation rates should give the same long-run information.

As noted in Johansen (1992a), the presence of I(2) components will change the empirical version of the specification of (2.10). He argues that if M and P are I(2), then the traditional I(1) specification of money demand in (2.10) omits a critical variable, namely the common stochastic trend. This I(1) trend is associated with the cointegration between the two I(2) variables, $\ln M$ and $\ln P$, which is theoretically imposed on the left-hand side of (2.10). Based on the loadings on the common trend, Johansen (1992a) identifies inflation rates as a proxy for the nominal growth variable. Given the theoretical framework of Lucas (1988), we can make strong statements about the nominal growth variable. Since prices inherit their long-run properties from the money evolution equation in (2.2), theoretically we could equivalently measure the common stochastic I(1) trend by the stochastic monetary growth rate. Then, assuming ϕ_1 is log-linear, the following two I(2) specifications for money demand would be equivalent:

$$\ln M_t - \ln P_t = \alpha_1 + \alpha_r \ln r_t + \alpha_y \ln y_t + \alpha_{\Pi} \Delta P_t\,, \tag{2.11a}$$

$$\ln M_t - \ln P_t = \alpha_1 + \alpha_r \ln r_t + \alpha_y \ln y_t + \alpha_{\Pi} \Delta \ln M_t\,, \tag{2.11b}$$

where (2.11a) is a version of the I(2) cointegrating relationship found in Johansen (1992a), and (2.11b) is the specification implied by a cash-in-advance framework.

3. Empirical results

The data are from Citibase covering 1959:1–1992:2, and are at a quarterly frequency. The monetary aggregates are non-seasonally adjusted and are adjusted for population changes. The income measure is the private GDP analogous to the one in Norrbin and Reffett (1993). Similarly, the price variable is the private GDP deflator. The interest rate is the three-month government bond rate. All variables are logged.

First, we need to address the order of integration in our data. Instead of relying on a statistical analysis, we-note that the theoretical model would be consistent with money and prices being I(2) if real balances, nominal interest rates, and real income are each I(1). In the spirit of Johansen (1992a) and Juselius (1993a, b), we assume that money and prices are I(2), and interest rates and real income are I(1). Stock and Watson (1993) suggest that more than one cointegration test should be used in testing potentially cointegrated equations, therefore the I(2) specifications in (2.11a) and (2.11b) are estimated using both the CCR procedure with prewhitening described in Park and Ogaki (1991) and the MLE procedure in Johansen (1988).

Table 1 presents the empirical estimates of the CCR procedure. This estimator basically uses the OLS estimates and transforms the variances to render an efficient estimate of the regression. Thus, the standard *t*-distribution can be used for inferences when a cointegrating regression exists. To test whether the relationship is a cointegrating regression we used Park's spurious regressor test, which adds polynomial trends of different orders to the regression and then tests if these trends can be detected. The null hypothesis of this test is that a cointegrating regression exists, and if the hypothesis cannot be rejected then the regression estimates are valid, and inferences efficient.

Columns 2–5, in Table 1, are the estimated coefficients. Columns 6–8 report Park's spurious regressor tests for different polynomial trend additions. Notice that for each monetary aggregate there are three sets of results. For the sake of comparison with the existing literature, the first line presents the results for a traditional I(1) representation. The second line reports the results for the I(2) specification using the inflation rate to proxy for the nominal common stochastic trend, while the third line presents the I(2) specification using the monetary growth rate as the common I(1) trend. As this table indicates, CCR finds no support for the I(2) representation for narrow measures of money such as MB and M1. Cointegration is found for M2, but neither inflation rates nor monetary growth rates provide any contribution to the long-run relationship that characterizes the money demand function. Note that in CCR the normalization on the real money variable implies that only a cointegration relationship involving the real money variable can be found, thus the coefficients associated with inflation and money growth are close to zero.[5]

To check the robustness of the above findings we also present the results from the MLE procedure that uses no cointegration as the null hypothesis. The results for the MLE procedure are presented in Table 2. Normalizing the coefficients by setting the coefficient on the measure of real balances to unity, Columns 2–5 are the estimated coefficients in the

[5] The OLS estimates are 1.270, −0.061, and 1.084 for the I(2) money growth equation. These coefficients are close to the CCR results because both of the regressions normalize by using the real monetary aggregate as the dependent variable, thus the potentially cointegrated regression must include the real monetary aggregate.

Table 1
Testing long-run money demand stability: CCR results

	Estimated elasticities[a]				Test statistics[b]		
	Y	i	π	ΔM_i[c]	$H(1,3)$	$H(1,4)$	$H(1,5)$
Monetary base							
I(1)	0.716	−0.243			21.413*	24.112*	28.783*
					(0.001)	(0.001)	(0.001)
I(2)	0.836	−0.209	−0.030		40.422*	43.893*	54.428*
					(0.001)	(0.001)	(0.001)
I(2)	1.012	−0.249		−1.115	25.099*	27.552*	30.232*
					(0.001)	(0.001)	(0.001)
M1							
I(1)	1.115	−0.397			8.239*	12.554*	12.555*
					(0.016)	(0.006)	(0.014)
I(2)	1.123	−0.204	−0.123		22.322*	29.725*	45.009*
					(0.001)	(0.001)	(0.001)
I(2)	1.099	−0.291		−0.154	13.069*	21.042*	30.114*
					(0.001)	(0.001)	(0.001)
M2							
I(1)	1.461*	−0.014			4.284	4.381	6.128
	(0.123)	(0.017)			(0.177)	(0.223)	(0.189)
I(2)	1.483*	0.001	−0.014		1.734	3.127	3.852
	(0.131)	(0.019)	(0.010)		(0.420)	(0.372)	(0.426)
I(2)	1.400	−0.031		0.006	0.509	9.609*	12.205*
					(0.775)	(0.022)	(0.016)

[a] These are the estimated elasticities with the standard errors in parentheses for the cointegrated regressions. An asterisk signifies a rejection of the null of a zero effect, at the 5% significance.
[b] These are χ^2 statistics with p-values in parentheses. An asterisk signifies a rejection of the null hypothesis of cointegration, at the 5% significance. Thus, a failure to reject the null hypothesis implies a stable money demand function.
[c] The ΔM_i represents the nominal monetary growth, where i represents the Monetary base, M1 and M2 respectively.

cointegrating vectors for the different specification we use.[6] Column 6 reports the maximum eigenvalue test statistic, while column 7 reports the value for the trace statistic.[7] Both statistics are for the null hypothesis of no cointegration, implying that a rejection shows evidence of at least one cointegrating vector.

As Table 2 indicates, there is little support for the hypothesis that there is a stable long-run I(2) representation of money demand for either of the money measures. Only in three cases

[6] Note that this normalization differs from the one used in the CCR test procedure. In this case, only the coefficients are normalized so that all coefficients are reported with respect to the real monetary aggregate. However, no causal direction is imposed in the MLE procedure. Furthermore, the MLE procedure allows subsets of the variables to form cointegrated regressions, even if the subset does not include the real monetary aggregate.
[7] See Johansen (1988) for a discussion of these statistics.

Table 2
Testing long-run money demand stability: MLE results[a]

	Estimated elasticities[b]				Test statistics[c]	
	Y	i	π	ΔM_i[d]	Maximum eigenvalue	Trace statistic
Monetary base						
I(1)	0.384	−0.259			17.83	22.70
I(2)	0.507	−0.206	−0.094		21.30	45.09
I(2)	−1.204	−0.639		56.05	22.36	47.92*
M1						
I(1)	0.204	−0.372			13.79	18.71
I(2)	0.230	−0.194	−0.166		17.57	35.66
I(2)	0.128	−0.309		−4.018	15.53	35.85
M2						
I(1)	0.962	0.019			17.05	26.37
I(2)	1.380	−0.288	0.240		28.26*	52.38*
I(2)	5.812	−3.430		244.034	29.87*	50.19*

[a] The MLE results are based on VECM models using the BIC criterion for determining the lag-length. The lag-length of the Monetary base, M1 and M2, equations are five, six and two respectively.
[b] The elasticities are found by normalizing the coefficient on the monetary aggregate to unity.
[c] The critical values at the 5% significance level for the maximum eigenvalue tests are 20.96 and 27.07 for the I(1) and I(2) cases, respectively. The corresponding critical values for the Trace statistics are 29.68 and 47.21 for the I(1) and I(2) cases, respectively.
[d] The ΔM_i represents the nominal monetary growth, where i represents the Monetary base, M1 and M2 respectively.

can the null hypothesis of no cointegration be rejected, namely: the money base equation using monetary growth, and the M2 variable using both inflation rates and monetary growth. However, testing coefficient restrictions using likelihood ratio tests showed that this cointegrating relationship involves only the inflation rate/monetary growth variables and the interest rate variable.[8] Thus, this relationship is more of a Fisher-type relationship than a stable money demand.[9] This relationship also accounts for the unanticipated coefficient estimates. For example, the interest elasticity for money demand has a large negative coefficient that is contrary to the anticipated zero coefficient, and the coefficients on the monetary growth and

[8] No regression normalization is made in MLE, and thus one or many cointegrating regression(s) can be detected among many variables in the system. Likelihood ratio tests can be used to restrict the specification of the system to interpret the cointegrating vectors. In this example, the null hypothesis that real monetary aggregates and income variables have zero coefficients could not be rejected at 95% confidence. Therefore, the cointegrating vector actually only involved the remaining two variables in the system. These likelihood ratio statistics are not reported due to space limitations.

[9] Note, also, that only one cointegrating vector is significant. If the money demand relationship were stable, a second cointegrating vector would have to be significant. However, no other vector is found to be significant, implying that the only potentially stationary relationship is between the interest rate–inflation rate and the interest rate–monetary growth rate.

interest rate variables are widely different from the coefficients in Table 1.[10] However, as a Fisher-type relationship the coefficients are, as expected, arguing that if inflation or monetary growth goes up then interest rates must go up as well.[11]

4. Conclusion

In the paper, we present a simple general equilibrium model of money demand. From this framework, we construct an I(2) representation for money demand following Johansen (1992a). We test the model on post-war US data. In contrast to the findings in Johansen (1992a) and Juselius (1993a, b), we find limited support for the I(2) specification. Using the CCR procedures, we find no support for the I(2) approach. As in the I(1) models, stable representations for long-run money demand are only found in the case of broad monetary aggregates such as M2. For narrow money measure, we find no support for cointegration. Also, using the MLE approach, we find no support for a stable money demand function, although we do find a marginally cointegrated Fisher-type relationship. Furthermore, we find no support for the theoretical argument that monetary growth rates and inflation rates should lead to an equivalent long-run money demand representation.

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[10] Because the coefficient on the real monetary aggregate is normalized to unity, the other coefficients adjust. The cointegrating vector does not involve the real monetary aggregate, and thus the coefficient should actually be zero. Therefore, the other coefficients adjust to large numbers to give the real monetary aggregate a small weight. If one renormalized picking the monetary growth as unity the other coefficients would be small, and the only coefficient of importance would be the interest rate coefficient because it is the only other coefficient that is significant.

[11] The Fisher-type relationship is very sensitive to the lag-length specification and is not very strong. In Table 2 the lag-length choice of five or six produces no cointegrating vector, whereas the lag-length of two produces such a vector between interest rates and inflation. Similarly, the interest rate–monetary aggregate relationship is marginally significant and sensitive to lag-length changes. Furthermore, we tested these Fisher-type relationships using the CCR procedure and found no evidence of cointegration. Thus, we conclude that only marginal support, if any, exists for a Fisher-type effect. It should be noted that the MLE procedure remains an efficient estimation technique even if a Fisher-type relationship exists, as the MLE procedure allows for multiple cointegrating vectors. This is, however, not the case using the CCR procedure, which allows only one cointegrating vector.

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