



UNDERSTANDING LIQUEFACTION THROUGH APPLIED MECHANICS

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ABSTRACT

Soil liquefaction is conventionally evaluated through an empirical framework based on accumulated experience from case histories - the "NCEER Method". However, the empirical framework contains inconsistent physics and characterizations that are unrelated to modern understanding of soil constitutive behaviour. This paper considers cyclic mobility within the context of a modern constitutive model, NorSand, to illustrate a proper approach to evaluating the effect of soil type ('fines content'), stress level and initial stress state on liquefaction. A two-pronged approach is used with soil state in situ being inferred from the CPT, while the cyclic strength-state relationship is computed using measurable, standard, soil properties (compressibility, etc). Computed liquefaction resistances are consistent with the case history record, but the approach now offers understanding as to how that experience should be extrapolated to other situations. And, contrary to what might be expected, the proposed approach turns out to be both straightforward and readily done in engineering practice - the calculations run in a spreadsheet. Of course, it now becomes necessary to measure soil compressibility and elastic shear modulus.

Keywords: State Parameter; Plasticity; Shear Modulus (G_{max}), Cyclic Mobility; CPT.

INTRODUCTION

Soil liquefaction is a phenomenon in which soil loses much of its strength or stiffness. These strength losses are generally of short duration, but nevertheless long enough for liquefaction to be the cause of many failures, deaths and major financial losses. Arguably liquefaction is the most difficult situation any geotechnical engineer will deal, with considerable importance from both public safety and financial standpoints.

The difficulty with liquefaction stems from three things. First, although the nature of the process is easy to understand – soil particles re-arranging to denser state under a stress perturbation and causing excess pore water pressure to develop because of insufficient time for the displaced water to escape – it is complicated to quantify these processes in constitutive models; it is only in the past twenty years of liquefaction experience that proper models have become available, and few (if any) are comprehensible to most geotechnical engineers. Implementation of such models in useable modelling packages (finite element codes or the like) is a further issue, with none of the good liquefaction models readily available for engineering practice. Second, the stress conditions that arise during liquefaction are difficult to simulate with standard laboratory tests: the triaxial test, the backbone of most understanding of soil behaviour, is a particularly poor analogue for earthquake induced stress paths; the simple shear apparatus is attractive at first sight, but leaves horizontal stress unmeasured and suffers from non-uniform

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conditions; the hollow cylinder test, while allowing very good simulation of appropriate loading paths, is complex and difficult to use with only a handful of test programs at very few research universities. Direct measurement of soil behaviour is not practical, and even calibrating those liquefaction models that do exist is problematic. Third, direct physical simulation using models in a centrifuge has limited use for most engineering projects because of the high cost involved. And, physical modelling is not exactly a good basis to develop understanding as far too few variables and soil properties are investigated.

Given these three broad areas of difficulty, it is unsurprising that geotechnical engineers have resorted to a 'case history' based approach in which experience is assimilated into charts indicating probable performance. In the case of static liquefaction, or the available strength at large deformation following a liquefaction event, back analysis of past failures (normally using familiar limit equilibrium methods) provides estimates of liquefied strength (s_r). In the case of earthquake induced liquefaction, earthquake ground motion is idealized as a vertically propagating shear wave in the soil from the underlying rock. The average horizontal cyclic shear stress (τ_{av}) induced by the vertically propagating wave is estimated by site response analysis for the recorded strong ground motion, and this stress is normalized by the initial vertical effective stress σ'_{v0} to give the cyclic stress ratio τ_{cyc}/σ'_{v0} (= 'CSR') as a measure of the imposed loading of the ground by the earthquake. Conditions of 'liquefaction' and 'no liquefaction' being distinguished as a GO/NO-GO criterion for design; Figure 1 shows an 'expert consensus' version of the cyclic resistance from Youd et al (2001). In either case, the soil condition corresponding to these strength estimates is characterized by its penetration resistance with a further slight indexing by fines content. Given sufficient time, and detailed back-analyses have been accumulating for some fifty years, surely this knowledge should provide the best-approach to liquefaction assessment? And, that is indeed the claim in the state-of-practice publication on this approach a decade ago (Youd et al, 2001).

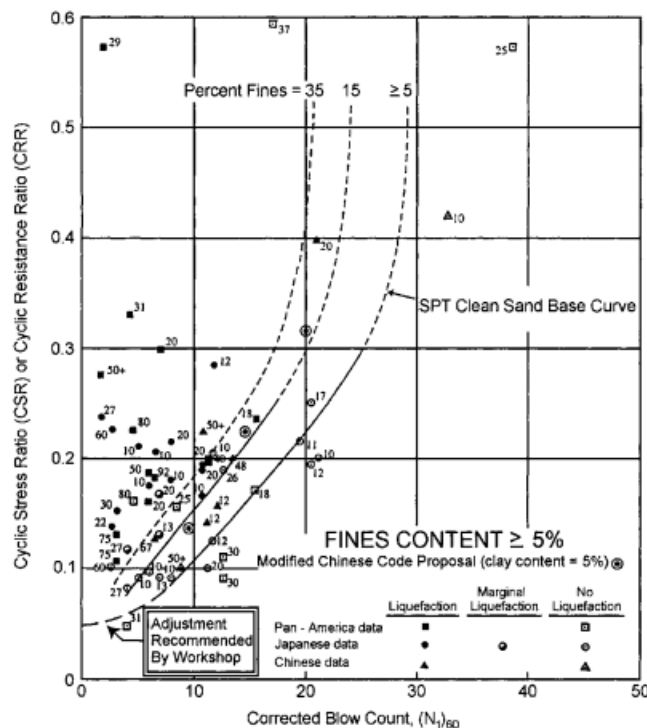


Figure 1: SPT Clean-Sand Base Curve for Magnitude 7.5 Earthquakes with Data from Liquefaction Case Histories (Youd et al, 2001). (Modified from Seed et al. 1985)

However, there are serious flaws in the present version of the case history approach. One of these flaws is readily appreciated from an example. The first evaluation of the s_r from the case history data was by Seed (1987) and correlated s_r directly to the normalized penetration resistance $(N_1)_{60}$, see Figure 2. This was despite exactly the same penetration resistance measure being correlated to a stress ratio (τ_{av}/σ'_{v0}) in the case of liquefaction triggering (Figure 1, a form of which was first proposed by Seed & Idriss, 1971). Both correlations cannot be correct, since the first correlation implies $(N_1)_{60}$ has the dimensions FL^{-2} while the second correlation implies that this penetration resistance measure is dimensionless. Almost unbelievably, advocacy that Figures 1 and 2 (or variations on the same) should be used even arose in invited workshops to progress the understanding of liquefaction sponsored by the National Science Foundation during the 1990's despite the mutually exclusive nature of the implied mechanical framework. The situation appears to be now resolved with acceptance that the post-liquefaction strength correlates to penetration resistance as a stress ratio (see for example Robertson, 2010). But, between 1987 and 2005 many clients paid for engineered construction/remediation based on a methodology that even the simplest mechanics showed to be wrong. Does this serve our customers well? Is this denial of elementary mechanics appropriate for a learned profession?

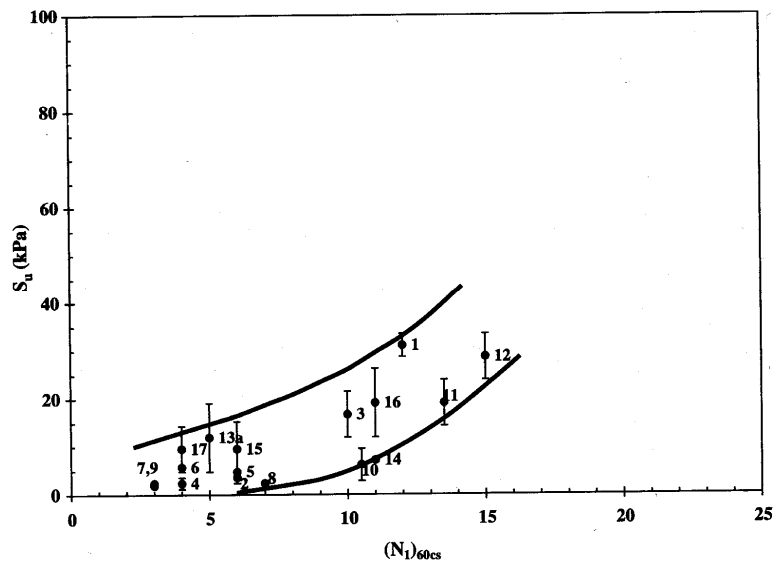


Figure 2: Correlation for post-liquefaction strength as modified by Seed & Harder (1990) from original proposal by Seed (1987)

There are further issues with the case history approach. First, penetration resistance depends on several soil properties, with theoretical analyses indicating the importance of compressibility in particular (apparently first suggested by Robertson & Campanella, 1983 and considered at length in Shuttle & Jefferies, 1998). But, the case history approach of Youd et al (2001) only admits one measure of soil type: fines content (the fraction finer than the #200 sieve). In reality, although fine grained soils tend to be more compressible than coarse grained soils, fines content on its own is poorly correlated to the compressibility that affects penetration resistance. Other properties that markedly affect penetration resistance (elastic shear modulus G_{max} and the critical friction angle are two) are not even admitted as factors in the case history method. Second, there is an effect of stress level, and a perceived effect of static shear stress, on the assessed strength when taking it forward into design; these issues are associated with 'correction' factors. But, what is the basis for these factors? Might they change with soil type? Might they change with initial geostatic stress ratio? Are they even dimensionally consistent? Is even the adjustment of penetration resistance for fines content correct?

These difficulties with design methods based entirely on correlations to experience are neither new nor limited to liquefaction. The proper way forward is to use case-history data within a framework of understanding based on applied mechanics – this is exactly what is done in other branches of engineering, including other branches of civil engineering. In the mechanics approach the case-history data is not neglected, but rather used to determine the effect of ‘model uncertainty’, thus calibrating a possibly idealized understanding to the practical reality of full-scale civil engineering works. In the case of soil liquefaction, however caused, liquefaction is a constitutive behaviour subject to the laws of physics, it becomes necessary to describe the mechanics mathematically – that is, the starting point to a mechanics-based understanding is an appropriate constitutive model. There are now several such constitutive models.

In this paper, we consider liquefaction induced by earthquakes because of cyclically varying stresses in the ground. In this situation the available soil strength to resist earthquake induced liquefaction is called the cyclic resistance ratio ($CRR = \text{cyclic shear stress} / \text{initial vertical effective stress}$) and the value to be used for an element of soil in the ground takes the form:

$$CRR = CRR_{15} K_M K_\sigma K_\alpha \quad (1)$$

where CRR_{15} follows from Figure 1 or similar and K_M , K_σ , K_α are factors to scale the design curve from Figure 1 for, respectively, earthquake magnitude, stress level, and pre-existing shear stress on a horizontal plane.

In assessing a proper framework for [1], the constitutive model used is general and transitions smoothly from cyclic mobility to outright large displacement static liquefaction as appropriate. We briefly describe the model (this is not a mechanics paper...) and then illustrate its calibration to a reasonably comprehensive set of laboratory data. We then move to compute a framework for the standard liquefaction charts in terms of measurable soil properties and determine the true nature of the K_σ and K_α ‘correction’ factors. We do not consider the magnitude correction factor K_M . All calculations, including the constitutive model, run within an Excel spreadsheet. The authors are happy to supply copies of this spreadsheet to interested workers under the terms of the GNU open software foundation license; all code is open-source, and there are no hidden or black-box routines. The applied mechanics involved is straightforward modern soil plasticity, and not much more complicated than the widely taught Cam-Clay model. The soil properties involved are largely conventional (only a couple of new soil properties will be introduced) and all are measured in standard laboratory tests. In situ conditions are determined using the combination of measured soil properties and measured penetration resistance; these aspects are also discussed in illustrating the correct form for liquefaction assessment.

ROLE OF CRITICAL STATE SOIL MECHANICS

Density affects the behaviour of all soils – crudely, dense soils are strong and dilatant, loose soils weak and compressible. Now, as any particular soil can exist across a wide range of densities it is unreasonable to treat any particular density as having its own properties. Instead what is needed is a framework that explains why a particular density behaves in a particular way. The aim is to separate the description of soil into true properties that are invariant with density (e.g. critical friction angle) and measures of the soils state (e.g. current void ratio or density). Soil behaviour should then follow as a function of these properties and state.

To date, all constitutive models for soil that are based on density-invariant soil properties fall within what is usually called ‘critical state theory’. The name critical state derives from anchoring the theory to a particular condition of the soil, called the critical void ratio by Casagrande in 1936. The critical state is the end state if the soil is deformed (sheared) continuously. The neat thing about the critical state, at least mathematically and philosophically, is that if the end-state is known it then becomes simple to construct well-behaved models. You always know where you are going. As a quick aside, some US workers have referred to the end-state condition of loose samples failed in rapid undrained loading as the *steady state*. However, there is no difference between the definitions of steady and critical states and they refer to the same soil behaviour. We retain the modern usage of the term critical state throughout (interested readers can find a history of this subject, and detailed discussion, in Jefferies & Been, 2006).

The critical void ratio varies with stress level, and this variation with mean effective stress (σ_m') is denoted as the critical state locus/line (CSL). It is common to treat the CSL as semi-logarithmic for all soils:

$$e_c = \Gamma - \lambda \ln(\sigma_m') \quad (2)$$

in which Γ and λ are intrinsic soil properties. Caution is needed when looking at quoted values of λ , as both log base 10 and natural logarithms are used. Natural logarithms are more convenient for constitutive modelling, whereas base 10 logarithms arise when plotting experimental data. We distinguish between the two using the notation λ and λ_{10} ($= 2.303\lambda$). The parameter Γ also has an associated stress level, which is $\sigma_m' = 1$ kPa by convention. More sophisticated representations of the CSL exist, but the validity of the CSL as a frame of reference does not depend on a semi-log approximation – it is only a modelling detail.

The CSL also involves continued shearing and the stresses to cause this shearing are denoted by the ratio M ($M = \eta_c$ where $\eta = \sigma_q/\sigma_m'$ where σ_q is the deviatoric stress invariant in the critical state and σ_m' the mean effective stress, with the subscript c denoting shearing at the critical state). The numerical value of M depends on the intermediate principal stress, and it is the value of M under triaxial compression, M_{tc} , that is taken as the soil property by convention. The variation of M with intermediate principal stress then becomes a feature of the constitutive model (see Jefferies & Shuttle, 2010 for further discussion).

One of the attractions of the CSL is that it is a large deformation state, and as such is independent of initial soil fabric. This leads to the great convenience that the CSL can be measured by testing reconstituted samples in the laboratory.

Constitutive models for soil based on the CSL were developed in the mid-1960's including Granta Gravel, Cam Clay and Modified Cam Clay. Granta Gravel and Cam Clay were presented in the book *Critical State Soil Mechanics* (Schofield and Worth, 1968; download at <http://www.geotechnique.info>) and there is a widespread tendency to infer the description "critical state soil mechanics" implies Cam Clay or a derivative of it. This tendency is unfortunate, as all good soil models adopt a CSL as their basic internal reference, but no other of the good models have the particular idealizations of Cam Clay. By "good" we mean models that predict the effect of void ratio on soil behaviour, these models having the feature that their properties are independent of soil density.

All these modern, good constitutive models invoke plasticity. Plasticity is based on their being a region in stress space in which small changes in stresses produce only small, recoverable changes in soil deformation – described by elasticity. The boundary to the region is a yield surface and stress changes directed outwards from this boundary cause large, irrecoverable deformations – ‘plastic’ strains.

Cam Clay and its variants are based on the idealization that the soil's yield surface intersects the CSL. This idealization, while simple and convenient, is neither necessary nor helpful in general (it is the reason why Cam Clay is a very poor model for sands). Modern constitutive models do not link the yield surface directly to the CSL, instead relating the yield surface to the offset of the soil's void ratio from the CSL – the 'state parameter' ψ , defined on Figure 3. Dense soils have negative ψ and loose contractive soils have positive ψ . Like void ratio, ψ is dimensionless.

The way modern models use the CSL is by making peak frictional strength or peak dilatancy a function of ψ (it does not matter greatly which approach is followed because dilatancy and strength are related through stress-dilatancy behaviour; again this is a modelling detail rather than something fundamental). Figure 4 shows test data from some twenty sands with a range of fines content to illustrate the relationship between the dilational component of strength $\phi - \phi_c$ and ψ . On average, the simple relationship emerges:

$$\phi_c = \phi_c - 50 \psi \quad (3)$$

where ϕ_c is the maximum frictional strength in triaxial compression and ϕ_c is the critical friction angle; both are measured in degrees and ψ is determined at peak strength, not the initial conditions of the test.

When this basic idea of relating soil strength to ψ was suggested (Been & Jefferies, 1985) it was thought that the coefficient ('50') in [3] was independent of soil type – an enormously attractive idea. Since then, with the testing of a wider range of soils and particularly well-graded soils, it has become apparent that greater accuracy is obtained in constitutive modelling if the coefficient is recognized as a soil property that is measured using drained triaxial compression tests.

Turning to cyclic loading of soils, the same idea as drained compressive strength continues. The actual cyclic behaviour depends on a strength/dilatancy that is related directly to ψ through an analogous relationship to equation [3]. It turns out this is not so very different to the liquefaction assessment chart. However, although ψ is known from the sample void ratio in laboratory testing, that does not work for field application because of the effective impossibility of getting undisturbed samples of sands and silts. The in situ ψ is therefore found from penetration testing (the methodology for this is given later in the paper).

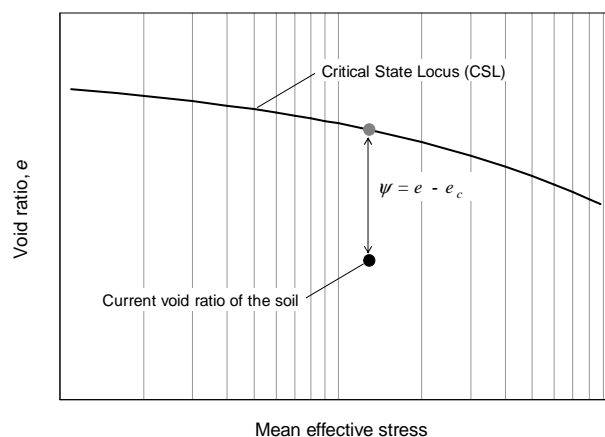


Figure 3: Definition of state parameter ψ

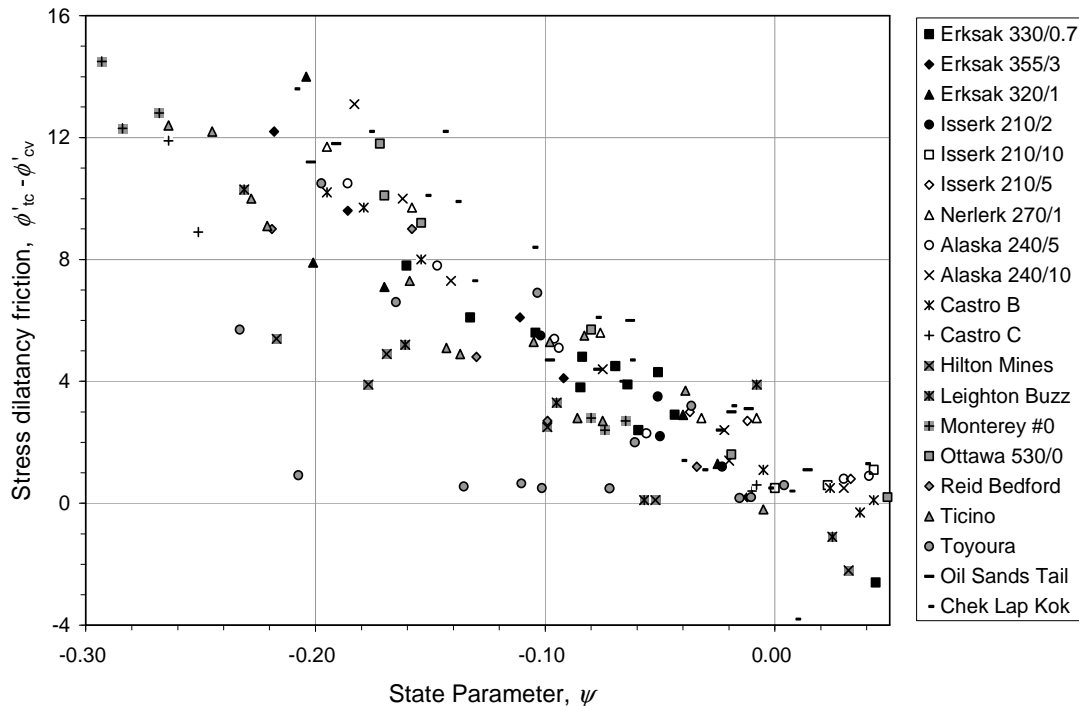


Figure 4: Stress-dilatancy component of peak strength of twenty soils in drained triaxial compression

When using a mechanics based approach, it is possible to eliminate ψ between the CPT evaluation methodology and the cyclic strength assessment to give something similar to the familiar liquefaction assessment chart (Figure 1). We prefer to not do that because soil properties affect cyclic strength and penetration resistance differently, and clearer understanding is found if these aspects are kept separate. There are also more parameters involved than fines content (or compressibility) and it is difficult to develop a clear chart that captures these multiple influences. Conversely, it is straightforward to set up the framework within a numerical solution scheme and this can even be done in a spreadsheet environment – which is the approach we have followed.

ROLE OF PRINCIPAL STRESS ROTATION

From a purely mechanics perspective, there are two objections to characterizing the earthquake loading by the CSR. First, characterization by the CSR implies a horizontal plane controls the soil response whereas all modern constitutive models, which are founded in laboratory element tests, are derived in terms of stress invariants with no attribution of a controlling plane within the element. Second, the CSR obscures the periodic change in principal stress direction that actually occurs and which appears the fundamental driver of soil response at the micromechanical (grain to grain contact) level.

Consider an element of soil lying at some depth below a horizontal ground surface, under geostatic (“at rest”) conditions - the principal stresses will be vertical and horizontal. Restricting our thoughts to plane strain loading, if we consider the case $\sigma_v > \sigma_h$, then $\sigma_1 = \sigma_v$ and this can also be viewed as an angle $\alpha = 0$ where α is the angle between σ_1 and the vertical direction. If a horizontal shear stress is imposed, as say

from a propagating shear wave in an earthquake, σ_v is no longer principal since the plane it now acts on has a shear stress. Because σ_v remains constant (controlled by self-weight) under constant K_0 conditions, the relationship between the shear stress on a horizontal plane τ_H and the angle α is

$$\text{TAN}(2\alpha) = \frac{2\tau_H}{\sigma'_v - \sigma'_H} = \frac{2(\tau_H / \sigma'_v)}{1 - (\sigma'_H / \sigma'_v)} \quad (4)$$

If we look at the maximum values, then substituting for τ_H / σ_{v0} (= CSR) in (4) shows there is a one-to-one relationship between CSR and the magnitude of periodic rotation in the principal stress direction α_c . But, notice that the initial geostatic stress state K_0 is now also a factor:

$$\text{TAN}(2\alpha_c) = \frac{2 \text{ CSR}}{1 - K_0} \quad (5)$$

The importance of principal stress rotation has been essentially neglected within geotechnical engineering. Even today, many of the advanced constitutive models for soils use changes in the stress invariants as the basis for how they simulate cyclic loading by invoking idealizations such as yield surfaces that move in stress space rather than changing their size. But, there is some forty year old data that indicates it is principal stress rotation that controls soil behaviour.

Arthur and co-workers (Arthur et al., 1979) developed a Directional Shear Cell (DSC, which is somewhat like a simple shear test except that all principal stresses are measured. Using the DSC, the principal stress was smoothly varied in a sine-wave at near constant stress ratio. The soil tested was Leighton Buzzard sand, and it was placed dense at approximately $D_r = 90\%$. When this dense sand was subjected to principal stress rotation, at near constant mobilized shear stress, strains accumulated readily with each loading cycle. The greater the principal stress rotation the greater the plastic straining induced. A more detailed study was reported by Wong and Arthur (1986), still using the DSC apparatus. This 1986 study used the same Leighton Buzzard sand, but now looked at two sand densities $D_r = 20\%$ and $D_r = 90\%$. Because the DSC could only impose rather low confining stress (maximum $\sigma'_3 = 20$ kPa), when sheared monotonically in plane strain with fixed principal stress direction, the loose sample ($e_0 \approx 0.73$) gave a peak friction angle of 40° and 5° dilation. The dense sample ($e_0 \approx 0.52$) gave a peak strength of 49° and 18° dilation. Figure 5 plots the relationship between volumetric and shear strain for differing levels of mobilized friction ϕ_m (i.e. the imposed ratio σ'_1 / σ'_3) and different imposed rotations of principal stress. It is apparent from even casual inspection that principal stress rotation has dominated the sand's behaviour and that even rather small stress rotation can suppress dilation. In their conclusion Wong and Arthur (1986) state *Cyclic rotation of principal stress directions in sand which causes strain radically alters the behaviour of the material from that seen in shear under constant directions of principal stress.*

The DSC equipment never became widely used, possibly because it was never able to test at stress levels of practical importance. Japanese work never suffered in this regard because the hollow cylinder was adopted from the outset, Ishihara and Towhata (1983) reporting on the effect of a pure cyclic rotation of principal stresses in drained tests on loose Toyoura sand at a mean effective stress of nearly 300 kPa. The direction of the principal stresses was rotated continuously from 0° to $\pm 45^\circ$ following a semi-circular stress path as shown in Figure 6. That is the deviator and mean stress were kept constant. Even though these familiar measurements of soil loading did not change, principal stress rotation on its own caused an irrecoverable volumetric strain but its increment gradually decreased as the number of cycles increased. The trend of decreasing volumetric strain increment from cycle to cycle is a hardening response.

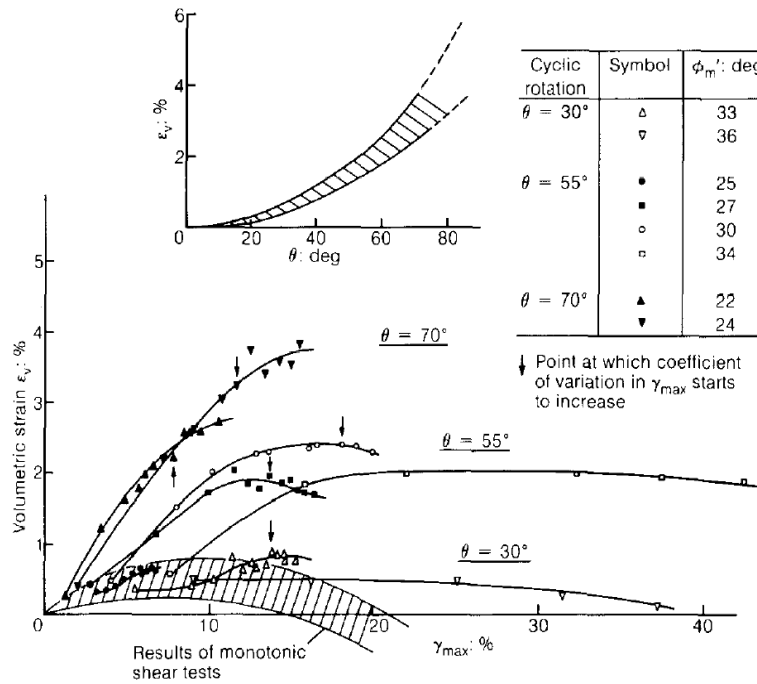


Figure 5: Cumulative volumetric strain in lightly dilatant Leighton Buzzard sand caused by principal stress rotation (Wong and Arthur, 1986)

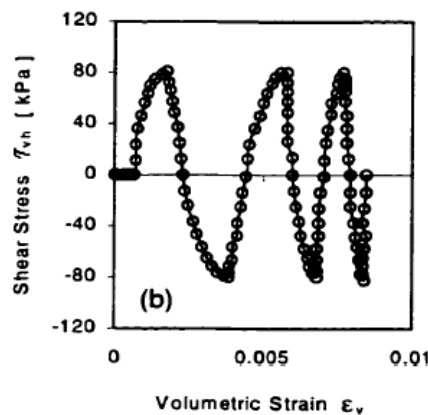
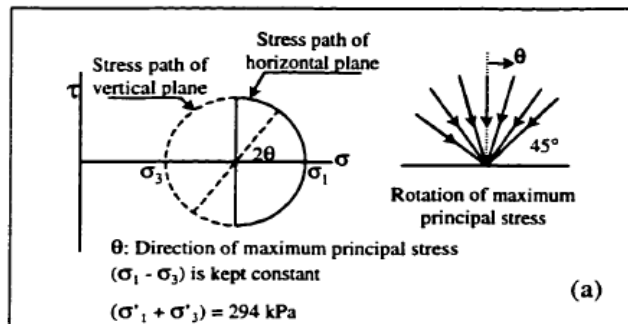


Figure 6: Result of a pure principal stress rotation test on loose Toyoura sand (Ishihara and Towhata, 1983)

These early studies of principal stress rotation were directed at illustrating the importance of this rotation, at a fundamental level, to sand behaviour (and for that matter soil in general). There can be no doubt that principal stress rotation is fundamental from this experimental data, since if stress invariants alone were sufficient then any constitutive model based solely on invariants would predict essentially no strains in the experiments of either Arthur et al. (1980) or Ishihara and Towhata (1983) - contrary to what is observed. Further experiments involving principal stress rotation to begin quantifying the effect of rotations in a range of situations have been carried out. For example, Tatsuoka et al. (1986) present a comprehensive set of data on Toyoura, Fuji and Sengenyama sands in which they compare results of hollow cylinder with cyclic triaxial tests. Sample preparation methods were also varied, and indicated the substantial effect of preparation method (i.e. initial soil fabric) on the results.

Despite these elegant experiments indicating that principal stress direction change is an important 'loading' for soil, to date, only one constitutive model for soils includes both a critical state framework and principal stress rotation: NorSand. This model is now described.

OVERVIEW OF NORSAND

In choosing a constitutive model to represent sands and silts, the goal is to predict the spectrum of behaviours caused by changes in the soil's void ratio and confining stress level. There are now several such models in the literature, all of which are based on the state parameter ψ . NorSand (Jefferies 1993; Jefferies and Shuttle 2002, 2005) was the first of such models. Falling within the general framework of critical state soil mechanics, NorSand is an idealized model in the sense that it is based on postulates about soil behaviour rather than trying to fit trends to experimental data. It is this idealized basis that gives NorSand its power, since it is always consistent mechanics. NorSand's validity derives from its ability to simulate soil behaviour under arbitrary loading paths, after calibration of its few properties for the soil in question.

Critical state soil mechanics is based on two axioms which define the framework: (1) a unique CSL exists; (2) soils move to the CSL with shear strain. Axiom 1 is still regarded as controversial by some, but invariably this traces back to workers who confuse what is variously called the phase transition condition or the pseudo steady state with the critical state. A detailed experimental investigation of the CSL was given in Been et al (1991, 1992), and which addresses issues of uniqueness in particular.

The key idealization introduced by NorSand is that there is a spectrum of normal compression loci (NCL), rather than a single locus that parallels the CSL. But, like realizing that principal stress rotation matters, a spectrum of NCL's was not a new idea - Ishihara et al (1975) identified this aspect of soil behaviour in laboratory test. It is this fact that a spectrum of NCL exist that requires ψ as an index of soil state, since a spectrum of NCL make the statement that a soil is 'normally consolidated' an insufficient description of the soil's condition.

Where NorSand differs from existing 'good' plasticity models for soil is that NorSand includes the effect of principal stress rotation. Since plasticity is an abstraction of the underlying micromechanical behaviour, and grain contacts align to carry the imposed principal stresses, it follows that any rotation of the major principal stress will result in load being applied at sub-optimal grain contact arrangements. This implies softening of the yield surface, as illustrated on Figure 7. The softening rule used in NorSand is that softening is directly proportional to the extent of principal stress rotation with the coefficient of proportionality being a material constant.

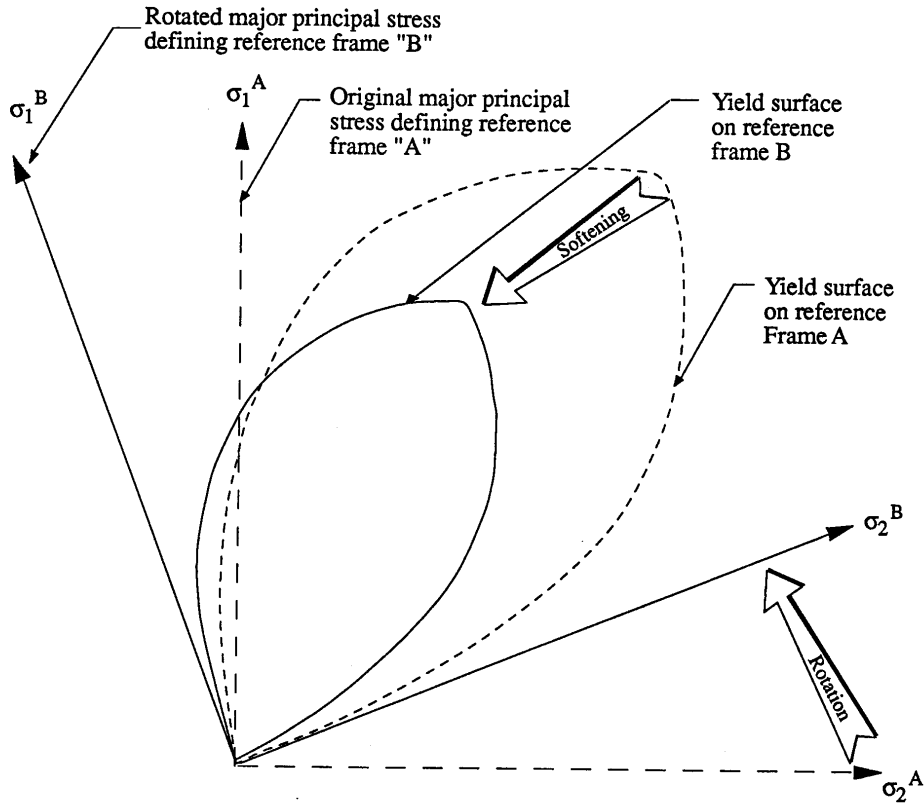


Figure 7: NorSand yield surface softening induced by principal stress rotation

NorSand is a sparse model with just eight soil model parameters, all of which are dimensionless. Three parameters (M_{ic} , Γ , λ) are used to define the critical state. Two parameters are associated with plastic hardening, χ determining the influence of ψ on maximum dilatancy and H being the plastic hardening modulus. Two properties, I_r and ν , define elasticity. The last property is the parameter Z which is the coefficient describing the softening (shrinking) of the yield surface caused by principal stress rotation.

The hardening modulus is required as a direct consequence of a spectrum of NCL's. Unlike Cam Clay in which the NCL and the CSL are parallel, making the parameter λ a general measure of compressibility, decoupling the NCL from the CSL means λ no longer relates to the soil compressibility - another soil property is needed, and this is H . However, there is a general sense that H and λ are related:

$$H \propto \frac{1+e}{\lambda - \kappa} \quad (6)$$

where the proportionality in (6) depends on both ψ and soil fabric (sample preparation method).

NorSand has seen progressive revisions and the version used here is an extension of Jefferies & Shuttle (2002). Further details, including procedures used to determine the properties, and validation examples

over a range of stress paths, can be found in Jefferies and Shuttle (2005). Using NorSand is not difficult. Although its equations cannot be solved in 'closed form', they are trivial to solve numerically - a spreadsheet is sufficient for 'element' tests where the stress or strain path is defined. This will now be illustrated by calibrating NorSand and then predicting cyclic strength behaviour.

EXAMPLE CALIBRATION

Fraser River Sand

Fraser River Sand (FRS), is an alluvial deposit widespread in the Fraser River Delta of the Lower Mainland of British Columbia, Canada. The area includes the city of Vancouver and is of considerable economic importance. Lying on the west coast of North America, the area is vulnerable to earthquakes and the FRS deposits are known to have liquefied in past earthquakes.

Given the wide distribution of FRS in the highly populated and seismically active area near Vancouver, FRS has been quite well researched, and FRS sites were included as part of the CANadian Liquefaction EXperiment (CANLEX) research initiative. Properties of FRS have been reported in the literature, including contributions from Wride & Robertson (1997), Chillarige et al. (1997), Konrad (1997), Sukumaran and Ashmawy (2001), and Ghafghazi & Shuttle (2010), among others. However, despite the number of contributions in the literature, the properties of FRS are not well defined. The problem is that FRS being natural sand, with varying proportion of fines, shows significant natural variation geographically. Therefore it is necessary to test the FRS gradation of interest.

The FRS gradation tested contains around 0.8% fines content and has D_{50} and D_{10} of 0.271 mm and 0.161 mm respectively. FRS is a uniform, angular to sub-angular with low to medium sphericity, medium grained clean sand (Figure 8) with $e_{\min} = 0.627$, $e_{\max} = 0.989$ and $G_s = 2.719$ (Shozen, 1991). The average mineral composition based on a petrographic examination is 25% quartz, 19% feldspar, 35% metamorphic rocks, 16% granites and 5% miscellaneous detritus.

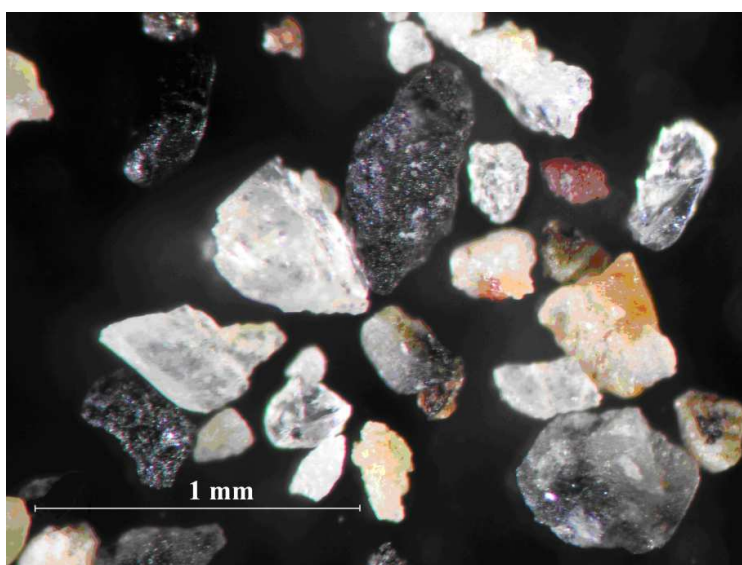


Figure 8: Microscopic picture of FRS grains

Monotonic Calibration to Fraser River Sand

Testing program

The testing program comprised 9 drained and 7 undrained triaxial compression tests prepared using the moist tamping technique. All tests were conducted on samples that were 142 mm in height and 71 mm in diameter, and using lubricated end platens to reduce stress non-uniformity. The samples were flushed with CO₂ and de-aired water and back pressurised until a *B* value of 0.95 or greater was obtained. They were then consolidated and sheared until a steady state was reached, apparent shear localisation was observed, or the equipment limitations were met. The strain controlled shearing was applied at a constant rate of 5% per hour. At the end of the shearing phase the drainage valves were closed. The sample was then removed from the cell and the cell base, membrane and cap were dried before putting the setting in a freezer for 24 hours. The frozen sample was then extracted carefully to ensure no water or grains were lost during the process. This technique effectively eliminated loss of water during sample extraction, enabling accurate determination of water contents. A repeatability of 0.01 or better was obtained for three pairs of tests that were targeted to start from identical conditions. Corrections were applied for membrane penetration (Vaid and Negussey, 1984) and membrane force (Kuerbis and Vaid, 1988).

Critical State Parameters

The stress paths of the 14 of the 16 available triaxial tests which did not include unload-reload loops are plotted in Figure 9. This figure also indicates the start of test, end of test, and whether the sample was dilating, contracting, or undergoing negligible volume change at the end of the test.

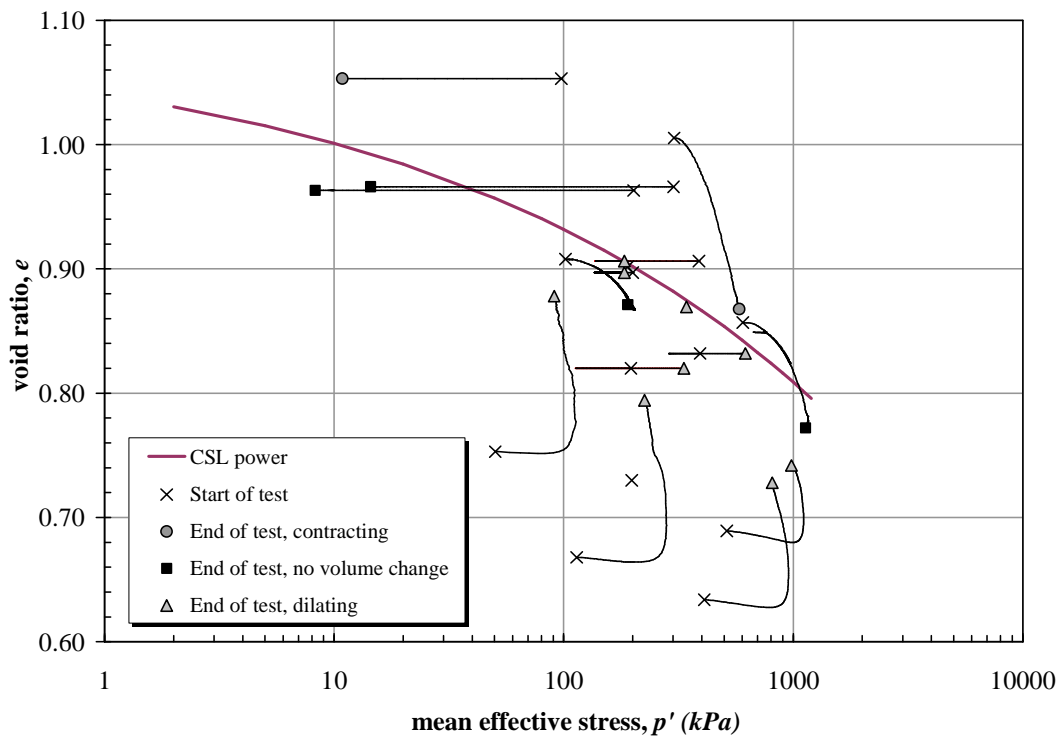


Figure 9: Power law CSL for Fraser River Sand

These data indicate a non-linear critical state locus in e - $\log(p')$ space for a range of stress from about 10 kPa to 1000 kPa. Because reasonably good resolution of the critical state is required at the low mean effective stresses often measured during liquefaction, for FRS it was decided to move beyond the familiar semi-log idealization of the CSL and to adopt a power-law for the CSL:

$$e_c = 1.09 - 0.05 p'^{0.25} \quad (7)$$

where e_c is the void ratio at the critical state.

The critical state friction ratio, M_{tc} , where 'tc' refers to triaxial compression, was obtained using Bishop's method (Bishop, 1971). In this approach the stress ratio at peak ($\eta_{max} = q/p'$) is plotted against the corresponding dilatancy ($D_{min} = \dot{\epsilon}_v / \dot{\epsilon}_q$ where the subscripts v and q refer to the volumetric and shear invariants of strain respectively). Making use of the requirement for a soil reaching peak at the critical state to have zero dilatancy (i.e. $D_{min} = 0$) the value of M_{tc} may be obtained from the 'y intercept'. The Bishop plot for FRS is shown in Figure 10 and gives $M_{tc} = 1.486$. Note that only dilatant samples are plotted.

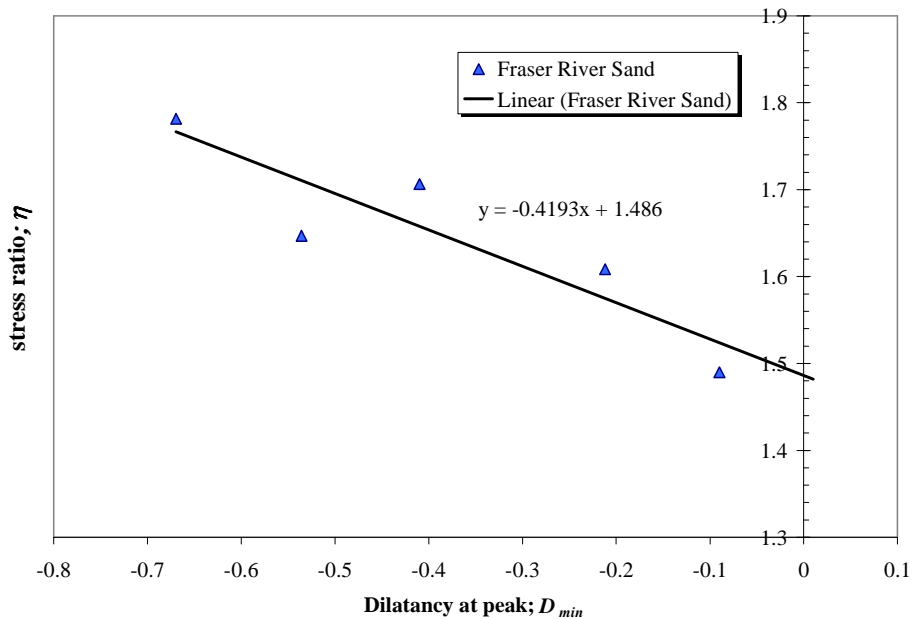


Figure 10: Peak triaxial stress-dilatancy (η_{max} vs. D_{min}) of FRS

Plasticity Parameters

The slope of the stress-dilatancy plot is also used to determine the volumetric coupling parameter, N ; the slope of the line in Figure 10 being equal to $(1 - N)$. This gives a value of $N = 0.5807$ for FRS.

The parameter χ_{tc} is the slope of the trend line for minimum dilatancy ($= D_{peak}$) versus the state parameter at peak. The χ determination for FRS is shown in Figure 11 and gives $\chi_{tc} = 3.17$.

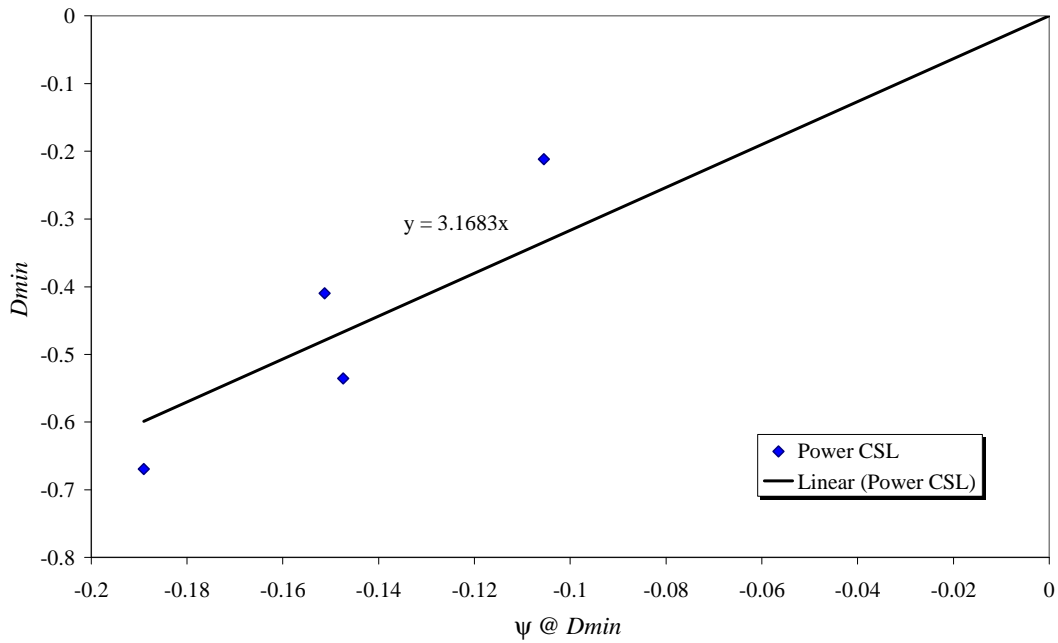


Figure 11: χ from dilatancy at peak (D_{min}) versus ψ at peak for FRS

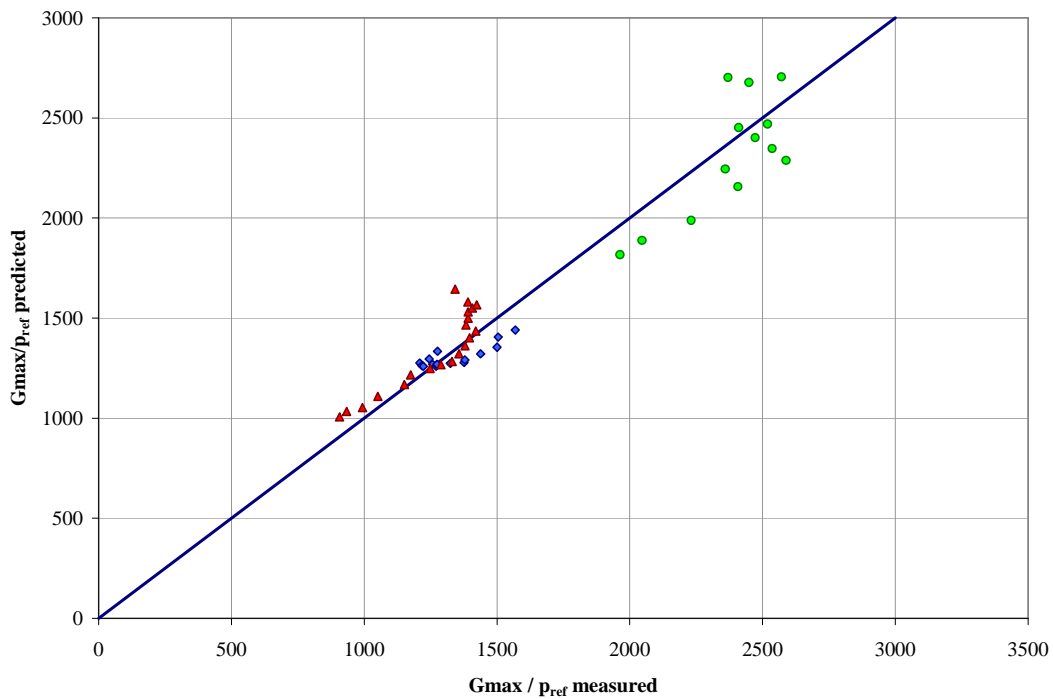


Figure 12: Comparison between measured and modelled FRS elasticity

Elasticity Parameters

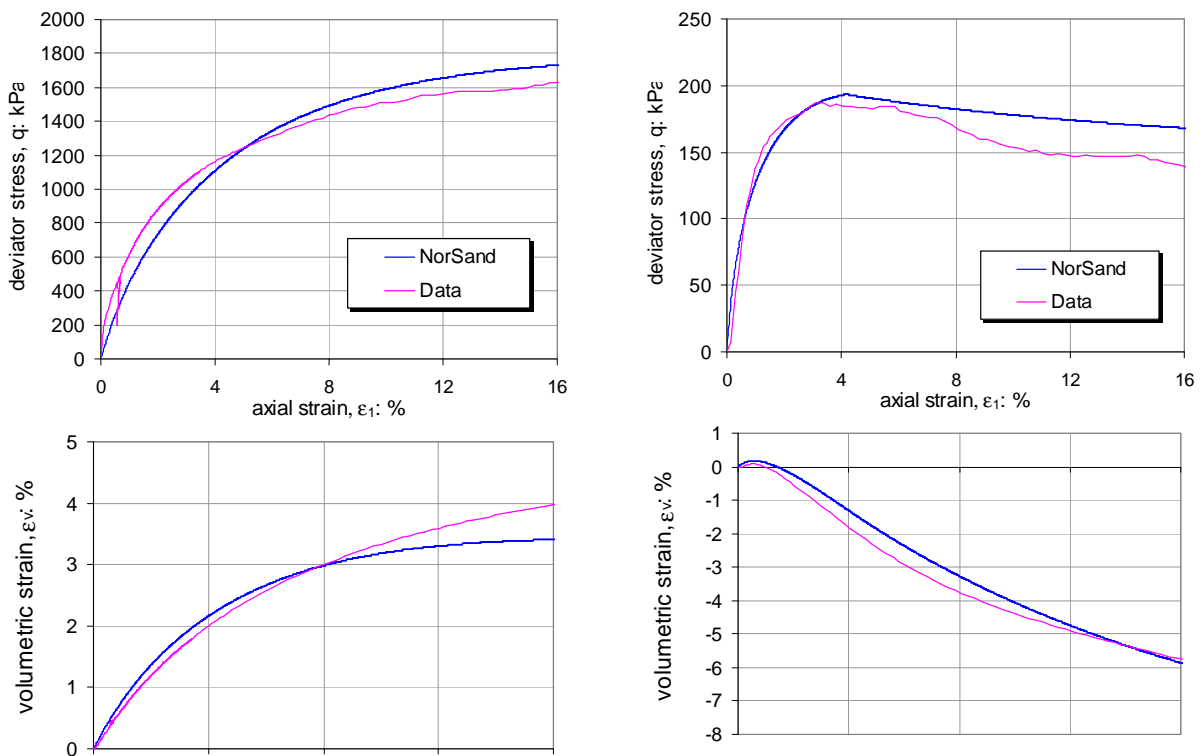
Elasticity during shearing was measured using bender elements. The data was modelled using:

$$\frac{G}{p_{ref}} = \frac{A}{(e - e_{min})} \left(\frac{p}{p_{ref}} \right)^b \quad (8)$$

A good fit of this elasticity idealization (Figure 12, previous page) to the measured data was obtained using $A = 416.6$, $e_{min} = 0.293$, and $b = 0.463$.

Hardening

With the basic soil properties determined as just described, the hardening parameter H is found by iterative forward modelling of entire drained triaxial tests (in a spreadsheet). In this process, an initial value of H is guessed and the theoretical stress-strain behaviour computed using NorSand. Visual comparison of the computed and actual data leads to an improved guess for H , the process being iterate to best align the computed and actual behaviour. Two example calibrations on completion of this iterative procedure are shown on Figure 13 for a loose and a dense sample of FRS.



Test: CID_L_600; $p' = 603$ kPa, $e_0 = 0.857$

Test: CID_D_50; $p' = 50$ kPa, $e = 0.753$

Figure 13: Example calibrations of NorSand for FRS

Calibration for Principal Stress Rotation in Fraser River Sand

Procedure

Soil properties do not change just because the mode of loadings imposed on it are now different. However, determining the effect of principal stress rotation on soil behaviour requires laboratory tests in which principal stress is rotated. Although much understanding about the fundamental influence of principal stress rotation has come from hollow cylinder tests, this equipment is too difficult and time consuming for engineering practice. But, the modern development of computerized cyclic simple shear (CSS) testing is adequate to assess the effect of principal stress rotation once other soil properties have been determined by triaxial compression testing. Computer controlled CSS testing is now routine.

Like the calibration to drained triaxial compression, iterative forward modelling is used in which the computed behaviour is compared to the measured behaviour, over the entire test, with model parameters optimized to best align model with data. The soil parameters already fitted using the drained triaxial data are largely unchanged, 'largely' because elastic moduli depend on fabric but the shear modulus in any CSS sample cannot be measured in the present CSS equipment using bender elements; accordingly, some modest changes in elastic moduli are reasonable. This leaves two parameters for optimizing the fit of NorSand to data: the initial geostatic stress ratio, K_0 , and the softening parameter, Z .

Undrained CSS loading is preferred because the initial geostatic stress is not measured in present CSS equipment (at least not on a routine basis that is available to most engineers). This makes the initial stress state in the test uncertain and leaves K_0 as somewhat of a 'free' parameter in the optimization, since there is quite a range of plausible values (and depending on the sample preparation method). However, the excess pore pressure induced during liquefaction is strongly related to the mean stress; K_0 is used to optimize this aspect of the fit.

Testing Program

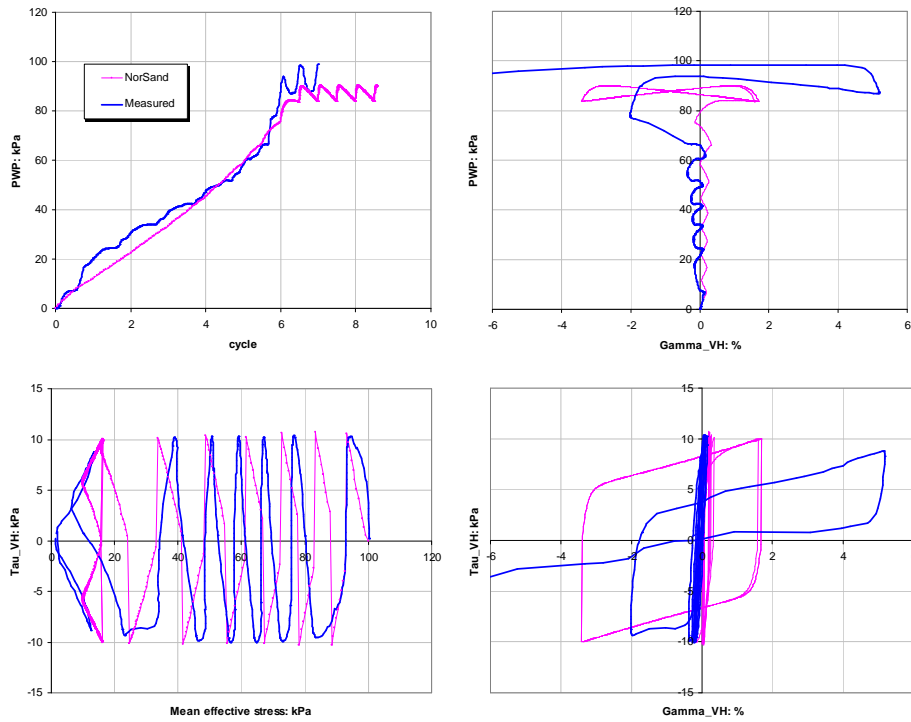
The cyclic simple shear tests on samples of FRS were undertaken at the University of British Columbia (UBC) between 2002 and 2003 as part of the 'Earthquake induced damage mitigation from soil liquefaction' initiative. The testing is described in detail in Sriskandakumar (2004). The UBC simple shear apparatus is of the NGI type (Bjerrum and Landva, 1966) and tests a cylindrical sample 70 mm in diameter and about 20 mm in height. Two relative densities were tested; about 40% and about 80%. All samples were air pluviated to about 40% relative density. The denser samples were then manually tamped prior to confinement being applied. The stress controlled cyclic tests were undertaken by enforcing a constant-volume boundary condition at applied strain rates of 10% or 20% strain per hour.

Calibration

The softening parameter Z controls the rate at which excess pore pressure accumulates and optimizing this parameter is straightforward. Figure 14 shows fits achieved to two of the FRS tests, being loose and dense samples respectively. It is unclear whether Z is a soil property or something more fundamental. We have carried out detailed calibration of NorSand to the cyclic behaviour of Nevada and Fraser River sands. Despite being rather different soils, the variation of Z with ψ appears common to both, Figure 15, with a trend that is given by:

$$Z = 134 + 93 \psi + 21 \psi^2 \quad (9)$$

a) Test FRS DSS40-100-0p1 (Relative density = 40%)



b) Test FRS DSS80-100-0p25 (Relative density = 80%)

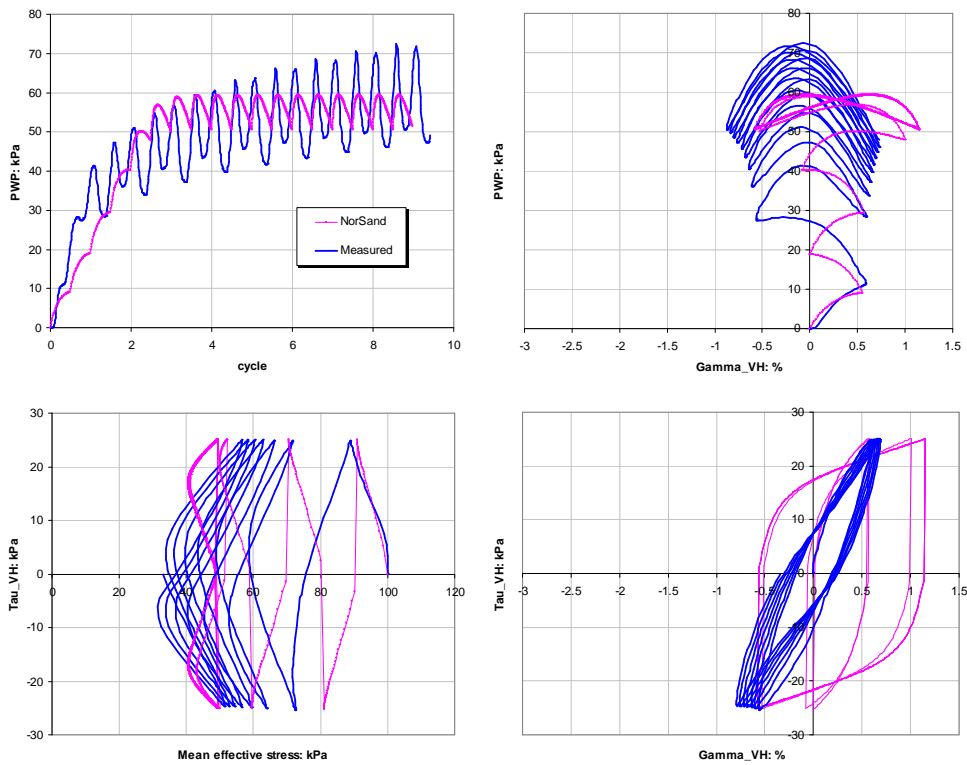


Figure 14: Measured FRS behaviour in CSS (blue) versus NorSand simulations (pink)

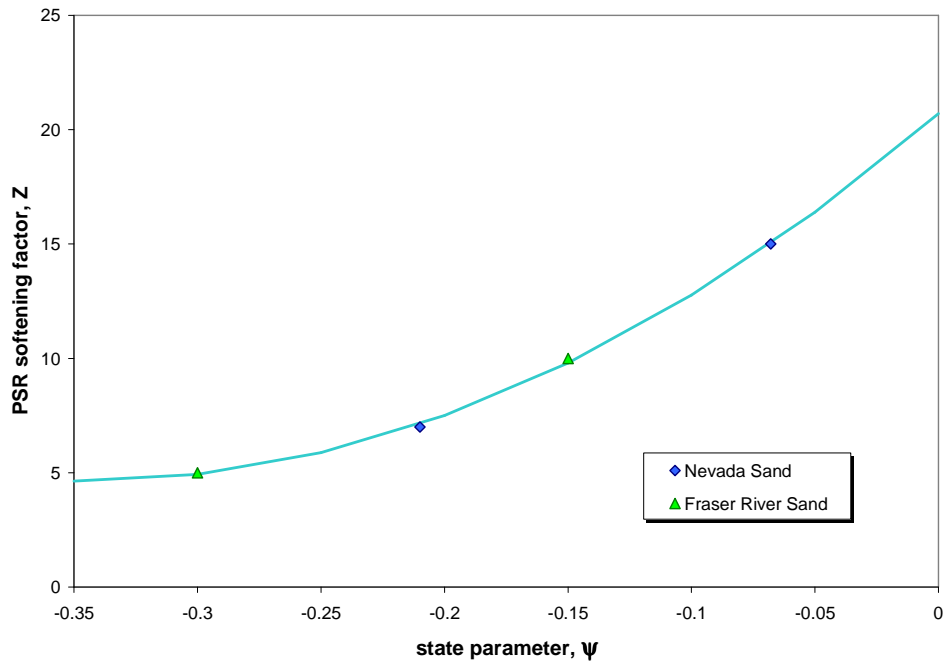


Figure 15: Apparent variation in principal stress rotation softening parameter Z with soil state parameter ψ

Summary of FRS Calibration

The calibrated parameter sets for this FRS gradation is provided in Table 1.

Table 1: NorSand Calibration for Fraser River Sand

Property	Value	Remark
<i>CSL</i>		
A	1.09	CSL is a power law function
B	.05	of the form
C	.25	$e_c = A - B (p')^C$
<i>Plasticity</i>		
M_c	1.49	Critical friction ratio
N	0.58	Volumetric coupling coefficient
H	40-550 ψ_0 with min. of 40	Plastic hardening modulus for loading, often $f(\psi)$
χ_{tc}	3.17	Relates minimum dilatancy to ψ Often taken as 3.5.
Z	Eqn (9)	Calibrated using CSS
<i>Elasticity</i>		
I_r	$\frac{G}{p_{ref}} = \frac{A}{(e - e_{min})} \left(\frac{p}{p_{ref}} \right)^b$	A=416.6, e_{min} =0.293, b=0.463
ν	0.2	Poisson's ratio, commonly 0.2 adopted

DETERMINING THE IN SITU STATE

What has been established so far is that the CRR depends on both soil properties and the state parameter. Of these inputs, soil properties are determined by laboratory tests on reconstituted samples as just described. This leaves the state parameter to be measured.

Sands and silts, the soils normally of interest for liquefaction assessment, are essentially impossible to sample and test undisturbed – some form of in situ testing must be used to determine ψ . In the case of the NCEER method (Youd et al, 2001) there appears to be a preference for the SPT. However, the SPT is a complex test to understand with dynamic energy losses, variable tip area as soil moves into the sampler, and poor repeatability. These issues do not arise with the static penetration soundings, and lead to the modern electronic piezocone penetration test (CPTu) becoming a basic reference test for assessing ψ in situ. But, because the CPTu measures tip resistance, shear force on the friction sleeve, and pore water pressure at the tip, the measured data must be “interpreted” to determine ψ . The interpretation methodology has evolved over the past twenty five years.

Penetration test resistance is very dependent on the stress level, all other factors being equal. This makes the *in situ* stress one of the key considerations in inversion. Within North American practice, it has become common to “correct” the measured data to a reference stress level. Correction is a misleading word, however, as there is nothing wrong with the original data. What happens in the reference stress level approach is that the measured data is mapped to what would have been measured at the reference stress level if nothing else (e.g. void ratio) were changed. We will return to this method later, as it is the reason why the factor K_σ has to exist. But, the reference stress approach is questionable mechanics and it is better to properly understand the nature of soil response to a penetration test before dealing with the reference stress method. The relevant dimensionless parameters for the CPT are presented in Table 2. The parameter group $Q(1-B_q)+1$ appears to be especially useful (Shuttle & Cuning, 2007) as it unifies both sands and silts into the same framework (for sands, $B_q = 0$).

Table 2: Dimensionless parameter groupings for CPT interpretation

<i>Dimensionless parameter group</i>	<i>Description</i>
$Q_p = (q_t - p_o) / p_o'$	Tip resistance normalized by mean stress
$Q = (q_t - \sigma_{vo}) / \sigma'_{vo}$	Tip resistance normalized by vertical stress
$F = f_s / (q_t - \sigma_{vo})$	Normalized friction ratio (usually expressed in %)
$B_q = (u - u_o) / (q_t - \sigma_{vo})$	Normalised excess pore pressure
$Q(1 - B_q) + 1 = (q_t - u) / \sigma'_{vo}$	Suggested by Houlsby (1988). Appears fundamental.

Although it may seem appealing to model the actual geometry of a CPT in finite element simulations, and a few workers have attempted this (e.g. van den Berg 1994; Yu et al. 2000; Lu et al. 2004), there are presently difficulties with large displacement formulations. The widely used approach has therefore been to rely on systematic calibration of the CPT in large chambers. Although these calibrations are sand-specific, over ten sands have now been systematically tested with the range of soil types for eleven of these illustrated on Figure 16 (Jefferies & Been, 2006, provides a useful summary of the data).

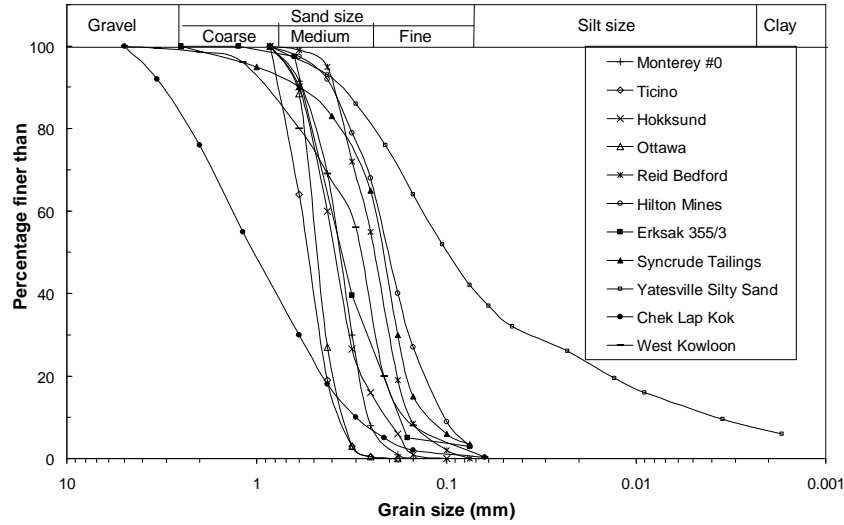


Figure 16: Grain size distributions of soils used for CPT calibration

Each of the tested sands in a calibration chamber (after correction for chamber boundary effects) shows a trend in the relation between CPT measurements and ψ that has the form:

$$\psi = \frac{-\ln(Q_p / k)}{m} \quad (10)$$

where k, m are soil-specific coefficients. Because mathematical or numerical modelling of the drained penetration of the CPT is extremely difficult, equation (10) was proposed on dimensional grounds and correlated to chamber test data. However, although equation (10) is both appealingly simple and dimensionless (as would be expected for something based in mechanics), it was criticized on the grounds that careful examination of the chamber test data indicates substantial bias with stress level (Sladen, 1989). At least k , and possibly m , were functions of mean effective stress. The existence of a stress level bias is curious given the dimensionless formulation. At the time of the original work (1986-7) there were not theoretical methods available to go further, but these became available some ten years later and showed how to proceed.

A large-scale numerical examination of equation (10) was undertaken by Shuttle and Jefferies (1998) using cavity expansion analysis and the NorSand model. This then developed into a universal framework for evaluating ψ from the CPT. An example of the results obtained by Shuttle and Jefferies is shown on Figure 17 for a single sand (computed using properties of Ticino sand). If Figure 17 is examined closely, a very small curvature can be detected in the trend line of the results, but this is much less than second order and for all practical purposes equation (10) is a very good representation of CPT resistance at constant modulus. However, if the mean stress was changed, and all other properties kept constant, different Q_p values were computed. Changing G , such that the ratio G/p_o was returned to its original value, put the computed results exactly back on the prior trend line. The explanation here is that the earlier work on determining the state parameter from the CPT missed a dimensionless group, $I_r = G/p_o$, the soil rigidity. The Q_p vs. ψ relation for Ticino Sand from Shuttle & Jefferies was subsequently independently verified by Russell & Khalili (2002).

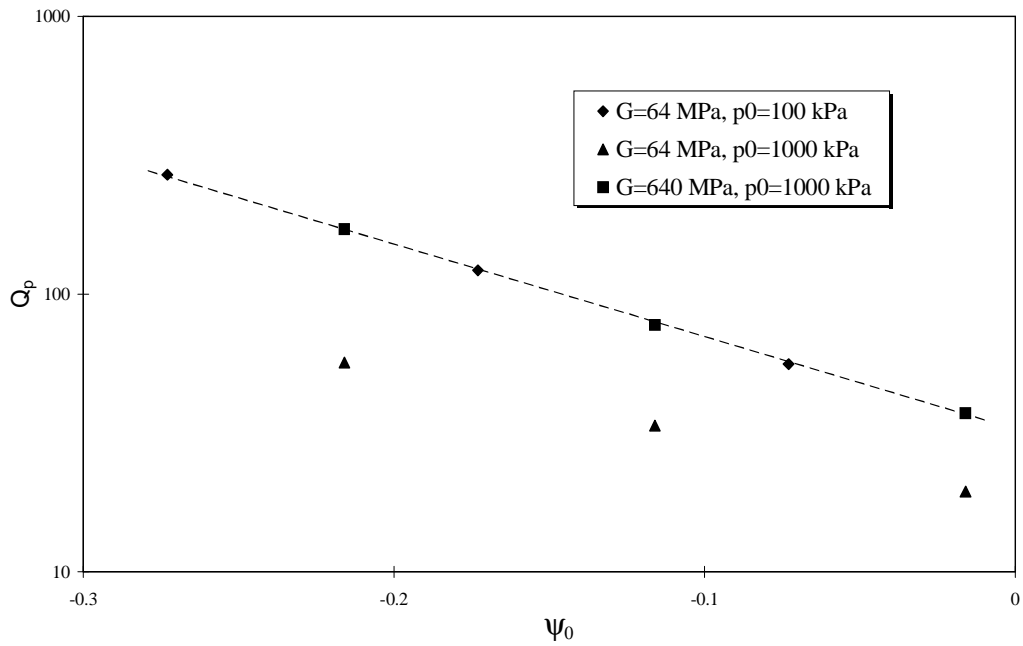


Figure 17: Numerical calculation of $Q_p - \psi_0$ relationship for Ticino sand, showing linearity and effect of elastic modulus as cause of stress level bias

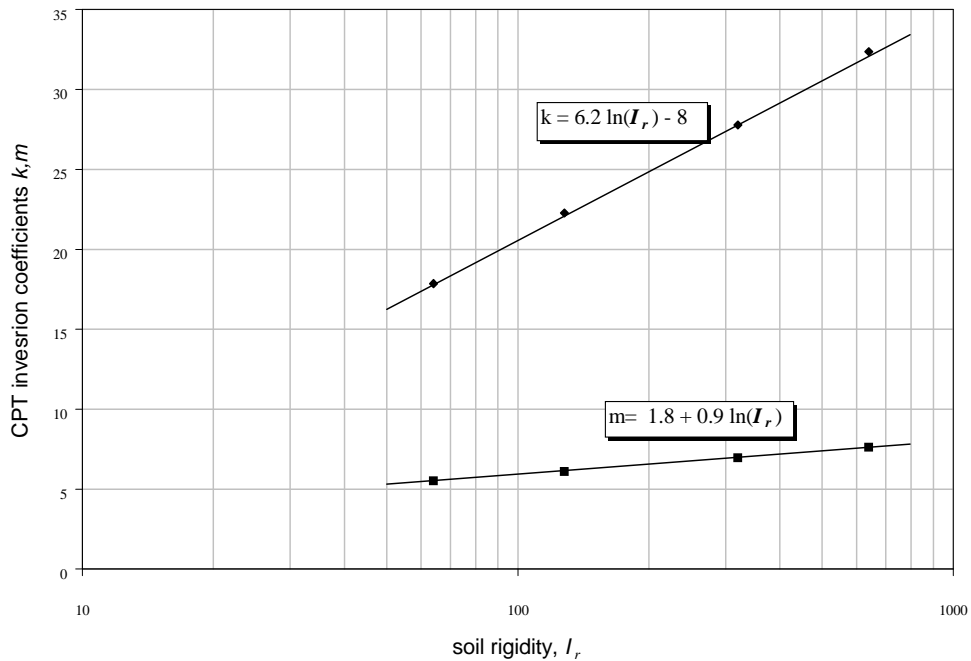


Figure 18: Computed effect of I_r on k, m coefficients for Ticino sand

In addition to G being part of the theoretically important dimensionless grouping, Ir , accounting for G also translates into better accuracy in practical ψ determination. Applying the NorSand cavity expansion methodology described above, Ghafghazi & Shuttle (2008) found that for a database of nine soils, the most accurate determination of ψ was obtained for the three soils where G has been measured. And the methodologies for determining ψ that account for a soil's actual properties, including G , do better than those that do not (Ghafghazi & Shuttle, 2010)

The implication of this understanding of the CPT for liquefaction assessments is straightforward: assessments requiring precision must be supported by in situ measurement of the elastic shear modulus. Penetration testing alone is not sufficient. Fortunately, this requirement is not onerous and involves using a seismic cone for at least a few of the soundings in any CPT investigation. Alternatively, cross-hole or downhole seismic survey can be made in the investigation borings. Figure 18 shows how the CPT coefficient in (10) maps with elastic shear rigidity $I_r (= G_{max}/p')$.

In order to evaluate the effect of soil properties in addition to rigidity on the penetration resistance, the simulations of Shuttle and Jefferies were extended to examine separately the effect of soil properties on the penetration resistance. In addition to the effect of rigidity, it was found that both intercept k and slope m were strong functions of the plastic hardening modulus H , as well as the critical friction ratio M . There was a much weaker influence of N and the soil compressibility λ . Poisson's ratio had essentially no effect. To determine the most accurate interpretation of the CPT in any soil, the methodology outlined by Shuttle and Jefferies requires detailed numerical simulations to ascertain the values of k and m as a function of rigidity and soil properties. This is fairly time consuming. However, Shuttle and Jefferies offered an approximate general inversion obtained by fitting trend lines to the computed results:

$$k = (f_1(G/p') f_2(M) f_3(N) f_4(H) f_5(\lambda) f_6(\nu))^{1.45} \quad (11a)$$

$$m = 1.45 f_7(G/p') f_8(M) f_9(N) f_{10}(H) f_{11}(\lambda) f_{12}(\nu) \quad (11b)$$

where the fitted functions $f_1 - f_{12}$ are given on Table3.

Table 3: Approximate expressions for general inverse form $\psi=f(Q_p)$

Function	Approximation
$f_1(G/p_0)$	$3.79 + 1.12 \ln(G/p')$
$f_2(M)$	$1 + 1.06 (M - 1.25)$
$f_3(N)$	$1 - 0.30 (N - 0.2)$
$f_4(H)$	$(H / 100)^{0.326}$
$f_5(\lambda)$	$1 - 1.55 (\lambda - 0.01)$
$f_6(\nu)$	Unity
$f_7(G/p_0)$	$1.04 + 0.46 \ln(G/p')$
$f_8(M)$	$1 - 0.40 (M - 1.25)$
$f_9(N)$	$1 - 0.30 (N - 0.2)$
$f_{10}(H)$	$(H / 100)^{0.15}$
$f_{11}(\lambda)$	$1 - 2.21 (\lambda - 0.01)$
$f_{12}(\nu)$	Unity

The performance of the proposed general inversion was verified by taking 10 sets of randomly generated soil properties/states and computing the Q_p value using the full numerical procedure. This computed Q_p was then input to the general inversion to recover the estimated value of ψ . The proposed general inversion recovers ψ with an accuracy of $\Delta\psi = \pm 0.02$.

What about silts? Recall that an issue with the NCEER method is its treatment of 'fines' as the only index of soil behaviour. Use of the CPT in siltier soils to interpret soil type is not uncommon in the literature (e.g. East et al 1988; Dasenbrock 2005; Yafrate and DeJong 2005), but such work has concentrated on silty sands (typically less than 20% silt). Even at 0-15% silt, problems have been noted with the standard liquefaction assessment approach (e.g. Carraro et al. 2005). Quantitative evaluation of CPT data in true silt is rarer, typically relying on the use of site specific correlations (e.g. East et al. 1988; Lou et al. 1991) and an appropriate framework for such correlations is unclear. Moreover, to date, CPT chamber calibration tests have not been undertaken using silt, and correspondingly there are no directly measured empirical behaviour trends to underpin a methodology.

Some initial steps for evaluating the CPT in silt using an effective stress approach have been presented by Been et al. (1988), Plewes et al. (1992) and Been and Jefferies (1992), but these methods are very much first approximations and indeed might be argued as speculative. On a theoretical level, effective stress solutions for undrained CPT soundings have been provided by Cao et al. (2001) and Chang et al. (2001) using the Modified Cam Clay model. However, Modified Cam Clay is not an appropriate model for silt as it cannot dilate properly with dense soils nor display liquefaction behaviour in loose soils.

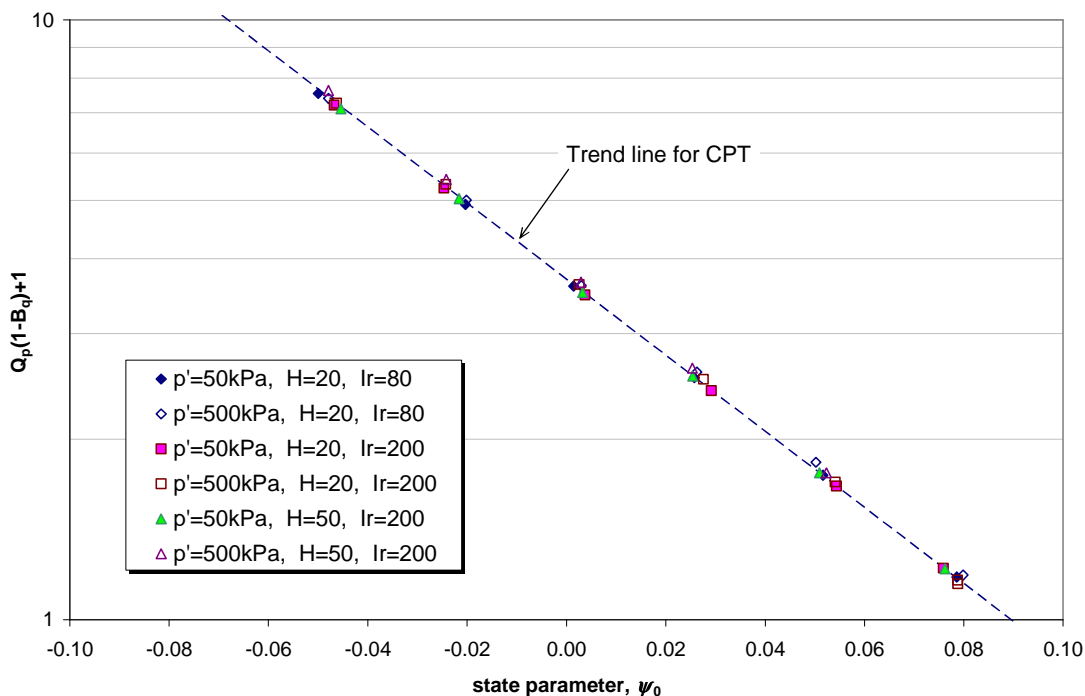


Figure 19: Normalized results of numerical simulations (all 36 simulation results plotted)

An effective stress CPT approach using the NorSand constitutive model was developed by Shuttle & Cunning (2007) for application to tailings with high silt content; this work was an extension of Shuttle & Jefferies described above, with similar idealizations and so forth. Just as with sands, Norsand was fitted to measured silt behaviour in the triaxial compression test and then used in a cavity expansion model. The difference for the silts was that the simulations were made undrained. The results are shown on Figure 19 for the calibration to silt-sized tailings from the Faro mine (Yukon, Canada). Houlsby's expectation of normalized behaviour is a remarkable fit to the results.

Work remains to link the silt and sand evaluations into a completely general method, presumably involving hydraulic conductivity as a further parameter. Equally, running the large displacement finite element model is quick and it is straightforward to create soil-specific CPT evaluation parameters. That is the state of our current work.

UNDERSTANDING CYCLIC MOBILITY

The Physics

The nature of cyclic mobility is evident from the computed behaviour illustrated on Figure 14. Both these samples were dense of the CSL, so that although principal stress rotation causes softening of the yield surface and concurrent compressive volumetric plastic strain, for the soil to achieve a failure state it has to dilate at some point. The amount of dilation needed depends on the CSR. So, a balance develops in the soil with "butterfly" like stress paths and a softening in shear stiffness.

Of course, this cyclic mobility is generally only possible if the soil is dense of the CSL, with correspondingly negative values of ψ . The exception is very low CSR where the peak shear stress is within the residual strength of even very loose soil, and in this case essentially elastic behaviour arises. It follows from this that the first step in any liquefaction assessment is to determine if the soil in question has negative ψ . That step is missing in the NCEER method, but it remains fundamental. The 'CPT + G_{\max} ' procedure outlined should be used, and this procedure has the additional merit of working for the spectrum of liquefiable soils of pure silts (often encountered as mine tailings) through typical holocene silty sands to the clean sands found with hydraulic fills.

Once it is established that a soil has negative ψ , then its available cyclic 'strength' can be discussed. However, strength is in many ways misleading. What is seen in both the test data and the numerical simulations is that the soil softens - deformations will depend on the extent that the "load follows the soil" and on the duration of the load cyclic. What is unclear in the NCEER approach is the tacit deformation limits in the field data - depending on the situation, the inferred strength could be useful or misleading. In all likelihood, it is deformations that matter and these are not dealt with in the NCEER method. That is one utility of a calibrated model like NorSand - it is capable of providing deformation simulations.

Mechanics vs Field Experience

Although the original work in evaluating the case history experience of liquefaction was based on the SPT, the many deficiencies of the SPT are widely known and several workers developed comparable liquefaction assessment charts to Figure 1 using the CPT as the input information. This effort started some twenty years ago with Robertson and Campanella (1985) and continued with contributions by Seed

and de Alba (1986), Olsen (1988), Olsen and Malone (1988), Shibata and Teparaska (1988), Suzuki et al. (1995), Stark and Olson (1995), Olsen and Koester (1995), and Robertson and Wride (1998).

To clarify the situation from the case-history data, it has been sorted to include only case histories with sands having less than 5% fines. Plausibly, this sorted sub-set of the case histories should have a restricted range of properties and reasonable "average" properties can be estimated ($M_{tc} \approx 1.25$, $\lambda_{10} \approx 0.05$, $\chi \approx 3.5$, $I_r = 600$). These case histories also had rather a limited range of initial *in situ* vertical effective stress, in the order of $\sigma_{v0} \approx 100$ kPa, and so it is reasonable to accept for them that $Q \approx q_{cIN}$. The individual Q values can then be mapped to ψ using the estimated average material properties (which give CPT coefficients $k = 31.5$, $m = 9.4$). Making the one further assumption that a representative stress ratio is $K_o = 0.7$ provides a relationship between CRR and ψ from the field case history data: Figure 20. Figure 20 has similarities to the original NCEER form (Figure 1) but is now transformed to something approaching a proper representation of that case history experience based on mechanics ('approaching' because each case history should have its own measured properties to obtain a fully mechanics-based figure).

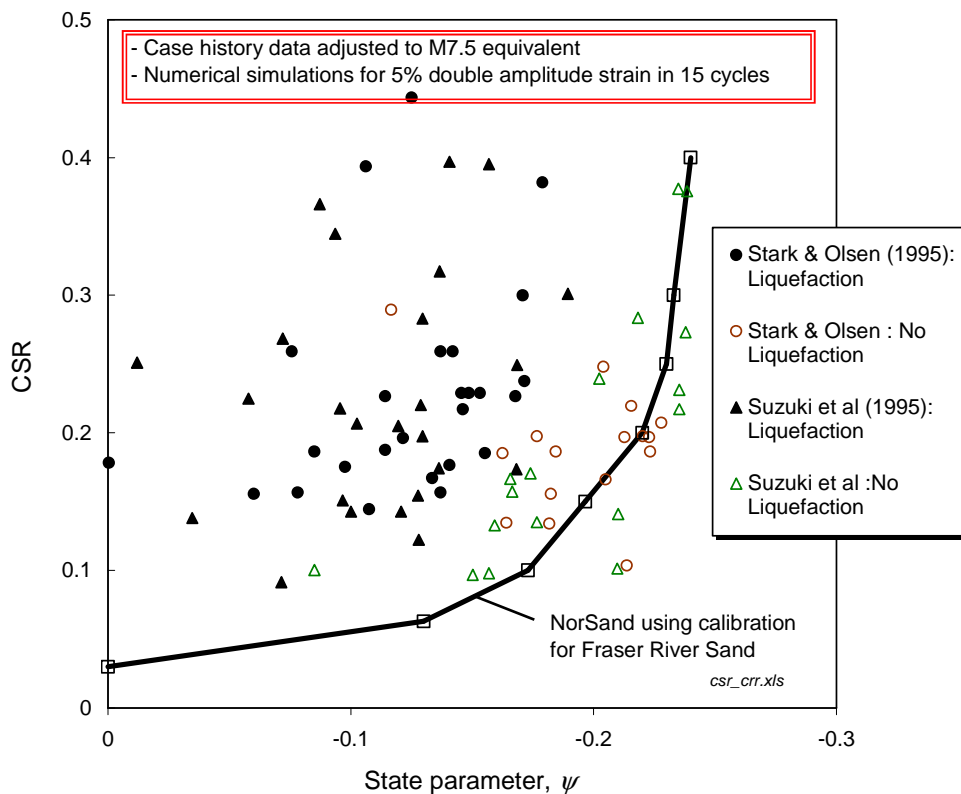


Figure 20: Field case history data on liquefaction expressed in terms of ψ

The calibration of NorSand to Fraser River Sand has been illustrated. This calibration can then be used to systematically simulate how changing ψ affects the available cyclic strength. In doing this a criterion defining the strains or pore pressures associated with 'strength' is needed. Here we adopt a double amplitude shear strain of 5%. Likewise, it is necessary to choose a reference number of stress cycles.

Here we have chosen the strength mobilized in fifteen uniform cycles of stress variation. The results are shown on Figure 20 and compared to experience from the case history record.

Looking at Figure 20, it is readily apparent that the cyclic NorSand simulations are under-predicting strength for lightly dilatant states. It is unclear whether this is caused by the CSS tests used for calibration or by the mis-match between the 'strength' criterion used for the case-histories and the simulations. On the other hand, the computed behaviour is based on model self-consistently derived from simple postulated physics and this degree of match could even be viewed as remarkable. And there is the possibility that Fraser River sand is unusually weak.

In the end though, what matters is getting the framework right. And it is evident from Figure 20, when considered in light of the calibrations to simple element tests presented earlier, that setting the case history data directly against the characteristic in situ state parameter is the correct way to capture full scale experience. The best-fit to the case history data on Figure 20 is the simple equation:

$$CRR = 0.03 \exp(-11\psi) \quad (12)$$

where CRR is the conventional available strength expressed as the cyclic resistance ratio analogously to the CSR. An important attraction of this equation is that there is no adjustment for stress level required when using the strength.

Insight from Mechanics: K_α

There is much confusion within Youd et al (2001) over the nature of K_α and how it may vary. In fact, this is a very easy topic to deal with using simple mechanics. Recall the elegant experiments discussed earlier that demonstrate it is principal stress rotation that controls (or at least dominates) the response of soils to cyclic loading. The relationship between shear stress on a horizontal plane and the principal stress direction, equation (4), is now written to break the horizontal shear stress into cyclic and static bias components, τ_{cyc} and τ_{st} , respectively:

$$TAN(2\alpha) = \frac{2(\tau_{cyc}/\sigma'_v + \tau_{st}/\sigma'_v)}{1 - (\sigma'_H/\sigma'_v)} \quad (13)$$

The effect of static bias on the rotation of principal stress is illustrated on Figure 21 for a CSR = 0.15 an analogous static bias ratio of 0.1, and $K_0 = 0.5$. It can be seen that although the direction of the principal stress changes, the magnitude of the cyclic variation is unchanged by static bias. Given that it is cyclic rotation of principal stress that drives soil behaviour, it follows that $K_\alpha=1$ under all circumstances.

Insight from Mechanics K_σ

The NCEER method is based on normalizing the penetration resistance to that which would have been measured for the soil at the same void ratio but at a stress level of $\sigma_v = 100$ kPa. This automatically then requires a compensating adjustment to map the inferred cyclic strength back to the actual stress level in the soil, K_σ . Like other aspects of the NCEER approach, the factor K_σ was estimated from soil testing associated with various case histories and, as illustrated on Figure 22, there is a wide range of trends and an equally wide range of recommendations as to how K_σ changes with soil conditions. Frequently density itself is cited as a parameter with different K_σ being suggested depending on the initial relative density. These suggestions cannot be substantiated by calculations.

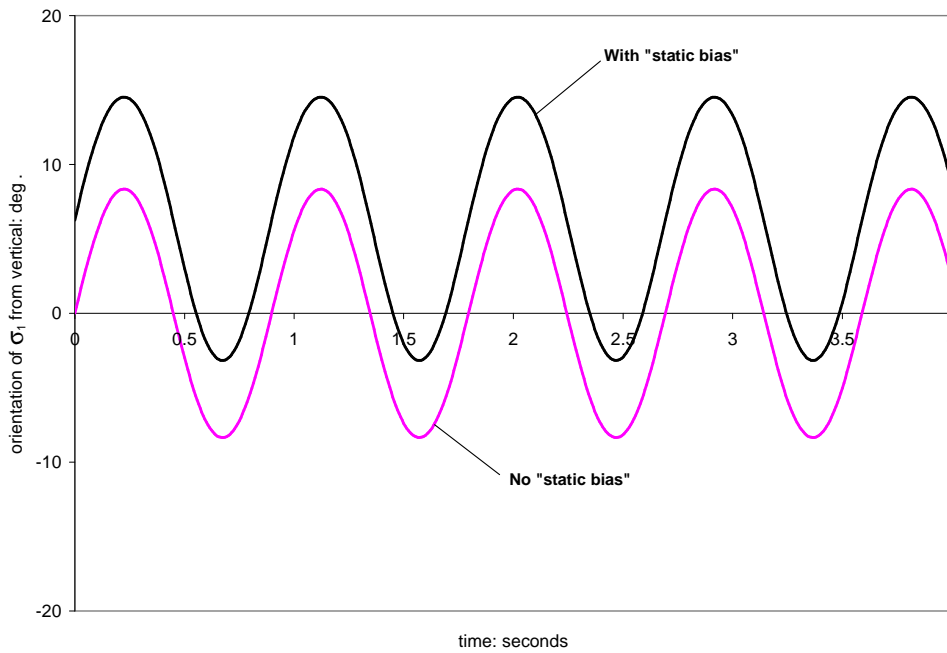


Figure 21: Effect of static bias on principal stress rotation

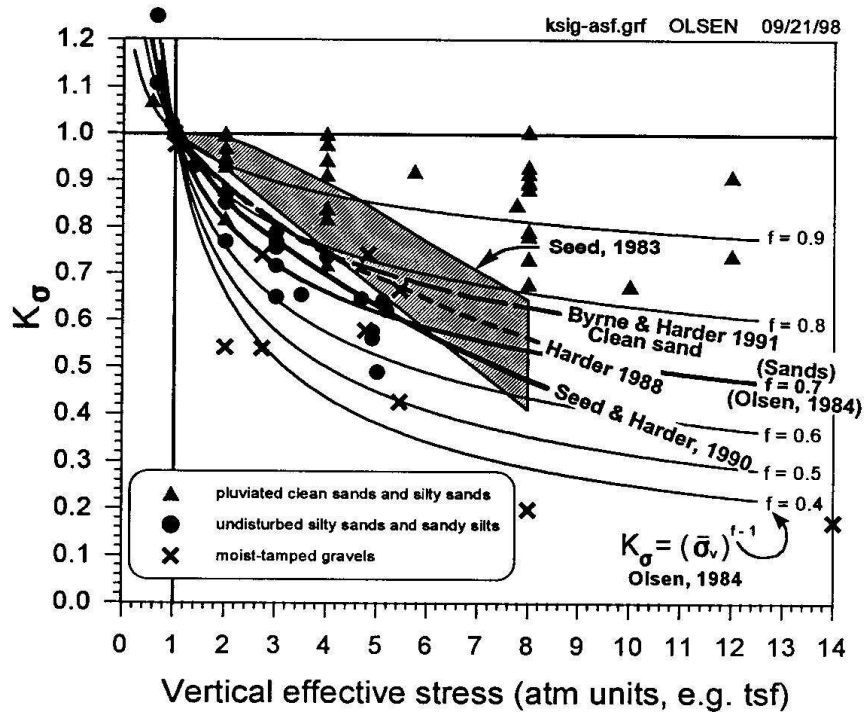


Figure 22: Illustration of the range of opinion about K_σ
 (after Hynes & Olsen, 1999)

With the understanding from laboratory tests that constant ψ implies constant CRR for constant intrinsic soil properties (and in particular fabric), we can calculate just how K_σ should vary. Recall that K_σ was intended to allow for the *in situ* stress conditions assuming that soil density remains unchanged. A constant density requirement implies progressively changing ψ as the confining stress changes. Equation (12) is the best-fit for CRR based on the sorted field data and can be used to compute K_σ as a function of stress level and soil properties. Figure 23 shows values of K_σ as a function of the slope of the critical state line λ (which is the soil property that determines how ψ varies with stress level for constant density). It is readily apparent that Figure 23 is a plausible explanation of Figure 22 - 'plausible' because the soil compressibility associated with the various data points in Figure 22 is not reported and appears unmeasured.

What is more important, however, is the simplicity obtained if the entire adjustment of the penetration resistance to a reference stress level is abandoned. If equation (12) is used as it is, and if ψ is determined using the combination of CPT and *in situ* G_{\max} as presented earlier, then the need for K_σ vanishes. Liquefaction assessment becomes a whole lot simpler as well as being based on measured soil properties rather than geologically-based speculation.

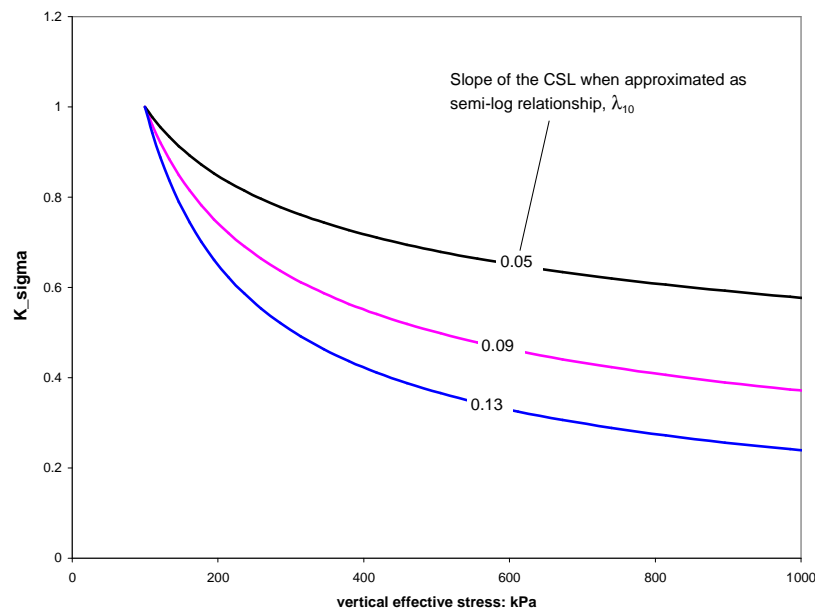


Figure 23: Computed K_σ based on soil properties

CONCLUSION

The nature of the NCEER method for assessing the resistance to cyclic mobility has been discussed from a standpoint of consistent applied mechanics. From this perspective the NCEER method has two basic flaws: (i) Soil properties are neglected; and (ii) there is no mechanistic basis for the proposed extrapolations.

However, an important feature of the NCEER method is that it is anchored to a now considerable body of case-history experience. But that does not mean engineers should, or have to, accept the proposed basis for how that experience is used. As illustrated in this paper, if a proper framework of applied mechanics is used based on the state parameter, ψ , the same case-history data is honoured but the trends within it now become far simpler to understand. Necessarily, this means a few soil properties must be measured, but that can hardly be seen as onerous given the scale of \$ involved with liquefaction failures and remediation to prevent them.

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