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# Real interest rates and endogenous growth in a monetary economy

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In endogenous growth models where the engine of growth is human capital acquired via formal education, inflation taxes may raise or lower real interest rates depending upon whether or not physical and/or human capital are liquidity constrained. Cases are examined.

## 1. Introduction

In a neoclassical, partial currency model with cash-in-advance imposed on capital, the inflation tax associated with positive nominal interest rates distorts steady-state allocations away from capital and toward consumption of the credit good. This effectively raises the marginal product of capital, and hence the steady-state real interest rate. In a model of endogenous growth with human capital as the engine of growth, Marquis and Reffett (1991) have shown that liquidity constraints on investment in human capital that is acquired through formal education, leads to a distortion due to inflation taxes along the balanced growth path that reduces the rate of acquisition of human capital, and hence the rate of growth of the economy. Households effectively choose to allocate more of their non-leisure time to production and less to education than is consistent with Pareto optimal allocations. In this model, real interest rates are positively related to the rate of growth and are therefore reduced by the inflation tax.

The purpose of this not is to drive more general conditions in a model of endogenous growth via formal education under which real interest rates are affected by the inflation tax when it impinges on the investment decisions for both physical and human capital. A sufficient condition for this distortion to raise real rates is that the rate of depreciation of physical capital exceed that of human capital. Otherwise, if the distortion is large and the quality of education is low, the inflation tax may lower real interest rates.

## 2. Theoretical model of endogenous growth via formal education

The model is similar to Marquis and Reffett (1991). Households maximize lifetime utility which is derived from the consumption of two goods: the credit good, $c_1(s_t)$, and the cash good, $c_2(s_t)$, where $s_t$ is the vector of state variables for the representative household (described below). The choice variables include the consumption goods, next period's stocks of physical and human capital, $k_{t+1}$ and $h_{t+1}$, the allocation of non-leisure time to production, $v$, and the stock of money balances carried forward to next period, $m^d_{t+1}$.

$$\max_{\{c_1(s_t),\, c_2(s_t),\, v,\, m^d_{t+1} k_{t+1} h_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U[c_1(s_t), c_2(s_t)], \quad 0<\beta<1. \tag{1}$$

Gross investments in physical and human capital, $x(s_t)$ and $z(s_t)$, are given by:

$$k_{t+1} \le (1-\delta_1)k_t + x(s_t), \quad x(s_t) \ge 0, \tag{2}$$

$$h_{t+1} \ge (1-\delta_2)h_t + z(s_t), \quad z(s_t) \ge 0, \tag{3}$$

where $\delta_1$ and $\delta_2$ are the rates of depreciation on physical and human capital. Physical capital is acquired by the transformation of output into capital goods rather than consumption goods via the production function, $F[K_t, VH_t]$, where the upper case letters denote economy-wide per capita averages that are taken as given by the representative agent. $VH_t$ is therefore the average skill-weighted hours worked per capita.

Human capital is acquired via formal education and requires that non-leisure time be diverted from production. Normalizing the total amount of non-leisure time to one, the amount of time used by the household to acquire formal education is $(1-v)$. The technology for acquiring the education is expressed in terms of units of human capital as:

$$z(s_t) \le (1-v)\gamma, \quad \gamma > 0, \tag{4}$$

where $\gamma$ measures the quality of the education in terms of number of units of human capital acquired per unit of time.

The household's budget constraint is given as:

$$\begin{aligned} &P(S_t)[c_1(s_t) + c_2(s_t) + x(s_t)] + Q(S_t)z(s_t) + m^d_{t+1} \\ &\quad \le (M_{t-1} + J_t) + P(S_t)\{F[K_t, VH_t] + (k_t - K_t)F_1[K_t, VH_t] \\ &\qquad + (vh_t - VH_t)F_2[K_t, VH_t]\}, \end{aligned} \tag{5}$$

where $P(S_t)$ and $Q(S_t)$ are the unit prices of output and human capital; $S_t$ is the vector of per capita state variables for the economy, and includes $K_t$, $H_t$, $V$, and $M_t$, which is the beginning of period per capita money supply and $J_t$, which is the per capita monetary transfer for the period. (The subscripts on the production function indicate partial derivatives.) The vector of state variables for the household, $s_t$, can now be defined as $S_t$, $v$, $m_t$, $h_t$, and $k_t$.

To allow for alternative sets of liquidity constraints, the following cash-in-advance constraint is imposed.

$$P(S_t)[c_1(s_t) + \phi_1 x(s_t)] + \phi_2 Q(S_t)z(s_t) \le m_{t-1} + J_t, \tag{6}$$

where $\phi_1$ and $\phi_2$ are indicator variables that take on a value of one when a liquidity constraint is imposed on investment in physical or human capital respectively, and zero otherwise. The money supply is governed by the following aggregate monetary rule:

$$M_t = G(S_{t-1}) M_{t-1}, \tag{7}$$

where $G(S_t)$ is the state dependent rule.

To complete the model, equilibrium pricing of investment units of human capital, $Q(S_t)$, in terms of output prices, $P(S_t)$, can be found by equating the marginal product of human capital in production with the marginal product of human capital in education. [See Marquis and Reffett (1991) for a more detailed discussion of the model.] This requires:

$$Q(S_t)\gamma = P(S_t) F_2(K_t, VH_t). \tag{8}$$

A stationary monetary equilibrium solution to this model consists of the price vector, $[P(S_t), Q(S_t)]$, the optimal vector of choice variables for the household, $[c_1(s_t), c_2(s_t), v, m^d_{t+1}, k_{t+1}, h_{t+1}]$, and the monetary rule, $G(S_t)$, that satisfy (details are contained in an appendix that is available from the authors):

*Household optimization.* The first-order conditions resulting from (1).

*Market-clearing conditions.* (i) goods market equilibrium: $c_1(s_t) + c_2(s_t) + x(s_t) = F(K_t, VH_t)$; (ii) money market equilibrium: $m_t = M_t$; (iii) capital market equilibria: $h_t = H_t$ and $k_t = K_t$; and (iv) $v = V$ such that eq. (8) holds.

*Constant monetary distortion.* Constant non-negative nominal interest rate, $r$.

## 3. Results

To obtain explicit solutions, the model is specialized with the following utility and production functions:

$$U[c_1(s_t), c_2(s_t)] = \ln c_1(s_t) + a \ln c_2(s_t), \quad a > 0. \tag{9}$$

$$F[K_t, VH_t] = K_t^{\alpha}(VH_t)^{(1-\alpha)}, \quad \alpha > 0. \tag{10}$$

Of interest are the effects of the alternative liquidity constraints on capital that are imbedded in (6) on the equilibrium real interest rate. The real interest rate, $i$ is defined in terms of the real return to physical capital net of depreciation, i.e., $1 + i = F_1[K_t, VH_t] + (1 - \delta_1)$. There are four cases.

*Case 1.* The partial currency model with no liquidity constraints on capital, i.e., $\phi_1 = \phi_2 = 0$. The growth rate of the economy (i.e., for all real variables), $\theta$, and the real interest rate are given by the following expressions:

$$\theta = \beta[(1 - \delta_2) + \gamma/2], \tag{11}$$

$$1 + i = (1 - \delta_2) + \gamma/2. \tag{12}$$

In this case, neither the growth rate nor the real interest rate are affected by the monetary distortion. This is a generalization to an endogenous growth context of Abel's (1985) result for the neoclassical growth model that the steady-state capital stocks are unaffected by the inflation tax when it impinges only on consumption decisions.

*Case 2.* Investment in physical capital is liquidity constrained; investment in human capital is not, i.e., $\phi_1 = 1$ and $\phi_2 = 0$.

$$\theta = \beta[(1-\delta_2) + \gamma/2], \tag{13}$$

$$1 + i = [(1-\delta_2) + \gamma/2] + r[(\delta_1 - \delta_2) + \gamma/2]. \tag{14}$$

When only physical capital is liquidity constrained, the monetary distortion does not affect growth, but it does affect levels. From (14),

$$\mathrm{d}i/\mathrm{d}r = (\delta_1 - \delta_2) + \gamma/2 \gtreqless \text{ as } \delta_2 \lesseqgtr \delta_1 + \gamma/2 \tag{15}$$

implying that positive nominal interest rates reduce the stock of physical capital, thereby raising the marginal product and increasing the real interest rate, provided that the depreciation rate on human capital is small relative to the depreciation rate on physical capital and to the quality of education indigenous to the economy.

*Case 3.* Liquidity constraints are imposed on human capital but not on physical capital, i.e., $\phi_1 = 0$ and $\phi_2 = 1$. [This is the case examined in Marquis and Reffett (1991).]

$$\theta = \beta[(1-\delta_2) + \gamma/(2+r)], \tag{16}$$

$$1 + i = (1-\delta_2) + \gamma/(2+r). \tag{17}$$

In this case, the monetary distortion raises the cost of acquiring human capital, and households reallocate time away from education and toward production. This reduces the growth rate of output, and hence reduces the real interest rate, i.e., $\mathrm{d}\theta/\mathrm{d}r < 0$ and $\mathrm{d}i/\mathrm{d}r < 0$.

*Case 4.* Liquidity constraints are imposed on investment in both physical and human capital, i.e., $\phi_1 = \phi_2 = 1$.

$$\theta = \beta[(1-\delta_2) + \gamma/(2+r)], \tag{18}$$

$$1 + i = [(1-\delta_2) + \gamma/(2+r)] + r[(\delta_1 - \delta_2) + \gamma/(2+r)]. \tag{19}$$

In this model, the monetary distortion has both growth and level effects. The growth rate is reduced by a positive nominal interest rate as in Case 3. By itself, this would tend to lower the real interest rate; however, the liquidity constraint on physical capital reduces the stock of capital along the balanced growth path from it otherwise would have been as in Case 2. This tends to raise the real interest rate. From (19),

$$\mathrm{d}i/\mathrm{d}r = (\delta_1 - \delta_2) + \gamma/(2+r)^2 \gtreqless 0 \quad \text{as} \quad \delta_2 \lesseqgtr \delta_1 + \gamma/(2+r)^2. \tag{20}$$

The criterion for signing the effect on the real interest rate to the inflation tax is similar to Case 2, viz., $\delta_2$ must be sufficiently small for the sign to be positive. However, the upper limit on $\delta_2$ beyond which there would be a sign reversal is lower in this case due to the lower returns to investment associated with the lower growth rates.

## 4. Conclusions

Pareto optimal allocations in a monetary economy are associated with a constant deflation that produces a zero nominal interest rate [Friedman (1969)]. In an endogenous growth model with human capital acquired via formal education as the engine of growth, the distortion associated with the inflation tax affects neither the growth rate nor the real interest rate unless capital is liquidity constrained. Whenever human capital acquisition is liquidity constrained, the inflation tax unambiguously lowers growth. In order that real interest rates rise in response to the monetary distortion, liquidity constrains must be imposed on physical capital. However, this result depends on the relative depreciation rates of physical and human capital as well as on the quality of education indigenous to the economy. A sufficient condition for an inflation tax to raise real interest rates in this case is that the depreciation rate on physical capital exceed that on human capital. The empirical evidence on this is scanty. For the U.S. economy, Mincer and Ofek (1982) have estimated the annual depreciation rates for human capital in a study of temporary dropouts from the workforce to be in the 3.3 to 7.6 percent range. Other studies have obtained estimates ranging from 0.6 to 13.3 percent [Johnson (1970), Johnson and Hebein (1974), Heckman (1975), Haley (1976), and Rosen (1976)]. This compares to approximately 10 percent for physical capital. [See, for example, Kydland and Prescott (1982).] While these empirical results are not conclusive, they do suggest that the distortion produced by the inflation tax is more likely to raise than to lower real rates, even though growth may have been reduced.

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