Capital in the Payments System
Author(s): Milton H. Marquis and Kevin L. Reffett
Source: *Economica*, New Series, Vol. 59, No. 235 (Aug., 1992), pp. 351-364
Published by: Wiley on behalf of The London School of Economics and Political Science and The Suntory and Toyota International Centres for Economics and Related Disciplines
Stable URL: http://www.jstor.org/stable/2554606
Accessed: 15/08/2013 13:57

Economica, **59**, 351–64

Capital in the Payments System

By Milton H. Marquis and Kevin L. Reffett

Florida State University

Final version received 17 July 1991. Accepted 17 September 1991.

Capital is required to support the payments system in modern economies with well developed financial markets. Financial innovations raise the marginal product of capital in this usage. This suggests that there are general equilibrium consequences associated with an optimal selection of a payments system that includes barter, money and a capital-based accounting system. In this paper, goods are differentiated with respect to the medium of exchange associated with their acquisition which is endogenously determined as a consequence of explicit trading frictions. The response of the economy to endowment, production and payments system shocks, including financial innovations, is examined.

Introduction

In a modern economy with well developed financial markets, capital is required to support the payments system. Much of this capital comes in the form of computer and information systems hardware, but it also involves structures and other supporting capital facilities. Given that this capital has alternative uses in the production of goods, the optimal selection of a capital-based payments system must be made within a general equilibrium context that ensures the overall efficient allocation of the economy's capital resources.

A capital-based payments system is designed around the existing level of technology (e.g. in the telecommunications field). Financial innovations that affect the payments system can come in the form of improved technology or simply more innovative financial arrangements, e.g. between commercial banks and their major corporate clients, which are better able to exploit the existing technology. In either case, these innovations raise the marginal product of capital in this usage, and induce a greater capital allocation to the system. As a result, the payments system becomes more capital-intensive as agents reduce their traditional reliance on maintaining what would otherwise have been larger money balances to affect given levels of transactions.[1]

This progression towards an ever-increasing capital-intensive payments system implies that 'money' is becoming less important as a medium of exchange. It is effectively being displaced by capital.[2] As this process continues, monetary policy becomes increasingly less potent under the current regulatory regime (see Marquis and Cunningham, 1990). Moreover, monetary policy itself can hasten the process of ever-increasing capital intensity in the payments system by rendering money an unattractive asset through a high-inflation policy. (See Hester (1981) for a discussion of this interaction during the 1970s.)

This paper examines an economy with multiple payments schemes that coexist.[3] One of the payments schemes is pure barter, where a 'double coincidence of wants' between trading partners exists. The second is a monetary (or pure cash) transactions market. This market emerges endogenously in a spatial environment as a result of the absence of a double coincidence of wants, and because of agent itineraries that render trade credit and private debt incentive incompatible.

The third payments scheme is similar to that envisioned by Fischer Black (1970). In his world, all transactions represent exchanges of claims on assets (or goods) that are conducted via a centralized system of accounting entries. Buyers' accounts are debited and sellers' accounts are credited. No monetary transfers are required. However, capital is needed in order to support the facility. To model this facility, we assume that these transactions accounts are effectively zero balance accounts as described below. This limits the role of the facility to one of making a transactions market that would otherwise be 'missing' because of the frictions imposed by the trading environment.[4] All agents who make use of this facility (which includes all agents in the economy) establish a line of credit at the beginning of each period by relinquishing claims to a predetermined quantity of goods produced in their home location. This line of credit is used to purchase goods produced in other locations and over which the agents have utility defined, but which can be acquired only through trade. Monetary transactions do not emerge to facilitate this set of trades because of the infinite search costs associated with locating the desired trading partners, i.e. one who has the good for which an agent wishes to trade. Markets clear each period, implying that all account balances associated with the line of credit are restored to zero. The operation of this facility involves the costly coordination or making of the transactions market. These costs are met by agents contributing equal shares of capital (which is fully depreciated each period) to run the facility. The capacity of this facility to produce transactions can be increased with a greater allocation of capital. A financial innovation raises the marginal product of capital in this usage.

In Section I, the spatial trading environment that includes the information structure of the economy is specified. The purpose of the section is to rationalize the payments system constraints that are imposed on the optimization problem, such that they emerge endogenously from an explicit set of trading frictions (as in Townsend 1987, and King and Plosser 1986). The spatial symmetry of the model allows the general equilibrium of the economy to be characterized by the representative household's intertemporal maximization problem which is detailed in Section II. This model generates pricing anomalies of money and capital in terms of consumption goods which produce the real rate-of-return dominance of capital over money. In Townsend (1987), this rate-of-return dominance arises from the additional marginal utility received by money holdings from the relaxation of the cash-in-advance constraint on consumption. Townsend's results fully generalize to a model economy with the more sophisticated payments system that is examined in this paper. Moreover, as shown in Section III, the pricing anomalies of capital arise even in a non-monetary economy provided only that it is costly to transact.

The response of the economy to real shocks, including financial innovations to the capital-based transactions facility, are discussed in Section IV. Positive shocks to a capital storage technology similar to Eichenbaum and Singleton (1986) effectively raise endowments in this model and cause output to rise, without affecting its composition across consumption goods. Positive production technology shocks also increase output and consumption across all goods. However, at the same time they alter the optimal mix of capital utilization between production and the capital-based transactions facility, and consumption shares adjust. Likewise, shocks to the payments system do generally alter the consumption shares of output. A financial innovation lowers the cost of transacting in the market for the trade good (which is acquired through the transactions facility), causing consumption of that good to rise, and to assume a larger share of output in the current period. Alternatively, a positive innovation to the monetary growth rate relaxes the cash-in-advance constraint in the next period and imposes an inflation tax on consumption of the cash good (which is acquired via a monetary exchange), thereby reducing its share of output for the current period. Overall, the response of fully informed households to a positive financial innovation or a negative monetary innovation increases output.

I. The Trading Environment

The spatial setting of this model is derived from that used by Townsend (1987). Refer to Figure 1. To identify locations, consider a countable infinity of horizontal parallel 'trading' lines indexed by j. On each trading line there is a countable infinity of local markets indexed by i. At each location (i, j) there is an uncountable infinity of identical three-person households which are positioned uniformly (and densely) around the circumference of a circle, i.e. at a point such as x. Each household is endowed with location (i, j, x)-specific capital that can be: (1) transformed into a location-specific consumption good from which the household derives utility; (2) consumed in the production of transaction services (of a type to be more fully specified below); or (3) stored

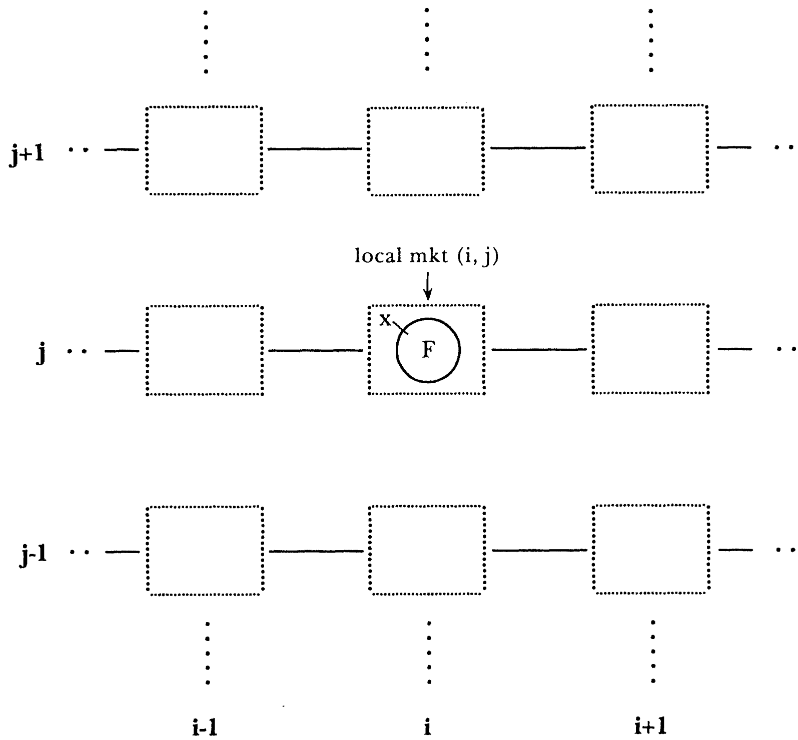

Figure 1. The trading environment.

for future use. The consumption good produced in the household's resident location is hereafter referred to as the 'home' good.[5]

Within each period, one member of each household at any location (i, j) can travel costlessly along the 'trading line' by an amount equal to one-half the distance to the adjacent location to its immediate left, while a second member of the household may travel costlessly a similar distance to its immediate right. Therefore, markets for trade exist at locations $[(i-\frac{1}{2}), j]$ and $[(i+\frac{1}{2}), j]$. Utility for the households at (i, j) is defined in part over the goods produced at the location $(i+1, j)$ to their immediate right, and which the household perceives to be perfect substitutes, but not for the goods produced at location $(i-1), j$ to their immediate left. This asymmetry of consumption preferences along the trading line produces a 'double coincidence of wants' problem that precludes pure barter exchanges from taking place in the markets at locations $[(i \pm \frac{1}{2}), j]$.

To prevent private debt or trade credit from emerging to facilitate these transactions, incentive-compatibility constraints are introduced via exogenously given inter-period itineraries. Between periods, households assume new resident locations for the next period. All households at an even i-integrated location move down to location $(i, j-1)$ on the next trading line; while all households at odd i-integrated locations remain resident at location (i, j). Under these conditions, money emerges to facilitate trade in adjacent markets. One member of each household location at (i, j) travels to the market at $[(i-\frac{1}{2}), j]$ to exchange the home good for money. In the subsequent period, this money is added to the monetary transfer from the economy's monetary authorities, and is used by the second member of the household, who travels along the trading line to the market to his immediate right at location $[(i+\frac{1}{2}), j]$ for odd i-integrated households and at location $[(i+\frac{1}{2}), (j-1)]$ for even i-integrated households, to exchange money for goods to be consumed within that period. Hereafter, this good is referred to as the 'cash' good. The infinite number of households participating in these markets ensures that they are perfectly competitive.

Within the local market (i, j), a household located at x on the circle has utility for the current period defined over consumption goods produced at a distance from x around the circle whose measure at the end of the previous period is characterized by a rational number. There is a countable infinity of such locations for any arbitrary x. These goods are hereafter referred to as the 'trade' goods. A household located at x does not have utility defined over goods produced at locations around the circle whose distance is measured at the end of the period by an irrational number. There is an uncountable infinity of such locations for any arbitrary x. At the beginning of each period, the relative position of these markets is perturbed by a small random event. For household to locate a market in which the good it demands is produced requires a search. However, the probability of locating a market with the desired good has measure zero.[6] Therefore, the search costs for individual households become infinite and, as a consequence, these transactions markets are 'missing'.

To fill this missing market, a centralized transactions facility emerges to solve the search problem of the agents by matching up desired trades. This transactions market is located in the centre of the circle at F, and can be costlessly reached by the third member of each household within the period. At the beginning of the period, agents travel to F and relinquish ownership to a quantity of their home good for which they receive a 'line of credit' or a positive account balance in exchange. The facility performs an anonymous matching of trades, and the markets clear, implying from symmetry that all lines of credit are completely exhausted and account balances are restored to zero at the end of the period. This coordination or making of the transactions market is costly and is supported by an equal commitment of capital by each agent who makes use of the facility. Each trade is a random bilateral matching of agents from an infinite population which ensures perfectly competitive markets.

II. The Representative Household's Maximization Problem

The spatial symmetry of the trading environment allows the general equilibrium of the model economy to be determined by the solution of a representative household's intertemporal maximization problem and market-clearing conditions. Each household is assumed to be infinitely lived and to derive utility from the consumption of three perishable goods: the home good, c_t^1, the cash good, c_t^2, and the trade good, c_t^3. Its objective becomes:

$$(1)\qquad \max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^1, c_t^2, c_t^3), \qquad 0<\beta<1,$$

where β is the rate of discount of future over current consumption, and where the utility function satisfies the Inada conditions; i.e. $U_i(\cdot)\to\infty$ as $c^i\to 0$ for any i, where $U_i(\cdot)$ is the first partial derivative of the utility function with respect to the ith argument.[7] The household's beginning-of-period endowments consist of a stock of (location-specific) capital, k_{t-1}, and a stock of money, M_t^d, which are carried over from the previous period. The initial stock of capital is allocated across three potential uses. For the first two uses, it is drawn by an amount i_t^s. This quantity is allocated towards the production of home goods, in an amount i_t^p, and towards the production of transaction services via the capital-based accounting system, in an amount i_t^b. This implies the following adding-up constraint, which is equivalent to capital market equilibrium:[8]

$$(2)\qquad i_t^s = i_t^p + i_t^b.$$

The third use is to store the remaining stock of capital, which is then subject to a storage technology that produces a stochastic return:

$$(3)\qquad k_t = \theta_t^1(k_{t-1} - i_{t-1}^s), \qquad \ln \theta_t^1 \sim \text{iid}\,(0, \sigma_1^2),$$

where θ_t^1 is a log-normal innovation to the storage technology.

The output of home good, y_t, is determined by a stochastic log-linear production function,

$$(4)\qquad y_t = \theta_t^2(i_t^p)^{\alpha_1}, \qquad \alpha_1>0, \qquad \ln \theta_t^2 \sim \text{iid}\,(0, \sigma_2^2),$$

where θ_t^2 is a log-normal innovation to the production technology. From symmetry, goods market equilibrium equates output to consumption:

$$(5)\qquad y_t = c_t^1 + c_t^2 + c_t^3.$$

Consumption of the trade good is constrained by the capacity of the capital-based transactions facility to effect transactions within the local markets. The capacity of this facility is measured in terms of the maximum possible volume of the trade good that can be exchanged within the period, g_t. The capacity of the system can be increased with an additional allocation of capital households, i.e. an increase in i_t^b. This gives rise to a production function for g_t that is specific as log-linear in i_t^b and subject to a stochastic technology:

$$(6)\qquad g_t = \theta_t^3 (i_t^b)^{\alpha_2}, \qquad \alpha_2 > 0, \qquad \ln \theta_t^3 \sim \text{iid}\,(0, \sigma_3^2),$$

where the log-normal random variable, θ_t^3, is the financial innovation. Equation (6) introduces a capacity constraint on the consumption of the trade good:

$$(7)\qquad g_t \geq c_t^3.$$

The household is also constrained in the consumption of the cash good by the stock of money that it carried over from the previous period, M_t^d, plus the monetary transfer, J_t. This produces the following 'cash-in-advance' constraint:

$$(8)\qquad M_t^d + J_t \geq p_t c_t^2,$$

where p_t is the unit price of a consumption good.[9]

The household's budget constraint can now be expressed in nominal terms:

$$(9)\qquad p_t y_t + M_t^d + J_t + W_t i_t^s \geq \sum_{i=1}^{3} p_t c_t^i + W_t (i_t^b + i_t^p) + M_{t+1}^d,$$

where W_t is the unit price of capital, and M_{t+1}^d is the household's demand for money for the upcoming period. Note that, since the monetary transfer is known prior to the household taking its consumption and production decisions, it will affect the *offering* of c_t^2. This gives rise to the short-run non-neutrality in the model that is discussed in Section V.

The general equilibrium for the model can be defined once the process governing the evolution of the money supply and money market equilibrium have been specified, and the household optimization conditions are given:

$$(10)\qquad M_t^s = (M_{t-1})^{\alpha_3} \theta_t^4, \qquad \alpha_3 > 0, \qquad \ln \theta_t^4 \sim \text{iid}\,(0, \sigma_4^2),$$

$$(11)\qquad M_t^s = M_t^d + J_t,$$

where M_t^s is the money supply, and θ_t^4 the monetary innovation.[10]

To obtain a closed-form analytical solution to the model, further specializations of the utility function, the production functions for output and the transactions facility, and the money supply process are required. The utility function is assumed to be time-separable and is specified as log-linear:

$$(12) \qquad U(c_t^1, c_t^2, c_t^3) = d_1 \ln c_t^1 + d_2 \ln c_t^2 + \ln c_t^3, \qquad d_1, d_2 > 0.$$

The production functions are specified to be linear; i.e. $\alpha_1 = \alpha_2 = 1$, implying constant returns to scale.[11] The money supply process is restricted to one containing a unit root; i.e. $\alpha_4 = 1$.[12] This implies that the monetary innovation, θ_t^4, alters the growth rate of the money supply.

The representative household's maximization problem can now be solved using Lagrange methods. The Lagrangean may be written in the following form:

$$(13) \qquad \mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ (d_1 \ln c_t^1 + d_2 \ln c_t^2 + \ln c_t^3)$$
$$+ \lambda_t^1 \left[p_t i_t^p \theta_t^2 + M_t^d + J_t - \sum_{i=1}^{3} p_t c_t^i - W_t(i_t^p + i_t^b - i_t^s) - M_{t+1}^d \right]$$
$$+ \lambda_t^2 (M_t^d + J_t - p_t c_t^2) + \lambda_t^3 (p_t i_t^b \theta_t^3 - p_t c_t^3)$$
$$+ \lambda_t^4 [W_t \theta_t^1 (k_{t-1} - i_{t-1}^s) - W_t k_t] \Bigg\},$$

given the initial endowments of k_0 and M_0, and subject to $k_t \geq 0$, $M_t \geq 0$, $i_t^s \geq 0$, $i_t^p \geq 0$, $i_t^b \geq 0$ and $c_t^i \geq 0$, $\forall i$, with λ_t^j, $j = 1, \ldots, 4$, sequences of non-negative Lagrange multipliers on the budget constraint, the 'cash-in-advance' constraint on the cash good, the transactions facility capacity constraint on the trade good and the capital resource constraint, respectively.[13] These multipliers have the interpretation of the marginal utility derived by relaxing the appropriate constraint by an amount equal to one unit of M_t.[14]

$$(14) \qquad \{c_t^1\}: \; d_1/c_t^1 = \lambda_t^1 p_t$$

$$(15) \qquad \{c_t^2\}: \; d_2/c_t^2 = (\lambda_t^1 + \lambda_t^2) p_t$$

$$(16) \qquad \{c_t^3\}: \; 1/c_t^3 = (\lambda_t^1 + \lambda_t^3) p_t$$

$$(17) \qquad \{k_t\}: \; \lambda_t^4 W_t = \beta E_t(\lambda_{t+1}^4 W_{t+1} \theta_{t+1}^1)$$

$$(18) \qquad \{i_t^s\}: \; \lambda_t^1 W_t = \beta E_t(\lambda_{t+1}^4 W_{t+1} \theta_{t+1}^1)$$

$$(19) \qquad \{i_t^p\}: \; W_t/p_t = \theta_t^2$$

$$(20) \qquad \{i_t^b\}: \; W_t/p_t = (\lambda_t^3/\lambda_t^1)\theta_t^3$$

$$(21) \qquad \{M_{t+1}^d\}: \; \lambda_t^1 = \beta E_t(\lambda_{t+1}^1 + \lambda_{t+1}^2).$$

This enables the following definition to be made.

Definition. The symmetric, stationary monetary equilibrium is the sequence of prices, $\{p_t, W_t\}_{t=0}^{\infty}$, the sequence of the choice vector of the representative household, $\{c_t^1, c_t^2, c_t^3, k_t, i_t^s, i_t^p, i_t^b, m_{t+1}^d\}_{t=0}^{\infty}$, and a monetary rule, (10), that satisfy:

household optimization: equations (14)-(21);
market-clearing conditions: equations (2), (5) and (11).

III. Pricing Anomalies of Capital Induced by the Payments System

Townsend (1987) has demonstrated that, in a monetary economy, the special role that money plays in facilitating transactions produces pricing anomalies of money and capital in terms of consumption goods. These pricing anomalies, in turn, lead to a real rate-of-return dominance of capital over money. This section demonstrates that the pricing anomalies of capital identified by Townsend can occur in a non-monetary economy, provided that there are costs to transacting.

The first-order conditions, equations (14)-(21), can be manipulated to yield the following expressions.[15]

$$(22)\qquad U_1(c_t^1, c_t^2, c_t^3) = \beta E_t[U_1(c_{t+1}^1, c_{t+1}^2, c_{t+1}^3)(\theta_{t+1}^1\theta_{t+1}^2/\theta_t^2)],$$

$$(23)\qquad U_2(c_t^1, c_t^2, c_t^3) = \beta E_t[U_2(c_{t+1}^1, c_{t+1}^2, c_{t+1}^3)(\theta_{t+1}^1\theta_{t+1}^2/\theta_t^2)] - \beta E_t[\lambda_{t+1}^2 p_{t+1}(\theta_{t+1}^1\theta_{t+1}^2/\theta_t^2)] + \lambda_t^2 p_t,$$

$$(24)\qquad U_3(c_t^1, c_t^2, c_t^3) = \beta E_t[U_3(c_{t+1}^1, c_{t+1}^2, c_{t+1}^3)(\theta_{t+1}^1\theta_{t+1}^2/\theta_t^2)] - \beta E_t[\lambda_{t+1}^3 P_{t+1}(\theta_{t+1}^1\theta_{t+1}^2/\theta_t^2)] + \lambda_t^3 p_t.$$

Equation (22) indicates that capital is efficiently priced with respect to the home good, with the expression $(\theta_{t+1}^1\theta_{t+1}^2/\theta_t^2)$ equal to the intertemporal marginal product of capital inclusive of the rate of return to storage. However, the last two terms in equations (23) and (24) indicate that capital is inefficiently priced with respect to the cash and trade goods owing to the constraints on consumption imposed by the payments system. The rationale for these results is as follows. Withdrawal of consumption of the cash good in the current period with the 'savings' carried forward in the form of capital relaxes the cash-in-advance constraint on the consumption of the cash good. This leads to an optimal reallocation of consumption across goods in the current period, which mitigates the current-period loss and adds to the return. This is captured by the last term on the right-hand side of (23). However, the higher consumption in the next period increases the extent to which the cash-in-advance constraint is binding during that period and lowers the return. This is captured by the second term on the right-hand side of (23). A completely symmetrical argument can be made to demonstrate that capital is also inefficiently priced with respect to the trade good. This indicates that the results of Townsend generalize to this economy with a more elaborate payments system.[16] Moreover, equation (24) indicates that the pricing anomaly of capital identified by Townsend can occur in a non-monetary economy, provided only that transactions are costly to perform.[17]

IV. Economic Fluctuations and the Role of the Payments System

To examine how the fully informed household in this model adjusts its consumption and production decisions to endowment shocks, to technological innovations and to changes in monetary growth, the model can be solved to obtain the household's optimal consumption bundle, output and optimal mix of capital utilization that accord with the symmetric, stationary monetary equilibrium defined above:[18]

$$(25) \qquad c_t^1 = \beta\theta_t^1\theta_t^2\eta_{t-1}^1 i_{t-1}^p$$

$$(26) \qquad c_1^2 = (\beta^2 d_2\theta_t^1\theta_t^2\eta_{t-1}^2 i_{t-1}^p)/[d_1\theta_t^4 \exp(\sigma_4^2/2)]$$

$$(27) \qquad c_t^3 = (\beta d_2\theta_t^1\theta_t^2\theta_t^3\eta_{t-1}^3 i_{t-1}^p)/[d_1(\theta_t^2 + \theta_t^3)]$$

$$(28) \qquad y_t = \beta\theta_t^1\theta_t^2\Psi_t i_{t-1}^p$$

$$(29) \qquad i_t^p = \beta\theta_t^1\Psi_t i_{t-1}^p$$

$$(30) \qquad i_t^b = (\beta d_2\theta_t^1\theta_t^2\eta_{t-1}^3 i_{t-1}^p)/[d_1(\theta_t^2 + \theta_t^3)],$$

where

$$\Psi_t \equiv \eta_{t-1}^1 + [\beta d_2\eta_{t-1}^2/d_1\theta_t^4 \exp(\sigma_4^2/2)] + [d_2\theta_t^3\eta_{t-1}^3/d_1(\theta_1^2 + \theta_t^3)].$$

The η_t^i are consumption shares of output such that $c_t^i = \eta_t^i y_t$, for $i = 1, 2, 3$, with $\sum_{i=1}^3 \eta_t^i = 1$, and are given by the following expressions:

$$(31) \qquad \eta_t^1 = d_1(\theta_t^2 + \theta_t^3)\theta_t^4 \exp(\sigma_4^2/2)/\Delta$$

$$(32) \qquad \eta_t^2 = \beta d_2(\theta_t^2 + \theta_t^3)/\Delta$$

$$(33) \qquad \eta_t^3 = d_2\theta_t^3\theta_t^4 \exp(\sigma_4^2/2)/\Delta,$$

where

$$\Delta \equiv [d_1(\theta_t^2 + \theta_t^3) + d_2\theta_t^3]\theta_t^4 \exp(\sigma_4^2/2) + \beta d_2(\theta_t^2 + \theta_t^3).$$

In analysing the endowment, technological and monetary innovations, it is first reiterated that households are fully informed of these innovations *prior to* taking their consumption and production decisions. As such, a positive innovation to the storage technology, θ_t^1, simply increases the household's capital endowment, and scales up consumption, capital utilization and output in the current period. However, equation (29) indicates persistence in the optimal rate of capital utilization in production, which is seen to evolve according to the following expression:[19]

$$(34) \qquad i_t^p = \beta^t \prod_{i=0}^{t-1} (\Psi_{t-i}\theta_{t-i}^i) i_0^p.$$

This suggests that the household chooses not to fully absorb the endowment shock in the current period, but rather to spread its effect through time. As is evident from equations (25)-(28) and (30), this is induced by 'consumption-smoothing', which also produces persistence in output, and in the rate of capital utilization in the transactions facility.

A positive innovation in the production technology, θ_t^2, increases output in the current period. However, the household chooses to spread optimally across all consumption goods the utility gains that are derived from this greater productivity of capital in one of its uses. It does so by marginally reducing the allocation of capital to production—i.e. i_t^p from (29) is lower, although output remains higher—while marginally increasing the allocation of capital towards expanding the capacity of the transactions facility—i.e. i_t^b from equation (30) is higher. This facilitates an increase in consumption of the trade good, although by a lesser amount than the increase in consumption of home and cash goods. This is evident from equations (32)-(33). The realignment of the consumption bundle gives higher shares to the home and cash goods—i.e. η_t^1 and η_t^2 increase—and a lower share to the trade good—i.e. η_t^3 falls. This is, of course, attributed to the fact that the production technology shock makes the utilization of capital in the transactions facility relatively less attractive.

Conversely, a positive financial innovation, i.e. $\theta_t^3 > 1$, makes the utilization of capital resources in the transactions facility relatively more attractive. The household responds by increasing output in the current period; i.e.

$$(\partial\Psi_t/\partial\theta_t^3) > 0.$$

However, from equations (25)-(27), it is seen that this higher output is completely absorbed in the current period by higher consumption of the trade good.[20] That is, a greater production of the home good is required to facilitate the exchange for a greater quantity of the trade good. This requires an additional allocation of capital resources to production; i.e. from equation (29), i_t^p increases. A portion of these capital resources is made available by a reduction in the allocation of capital to the transactions facility due to financial innovation; i.e. i_t^b is lower, although the capacity of the system, g_t, is higher.[21] In subsequent periods, consumption of the home and cash goods also rise. On balance, equations (31)-(32) indicate that the realignment of the consumption bundle gives a greater share of output to the trade good—i.e. η_t^3 increases—and lower shares to the home and cash goods—i.e., η_t^1 and η_t^2 are lower.

A positive monetary innovation, θ_t^4, is known by the household at the beginning of the period prior to taking consumption and production decisions. However, in this model the monetary transfer cannot be used to relax the current-period 'cash-in-advance' constraint on the purchase of the cash good. In the spatial trading environment described in Section I, an agent of the household trades home good for money which is to be used for purchasing the cash good in the *next* period. The monetary transfer therefore relaxes next period's 'cash-in-advance' constraint, and thereby reduces the *current*-period offering of home good in the adjacent market, owing to the associated inflation tax. Output is therefore reduced by the full amount of the reduction in the consumption of the cash good.[22] Current-period consumption of the home and cash goods are unaffected, although in subsequent periods consumption of the home and trade goods also decline, but by a lesser amount than the decline in the cash good. The monetary distortion thereby causes home and trade goods to assume larger shares of output; i.e. η_t^1 and η_t^3 increase and η_t^2 falls.

V. Conclusions

In a (hypothetical) primitive, non-monetary pure barter economy with many goods, trade takes place via a sequence of bilateral exchanges that is necessitated by 'double coincidence of wants' problems. Coordination of these trades is, in general, costly. If the duration of the trading period is finite and the spatial separation of the transactions markets is sufficiently great, certain desired exchanges may not take place. That is, the transactions costs may become prohibitively high, and the transactions market may be 'missing'. The introduction of money into the economy as a common medium of exchange can substantially reduce these coordination costs by enabling money to be exchanged for good whenever a double coincidence of wants problem emerges in a bilateral exchange. Therefore, many of the markets that may otherwise be missing in a pure barter economy could exist in a monetary economy.

However, missing transactions markets may continue to exist in monetary economies provided that the search problem necessitated by the spatial separation of markets is sufficiently costly to solve. To fill these missing transactions markets, further reductions in the coordination costs are required. In modern economies, this is achieved in part by financial institutions that employ real resources to coordinate the transfer of ownership of claims on assets between agents. Not all of these transfers involve financial assets whose supply is subject to regulatory control by the central bank. Many of these transactions are consummated by a series of accounting entries that, of course, net out to zero, less the costs of providing the transactions services. That is, the institutions that effect these transactions constitute a system of non-monetary exchanges which are supported by the dedication of real resources from the economy that could otherwise be used to produce goods or effect transactions, but are instead more highly valued as a means of supporting a centralized, coordinated transactions facility. Therefore, these institutions emerge endogenously to fill missing transactions markets and, more generally, to displace the less efficient of the monetary exchange markets (although this latter condition was not explicitly modelled in this paper).

With capital as a factor input in the economy's payments system, there are significant general equilibrium consequences associated with the absorption and translation of real and monetary shocks into economic fluctuations. Of particular interest is the recognition of an additional channel that this introduces by which monetary innovations in the form of anticipated inflation affect the real economy. An increase in the inflation tax on a subset of consumption goods induces a reallocation of capital away from production and towards the payments system. The effects of the distortion are to alter the consumption bundle (with too little consumption of the cash good) and lower output. The latter effect is a direct consequence of the model's prediction that the inflation tax alters the structure of the payments system by rendering it more capital-intensive.

ACKNOWLEDGMENTS

An earlier version of this paper was presented in April 1990 at the Spring 1990 Academic Conference at the Federal Reserve Bank of Atlanta under the title, 'Efficient Barter in an Equilibrium Monetary Economy'.

NOTES

1. Brunner and Meltzer (1971) recognized the trade-off that 'money is a substitute for investment in information' (p. 799). However, the investment that they had in mind is in terms of *human* capital. King and Plosser (1986) have pursued this insight to examine the medium of exchange property of money under various information structures. Our paper focuses instead on investment in *physical* capital, which we conjecture more adequately explains the secular movements of the GNP velocity of M1 since the Second World War.
2. With the exception of the early 1980s, the M1 velocity in the United States has exhibited a secular rise over the past 45 years, reflecting this move towards a more capital-intensive payments system. The decline of velocity in the 1980s is attributable to the nationwide legalization of NOW accounts, and has been concentrated in the household sector. See Marquis and Witte (1989).
3. The payments system that is modelled here represents a generalization of the payments system normally associated with a 'cash-in-advance' economy; see e.g. Lucas and Stokey (1987). It also differs from King and Plosser's (1984) approach to modelling a transactions technology which ignores capital as an input into the production technology for transactions.
4. The facility that Black has in mind is obviously a financial institution that engages in the various aspects of financial intermediation that modern economies require. One feature of Black's system that is relevant to the economy's payments system, and that our model does not capture, is the ability of the financial institution to provide trade credit via the same accounting mechanism described in the text. Rather than requiring all agents to effectively possess zero balance accounts that clear simultaneously each 'period', his scheme would allow individuals to run positive or negative account balances which earn positive and negative interest rates, respectively, with the accounts being debited to pay for the transaction services rendered by the financial institution. This suggests an alternative means of introducing trade credit into a monetary or pure exchange economy that is not restricted to bilateral contracts between individual trading agents, as is the current practice in, e.g. Townsend (1989), Lacker and Schreft (1991) and Ireland (1991). Fama (1980) explores a similar institutional structure. Our current research seeks to incorporate this feature of a more general payments system into a detailed model of the financial institution. However, our purpose in writing this paper is to examine the general equilibrium aspects associated with an optimal selection of a capital-based payments system.
5. In this setup, there is no opportunity for households to trade for their respective home goods; moreover, it gives each household a monopoly over the production of their home good. This could be avoided by assuming a large (infinite) number of identical households at each location (i, j, x). However, this assumption is unnecessary for the purposes of the model. It is not important that opportunities for barter in the home good exist, only that there are no payments system constraints that would impose costs on the acquisition of the good. The monopoly over production of the home goods is dissipated by assuming that potential demanders of the good may choose from a large (infinite) number of goods that they perceive to be perfect substitutes.
6. This is an extreme form of the Kiyotaki–Wright (1989) model, where households are willing to trade for a consumption good that is 'close enough' along a continuum of goods to their desired good. The analogy that they draw is to a continuum of colours along a spectrum, as suggested by Lucas (1980). This produces some positive measure to the event that the household will locate an acceptable good.
7. Engineer and Bernhardt (1991) choose a utility function for which Inada conditions do not require consumption to be positive in all goods. This allows them to illustrate the dependence of the payments system on the monetary distortion such that if the inflation tax is high enough agents have an incentive to cash in their monetary assets, and thereafter conduct trade via costly barter arrangements. In this case money becomes valueless, and the monetary equilibrium collapses.
8. This is equivalent to a 'putty-clay' model of capital with 100 per cent depreciation. It is a well-known problem that the latter is, in general, required in order to obtain a closed-form solution. To avoid the usual unsatisfying assumption that all capital in the economy is fully depreciated each period (see e.g. Long and Plosser, 1983, we have adapted the storage technology specification of Eichenbaum and Singleton (1986) to our problem, which allows

the 'unused' endowment of capital ('putty') to receive a stochastic return each period. It should be noted that such a specification lends itself well to introducing a 'time-to-build' technology into the model.

9. In this model, all three consumption goods have the same unit price because of the symmetry of the model; i.e. what is a home good for a household at one location is a cash good or a trade good for households at other locations.
10. The monetary transfer in the current period, J_t, is trivially related to the monetary innovation in the current period by expression $J_t = M_{t-1}[(M_{t-1})^{(\alpha_4-1)}\theta_t^4 - 1]$, which reduces to $J_t = M_{t-1}[\theta_t^4 - 1]$ for $\alpha_4 = 1$, which is the specialization used in this model.
11. These latter restrictions on the production technologies do have a limited qualitative effect on the model. A financial innovation affects only the consumption of the trade good in the current period, whereas a monetary innovation affects only the consumption of the cash good in the current period. However, the penalty of this assumption is relatively small. The questions that cannot be answered are: How would households optimally distribute the decline in the consumption of the home good and the cash (trade) good in the current period in response to a positive financial innovation (negative monetary innovation) that increases consumption of the trade (cash) good?
12. This is consistent with the actual time series of virtually all monetary series.
13. The transversality condition on the evolution of the money endowment is that the expected monetary growth rate exceed $(1/\beta)$. The transversality condition on the evolution of the capital endowment, i.e. on the storage technology, is noted in n. 18 below.
14. This interpretation results from having written the constraints in nominal terms. Had they all been deflated by the price level, the λ would have the interpretation of the marginal utility of relaxing the appropriate constraint by one unit of c_t^3.
15. Note that $U_1(\cdot)$, $U_2(\cdot)$ and $U_3(\cdot)$ equal the left-hand side of equations (14), (15) and (16), respectively.
16. Compare equations (22)-(24) with equations (5.7) and (5.8), p. 231, in Townsend (1987).
17. It can also be demonstrated that the inefficient pricing of money as an asset in terms of consumption, and the real rate-of-return dominance of capital over money that Townsend identified, carry over to this model.
18. The key to the solution is to 'guess-and-verify' that the expression $c_t^1 = \beta(\theta_t^1\theta_t^2/\theta_{t-1}^2)c_{t-1}^1$ solves the equation $(\theta_t^2/c_t^1) = \beta E_t(\theta_{t+1}^1\theta_{t+1}^2/c_{t+1}^1)$. This latter expression can be trivially discovered by manipulating equations (14), (17), (19) and (20).
19. The transversality condition on the evolution of the capital stock is seen from either equation (29) or (34) to be that the expected value of (Ψ_t, θ_t^1) is strictly less than $(1/\beta)$.
20. This is an artefact of the assumptions of linear production and capital-based transactions technologies that were required to obtain a closed-form analytical solution. In general, the utility gains from a financial innovation would be spread across all consumption goods. This would include the cash good, which is the market that determines the price level. In the case where the production decisions are taken by fully informed agents, the price level would fall to facilitate the higher real demand for money. If the financial innovation were realized after the production decisions were made but prior to the consumption decisions, then the relaxation of the transactions facility's capacity constraint would induce a shift in the consumption bundle towards the trade good and away from the home and cash goods. The latter would reduce the current-period real demand for money that would be satisfied by an increase in the price level. In this case, the financial innovation has a similar short-run effect on the price level as a monetary innovation. This result is also obtained in Marquis and Cunningham (1990) in a model where the medium of exchange is not good-specific.
21. This freeing up of real resources for production of goods is an issue that Black (1970) has emphasized in his discussion of the evolution of the payments system towards a cashless society.
22. This result, that consumption of the cash good completely absorbs this shock, is also due to the assumption of linear production and capital-based transactions technologies.

REFERENCES

BLACK, FISCHER (1970). Banking and interest rates in a world without money. *Journal of Bank Research*, **1**, 9–20.

BRUNNER, KARL and MELTZER, ALLAN H. (1971). The uses of money: money in the theory of an exchange economy. *American Economic Review*, **61**, 784–805.

EICHENBAUM, MARTIN and SINGLETON, KENNETH J. (1986). Do equilibrium real business cycle theories explain postwar US business cycles? NBER Working Paper no. 1932, May.

ENGINEER, MERWIN and BERNHARDT, DAN (1991). Money, barter, and the optimality of legal restrictions. *Journal of Political Economy*, **99**, 743–73.

FAMA, EUGENE (1980). Banking in a theory of finance. *Journal of Monetary Economics*, **6**, 39–57.

HESTER, DOBALD D. (1981). Innovations and monetary controls. *Brookings Papers on Economic Activity*, **1**.

IRELAND, PETER N. (1991). Endogenous financial innovation and the ratchet effect on money demand. Unpublished manuscript, University of Chicago, May.

KING, ROBERT G. and PLOSSER, CHARLES I. (1984). Money, credit and prices in a real business cycle. *American Economic Review*, **74**, 363-80.

—— and —— (1986). Money as the medium of exchange. *Journal of Monetary Economics*, **17**, 93-115.

KIYOTAKI, NOBUHIRO and WRIGHT, RANDALL (1989). A contribution to the pure theory of money, Federal Reserve Bank of Minneapolis Staff Report no. 123, August.

LACKER, JEFFREY M. and SCHREFT, STACY L. (1991). Money, trade credit, and asset prices. Unpublished manuscript, Federal Reserve Bank of Richmond.

LONG, JOHN B. and PLOSSER, CHARLES I. (1983). Real business cycles. *Journal of Political Economy*, **91**, 39-69.

LUCAS, ROBERT E. (1980). Equilibrium in a pure exchange economy. *Economic Inquiry*, **18**, 203-20.

—— and STOKEY, NANCY L. (1987). Money and interest in a cash-in-advance economy. *Econometrica*, **55**, 491-514.

MARQUIS, MILTON H. and CUNNINGHAM, STEVEN R. (1960). Financial innovation, price smoothing, and monetary policy. *Economic Inquiry*, **28**, 831-50.

——, —— and WITTE, WILLARD E. (1989). Cash management and the demand for money by firms. *Journal of Macroeconomics*, **11**, 333-50.

TOWNSEND, ROBERT (1987). Asset-return anomalies in a monetary economy. *Journal of Economic Theory*, **41**, 219-47.

—— (1989). Currency and credit in a private information economy. *Journal of Political Economy*, **97**, 1323-44.