AN EAR FOR GEARS - UNDERSTANDING GEARBOX SIGNATURES
Speed reducers and other gearboxes are common industrial components. Monitoring their case vibration with accelerometers is an effective means of detecting problems within. To effectively diagnose these important mechanical elements, you need to understand the geometric details of their construction. Most importantly, you need to know the arrangement of shafts and the number of teeth on every gear within your transmission.

Consider the simplest system, two mating spur gears. Presuming the gearbox is a speed reducer, the input shaft has the smaller diameter of the two gears – we will call this the pinion. The pinion has $T_p$ teeth on its perimeter, and these mate with the $T_g$ teeth of the larger gear on the slower output shaft. The gear ratio (the speed reduction ratio), $R$, is defined as:

$$R = \frac{T_g}{T_p} \quad (1)$$

But knowing the number of teeth on the gear and pinion tell us much more than simply the speed reduction (torque multiplication) provided by the gearbox. Figure 1 illustrates the conventional numbering sequence of gear and pinion teeth as they enter mesh with one another. Now consider which gear teeth engage which pinion teeth during operation. We will look at three different sized gears mated to a 6-tooth pinion.

![Figure 1: Tooth numbering convention](image)

If the gear is of the same size (6 teeth), each pinion tooth only engages its numerically equal gear tooth as shown in figure 2. Thus pinion tooth #4 always transfers its load to gear tooth #4 (and no other) and so on. Since only like-numbered gear and pinion tooth-pairs are mated throughout their service life, it is likely they will develop complimentary wear patterns to compensate for any minute imperfections of manufacture. Should this gear pair be separated during a repair, the odds are 5 to 1 that this same arrangement of parts will not be restored unless the mating parts were marked at disassembly. In subsequent operation, the mating teeth will no longer have their hard-won complimentary wear patterns; this can cause a signature change.

Now consider the entirely different situation illustrated by figure 3. Here, the same 6-tooth pinion drives a 7-tooth gear. Note that every pinion tooth eventually drives every gear tooth. All 42 mating combinations are encountered in seven pinion revolutions (6 gear revolutions). That is, this gear set can only be assembled in one way and there is no benefit to marking its parts at disassembly. Such a gear set is said to have one assembly phase in contrast to the gear set of figure 2 which has six assembly phases.

![Figure 3: Tooth mating sequence for 6-tooth pinion and 7-tooth gear](image)
Adding another tooth to our gear produces a situation between these extremes. As shown in figure 4, when a 6-tooth pinion drives an 8-tooth gear, every pinion tooth only comes into contact with four gear teeth and every gear tooth only engages three pinion teeth. By following the path lines between tooth-pair intersection points, you will note a given pair of teeth enters the mesh every 3 gear rotations (every 4 pinion rotations).

![Figure 4: Tooth mating sequence for 6-tooth pinion and 8-tooth gear](image)

Figure 4: Tooth mating sequence for 6-tooth pinion and 8-tooth gear

We can summarize these observations by rewriting the gear ratio equation. Specifically:

\[
R = \frac{T_g}{T_p} = \frac{N_{gp} \cdot t_g}{N_{ap} \cdot t_p} \quad (1a)
\]

\(N_{ap}\) is the largest (integer) common factor between \(T_p\) and \(T_g\). This common factor is the number of assembly phases possible between the pinion and gear, while \(t_p\) is the number of gear teeth in the assembly phase and \(t_p\) is the number of pinion teeth in the assembly phase.

Figure 5 makes these points graphically. It illustrates one of the 12 possible assembly phases between a 108-tooth pinion and a 120-tooth gear. In this case:

\[
R = \frac{120}{108} \times \frac{12 \times 10}{12 \times 9} = 1.111 \quad (1b)
\]

While all of the gear and pinion teeth participate in mesh, any tooth within this assembly phase can only contact the mate shown. Clearly, this gear pair may be assembled with twelve different tooth-mating sequences.

The frequencies defined by Equations (2) through (6) can all be expected to be present (with varying intensities) in any gearing signature. Invariably, \(f_{mesh}\) will be a dominant component in any noise or vibration spectrum measured from a gearbox. Note that gear mesh is a much higher frequency than the shaft speeds, the assembly phase passage frequency and the “hunting tooth” frequency. In fact, all of these frequencies are integer sub-harmonics of \(f_{mesh}\). The hunting-tooth frequency is the lowest characteristic frequency of the gear-pair and the remaining characteristic frequencies are all integer harmonics of \(f_{hunting\ tooth}\)

\[
f_{gear} = \frac{\text{RPM}_{gear}}{60} \quad (2)
\]

where: \(\text{RPM}_{gear}\) = the gear-shaft turning speed in RPM and \(f_{gear}\) = the (same) gear-turning frequency in Hertz

\[
f_{pinion} = R f_{gear} \quad (3)
\]

where: \(f_{pinion}\) = the turning speed of the pinion shaft in Hertz

\[
f_{mesh} = T_g f_{gear} = T_p f_{pinion} \quad (4)
\]

where: \(f_{mesh}\) = the frequency at which teeth-pairs enter the gear mesh

\[
f_{assembly\ pass} = T_g f_{gear} = T_p f_{pinion} = \frac{f_{mesh}}{N_{ap}} \quad (5)
\]

where: \(f_{assembly\ pass}\) = the frequency at which an assembly phase passes through mesh

\[
f_{hunting\ tooth} = \frac{f_{gear}}{t_p} = \frac{f_{pinion}}{t_g} = \frac{f_{mesh}}{N_{ap}} = \frac{N_{ap} f_{mesh}}{T_g T_p} \quad (6)
\]

where: \(f_{hunting\ tooth}\) = the frequency at which a specific tooth-pair mates in mesh.
Gears function to transmit torque from one shaft to another. In a perfect world, they would do so with the quiet smoothness of a pair of rolling elements in perfect contact with infinite friction at their juncture. Gear tooth profiles are carefully designed to pass load from one tooth to the next as smoothly as possible. They engage one another with an intended rolling contact and as little sliding as possible to preserve high efficiency. Unfortunately, perfection has eluded us and real gears perform this load hand-off between adjacent tooth-pairs with a certain amount of impacting, sliding, bending and other socially undesirable behavior. This “mating activity” repeats at the mesh frequency. \( f_{\text{mesh}} \) and its harmonics are always present in the noise and vibration spectra, most particularly when the gears are lightly loaded. The shaft speeds \( f_{\text{gear}} \) and \( f_{\text{pinion}} \) may appear in vibration spectra due to unbalance of the driver or driven element. Second harmonics of these terms often accompany misalignment of shafts (or loose mounting of a drivetrain component). Generally, these frequencies indicate problems external to the gear-set.

In contrast, a small eccentricity of the pinion on its shaft will produce vibration at the sum and difference frequencies - in contrast, a small eccentricity of the pinion on its shaft will in these frequencies indicate problems external to the gear-set. In practice, both the mesh frequency carrier and the (pinion or) gear-shaft frequency modulator can have many harmonics (of various phasing) present. This means that the resulting modulated signal can be more complicated with distinct tones appearing at frequencies of \( nf_{\text{mesh}} \pm kf_{\text{gear}} \), where \( n \) and \( k \) are integer constants. Normally, the terms symmetrically disposed about \( f_{\text{mesh}} \) (where \( n \) equals 1) will dominate. Sidebands associated with low \( k \) value are usually the most detectable. Gear wear and clearance changes in plain bearings can cause a change in the \( f_{\text{gear}} \) and \( f_{\text{pinion}} \) sidebands. So can an unrelated repair that causes the gears to be re-mated with “new” assembly phasing. Such unintended signature change may also be accompanied by a rise in the \( \pm f_{\text{assembly pass}} \) sidebands of \( f_{\text{mesh}} \) and its harmonics (when these terms are unique).

In a 1:1 gearing, \( N_{\text{sp}} = T_p = T_g \), so that \( f_p = f_g = 1 \). Hence, \( f_{\text{assembly pass}} = f_{\text{gear}} = f_{\text{pinion}} \) and the assembly phase passage frequency is not a unique characteristic. The gear-set is designed for uniform distribution of tooth wear, the gear and pinion tooth-counts share no prime numbers in common, save 1. In this circumstance, \( N_{\text{sp}} = 1 \), resulting in \( f_{\text{assembly pass}} = f_{\text{mesh}} \). Hence any \( \pm f_{\text{assembly pass}} \) sidebands of \( f_{\text{mesh}} \) and its harmonics will appear as either an harmonic of \( f_{\text{mesh}} \) or at DC; they will not be uniquely identifiable. In all other situations, \( f_{\text{assembly pass}} \) is a unique characteristic frequency. Since \( f_{\text{assembly pass}} \) is typically much higher than either \( f_{\text{gear}} \) or \( f_{\text{pinion}} \), the \( \pm f_{\text{assembly pass}} \) sidebands of \( f_{\text{mesh}} \) will be widely spaced from their carrier and are not likely to be confused as “high \( k \) images” of either \( f_{\text{gear}} \) or \( f_{\text{pinion}} \).

Presence of the very low frequency \( f_{\text{hunting tooth}} \) is a clear indicator of severe local tooth-pair damage such as that encountered when the mesh has “digested” a significant solid contaminant. This term also shows up as a pair of symmetric sidebands centered upon the mesh frequency. It is difficult to detect these \( f_{\text{mesh}} \) \( \pm f_{\text{hunting tooth}} \) components and differentiate them from gear mesh (particularly in uniform tooth wear gearing) owing to the close frequency spacing. An order normalized analysis of at least \( T_p, T_g / N_{\text{sp}} \) lines of spectral resolution is required to detect these indicators. They are worth separating; when they are observed, they typically indicate local tooth damage that can be seen with the unaided eye.

If the gearbox you are diagnosing operated at a tightly regulated speed, all of these terms may be discernible in an ordinary FFT spectrum. However, if the machine speed varies (albeit slightly) with load, the resulting spectral peaks you wish to characterize will likely be blurred or smeared into the background across many frequency bins. The answer to this problem is to run the Order Tracking software on your handheld CoCo-80/90 or modular Spider-80X front end. This optional software phase-locks the sample rate of your analyzer to the speed of a shaft by synchronizing on a pulse tachometer signal. Digital re-sampling of the data provides a matching speed-tracking anti-aliasing filter.

![Figure 6: Block diagram of the Order Tracking measurement process](image-url)
In fixed-bandwidth operation, an analyzer collects N successive samples from an analog time-history at a constant sample rate, fs. The analog signal is pre-filtered by a low-pass anti-aliasing filter set to the desired analysis frequency range, F\text{span}, and the sample rate is set to k F\text{span}, where k is a constant specific to the analyzer. Each captured time-history is transformed to yield a spectrum. The following spans and resolutions result:

\[ \Delta t = \frac{1}{f_s} = \frac{1}{k F_{\text{span}}} \text{ time between adjacent time points (S)} \]
\[ T_{\text{span}} = N \Delta t \text{ duration of each time capture or memory load period (S)} \]
\[ \Delta F = \frac{1}{T_{\text{span}}} \text{ difference between adjacent frequency points (Hz)} \]
\[ F_{\text{span}} = N \Delta F / k \text{ frequency range presented (Hz)} \]

In order-normalized (order-tracked) analysis, both the frequency range and sample rate must vary in proportion to the machine speed. This is accomplished by measuring the shaft speed with a tachometer and deriving a sample rate equal to \( k O_{\text{span}} \) times the instantaneous shaft speed. \( O_{\text{span}} \) is the maximum number of shaft-speed orders (multiples) to be measured in a spectrum. The effective anti-aliasing filter must constantly adjust to limit the incoming signal bandwidth to \( O_{\text{span}} \) times the shaft-turning frequency. This results in the following spans and resolutions:

\[ \Delta R = \frac{1}{f_s} = \frac{1}{k O_{\text{span}}} \text{ shaft-angle between adjacent signal samples (Revolution)} \]
\[ R_{\text{span}} = N \Delta R \text{ number of turns in each memory capture (Revolution)} \]
\[ \Delta O = \frac{1}{R_{\text{span}}} \text{ difference between adjacent order points (Order)} \]
\[ O_{\text{span}} = N \Delta O / k \text{ order span presented (Order)} \]

Typical analyzers require between 2.56 and 4 samples per maximum order spanned. This is the same k multiple relating the analyzer’s sample-rate to the frequency band studied in normal fixed-bandwidth analysis. The exact numeric value is determined by the analyzer’s design specifics.