Dynamic Regulation Design Without Payments:
The Importance of Timing[*]

Ralph Boleslavsky
Department of Economics
University of Miami
Box 248126
Coral Gables, FL 33124
r.boleslavsky@miami.edu

and

David L. Kelly
Department of Economics
University of Miami
Box 248126
Coral Gables, FL 33124
dkelly@miami.edu

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Abstract

We consider a two period model of optimal regulation of a firm subject to marginal compliance cost shocks. The regulator faces an asymmetric information problem: the firm knows current compliance costs, but the regulator does not. Both the regulator and the firm are uncertain about future costs. In our basic framework, the regulator may not offer payments to the firm; we show that the regulator can vary the strength of regulation over time to induce the firm to reveal its costs and increase welfare. In the optimal mechanism, the regulator offers stronger (weaker) regulation in the first period and weaker (stronger) regulation in the second period if the firm reports low (high) compliance costs in the first period. Low cost firms expect compliance costs to rise in the future, and thus prefer weaker regulation in the second period. High cost firms expect costs to fall in the future and thus prefer regulation which becomes more strict over time. Thus the regulator offers the low (high) cost firms *slightly* weaker (stronger) regulation in the second period in exchange for *much* stronger (weaker) regulation in the first period, thereby "timing" the regulation. If the regulator can make payments, then the optimal mechanism to some degree times the regulation as long as a positive marginal cost of funds exists. If the marginal cost of funds is high enough, then under the optimal mechanism the regulator will not use payments and use our timing mechanism exclusively.

1 Introduction

We consider a two period model of optimal regulation of a firm subject to marginal compliance cost shocks. The regulator faces an asymmetric information problem: the firm knows the current compliance cost, but the regulator does not. Both the regulator and the firm are uncertain about future compliance costs. Standard economic theory suggests making payments or rebates conditional on the benefits or costs of regulation. Frequently, however, regulators are unable to make monetary payments to firms. Regulators do typically have considerable latitude on how regulations are implemented: they may interpret vague statutes weakly or strictly, grant waivers to delay implementation of the regulation, shape future legislation so that regulations become more strict or weak, and/or vary enforcement. We show that the regulator can vary the strength of regulation over time to induce the firm to reveal the cost of compliance and increase welfare by explicitly characterizing the optimal regulatory policy.

In the optimal mechanism the regulator offers stronger regulation in the current period and weaker regulation in the next period if a firm reports low compliance costs in the current period. Conversely, firms reporting high costs receive regulation that becomes stronger over time. We refer our mechanism as "timing" the regulation. At first glance, timing the regulation may seem counterintuitive. Since compliance costs are convex, a policy that strengthens regulation in the current period and weakens regulation in the next period by an equal amount is more costly than an average level of regulation in both periods. However, the regulator need only offer slightly weaker regulation in the future in exchange for much stronger regulation today to induce the low cost firms to reveal their type. This is because a firm that receives a below average compliance cost shock in the current period expects higher costs in the next period. Thus, low cost firms prefer to be regulated lightly in the future, and so the regulator need only offer slightly weaker future regulation to induce the low cost firms to reveal their types today. Similarly, firms receiving a higher than average cost shock expect costs to fall over time, and thus prefer regulation that is initially weaker. As will be clear in the paper, timing the regulation not only improves welfare by making regulation stronger when compliance costs are low, but also improves welfare by inducing firms to reveal cost shocks.

A large literature develops mechanisms that induce firms to reveal compliance cost shocks and raise welfare. Standard economic theory (see for example, Roberts and Spence 1976, Kwerel 1977) suggests the first best (full information) level of regulation may be achieved in competitive environments via hybrid tax/subsidy or permit/subsidy mechanisms. These mechanisms, however, require firms to be competitive price takers. Dasgupta, Hammond, and Maskin (1980), Kim and Chang (1993), Montero (2008), and Spulber (1988) consider regulation of potentially non-competitive firms via tax/subsidy or permit/subsidy mechanisms. For example, Montero (2008) proposes an elegant first best mechanism whereby firms first bid for permits via a uniform-price sealed-bid auction. The regulator then rebates a fraction of the auction revenue to the firm conditional on the residual marginal benefit of regulating each firm. In this way, the benefits of regulation are transferred to the firm, and the firm's problem becomes identical to the regulator's. Firms then optimally choose the first best (full information) level of regulation as a dominant strategy.[1]

The degree to which each of these mechanisms are used, or could be used, in practice varies. Mechanisms that rely on perfect competition rule out a host of highly regulated industries, such as electricity. Similarly, firms are not typically asked to report each other's costs since cost information is likely private (Wiggins and Libecap 1985). The mechanism proposed by Montero (2008) is consistent with some regulations.[2]

Nearly all mechanisms characterized in this literature require monetary transfers between the regulator and firm (typically the regulator extracts payments from the firm, which are then rebated back to the firm).[3] If the regulator has access to a revenue stream and legal authority to make payments from that revenue stream, then such payments are plausible. For example, sulfur dioxide permit auction revenue provides a funding source and the EPA has the authority to design an auction with a rebate. Mason and Plantinga (2013) also propose a plausible mechanism whereby payments for carbon offsets are subject to a "clawback," which allows the regulator to revoke some payments ex post.[4]

Most regulatory environments (examples include all command-and-control regulation and permit based regulation in which permits are grandfathered or otherwise freely allocated) do not feature payments from the firm to the regulator, nor monetary subsidies to firms that report low compliance costs. Instead, regulators typically have considerable discretion over the interpretation of vague statutes, enforcement of existing regulations, the granting of

[1]Other mechanisms (Varian 1994, Duggan and Roberts 2002) rely on the assumption that firms know each other's marginal costs. Given this unlikely assumption, however, the regulator can simply require firm's to report all other firm's costs, and punish firms if the results do not agree (Cremer and McLean 1988).

[2]NO_x permit allocations in Sweden have a rebate based on market share (Gersbach and Requae 2004). In the US, the EPA holds back 2.8% of grandfathered SO_2 allowances from firms, and then auctions them, rebating the revenue back to the firms (Joskow and Schmalensee 1998).

[3]The exception are those mechanisms requiring firms to know and report each other's costs. The mechanism of Kwerel (1977) does not use payments to the firm in equilibrium.

[4]A legislator may have the freedom to design a bill with a payment of an initial allocation of permits. The allocation would have to be tied to the residual marginal benefits of regulating each firm, however.

waivers,[5] and other decisions affecting the strength of regulation. For example, "New Source Review" regulation requires that, with the exception of "routine maintenance," modifications to a plant which cause a "significant increase" in a regulated pollutant receive an EPA review that typically forces the plant to adopt the best available pollution control technology. Both "routine maintenance" and "significant increase" are terms that are not precisely defined, and indeed interpretations of this statute by the EPA has varied over time (Stavins 2006, footnote 90). While New Source Review and similar command and control regulations do not give the regulator discretion to set up a permit or tax/subsidy mechanism, our results show that the regulator can improve welfare by timing the regulation: offering firms a choice of regulations that become either stronger or weaker over time.

Although our paper is theoretical, in practice regulators sometimes implement dynamic regulations that resemble our mechanism, allowing firms a choice of regulation that either becomes stronger or weaker over time. Joskow and Schmalensee (1998) provide a detailed examination of the rules of the sulfur dioxide permit trading system created by the 1990 Clean Air Act. One provision gives utilities that install scrubbers future "bonus" permit allocations. Firms that install scrubbers clearly face more costly regulation up front, but are rewarded with weaker regulation in the future, since (at a minimum) their allocation of permits rises over time. Conversely, by declining the option, firms save the initial cost of scrubbers, but do not gain bonus permits later. Thus declining the option results in regulation which becomes stronger over time.[6] Section 6 discusses additional practical examples.

Even if the regulatory framework allows for payments to and from the firm, the absence of lump sum taxes implies payments to the firm could instead be used to reduce labor or other distortionary taxes (Bovenberg and Goulder 1996). Thus, a regulator using payments faces a tradeoff between information revelation and the distortionary cost of government funds (Montero 2008). Therefore, with a distortionary cost of funds, payment-based mechanisms no longer achieve the first best. Our mechanism, which trades off current and future distortions, also does not achieve the first best. Nonetheless, we show that with any positive marginal cost of funds, the optimal regulation involves some degree of timing, even when payments are available. Further, we show that for marginal costs of funds above a threshold, characterized

[5] The provision of the Patient Protection and Affordable Care Act phasing out annual payment limits has been temporarily waived for 729 companies (Department of Health and Human Services 2011).

[6] The Clean Air Act allows pollution permit "banking" (Ellerman and Montero 2007), which also gives firms some control over the strength of regulation over time. However, we show in section 2.2 that our timing mechanism yields higher welfare than permit banking, since the timing mechanism induces firms to reveal cost shocks, while banking does not.

explicitly in Proposition 4, the regulator uses the timing mechanism exclusively.[7]

The timing mechanism takes advantage of firm uncertainty over the realization of future cost shocks. Many authors consider regulation with time varying compliance cost shocks, which fit naturally into our framework. Newell and Pizer (2003) and Karp and Zhang (2005) consider pollution regulation with time varying abatement cost shocks. Heutel (2012) and Fischer and Springborn (2011) consider climate change regulation when firms experience autoregressive productivity shocks. Productivity shocks fit naturally into our framework since firms know current, but not future, shocks. Other natural interpretations of time-varying compliance costs include input prices which vary randomly over time and the uncertain discovery of cost saving innovations. Section 4 extends our mechanism to general cost-shock processes, allowing for costs that are correlated over time, such as productivity shocks.

Our mechanism relies on commitment: the ability of the regulator to commit to weak (strong) regulation in the future for firms that reports low (high) costs today.[8] A number of papers (e.g. Freixas, Guesnerie, and Tirole 1985, Yao 1988) study models in which marginal costs are fixed and not subject to shocks. In this case, the regulator who learns a firm has permanently low costs has an incentive to renege on a commitment to weak regulation and instead impose the optimal regulation given the known low compliance costs in the second period (the "ratchet effect"). In contrast, the incentive to renege is relatively minor in our mechanism. If costs are i.i.d. over time, then the regulator who learns the firm has low costs in period one has only prior information about the firm's costs in period two. The regulator thus does not desire to ratchet the second period regulation up to the optimal level for a firm known to have a low compliance costs, but instead only desires to strengthen the regulation to the optimal level given the prior belief.

One way to solve the regulator's commitment problem is through contracts. Baron and Besanko (1987) argue that regulators and public utilities have successfully maintained commitments to implicit contractual arrangements.[9] In the absence of a formal institution

[7]Our result should not be confused with the dynamic moral hazard literature, in which it is optimal for the principal to use both payments and continuation values to reward agents. Here, the gains to the principal from using the continuation value as compensation are not driven by "payment smoothing." In our mechanism, payments in the form of weaker regulation are not perfect substitutes across time to the agent, which the principal exploits to gain information.

[8]All permit-subsidy schemes require commitment at some level, since otherwise the regulator would renege on the subsidy.

[9]Hahn (1989) notes that some permit regulations are written specifically so that the regulator may devalue existing permits without compensation. For example, the sulfur dioxide permit system legislation states that the EPA may abandon the permit system without compensation at any time. However, Joskow and Schmalensee (1998, footnote 4) note that the EPA issued permits several years ahead as a commitment device, making it politically difficult to renege (indeed, the sulfur permit system has now been in place for

allowing the regulator to commit, if the discount factor is sufficiently high and the regulator interacts with (possibly different) firms over time, then commitment power in each period can be generated by the regulator's concern for his reputation, which influences his future payoffs (e.g. Yao 1988). For this reason, we have in mind repeated interactions between a career regulator and firm, rather than a more temporary political appointee.[10]

Many regulations involve long run interactions between the firm and regulator. Some studies of such long run relationships argue the result is regulatory capture: because the regulator and the firm have repeated interactions, the regulator is more responsive to the firm's needs and regulation tends to be weak (Besley and Coate 2003). Our model provides an alternative explanation of this behavior. Regulation which becomes more lax over time may result from the regulator following through on a (ex ante welfare increasing) commitment to the firm. These two hypothesis can be resolved empirically using the model's testable predicted relationship between past and future regulation and the firm's costs.

Section 2 solves for the optimal mechanism in the basic model with one firm and determines the properties of the mechanism. Section 3 does the same when the government has a cost of funds. Section 4 characterizes the optimal regulatory policy for more general marginal cost processes, including correlated cost shocks. Section 5 extends the mechanism to n firms. Section 6 discusses practical examples of our mechanism. Section 7 concludes. All proofs are in the Appendix.

2 Model: Two period problem with a single firm

Consider a regulator imposing a level of regulation $q \in \mathcal{R}^+$ on a firm in each of two periods, $t = 1,2$. The function $B(q)$ is the (gross) benefit of regulation in a period—for example, a cleaner environment in the case of pollution regulation or a reduction in systemic risk in the case of banking regulation. We assume that $B(q)$ is differentiable, increasing, and concave in q.

The benefits of regulation increase welfare, but these benefits do not affect the profits of the firm (a production externality exists). In contrast, stronger regulation results in higher compliance costs, which reduce the firm's profit. When regulation level q is imposed in

20 years and the EPA has not reneged).

[10]Guasch, Laffont, and Straub (2008) show the probability of contract renegotiation between regulators and firms in Latin America decreases significantly when a regulatory agency negotiates the original contract. Besley and Coate (2003) show firms extract more rents from elected than appointed regulators.

period t, the firm's period t payoff is equal to its profit:

$$V(q,\pi) \equiv V_0 - C(q,\pi). \tag{2.1}$$

Here $C(q,\pi)$ represents the cost of complying with regulation level q—the cost of installing scrubbers on smokestacks to reach a required level of emissions or foregone interest associated with requiring banks to hold a minimum level of risk free assets, for example.[11] In the absence of regulation, $q = 0$ (*laissez faire*), the firm's profit is V_0, so that $C(0,\pi) = 0$. We assume that $C(q,\pi)$ is twice differentiable, increasing and weakly convex in q, and increasing in π. In addition to the strength of regulation, the firm's compliance cost in period t also depends on a privately known shock, π_t. This compliance cost shock follows an iid Bernoulli process: for $t = 1{,}2$, $\Pr(\pi_t = \pi_L) = \gamma$ and $\Pr(\pi_t = \pi_H) = 1 - \gamma$.[12] Let $\pi_L < \pi_H$, so that π_L represents low compliance costs.

We assume that V_0 is sufficiently high so that the firm does not shut down for all of the regulation levels that arise in equilibrium. Because the firm always participates, it is without loss of generality for the firm and the regulator to calculate firm profits relative to *laissez faire*, $v(q,\pi) = V(q,\pi) - V(0,\pi) = -C(q,\pi)$. The firm's realized cost shock in each period is unknown to the regulator, but the firm knows π_1 at the beginning of period one, and learns π_2 at the beginning of period two. The expected change in lifetime profit of a firm subject to regulation levels (q_1,q_2) is:

$$\mathcal{V}(q_1,q_2,\pi_1) = v_1(q_1,\pi_1) + \delta \mathrm{E}[v_2(q_2,\pi_2)] = -C(q_1,\pi_1) - \delta \mathrm{E}[C(q_2,\pi)]. \tag{2.2}$$

Both the regulator and the firm have commonly known discount factor δ.

The regulator maximizes expected welfare, $\mathcal{W}$, accounting for the gross benefits of regulation and firm profits.

$$\mathcal{W} = \mathrm{E}[w(q_1,\pi_1)] + \delta \mathrm{E}[w(q_2,\pi_2)], \text{ where} \tag{2.3}$$

$$w(q,\pi) = B(q) - C(q,\pi). \tag{2.4}$$

Throughout the paper, subscripts on functions denote partial derivatives with respect to the subscript variable. We assume $C_q(0,\pi_H) < B_q(0)$, so that nonzero regulation is optimal

[11] The regulation strength may also affect the firm's subsequent equilibrium pricing or supply decisions. To increase the applicability of the model, we do not model these decisions directly, but our specification is consistent with a variety of possible industry configurations.

[12] We relax the iid assumption in section 4.

even if compliance costs are high.

One example of this setting is environmental regulation. If E represents emissions, with uncontrolled emissions equal to E_0, then $q = E_0 - E$ can be interpreted as regulation implementing an emissions standard of E or a supply of E emissions permits. In this case, $C(E_0 - E, \pi)$ is the convex cost of reducing emissions to meet the target. If $D(E)$ is a convex function that represents the damages from emission level E, then the gross benefit of regulation q is given by $B(q) = D(E_0) - D(q)$.[13]

2.1 Two Period Regulatory Mechanism Without Payments

The regulator requires the firm to report either low ($\hat{\pi} = \pi_L$) or high ($\hat{\pi} = \pi_H$) compliance costs in period one. The regulator commits to a set of policies $q(\hat{\pi})$, based on the firm's report. In particular, the regulator implements $\{q_{1i}, q_{2i}\}$ when the firm reports $\hat{\pi} = \pi_i$, $i = L,H$. The regulator cannot condition regulation in period two on the firm's report in period two, because the firm would always report the type associated with the smallest regulation level.[14] Clearly a firm with low compliance costs has an incentive to report high compliance costs to induce the regulator to implement weaker regulation. We assume that if the firm is indifferent between reporting truthfully or not, the firm reports truthfully.

Incentive compatibility requires that truthful reporting maximizes expected profits for both types of firms.

$$\mathcal{V}(q_{1L}, q_{2L}, \pi_L) \geq \mathcal{V}(q_{1H}, q_{2H}, \pi_L), \tag{2.5}$$

$$\mathcal{V}(q_{1H}, q_{2H}, \pi_H) \geq \mathcal{V}(q_{1L}, q_{2L}, \pi_H). \tag{2.6}$$

Our strategy is to compute the regulatory mechanism which maximizes welfare subject to the constraint that the low cost firm not misrepresent itself as a high cost firm (2.5). We will then verify that, under mild conditions, the solution to the relaxed problem satisfies (2.6). The Lagrangian for the relaxed problem anticipates truth telling on the part of the firm:

$$\mathcal{L} = \gamma \cdot \Big[w(q_{1L}, \pi_L) + \delta \mathrm{E}[w(q_{2L}, \pi)] \Big] + (1-\gamma) \cdot \Big[w(q_{1H}, \pi_H) + \delta \mathrm{E}[w(q_{2H}, \pi)] \Big] +$$

[13] This example also requires $B_q(E_0, \pi_L) < C_q(E_0, \pi_L)$, since the maximum abatement q is finite.

[14] This result depends on the sequential nature of the regulation: working backward, in the second period the first period regulations have already been implemented, so there is no way to use the first period to motivate truthful reporting in the second. Note also that if monetary payments are allowed, the regulator can use payments to induce the firm to report truthfully in the second period, see section 3.

$$\lambda \cdot \left[\mathcal{V}\left(q_{1L}, q_{2L}, \pi_L\right) - \mathcal{V}\left(q_{1H}, q_{2H}, \pi_L\right) \right] \tag{2.7}$$

The first order conditions for the relaxed problem are:

$$B_q\left(q_{1L}\right) = C_q\left(q_{1L}, \pi_L\right)\left(1 + \frac{\lambda}{\gamma}\right) \tag{2.8}$$

$$B_q\left(q_{1H}\right) = C_q\left(q_{1H}, \pi_H\right)\left(1 - \frac{\lambda}{1-\gamma}\frac{C_q\left(q_{1H}, \pi_L\right)}{C_q\left(q_{1H}, \pi_H\right)}\right) \tag{2.9}$$

$$B_q\left(q_{2L}\right) = \mathrm{E}\left[C_q\left(q_{2L}, \pi\right)\right]\left(1 + \frac{\lambda}{\gamma}\right) \tag{2.10}$$

$$B_q\left(q_{2H}\right) = \mathrm{E}\left[C_q\left(q_{2H}, \pi\right)\right]\left(1 - \frac{\lambda}{1-\gamma}\right) \tag{2.11}$$

$$C\left(q_{1H}, \pi_L\right) - C\left(q_{1L}, \pi_L\right) + \delta\left(\mathrm{E}\left[C\left(q_{2H}, \pi\right)\right] - \mathrm{E}\left[C\left(q_{2L}, \pi\right)\right]\right) = 0 \tag{2.12}$$

Provided constraint (2.6) holds, equations (2.8)-(2.12) define the optimal regulatory structure. Since the first best mechanism (with $\lambda = 0$) regulates the low cost firm more strongly than the high cost firm in both periods, it violates (2.5). Hence, in the optimal second best mechanism $\lambda > 0$: the regulator must move marginal benefits away from marginal costs to induce truth telling. Proposition (1) shows constraint (2.6) is satisfied, which completes the solution.

PROPOSITION 1 *Suppose C is supermodular in $[q, \pi]$. Then the solution to problem (2.7) satisfies condition (2.6).*

A twice differentiable function is super modular if and only if the cross partial derivative is positive. Thus we assume $C_{q\pi} > 0$, or that π is a positive shock to the firm's marginal cost, which is standard.

2.2 Properties of the Timing Mechanism

We first derive some properties of the solution, and then use these properties to develop an intuition of the results. The spread between high and low marginal costs is:

$$R\left(q, \pi_H, \pi_L\right) \equiv \frac{C_q\left(q, \pi_H\right)}{C_q\left(q, \pi_L\right)}. \tag{2.13}$$

Two natural benchmarks for comparison are the (first best) full information regulation, which assumes the regulator observes firm costs, and the prior-information regulation, which assumes that the regulator cannot vary regulation by the firm's report.

The full information regulations, $\{q_L^*, q_H^*\}$, equate the marginal benefits of regulation with the realized marginal costs, while the prior information regulation policy, $\bar{q}$, equates the marginal benefit of regulation with the expected marginal cost:

$$B_q\left(q_i^*\right) = C_q\left(q_i^*, \pi_i\right), \quad i = L{,}H, \tag{2.14}$$

$$B_q\left(\bar{q}\right) = \mathrm{E}\left[C_q\left(\bar{q}, \pi\right)\right]. \tag{2.15}$$

Clearly, $q_H^* < \bar{q} < q_L^*$.

Proposition 2 describes the relationship between the optimal dynamic mechanism and these benchmarks.

PROPOSITION 2 *The solution to the two period problem has the following properties:*

2.1. $q_H^* < q_{1H} < \bar{q}$ *and* $q_{1L} < q_L^*$*, and if* R *is constant in* q*, then* $\bar{q} < q_{1L}$.

2.2. $q_{2L} < \bar{q} < q_{2H}$.

2.3. $0 \leq \lambda \leq 1 - \gamma$.

Proposition 2 shows that the optimal period one regulation levels lie between the prior information regulation level and their full information counterparts. Thus, first period welfare is higher under the mechanism than under $\bar{q}$, regardless of firm type. In period two, the ex ante optimal level of regulation for both types is $\bar{q}$, but the low type receives $q_{2L} < \bar{q}$ and the high type receives $q_{2H} > \bar{q}$. These distortions provide the low cost firm with incentives to report low costs and accept stronger regulation ($q_{1L} > q_{1H}$) in period one. The low cost firm expects higher costs in period two, and thus values weaker regulation more in period two. Thus, the welfare cost of the period two distortions is smaller than the period one gain.

Figure 1 illustrates the graphical intuition when $\delta = 1$.[15] The expected period one welfare loss arising when the regulator has only prior information about firm costs is equal to the average of the areas in regions L_1, L_2, H_1, and H_2, where the area in $L_1 \cup L_2$ is weighted by γ and $H_1 \cup H_2$ is weighted by $1 - \gamma$. Indeed, suppose the regulator sets $\bar{q}$ in period one, equating marginal benefits with *expected* marginal cost. With probability γ, the firm realizes low marginal costs, and thus marginal benefits exceed marginal costs, creating a welfare

[15]Discounting means the low cost firm requires weaker regulation in period two. However, the regulator discounts the welfare loss of weaker regulation for the low cost type in period two more. So the qualitative properties of the mechanism do not depend on δ.

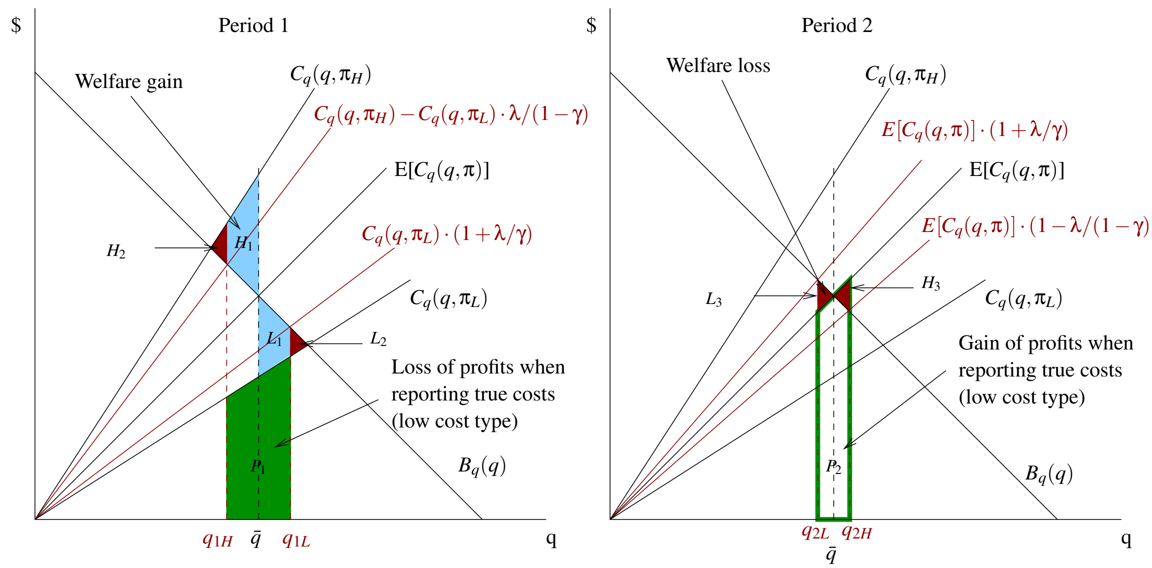

Figure 1: Intuition for the regulator's problem.

loss equal to the area of $L_1 \cup L_2$. With probability $1-\gamma$, marginal costs exceed marginal benefits, and welfare loss is the area of $H_1 \cup H_2$. Now suppose the regulator imposes q_{1L} or q_{1H} depending on the firm's report. Given truthful reporting, expected welfare loss falls to the weighted average of the areas of H_2 and L_2. However, a low cost firm now gets higher period one profits by claiming to be the high cost type. The gain in profits for a low cost firm claiming to be the high cost type in period one is the area of P_1. Thus the regulator must offer the firm reporting low costs weaker regulation in period two to offset the loss in profits in period one. Further, the marginal loss to the low cost firm from a marginal increase in q_{1L} is $C_q(q_{1L}, \pi_L)$.

Looking forward to period two, all firms expect marginal costs equal to $\mathrm{E}[C_q(q,\pi)]$. Thus, the low cost firm expects to have higher costs in period two and values lenient regulation in period two more than in period one (the opposite is true for the high cost firm, which is why the high cost firm prefers not to report low costs). Conversely, the regulator has no knowledge of firm costs in either period and is thus indifferent as to which period has the stronger regulation. Given $q_{2L} < q_{2H}$, the low cost firm expects to gain profits in period two equal to the area of P_2 by reporting truthfully. Thus the regulator must equalize the area of P_1 and P_2 to induce truthful reporting. A marginal decrease in q_{2L} raises expected profits in the second period by $\mathrm{E}[C_q(q_{2L},\pi)]$, whereas a marginal decrease in q_{1L} raises expected profits by only $C_q(q_{1L},\pi_L)$. Therefore, the firm reports truthfully even though the difference in period one regulation, $q_{1L} - q_{1H}$, is larger than the difference in period 2.

The regulator therefore achieves welfare gains in period one at a smaller welfare cost in period two (the weighted average of $H_3 \cup L_3$). The regulator continues to raise $q_{1L} - q_{1H}$ and $q_{2H} - q_{2L}$ until the weighted average of the areas of the H_2 and L_2 in period one equal the weighted average of H_3 and L_3 in period two.

Note that the incentive to renege is weaker here than the typical ratchet effect. If the regulator has the opportunity to renege in period two, the regulator will optimally set regulation to $\bar{q}$. In a model with a fixed type, the regulator who can renege prefers to set q_L^* if the firm reports low cost in period one. Because $q_{2L} < \bar{q} < q_L^*$, the gains in period two from reneging in Figure 1 is the area of L_3, rather than the much larger area of the triangle defined by the difference between marginal benefits and marginal costs between q_{2L} and q_L^*.

2.3 Quadratic Example

To further illustrate the mechanism, consider the following quadratic example:

$$B\left(q\right) = \theta q - \frac{1}{2}q^2, \tag{2.16}$$

$$C\left(q,\pi\right) = \pi q. \tag{2.17}$$

Solving equations (2.8)-(2.12) given the functions (2.16)-(2.17) results in:

$$\begin{aligned}
q_{1L} &= \bar{q} + \left(1-\gamma\right)\Delta\pi\frac{\delta\rho^2}{1+\delta\rho^2}, && (2.18)\\
q_{1H} &= \bar{q} - \gamma\Delta\pi\frac{\delta\rho^2}{1+\delta\rho^2}, && (2.19)\\
q_{2L} &= \bar{q} - \left(1-\gamma\right)\Delta\pi\frac{\rho}{1+\delta\rho^2}, && (2.20)\\
q_{2H} &= \bar{q} + \gamma\Delta\pi\frac{\rho}{1+\delta\rho^2}, && (2.21)\\
\lambda &= \gamma\left(1-\gamma\right)\frac{\Delta\pi}{\pi_L}\frac{1}{1+\delta\rho^2}, && (2.22)\\
\Delta\pi &\equiv \pi_H - \pi_L, \quad \rho \equiv \frac{\bar{\pi}}{\pi_L}, \quad \bar{q} = \theta - \bar{\pi}. && (2.23)
\end{aligned}$$

The degree to which the regulations differ from the no-information benchmark, $\bar{q}$, depends critically on the variability of π in two different ways. First, an increase in $\Delta\pi$ moves all four optimal regulations away from $\bar{q}$. As $\Delta\pi$ increases, implementing $\bar{q}$ results in large welfare losses, because q_L^* and q_H^* are far from $\bar{q}$. Therefore, q_{1L} and q_{1H} move away from $\bar{q}$, and thus

q_{2L} and q_{2H} also move away from $\bar{q}$ to maintain incentive compatibility. Second, for large values of ρ,[16] the low cost firm anticipates high costs in period two, and so the regulator need only offer slightly weaker regulation in period two to induce the low cost firm to report truthfully. Thus for large ρ implementing the mechanism is less costly. The effect of ρ on q_{2L} and q_{2H} is ambiguous. On one hand the regulator needs to weaken (strengthen) period two regulation less for the low (high) cost firm to satisfy the incentive constraint, which moves q_{2L} and q_{2H} towards $\bar{q}$. But because implementing the mechanism is less costly, the regulator widens the spread in period one, which tends to widen the spread in period two.

The variance of the cost shock increases the value of the information gained through the mechanism. With only prior information, expected welfare is:

$$\mathcal{W}(\bar{q}, \bar{q}) = \frac{1}{2}(1+\delta)\bar{q}^2. \tag{2.24}$$

Whereas, using (2.7), the expected welfare of the optimal mechanism is:

$$\mathcal{L} = \frac{1}{2}(1+\delta)\bar{q}^2 + \frac{1}{2}\gamma(1-\gamma)\left(\frac{\delta\rho^2}{1+\delta\rho^2}\right)\Delta\pi^2. \tag{2.25}$$

Thus the second term of (2.25) represents the regulator's gains from using the optimal mechanism. Welfare is proportional to the prior variance, $\gamma(1-\gamma)\Delta\pi^2$, since an increase in the prior variance increases the returns to acquiring information. Welfare is also increasing in ρ, as an increase in ρ makes implementing the mechanism less costly.

The example illustrates that the expected welfare gains achieved by the mechanism (over the prior mechanism) are not driven by increases or decreases in the average levels of regulation. Indeed, in this example, the expected level of regulation in each period under the mechanism equals the prior information regulation level: $\gamma q_{iL} + (1-\gamma) q_{iH} = \bar{q}$ for $i = 1{,}2$, yet welfare increases by offering the mechanism. The gains are driven by the ability to use information: relative to $\bar{q}$, the first period regulation is significantly better suited to the firm's true cost (higher for low cost firms, lower for high cost firms). Meanwhile, in the second period, the distortions from $\bar{q}$ needed to incentivize information provision are relatively small.

The online appendix (section 9.1) shows that many of the welfare results from the quadratic case extend to more general settings. In particular, we show that welfare under the mechanism increases with a mean preserving increase in the spread of π, because firms have stronger preferences for which period has strict regulation, reducing the cost of

[16]Note that $\rho = \gamma + (1-\gamma)R$ so the intuition for ρ and R are the same.

implementing the mechanism. In contrast, welfare under the prior information policy is constant with a mean preserving increase in the spread. Similarly, the online appendix shows that welfare under both the mechanism and the prior policy approaches first best as $\gamma \to 0,1$ and the welfare difference is maximized for some γ on the interior of (0,1). Therefore, more uncertainty increases the welfare gain from implementing the mechanism.

2.4 Timing Versus Banking

Other regulation systems also give firms some discretion to choose the strength of regulation over time, but are not designed to reveal firm costs and therefore generate lower welfare than our mechanism. Consider, for example, pollution permit "banking." In our framework, permit banking regulation requires that firms implement a minimum lifetime level of regulation $\hat{q}$, but gives firms the discretion to choose the level of regulation in each period. The type i firm's banking problem is therefore:

$$\max_{\hat{q}_{1i},\hat{q}_{2i}} \mathcal{V}\left(\hat{q}_{1i}, \hat{q}_{2i}, \pi_i\right), \quad \text{s.t. } \hat{q}_{1i} + \hat{q}_{2i} \geq \hat{q}, \; i = L,H. \tag{2.26}$$

The regulations arising under banking do not coincide with those of the optimal regulation mechanism characterized by (2.8)-(2.11). First, under banking, the firm may transfer regulation across time at an intertemporal price of one, which is not generally optimal.[17] Second, with banking the regulator does not anticipate the information revealed by the firm's choice in the policy design (to do so requires $\hat{q}$ and the intertemporal price to vary according to the firm's choice of $\hat{q}_{1i}$). Suppose, for example, that the cost function is strictly convex. Because any solution to firm i's banking problem is feasible in firm j's banking problem, it must be that i strictly prefers the solution to its banking problem over the solution to j's banking problem:

$$\mathcal{V}\left(\hat{q}_{1L}, \hat{q}_{2L}, \pi_L\right) > \mathcal{V}\left(\hat{q}_{1H}, \hat{q}_{2H}, \pi_L\right), \tag{2.27}$$

$$\mathcal{V}\left(\hat{q}_{1H}, \hat{q}_{2H}, \pi_H\right) > \mathcal{V}\left(\hat{q}_{1L}, \hat{q}_{2L}, \pi_H\right). \tag{2.28}$$

Hence, the regulations ($\hat{q}_{1L}$,$\hat{q}_{2L}$) and ($\hat{q}_{1L}$,$\hat{q}_{2L}$) are feasible in the general regulatory design problem (2.7). Yet, the optimal regulations arising under banking cannot be the solutions to (2.7), because (for example) the incentive constraint for the low cost type does not bind

[17] Too see this contrast the first order condition of (2.26) for $i = L$ with (2.8) and (2.10). See (Kling and Rubin 1997) for a formal argument and solution with one type of firm under certainty.

under banking, an essential feature of the timing mechanism.

The main drawback of banking is that the regulator does not make use of the information provided in the firm's choices. While both banking and the timing mechanism allow firms to choose a regulatory policy, the timing mechanism restricts the set of choices so that firms willingly select the regulatory policy that the regulator has targeted to their type. Hence, the different regulatory policies from which the firms can choose are designed to be selected by different types of firm, allowing the regulator to anticipate the information revealed by the firm's choice in the policy design. Because $\hat{q}$ and the intertemporal price are independent of firm's choice $\hat{q}_{1i}$, the regulator uses less information, resulting in lower welfare.

3 Marginal Cost of Public Funds

Suppose that the regulator may offer payments to the firm conditional on the firm's reported type, but such payments are costly for the regulator. Such a marginal cost of funds arises naturally if lump sum taxes are not possible, and the regulator/government obtains funds via distortionary taxation, see for example (Bovenberg and Goulder 1996).

With payments, the regulator may elicit the firm's type in period two. Therefore, the regulator requires the firm to give a cost report in each period, $\{\hat{\pi}_1, \hat{\pi}_2\}$, and may condition regulation in period two on both reports. Let $q_{2ij} \equiv q\left(\hat{\pi}_i, \hat{\pi}_j\right)$ denote the level of regulation in period two imposed on a firm that reported type i in period 1 and j in period 2. Because the regulator imposes the period one regulation before the firm learns the cost shock in period two, regulation in period one, $q_{1i} \equiv q_1(\hat{\pi}_i)$ for $i \in \{H,L\}$, depends only on the period one report. Similarly, let $t_{1i} \equiv t_1\left(\hat{\pi}_i\right)$ and $t_{2ij} \equiv t_2\left(\hat{\pi}_i, \hat{\pi}_j\right)$ represent the payments from the regulator to the firm in periods one and two, respectively, if the firm reports type i in period one and type j in period two.

Incentive compatibility in period two requires that truthful reporting maximizes profits in period two. Thus, the second period incentive constraints are:

$$-C\left(q_{2iL}, \pi_L\right) + t_{2iL} \geq -C\left(q_{2iH}, \pi_L\right) + t_{2iH}\ ,\ i = L,H, \tag{3.1}$$

$$-C\left(q_{2iH}, \pi_H\right) + t_{2iH} \geq -C\left(q_{2iL}, \pi_H\right) + t_{2iL}\ ,\ i = L,H. \tag{3.2}$$

Similarly, the first period incentive constraints are:

$$\begin{aligned} &\mathcal{V}\left(q_{1L}, q_2\left(\pi_L, \pi\right), \pi_L\right) + t_{1L} + \delta \mathrm{E}\left(t_2\left(\pi_L, \pi\right)\right) \geq \\ &\qquad \mathcal{V}\left(q_{1H}, q_2\left(\pi_H, \pi\right), \pi_L\right) + t_{1H} + \delta \mathrm{E}\left(t_2\left(\pi_H, \pi\right)\right), \end{aligned} \tag{3.3}$$

$$\mathcal{V}\left(q_{1H}, q_2\left(\pi_H, \pi\right), \pi_H\right) + t_{1H} + \delta \mathrm{E}\left(t_2\left(\pi_H, \pi\right)\right) \geq$$
$$\mathcal{V}\left(q_{1L}, q_2\left(\pi_L, \pi\right), \pi_H\right) + t_{1L} + \delta \mathrm{E}\left(t_2\left(\pi_L, \pi\right)\right). \tag{3.4}$$

The regulator can achieve unbounded welfare by imposing an arbitrarily large lump sum tax on the firm. Since we assume the marginal cost of funds is a constant ϕ, by imposing a lump sum tax $\bar{T}$, the regulator receives welfare gains equal to $\phi\bar{T}$. Since lump sum taxes are not distortionary, the regulator achieves unbounded welfare by increasing $\bar{T}$. Therefore, we impose a participation constraint that lifetime payments are positive, regardless of the firm's reports:[18]

$$t_{1i} + \delta t_{2ij} \geq 0 \text{ , } i,j = L, H. \tag{3.5}$$

With full commitment, the timing of payments is irrelevant: the regulator and firm are indifferent between a payment in period one and deferring the payment to period two, provided the amount of the deferred payment increases to compensate for discounting. Therefore, we combine both payments into a single second period payment:

$$t_{ij} = \frac{1}{\delta} t_{1i} + t_{2ij} \geq 0, \tag{3.6}$$

which represents the total payment associated with report history $(\hat{\pi}_i, \hat{\pi}_j)$.

Given a marginal cost of funds $\phi > 0$, the regulator's problem is:

$$\max_{q_{ij}, t_{ij}} \quad \gamma \cdot \left[w\left(q_{1L}, \pi_L\right) + \delta \mathrm{E}\left[w\left(q_2\left(\pi_L, \pi_2\right), \pi_2\right) - \phi t\left(\pi_L, \pi_2\right)\right]\right] +$$
$$(1-\gamma) \cdot \left[w\left(q_{1H}, \pi_H\right) + \delta \mathrm{E}\left[w\left(q_2\left(\pi_H, \pi_2\right), \pi_2\right) - \phi t\left(\pi_H, \pi_2\right)\right]\right], \tag{3.7}$$

subject to the incentive compatibility constraints (3.1), (3.2), (3.3), and (3.4), and the constraint on transfers (3.5).

Similar to section 2.1, we solve a relaxed problem where only some constraints bind. We then show that the solution satisfies the remaining constraints. The non-binding constraints are (3.2) $i = L,H$, and (3.4), the incentive constraints for the high cost firm in each period.

[18] If $t_{1i} + \delta t_{2ij} \geq -\bar{T}$ replaces (3.5), the regulator would be permitted to collect a maximum lump sum tax $\bar{T}$, as part of the mechanism. In this case, the regulator would simply reduce every payment that he issues under our mechanism by $\bar{T}$ (sometimes collecting payments from the firm instead of paying to the firm), leaving the mechanism otherwise unchanged. Therefore, our assumption rules out lump sum taxes on the firm. Montero (2008), footnote 18, also rules out lump sum taxes. This is sensible because a "lump sum" tax on firms would cause distortions not modeled in our analysis: households would reduce savings and increase consumption, and some low profit firms would exit the market.

The online appendix gives the first order conditions for problem (3.7), which show that the regulator can use payments to reduce the multipliers on the incentive constraints, but at a marginal cost of funds ϕ.

The next three propositions characterize the optimal regulation. In general, we show that a firm reporting low costs faces stricter regulation in the first period, $q_{1L} > q_{1H}$. The mechanism must therefore offer second period compensation—either in the form of payments or weaker regulation—to incentivize truthful reporting by the low cost firm. We determine to what extent the regulator uses payments or weaker future regulation (timing) to provide this incentive. Future regulation is weaker if $q_{2Li} \leq q_{2Hi}$, with strict inequality for at least one $i \in \{H,L\}$. That is, a firm reporting low costs in period one receives weaker regulation in period two with positive probability, and no worse in any event.[19] Further, if all payments are zero then the regulator is timing the regulation exclusively. Conversely, if payments are positive and $q_{2Li} = q_{2Hi}$ then the regulator is using payments to elicit information but is not timing the mechanism.

If the marginal cost of funds is zero, the regulator achieves first best using only payments.

PROPOSITION 3 *Let C be super modular in $[q, \pi]$ and $\phi = 0$. Then the solution to (3.7) has $q_{1L} = q_{2LL} = q_{2HL} = q_L^*$ and $q_{1H} = q_{2LH} = q_{2HH} = q_H^*$. That is, the regulator achieves first best using payments and does not use the timing mechanism.*

Proposition 3 says that with zero marginal cost of funds, the regulator assigns first best regulation q_i^* to a firm that reports current cost $\hat{\pi}_i$. Further, since $q_{2LH} = q_{2HH}$ and $q_{2LL} = q_{2HL}$, period two regulation does not depend on the period one report, that is, no timing is used. Instead, compensation takes the form of payments. If the marginal cost of funds is sufficiently high, however, the regulator uses the timing mechanism and does not use payments.

PROPOSITION 4 *Let C be super modular in $[q, \pi]$ and R be constant in q. Then if:*

$$\phi \geq \left(\frac{1-\gamma}{\gamma}\right) R, \tag{3.8}$$

then the solution to (2.7) solves problem (3.7), with $t_{ij} = 0$ for all i,j. That is, the regulator relies only on the timing mechanism and does not use payments.

As $\gamma \to 1$, the critical threshold (3.8) approaches zero. Intuitively, as $\gamma \to 1$, timing the regulation becomes less costly since the regulator can simply impose very high regulation in

[19] Clearly, this also implies that a firm reporting low costs receives weaker regulation in expectation.

period two if the firm reports it is the high cost type in period one. Low cost firms are then motivated to report truthfully, but because firms are unlikely to be the high type in period 2, the regulator is unlikely to bear the cost of overly stringent regulation of the high cost type in the second period.[20] In contrast, if the regulator uses payments, the firm reporting low costs in period one must receive a higher payment than the firm reporting high costs. For γ near one, almost all firms are the low type, so the regulator incurs the marginal cost of funds with high probability. Therefore, payments become more costly (and therefore less effective) as the regulator pays the low cost firm more often. The parameter R appears in equation (3.8) because a large spread between marginal costs of the high and low type implies a large welfare gain from moving to the first best regulation. The regulator is therefore more motivated to use payments even if ϕ is large. Thus, the timing mechanism tends to work well when payments do not and vice versa. The choice of payments versus timing may vary across industries depending on γ and R. However, the regulator uses the timing mechanism to some degree, as long as $\phi > 0$:

PROPOSITION 5 *Let C be super modular in $[q, \pi]$ and $\phi > 0$. In the optimal mechanism with transfers, a firm reporting low cost is regulated more strongly in period one $q_{1L} > q_{1H}$. Further, $q_{2Li} \leq q_{2Hi}$, with strict inequality for at least one $i \in \{H,L\}$. That is, it is optimal for the regulator to use the timing mechanism.*

Proposition 5 is interesting because, in practice, the absence of lump sum taxes implies $\phi > 0$. Therefore, some amount of regulatory timing is likely to be a robust feature of an optimal regulatory mechanism, even if the regulator can also use payments.

4 Correlated and Productivity Shocks, and General Marginal Cost Processes

The previous sections assume iid marginal cost shocks. In this section, we explicitly characterize the optimal regulatory policy for more general marginal cost processes, including correlated shocks. Independent shocks are appropriate if, for example, input prices fluctuate around a stationary value. However, for some applications (like productivity shocks), correlated costs are more appropriate (Stavins 1996, Heutel 2012). In addition, a more general marginal cost process yields additional insights into the nature of the optimal regulatory mechanism. In this section we assume that transfers are infeasible or are prohibitively costly.

[20] This intuition can also be seen in that the bound of shadow cost of the incentive constraint $\lambda < 1 - \gamma$ approaches zero as $\gamma \to 1$. Equations (2.20) and (2.21) also show that as $\gamma \to 1$, the mechanism only punishes the high type in period 2, but does not reward the low type.

We also assume (for ease of exposition) that $C(q,\pi) = \pi c(q)$, so that π is a multiplicative shock to the cost function.

The first period shock distribution remains $Pr(\pi_1 = \pi_L) = \gamma$. However, the expectation of the second period shock can now depend on the period one shock realization:

$$\mathrm{E}(\pi_2|\pi_1 = \pi_L) = \bar{\pi}_L \text{ and } \mathrm{E}(\pi_2|\pi_1 = \pi_H) = \bar{\pi}_H. \tag{4.9}$$

Thus, $\bar{\pi}_i$ represents the firm's expected cost in period two, given that period one's cost realization is π_i. Equation (4.9) allows for the most general dependent cost structures possible in a two period model. Because the regulator offers the mechanism in period one, from the perspective of both regulator and firm only the conditional expectations $\bar{\pi}_i = E[\pi_2|\pi_1]$ enter into the objective function and the constraints. Therefore, equation (4.9) supports any dependent distribution $\pi_2 \sim F(x|\,\pi_1)$. Common special cases include (1) permanent shocks, $\bar{\pi}_1 = \pi_1$,[21] (2) persistent shocks, $\bar{\pi}_L < \bar{\pi}_H$, where the low cost firm in period one is more likely to have low costs in period two, (3) multiplicative marginal cost shocks (let $\bar{\pi}_i = \beta_i \chi_i \pi_i + (1-\beta_i)\pi_i$, then with probability β the firm experiences a multiplicative marginal cost shock χ,[22] (4) productivity shocks (if costs are a fraction of GDP, πy, so that $C(q) = \hat{c}(q)\pi y$, then π is a productivity shock).[23] The ex ante expectation of the second period shock is now $\mathrm{E}[\pi_2] = \gamma\bar{\pi}_L + (1-\gamma)\bar{\pi}_H$. Optimal regulation given prior information may now be different in period two. Let $\bar{q}_1 = \bar{q}$ be the prior information optimal regulation in period one and let $\bar{q}_2$ be the prior information level of regulation in period two, which satisfies:

$$B_q(\bar{q}_2) = \mathrm{E}[\pi_2]\, c_q(\bar{q}_2), \tag{4.10}$$

The regulator's objective function is

$$\gamma \cdot \Big[w(q_{1L}, \pi_L) + \delta w(q_{2L}, \bar{\pi}_L) \Big] + (1-\gamma) \cdot \Big[w(q_{1H}, \pi_H) + \delta w(q_{2H}, \bar{\pi}_H) \Big]. \tag{4.11}$$

Note that the assumption on the cost function guarantees that $\mathrm{E}[w(q,\pi)] = w(q,\mathrm{E}[\pi])$. The expected change in firm profits from the regulation policy is:

$$\mathcal{V}(q_1, q_2, \pi_1, \bar{\pi}_1) = -\pi_1 c(q_1) - \delta\bar{\pi}_1 c(q_2). \tag{4.12}$$

[21] Given that only conditional expectations matter, the optimal regulation policy is identical for permanent shocks and shocks which are only expected not to change.

[22] If χ represents the discovery of a cost reducing innovation, the probability and size of the innovations naturally depend on the firms current technology/type.

[23] If productivity follow a discrete Markov process where γ^i, $i = 1{,}2$, is the probability of state i in period two conditional on state i in period one, then $\pi_0 = \pi_L$, $\gamma = \gamma^1$, $\bar{\pi}_L = \bar{\pi}$ and $\bar{\pi}_H = (1-\gamma^2)\,\pi_L + \gamma^2\pi_H$.

Thus incentive constraints are:

$$\mathcal{V}\left(q_{1L}, q_{2L}, \pi_L, \bar{\pi}_L\right) \geq \mathcal{V}\left(q_{1H}, q_{2H}, \pi_L, \bar{\pi}_L\right), \tag{4.13}$$

$$\mathcal{V}\left(q_{1H}, q_{2H}, \pi_H, \bar{\pi}_H\right) \geq \mathcal{V}\left(q_{1L}, q_{2L}, \pi_H, \bar{\pi}_H\right), \tag{4.14}$$

Proposition 6 shows that the mechanism is qualitatively unchanged, given two conditions,

$$\frac{\bar{\pi}_H}{\pi_H} \leq \frac{\bar{\pi}_L}{\pi_L}, \tag{4.15}$$

and $\bar{\pi}_H > \bar{\pi}_L$, which we discuss below.

PROPOSITION 6 *Suppose $C(q, \pi) = \pi c(q)$, $\bar{\pi}_H > \bar{\pi}_L$, and (4.15) holds. Then the optimal regulatory mechanism has the following properties:*

6.1. $\gamma(1-\gamma)\left(\frac{\bar{\pi}_H}{\bar{\pi}_L} - 1\right) = \lambda_L < \lambda < \lambda_R = \gamma(1-\gamma)\left(\frac{\pi_H}{\pi_L} - 1\right)$

6.2. $q_{1H} < \bar{q}_1 < q_{1L}$.

6.3. $q_{2L} < \bar{q}_2 < q_{2H}$.

If condition (4.15) holds with equality, then the solution is the prior information solution: $q_{1L} = q_{1H} = \bar{q}_1$ and $q_{2L} = q_{2H} = \bar{q}_2$.

The regulator prefers to move regulation closer to first best in the period where the expected difference in marginal costs is widest. The regulator uses the other period to satisfy incentive compatibility. If the inequality in (4.15) is strict, then marginal costs are wider in the period one, and the mechanism is qualitatively unchanged from the previous sections.[24] If (4.15) holds with equality, then the difference in marginal costs is identical in the periods one and two. In this case, moving regulation toward first best in one period requires an equal move away from first best in the other period. Thus here the regulator does not gain from varying the regulation over time, and uses the prior information level of regulation.[25] Finally, if (4.15) does not hold, then the timing of the mechanism reverses: regulation is closer to first best in period two, and period one satisfies incentive compatibility. Therefore, in all but the knife edge case of identical expected growth rates, the mechanism moves regulation closer to the first best in one period, and varies the strength of regulation in

[24] Condition (4.15) also implies the expected growth rate of π_H is less than that of π_L.

[25] Since costs are convex, the regulator prefers a constant level of regulation over time relative to regulation that varies by equal amounts in each period.

the other to provide incentives. We focus on $\bar{\pi}_H \geq \bar{\pi}_L$ to ensure that the first best mechanism always violates incentive compatibility for the low type.[26]

Let $\rho_L \equiv \bar{\pi}_L/\pi_L$. Resolving the quadratic model with general shocks gives:

$$q_{1L} = \bar{q} + (1-\gamma)\frac{\delta\rho_L^2}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.16}$$

$$q_{1H} = \bar{q} - \gamma\frac{\delta\rho_L^2}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.17}$$

$$q_{2L} = \bar{q}_2 - (1-\gamma)\frac{\rho_L}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.18}$$

$$q_{2H} = \bar{q}_2 + \gamma\frac{\rho_L}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.19}$$

$$\lambda = \frac{1}{1+\delta\rho_L^2}\lambda_R + \frac{\delta\rho_L^2}{1+\delta\rho_L^2}\lambda_L, \tag{4.20}$$

$$W = \frac{1}{2}\left(\bar{q}_1^2 + \delta\bar{q}_2^2\right) + \frac{1}{2}\gamma(1-\gamma)\frac{\delta\rho_L^2}{1+\delta\rho_L^2}\pi_L^2\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right)^2. \tag{4.21}$$

As derived in Proposition 6, the optimal mechanism reduces to the prior information case if the growth rates are identical, and reduces to the solution of section 2.3 if $\bar{\pi}_H = \bar{\pi}_L = \bar{\pi}$. Further, the solution moves closer to first best in the first period if and only if (4.15) holds strictly. With a bit of algebra, we can see that the welfare gains are convex in the expected difference of the growth rates, $\bar{\pi}_L/\pi_L - \bar{\pi}_H/\pi_H$. Compared with iid shocks, persistent shocks ($\bar{\pi}_H > \bar{\pi}_L$) reduce the welfare gains of the mechanism.

5 Multiple firms

In this section, we allow for multiple firms and show that the qualitative properties of the timing mechanism continue to hold. Suppose that n firms exist, the cost function is strictly convex, and cost shocks are iid Bernoulli (as in section 2.1) across firms. We assume the regulator collects all reports, and then assigns regulation to each firm in each period based on all reports. All firms that report low costs are identical to the regulator, and thus receive identical regulation. Let $0 \leq m \leq n$ be the number of firms reporting the low cost shock. If m firms report low costs in period one, a firm reporting cost shock $\hat{\pi}_j$ receives regulation $q_{ij,m} = q_i(\pi_j, m)$, $i = 1{,}2$, $j = L{,}H$. We assume that the benefit of regulating one firm is a

[26] If $\bar{\pi}_H < \bar{\pi}_L$ the preference for more strict regulation in period two for low cost firm can be so strong that the regulator achieves first best (a trivial example has $\delta = 1$, $\pi_L = \bar{\pi}_H$, and $\pi_H = \bar{\pi}_L$).

perfect substitute for regulating another:

$$B_i = B\left(mq_{iL,m} + (n-m)\, q_{iH,m}\right), \quad i = 1{,}2. \tag{5.1}$$

Let $Pr\,(m|i)$ denote the probability that m firms received the low cost shock, conditional on one firm receiving shock $i \in \{L,H\}$. We assume firms do not observe each other's cost shock.[27] The incentive constraints are therefore:

$$\sum_{m=1}^{n} Pr\,(m|L)\left[\mathcal{V}\left(q_{1L,m}, q_{2L,m}, \pi_L\right)\right] \geq \sum_{m=1}^{n} Pr\,(m|L)\left[\mathcal{V}\left(q_{1H,m-1}, q_{2H,m-1}, \pi_L\right)\right]. \tag{5.2}$$

$$\sum_{m=0}^{n-1} Pr\,(m|H)\left[\mathcal{V}\left(q_{1H,m}, q_{2H,m}, \pi_H\right)\right] \geq \sum_{m=0}^{n-1} Pr\,(m|H)\left[\mathcal{V}\left(q_{1L,m+1}, q_{2L,m+1}, \pi_H\right)\right]. \tag{5.3}$$

Constraints (5.2) and (5.3) require that truthful reporting maximize expected profits for both types of firms. The regulation the firm faces depends on the unknown reports of other firms. Therefore, each type has a single incentive constraint, requiring that the *expected* payoff of reporting honestly is no lower than the expected payoff of lying (anticipating truthful reporting by other firms). Mechanisms that satisfy (5.2) and (5.3) generate truthful revelation of information as a Bayesian-Nash equilibrium. The regulator's objective function is:

$$\sum_{m=0}^{n} Pr\,(m)\left(B\left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) - mC\left(q_{1L,m}, \pi_L\right) - (n-m)\, C\left(q_{1H,m}, \pi_H\right) + \right.$$
$$\left. \delta B\left(mq_{2L,m} + (n-m)\, q_{2H,m}\right) - m\delta \mathrm{E}\left[C\left(q_{2L,m}, \pi\right)\right] - (n-m)\,\delta \mathrm{E}\left[C\left(q_{2H,m}, \pi\right)\right]\right), \tag{5.4}$$

subject to (5.2) and (5.3). The first order conditions presented in the Appendix reduce to:

$$B_q\left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) = C_q\left(q_{1L,m}, \pi_L\right)\left(1 + \frac{\lambda}{\gamma}\right), \; m = 1, \ldots, n \tag{5.5}$$

$$B_q\left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) = C_q\left(q_{1H,m}, \pi_H\right)\left(1 - \frac{\lambda}{(1-\gamma)}\frac{C_q\left(q_{1H,m}, \pi_L\right)}{C_q\left(q_{1H,m}, \pi_H\right)}\right), \; m = 0, \ldots, n-1 \tag{5.6}$$

$$B_q\left(mq_{2L,m} + (n-m)\, q_{2H,m}\right) = \mathrm{E}\left[C_q\left(q_{2L,m}, \pi\right)\right]\left(1 + \frac{\lambda}{\gamma}\right), \; m = 1, \ldots, n \tag{5.7}$$

[27] Since shocks are iid across firms, in period two each firm has identical knowledge about the other firm's shocks as the regulator. Therefore, the regulator cannot use a period two report to learn the second period shocks by punishing firms that report shocks which are different than other firm's reports.

$$B_q\left(mq_{2L,m}+(n-m)\,q_{2H,m}\right)=\mathrm{E}\left[C_q\left(q_{2H,m},\pi\right)\right]\left(1-\frac{\lambda}{(1-\gamma)}\right),\ m=0,\ldots,n-1. \tag{5.8}$$

The first order conditions revert back to those of section 2.1 for $n = 1$. Indeed, the results change only in that the marginal benefits of regulation are lower with more firms since costs increase linearly with the number of firms but benefits are concave.[28] Equations (5.5)-(5.6), and (5.7)-(5.8) imply that the equi-marginal principle is violated in both periods. The regulator cannot equalize marginal costs across types without violating the incentive constraint.

Let the optimal regulation with only prior information for n firms, $\bar{q}_n$ satisfy:

$$B_q\left(n\bar{q}_n\right)=\mathrm{E}\left[C_q\left(\bar{q}_n,\pi\right)\right]. \tag{5.9}$$

The first best policies, $q^*_{L,m}$ and $q^*_{H,m}$, satisfy:

$$B_q\left(mq^*_{L,m}+(n-m)\,q^*_{H,m}\right)=C_q\left(q^*_{L,m},\pi_L\right)=C_q\left(q^*_{H,m},\pi_H\right). \tag{5.10}$$

Proposition 7 shows that optimal mechanism with n firms has similar properties to the case of a single firm.

PROPOSITION 7 *Let C be super modular and R be constant in q. Then the optimal regulatory mechanism with n firms has the following properties:*

7.1. $q_{1H,m} < \bar{q}_n < q_{1L,m}$.

7.2. $q_{2L,m} < \bar{q}_n < q_{2H,m}$.

7.3. $q_{1H,m}$ *and* $q_{1L,m}$ *are increasing functions of m.*

6 Further Discussion and Practical Examples

One practical implementation of the timing mechanism combines a default regulation that becomes stronger over time (as is the case for most regulations), with a program whereby (low cost) firms exceeding the regulatory standard in the current period are granted waivers or credits for use in the future.

[28] For the limiting case, normalize the size of each firm to $1/n$, then as $n \to \infty$, total regulation approaches $\gamma q_{jL} + (1-\gamma)\,q_{jH},\ j = 1{,}2$. This case differs from section 2.1 only in that here the regulator faces no aggregate uncertainty.

Such waiver and credit programs are common. For example, a provision of the corporate average fuel economy (CAFE) standards gives credits to companies that exceed the fuel economy standard in the current period, which permit the companies to be below the standard in the future. Another example is a provision of the 1990 Clean Air Act, which granted "bonus" pollution permits (which could be redeemed in future periods) to utilities that chose to initially install scrubbers. Rewarding firms that take costly efforts to reduce current emissions with future pollution permits is a natural way for regulators to weaken future regulation for the low cost type. These programs are more closely to related to the timing mechanism than to banking, because the firms are assigned permits, credits, or waivers as a reward for a high level of initial compliance.

Still, regulators may wish to offer rewards for early compliance for other reasons: to compensate firms for the risk of adopting compliance technologies that may not become standard, for example. Verifying that regulators use a menu of regulations to discern firm types and tailor regulation to each type requires detailed information about the rationale of the regulator. Nichols (1997) provides an insider's account of the EPA decision to lower the lead standard in gasoline from 1.1 grams per leaded gallon (gplg) to 0.1.

In August 1984, cost benefit analysis performed by the EPA indicated that refiners could meet an intermediate target of 0.5 gplg in July, 1985 and the 0.1 standard by January 1, 1986. We will refer to the second half of 1985 as period one and 1986 as period two. In public commentary, some refiners expressed concern that the policy was infeasible in such a short period of time. Simulations by the EPA gave some credibility to the refiners case, revealing a small, though significant, probability that the refiners were correct. The EPA also believed that the compliance cost of the regulation varied across firms, with smaller firms thought to have more difficulty. The EPA was worried that if compliance costs were too large, imposing the policy could lead to supply disruptions. Still, the EPA believed most firms could meet the 0.1 standard immediately.

The EPA implemented a regulation reminiscent of the timing mechanism. The EPA announced the policy, as originally proposed. However, the regulation granted extra permits for use in period two for firms that exceeded the standard in period one. We argue that the EPA in essence used the information in the firm's choice to tailor the regulation. The EPA was essentially able to strengthen the regulation period one (below 0.5) for most firms, because only refiners with a low cost of compliance would accept this option. Low cost refiners were rewarded with waivers, leading to weaker regulation for them in the future, which insured incentive compatibility.[29] We expect γ was relatively close to one, as the EPA believed most firms could comply immediately.[30] Indeed, most firms chose to comply early. High compliance costs refiners, by declining the option, faced weaker regulation prior to January 1, and stronger regulation after.

The estimated additional savings from the flexible policy was between \$173-\$226 million. The following quote helps to understand the motivations of some of the regulators: “These estimated net savings arose from the fact that it was relatively easy for at least some [low cost] refiners to reduce lead below 1.1 gplg in early 1985; the extra cost of those early reductions was more than offset by later savings from being allowed to use more than 0.1 gplg” (Nichols 1997). In other words, the flexibility allowed the EPA to tailor an aggressive emissions reduction for low cost firms knowing that only low cost firms would choose this option.

7 Conclusions

In an environment where marginal compliance costs are subject to random shocks, we show that the regulator can induce firms to reveal their costs shocks and increase welfare using a regulatory mechanism that varies the strength of regulation over time. In particular, the optimal mechanism is to offer the firm two regulation choices. The first starts out weak and becomes stronger, while the second does the opposite. Firms currently facing high cost shocks know that their costs are likely to decline over time, and chose regulation which is initially weak. Firms with low cost shocks choose the opposite. In this way, firms reveal their cost shocks to the regulator. Welfare improves both because firms choose strict regulation only when marginal costs are low, *and* because doing so reveals information to the regulator, resulting in regulation tailored to the firm’s type.

Our mechanism is robust to a number of extensions, including general cost shock processes and multiple firms. If the regulator may make payments to the firm, then for any positive marginal cost of funds, the optimal mechanism varies regulation over time to some degree. Further, if the marginal cost of funds is high enough, the optimal mechanism does not use payments, but instead relies exclusively on the timing of regulation. In general, varying regulation over time is more effective than payments when the probability of receiving a low

[29] The EPA also noted that the waivers allowed refiners “extra flexibility to deal with unexpected problems (e.g. equipment breakdowns or a sudden increase in summertime demand)”, indicating future compliance costs had a random component and may increase.

[30] Note from Proposition 4 that for γ close to one, regulation timing is relatively more effective than payments.

cost shock is high. If firms can make a dynamic capital investment that complies with the regulation (e.g. install scrubbers) at lower cost than a static method (e.g. switch from high to lower sulfur coal), then we can show (see the online appendix) that firms undertake the socially optimal level of investment. Further, our mechanism is unchanged except that firms are now either above and then below a baseline increasing trend in regulatory stringency, or the reverse, depending on the cost shock.

Our results have several caveats. First, our mechanism essentially trades off current for future distortions, and thus cannot achieve first best. Using payments results in the first best allocation, but only if no marginal cost of funds exists (that is, that lump sum taxes are available).

Second, our mechanism relies on commitment. The regulator has an incentive to renege on promised regulation in period two. Nonetheless, the incentive to renege here is more mild than in models with fixed firm types, since the regulator only prefers the prior-information level of regulation (that is, no "ratchet effect" exists). For the example of the 1990 amendment to the Clean Air Act, the EPA offered firms bonus permits in the future for installing scrubbers. The EPA kept the commitment, despite clauses in the law stating that the EPA could revoke any permits at any time.

Third, the mechanism breaks down if shocks have identical expected growth rates across time. The mechanism relies on differentiating the regulations in the period where marginal costs are most different, and choosing regulations to satisfy incentive compatibility in the other period. A deeper issue arises, however, here and in the literature with constant, unknown marginal costs. In the long run only firms with the lowest cost technologies survive in a competitive market. Shocks which are constant over time are not consistent with a long run competitive equilibrium.[31] In contrast, differential expected growth rates have natural interpretations.

The most interesting extension is to make the number of periods infinite. In this case, we expect the regulator would start with some promised level of total future profits generated from past reports, and then offer a promise of future profits that are either higher or lower depending on the firm's report. Still, an infinite horizon with commitment may be less realistic than our two period model, since regulators serve for a finite time.

Despite these caveats, a robust result is that timing improves regulatory efficiency. In recent years the public's appetite for increased regulation has grown. Regulations are becoming increasingly complex, with compliance costs that are increasingly difficult to forecast, for

[31] In addition, it is not clear why a regulator is uncertain about constant marginal costs, since the regulator could simply invert the cost function after one observation and learn the unobserved cost parameter.

both firms and regulators. Therefore, it is clear that more efficient regulation is an important policy goal, and it will only become more so in the future.

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8 Appendix: Proof of Main Theorems

8.1 Proof of Proposition 1

Substituting equation (2.12) into (2.6) implies (2.6) holds if and only if:

$$C\left(q_{1H}, \pi_L\right) - C\left(q_{1L}, \pi_L\right) \geq C\left(q_{1H}, \pi_H\right) - C\left(q_{1L}, \pi_H\right), \tag{8.1}$$

Next, let $a = [q_{1H}, \pi_H]$ and $b = [q_{1L}, \pi_L]$, then since $q_{1L} > q_{1H}$ (see Proposition 2) and $\pi_H > \pi_L$:

$$C\left(a \wedge b\right) - C\left(b\right) \geq C\left(a\right) - C\left(a \vee b\right). \tag{8.2}$$

Here $\wedge$ and $\vee$ the component-wise minimum and maximum, respectively. For example, $a \wedge b = \left[\min\left\{q_{1H}, q_{1L}\right\}, \min\left\{\pi_H, \pi_L\right\}\right]$. Condition (8.2) holds if and only if C is supermodular.

8.2 Proof of Proposition 2

2.2. For the second period regulations, since B is concave and C is convex, $q_{2L} < \bar{q}$ if and only if:

$$\frac{B_q(q_{2L})}{\mathrm{E}[C_q(q_{2L},\pi)]} > \frac{B_q(\bar{q})}{\mathrm{E}[C_q(\bar{q},\pi)]} = 1. \tag{8.3}$$

Here the equality follows from the definition of $\bar{q}$, equation (2.15). Using equation (2.10) to eliminate B_q, the above inequality reduces to:

$$\mathrm{E}[C_q(q_{2L},\pi)]\left(1+\frac{\lambda}{\gamma}\right) > \mathrm{E}[C_q(q_{2L},\pi)], \tag{8.4}$$

which holds since $\lambda > 0$. Similarly, $q_{2H} > \bar{q}$ if and only if:

$$\frac{B_q(q_{2H})}{\mathrm{E}[C_q(q_{2H},\pi)]} < \frac{B_q(\bar{q})}{\mathrm{E}[C_q(\bar{q},\pi)]} = 1. \tag{8.5}$$

Equation (2.11) implies the above inequality holds since $\lambda > 0$.

2.1. Given B is concave and C is convex, $q_{1L} < q_L^*$ if and only if:

$$\frac{B_q(q_{1L})}{C_q(q_{1L},\pi_L)} > \frac{B_q(q_L^*)}{C_q(q_L^*,\pi_L)} = 1. \tag{8.6}$$

Here the equality follows from the definition of q_L^* (2.14). Using equation (2.8), the above inequality holds since $\lambda > 0$. Similarly, $q_{1H} > q_H^*$ if and only if:

$$\frac{B_q(q_{1H})}{C_q(q_{1H},\pi_H)} < \frac{B_q(q_H^*)}{C_q(q_H^*,\pi_H)} = 1. \tag{8.7}$$

Equation (2.9) implies the above inequality holds since $\lambda > 0$.

For $q_{1H} < \bar{q}$, we first show that $q_{1L} > q_{1H}$. Suppose not. Suppose $q_{1H} \geq q_{1L}$, then since $q_{2H} > q_{2L}$, the incentive constraint (2.5) does not bind, which implies $\lambda = 0$. But from the first order conditions, $\lambda = 0$ implies $q_{2L} = q_L^*$ which contradicts $q_{2L} < q_{2H}$, for example. Thus $q_{1H} < q_{1L}$.

To show $q_{1H} < \bar{q}$, we suppose not and then construct a regulation set which is feasible and provides higher welfare, thus contradicting that $q_{1H} \geq \bar{q}$ is an optimum. Suppose $\{q_{1L}, q_{1H}, q_{2L}, q_{2H}\}$ is optimal with $q_{1H} \geq \bar{q}$. Consider an alternative policy $\{q_{1L} - \epsilon, q_{1H} - \epsilon, q_{2L}, q_{2H}\}$, with $\epsilon > 0$ sufficiently small (i.e. small enough to make a first order approximation of B and C accurate enough so as to not change the signs of any of the inequalities). The alternative policy is feasible if and only if:

$$C\left(q_{1H} - \epsilon, \pi_L\right) - C\left(q_{1L} - \epsilon, \pi_L\right) \geq X = C\left(q_{1H}, \pi_L\right) - C\left(q_{1L}, \pi_L\right). \tag{8.8}$$

Approximating $C\left(q_{1L} - \epsilon, \pi_L\right)$ around q_{1L} and $C\left(q_{1H} - \epsilon, \pi_L\right)$ around q_{1H}, implies for ϵ small the inequality reduces to:

$$C_q\left(q_{1H}, \pi_L\right)(-\epsilon) - C_q\left(q_{1L}, \pi_L\right)(-\epsilon) \geq 0, \tag{8.9}$$

Equation (8.9) holds since $q_{1L} > q_{1H}$. Thus the alternative policy is feasible.

Since the period two policies are identical, the alternative policy generates higher welfare if and only if:

$$\begin{aligned} &\gamma\left[B\left(q_{1L} - \epsilon\right) - C\left(q_{1L} - \epsilon, \pi_L\right)\right] + (1-\gamma)\left[B\left(q_{1H} - \epsilon\right) - C\left(q_{1H} - \epsilon, \pi_H\right)\right] > \\ &\quad \gamma\left[B\left(q_{1L}\right) - C\left(q_{1L}, \pi_L\right)\right] + (1-\gamma)\left[B\left(q_{1H}\right) - C\left(q_{1H}, \pi_H\right)\right]. \end{aligned} \tag{8.10}$$

Performing first order approximations reduces the inequality to:

$$\gamma\left[B_q\left(q_{1L}\right) - C_q\left(q_{1L}, \pi_L\right)\right] + (1-\gamma)\left[B_q\left(q_{1H}\right) - C_q\left(q_{1H}, \pi_H\right)\right] < 0. \tag{8.11}$$

Next, since we assumed $q_{1H} \geq \bar{q}$, and B is concave and C is convex:

$$B_q\left(q_{1H}\right) - C_q\left(q_{1H}, \pi_H\right) \leq B_q\left(\bar{q}\right) - C_q\left(\bar{q}, \pi_H\right). \tag{8.12}$$

Further, since $q_{1L} > q_{1H} \geq \bar{q}$,

$$B_q\left(q_{1L}\right) - C_q\left(q_{1L}, \pi_L\right) < B_q\left(\bar{q}\right) - C_q\left(\bar{q}, \pi_L\right). \tag{8.13}$$

We multiply (8.12) by $1-\gamma$ and (8.13) by γ, and sum the two resulting equations. Comparing the result with (8.11), it is sufficient to show:

$$\gamma\left[B_q\left(\bar{q}\right)-C_q\left(\bar{q},\pi_L\right)\right]+\left(1-\gamma\right)\cdot\left[B_q\left(\bar{q}\right)-C_q\left(\bar{q},\pi_H\right)\right]\leq 0, \tag{8.14}$$

The left hand side of (8.14) is zero using the definition of $\bar{q}$. Thus we have a contradiction that $\{q_{1L}, q_{1H}, q_{2L}, q_{2H}\}$ is optimal as the alternative policy is feasible and generates higher welfare.

To show $q_{1L} > \bar{q}$, we use the previous results. First, since $q_{1H} < \bar{q}$, we have:

$$\frac{B_q\left(q_{1H}\right)}{\mathrm{E}\left[C_q\left(q_{1H},\pi\right)\right]}>\frac{B_q\left(\bar{q}\right)}{\mathrm{E}\left[C_q\left(\bar{q},\pi\right)\right]}=1. \tag{8.15}$$

Using equation (2.9):

$$\left(1-\frac{\lambda}{1-\gamma}\frac{1}{R}\right)C_q\left(q_{1H},\pi_H\right)>\gamma C_q\left(q_{1H},\pi_L\right)+\left(1-\gamma\right)C_q\left(q_{1H},\pi_H\right), \tag{8.16}$$

$$\lambda<\gamma\left(1-\gamma\right)\left(R-1\right). \tag{8.17}$$

Thus λ cannot be too big. Finally, $q_{1L} > \bar{q}$ if and only if:

$$\frac{B_q\left(q_{1L}\right)}{\mathrm{E}\left[C_q\left(q_{1L},\pi\right)\right]}<\frac{B_q\left(\bar{q}\right)}{\mathrm{E}\left[C_q\left(\bar{q},\pi\right)\right]}=1. \tag{8.18}$$

Using equation (2.8), a similar argument implies (8.18) holds if and only if (8.17) holds. Since we have shown (8.18) holds, $q_{1L} > \bar{q}$.

2.3. For $\lambda < 1-\gamma$, suppose not, suppose $\{q_{1L}, q_{1H}, q_{2L}, q_{2H}\}$ is an optimum with $\lambda \geq (1-\gamma)$. Then from (2.11), we have a corner solution of $q_{2H} = 0$, since for all $q_{2H} > 0$,

$$B_q\left(q_{2H}\right)>0\geq\mathrm{E}\left[C_q\left(q_{2H},\pi\right)\right]\left(1-\frac{\lambda}{1-\gamma}\right), \tag{8.19}$$

Next, the incentive constraint (2.5) with $q_{2H} = 0$ implies:

$$C\left(q_{1H}, \pi_L\right) - C\left(q_{1L}, \pi_L\right) \geq \delta \mathrm{E}\left[C\left(q_{2L}, \pi\right)\right]. \tag{8.20}$$

Thus $q_{1H} \geq q_{1L}$ is required for incentive compatibility, which is a contradiction since we have already shown $q_{1L} > q_{1H}$. Thus $\lambda < 1 - \gamma$.

9 Proofs of Remaining Propositions and Remaining Calculations

Please see the online appendix, `http://WebAddressOfOnlineAppendix`.