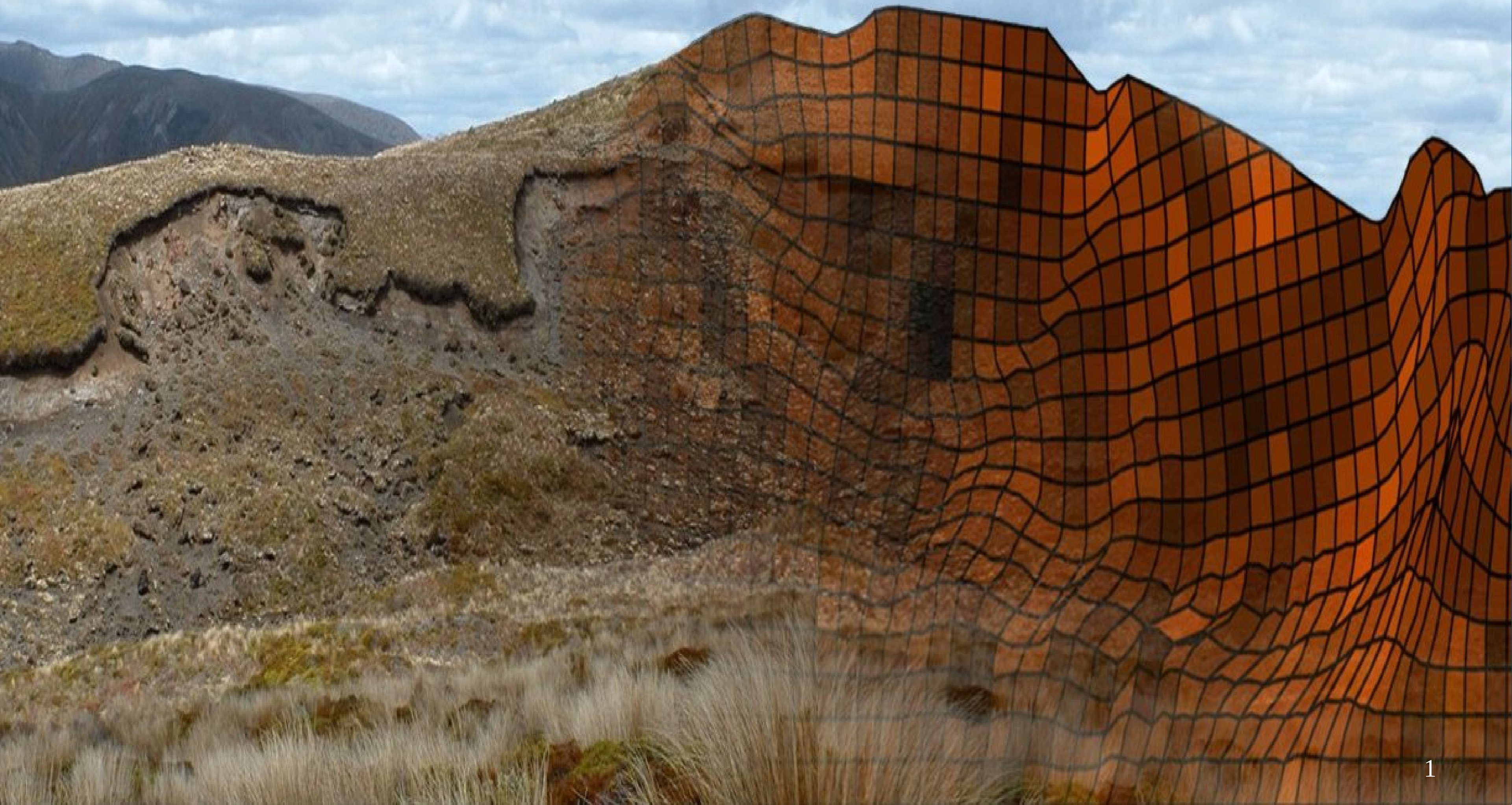
Reliability-Based Geotechnical Design:
From Theory to Practice

Presented by

Gordon A. Fenton

CGS Cross-Canada Lecture Tour, Fall 2015



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The Problem

- The ground is a highly uncertain and site specific engineering material
- Uncertainty is reduced with increasing site investigation and modeling effort
- How should the level of site and model understanding be rationally accounted for in our geotechnical designs?
- How should our design codes of practice account for the level of site and model understanding and severity of failure consequences?

Possible Solutions

Experimental Databases:

- need records of multiple 'nominally similar' sites with installed geotechnical systems having varying levels of site and model understanding and varying designs
- need sufficient number of "realizations" to estimate failure probability of each design
- unfortunately, we usually have only one realization of each geotechnical system
- in other words, such databases do not generally exist

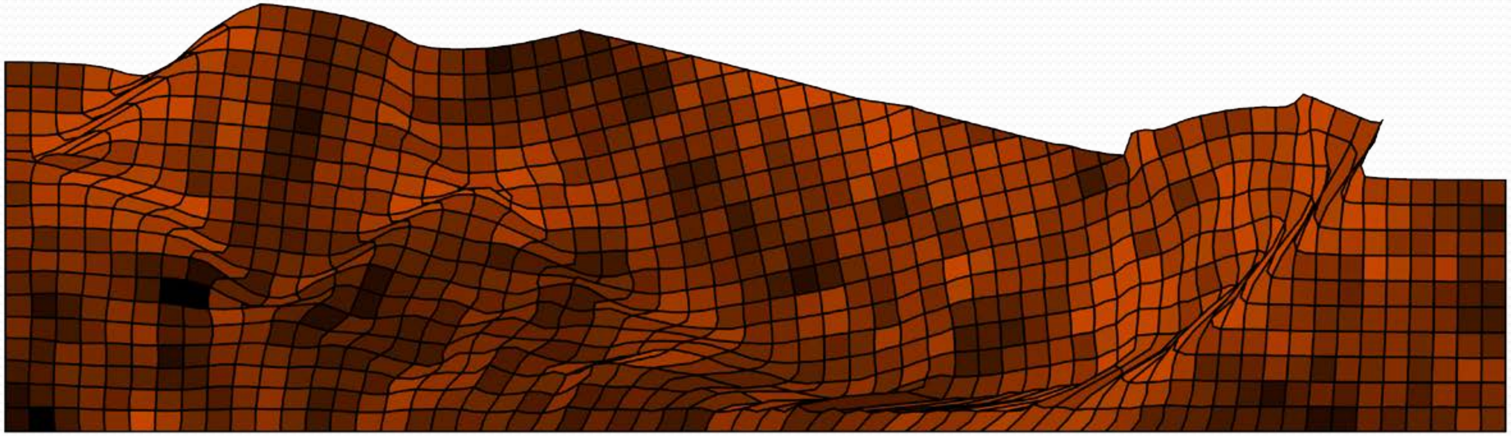
Possible Solutions

Monte Carlo Simulation:

- model the ground as a spatially varying random field
- virtually install a designed geotechnical system and predict its performance
- degree of site understanding for design can be modeled
- multiple realizations are possible so that failure probability can be estimated
- failure probabilities of different designs can be compared to determine optimal design for target failure probability.

Modeling the Ground: Random Fields

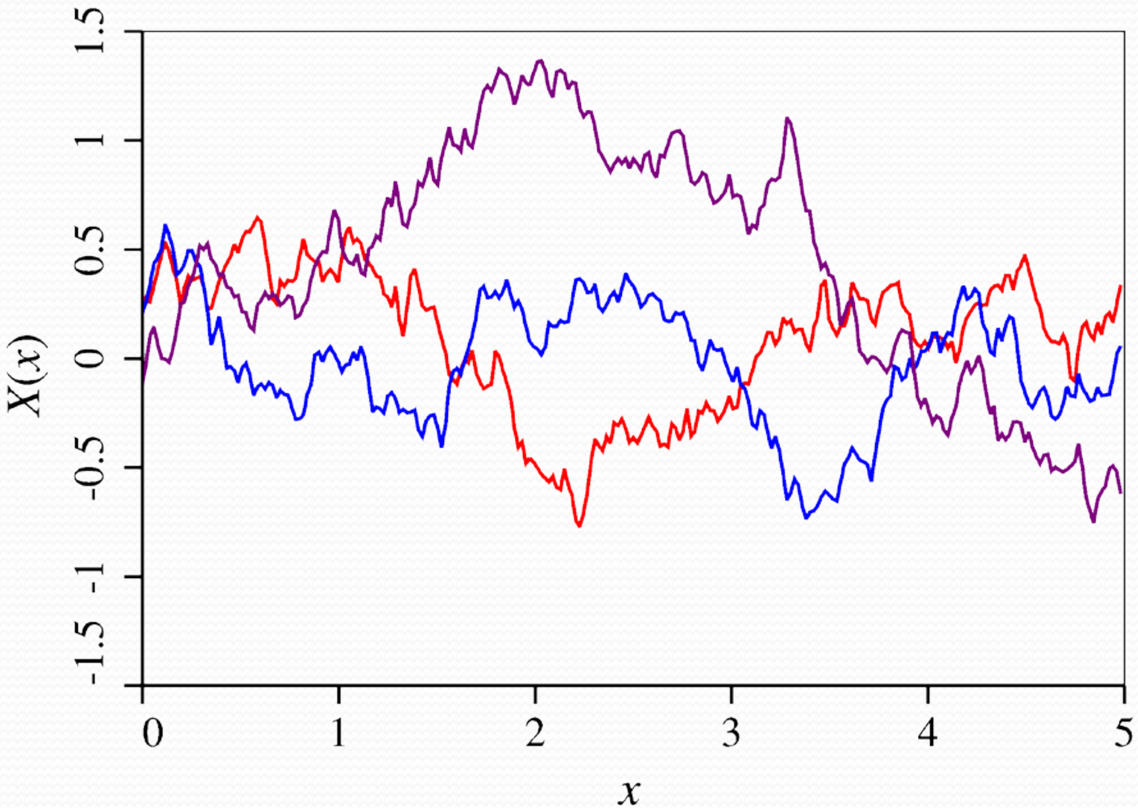
- Soils are spatially variable → random fields



- How does spatial variability affect the probability of failure?
- Is failure dependent on a "weakest path" or on an average?
- Can different types of averages address these questions?

What is a Random Field?

A random field is a collection of random variables, $X(\boldsymbol{x}_1)$, $X(\boldsymbol{x}_2)$, …, one for each point in the field. Each $X(\boldsymbol{x}_i)$ is uncertain (i.e. random) until it is observed.



Three realizations of $X(\boldsymbol{x})$. The value of X at each $\boldsymbol{x}$ follows a distribution, $f_X(\boldsymbol{x})$, which might change with $\boldsymbol{x}$.

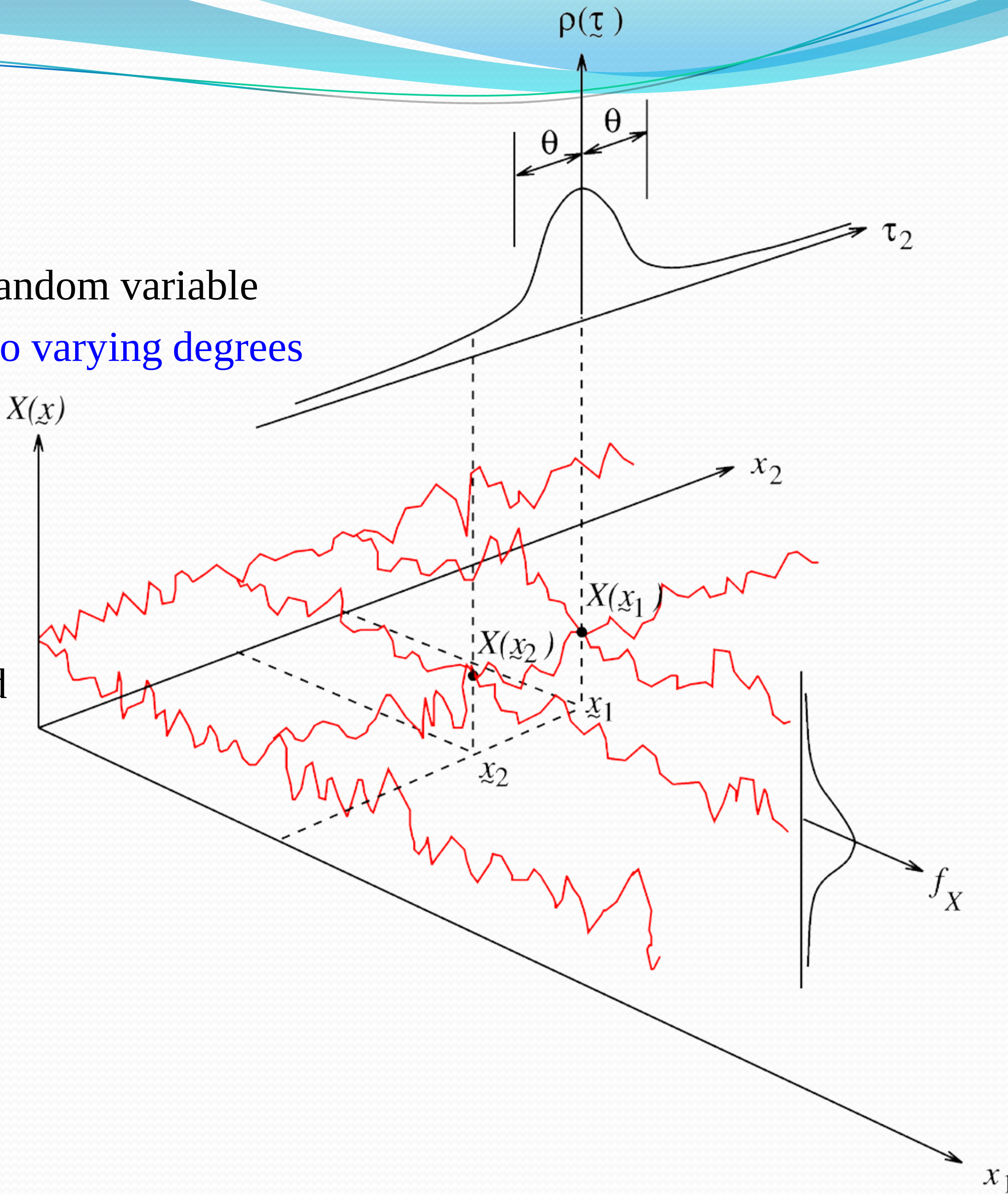
Random Fields

every point in the field (site) is a random variable

all points are mutually correlated to varying degrees

small θ → rough fields

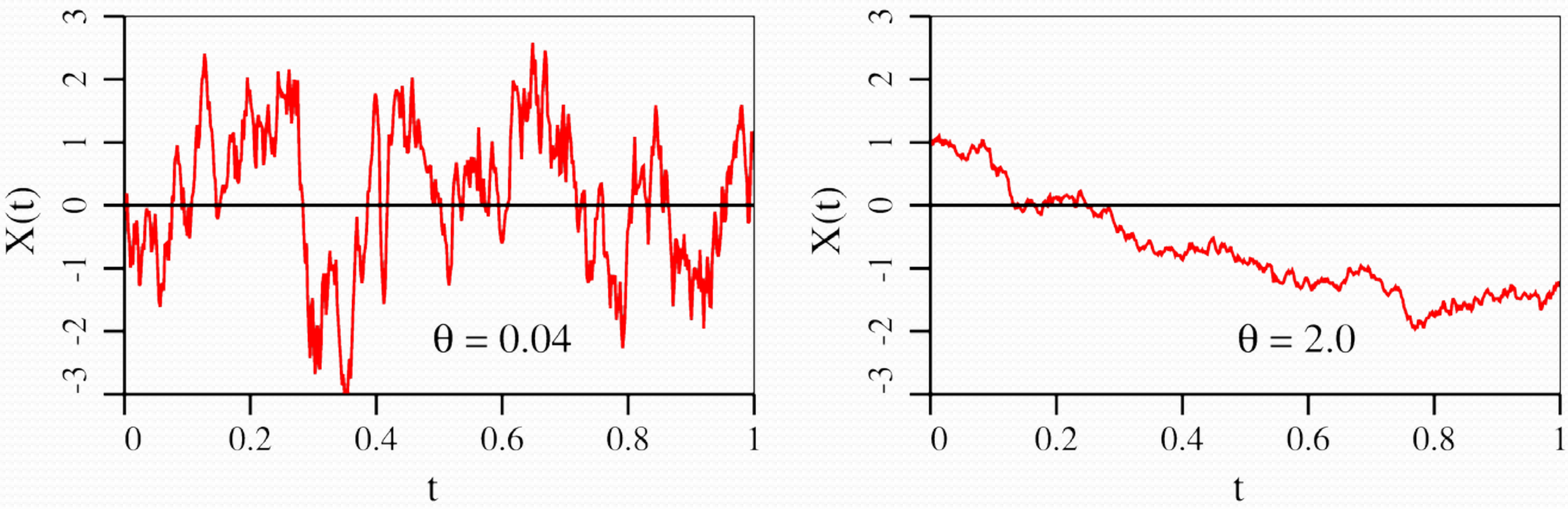
large θ → smoother fields



If the random field is stationary and normally distributed, then we need;

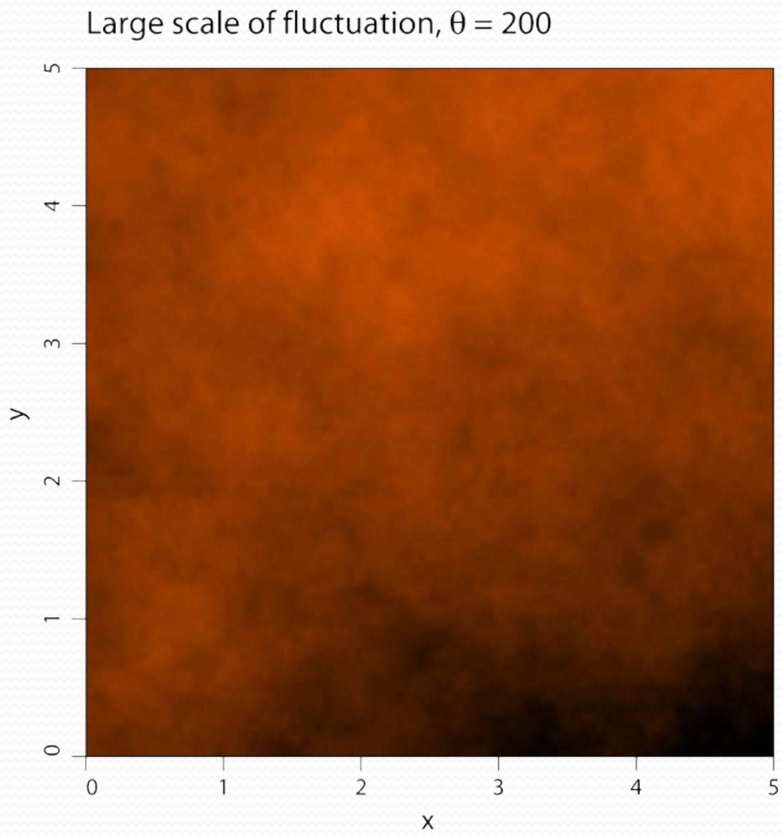
1. the field mean, μ_X ,
2. the field variance, σ_X^2 ,
3. the field correlation structure, commonly parameterized by the correlation length, θ_X

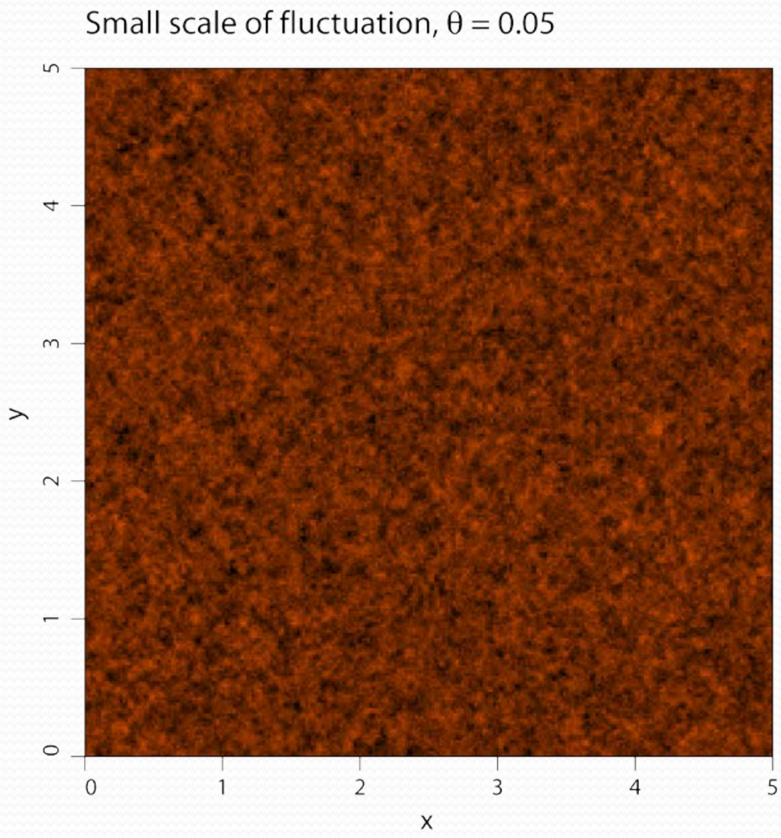
Effect of Correlation Length



- large correlation lengths have long wavelength components
- realizations appear to have a trend when viewed at scales less than the correlation length (both plots above are of *stationary* random processes).

Effect of Correlation Length





Monte Carlo Simulation

- Monte Carlo Simulation (MCS) involves
  - generating a realization of a random variable (or set of random variables) according to its prescribed distribution
  - computing the 'system' response
  - repeating the above two steps many times to assess probabilities/statistics (accuracy increases as the number of realizations increases)
- MCS is sometimes computationally intensive
- MCS has the advantage over first- and second-order methods of being able to estimate the entire distribution of the response.

Modeling the Ground

The Random Finite Element Method (RFEM)

The Random Finite Element Method involves a combination of Random Field Simulation (e.g. Fenton and Vanmarcke 1990) and the Finite Element Method (e.g. Smith and Griffiths 2004)

Random fields represent spatial variability of the ground.

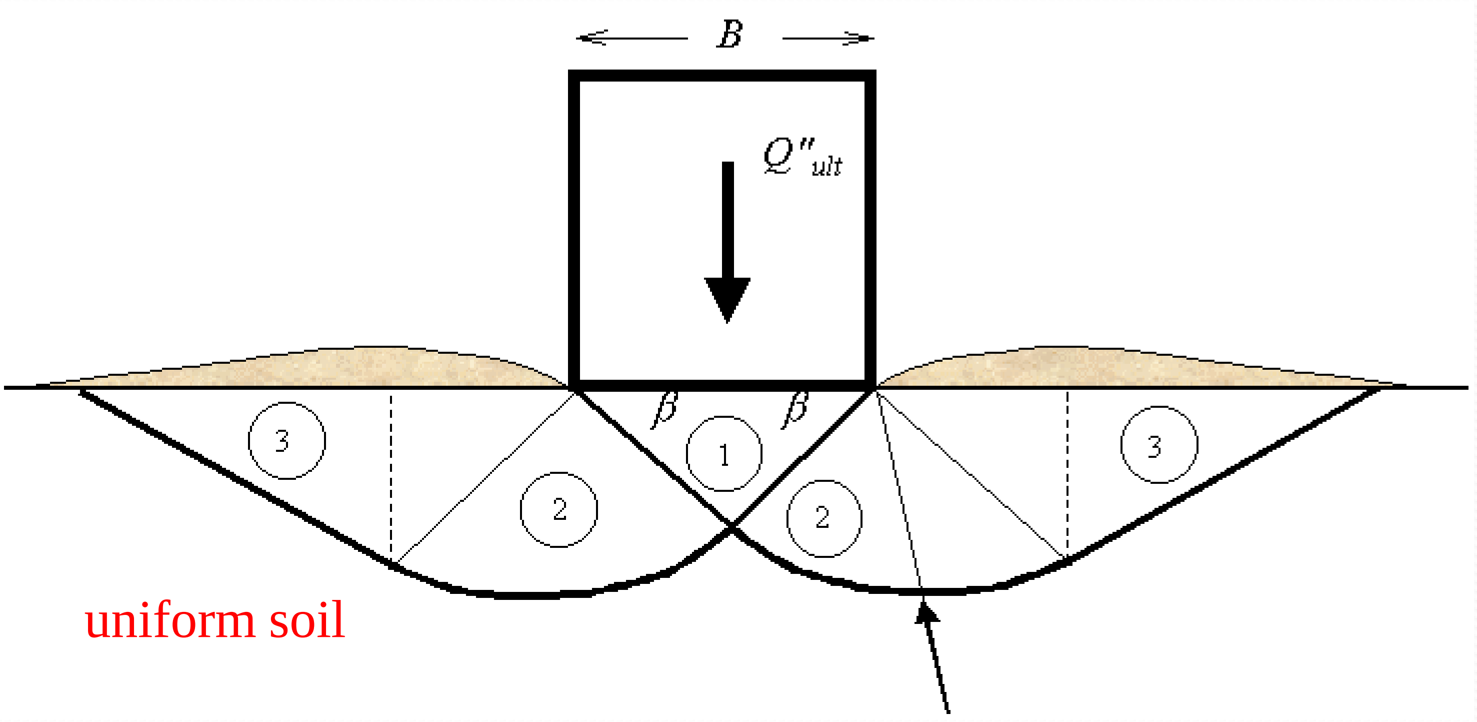
The finite element method naturally seeks out the weakest failure path through the ground

The method allows the investigation of design code provisions, *including* the effect of the level of site understanding.

The method is applied in a Monte-Carlo framework.

RFEM for Bearing Capacity

Classical Solution

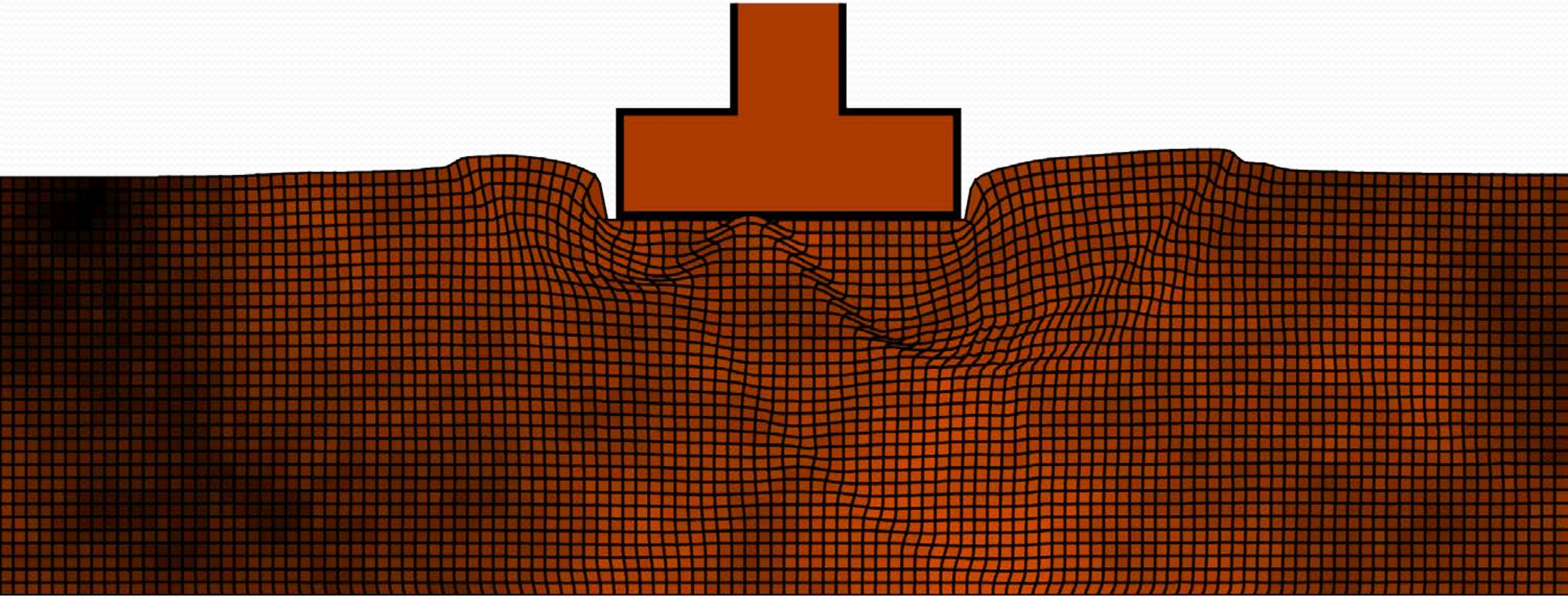


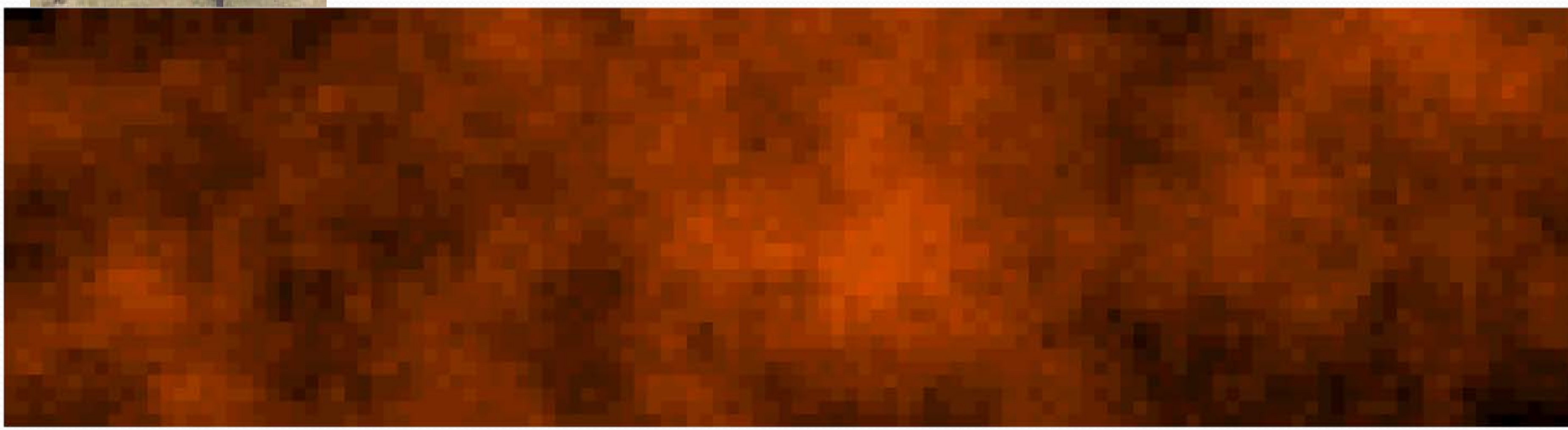
symmetric log-spiral failure surface

(Courtesy, CE 3600 course notes, Univ. Wyoming)

RFEM for Bearing Capacity

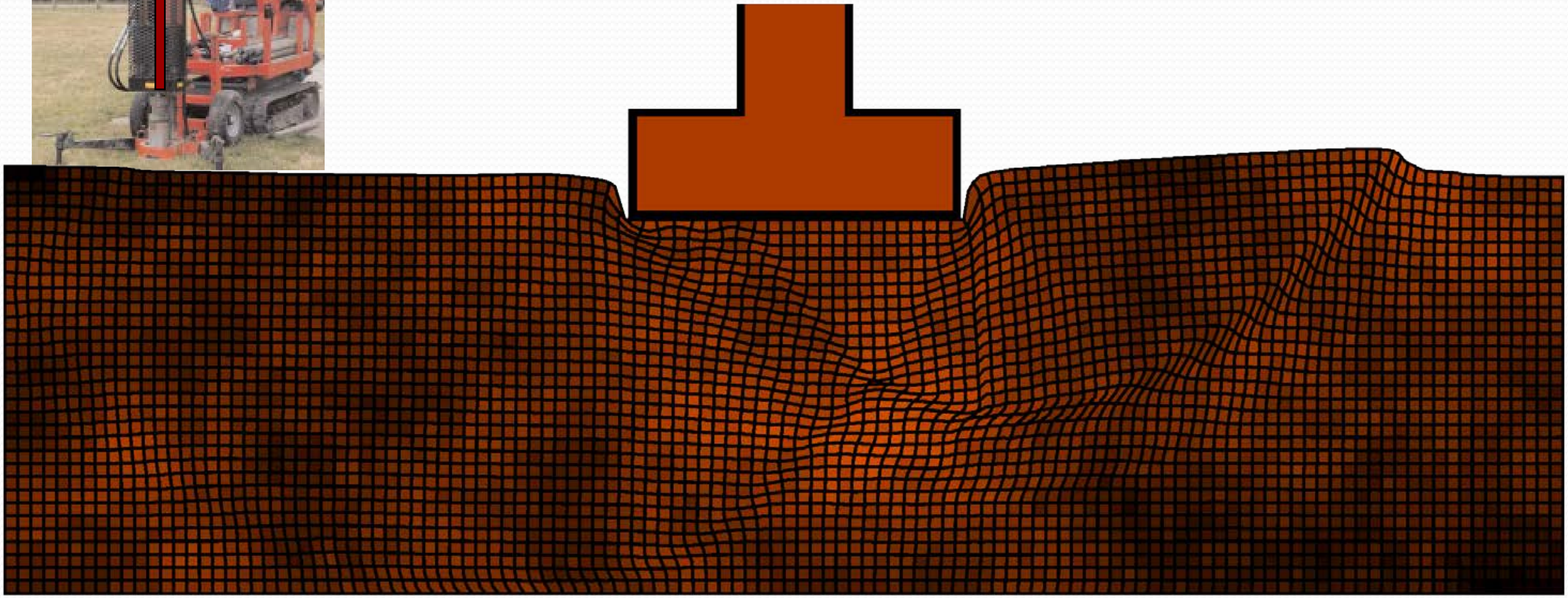
Spatially Variable Failure Surface





Degree of Site Understanding

- use sample results to design the foundation



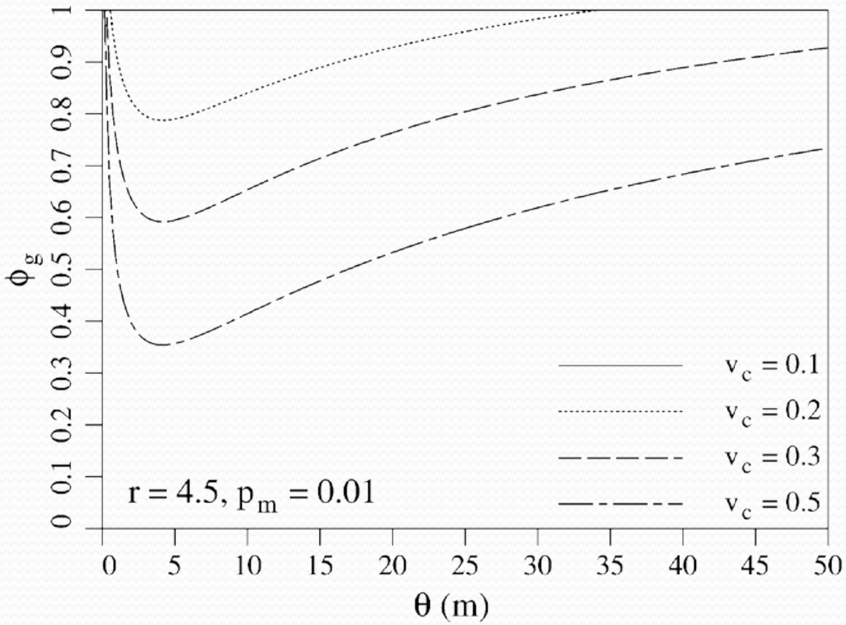
- if bearing failure occurs too often, design is unconservative
- reduce resistance factor and repeat simulations to assess reliability of design

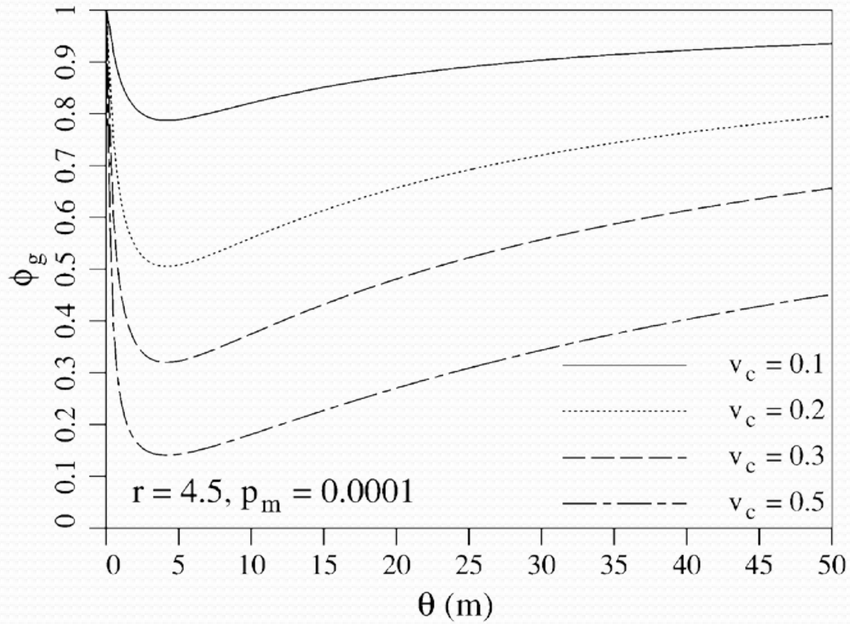
Resistance Factors For Bearing Capacity

Resistance factors can be estimated by simulation (or theory);

- for various failure consequence levels (e.g. low, p_m= 0.01, or high, p_m = 0.0001)
- for various levels of site understanding.

Note the worst case correlation length.

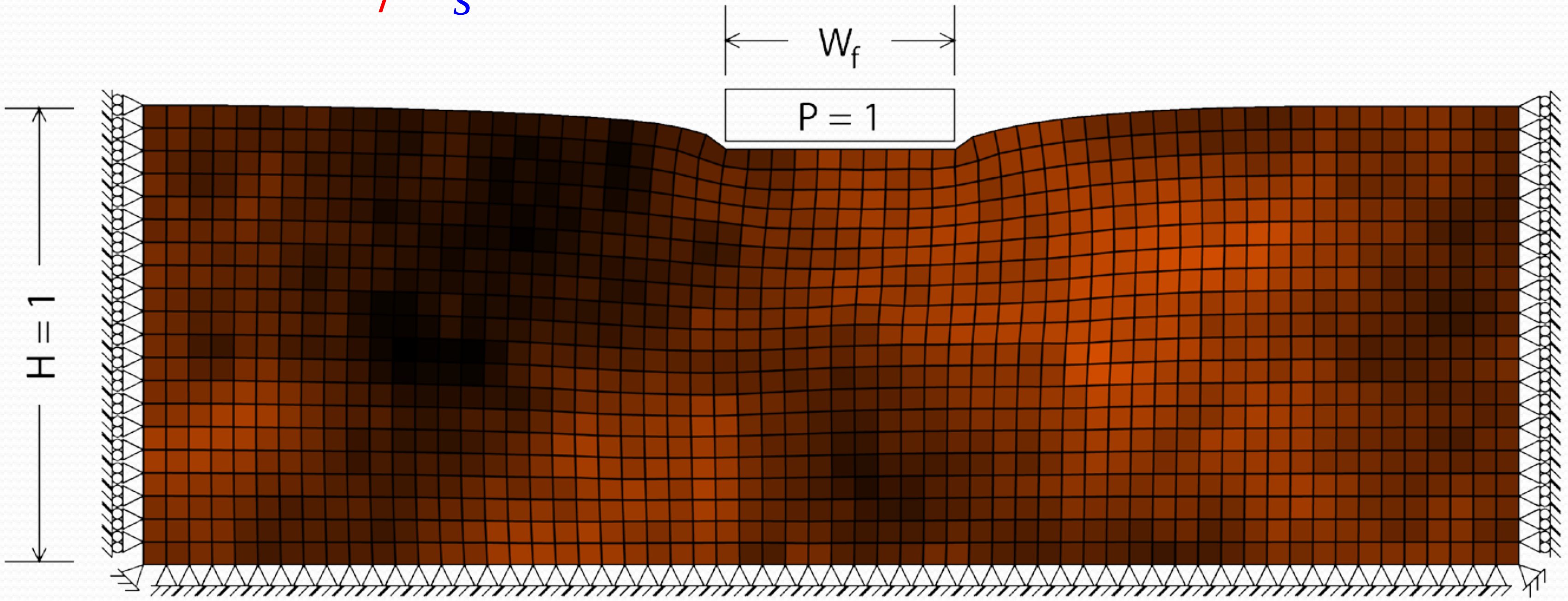




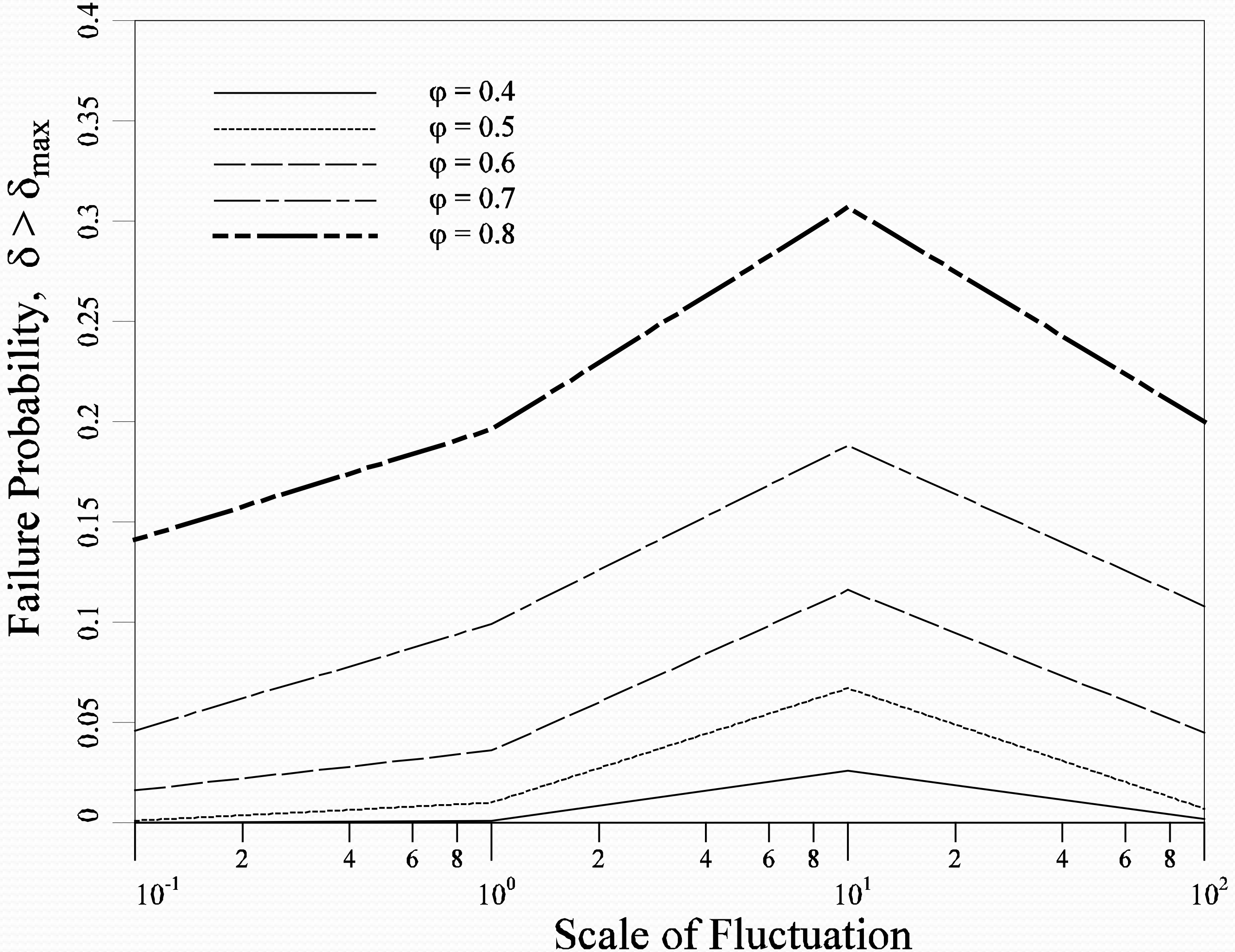
RFEM for Settlement

Determine the reliability of a strip footing designed using Janbu's (1956) settlement prediction

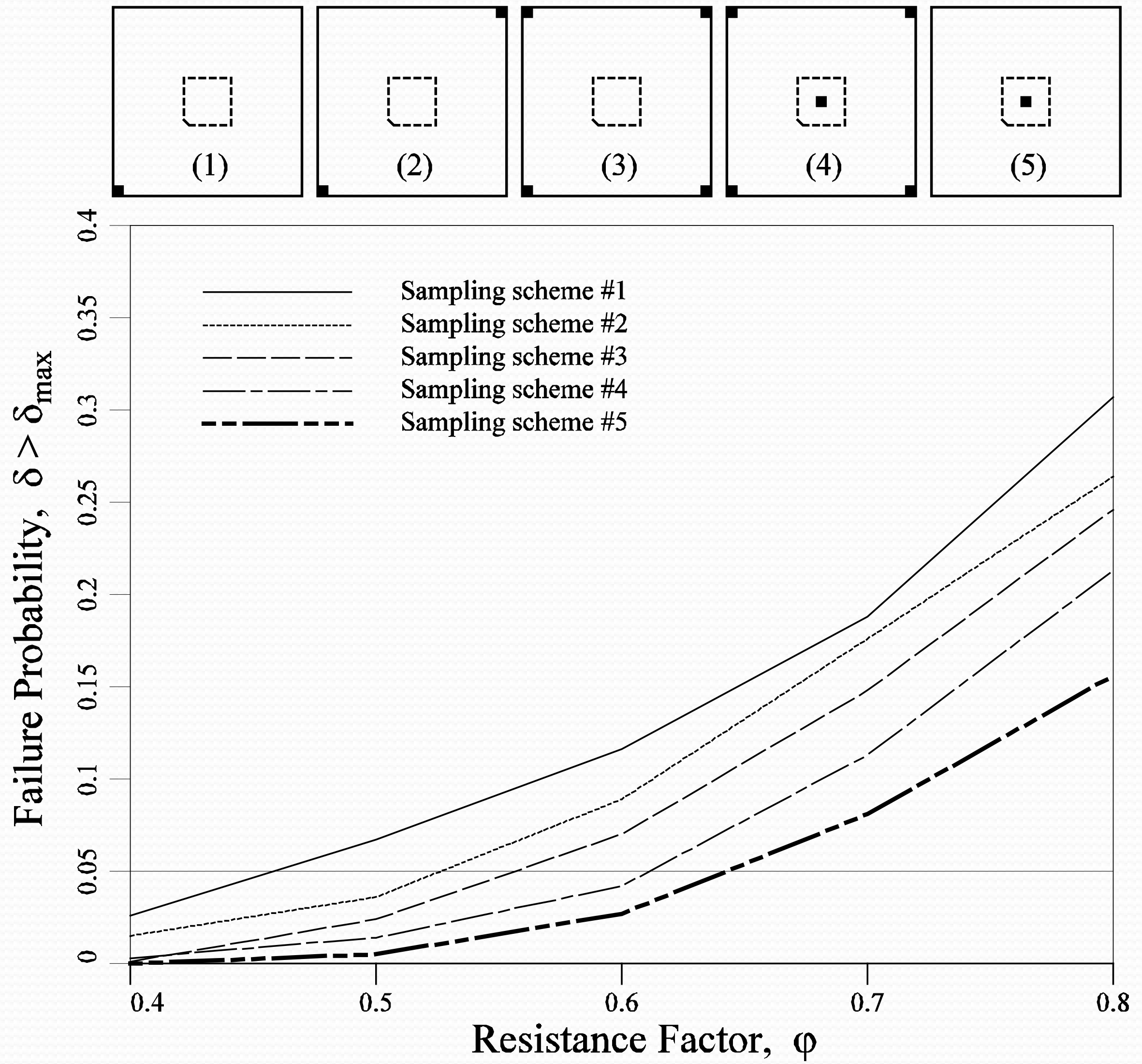
$$\delta_{max} = \mu_0 \cdot \mu_1 \cdot \frac{qW_f}{\varphi E_s^*}$$



RFEM for Settlement



RFEM for Settlement



sampling schemes

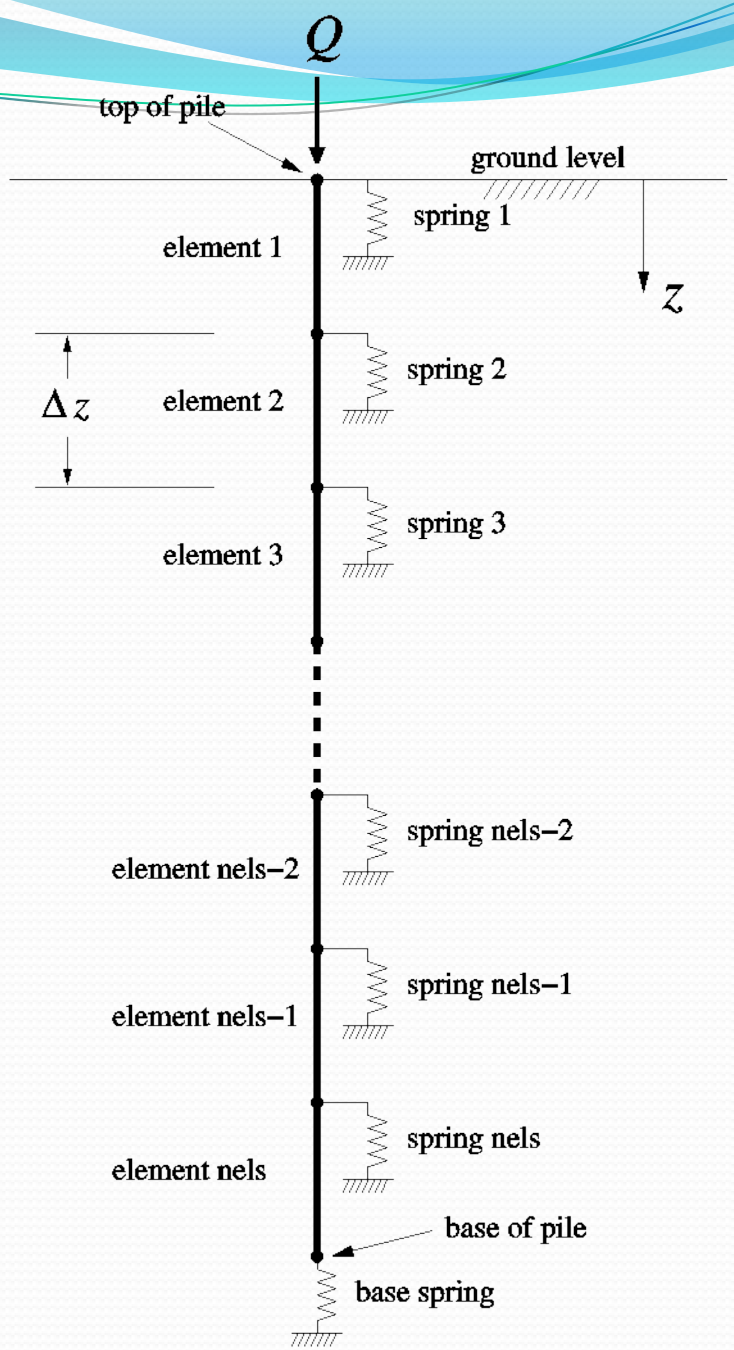
probability of excessive settlement

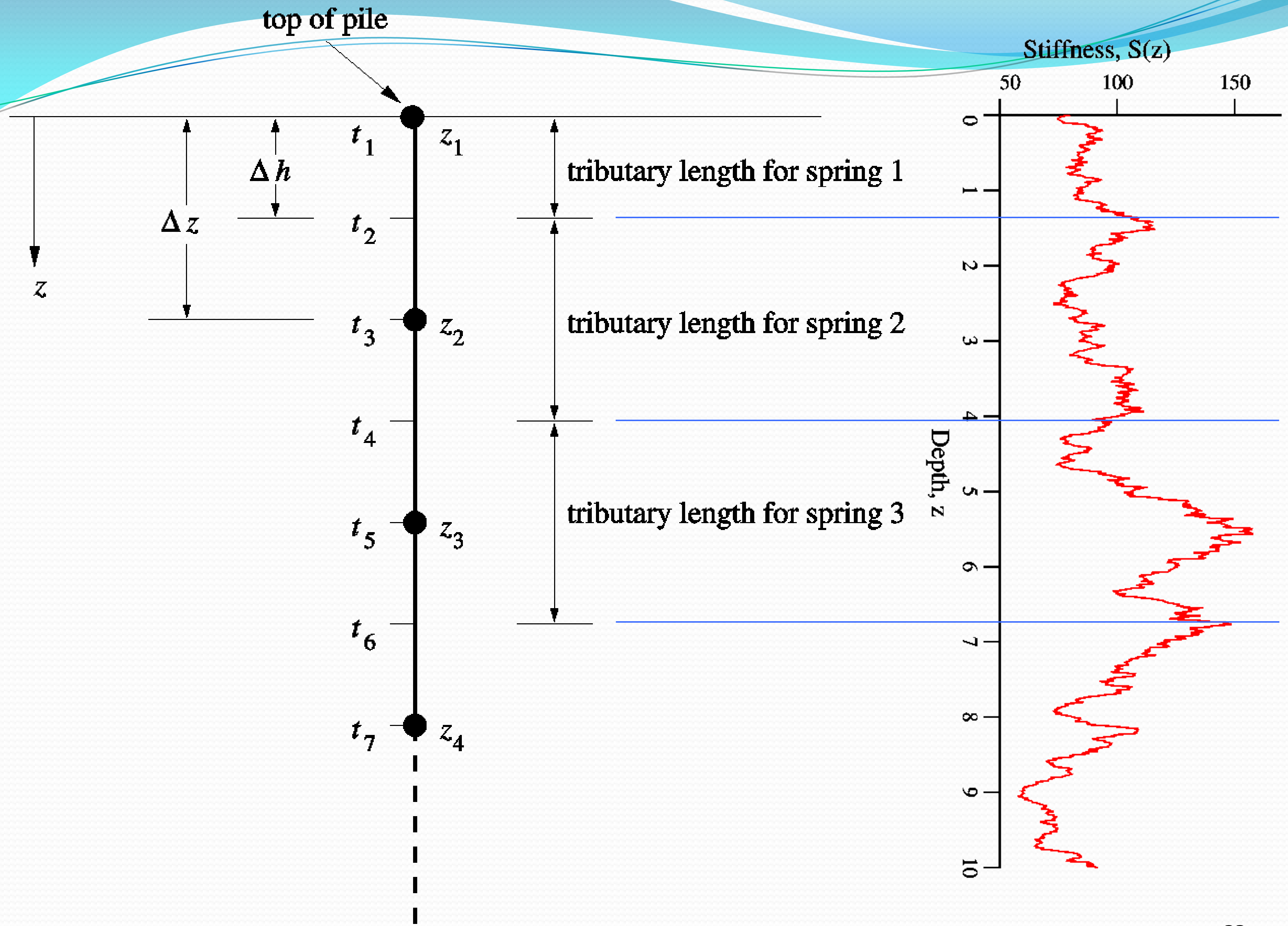
RFEM for Deep Foundations (SLS)

Finite Element Model of Pile Foundation

- soil resistance modeled using bi-linear spring elements with spatially variable properties,

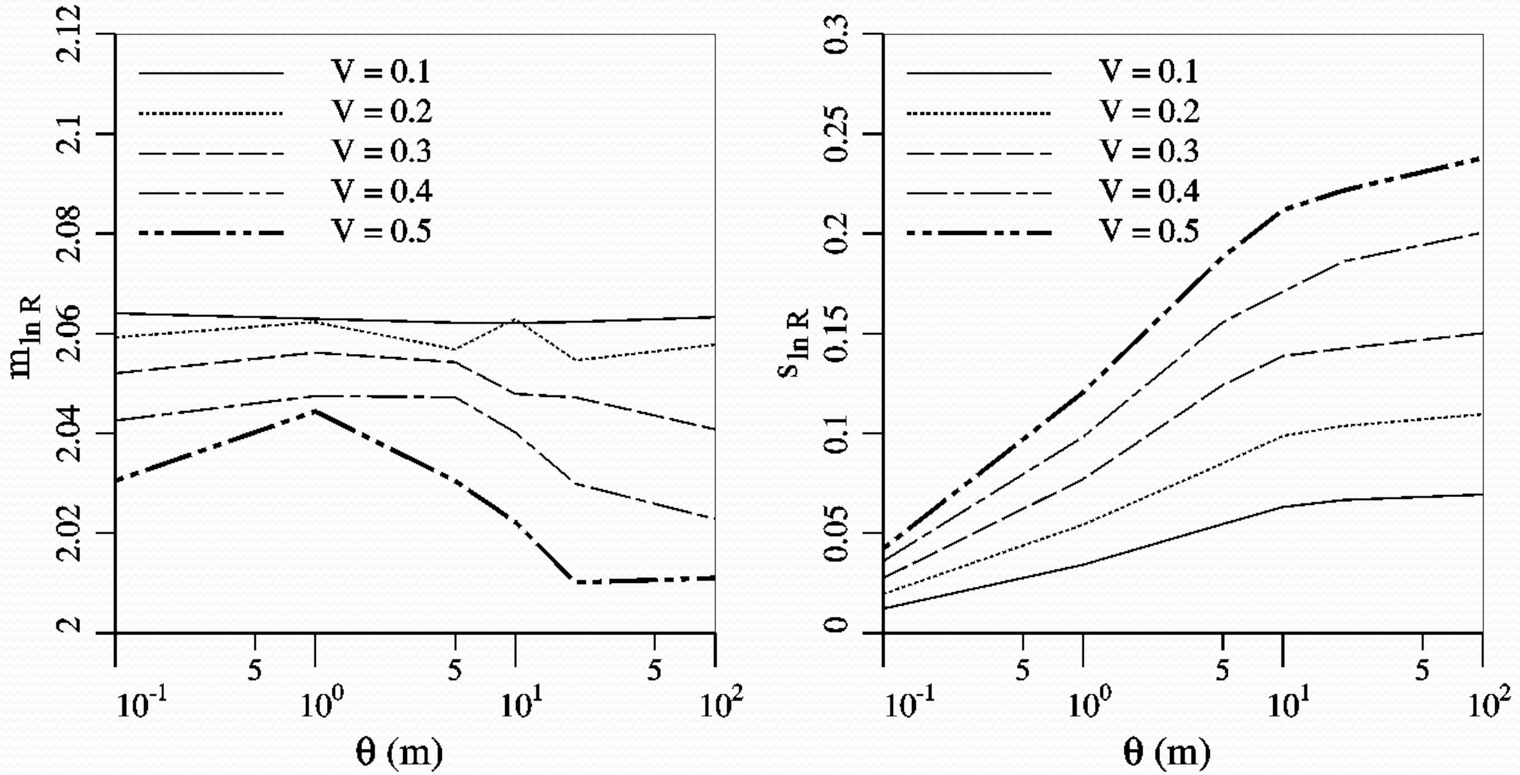
- pile discretized into elements having spatially variable stiffness, AE_p




top of pile
t_1
z_1
Δh
Δz
z
t_2
t_3
z_2
t_4
t_5
z_3
t_6
t_7
z_4
tributary length for spring 1
tributary length for spring 2
tributary length for spring 3
Stiffness, S(z)
50
100
150
Depth, z
0
1
2
3
4
5
6
7
8
9
10

RFEM for Deep Foundations (SLS)

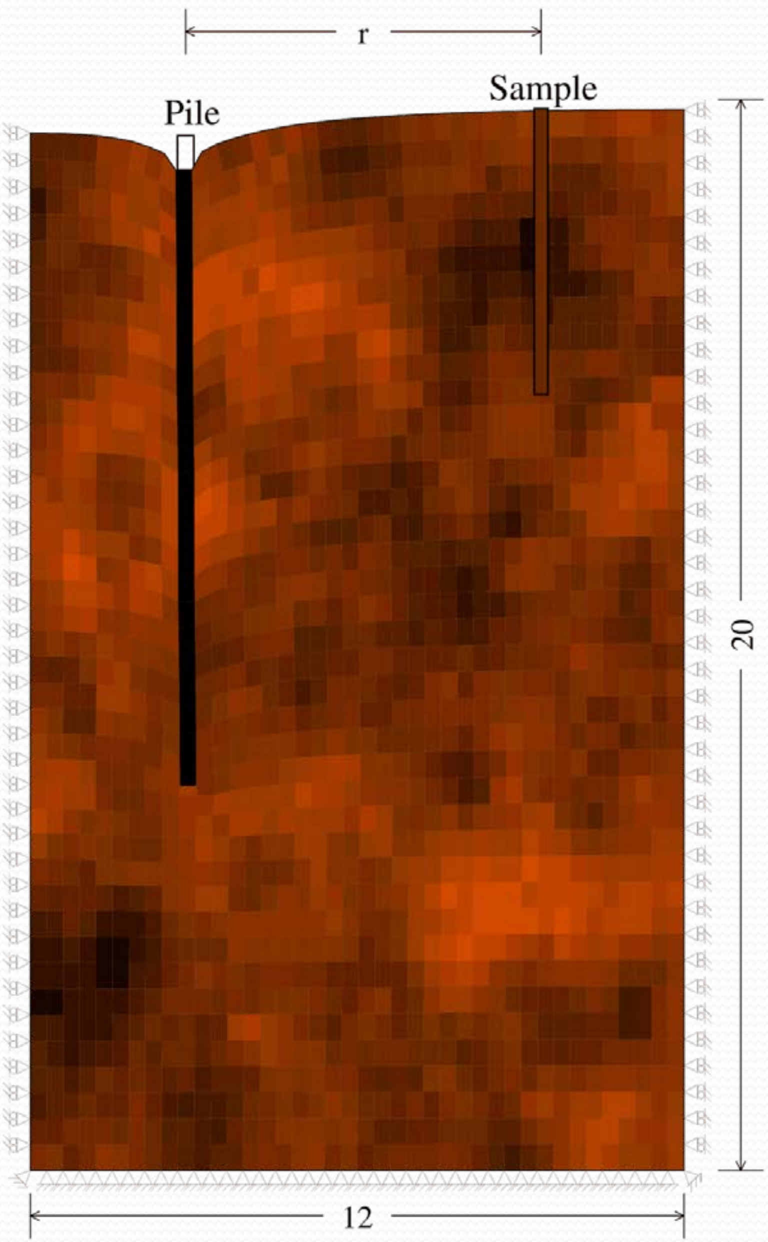
Serviceability Limit State

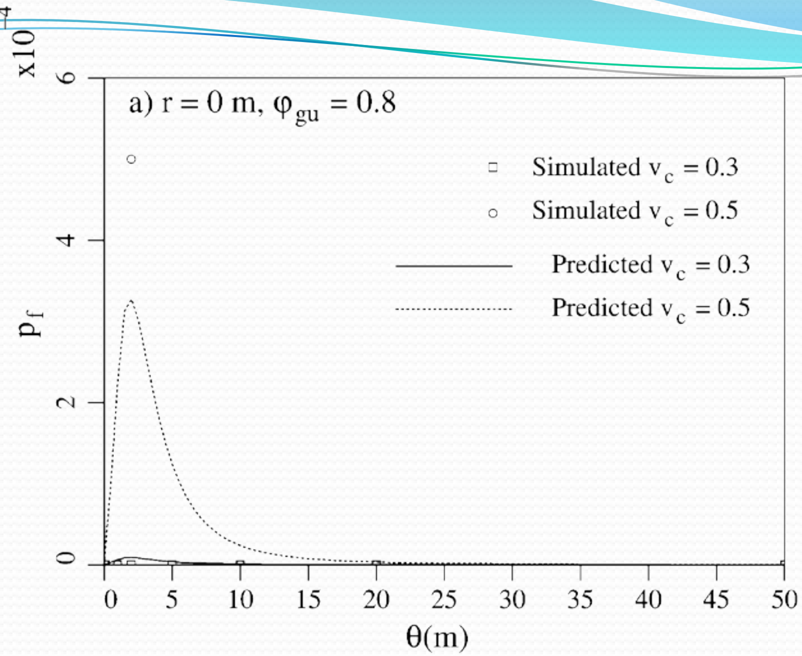


- mean resistance at SLS is little affected by V and θ (due to averaging along pile) although worst case θ does exist.

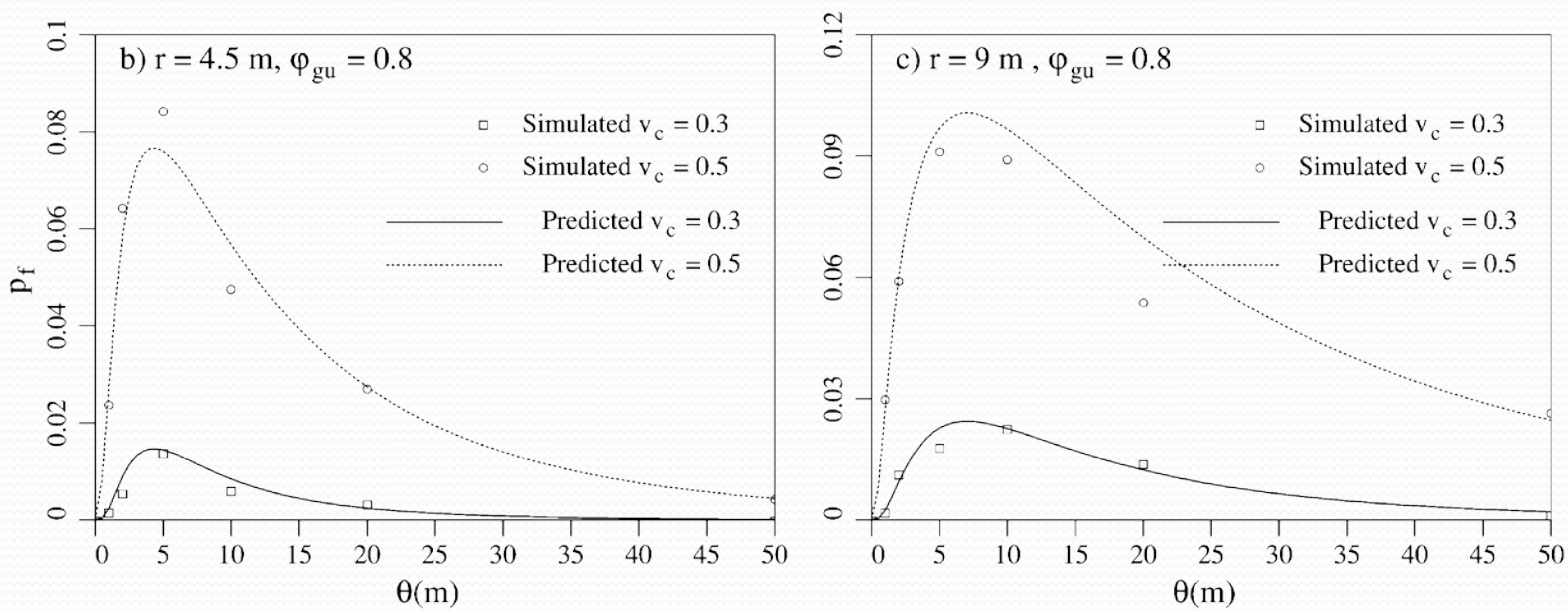
RFEM for Deep Foundations (ULS)

1) virtually sample the simulated soil,
2) choose a resistance factor and design the pile required to avoid ULS,
3) place pile in simulated soil and check, via FEM, if failure occurs,
4) repeat many times to estimate failure probability,
5) if necessary, adjust resistance factor used in design.

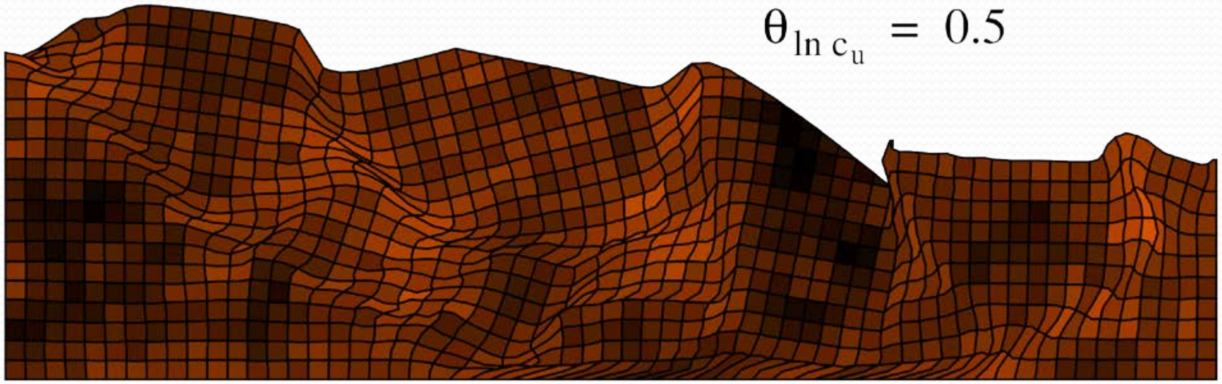


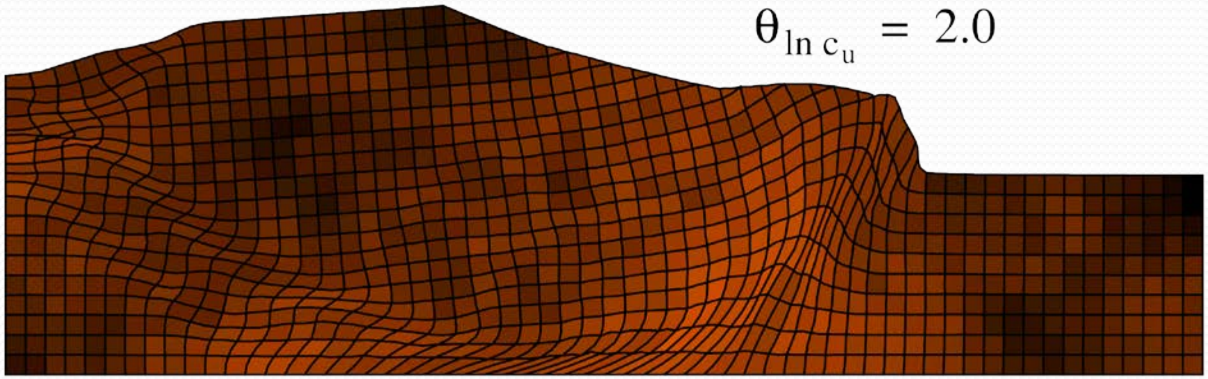


r is the distance between sample and pile (proxy for site under-standing)



RFEM for Slope Stability





Remarks

- The Random Finite Element Method reduces model error by accounting for spatial variability in the ground and allowing for the seeking out of the weakest failure path.

- In a practical sense, the correlation length will almost always be unknown at most sites. Fortunately, a worst case correlation length usually exists which can be used conservatively.

- RFEM is reasonably easily extended to site specific cases (e.g. anisotropic correlation lengths, layering, etc.)

- Analytical probabilistic solutions are often possible through the use of a properly selected average.

Remarks

The Random Finite Element Method also allows the investigation of code provisions by;

1) simulating the ground using a random field (or multiple fields),
2) sampling the field at some location(s), as would be done in practice,
3) designing the foundation using code provisions,
4) placing the foundation on/in simulated ground and applying loads,
5) if foundation fails, record the failure, and repeat *n* times,
6) estimate probability of design failure. If excessive (or too low), reduce (or increase) resistance factor and repeat from step 1 until best resistance factor is determined.

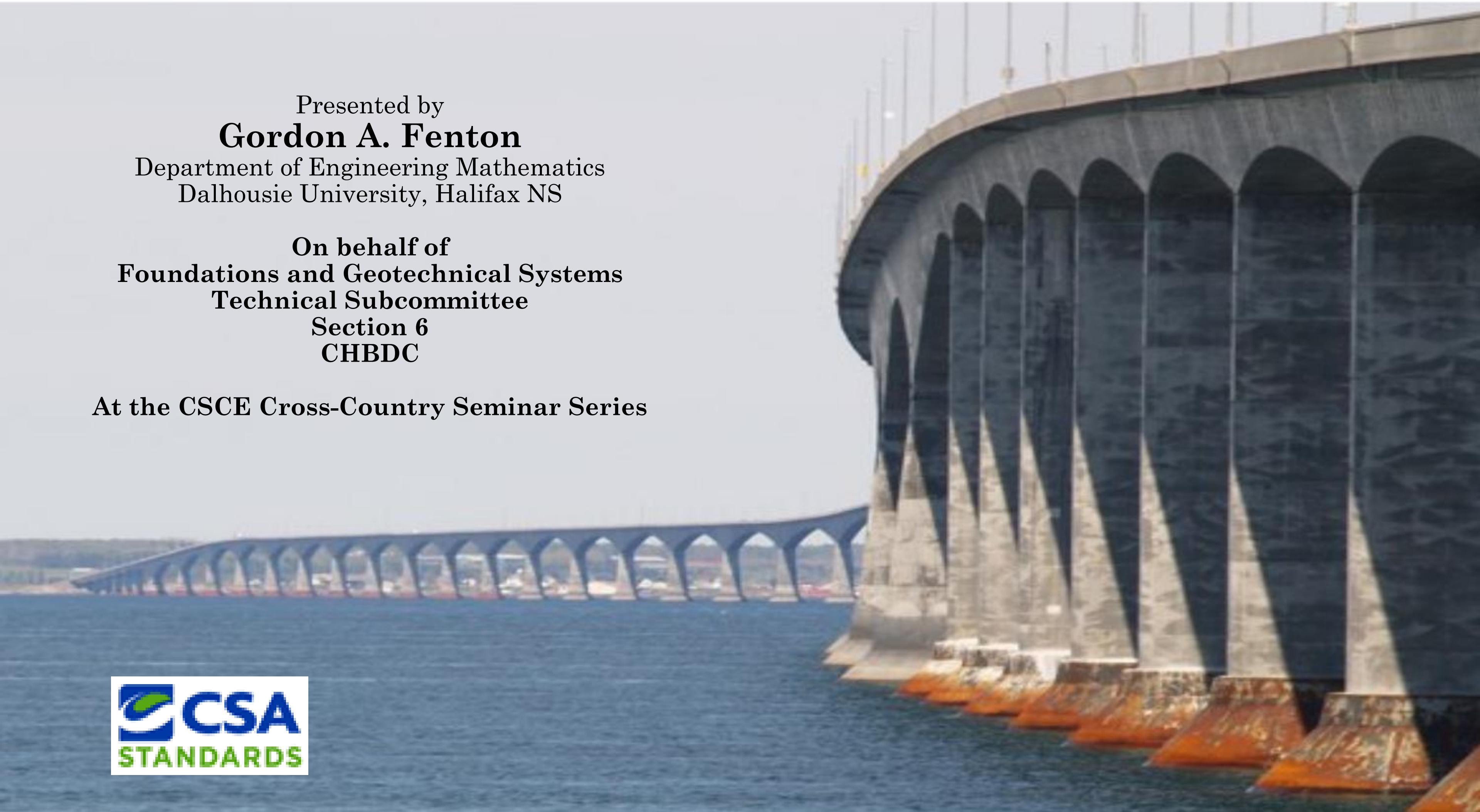
How to Use Theoretical Results

- Just because a theoretical model suggests $p_f = 10^{-5}$ doesn't mean that the true probability of failure is 10^{-5}.
- Theoretical results must be used in a relative sense, i.e. if design A has $p_f = 10^{-5}$ and design B has $p_f = 10^{-4}$, then design A is safer (and is approximate 10 times safer).
- "Typical" resistance factors should be aimed at current "acceptable" values.
- Theory tells us how to adjust the factors up or down from "typical" to achieve lower or higher reliabilities or accommodate higher or lower levels of site understanding.

CHBDC Section 6

Foundations and Geotechnical Systems

Reliability-Based Design Code Provisions



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On behalf of
Foundations and Geotechnical Systems
Technical Subcommittee
Section 6
CHBDC

At the CSCE Cross-Country Seminar Series

CSA STANDARDS

Canadian Highway Bridge Design Code 2006

Table 6.1
Geotechnical resistance factors
(See Clauses 6.6.2.1, 6.6.2.3, 6.10.2.2, and 6.13.2.3.)

Application	Resistance factor
Shallow foundations	
Bearing resistance	0.5
Passive resistance	0.5
Horizontal resistance (sliding)	0.8
Ground anchors (soil or rock)	
Static analysis — Tension	0.4
Static test — Tension	0.6
Deep foundations — Piles	
Static analysis	
Compression	0.4
Tension	0.3
Static test	
Compression	0.6
Tension	0.4
Dynamic analysis — Compression	0.4
Dynamic test — Compression (field measurement and analysis)	0.5
Horizontal passive resistance	0.5

Reliability-Based Design Concepts

- allow quantification of reliability,
- allow designs to target a specified reliability level,
- reward better site investigation by permitting a higher factor to be used in design, thus permitting a more economical design while ensuring acceptable reliability,
- allow quality to be maintained while achieving a less expensive total project cost, depending on the risk that the owner is willing to accept,
- lead to harmonization with other structural codes by establishing a common conceptual framework to address reliability issues.

Reliability-Based Code Objectives

"You pay for a site investigation whether you have one or not" (Institution of Civil Engineers, Inadequate Site Investigation, 1991)

There is a desire in the geotechnical community to:

- provide a means to adjust design/construction economies based on level of site understanding
  - take site investigation/modeling intensity into account in the design process
  - provide rationale for increased investigation/modeling effort
- provide a means to adjust target geotechnical system reliability based on potential failure consequences
  - higher reliability for more important structures/systems by properly accounting for the high variability of the ground

Reliability-Based Code Provisions

Basic idea is to split traditional F_s into

1. Load factors – from load section of code
2. Resistance factors, φ_{gu} and φ_{gs} : capture "resistance" uncertainty
   - Depend on level of site and prediction model understanding
   - Three levels of understanding considered: high, typical, and low
   - Consider SLS and ULS resistance factors separately (different models and target maximum acceptable failure probability)
3. Consequence factor, Ψ : captures system importance
   - Three consequence levels considered ;
     - High: $\beta = 3.7$ ($p_f = 1/10{,}000$) at ULS, $\beta = 3.1$ ($p_f = 1/1000$) at SLS
     - Typical: $\beta = 3.5$ ($p_f = 1/5{,}000$) at ULS, $\beta = 2.9$ ($p_f = 1/500$) at SLS
     - Low: $\beta = 3.1$ ($p_f = 1/1{,}000$) at ULS, $\beta = 2.3$ ($p_f = 1/100$) at SLS

Floating Resistance Factor Concept: $\Psi\varphi_g$

HIGH Consequence	1.0	0.8	0.6
Ψ	1.2	1.0 DEFAULT VALUE	0.8
LOW Consequence	1.4	1.2	1.0
	LOW Uncertainty	φ_g	**HIGH Uncertainty**

Load and Resistance Factors

- load factors, $\alpha_i > 1$, account for variability in loads (risk of extreme loads)
- resistance factor, $\varphi_g < 1$, accounts for variability in soil properties, variability in construction, and model error (risk of low resistance)
- consequence factor, Ψ, accounts for failure consequences

$$\Psi \varphi_g R \geq \sum I_i \alpha_i F_i \qquad \text{(LRFD)}$$

Load and Resistance Factor Design

- Ultimate Limit State (ULS)

Factored ultimate geotechnical resistance ≥ effect of factored ULS loads

$$\Psi \varphi_{gu} R_u \geq \sum_i \alpha_{ui} F_{ui}$$

where

Ψ = consequence factor,

φ_{gu} = ultimate geotechnical resistance factor,

R_u = ultimate characteristic geotechnical resistance,

α_{ui} = i'th ULS load factor,

F_{ui} = i'th load effect for a given ULS.

Load and Resistance Factor Design

- Serviceability Limit State (SLS)

Factored serviceability geotechnical resistance ≥ effect of factored SLS loads

$$\Psi \varphi_{gs} R_s \geq \sum_i \alpha_{si} F_{si}$$

where

Ψ = consequence factor,

φ_{gs} = serviceability geotechnical resistance factor,

R_s = serviceability characteristic geotechnical resistance,

α_{si} = i'th SLS load factor, and

F_{si} = i'th load effect for a given SLS.

Degree of Site and Model Understanding $\left(\varphi_g\right)$

Motivation:

- Differentiating between levels of site understanding allows for design economies – the greater the level of understanding, the lower the risk of failure and the greater the economy of the final design.
- Allows the designer to show "proof" (thus justifying higher design phase costs) that increased understanding (e.g. increased site investigation) leads to construction savings and lower total project costs.

Degree of Site and Model Understanding $\left(\varphi_g\right)$

Site and model understanding includes;

- understanding of the ground and the geotechnical properties throughout the site,
- the type and degree of confidence about the numerical prediction models to be used to estimate serviceability and ultimate geotechnical resistances, and
- observational (monitoring) methods for confirmation.

Degree of Site and Model Understanding $\left(\varphi_g\right)$

Three levels of site understanding are addressed in the CHBDC Code:

- ***High understanding***: Extensive project-specific investigation procedures and/or knowledge is combined with prediction models of demonstrated (or proven) quality to achieve a high level of confidence with performance predictions.
- ***Typical understanding***: Typical project-specific investigation procedures and/or knowledge is combined with conventional prediction models to achieve a medium level of confidence with performance predictions.
- ***Low understanding***: Understanding of the ground properties and behaviour are based on limited representative information (e.g. previous experience, extrapolation from nearby and/or similar sites, etc.) combined with conventional prediction models to achieve a lower level of confidence with the performance predictions.

Geotechnical Resistance Factors (For example only) $\left(\varphi_g\right)$

Application	Limit State	Test Method/Model	Degree of Understanding		
			Low	Typical	High
Shallow Foundations	bearing, φ_{gu}	analysis	0.45	0.50	0.60
		scale model test	0.50	0.55	0.65
	sliding, φ_{gu} frictional	analysis	0.70	0.80	0.90
		scale model test	0.75	0.85	0.95
	sliding, φ_{gu} cohesive	analysis	0.55	0.60	0.65
		scale model test	0.60	0.65	0.70
	passive resistance, φ_{gu}	analysis	0.40	0.50	0.55
	settlement or lateral movement, φ_{gs}	analysis	0.7	0.8	0.9
		scale model test	0.8	0.9	1.0

Consequence Factor

Motivation:

- Different bridges will have different consequences of failure. For example, the failure of an expressway bridge in a major city has far higher consequences (life threat, economic, etc.) than does the failure of a bridge on a low volume rural bridge (such as for a country back road).
- The target maximum acceptable failure probability of a bridge with high failure consequence should be significantly lower than that for a bridge with low failure consequence.
- Rational assessment on the basis of risk and consequence of failure will allow for more realistic allocation of infrastructure budgets.

Consequence Factor

Bridges and geotechnical systems (such as approach embankments) can be assigned consequence levels associated with exceeding limit states;

- ***High consequence*** – bridge is designed to be essential to post-disaster recovery (e.g. lifeline), and/or has large societal and/or economic impacts,
- ***Typical consequence*** – bridge is designed to carry medium to large volumes of traffic and/or having potential impacts on alternative transportation corridors or structures. This is the default consequence level.
- ***Low consequence*** – bridge carries low volumes of traffic with limited impacts on alternative transportation corridors or structures, including temporary structures.

Consequence Factor

Consequence Level	**Reliability Index, β (SLS in parentheses)**	**Example**	**Consequence Factor, Ψ**
High	3.7 (3.1)	Lifelines, Emergency	0.9
Typical	3.5 (2.8)	Highway bridges	1.0
Low	3.1 (2.3)	Secondary bridges	1.15

Summary of Philosophical Changes to CHBDC

- introduced three levels of site understanding – high, typical (default), and low – through the resistance factor
  - resistance factors vary with site understanding – higher for better understanding
  - this approach allows for greater economies in the tradeoff between design/investigation effort and overall construction costs
- introduced three levels of failure consequence – high, typical (default), and low – through the consequence factor
  - consequence factor, which modifies the factored resistance, varies with consequence level – lower for higher consequences
  - this also allows for greater economies in the tradeoff between target reliability and construction costs

Summary

- Geotechnical design codes are migrating towards reliability-based design to allow;
  - harmonization with structural codes
  - quantification of reliability
- Ground properties are typically both site specific and highly (spatially) variable. The development of reliability-based design in geotechnical engineering is a significant challenge.
- Reliability-based design codes are currently largely developed through calibration with WSD but theoretical random field models are coming along.
- Design codes should allow for varying degrees of site understanding and take failure consequence into account.
- Sophisticated probabilistic tools exist to assess risk and develop required resistance and consequence factors (e.g. RFEM).
- Much work is still required, but efforts are ongoing world-wide.

Thanks for listening!

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