

# Preface: Special Issue on Dynamic Games in Macroeconomics

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Many important questions in macroeconomics can be formulated as a dynamic game. For example, in dynamic general equilibrium models with public policy, where policymakers face a common agreed upon social objective, the optimal plans of the current government can fail to be time consistent (e.g., Kydland and Prescott ([15]) and Barro and Gordon ([8])). This time inconsistency problem can be studied as a constrained optimization problem with forward or backward looking endogenous constraints on an enlarged state space (e.g., see Kydland and Prescott ([16]), Rustichini ([27]), Marcet and Marimon ([21]), or Feng, Miao, Peralta-Alva, and Santos ([13])), or as a dynamic game (i.e., Pollak ([26]), Peleg and Yaari ([23]), Phelan and Stacchetti ([24])). A related situation arises when households make consumption-savings decisions and their intertemporal preferences are dynamically inconsistent. This problem was first studied in Strotz ([29]) and Pollak ([26]), and has been the focus of an extensive literature in macroeconomics (e.g., see Laibson ([17]) and Bernheim, Ray, and Yeltekin ([10])). Yet another prototype dynamic game in macroeconomics arises in altruistic models of economic growth where the dynastic choice problem between generations is a strategic one. Models in this spirit were first introduced in Phelps and Pollak ([25]), and subsequently studied in Bernheim and Ray ([9]), Leininger ([18]), Amir ([3]), and Balbus, Reffett, and Wozny ([6], [5]), among others. One additional prototype of a dynamic games and strategic interactions in macroeconomics occurs

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in models of endogenous borrowing constraints and sustainable debt (e.g., Chari and Kehoe ([11]) and Alvarez and Jermann ([4])).

This special issue of *Dynamic Games and Applications* collects recent papers providing new results on the existence, characterization, and the computation of dynamic equilibria in macroeconomic models with strategically interacting agents. The volume contains work in both dynamic and stochastic games. In the paper by Messner and Pavoni ([22]), the authors reconsider the nature of the solutions to incentive constrained dynamic programs generated by recursive saddlepoint methods pioneered in the work of Marcat and Marimon ([21]). These methods have found extensive application in the literature over the last two decades on dynamic models with limited commitment and strategic interaction. The paper shows that even under the best of case (i.e., the incentive-constrained dynamic programs remain convex, saddlepoints exist, and strong duality is present), the recursive saddlepoint method can provide solutions that are not primal feasible optimal and suboptimal.

Two papers in the volume consider strategic dynamic programming as an alternative to recursive saddlepoint methods for computing dynamic equilibria in games. These methods can be viewed as set-valued primal formulations of incentive-constrained dynamic programming methods, and were pioneered for repeated games by Abreu, Pearce, and Stacchetti ((henceforth APS, [1], [2])). In this volume, the methods are extended to the setting of dynamic and/or a stochastic games. When these methods can be applied, one obtains characterization of equilibrium values of all the subgame perfect equilibria in the games relative to the equilibrium concept imposed (e.g., subgame perfect, Markov perfect, etc.)<sup>1</sup>

It is well-known that computing the equilibrium value set can be a serious complication of implementing strategic dynamic programming methods. In the paper of Sleet and Yeltekin ([28]), the authors discuss how APS methods can be applied to interesting classes of dynamic games, and then provide a new method for computing equilibrium value correspondences. The authors new method is based on outer and inner approximations of the equilibrium value correspondence via step-valued set functions. The new methods are related to the methods used in the paper by Feng and Dominguez ([12]) to compute optimal time consistent solutions for dynamic general equilibrium models with public policy.

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<sup>1</sup>In the original APS methods, the value correspondence was constructed for subgame perfect equilibrium. But as the paper of Balbus and Woźny ([7]) in this volume shows, one can consider shorter memory constructions (e.g., using nonstationary minimal state space Markov perfect equilibria).

In the paper of Balbus and Woźny ([7]), the authors extend APS methods to the study of the equilibrium value set generated by Markov Perfect Nash equilibrium (MPNE) in a class of stochastic games with uncountable state spaces. In their methods, the equilibrium value correspondence is generated by short-memory nonstationary equilibrium Markov decision processes. The authors then show how to apply these methods to the problem of subgame perfect equilibrium in dynamic models where agents possess dynamically inconsistent preference (e.g., models with quasihyperbolic agents).

In the paper by Fesselmeier, Mirman, and Santugini ([14]), the authors study a Cournot-Nash equilibrium in an important class of one-sector growth models with strategic interactions and externalities. Their model studies the case of strategic interaction between agents over dynamic consumption and investments in a world with technical progress. Specializing the primitives of their model in a similar vein as used in the seminal paper of Levhari and Mirman ([19]), the authors are able to obtain sharp closed-form characterizations of Markovian equilibrium in this class of models.

The last two papers of the volume study prototype time consistency problems as they arise in dynamic general equilibrium models in macroeconomics. In the paper by Feng and Dominguez ([12]), the authors study the time-consistency problem of labor taxes in a dynamic economy without capital taxes. The environment is very closely related to the Ramsey taxation setting studied in the paper of Phelan and Stachetti ([24]), a prototype time consistency problem in the spirit of Kydland and Prescott ([15], [16]). But in their paper, the emphasis is on the structure of time consistent labor taxes (as opposed to capital taxes).

In the paper by Malair and Malair ([20]), the authors consider the question of computing time consistent equilibrium solutions for models where agents possess dynamically inconsistent preferences (the prime example being the so called " $\beta - \delta$ " models with quasihyperbolic agents). In particular, they show that generalized Euler equation methods that have been applied extensively in this literature generate a continuum of equilibrium solutions, each characterized by a different constant of integration. Then, they develop an endogenous grid method (based on backward value function induction) where the limit of the finite horizon solution converges to a limiting solution for the infinite horizon case.

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