Evolving Influence: Mitigating Extreme Conflicts of Interest in Advisory Relationships

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Abstract

An advocate for a special interest provides advice to a planner, who subsequently makes a sequence of decisions. The advocate is interested only in advancing his cause and will distort his advice to manipulate the planner’s choices. Each time she acts the planner observes the result, providing a signal that corroborates or contradicts the advocate’s recommendation. Without commitment, no influential communication takes place. With commitment, the planner can exploit the information that is revealed over time to mitigate the advocate’s incentive to lie. We derive the optimal mechanism for eliciting advice, characterizing the evolution of the advocate’s influence. We also consider costly information acquisition, the use of transfers, and a noisy private signal.

JEL codes: D80, D82, D83, D86

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1 INTRODUCTION

A policymaker allocates funding over time to an initiative designed to help the homeless. Although the initiative’s exact consequences in each period are uncertain, the policymaker seeks the guidance of an advisor who has private information about the program’s likely future benefits. Because the advisor’s assessment of the program is unverifiable, he can distort his recommendation without worrying that self-serving behavior will be instantly exposed. To complicate matters, the advisor is a passionate advocate for the homeless—he always prefers that the initiative receives more funding. Consequently, the advocate’s advice is only meant to advance his cause, and it has no influence over the policymaker’s decisions. Even though the advocate has valuable information, his ability to communicate it is undermined by his extreme bias.

Over time, however, the policymaker can observe the initiative’s performance, which provides an exogenous signal of its merit. If she has the ability to commit to future actions (that depend on the advocate’s recommendation and on the program’s performance), then the policymaker can exploit the information revealed over the course of the relationship to counteract the advocate’s incentive to exaggerate the program’s likely benefit at the outset. By making such a commitment, the policymaker determines the advocate’s influence over the actions that she will choose. The advocate’s influence over actions evolves over the course of the relationship, as the initiative’s historical performance confirms or contradicts the advocate’s initial claims. Our analysis incorporates commitment power and the gradual arrival of exogenous information into a model of communication, characterizing the optimal evolution of influence in advisory relationships plagued by severe conflicts of interest.

In our model, a planner makes a sequence of decisions. In each period, she would like to adjust her action to match an unknown state of the world. The state in each period is stochastic, and it is publicly revealed after the planner has selected an action. The planner is uncertain about the distribution of the state. An advisor knows the distribution from which the states are drawn, either \( \phi_H \) or \( \phi_L \), but, like the planner, is unable to observe the state until after the planner selects an action. The advisor is known to be an advocate; he always prefers that the planner increases her action, independent of the true state. If it is common knowledge that the distribution is \( \phi_H \), then the planner’s will select higher actions. Therefore, the advocate would like to manipulate the planner into thinking that the distribution is \( \phi_H \), even if it is truly \( \phi_L \).

The advocate’s strong conflict of interest impedes communication: without a commitment by the planner, no influential communication takes place in equilibrium. If the advocate’s recommendation suggests that the high distribution is more likely, it is sequentially rational for the planner to choose a larger action in each period regardless of the history of states. Therefore, if the advocate expects the planner’s belief to respond (at all) to his recommendation then he will always issue the recommendation that makes the high distribution seem more likely. However, if the advocate is always expected to issue the same recommendation (regardless of the true distribution) then the planner’s equilibrium beliefs will not be affected by the recommendation—communication has no influence. This result is not surprising given the parties’ disparate preferences, but it allows us to
highlight the critical role of commitment power in supporting credible communication.

We begin by analyzing commitment power in a benchmark model with a single decision and a public signal. This benchmark allows us to explore how the combination of commitment and an exogenous signal facilitate influential communication, abstracting away from the dynamics of the multi-period model. Under the optimal mechanism, when the advocate reports \( L \), the planner commits to an action that is independent of the signal realization and exceeds her optimal action under the low distribution. We interpret this as a compromise the planner makes in return for the advocate’s report of unfavorable news. When the advocate reports \( H \) (i.e. supporting the distribution he finds favorable) the planner commits to an action that depends on the signal realization. The action is always smaller than the planner’s optimal action under the high distribution, but the gap depends on the realization of the public signal. If the realization is consistent (in a Bayesian sense) with the high distribution, then the action is close to the planner’s optimal action under \( \phi_H \), but if the realization is inconsistent, then it is far below the optimal action under \( \phi_H \). Thus, the optimal mechanism rewards the advocate when he reports personally unfavorable information or personally favorable information that is supported by the realization of the public signal. If the advocate reports personally favorable information that is inconsistent with the signal realization, the advocate is punished by the mechanism. The planner’s ability to distinguish the two distributions by observing the public signal (measured by a divergence statistic) determines the magnitude of the second best distortions and the welfare gains from commitment.

We then consider commitment in the multi-period model. Within a period, the structure of the optimal evolving influence mechanism is identical to the single period benchmark. When the advocate reports against his bias, the planner commits to a compromise action that is independent of the history of realizations; in the multi-period model, the compromise action is also independent of time. When the advocate reports personally favorable information, the planner commits to “cross-check” the recommendation with the public signal, rewarding him when the history of observed states is consistent with the recommendation and punishing him when the history is inconsistent with the recommendation. In the multi-period model, this generates a plan of action with dynamics that are qualitatively different from the ones without commitment. In addition, in the evolving influence mechanism the planner exploits the advocate’s intertemporal preference for actions by “back-loading” the provision of incentives—the history of realizations in late periods is more informative than in early periods, allowing the planner to more easily distinguish the two distributions. It is therefore optimal for the later periods to play a larger role in achieving incentive compatibility. Consequently, an increase in the time horizon reduces the distortions from the first best in all periods, increasing the planner’s payoff. Provided the planner can commit, the advocate’s “long view” for supporting his cause increases his value as an advisor.

Our analysis also provides some insights into the prevalence of advocacy by describing situations in which advocates are desirable sources of unverifiable information. Most analyses of advocacy presume that planners rely on advocates for advice because they must be informed about issues that they care so much about. While this is often true, it doesn’t explain why planners don’t prefer
to consult other informed, but less biased, sources for advice. This explanation is also weakened if the advocate is initially uninformed, and must exert effort to acquire the information sought by the planner. It may seem that if the planner could pay a sufficiently small cost to consult an impartial advisor (or learn on her own), the planner would prefer to do so. Paradoxically, we show that if she can offer the optimal evolving influence mechanism, then for any positive cost of consulting an impartial advisor (or learning on her own), the planner may prefer an advocate instead. If the planner seeks out an impartial advisor, she bears a (potentially small) cost of information acquisition but makes the best possible decisions thereafter. If she consults an advocate, however, she does not bear the cost of information acquisition directly but (optimally) distorts her decisions to provide incentives for information acquisition and truthful communication. As the planner’s ability to distinguish the distributions from the history grows, the distortions in the optimal actions necessary to induce learning vanish, as does the payoff cost of these distortions. Ironically our theory shows that it is precisely the advocate’s extreme preference that sometimes makes him an attractive source of information.

The plan for the rest of the paper is as follows. We discuss the literature in Section 2. We present our model in Section 3. In Section 4 we analyze commitment in the static benchmark and in the multi-period model. In Section 5 we extend our analysis to a setting in which advocates must exert effort to learn. In Section 6 we consider a model in which transfers are permitted. In Section 7 we analyze the case in which the advocate’s private signal is partially revealing. Section 8 concludes with a summary of results and directions for further research. Proofs and calculations are in the appendix.

2 LITERATURE

When dealing with an advocate, it would be natural for the planner to demand that he supports his recommendation with verifiable evidence (Grossman 1981, Milgrom and Roberts 1986, Dewatripont and Tirole 1999, Bull and Watson 2004, 2007, Che and Kartik 2009, Cotton 2011, 2012, Dziuda 2011)). In our analysis the advocate does not possess verifiable evidence, nor can he conduct an experiment to generate it. The advocate’s initial information is private and unverifiable. One natural interpretation is to think of this information as an accurate subjective impression on the part of the advocate, based on the advocate’s expertise in the area rather than an exhaustive process of gathering and analyzing evidence. While the advocate’s initial information is unverifiable, in our analysis verifiable evidence of the true distribution arrives over time, in the form of realizations of the state. Our analysis focuses on the way that this verifiable history of state observations changes the advocate’s incentives to report the unverifiable information at the beginning of the interaction.\(^1\)

\(^1\)In an emerging literature on the “persuasion of a Bayesian” (Kamenica and Gentzkow 2011, Gentzkow and Kamenica 2012, Boleslavsky and Cotton 2011, Alonso and Cámara 2016, Kolotilin et al. 2015)), the advisor searches for evidence by choosing a test or experiment, whose realization is informative about the underlying state. Both the advisor’s chosen test and its realization are observed by the decision maker. This literature focuses on characterizing the structure of this “endogenous” evidence, particularly on its value for the decision maker.

\(^2\)In its focus on the role of “data” (the history of state observations) as an incentive instrument, our paper is connected with a literature on the testing of experts, see for example Olszewski and Sandroni (2008, 2009, 2011),
If demanding evidence is infeasible, the planner could simply ask the advocate for a recommendation; she would then evaluate both the advocate’s motives and his message when subsequently choosing a course of action. This is the paradigm of strategic communication or cheap talk, with seminal contributions by Crawford and Sobel (1982) and Green and Stokey (2007). The central idea in this literature is that the amount of information conveyed in equilibrium is inversely related to the conflict of interest between the parties. For example, in Crawford and Sobel (1982) both decision maker and advisor have quadratic preferences over actions. The decision maker’s ideal action is equal to the true state, but the advisor’s ideal action is shifted relative to the decision maker’s by a parameter, representing the intensity of the conflict of interest. The smaller this parameter, the more information is conveyed in equilibrium. In our framework the advocate’s preferences are increasing in the policymaker’s action. His “ideal point” is infinite and the conflict of interest is (in this sense) unbounded, impeding influential communication.3

If demanding evidence is infeasible and simply asking for advice is ineffective, the planner could try to let the advocate decide on his own, subject to some rules that she commits to initially. This class of models is studied in the delegation literature, pioneered by Holmstrom (1977, 1984), with contributions by Dessein (2002), who compares delegation and strategic communication, Alonso and Matouschek (2008) who characterize optimal delegation for arbitrary distributions and preference structures, and Szalay (2005) who incorporates information acquisition into a delegation model. While the extreme preference conflict inherent in our advisory relationship undermines the effectiveness of delegation in a static setting, our evolving influence mechanism could be interpreted as a multi-period delegation mechanism. The mechanism specifies the maximum action that the advocate can select in each period as a function of his initial message and the history of previous state observations. The optimal dynamic influence mechanism shares elements of Strausz (2006) theory of interim information in employment contracts and Cooper and Hayes (1987) description of price discrimination in long term insurance contracts.

A handful of papers consider communication games with extreme conflicts of interest—the sender’s payoff is monotonic in the receiver’s action, regardless of the true state. Chakraborty and Harbaugh (2010) analyze a static cheap talk model with multiple dimensions, and show that because of tradeoffs across dimensions, influential communication can take place despite the advi-

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3Sobel (1985) and Morris (2001) construct equilibria with informative communication in repeated settings without commitment, focusing on a biased advisor’s incentive to establish a reputation for credibility, which he may profitably exploit. Also relevant is a literature on strategic communication that focuses on the timing of the communication protocol. Krishna and Morgan (2004) show that introducing an initial round of simultaneous communication, followed by a message by the expert leads to improved information transmission in the model of Crawford and Sobel (1982). Aumann and Hart (2003) provide a general characterization of the set of payoffs that can be achieved by incorporating cheap talk communication in a two-by-two game where one side is better informed, allowing an infinite number of communication rounds. In our model, the planner can take a continuum of possible actions; hence, these results do not apply directly. Furthermore, the extreme conflict of interest at the heart of our paper can also undermine communication in their setting (see example 2.2 of Aumann and Hart (2003)). Golosov et al. (2014) construct fully-revealing equilibria in a repeated version of the Crawford and Sobel (1982) model with a persistent state. In this model, a decision is taken in each period, but the decision maker does not observe any information beyond the advisor’s message.
sor’s extreme bias. Although the planner makes multiple decisions in our model, their sequential timing rules out the tradeoff at the heart of Chakraborty and Harbaugh (2010)’s analysis. Dziuda (2012) considers a static cheap talk model in which the receiver may detect that the sender has lied with an exogenous probability, showing that this type of detectability can facilitate influential communication. In our model, the nature of the public signal does not allow for this type of equilibrium.

Watson (1996) also considers a cheap talk model with an extreme conflict of interest. In this model, the sender has access to a private signal but is confused about its meaning—he is unsure which realization of his signal is likely to induce the receiver to select an action that he prefers. Meanwhile, the receiver knows how she would interpret the sender’s information—she knows which action she would choose given a realization of the sender’s signal—but she cannot observe the sender’s realization directly. The key result is that, if the sender is sufficiently confused, and his preferred action is sufficiently likely to also be in the receiver’s best interest, then truthful communication arises in equilibrium without commitment.\footnote{In one model from Watson (1996), the sender and receiver privately observe the realizations of positively correlated Bernoulli random variables. The state is represented by the four possible combinations of the realizations. Two actions are possible. If the realizations match, it is sequentially rational for the receiver to select the action preferred by the sender, if the realizations mismatch, then it is sequentially rational for the receiver to select the action that is worse for the sender. Thus, if the receiver anticipates truthful reporting, she will choose the action that the sender likes whenever the sender’s reported realization matches the receiver’s. Because the random variables are positively correlated, then (conditional on the sender’s realization) it is more likely that the receiver’s realization matches the sender’s realization. Therefore, if the sender reports truthfully, the receiver is more likely to take the action that the sender likes; if he secretly deviates from truthful reporting, he increases the chance that his report fails to match the receiver’s realization, leading to a worse action.}

Our notion of advocacy, however, does not allow for confusion on the part of the sender: the advocate knows that by manipulating the planner into believing that the distribution is more likely to be high, he can induce the planner to select an action that he finds more desirable for every possible realization of the future states. Because it is common knowledge that the sender always benefits from creating the perception that the distribution is high, influential communication is impeded.

A number of papers consider the role of advocates in uncovering or communicating information (Aghion and Tirole 1997, Dewatripont and Tirole 1999). Most closely related to our analysis is a concurrent paper by Boleslavsky, Chatterji and Lewis (2013), which develops a static model of advocacy and explores the role of advocates in promoting corporate social responsibility. The central focus of our analysis, the optimal evolution of influence over time, is not addressed (to the best of our knowledge) by any of the contributions in the advocacy literature.

\section{Model}

A planner faces a decision or sequence of decisions. In each period, the planner’s choice, called the action, is represented by a real number $q$. The planner’s payoff in each period depends on the action she chooses and an unknown state of the world, also a real number, $x$. Given state $x$ and...
action $q$, her payoff is a quadratic loss function:

$$u_p(q, x) = -(q - x)^2$$

The planner cares only about matching her action as closely as possible to the state and does not derive utility independently from either the state or her action.

In each period, the true state is independently and identically distributed, but the exact distribution of the state in all periods is unknown to the planner. Two distributions for the state are possible, a high distribution $F(x|\phi_H)$ and a low distribution $F(x|\phi_L)$, with common support $X$. Both distributions admit continuous densities, $f(x|\phi_L)$, and $f(x|\phi_H)$ that are strictly positive inside $X$. The expected value of the state under the high distribution, $\mu_H$, is larger than the expected value of the state under the low distribution, $\mu_L$.

$$E[x|\phi_H] = \mu_H \quad E[x|\phi_L] = \mu_L$$

$$\mu_H > \mu_L$$

Given her information at the beginning of the interaction, the planner believes the distribution of the state to be the low distribution with probability $\gamma \in (0, 1)$.

Faced with a choice of action in each period, the planner would clearly benefit from knowing the true distribution of the state at the beginning of the interaction. With quadratic preferences, choosing an action equal to the mean of the true state distribution maximizes the planner’s expected utility. However, if she doesn’t know the true distribution, then her sequentially rational action in any period is her Bayesian update of the true mean, based on the observed history. The planner’s sequentially rational action therefore incorporates all available information, but it is never equal to the mean of the true distribution (which she does not know). Thus, learning the true state distribution immediately is valuable. We refer to situations in which the planner can directly observe the advocate’s private information as the “first best.” With the exception of Section 7 (where the advocate’s private information is noisy), the first best implies that the planner knows the true distribution of the underlying state. If she knows the true distribution, then in each period the planner achieves her first best payoff $\tilde{V} = \gamma \sigma_{\phi_L}^2 + (1 - \gamma) \sigma_{\phi_H}^2$, where $\sigma_{\phi_i}^2 \equiv \text{Var}[x|\phi_i]$ for $i \in \{H, L\}$.

Fortunately, an advisor knows the true distribution of the state (though he has no additional information about the realized state in each period). Unfortunately, the advisor is an advocate whose payoff differs from the planner’s:

$$u_a(q, x) = q.$$ 

Unlike the planner, who would like to match the action to the state, the advocate would always like the planner to choose a higher action, regardless of the true state of the world. The advocate therefore has an incentive to claim that the distribution is $H$, to induce her to choose higher actions, hindering influential communication.
In the main model, we focus on situations in which transfers are prohibited for institutional or legal reasons. Alternatively, we could assume that advocates are not motivated by financial incentives, drawing utility only from promoting their cause. Without payments, the planner can reward or punish the advocate only through her choice of action. In Section 6 we relax the restriction on transfers.

We model the interaction between planner and advocate as a game in which advice is exchanged for influence over actions. The game takes place over periods \( k = \{0, 1, \ldots, N\} \) according to the following sequence:

- **Period 0.** The advocate observes the true state distribution, \( F(x|\phi_L) \) or \( F(x|\phi_H) \), and issues a recommendation (or sends a message) \( M \in \{H, L\} \).
- **Periods 1 : N.** At the beginning of period \( k \), the planner selects an action \( q_k \). Once the action is selected, the true state \( x_k \) is revealed, and both planner and advocate realize their period \( k \) payoffs.

The advocate’s strategy is a pair of probabilities \((r_H, r_L) \in [0, 1]^2\); \( r_i \) represents the probability that the advocate reports \( L \) when the true distribution is \( i \). In each period the planner selects her action based on the message she receives from the advocate and on the history of past states that she observes. Let \( X_{k-1} = (x_{i=1}^{k-1}X) \), represent the set of possible histories at time \( k - 1 \), and \( h_{k-1} = (x_1, \ldots, x_{k-1}) \in X_{k-1} \) represents a particular history of past states. An information set for the planner is a combination of message and a history of states, \((M, h_{k-1})\). The planner’s strategy is a family of functions, \( \{q_M(h_{k-1})\}_{k=1}^N \), which maps information set \((M, h_{k-1})\) into an action \( q_k \). Let \( \gamma_M(h_{k-1}) \) be the planner’s belief that the state distribution is \( \phi_L \) at information set \((M, h_{k-1})\), and let \( \tilde{\gamma}_M \) be the planner’s belief that the state distribution is \( \phi_L \) immediately following message \( M \); that is, \( \tilde{\gamma}_M \equiv \gamma_M(h_0) \). From an ex ante perspective, the history of states in period \( k \) is a random variable with probability density function \( \prod_{j=1}^{k-1} f(x_j|\phi_i) \) given true state distribution \( \phi_i \in \{\phi_H, \phi_L\} \). Planner and advocate have a common discount factor \( \delta \).

Throughout the paper, we focus on Perfect Bayesian Equilibrium. Without an ex ante commitment from the planner, the planner and advocate select strategies that are sequentially rational given the strategy of the other player, and the planner’s beliefs at each information set are updated using Bayes’ rule whenever it is possible to do so. In particular, if either type of advocate sends message \( M \) with positive probability, then at any subsequent information set \((M, h_{k-1})\) the planner’s belief is the Bayesian update of her prior based on the observed history of state realizations, the advocate’s message, and the advocate’s strategy. If both types of advocate send message \( M \) with probability zero, then the equilibrium places no restriction on \( \tilde{\gamma}_M \), the planner’s belief at the information set immediately following this message. However, even if the advocate’s message was a

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5 By convention, \( h_0 \) is the null set and \( f(h_0|\phi_H) = f(h_0|\phi_L) = 1 \). With this convention, any formula or equation for period \( k \geq 2 \) also applies correctly to period one.

6 Throughout the paper we ignore issues related to variations in the decision maker’s strategy on sets of measure zero without any significant impact on the analysis or results. Readers uncomfortable with this can assume that \( X \) is discrete with no substantive changes.

7 We denote the random variable representing the history as \( h_{k-1} \), being careful to avoid confusing the random variable with the realization.
surprise, at subsequent information sets the planner’s beliefs update in response to the observed history according to Bayes’ rule (we provide a formal definition in the Appendix). In situations where the planner can commit, we allow the planner to select her strategy first, relaxing the restriction that her strategy must be sequentially rational. From the Revelation Principle, it is without loss of generality to represent the equilibrium with commitment as an optimization problem in which the planner chooses her strategy to maximize her ex ante expected payoff, subject to inducing a truthful message from the advocate.

The information structure of the advice for influence game differs from the standard information structure in standard agency settings in the following key respect: in standard settings the agent’s private information is about some attribute of the agent (e.g. his cost for executing a task, or his valuation for a good), and the only way for the principal to learn this information is to provide the agent with incentives to reveal it. In contrast, in the advice for influence game, the advocate’s private information is about an exogenous feature of the planner’s problem—the state distribution. Each time she repeats the problem, she observes the previous period’s state realization, which acts as a public signal of the true distribution. In the long run, she can learn the true distribution on her own by observing the sequence of public signals.

However, on its own, the public signals are not enough to overcome the advocate’s strong incentive to manipulate the planner: if the planner cannot commit to her strategy, then no influential communication takes place in equilibrium. With quadratic preferences, the planner’s sequentially rational action at information set \((M, h_{k-1})\) is the expected value of the state, given her belief about the true distribution, \(\gamma_M(h_{k-1})\). In equilibrium, belief \(\gamma_M(h_{k-1})\) is derived from \(\hat{\gamma}_M\) by applying Bayes’ rule, incorporating the information inherent in the realization of the history. Thus, without commitment the planner selects

\[
q_M(h_{k-1}) = \frac{\hat{\gamma}_M f(h_{k-1}|\phi_L)\mu_L + (1 - \hat{\gamma}_M)f(h_{k-1}|\phi_H)\mu_H}{\hat{\gamma}_M f(h_{k-1}|\phi_L) + (1 - \hat{\gamma}_M)f(h_{k-1}|\phi_H)}.
\]

Note that for every possible history, \(q_M(h_{k-1})\) is decreasing in \(\hat{\gamma}_M\)—the larger her initial belief that the distribution is \(\phi_L\), the smaller her update of the mean for every possible realization of the history. Hence, if \(\hat{\gamma}_H \neq \hat{\gamma}_L\), then the advocate always sends the message associated with the smaller value, inducing a larger action for every possible realization of the history. But because the message is always the same, in equilibrium it has no effect on the planner’s belief or subsequent actions. If instead \(\hat{\gamma}_H = \hat{\gamma}_L\), then the Law of Iterated Expectation implies that \(\hat{\gamma}_H = \hat{\gamma}_L = \gamma\), and hence, both messages have no effect on the planner’s belief or subsequent actions.

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8The definition of Perfect Bayesian Equilibrium that we use incorporates belief updating similar to Definition 8.2 of Fudenberg and Tirole (1991) (point B(ii)) and Watson (2016), requiring that belief updating happens in a manner consistent with Bayes’ rule at consecutive information sets. This is a natural assumption, but it is not essential for the results.

9We say that communication is influential if for some message \(M \in \{H, L\}\) that is sent with positive probability in equilibrium, the planner’s belief about the state distribution differs from the prior.

10Given our assumption about belief updating this is true even if message \(M\) is a surprise.

11From the Law of Iterated Expectations, \(\Pr(\phi_L) = \Pr(\phi_L|M = L)\Pr(M = L) + \Pr(\phi_L|M = H)\Pr(M = H)\), and hence, \(\gamma = \hat{\gamma}_L\Pr(M = L) + \hat{\gamma}_H\Pr(M = H)\). Therefore, \(\hat{\gamma}_L = \hat{\gamma}_H\) implies \(\hat{\gamma}_L = \hat{\gamma}_H = \gamma\).
This result, while standard, establishes a simple baseline for comparison: in the absence of commitment, no influential communication takes place, and the planner’s action in each period is her Bayesian update of the mean. It provides a context for our study of environments with commitment in Section 4: influential communication, the optimal evolution of influence, and all associated welfare gains are a direct consequence of the planner’s commitment power.

**Additional Notation.** The optimal influence mechanism depends critically on two objects that we introduce now in anticipation of the results to follow. First, the realization of period \( k \)’s history, \( h_{k-1} \), influences the planner’s actions through its likelihood ratio:

\[
\Lambda(h_{k-1}) = \frac{f(h_{k-1} | \phi_L)}{f(h_{k-1} | \phi_H)} = \prod_{i=1}^{k-1} \frac{f(x_i | \phi_L)}{f(x_i | \phi_H)},
\]

which indicates which of the two possible distributions are more consistent with the observed state realizations: high values of \( \Lambda(h_{k-1}) \) support the inference that the true distribution is \( \phi_L \), while low values support \( \phi_H \). Second, the optimal influence mechanism also depends on parameter \( \alpha \), where

\[
\alpha = \int_X \frac{f(x | \phi_L)^2}{f(x | \phi_H)} dx = E[\Lambda(x) | \phi_L].
\]

Parameter \( \alpha \) is a statistical measure of the “divergence” (or dissimilarity) of the possible state distributions. While it is not a true metric (as it is neither symmetric nor satisfies the triangle inequality) this divergence measure is encountered in a variety of statistical settings;\(^{12}\) it is weakly larger than one (with equality only if the distributions are identical almost everywhere) and larger values indicate that the distributions are more divergent (or less similar). As we shall see, the divergence of the state distributions is directly connected to the strength of the incentives in the optimal mechanism.

4 COMMUNICATION WITH COMMITMENT

In this section we analyze the advice for influence game in environments in which the planner can commit to her strategy. Depending on the application, commitment power can arise from a number of sources. First, the planner could derive commitment power by writing explicit contracts or rules that are legally enforced, specifying future actions as a function of the realization of the public signal. Chu and Sappington (2010) present examples of regulatory contracts in which the regulator’s action depends explicitly on the realization of a public signal of the firm’s private information.\(^{13}\) Commitment to a contract covering multiple periods is also commonly encountered in the literature on optimal dynamic sales mechanisms (Battaglini 2005, Boleslavsky and Said

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\(^{12}\)See Pearson (1904), Lancaster (1986), or Csizsar and Shields (2004).

\(^{13}\)In Chu and Sappington (2010), the firm’s private information is about the quality of the public signal—that is, the firm has private information about the sensitivity of its cost to the public signal realization. Meanwhile, in our analysis, the realization of the public signal (the history of past states) is not directly payoff relevant for the advocate, but it allows the planner to “cross-validate” the advocate’s private information.
2013, Pavan, Segal and Toikka 2014). More generally, commitment power for the planner could be generated by a cost that is imposed on the planner whenever she deviates from her promised action profile. As in Alonso and Matouschek (2007) and MacLeod (2003), we can interpret this cost as the damage that the advocate imposes on the planner through manipulative behavior in a long term advisory relationship. That is, if the advocate advises the planner on many issues over time, then by deviating from a promised action she loses the ability to make credible promises, and with it, the ability to elicit influential advice in the future. Even if she interacts with the advocate only once, the planner could benefit \textit{ex ante} by creating an independent “watchdog” organization that generates negative publicity or other adverse consequences whenever the planner deviates from her promise. Alternatively, she could post a bond that she would forfeit if she deviates from the promised action profile. In the context of interpersonal relationships, adhering to the planner’s optimal action profile could be codified as a social norm, imposing significant social costs on deviating individuals (Elster 1989).

A STATIC BENCHMARK

We begin our analysis by considering a static benchmark that has both of the main features of the evolving influence setting: a public signal of the true distribution and planner commitment. This setting allows us to illustrate the main features and intuition of the evolving influence mechanism, abstracting away from the dynamics of the multi-period model. We present the multi-period model in the next section.

Consider the advice for influence game with a single period, \( N = 1 \). Imagine that after the advocate selects a message, but before the planner selects her action \( q_1 \), the planner observes a public draw of \( K \) independent realizations from the true state distribution. This realization, \( h_K \equiv (x_1, \ldots, x_K) \) provides a signal of the true distribution. The informativeness of the signal is parametrized by \( K \), the number of draws: as more draws are generated, more information about the true distribution is revealed. Modeling the public signal as a draw of realizations from the true distribution allows for a straightforward comparison of the benchmark with the evolving influence mechanism, but this choice is merely for ease of exposition.

We consider the possibility of influential communication when the planner commits to an action that depends on both the advocate’s report and the realization of the public signal. Specifically,

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we assume that at the beginning of the game, the planner commits to a mechanism, \( m \), that specifies her action, \( q_1 \), as a function of the advocate’s report, \( M \in \{H, L\} \), and the signal realization, \( h_K \):

\[
m \equiv \{q_L(h_K), q_H(h_K)\}
\]

The advocate observes the true distribution and issues a recommendation. The public signal is realized, and the planner’s promised action is implemented. The Revelation Principle guarantees no loss of generality in restricting attention to mechanisms that induce the advocate to issue a truthful recommendation. Therefore, incentive compatibility for the advocate requires that he cannot induce a higher expected action by manipulating his report of the state distribution:

\[
E[q_L(h_K)|\phi_L] \geq E[q_H(h_K)|\phi_L] \quad \text{(ICL)}
\]

\[
E[q_H(h_K)|\phi_H] \geq E[q_L(h_K)|\phi_H]. \quad \text{(ICH)}
\]

Because the planner anticipates truthful reporting, her expected payoff of offering mechanism \( m \) is:

\[
P(m) = -\gamma E[(q_L(h_K) - x)^2|\phi_L] - (1 - \gamma)E[(q_H(h_K) - x)^2|\phi_H].
\]

To ensure that the advocate will participate in the mechanism, it is enough that the planner commits to interpret the absence of a message as message \( L \). Constraint (ICH) then ensures that the advocate (weakly) prefers to send a message than no message at all.

In the benchmark model, the optimal mechanism depends on parameter \( \omega_K^* \in (0, 1) \), which affects the optimal distortion from the first best actions:

\[
\omega_K^* \equiv \frac{1}{(1 - \gamma) + \gamma \alpha K}.
\]

We refer to this parameter as the magnitude of the distortions. We are now ready to characterize the optimal mechanism for the static benchmark:

**Proposition 4.1 (Optimal Static Mechanism With Public Signal).** The mechanism which maximizes \( P(m) \) subject to (ICL) and (ICH) is characterized as follows:

- If the advocate reports \( L \), the planner’s action does not depend on the realization of the public signal and is greater than her first best action \( \mu_L \):

\[
q_L(h_K) = \mu_L + (1 - \gamma)\omega_K^*(\mu_H - \mu_L)
\]

- If the advocate reports \( H \), the planner’s action depends on the realization of the public signal.
and is always less than her first best action $\mu_H$:

$$q_H(h_K) = \mu_H - \gamma \omega^{*}_K \Lambda(h_K)(\mu_H - \mu_L)$$

- The planner payoff $V$ and advocate payoff $U$ are given by:

$$V = \bar{V} - \omega^{*}_K (\gamma - \gamma^2)(\mu_H - \mu_L)^2 \quad U = \gamma \mu_L + (1 - \gamma) \mu_H$$

Because payments are infeasible or ineffective, the planner provides incentives for truthful reporting by committing to vary her actions in response to the realization of the public signal: incentive compatibility requires the planner to distort her action away from the first best ($\mu_L$ or $\mu_H$). Truth-telling is optimally induced by rewarding the advocate when he issues recommendations that he finds unfavorable and punishing the advocate when he reports information that he finds favorable, if his report appears to be inconsistent with the observed realization of the public signal. When the advocate reports $L$, the distortion from the first best action is a positive constant. By choosing an action higher than the first best action $\mu_L$, the planner makes a report of $L$ more attractive for the advocate, which reduces his incentive to lie when the true distribution is $\phi_L$. We interpret this fixed reward as a commitment to compromise.

The planner commits to evaluate self-serving advice by “cross-checking” it with the public signal: following report $H$, the planner commits to a realization-dependent distortion that depends on the likelihood ratio. If the likelihood ratio is small, the realization of the public signal supports the inference that the distribution is $\phi_H$. The advocate’s advice appears valid, the advocate “gains influence,” and the action is close to the first best action $\mu_H$. If the signal realization suggests that the true distribution is $\phi_L$, the advice appears invalid, and the action is reduced. This punishes the advocate but also hurts the planner’s payoff. To optimally provide incentives, the planner acts “as if” she is skeptical of the advocate’s advice, even though she knows that the advocate’s report is truthful. In the optimal mechanism, the planner is not learning about the true distribution from the public signal. Rather, the public signal allows the planner to distribute the distortions required by incentive compatibility across signal realizations in a less-costly way.

To understand the structure of the optimal distortions, recall that the first best mechanism violates incentive compatibility for the low type. Therefore, in order to provide incentives for truthful reporting, the planner must make report $L$ (weakly) more attractive and report $H$ (weakly) less attractive when the true distribution is $\phi_L$. Hence, the planner must introduce a distortion into the actions following report $L$ that is positive in expectation when the true distribution is $\phi_L$ and, conversely, a distortion following the high report that is negative in expectation when the true distribution is $\phi_L$. Because his payoff is equal to the planner’s action, from the advocate’s perspective, the way that distortions are assigned to signal realizations is irrelevant—only the expected distortion (given distribution $\phi_L$) affects the advocate’s payoff.

However, the planner is not indifferent to the way that distortions are assigned to realizations of the public signal. First, consider the low report. In order to make report $L$ more attractive under $\phi_L$,
the planner would like to introduce a distortion that increases the advocate’s expected payoff by a target amount, $D_L$. Because the advocate reports truthfully in equilibrium, the planner anticipates that the public signal is actually generated from $\phi_L$. Hence, she seeks a distortion function, $d_L(h_K)$, mapping a signal realization into a distortion from the first best action ($\mu_L$), that achieves the target (in expectation under $\phi_L$) while minimizing the distortion’s expected cost:

$$\min E[d_L(h_K)^2|\phi_L] \text{ subject to } E[d_L(h_K)|\phi_L] = D_L.$$ 

The solution to this problem is a constant distortion, $d_L(h_K) = D_L$. Intuitively, from the planner’s perspective, selecting a distortion function amounts to a choice of lottery whose mean is constrained. Because her payoff function is concave, the planner dislikes variation in her payoff. Therefore, she eliminates all variation by selecting a constant distortion.

When assigning the distortions associated with the high report an additional effect arises, which makes it optimal for the planner to connect the distortion to the signal realization. Imagine, as above, that the planner introduces a distortion into the first best mechanism in order to make report $H$ less attractive under $\phi_L$, reducing the advocate’s expected payoff by a target amount, $D_H$. Because the advocate reports truthfully in equilibrium, when she receives report $H$ the planner anticipates that the public signal is actually generated from $\phi_H$. Therefore, to provide incentives, the planner reduces the attractiveness of report $H$ given distribution $\phi_L$, anticipating that on the equilibrium path the true distribution of the public signal is $\phi_H$. Hence, she seeks a distortion function, $d_H(h_K)$ whose expected value under $\phi_L$ is equal to the target, $D_H$, while minimizing the distortion’s expected cost under the true distribution, $\phi_H$:

$$\min E[d_H(h_K)^2|\phi_H] \text{ subject to } E[d_H(h_K)|\phi_L] = D_H.$$ 

The solution to this problem is $d_H(h_K) = \frac{D_H}{\Lambda(h_K)}$. Hence, large distortions are optimally assigned to signal realizations that are likely under $\phi_L$ and unlikely under $\phi_H$ (and vice versa). Because the advocate reports truthfully in equilibrium, the planner only pays the cost of distortion $d_H(h_K)$ when the signal realization $h_K$ arises under $\phi_H$. It is therefore less costly, in expectation, to attach large distortions to realizations that are unlikely under $\phi_H$. Simultaneously, by reporting $H$ when the distribution is $\phi_L$, the advocate experiences distortion $d_H(h_K)$ whenever signal realization $h_K$ arises under $\phi_L$. Consequently, distortions associated with realizations that are likely under $\phi_L$ contribute most to meeting the expected distortion target, $D_H$. Taken together, these observations imply that distortions are simultaneously less costly for the planner and more beneficial in providing incentives when they are attached to signal realizations with large likelihood ratios. Hence, optimal

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19If the true distribution is $\phi_i$, for $i \in \{H, L\}$, and she always selects the first best action, $\mu_i$, then the planner’s expected payoff is $-E[(\mu_i - x)^2|\phi_i]$. If she adopts distortion function $d_i(h_K)$, resulting in actions $q_i(h_K) = \mu_i \pm d_i(h_K)$, then her expected payoff is $-E[(q_i(h_K) - x)^2|\phi_i] = -E[(\mu_i \pm d_i(h_K) - x)^2|\phi_i]$. Hence, the cost of the distortions is $E[(\mu_i \pm d_i(h_K) - x)^2|\phi_i] - E[(\mu_i - x)^2|\phi_i] = E[d_i(h_K)^2] + 2d_i(h_K)(\mu_i - x)|\phi_i] = E[d_i(h_K)^2|\phi_i]$, where the last equality follows because $\mu_i = E[x|\phi_i]$.

20To see that the expected cost of the distortion is $E[d_H(h_K)^2|\phi_H]$, consult the previous footnote.
distortions are large (small) for realizations with large (small) likelihood ratios.\textsuperscript{21}

It is also important to point out that the optimal distortion magnitude is decreasing—and the planner payoff increasing—in both the number of realizations, $K$, and the divergence of the distributions, $\alpha$. Under the optimal mechanism, if the advocate manipulates the planner by reporting $H$ when the distribution is $\phi_L$, then the advocate expects a penalty equal to $E[\Lambda(h_{K-1})|\phi_H] = \alpha^{K-1}$. The more divergent the distributions, or the more observations in the public signal, the larger the penalty. If either $K$ or $\alpha$ increases, then the planner can reduce the distortion magnitude while preserving incentive compatibility, increasing her payoff. Intuitively, the more divergent the state distributions, the easier for the planner to resolve the underlying distribution from the signal realization. Consequently, it is more difficult for the advocate to manipulate the planner, increasing the planner payoff.

**Evolving Influence**

In this section we consider the impact of planner commitment in the advice for influence game with multiple periods, $N \geq 2$. Once she selects an action in period $k$, the planner observes the period’s true state. The planner can condition her action in period $k$ on the entire history of past state realizations, $h_{k-1}$, which acts as a public signal of the true distribution. As in the static benchmark, commitment power allows the planner to leverage the information revealed by the history of states to dissuade the advocate from issuing self-serving recommendations. Unlike the static benchmark, in this section the planner makes a sequence of decisions, and anticipates the arrival of a new signal (a realization of the state) after each decision is made. These features interact in the provision of incentives, giving the planner further leverage over the advocate.

At the beginning of the interaction the planner commits to an evolving influence mechanism, $m$, that specifies an action in each period conditional on the advocate’s message and the observed history:

$$m_N \equiv \{q_L(h_{k-1}), q_H(h_{k-1})\}_{k=1}^{N}$$

The Revelation Principle guarantees no loss of generality in restricting attention to mechanisms that induce the advocate to issue a single truthful recommendation at the beginning of the interaction. In the multi-period setting, incentive compatibility requires that the advocate cannot induce a

\textsuperscript{21}The reasoning of the last two paragraphs does not rely on the quadratic specification of the planner’s payoff function. In particular, if the planner’s payoff function is $u_p(q, x) = v_p(q - x)$ where $v_p(\cdot)$ is strictly concave with a maximum at zero, then the distortion following the low report is independent of the realization of the public signal, and the distortion following the high report is large (small) for realizations with large (small) likelihood ratios (see the Supplemental Material for details).
higher discounted sum of expected actions by misreporting the distribution of the state:
\[
\sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1})|\phi_L] \geq \sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1})|\phi_L] \quad \text{(ICL-N)}
\]
\[
\sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1})|\phi_H] \geq \sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1})|\phi_H]. \quad \text{(ICH-N)}
\]

The planner’s payoff must also be adjusted to account for the sequence of decisions:
\[
P_N(m_N) = -\gamma \sum_{k=1}^{N} \delta^{k-1} E[(q_L(h_{k-1}) - x)^2|\phi_L] - (1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} E[(q_H(h_{k-1}) - x)^2|\phi_H].
\]

As in the static benchmark, participation is ensured by a commitment to interpret the absence of
a report as message $L$. Finally, define
\[
s_\delta \equiv \sum_{k=1}^{N} \delta^{k-1}, \quad s_{\alpha \delta} \equiv \sum_{k=1}^{N} (\alpha \delta)^{k-1}, \quad \omega_N \equiv \frac{s_\delta}{(1 - \gamma)s_\delta + \gamma s_{\alpha \delta}},
\]
(and note that $s_{\alpha \delta} > s_\delta$ and $\omega_N \in (0, 1)$). Parameter $\omega_N$ represents the adjusted distortion
magnitude for the optimal mechanism. We are now ready to characterize optimal evolving influence.

**Proposition 4.2 (Optimal Evolving Influence).** The mechanism which maximizes $P_N(m_N)$ subject
to (ICL) and (ICH) is characterized as follows:

- If the advocate reports $L$, the planner’s action does not depend on the history of states and is
greater than her first best action $\mu_L$:
  \[
  q_L(h_{k-1}) = \mu_L + (1 - \gamma)\omega_N(\mu_H - \mu_L)
  \]

- If the advocate reports $H$, the planner’s action depends on the observed history and is always
smaller than her first best action $\mu_H$:
  \[
  q_H(h_{k-1}) = \mu_H - \gamma\Lambda(h_{k-1})\omega_N(\mu_H - \mu_L)
  \]

- The planner payoff $V_N$ and advocate payoff $U_N$ are given by:
  \[
  V_N = s_\delta \left( V - \omega_N(\gamma - \gamma^2)(\mu_H - \mu_L)^2 \right) \quad U_N = s_\delta (\gamma \mu_L + (1 - \gamma)\mu_H)
  \]

The structure of the optimal evolving influence mechanism is identical to the static benchmark:
the planner commits to compromise when the advocate reports $L$ and to cross-check against
the history of past realizations when the advocate reports $H$. Nevertheless, both the the dynamics of
decisions and the pattern of distortions exhibit noteworthy features.
Under the optimal influence mechanism, actions exhibit qualitatively different dynamics from the case in which the planner cannot commit. Without commitment, the planner’s optimal action in each period is the expected value of the state, conditional on the observed history (see Section 3). This action always lies between the two first-best actions, $\mu_L$ and $\mu_H$, and it adjusts in every period to reflect the additional information acquired from the realization of the previous period’s state. When influence evolves optimally, neither of these properties holds. If the advocate reports $H$, the optimal action is $\mu_H$ adjusted down by a penalty proportional to the likelihood ratio. If certain states, unlikely under $\phi_H$, are likely under $\phi_L$, then the associated likelihood ratio can be very large. If one of these states occurs under $\phi_H$, the optimal actions associated with the report of $H$ could be smaller than $\mu_L$ (and could conceivably be arbitrarily negative). Also observe that if the true distribution is $\phi_L$, the sequence of actions is fixed.

In the multi-period model, the advocate derives utility from the planner’s entire sequence of decisions, not just one. Consequently, the planner has greater freedom to distribute the distortions required by incentive compatibility, not only across signal realizations in a single period (as in the static benchmark) but also across time periods. To see how the planner uses this flexibility in the optimal mechanism, recall binding constraint (ICL-N):

$$\sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1}) - q_H(h_{k-1})|\phi_L] = 0.$$  

In a mechanism that does not assign distortions across time periods, each term in the sum is the same (and is equal to zero). However, the optimal evolving influence mechanism does not exhibit this feature. A straightforward argument reveals that $E[q_L(h_{k-1}) - q_H(h_{k-1})|\phi_L]$ is an increasing function of $k$, and for some $k^* \in \{2, ..., N-1\}$,

$$E[q_L(h_{k-1}) - q_H(h_{k-1})|\phi_L] \leq 0 \iff k \leq k^*.$$  

Therefore, in the optimal mechanism the distortions in early periods are too small to achieve incentive compatibility on their own—conversely, distortions in later periods are too large to be optimal on their own. Intuitively, the history in an early period contains relatively few realizations, and it is therefore less useful as a public signal when assigning distortions across signal realizations within the period. The planner optimally shifts the burden of providing incentives onto later periods, where the longer history allows her to distribute a distortion across signal realizations within the period in a more cost-effective way. Consequently, a longer time horizon benefits the planner: the advocate’s “long view” toward supporting his cause increase his desirability as an advisor (assuming, of course that the planner can sustain a long-term commitment).

In a long interaction ($N \to \infty$), the planner is able to eventually learn the true distribution

$$E[q_L(h_{k-1}) - q_H(h_{k-1})|\phi_L] = (\mu_H - \mu_L)\gamma \omega_N (\alpha^{k-1} - (1 + \frac{1 - \omega_N}{\omega_N})).$$  

Because $\omega_N \in (0, 1)$, the difference is negative for $k = 1$. In order for (ICL-N) to hold with equality, there must be some periods in which it is positive. Because the difference in question is strictly increasing in $k$, there must be a period greater than 1 but less than $N$ at which it switches from negative to positive.
EVOLVING INFLUENCE

(with virtual certainty) from the observed history of states. This raises two issues that would not arise with a short time horizon. First, to what extent does the planner’s ability to perfectly learn in the long run mitigate the information asymmetry that exists at the beginning of the interaction? Second, does the advocate’s recommendation impact the planner’s action in the long run? Both of these issues are addressed by the following corollary.

**Corollary 4.3 (Evolving influence in Long Interactions)**

- **(High Divergence)** When \( \alpha \geq \frac{1}{\delta} \), as \( N \to \infty \), \( \omega_N \to 0 \), the optimal mechanism approaches the first best mechanism, and the planner’s payoff approaches the lifetime first best payoff, \( V_N \to \bar{V}/(1 - \delta) \).

- **(Low Divergence)** When \( \alpha < \frac{1}{\delta} \), as \( N \to \infty \), \( \omega_N \to \omega_\infty \in (0, 1) \), and the planner’s payoff is bounded away from the lifetime first best payoff, \( V_N \to V_\infty < \bar{V}/(1 - \delta) \). When \( N = \infty \), the sequence of actions \( q_{H}(h_{k-1}) \xrightarrow{a.s.} \mu_H \).

To understand this corollary, recall that if the advocate reports \( H \), but the true distribution is \( \phi_L \), he expects a future penalty proportional to \( \alpha^{k-1} \) in period \( k \). However, the long term growth of this penalty may not be enough to completely eliminate his incentive to manipulate the planner; the advocate’s lifetime penalty for manipulating the planner is the discounted sum of the expected distortions, which may either converge or diverge. If the distributions are sufficiently divergent, this discounted sum diverges. In the limit, the ability to learn in the long run eliminates the advocate’s incentive to manipulate the planner, the distortions vanish, and the optimal mechanism approaches the first best.

When distributions are sufficiently divergent, in a long interaction, there is virtually no difference between information provided by an advocate, and information that the planner acquires herself. This result does not merely hold in the limit as \( \delta \to 1 \) but is valid for any value of \( \delta \geq 1/\alpha \). If the underlying distributions are similar (divergence is low), the discounted sum of expected distortions converges. The planner cannot shrink the magnitude of the distortions to zero without violating incentive compatibility, and the planner’s payoff is bounded away from the first best.

When the true distribution is \( \phi_H \), the planner’s action converges almost surely to the first best action \( \mu_H \). Therefore, the damaging distortions associated with a report of \( H \) are inherently a short run phenomenon. However, even in the absence of influential communication, the planner’s action would converge to \( \mu_H \) in the long term; thus, when the true distribution is \( \phi_H \), the impact of the advocate’s recommendation vanishes in the long run, as the optimal action with and without commitment both approach the first best action. In contrast, when the true distribution is \( \phi_L \), the advocate’s impact on the planner’s action never diminishes. Paradoxically, when the advocate

\[ ^{23} \text{Although the first best mechanism is not incentive compatible (even for } N = \infty \text{), by choosing an arbitrarily large } T, \text{ and repeating the optimal } T \text{ period mechanism an infinite number of times, the planner can construct a mechanism with infinite time horizon that is incentive compatible and approximates her first best payoff with arbitrary degree of accuracy.} \]

\[ ^{24} \text{The asymptotic behavior of the distortions is driven by the asymptotic behavior of the likelihood ratio under distribution } \phi_H, \text{ which converges almost surely to zero.} \]
makes the seemingly more believable report, the distortion from the first best action \( \mu_L \) persists forever; when the advocate reports in a seemingly biased way, the distortion from the first best action \( \mu_H \) is likely to be small after a large number of periods. In the long run, the advocate’s recommendation only impacts the planner’s behavior when he reports information that he finds unfavorable.

5 INCENTIVES FOR INFORMATION ACQUISITION

The theory of evolving influence we have developed so far explores how a planner can elicit a truthful recommendation from an informed but extremely biased advocate. In analyzing this question, we assumed that the advocate was already informed before encountering the planner. It is easy to imagine that becoming informed is costly; for example, evaluating the policy or program in question may require time or effort. It is also possible that the planner is not entirely clueless about the issue at hand and can learn by expending her own resources. Perhaps she can evaluate the alternatives on her own, or with some effort, she may be able to locate an impartial advisor who will honestly communicate the best course of action.

In light of these issues, we consider how the advocate might be motivated to gather the information required to make decisions. In addition, we relax the assumption that the planner is only able to learn the true distribution from the observed history, allowing both planner and advocate to obtain an unverifiable \( \text{ex ante} \) signal revealing the true distribution at a cost. Specifically, in order to learn the true state (in an unverifiable manner) the advocate must expend \( c \geq 0 \) resources. The advocate’s decision to become informed cannot be observed or verified by the planner; he not only has the ability to lie when issuing an informed recommendation, he can issue a recommendation without exerting effort to acquire any information at all. The planner also has the capability to draw an unverifiable signal of the state by expending cost \( c_p \), which may be greater, less, or equal to the advocate’s cost of learning the true state distribution. Throughout the section we focus on small values of \( c \).\(^{25}\) As is the case for the advocate, both the planner’s decision to acquire \( \text{ex ante} \) information and any information uncovered are unobservable and unverifiable.

In such a situation one rationale for consulting an advocate is expertise. If the advocate is also an expert, his cost of learning may be lower than the planner’s, leading to natural efficiency gains that could be realized in the absence of incentive costs associated with transmission of information. In the presence of such costs the advocate is not always consulted, even when his cost of learning is smaller.

In this section we present a more-surprising result: the planner may prefer to consult the advocate even if her cost of learning is smaller than the advocate’s. Because the advocate draws utility from decisions, the planner can design an evolving influence mechanism to leverage the interim information revealed during the interaction to provide the advocate with incentives to learn \( \text{ex ante} \). As we will demonstrate below (and saw above), when the distributions are sufficiently

\(^{25}\) The threshold between small and large costs is given in the appendix.
divergent, the cost of providing these incentives can be arbitrarily close to zero, smaller than her own cost of information acquisition. Even if the advocate has a higher cost of \textit{ex ante} learning, the evolving influence mechanism may make him a desirable source of information for the planner.

We start by deriving the optimal mechanism that provides incentives for the advocate to acquire information and report it honestly. This will help us understand how the planner motivates the advocate to acquire information when the decision to do so is unobservable. We will then compare the incentive cost of information acquisition through the advocate with the direct cost of information acquisition using the planner’s technology to help us understand why an advocate may be a desirable source of information.

If the planner would like to motivate the advocate to become informed, two additional constraints appear in the mechanism design problem. If an advocate decides to remain uninformed, he could just issue an uninformed recommendation, either \( H \) or \( L \).\footnote{Unless the uninformed advocate is indifferent between reporting \( H \) all the time and reporting \( L \) all the time, he would never choose to randomize. In the cases we consider in the body of the paper, an uninformed advocate always strictly prefers to report \( H \).} If the advocate chooses to acquire information, he anticipates that with probability \( \gamma \) he will learn that the distribution is \( \phi_L \), and with probability \( 1 - \gamma \) he will learn that the distribution is \( \phi_H \). Whatever he learns, constraints (ICH) and (ICL) ensure that he will report truthfully. Therefore, for the advocate to choose to learn, exerting effort and reporting truthfully should be preferred by an uninformed advocate to not exerting effort and either reporting \( H \) all the time (AICH) or reporting \( L \) all the time (AICL).

Formulating and simplifying these constraints leads to the following conditions:

\[
\sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1}) - q_H(h_{k-1})|\phi_L] \geq \frac{c}{\gamma} \quad \text{(AICH)}
\]

\[
\sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1}) - q_L(h_{k-1})|\phi_H] \geq \frac{c}{1 - \gamma} \quad \text{(AICL)}
\]

Of course, the optimal mechanism must also satisfy the constraints for truthful reporting:

\[
\sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1}) - q_H(h_{k-1})|\phi_L] \geq 0 \quad \text{(ICL)}
\]

\[
\sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1}) - q_L(h_{k-1})|\phi_H] \geq 0 \quad \text{(ICH)}
\]

Writing the constraints in this way makes it apparent that if \( c = 0 \), the constraints for reporting truthfully and the constraints for acquiring it are identical. Furthermore, because the optimal mechanism of Proposition 4.2 satisfies (ICL) as an equality, it violates constraint (AICH). If information acquisition is costly and subject to moral hazard, then confronted with the mechanism of 4.2, an advocate would never acquire information and would always report \( H \). Finally, it is clear that the constraints for incentive compatible information acquisition imply the incentive constraints
for truthful reporting; constraints (ICL) and (ICH) can therefore be ignored in the planner’s problem. As before, participation is ensured by the planner’s commitment to interpret the absence of a message as a report of \( L \). If the planner makes such a commitment, constraint (AICL) would ensure that the advocate prefers to learn rather than report nothing to the planner. In the appendix, we derive the optimal mechanism that induces information acquisition by maximizing (P) subject to (AICH) and (AICL).

**Proposition 5.1 (Optimal Evolving Influence With Information Acquisition).** For small costs, the mechanism that maximizes \( P_N(m_N) \) subject to (AICH) and (AICL) is identical to the optimal mechanism of Proposition 4.2 with an increased magnitude of distortions:

\[
\hat{\omega}_N \equiv \left(1 + \frac{c}{s_\delta \gamma (\mu_H - \mu_L)}\right) \omega_N.
\]

- The planner’s actions are:

\[
q_L(h_{k-1}) = \mu_L + (1 - \gamma)\hat{\omega}_N(\mu_H - \mu_L) \text{ and } q_H(h_{k-1}) = \mu_H - \gamma \Lambda(h_{k-1})\hat{\omega}_N(\mu_H - \mu_L)
\]

- The planner payoff \( V_N^c \) and advocate payoff \( U_N^c \) are given by:

\[
V_N^c = s_\delta \left( \bar{V} - \left(1 + \frac{c}{s_\delta \gamma (\mu_H - \mu_L)}\right)^2 \omega_N(\gamma - \gamma^2)(\mu_H - \mu_L)^2 \right), \quad U_N^c = U_N - c.
\]

Proposition 5.1 indicates that when the cost of acquiring information is strictly positive the planner “raises the stakes” of the mechanism, rewarding the advocate more when he reports personally unfavorable information and proportionally increasing the downward distortion (for every history) when the advocate reports a personally favorable distribution. Intuitively, two types could profitably deviate by reporting the high distribution: the type that learned that the true distribution is \( \phi_L \), and the uninformed advocate who is pretending to be informed. Because a report of \( L \) is still credible, the structure of the mechanism is similar to the zero-cost case. However, the distortions need to be increased because the mechanism also needs to motivate the advocate to exert effort to learn, rather than “gamble” by reporting \( H \) without learning anything. This result is reminiscent of Lewis and Sappington (1997) who find that, in an environment with hidden information and hidden actions, information acquisition is optimally induced by changing the sensitivity of the agent’s reward to the public realization of the project’s cost: in the low-cost environment
the sensitivity is increased, and in the high-cost environment sensitivity is decreased. Similarly, in our model, the planner increases the distortion magnitude, which makes the sequence of actions associated with the “suspicious” report, \( H \), more sensitive to the realization of the history.

In the optimal mechanism the social cost of information acquisition exceeds the advocate’s direct cost \( c \). Although the advocate bears the direct cost \( c \) himself, in order to motivate learning and honest reporting, the planner must increase the magnitude of the distortions from the first best, reducing her payoff. If the planner’s cost of learning the true distribution \( c_p \) does not exceed the advocate’s, then it is socially wasteful for the planner to rely on the advocate.

Although doing so is socially inefficient whenever \( c_p < c \), the planner may prefer to deal with the advocate rather than learn herself. If she chooses to acquire information herself, the planner can implement the first best actions, which leads to a payoff of \( s_\delta \bar{V} - c_p \). If she offers the advocate the optimal evolving influence mechanism, her payoff is \( V_N^c \) (see Proposition 5.1). Comparing these expressions it is clear that the planner prefers information via the advocate whenever her direct cost of information acquisition \( c_p \) is greater than the incentive cost of interacting with the advocate:

\[
 c_p \geq s_\delta \left( 1 + \frac{c}{s_\delta \gamma (\mu_H - \mu_L)} \right)^2 \omega_N (\gamma - \gamma^2)(\mu_H - \mu_L)^2
\]

A small value of \( \omega_N \) ensures that this inequality is satisfied; as discussed in the previous sub-section, if \( \alpha > 1/\delta \), then as the time horizon increases \( \omega_N \) approaches zero. Alternatively, keeping the time horizon fixed, as the divergence of the distributions increases, \( \omega_N \) approaches zero.

**Corollary 5.2 (Planner Preference for Advocates).** For any \( c_p > 0 \) (no matter how small), if the state distributions are sufficiently divergent (\( \alpha \) is sufficiently large), then the planner will prefer to consult the advocate rather than learn on her own. Whenever \( c_p < c \), consulting the advocate is socially inefficient.

This result may seem surprising, but the rationale for the planners’ preference to consult an advocate for advice is intuitive. If the planner learns on her own, she doesn’t need to worry about truthful revelation of information, but pays her cost of learning \( c_p \). Because of his extreme bias, if she deals with an advocate, the planner bears an incentive cost to induce the advocate to truthfully learn and report his information. When the distributions are sufficiently divergent (\( \alpha \) is large), the advocate expects a large lifetime penalty from reporting \( H \) when the true distribution is \( \phi_L \); therefore the planner does not need to increase the magnitude of distortions very much in order to induce learning. If this effect is strong enough, it can always reduce her incentive cost below any positive cost of information acquisition. In this case the planner’s incentive cost offering the optimal mechanism is smaller than the cost of direct information acquisition. It is important to note that we focus on a setting in which the advocate cannot “stop caring” about the planner’s actions. Consequently, the advocate’s participation in the mechanism is guaranteed. If the mechanism must also satisfy a participation constraint, then the planner may need to introduce additional distortions into her actions, which would reduce the mechanism’s payoff, increasing the
attractiveness of direct learning. With this qualification, Corollary 5.2 suggests an explanation for the use of advocates as advisors on regulatory, organizational, or personal decisions, complementing the results on acquisition of verifiable information presented by Dewatripont and Tirole (1999) and Che and Kartik (2009).

6 EVOLVING INFLUENCE WITH MONETARY TRANSFERS

To this point, we have focused on the way that “influence” can substitute for monetary transfers when providing incentives to advisors with strong conflicts of interest, considering only environments in which transfers are either infeasible or ineffective. In this section, we consider the interaction of monetary transfers and evolving influence as incentive instruments. We ignore issues related to information acquisition, assuming that acquiring ex ante information is costless for the advocate and infeasible for the planner.

In order for monetary transfers to play a significant role in the analysis, planner and advocate must derive utility from money. We extend our analysis by incorporating preferences for transfers for planner and advocate. Given an action \( q \), realized state \( x \), and a monetary transfer \( t \) from planner to advocate, the parties' payoffs are:

\[
\begin{align*}
    u_p(q, x, t) &= -(q - x)^2 - v_p t \\
    u_a(q, t) &= q + v_a t
\end{align*}
\]

As is standard, monetary transfers allow a linear exchange of utility between the parties. Parameter \( v_p \) gives the planner's marginal disutility of paying the advocate. A high value of \( v_p \) indicates that reducing transfers is relatively more important for the planner than making accurate decisions; a small value of \( v_p \) indicates that the planner places significantly more weight on making the correct decision, making her more-inclined to offer monetary transfers to the advocate. Similarly, parameter \( v_a \) represents the advocate's marginal rate of substitution between actions and transfers.

As in section 4, the planner commits to a mechanism, which now specifies an action in each period conditional on history, and a lifetime transfer from planner to advocate conditional on the entire history of the interaction.\(^{29}\) Thus, an evolving influence mechanism with transfers \( m^t \) is defined as follows:

\[
m^t \equiv \{q_L(h_{k-1}), t_L(h_N), q_H(h_{k-1}), t_H(h_N)\}_{k=1}^{N}
\]

where the planner's period \( k \) action conditional on history \( h_{k-1} \) and message \( M \) is \( q_M(h_{k-1}) \) and the lifetime transfer to the advocate following message \( M \) and final history \( h_N \) is denoted \( t_M(h_N) \). We focus on situations in which the advocate is protected by limited liability: lifetime transfers from the planner to the advocate must be non-negative.\(^{31}\) When transfers are possible the planner's

\(^{29}\)The analysis of the evolving influence mechanism presented in section 4 is a special case in which the advocate does not respond to monetary incentives, \( v_a = 0 \).

\(^{30}\)Because commitment to transfers is possible, the assumption of a single lifetime payment conditional on the history in the final period is without loss of generality.

\(^{31}\)If negative transfers were allowed, the planner could commit to “sell” favorable action profiles to the advocate in exchange for transfers, a practice that has the appearance of corruption. Negative transfers also go against the spirit
Proposition 6.1 (Optimal Evolving Influence With Transfers). The mechanism which maximizes the ratio of $\omega$ for the planner.

Objective function is augmented,

$$P^t(m^t) = P_N(m_N) - v_p(\gamma E[t_L(h_N)|\phi_L] + (1 - \gamma)E[t_H(h_N)|\phi_H])$$

as are the advocate’s incentive constraints:

$$v_aE[t_L(h_N)|\phi_L] + \sum_{k=1}^{N} \delta^{k-1}E[q_L(h_{k-1})|\phi_L] \geq v_aE[t_H(h_N)|\phi_L] + \sum_{k=1}^{N} \delta^{k-1}E[q_H(h_{k-1})|\phi_L] \quad \text{(ICLT)}$$

$$v_aE[t_H(h_N)|\phi_H] + \sum_{k=1}^{N} \delta^{k-1}E[q_H(h_{k-1})|\phi_H] \geq v_aE[t_L(h_N)|\phi_H] + \sum_{k=1}^{N} \delta^{k-1}E[q_L(h_{k-1})|\phi_H] \quad \text{(ICHT)}$$

The optimal evolving influence mechanism with transfers maximizes $P_t(m)$ subject to constraints (ICLT), (ICHT), and limited liability for the advocate. The characterization depends on the following model parameter:

$$\omega_t \equiv \frac{v_p}{2v_a} \left( \frac{1}{(1-\gamma)(\mu_H - \mu_L)} \right).$$

The ratio of $\omega_t$ to $\omega_N$ has significant impact on the structure of the optimal mechanism.

**Proposition 6.1** (Optimal Evolving Influence With Transfers). The mechanism which maximizes $P^t(m^t)$ subject to (ICLT) and (ICHT) is characterized as follows:

- If $\omega_t \leq \omega_N$, then the optimal mechanism with transfers, has a similar structure to the optimal mechanism without transfers, with a smaller distortion magnitude. Transfers associated with report $L$ are positive, but transfers associated with report $H$ are zero:

  $$q_L(h_{k-1}) = \mu_L + (1 - \gamma)\omega_t(\mu_H - \mu_L) \text{ and } q_H(h_{k-1}) = \mu_H - \gamma\Lambda(h_{k-1})\omega_t(\mu_H - \mu_L)$$

  $$E[t_L(h_N)|\phi_L] = s_\delta \frac{\mu_H - \mu_L}{v_a}(1 - \frac{\omega_t}{\omega_N}) \text{ and } t_H(h_N) = 0.$$  

  The planner payoff $V_N^t$ and advocate payoff $U_N^t$ are given by:

  $$V_N^t = s_\delta \left( \bar{V} - \omega_t(\gamma - \gamma^2)(\mu_H - \mu_L)^2 - \frac{v_p}{v_a}(1 - \frac{\omega_t}{\omega_N})(\mu_H - \mu_L) \right)$$

  $$U_N^t = s_\delta (\gamma\mu_L + (1 - \gamma)\mu_H) + s_\delta \gamma(\mu_H - \mu_L)(1 - \frac{\omega_t}{\omega_N}).$$

- If $\omega_t > \omega_N$, then optimal transfers are all zero; the mechanism is identical to the one described by Proposition 4.2.

A number of intriguing observations on the interplay of transfers and evolving influence follow from this characterization. First, whether transfers are explicitly ruled out or not, the structure of the optimal actions is unchanged: qualitatively, the optimal evolution of influence does not depend of our analysis, which focuses on the role of transfers as an instrument for aligning incentives, not as a source of rent for the planner.
on whether transfers are permitted by the institutional arrangement. Second, from the perspective of the policymaker, the role of optimal transfers is to substitute for the compromise portion of the evolving influence mechanism. Transfers are paid only if the advocate reports $L$, making this report more attractive. The planner can therefore move the fixed profile of actions (associated with report $L$) closer to the first best action. This also allows her to reduce the magnitude of the distortions associated with report $H$ for any history. All of the advocate’s information rent comes in the form of the transfers associated with the low report. Third, for a significant range of parameters, the optimal mechanism does not utilize transfers, even though it is feasible to do so.

7 PARTIALLY REVEALING PRIVATE INFORMATION

In the analysis of the preceding sections, the advocate’s private signal perfectly reveals the state distribution. Therefore, if an evolving influence mechanism is used to elicit this information, then the state realizations do not reveal any new information about the underlying distribution. While the history of states plays a critical role in providing ex ante incentives to the advocate, it does not convey any relevant information. In this section, we relax the assumption that the advocate is perfectly informed about the distribution of the state. Here, the history of states plays a dual role: it allows the planner to cross-check the advocate’s message, providing him with incentives, and it conveys new information about the true state distribution. In this section, we extend our theory of evolving influence to this scenario, providing an analog of Proposition 4.2 in this environment. Proposition 7.2 describes how changes in the informativeness of the advocate’s private signal affect the planner’s payoff.

To incorporate partially revealing information into the model, we assume that the advocate privately observes the realization of a binary signal, $S$, that conveys information about the underlying state distribution. One signal realization, $G$, reveals good news for the advocate, reducing the posterior belief that the distribution is $\phi_L$; the other signal realization, $B$, reveals bad news for the advocate, increasing the posterior belief that the true distribution is $\phi_L$:

$$
\gamma_G \equiv \Pr(\phi_L|S = G) \quad \gamma_B \equiv \Pr(\phi_L|S = B) \quad \gamma_G < \gamma < \gamma_B.
$$

The Law of Iterated Expectations requires that the expected value of the posterior belief is equal to the prior belief. Hence, given the prior $\gamma$, and Bayesian updates $(\gamma_B, \gamma_G)$, the Law of Iterated Expectations implies:

$$
\pi \equiv \Pr(S = B) = \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G}.
$$

The planner knows the distribution of the advocate’s signal (that is, she knows that a good realiza-
tion generates posterior $\gamma_G$ and a bad realization generates posterior $\gamma_B$), but she does not observe the signal’s realization ($G$ or $B$).

The Bayesian updates $(\gamma_G, \gamma_B)$ can be used to rank the informativeness of different signals. Given the same prior, $\gamma$, if two signals $(S, S')$ generate Bayesian updates $(\gamma_G, \gamma_B)$ and $(\gamma_G', \gamma_B')$ that are ordered, $\gamma_G' \leq \gamma_G$ and $\gamma_B \leq \gamma_B'$ (and at least one inequality is strict), then $S'$ is more informative than $S$ in the sense of Blackwell.\(^{34}\) Intuitively, the more informative signal has a bigger impact on the posterior belief, moving both of its possible realizations further away from the prior.

In this section, the advocate’s type is identified by the signal realization he has observed. A type $B$ advocate believes that the distribution is $\phi_L$ with probability $\gamma_B > \gamma$, while a type $G$ advocate believes that the distribution is $\phi_L$ with probability $\gamma_G < \gamma$. The planner believes that the advocate is a type $B$ with probability $\pi$, and type $G$ with probability $1 - \pi$. As in previous sections, the planner designs a mechanism $m \equiv \{q_B(h_{k-1}), q_G(h_{k-1})\}_{k=1}^N$, specifying an action in period $k$ as a function of history $h_{k-1}$ and the advocate’s report.

To formulate the planner’s maximization problem, we first modify the incentive constraints to reflect that the advocate’s signal is only partially revealing.\(^{35}\) When the advocate observes realization $j \in \{G, B\}$, he infers that the history of states is generated from $\phi_L$ with probability $\gamma_j$ and from $\phi_H$ with complementary probability. Therefore, by reporting $i \in \{G, B\}$ when the true signal realization is $j \in \{G, B\}$, the advocate expects a lifetime payoff of:

$$
\sum_{k=1}^{N} \delta^{k-1} \{ \gamma_j E[q_i(h_{k-1})|\phi_L] + (1 - \gamma_j) E[q_i(h_{k-1})|\phi_H] \}.
$$

Therefore, the incentive compatibility constraints are:

\[\sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[q_B(h_{k-1}) - q_G(h_{k-1})|\phi_L] + (1 - \gamma_B) E[q_B(h_{k-1}) - q_G(h_{k-1})|\phi_H] \} \geq 0, \quad \text{(ICB)}\]

\[\sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[q_G(h_{k-1}) - q_B(h_{k-1})|\phi_L] + (1 - \gamma_G) E[q_G(h_{k-1}) - q_B(h_{k-1})|\phi_H] \} \geq 0. \quad \text{(ICG)}\]

The planner’s expected payoff must also be modified to account for the advocate’s partially informative signal. The planner knows that with probability $\pi$ (resp. $1 - \pi$) the signal realization is $B$ (resp. $G$). Because reporting is truthful, if she observes report $B$ (resp. $G$) she believes that $h_{k-1}$ is generated by $\phi_L$ with probability $\gamma_B$ (resp. $\gamma_G$), and she is committed to follow actions $q_B(h_{k-1})$.

\(^{34}\)See, for example, Ganuza and Penalva (2010).

\(^{35}\)An explicit derivation of the incentive constraints and planner payoff function can be found in the Appendix preceding the proof of Proposition 7.1.
Meanwhile, the left hand side of (A) is the posterior belief that the distribution is \( \phi \) is the largest possible Bayesian update that the distribution is \( \phi \) of (A) is \( \max \Pr( L = x | h_{k-1}) \). Therefore, if \( L = x \) is realized from a single draw; hence, (A) requires that \( L > 0 \) for all states inside the support, the likelihood ratio is bounded away from zero. In this case (A) restricts the planner’s ability to learn that the distribution is \( \phi \) by

\[
P(m, S) = -\pi \sum_{k=1}^{N} \delta^{k-1}\{ \gamma_B \mathbb{E}[ (q_B(h_{k-1}) - x)^2 | \phi_L ] + (1 - \gamma_B) \mathbb{E}[ (q_B(h_{k-1}) - x)^2 | \phi_H ] \}
- (1 - \pi) \sum_{k=1}^{N} \delta^{k-1}\{ \gamma_G \mathbb{E}[ (q_G(h_{k-1}) - x)^2 | \phi_L ] + (1 - \gamma_G) \mathbb{E}[ (q_G(h_{k-1}) - x)^2 | \phi_H ] \}.
\]

Note that because the advocate’s signal, \( S \), determines \( (\pi, \gamma_G, \gamma_B) \), we have explicitly included the signal as an argument of the planner’s payoff function. The optimal mechanism for evolving influence maximizes \( P(m, S) \), subject to (ICB) and (ICG).

It is straightforward to see that the mechanism that the planner would select if she observes the signal realization directly—that is, the first best mechanism—violates incentive compatibility. Indeed, if she did not need to provide incentives to the advocate, the planner would select an action equal to the expected value of the state, conditional on the observed signal realization and the history:

\[
q_j(h_{k-1}) = \hat{\mu}(\gamma_j, h_{k-1}), \text{ where,}
\]

\[
\hat{\mu}(\gamma_j, h_{k-1}) \equiv \mathbb{E}[x | S = j, h_{k-1}] = \frac{\gamma_j f(h_{k-1} | \phi_L) \mu_L + (1 - \gamma_j) f(h_{k-1} | \phi_H) \mu_H}{\gamma_j f(h_{k-1} | \phi_L) + (1 - \gamma_j) f(h_{k-1} | \phi_H)}, \quad j \in \{G, B\}.
\]

However, for every possible realization of the history, the expected value of the state is smaller if realization \( B \) is observed than if realization \( G \) is observed. Therefore, if the planner offered the first best mechanism, the advocate would always report the good signal realization, violating (ICB). Even if the advocate’s private information is noisy, the main conflict of interest at the heart of the paper remains.

Before presenting the characterization of optimal evolving influence with partially revealing private information, we explain a technical assumption imposed in the proposition.

**Assumption (A):** \( \gamma_B \geq \frac{1 - \gamma}{\gamma L^{N-1} + (1 - \gamma)}, \) where \( L \equiv \min_{x \in X} \Lambda(x) \).

In the analysis, (A) is a sufficient condition for constraint (ICG) to be non-binding in the optimal mechanism. Qualitatively, (A) requires that the advocate’s private signal is sufficiently informative that a bad realization generates a posterior belief larger than a threshold.\(^{36}\) Note that if \( \gamma_B < 1 \), then (A) requires that \( L > 0 \): for all states inside the support, the likelihood ratio is bounded away from zero. In this case (A) restricts the planner’s ability to learn that the distribution is \( \phi_H \) by

\(^{36}\)The threshold in (A) has an interesting interpretation. \( L < 1 \) is the smallest value of the likelihood ratio that can be realized from a single draw; hence, \( L^{N-1} \) is the smallest value of the likelihood ratio that can be realized over the course of the planner’s decision’s problem i.e. over \( N - 1 \) draws. Therefore, the threshold on the right hand side of (A) is \( \max \Pr(\phi_H | h_{k-1}) \) where the max is over all \( h_{k-1} \in X_{k-1} \) and all \( k \leq N \). That is, the right hand side of (A) is the largest possible Bayesian update that the distribution is \( \phi_H \), that is based only on the history of realizations. Meanwhile, the left hand side of (A) is the posterior belief that the distribution is \( \phi_L \), based only on signal realization \( B \). Condition (A) therefore requires that \( \Pr(\phi_L | S = B) = \max \Pr(\phi_H | h_{k-1}) \) for all \( h_{k-1} \in X_{k-1} \) and \( k \leq N \).
observing the history. Note further that the class of signals in which $\gamma_B = 1$ always satisfies (A).\textsuperscript{37}

The following proposition characterizes the optimal evolving influence mechanism with partially revealing private information.

**Proposition 7.1 (Optimal Evolving Influence With Partially Revealing Information).** Assume (A). The mechanism which maximizes $P(m, S)$ subject to (ICB) and (ICG) is characterized as follows. An $\omega_r \in (0, \gamma_B - \gamma_G)$ exists such that:

- If the advocate reports $B$, the planner’s action depends on the observed history and is greater than her optimal action in the absence of information asymmetry, $\hat{\mu}(\gamma_B, h_{k-1})$:

  $$q_B(h_{k-1}) = \hat{\mu}(\gamma_B, h_{k-1}) + (1 - \pi)\omega_r(\mu_H - \mu_L).$$

- If the advocate reports $G$, the planner’s action depends on the observed history and is smaller than her optimal action in the absence of information asymmetry, $\hat{\mu}(\gamma_G, h_{k-1})$:

  $$q_G(h_{k-1}) = \hat{\mu}(\gamma_G, h_{k-1}) - \frac{\gamma_B}{\gamma_G}A(h_{k-1}) + (1 - \gamma_B)\omega_r(\mu_H - \mu_L).$$

The optimal mechanism can be decomposed into two parts: (i) the first best mechanism with partially revealing information, (i.e. $q_i(h_{k-1}) = \hat{\mu}(\gamma_i, h_{k-1})$ for $i \in \{G, B\}$), and (ii) distortions that ensure incentive compatibility. Unlike the case in which the advocate knows the distribution for certain (in which the first best action is equal to the mean of the true distribution), with partially revealing information the first best action changes over the course of the interaction as the planner continues to learn about the underlying distribution from the history. At the same time, the overall structure of the distortions is similar to the case of fully revealing private information (characterized in Proposition 4.2). When the advocate reports against his bias (report $B$), the planner introduces a constant, positive distortion that rewards the advocate for honesty. When the advocate reports personally favorable information (report $G$), the planner introduces a downward distortion that depends on the likelihood ratio—like its fully revealing counterpart, this distortion is largest when $A(h_{k-1})$ is high, which suggests that the true distribution is $\phi_L$.

While the overall structure of the distortions is similar to the case of fully revealing information, with partially revealing information the distortion following the good report, $\hat{\mu}(\gamma_G, h_{k-1}) - q_G(h_{k-1})$, exhibits an interesting difference. Unlike its counterpart in Proposition 4.2, which approaches infinity as the likelihood ratio grows, and approaches zero as the likelihood ratio vanishes, the distortion in question approaches a finite limit as the likelihood ratio grows, and is generically

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\textsuperscript{37}The class of signals for which $\gamma_B = 1$ can be interpreted as a search for a “bad attribute.” Imagine that if the planner’s decision environment has a certain attribute, then the state distribution is $\phi_L$, and if it lacks this attribute, then the distribution is $\phi_H$. If the advocate comes across this attribute during his evaluation of the planner’s decision problem, then he is confident that the distribution is $\phi_L$, i.e. $\gamma_B = 1$. However, failing to find the attribute makes the advocate more optimistic, but it does not mean that the distribution is necessarily $\phi_H$. Perhaps the advocate simply lacks the expertise to always uncover the bad attribute.
bounded away from zero as the likelihood ratio vanishes. To understand why the distortion behaves differently, recall that large values of the likelihood ratio are more likely to be realized when the true distribution is $\phi_L$ and small values are more likely when the distribution is $\phi_H$. When the advocate’s signal is noisy, the planner believes that the true distribution may actually be $\phi_L$, even if she observes a report of $G$ that she believes is honest. Consequently, the planner believes that large values of the likelihood ratio are more likely than under a perfect signal, and small values are less likely. Therefore, attaching large distortions to histories with large likelihood ratios is more costly for the planner with a noisy signal than with a perfect signal, and attaching larger distortions to histories with small likelihood ratios is less costly. Hence, after a report of $G$, a noisy signal “compresses” the distortions: a large likelihood ratio is penalized less, a small likelihood ratio is rewarded less.

The preceding discussion suggests that increases in informativeness of the advocate’s signal affect the planner’s payoff in two ways. First, the planner benefits because the advocate has better information for the planner to elicit: with a more informative signal the first best action ($q_i(h_{k-1}) = \hat{\mu}(\gamma_i, h_{k-1})$) is closer to the mean of the true distribution. Second, changes in informativeness affect the distortions from the first best needed to achieve incentive compatibility, with competing effects. On one hand, as informativeness increases, cheating becomes more tempting for type $B$ in the first best mechanism, making it more difficult to provide incentives. On the other hand, as informativeness increases, it is easier for the planner to cross-check the advocate’s report with the history, so that incentives are less costly to provide. It is interesting that despite these confounding effects, the planner’s payoff always increases when the advocate’s signal is more informative.

**Proposition 7.2 (Value of Expertise).** Suppose that for a given prior, $\gamma$, signal $S'$ is more informative than $S$ in the sense of Blackwell: $\gamma'_G \leq \gamma_G$ and $\gamma'_B \leq \gamma_B$, and at least one of these inequalities is strict. The optimal evolving influence mechanism with a noisy signal delivers the planner a strictly higher payoff when the signal is $S'$ than when it is $S$.

The proof is not based on the preceding characterization (and does not require assumption (A)). Instead, the argument constructs a mechanism that is incentive compatible under signal $S'$ and delivers the planner a strictly higher payoff than the optimal mechanism under signal $S$—the result immediately follows. This result is significant on its own, but it also provides additional justification for our focus on fully revealing private information in previous sections of the paper.

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38 A simple calculation reveals that $\hat{\mu}(\gamma_G, h_{k-1}) - q_G(h_{k-1}) \in (\pi\omega_r(\mu_H - \mu_L)(1 - \gamma_B)/(1 - \gamma_G), \pi\omega_r(\mu_H - \mu_L)\gamma_B/\gamma_G)$.  
39 In the first best with a noisy signal, by reporting $G$ instead of $B$ the advocate induces $\hat{\mu}(\gamma_G, h_{k-1})$ instead of $\hat{\mu}(\gamma_B, h_{k-1})$ following $h_{k-1}$. The difference between these actions, $\hat{\mu}(\gamma_G, h_{k-1}) - \hat{\mu}(\gamma_B, h_{k-1})$, is increasing in $\gamma_B$ and decreasing in $\gamma_G$. Hence, increases in $\gamma_B$ or reductions in $\gamma_G$ increase this difference for every history, making it more tempting for the advocate to manipulate the planner.
40 Conceptually, the planner commits to mechanism under $S'$ that garbles the advocate’s report. By selecting a particular garbling and subsequent action profile, the planner generates a stochastic mechanism that is incentive compatible and recreates her payoff from the optimal mechanism under signal $S$. If the planner then replaces the stochastic mechanism, in which planner’s action is the outcome of a specified lottery, with its expected value, she maintains incentive compatibility but strictly increases her payoff.
8 CONCLUSIONS

In this paper we illustrate the power of commitment and interim information to mitigate extreme conflicts of interest. We consider a planner who receives advice from an advocate. The advocate’s preferences are increasing in the action chosen by the planner and are independent of the true state. We first demonstrated that for an advocate to have influence, the planner must be able to commit to future actions and learn from her experience; under either strategic communication or static delegation no useful information is conveyed. These results demonstrate the negative consequences of the severe interest conflict that we consider in the paper. If both of these conditions are met, we find that the planner can elicit influential information from the advocate. The optimal evolving influence mechanism is characterized by compromise with an advocate who reports against his bias, and by a commitment to evaluate the validity of seemingly self-serving advice. Moreover, we show that planners may prefer the advice of a biased advocate to the recommendation of an indifferent adviser in instances where information about the planner’s problem is costly to acquire. Finally our theory illustrates that the possibility of offering transfers does not completely crowd out the benefits of evolving influence.

Our theory of evolving influence is special in a number of respects and some of the predictions of our model should be interpreted with care. One set of special assumptions is the linear-quadratic preference structure. If the advocate has preferences that are state-independent, then the assumption of a linear payoff function for the advocate is without loss of generality: simply “rescale” the planner’s action so that the decision variable is equal to the advocate’s utility. In a more general model, however, the planner would have a concave single-peaked payoff function; virtually all of the qualitative results of the current specification carry over to such an environment. If the conflict of interest were more mild, perhaps because the advocate’s preference puts some weight on the planner’s payoff, both strategic communication and static delegation could be effective means of eliciting information. Evaluating the relative merits of evolving influence in such settings could be an interesting subject of future research. We also assume only two possible underlying distributions. If we extend the model to allow for more possible distributions and assume that only local incentive constraints bind, then the distortion for each possible interior type consists of a compromise component, which makes it more attractive to the immediately higher type, and a “cross-checking” component, which makes it less attractive to the immediately lower type (the highest type will have only the “cross-checking” component and the lowest type will only have the compromise component). Thus, the advocate’s payoff of exaggerating depends on the expected value of the “local” likelihood ratio at the misreport, evaluated under the true distribution. Thus, the structure of distortions is more complicated than with two types, and establishing conditions under which the local incentive constraints are sufficient for global incentive compatibility would be valuable for

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41 As mentioned earlier, the result that the advocate gets the same ex ante payoff under the optimal mechanism as under no communication is unlikely to generalize to more general specifications.

42 That is, if the advocate knows that the true distribution is $\phi_i$, but is considering reporting $\phi_j$, his payoff would involve the following expectation: $E[f(h_{k-1}|\phi_{j-1})/f(h_{k-1}|\phi_j)|\phi_i]$. 
further study. Our analysis also assumes that the planner interacts with only one advocate. This assumption makes the planner’s problem as difficult as possible and therefore underscores the power of evolving influence as an incentive instrument. However, in many applications, it is common for decision makers to consult two or more advocates representing different interests; correspondingly, it is not uncommon for an advocate to advise multiple decision makers. One fruitful direction for future research would be to extend our analysis to consider how multiple advocates representing a range of possible points of view may be most effectively paired with decision makers to provide informative advice. All of these extensions are unlikely to change the main message of this paper: the optimal evolution of influence can mitigate or overcome even the most extreme interest conflicts.
9 APPENDIX

THE GAME WITHOUT COMMITMENT.

Equilibrium Definition. Let $\gamma_M(h_{k-1})$ denote the planner belief that the state distribution is $\phi_L$ at information set $(M, h_{k-1})$. The advocate strategy $(r_H, r_L)$, the planner strategy $\{q_H(h_{k-1}), q_L(h_{k-1})\}^N_{k=1}$ and the planner beliefs $\{\gamma_H(h_{k-1}), \gamma_L(h_{k-1})\}^N_{k=1}$ constitute a Perfect Bayesian Equilibrium if and only if the following three conditions are met.

(i) Advocate sequential rationality. For $i \in \{H, L\}$,

$$r_i \in \text{argmax}_{r \in [0,1]} \sum_{k=1}^N \delta^{k-1} E[q_L(h_{k-1})|\phi_i] + (1-r) \sum_{k=1}^N \delta^{k-1} E[q_H(h_{k-1})|\phi_i].$$

(ii) Planner sequential rationality. Let $U(q, g)$ represent the planner’s expected flow payoff if she selects action $q$ and her belief that the distribution is $\phi_L$ is $g$:

$$U(q, g) \equiv -gE[(q-x)^2|\phi_L] - (1-g)E[(q-x)^2|\phi_H].$$

Let $V_M(h_k)$ represent the planner continuation value of following her strategy starting from information set $(M, h_k)$:

$$V_M(h_k) \equiv \sum_{j=k+1}^N \delta^{j-(k+1)} E[U(q_M(h_{j-1}), \gamma_M(h_{j-1}))|h_k].$$

We require that

$$q_M(h_{k-1}) \in \text{argmax}_q U(q, \gamma_M(h_{k-1})) + \delta\{\gamma_M(h_{k-1})E[V_M(h_{k-1}, x)|\phi_L] + (1 - \gamma_M(h_{k-1}))E[V_M(h_{k-1}, x)|\phi_H]\},$$

where the last two expectations are taken with respect to $x$. Note that $q$ only appears in the expected flow payoff, and hence, this condition simplifies to

$$q_M(h_{k-1}) \in \text{argmax}_q U(q, \gamma_M(h_{k-1})).$$

(iii) Belief Consistency. The condition has two parts. The first part specifies how the planner’s belief is formed at information set $(M, h_0)$, i.e. at the beginning of Period 1.

B1: if $r_L > 0$ or $r_H > 0$, then $\gamma_L(h_0) = \frac{\gamma r_L}{\gamma r_L + (1-\gamma) r_H}$

if $r_L < 1$ or $r_H < 1$, then $\gamma_H(h_0) = \frac{\gamma (1-r_L)}{\gamma (1-r_L) + (1-\gamma)(1-r_H)}$. 


The second condition specifies how the planner’s belief about the state distribution evolves over the course of the planner’s decision problem.

**B2:** For \( k \geq 2 \) and \( h_{k-1} = (h_{k-2}, x_{k-1}) \),

\[
\gamma_M(h_{k-1}) = \frac{\gamma_M(h_{k-2}) f(x_{k-1} | \phi_L)}{\gamma_M(h_{k-2}) f(x_{k-1} | \phi_L) + (1 - \gamma_M(h_{k-2})) f(x_{k-1} | \phi_H)}.
\]

**Discussion.** Condition (i) requires that the advocate selects the message that delivers him the highest expected payoff, given his knowledge of the true distribution and the planner strategy.

Condition (ii) requires that the planner acts optimally at each of her information sets given her beliefs about the state distribution, anticipating her own behavior at subsequent information sets. The game has a simple structure that ensures that the planner continuation value is independent of her current action. Therefore, sequential rationality implies that the planner chooses the action that maximizes her flow payoff given her belief about the state distribution.

The first part of Condition (iii) (B1) requires that the planner updates her beliefs according to Bayes’ rule at the information set immediately following message \( M \) (information set \((M, h_0)\)) if it is possible to do so; that is, if the probability of the advocate sending message \( M \) is non-zero. If the planner observes a message that is sent with probability zero given the advocate’s strategy, then the equilibrium places no restriction on the planner’s belief at the initial information set.

The second part of Condition (iii) (B2), requires that the planner’s belief evolves over the course of her decision problem (periods 2 through \( N \)) in a manner that is consistent with Bayesian updating based on the realizations of the state. To see this, note that \( h_{k-1} = (h_{k-2}, x_{k-1}) \), which implies that information set \((M, h_{k-1})\) is the successor of \((M, h_{k-2})\) when state \( x_{k-1} \) is realized. (B2) requires that Belief \( \gamma_M(h_{k-1}) \) is derived from the preceding belief \( \gamma_M(h_{k-2}) \) by applying Bayes’ rule, reflecting the information generated by the realization \( x_{k-1} \). Note that this restriction holds whether or not message \( M \) is sent with positive probability in equilibrium, i.e. whether or not \((M, h_{k-1})\) and \((M, h_{k-2})\) are on the path of play. This is essentially condition B(ii) of Definition 8.2 in Fudenberg and Tirole (1991) adapted to our framework, and it is qualitatively similar to the conditions imposed by Watson (2016).

Condition (ii) immediately implies that in equilibrium, \( q_M(h_{k-1}) = \gamma_M(h_{k-1}) \mu_L + (1 - \gamma_M(h_{k-1})) \mu_H \).

Note that iterating condition (B2) reveals that for \( k \geq 2 \),

\[
\gamma_M(h_{k-1}) = \frac{\gamma_M(h_0) f(h_{k-1} | \phi_L)}{\gamma_M(h_0) f(h_{k-1} | \phi_L) + (1 - \gamma_M(h_0)) f(h_{k-1} | \phi_H)},
\]

and that with the convention that \( f(h_0 | \phi_i) = 1 \), this formula also applies to \( k = 1 \). Recalling that \( \hat{\gamma}_M \equiv \gamma_M(h_0) \), it follows that

\[
q_M(h_{k-1}) = \frac{\hat{\gamma}_M f(h_{k-1} | \phi_L) \mu_L + (1 - \hat{\gamma}_M) f(h_{k-1} | \phi_H) \mu_H}{\hat{\gamma}_M f(h_{k-1} | \phi_L) + (1 - \hat{\gamma}_M) f(h_{k-1} | \phi_H)},
\]

as in the text.
THE GAME WITH COMMITMENT.

Proof of Corollary 4.3 is standard and is therefore omitted.

Proposition 4.2 is a special case of Proposition 5.1.

Below we prove Propositions 4.1 and 5.1. Propositions 6.1, 7.1, and 7.2 are proved in Section A.

**Proof. Proposition 4.1**
The optimal mechanism described in the proposition maximizes \( P(m) \) subject to (ICH) and (ICL). We then solve a relaxed problem in which only constraint (ICH) is imposed. We then establish that (ICL) is satisfied. Finally we calculate the planner’s and advocate’s payoffs.

Consider a relaxed problem in which only constraint (ICH) is imposed. Because the objective function is concave in the choice variables and the constraint is linear in the choice variables, the solution of this relaxed problem is the unique combination of controls that satisfies the stationarity and primal feasibility condition. The objective function of the relaxed problem is given by:

\[
-\gamma \int_{X_K} f(h_K|\phi_L) [\int_X f(x|\phi_L)(q_L(h_K) - x)^2 dx] dh_K \\
-(1 - \gamma) \int_{X_K} f(h_K|\phi_H) [\int_X f(x|\phi_H)(q_H(h_K) - x)^2 dx] dh_K
\]

Evaluating the integrals in brackets gives the following expression for the objective function:

\[
-\gamma \int_{X_K} f(h_K|\phi_L) [q_L(h_K)^2 - 2\mu_L q_L(h_K) + \eta_L] dh_K \\
-(1 - \gamma) \int_{X_K} f(h_K|\phi_H) [q_H(h_K)^2 - 2\mu_H q_H(h_K) + \eta_H] dh_K
\]

where \( \eta_i \) represents the uncentered second moment of distribution \( \phi_i \). Constraint (ICH) can be written as

\[
\int_{X_K} f(h_K|\phi_L) [q_L(h_K) - q_H(h_K)] \geq 0
\]
Clearly, the first best mechanism \( q_H(h_K) = \mu_H, q_L(h_K) = \mu_L \) violates this condition, and it must therefore bind. The Lagrangian for the relaxed problem is therefore given by:

\[
\mathcal{L} = -\gamma \int_{X_K} f(h_K|\phi_L)[q_L(h_K)^2 - 2\mu_Lq_L(h_K) + \eta_L]dh_K \\
-(1 - \gamma) \int_{X_K} f(h_K|\phi_H)[q_H(h_K)^2 - 2\mu_Hq_H(h_K) + \eta_H]dh_K \\
+ \lambda \gamma \{ \int_{X_K} f(h_K|\phi_L)[q_L(h_K) - q_H(h_K)]dh_K \}
\]

For algebraic convenience, constraint (ICH) has been multiplied by \( \gamma \) in the Lagrangian. The stationarity conditions for this problem are:

\[
-\gamma f(h_K|\phi_L)[2(q_L(h_K) - \mu_L) - \lambda] = 0 \iff q_L(h_K) = \mu_L + \frac{\lambda}{2}
\]

\[
-(1 - \gamma)f(h_K|\phi_H)2(q_H(h_K) - \mu_H) - \lambda \gamma f(h_K|\phi_L) = 0 \\
\iff q_H(h_K) = \mu_H - \frac{\lambda}{2} \frac{\gamma}{1 - \gamma} \Lambda(h_K)
\]

To simplify the notation, let

\[
w \equiv \frac{\lambda}{2(1 - \gamma)(\mu_H - \mu_L)} \iff \frac{\lambda}{2} = w(1 - \gamma)(\mu_H - \mu_L)
\]

Hence, the stationarity conditions become:

\[
q_L(h_K) = \mu_L + w(1 - \gamma)(\mu_H - \mu_L)
\]

\[
q_H(h_K) = \mu_H - w\gamma \Lambda(h_K)(\mu_H - \mu_L)
\]

where

\[
\Lambda(h_K) \equiv \frac{f(h_K|\phi_L)}{f(h_K|\phi_H)} = \prod_{i=1}^{K} \frac{f(x_i|\phi_L)}{f(x_i|\phi_H)}
\]

Next, we solve for the value of \( w \) which satisfies constraint (ICH).

\[
\int_{X_K} f(h_K|\phi_L)[q_L(h_K) - q_H(s_K)]dh_K = 0
\]

\[
\mu_L + w(1 - \gamma)(\mu_H - \mu_L) - (\mu_H - w\gamma \alpha^K(\mu_H - \mu_L)) = 0 \\
w[(1 - \gamma) + \gamma \alpha^K] = 1 \\
w = \frac{1}{(1 - \gamma) + \gamma \alpha^K}
\]

Hence, \( w = \omega^*_K \), as defined in the text.
Next we show that constraint (ICL) is satisfied. The constraint requires:

\[ E[q_H(h_K) - q_L(h_K)]|\phi_H] \geq 0 \]

Because \( E[\Lambda(h_K)|\phi_H] = 1 \) this expression simplifies:

\[
\mu_H - \omega_K^*\gamma(\mu_H - \mu_L) - \mu_L - (1 - \gamma)\omega_K^*(\mu_H - \mu_L) \geq 0
\]

\[
(\mu_H - \mu_L)(1 - \omega_K^*) \geq 0 \iff \omega_K^* \leq 1
\]

which obviously holds, because \( \alpha > 1 \).

Finally, we calculate the parties’ expected payoffs. For the planner:

\[
-\gamma \int_{X_K} f(h_K|\phi_L)[q_L(h_K)^2 - 2\mu_L q_L(h_K) + \eta_L]dh_K
\]

\[
-(1 - \gamma) \int_{X_K} f(h_K|\phi_H)[q_H(h_K)^2 - 2\mu_H q_H(h_K) + \eta_H]dh_K
\]

Substituting the solution and simplifying gives:

\[
-\gamma[\sigma_L^2 + (1 - \gamma)^2(\omega_K^*)^2(\mu_H - \mu_L)^2]
\]

\[
-(1 - \gamma) \int_{X_K} f(h_K|\phi_H)[\sigma_H^2 + \gamma^2(\omega_K^*)^2(\mu_H - \mu_L)^2\Lambda(h_K)^2]dh_K
\]

Recalling the definition of \( \bar{V} \) gives:

\[
\bar{V} - \gamma(1 - \gamma)^2(\omega_K^*)^2(\mu_H - \mu_L)^2 - (1 - \gamma)\gamma^2(\omega_K^*)^2(\mu_H - \mu_L)^2 \alpha_K
\]

\[
\bar{V} - (\gamma - \gamma^2)(\omega_K^*)^2[(1 - \gamma) + \gamma \alpha_K](\mu_H - \mu_L)^2
\]

\[
\bar{V} - \omega_K^*(\gamma - \gamma^2)(\mu_H - \mu_L)^2
\]

To calculate the advocate payoff:

\[
\int_{X_K} \gamma f(h_K|\phi_L)q_L(h_K) + (1 - \gamma) f(h_K|\phi_H)q_H(h_K)dh_K = \\
\gamma(\mu_L + (1 - \gamma)\omega_K^*(\mu_H - \mu_L)) + (1 - \gamma)(\mu_H - \gamma\omega_K^*(\mu_H - \mu_L)) = \\
\gamma\mu_L + (1 - \gamma)\mu_H
\]

This is the expression presented in the text, completing the proof.

**Proof. Proposition 5.1**

The optimal mechanism described in the proposition maximizes \( P_X(m_N) \) subject to (AICH) and
(AICL). We begin by showing that the constraints can be written as in the body of the paper. We then consider a relaxed problem in which only constraint (AICH) is imposed. We then derive a condition on the cost of information acquisition for the unimposed constraint (AICL) to be satisfied. This condition is satisfied whenever \( c \) is sufficiently small. Finally we calculate the planner’s and advocate’s payoffs.

We first show that the incentive compatibility constraints can be written as in the text. Consider constraint (AICH) that requires that acquiring information followed by truthful reporting dominates not acquiring information and reporting \( H \).

\[
\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \gamma f(h_{k-1} | \phi_L)q_L(h_{k-1}) + (1 - \gamma) f(h_{k-1} | \phi_H)q_H(h_{k-1})dh_{k-1} - c \geq (AICH)
\]

Rearranging and simplifying gives

\[
\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \gamma f(h_{k-1} | \phi_L)q_L(h_{k-1}) - q_H(h_{k-1})dh_{k-1} \geq c
\]

Dividing both sides by \( \gamma \) gives the expression presented in the text.

Next, we consider a relaxed problem in which only constraint (AICH) is imposed. Because the objective function is concave in the choice variables and the constraint is linear in the choice variables, the solution of this relaxed problem is the unique combination of controls that satisfies the stationarity and primal feasibility condition. The objective function of the relaxed problem is given by:

\[
-\gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L)\left[ \int_X f(x | \phi_L)(q_L(h_{k-1}) - x)^2dx \right]dh_{k-1}
\]

\[
-(1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H)\left[ \int_X f(x | \phi_H)(q_H(h_{k-1}) - x)^2dx \right]dh_{k-1}
\]

Evaluating the integrals in brackets gives the following expression for the objective function:

\[
-\gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L)(q_L(h_{k-1})^2 - 2\mu_L q_L(h_{k-1}) + \eta_L)dh_{k-1}
\]

\[
-(1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H)(q_H(h_{k-1})^2 - 2\mu_H q_H(h_{k-1}) + \eta_H)dh_{k-1}
\]
where \( \eta \) represents the uncentered second moment of distribution \( \phi_i \). The Lagrangian for the relaxed problem is therefore given by:

\[
\mathcal{L} = -\gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L)[q_L(h_{k-1})]^2 - 2\mu_L q_L(h_{k-1}) + \eta_L]dh_{k-1} \\
- (1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)[q_H(h_{k-1})]^2 - 2\mu_H q_H(h_{k-1}) + \eta_H]dh_{k-1} \\
+ \lambda \left\{ \delta^{k-1} \int_{X_{k-1}} \gamma f(h_{k-1}|\phi_L)[q_L(h_{k-1}) - q_H(h_{k-1})]dh_{k-1} \right\} - c\}
\]

The inequality constraint (AICH) in the relaxed problem must bind, because the first-best violates it. Hence, the stationarity conditions for this problem are:

\[-\gamma \delta^{k-1} f(h_{k-1}|\phi_L)[2(q_L(h_{k-1}) - \mu_L) - \lambda] = 0 \Leftrightarrow q_L(h_{k-1}) = \mu_L + \frac{\lambda}{2} \]

\[-(1 - \gamma) \delta^{k-1} f(h_{k-1}|\phi_H)[2(q_H(h_{k-1}) - \mu_H) - \lambda \delta^{k-1} \gamma f(h_{k-1}|\phi_L) = 0 \]

\[\Leftrightarrow q_H(h_{k-1}) = \mu_H - \frac{\lambda}{2} \frac{\gamma}{1 - \gamma} \Lambda(h_{k-1}) \]

To simplify the notation, let

\[w \equiv \frac{\lambda}{2(1 - \gamma)(\mu_H - \mu_L)} \Leftrightarrow \frac{\lambda}{2} = w(1 - \gamma)(\mu_H - \mu_L)\]

Hence, the stationarity conditions become:

\[q_L(h_{k-1}) = \mu_L + w(1 - \gamma)(\mu_H - \mu_L)\]

\[q_H(h_{k-1}) = \mu_H - w\gamma \Lambda(h_{k-1})(\mu_H - \mu_L)\]

Next, we solve for the value of \( w \) which satisfies constraint (AICH).

\[\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \gamma f(h_{k-1}|\phi_L)[q_L(h_{k-1}) - q_H(h_{k-1})]dh_{k-1} = c\]

\[\gamma s_\delta(\mu_L + w(1 - \gamma)(\mu_H - \mu_L)) - \gamma \sum_{k=1}^{N} \delta^{k-1}(\mu_H - \mu_L)w\gamma \alpha^{k-1}(\mu_H - \mu_L) = c\]

\[\gamma s_\delta(\mu_L + w(1 - \gamma)(\mu_H - \mu_L)) - \gamma s_\delta \mu_H + w\gamma^2 s_\alpha(M_H - \mu_L) = c\]

\[w[1 - \gamma + \frac{s_\alpha s_\delta}{s_\delta}] = 1 + \frac{c}{\gamma s_\delta(M_H - \mu_L)}\]

\[w = \left( 1 + \frac{c}{\gamma s_\delta(M_H - \mu_L)} \right) \left( \frac{1}{1 - \gamma + \gamma \frac{s_\alpha s_\delta}{s_\delta}} \right)\]
\[ w = \left(1 + \frac{c}{\gamma s_\delta (\mu_H - \mu_L)}\right) \left(\frac{s_\delta}{(1 - \gamma) s_\delta + \gamma s_\alpha s}\right) \]

Substituting the definition of \( \omega_N \) shows that \( w = \hat{\omega}_N \) defined in the text.

Next we show that for \( c \in [0, \hat{c}] \), constraint (AICL) is satisfied. The constraint requires:

\[
(1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1}) - q_L(h_{k-1}) | \phi_H] \geq c \quad \text{(AICL)}
\]

Because \( E[\Lambda(h_{k-1}) | \phi_H] = 1 \) this expression simplifies:

\[
(1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} [\mu_H - \hat{\omega}_N \gamma (\mu_H - \mu_L) - \mu_L - (1 - \gamma) \hat{\omega}_N (\mu_H - \mu_L)] \geq c
\]

\[
(1 - \gamma)(\mu_H - \mu_L)s_\delta(1 - \hat{\omega}_N) \geq c
\]

\[
\hat{\omega}_N \leq 1 - \frac{c}{(1 - \gamma)(\mu_H - \mu_L)s_\delta}
\]

Observe that for \( c = 0 \) this inequality reduces to

\[
\frac{s_\delta}{(1 - \gamma)s_\delta + \gamma s_\alpha s} \leq 1
\]

which is obviously satisfied because \( \alpha > 1 \Rightarrow s_\alpha s > s_\delta \). Observe also that the left hand side of the required inequality is an increasing linear function of \( c \), while the right hand side is a decreasing linear function of \( c \). Let the unique intersection of these linear functions be \( \hat{c} \). Because the inequality is satisfied at \( c = 0 \), it is also satisfied in the interval \( c \in [0, \hat{c}] \).

Finally, we calculate the parties’ expected payoffs. For the policymaker:

\[
-\gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L)[q_L(h_{k-1})^2 - 2\mu_L q_L(h_{k-1}) + \eta_L] dh_{k-1}
\]

\[
-(1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H)[q_H(h_{k-1})^2 - 2\mu_H q_H(h_{k-1}) + \eta_H] dh_{k-1}
\]

Substituting the solution and simplifying gives:

\[
-\gamma \sum_{k=1}^{N} \delta^{k-1} [\sigma_L^2 + (1 - \gamma)^2 \hat{\omega}_N^2 (\mu_H - \mu_L)^2]
\]

\[
-(1 - \gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H)[\sigma_H^2 + \gamma^2 \hat{\omega}_N^2 (\mu_H - \mu_L)^2 \Lambda(h_{k-1})^2] dh_{k-1}
\]
Recalling the definition of $\bar{V}_N$ gives:

$$\bar{V}_N - \gamma(1 - \gamma)^2 \bar{V}_N^2 (\mu_H - \mu_L)^2 \sum_{k=1}^{N} \delta^{k-1} - (1 - \gamma) \gamma^2 \bar{V}_N^2 (\mu_H - \mu_L)^2 \sum_{k=1}^{N} (\alpha \delta)^{k-1}$$

$$\bar{V}_N - (\gamma - \gamma^2) \bar{V}_N |(1 - \gamma)s_\delta + \gamma s_\alpha| (\mu_H - \mu_L)^2$$

Substituting the definition of $\hat{\omega}_N$, canceling one power of the denominator, and factoring one power of $s_\delta$ to the front gives the expression presented in the text. To calculate the advocate payoff:

$$\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \gamma f(h_{k-1}|\phi_L) q_L(h_{k-1}) + (1 - \gamma) f(h_{k-1}|\phi_H) q_H(h_{k-1}) dh_{k-1} - c =$$

$$s_\delta \gamma (\mu_L + (1 - \gamma) \bar{\omega}_N (\mu_H - \mu_L)) + (1 - \gamma)(\mu_H - \gamma \bar{\omega}_N (\mu_H - \mu_L)) - c =$$

$$s_\delta [\gamma \mu_L + (1 - \gamma) \mu_H] - c$$

This is the expression presented in the text, completing the proof. ■
REFERENCES


A SUPPLEMENTARY MATERIAL

Further discussion of footnote 21. Suppose that the advocate’s payoff function is linear, but the planner’s payoff function is $u_p(q, x) = v_p(q - x)$ where $v_p(\cdot)$ is differentiable, strictly concave, and achieves a maximum at zero. Let $q^*_i$ for $i \in \{H, L\}$ represent the first best action, i.e. $q^*_i = \arg\max q E[u_p(q, x)|\phi_i]$, with the labeling convention that $q^*_H > q^*_L$. Consider the following optimization, analogous to the one in Section 4.

$$\max_{d_L(h_K)} E[v_p(q^*_L + d_L(h_K) - x)|\phi_L] \text{ subject to } E[d_L(h_K)|\phi_L] = D_L.$$  

The Lagrangian for this problem is

$$\int_{X_K} \int_X v_p(q^*_L + d_L(h_K) - x)f(x|\phi_L)f(h_K|\phi_L)dx\,dh_K - \lambda\left(\int_{X_K} d_L(h_K)f(h_K|\phi_L)dh_K - D_L\right).$$  

The stationarity condition is therefore

$$f(h_K|\phi_L)\int_X v'_p(q^*_L + d_L(h_K) - x)f(x|\phi_L)dx - \lambda f(h_K|\phi_L) = 0,$$

and hence,

$$\int_X v'_p(q^*_L + d_L(h_K) - x)f(x|\phi_L)dx = \lambda.$$  

Because $v'_p(\cdot)$ is strictly decreasing, a unique value $d_L(h_K)$ solves this equation. Hence, $d_L(h_K)$ is independent of $h_K$. Constraint $E[d_L(h_K)|\phi_L] = D_L$ implies $d_L(h_K) = D_L$.

Consider the following optimization, analogous to the one in Section 4.

$$\max_{d_H(h_K)} E[v_p(q^*_H + d_H(h_K) - x)|\phi_H] \text{ subject to } E[d_H(h_K)|\phi_L] = D_H.$$  

The Lagrangian for this problem is

$$\int_{X_K} \int_X v_p(q^*_H + d_H(h_K) - x)f(x|\phi_H)f(h_K|\phi_H)dx\,dh_K - \lambda\left(\int_{X_K} d_H(h_K)f(h_K|\phi_L)dh_K - D_H\right).$$  

The stationarity condition is therefore

$$f(h_K|\phi_H)\int_X v'_p(q^*_H + d_H(h_K) - x)f(x|\phi_H)dx - \lambda f(h_K|\phi_L) = 0,$$
and hence,

\[
\int_X v_p(q_H^* + d_H(h_K) - x)f(x|\phi_H)dx = \lambda \Lambda(h_K).
\]

Because \(v_p(\cdot)\) is concave, an increase in the expected distortion target \(D_H\) reduces the indirect objective function, and hence, \(\lambda < 0\). Therefore realizations of \(h_K\) associated with large \(\Lambda(h_K)\) generate a smaller value of the right hand side, implying a larger \(d_H(h_K)\). Hence, a large (small) value of the likelihood ratio \(\Lambda(h_K)\) is optimally associated with a large (small) distortion \(d_H(h_K)\).

**Proof. Proposition 6.1**

The optimal mechanism described in the proposition maximizes \(P_t(m)\) subject to (ICTH) and (ICLT). We consider a relaxed problem in which only constraint (ICLT) is imposed. We then show that the relaxed problem satisfies (ICLH). We also derive conditions under which the solution of the relaxed problem satisfies the non-negativity constraint on transfers. Finally we calculate the planner’s and advocate’s payoffs.

Consider the relaxed problem in which only constraint (ICLT) is imposed.

\[
v_a E[t_L(h_N)|\phi_L] + \sum_{k=1}^{N} \delta^{k-1}E[q_L(h_{k-1})|\phi_L] \geq v_a E[t_H(h_N)|\phi_L] + \sum_{k=1}^{N} \delta^{k-1}E[q_H(h_{k-1})|\phi_L] \quad \text{(ICLT)}
\]

We first show that over any non-zero probability subset of terminal histories inside the support \(X_N\), the transfer associated with report \(H\) must be zero. Suppose that \(t_H(h_N) > 0\) for some non-zero measure set of histories. Because the support of the possible state distributions is identical, \(\tau \equiv E[t_H(h_N)|\phi_L] > 0\). Consider a new mechanism \(m'\) in which actions are identical, but transfers are different: \(t'_H(h_N) = 0\) and \(t'_L(h_N) = t_L(h_N) - \tau\). For the new mechanism the incentive constraint is:

\[
v_a E[t_L(h_N) - \tau|\phi_L] + \sum_{k=1}^{N} \delta^{k-1}E[q_L(h_{k-1})|\phi_L] \geq \sum_{k=1}^{N} \delta^{k-1}E[q_H(h_{k-1})|\phi_L] \quad \text{(ICLT)}
\]

Obviously moving \(\tau\) to the right hand side gives the same value of both left and right hand sides of the inequality, Hence, if the original mechanism satisfied (ICLT) then so does the new mechanism. However, the new mechanism generates a larger value of the objective function, because all transfers (for every possible history and message) are smaller.

Next, observe that if constraint (ICLT) were non-binding in the relaxed problem, the solution would be the first best mechanism with zero transfers, which is not incentive compatible. Therefore, it must be that (ICLT) is binding at the solution to the relaxed problem, hence:

\[
E[t_L(h_N)|\phi_L] = \frac{1}{v_a} \sum_{k=1}^{N} \delta^{k-1}E[q_H(h_{k-1}) - q_L(h_{k-1})|\phi_L] \quad \text{(ICLT)}
\]
Because only the expression $E[t_L(h_N)|\phi_L]$ appears in both the objective function and the constraint of the relaxed problem, focusing on lifetime payments that are independent of the realized history is without loss of generality, i.e. replacing history dependent transfers with the expected value of the transfer under the low distribution is without loss of generality. Substituting this into the objective function gives:

$$-\gamma \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_L)[q_L(h_{k-1})^2 - 2\mu_L q_L(h_{k-1}) + \eta_L + \frac{v_p}{v_a}(q_H(h_{k-1}) - q_L(h_{k-1}))]dh_{k-1}$$

$$-(1-\gamma) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1}|\phi_H)[q_H(h_{k-1})^2 - 2\mu_H q_H(h_{k-1}) + \eta_H]dh_{k-1}$$

The first order conditions for this problem are:

$$-\gamma \delta^{k-1} f(h_{k-1}|\phi_L)[2(q_L(h_{k-1}) - \mu_L) - \frac{v_p}{v_a}] = 0 \Leftrightarrow q_L(h_{k-1}) = \mu_L + \frac{v_p}{2v_a}$$

$$-(1-\gamma) \delta^{k-1} f(h_{k-1}|\phi_H)[2(q_H(h_{k-1}) - \mu_H) - \delta^{k-1} \gamma \frac{v_p}{v_a} f(h_{k-1}|\phi_L) = 0$$

$$\Leftrightarrow q_H(h_{k-1}) = \mu_H - \frac{v_p}{2v_a} \frac{\gamma}{1-\gamma} \Lambda(h_{k-1})$$

Following the analysis without transfers, define

$$\omega_t \equiv \frac{v_p}{2v_a} \left( \frac{1}{(1-\gamma)(\mu_H - \mu_L)} \right)$$

Therefore the solution of the relaxed problem in which only constraint (ICLT) is imposed is given by:

$$q_L(h_{k-1}) = \mu_L + (1-\gamma)\omega_t(\mu_H - \mu_L)$$

$$q_H(h_{k-1}) = \mu_H - \gamma \omega_t \Lambda(h_{k-1})(\mu_H - \mu_L)$$
We use this to calculate the lifetime expected transfer associated with the report $L$. 

$$E[t_L(h_N)|\phi_L] = \frac{1}{v_a} \sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1}) - q_L(h_{k-1})|\phi_L]$$

$$= \frac{1}{v_a} \sum_{k=1}^{N} \delta^{k-1} [\mu_H - \gamma \omega_t \alpha^{k-1} (\mu_H - \mu_L) - \mu_L - (1 - \gamma) \omega_t (\mu_H - \mu_L)]$$

$$= \frac{(\mu_H - \mu_L)}{v_a} \sum_{k=1}^{N} \delta^{k-1} [1 - \gamma \omega_t \alpha^{k-1} - (1 - \gamma) \omega_t]$$

$$= \frac{(\mu_H - \mu_L)}{v_a} [s_{\delta} - \gamma \omega_t s_{\alpha \delta} - (1 - \gamma) \omega_t s_{\delta}]$$

$$= \frac{(\mu_H - \mu_L)}{v_a} [\gamma s_{\alpha \delta} + (1 - \gamma) s_{\delta}]$$

Clearly, the solution of the relaxed problem satisfies the non-negativity constraint on transfers whenever $\omega_t \leq \omega_N$.

To complete the characterization, we show that the solution of the relaxed problem always satisfies constraint (ICHT).

$$v_a E[t_H(h_N)|\phi_H] + \sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1})|\phi_H] \geq v_a E[t_L(h_N)|\phi_H] + \sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1})|\phi_H] \quad \text{(ICHT)}$$

Because no transfers are associated with the high report in the candidate solution, this constraint can be rewritten:

$$\sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1})|\phi_H] \geq v_a E[t_L(h_N)|\phi_H] + \sum_{k=1}^{N} \delta^{k-1} E[q_L(h_{k-1})|\phi_H] \quad \text{(ICHT)}$$

$$E[t_L(h_N)|\phi_H] \leq \frac{1}{v_a} \sum_{k=1}^{N} \delta^{k-1} E[q_H(h_{k-1}) - q_L(h_{k-1})|\phi_H]$$

$$\leq \frac{1}{v_a} \sum_{k=1}^{N} \delta^{k-1} [\mu_H - \gamma \omega_t (\mu_H - \mu_L) - \mu_L - (1 - \gamma) \omega_t (\mu_H - \mu_L)]$$

$$\leq \frac{(\mu_H - \mu_L)}{v_a} \sum_{k=1}^{N} \delta^{k-1} [1 - \omega_t]$$

$$\leq \frac{(\mu_H - \mu_L)}{v_a} s_{\delta} (1 - \omega_t)$$
Hence constraint (ICHT) becomes

\[
\frac{(\mu_H - \mu_L)(\gamma s_{\alpha \delta} + (1 - \gamma)s_{\delta})}{v_a} [\omega_N - \omega_t] \leq \frac{(\mu_H - \mu_L)}{v_a} s_\delta (1 - \omega_t)
\]

\[
(\gamma s_{\alpha \delta} + (1 - \gamma)s_{\delta})(\omega_N - \omega_t) \leq s_\delta (1 - \omega_t)
\]

\[
\omega_N - \omega_t \leq \omega_N (1 - \omega_t)
\]

Which is satisfied because \(\alpha > 1 \Rightarrow \omega_N < 1\).

Calculating the parties expected payoffs follows the calculation without transfers presented above.

\[
V^t_N = \bar{V}_N - s_\delta \omega_t (\gamma - \gamma^2)(\mu_H - \mu_L)^2 - s_\delta \frac{v_p}{v_a} (1 - \frac{\omega_t}{\omega_N})(\mu_H - \mu_L)
\]

\[
U^t_N = s_\delta (\gamma \mu_L + (1 - \gamma)\mu_H) + s_\delta \gamma (\mu_H - \mu_L)(1 - \frac{\omega_t}{\omega_N})
\]

This completes the proof. \(\blacksquare\)
A.1 ANALYSIS OF MODEL WITH NOISY PRIVATE INFORMATION

Incentive constraints and payoff function with partially revealing private information. Consider the advocate’s expected lifetime payoff from reporting \( i \in \{G, B\} \) when the true signal realization is \( j \in \{G, B\} \):

\[
U(i|j) = \sum_{k=1}^{N} \delta^{k-1} E[q_i(h_{k-1})|S = j].
\]

Using the law of total expectation:

\[
E[q_i(h_{k-1})|S = j] = \Pr(\phi_L|S = j)E[q_i(h_{k-1})|\phi_L, S = j] + \Pr(\phi_H|S = j)E[q_i(h_{k-1})|\phi_H, S = j].
\]

By construction, the distribution of \( h_{k-1} \), depends on \( \phi \in \{\phi_H, \phi_L\} \), but it does not depend directly on the realization of the signal; that is, the history \( h_{k-1} \) and the signal, \( S \), are independent conditional on the true \( \phi_i \in \{\phi_H, \phi_L\} \). Hence,

\[
E[q_i(h_{k-1})|\phi_L, S = j] = E[q_i(h_{k-1})|\phi_L] \quad \text{and} \quad E[q_i(h_{k-1})|\phi_H, S = j] = E[q_i(h_{k-1})|\phi_H],
\]

and hence,

\[
E[q_i(h_{k-1})|S = j] = \Pr(\phi_L|S = j)E[q_i(h_{k-1})|\phi_L] + \Pr(\phi_H|S = j)E[q_i(h_{k-1})|\phi_H],
\]

\[
E[q_i(h_{k-1})|S = j] = \gamma_j E[q_i(h_{k-1})|\phi_L] + (1 - \gamma_j)E[q_i(h_{k-1})|\phi_H].
\]

Therefore, the incentive constraints are:

\[
U(B|B) \geq U(G|B) \iff \sum_{k=1}^{N} \delta^{k-1} \{\gamma_B E[q_B(h_{k-1})|\phi_L] + (1 - \gamma_B)E[q_B(h_{k-1})|\phi_H]\} \geq 0 \quad (ICB)
\]

\[
U(G|G) \geq U(B|G) \iff \sum_{k=1}^{N} \delta^{k-1} \{\gamma_G E[q_G(h_{k-1})|\phi_L] + (1 - \gamma_G)E[q_G(h_{k-1})|\phi_H]\} \geq 0 \quad (ICG)
\]

as in the text.
To derive the planner’s payoff function, recall that \( \pi = \Pr(S = B) \). Because the planner’s mechanism is incentive compatible, whenever \( S = B \) sequence of actions \( q_B(h_{k-1}) \) is chosen, and whenever \( S = G \) sequence of actions \( q_G(h_{k-1}) \) is chosen. Hence, the planner’s expected payoff from offering mechanism \( m \) when the signal is \( S \) is the following:

\[
P(m, S) = -\pi \sum_{k=1}^{N} \delta^{k-1} E[(q_B(h_{k-1}) - x)^2 | S = B] - (1 - \pi) \sum_{k=1}^{N} \delta^{k-1} E[(q_G(h_{k-1}) - x)^2 | S = G].
\]

From the law of total expectation:

\[
E[(q_i(h_{k-1}) - x)^2 | S = i] = \Pr(\phi_L | S = i) E[(q_i(h_{k-1}) - x)^2 | \phi_L, S = i] + \Pr(\phi_H | S = i) E[(q_i(h_{k-1}) - x)^2 | \phi_H, S = i]
\]

for \( i \in \{G, B\} \). By construction, the distribution of \( h_{k-1} \), depends on \( \phi \in \{\phi_H, \phi_L\} \), but it does not depend directly on the realization of the signal; that is, the history \( h_{k-1} \) and the signal, \( S \), are independent conditional on \( \phi \). Hence,

\[
E[(q_i(h_{k-1}) - x)^2 | S = i] = \Pr(\phi_L | S = i) E[(q_i(h_{k-1}) - x)^2 | \phi_L] + \Pr(\phi_H | S = i) E[(q_i(h_{k-1}) - x)^2 | \phi_H] = \gamma_i E[(q_i(h_{k-1}) - x)^2 | \phi_L] + (1 - \gamma_i) E[(q_i(h_{k-1}) - x)^2 | \phi_H],
\]

and hence,

\[
P(m, S) = -\pi \sum_{k=1}^{N} \delta^{k-1}\{\gamma_B E[(q_B(h_{k-1}) - x)^2 | \phi_L] + (1 - \gamma_B) E[(q_B(h_{k-1}) - x)^2 | \phi_H]\}
\]

\[
-(1 - \pi) \sum_{k=1}^{N} \delta^{k-1}\{\gamma_G E[(q_G(h_{k-1}) - x)^2 | \phi_L] + (1 - \gamma_G) E[(q_G(h_{k-1}) - x)^2 | \phi_H]\},
\]

as in the text.

**Proof. Proposition 7.1**

We first consider a relaxed problem in which only constraint (ICB) is imposed. We then show that the solution to the relaxed problem satisfies constraint (ICG) and is therefore optimal.

Consider a relaxed problem in which only constraint (ICB) is imposed. Because the objective function is concave in the choice variables and the constraint is linear in the choice variables, the solution of this relaxed problem is the unique combination of controls that satisfies the stationarity and primal feasibility condition.

**Step I.** We formulate the Lagrangian and derive the stationarity conditions. The objective function
of the relaxed problem is given by:

\[ -\pi \sum_{k=1}^{N} \delta^{k-1} \left\{ \gamma_B E[|q_B(h_{k-1}) - x|^2|\phi_L] + (1 - \gamma_B) E[(q_B(h_{k-1}) - x)^2|\phi_H] \right\} \]

\[ -(1 - \pi) \sum_{k=1}^{N} \delta^{k-1} \left\{ \gamma_G E[|q_G(h_{k-1}) - x|^2|\phi_L] + (1 - \gamma_G) E[(q_G(h_{k-1}) - x)^2|\phi_H] \right\} = \]

\[ = -\pi \gamma_B \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_L)[\int_{X} f(x|\phi_L)(q_B(h_{k-1}) - x)^2dx]dh_{k-1} \right. \]

\[ \left. - \pi (1 - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_H)[\int_{X} f(x|\phi_H)(q_B(h_{k-1}) - x)^2dx]dh_{k-1} \right. \right. \]

\[ \left. \left. - (1 - \pi) \gamma_G \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_L)[\int_{X} f(x|\phi_L)(q_G(h_{k-1}) - x)^2dx]dh_{k-1} \right. \right. \]

\[ \left. \left. \left. \left. - (1 - \pi)(1 - \gamma_G) \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_H)[\int_{X} f(x|\phi_H)(q_G(h_{k-1}) - x)^2dx]dh_{k-1} \right. \right. \right. \right. \]

Evaluating the integrals in brackets gives the following expression for the objective function:

\[ -\pi \gamma_B \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_L)[q_B(h_{k-1})]^2 - 2\mu_L q_B(h_{k-1}) + \eta_L]dh_{k-1} \right. \]

\[ \left. - \pi (1 - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_H)[q_B(h_{k-1})]^2 - 2\mu_H q_B(h_{k-1}) + \eta_H]dh_{k-1} \right. \right. \]

\[ \left. \left. - (1 - \pi) \gamma_G \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_L)[q_G(h_{k-1})]^2 - 2\mu_L q_G(h_{k-1}) + \eta_L]dh_{k-1} \right. \right. \]

\[ \left. \left. \left. \left. - (1 - \pi)(1 - \gamma_G) \sum_{k=1}^{N} \delta^{k-1} \left[ \int_{X_{k-1}} f(h_{k-1}|\phi_H)[q_G(h_{k-1})]^2 - 2\mu_H q_G(h_{k-1}) + \eta_H]dh_{k-1} \right. \right. \right. \right. \]

Consider (ICB):

\[ \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[q_B(h_{k-1}) - q_G(h_{k-1})|\phi_L] + (1 - \gamma_B) E[q_B(h_{k-1}) - q_G(h_{k-1})|\phi_H] \} \geq 0 \iff \text{(ICB)} \]

\[ \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[q_G(h_{k-1}) - q_B(h_{k-1})|\phi_L] + (1 - \gamma_B) E[q_G(h_{k-1}) - q_B(h_{k-1})|\phi_H] \} \leq 0 \iff \]

\[ \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \}[q_G(h_{k-1}) - q_B(h_{k-1})]dh_{k-1} \leq 0. \]
Hence the Lagrangian for the relaxed problem is:

\[
L = -\pi \gamma_B \sum_{k=1}^{N} \delta^{k-1} \int_{X_{h-1}} f(h_{k-1} | \phi_L) [q_B(h_{k-1})^2 - 2\mu_L q_B(h_{k-1}) + \eta_L] dh_{k-1}
\]

\[-\pi (1 - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) [q_B(h_{k-1})^2 - 2\mu_H q_B(h_{k-1}) + \eta_H] dh_{k-1}\]

\[-(1 - \pi) \gamma_G \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_L) [q_G(h_{k-1})^2 - 2\mu_L q_G(h_{k-1}) + \eta_L] dh_{k-1}\]

\[-(1 - \pi) (1 - \gamma_G) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} f(h_{k-1} | \phi_H) [q_G(h_{k-1})^2 - 2\mu_H q_G(h_{k-1}) + \eta_H] dh_{k-1}\]

\[-\pi \lambda \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{\gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)\} [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} \]

The stationarity conditions for the relaxed problem are therefore:

\[-\pi \gamma_B \delta^{k-1} f(h_{k-1} | \phi_L) [2q_B(h_{k-1}) - 2\mu_L] - \pi (1 - \gamma_B) \delta^{k-1} f(h_{k-1} | \phi_H) [2q_B(h_{k-1}) - 2\mu_H] + \pi \lambda \delta^{k-1} \{\gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)\} = 0\]

\[\iff \gamma_B f(h_{k-1} | \phi_L) [q_B(h_{k-1}) - \mu_L] + (1 - \gamma_B) f(h_{k-1} | \phi_H) [q_G(h_{k-1}) - \mu_H] - \frac{\lambda}{2} \{\gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)\} = 0\]

\[q_B(h_{k-1}) = \frac{\gamma_B f(h_{k-1} | \phi_L) \mu_L + (1 - \gamma_B) f(h_{k-1} | \phi_H) \mu_H}{\gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)} + \frac{\lambda}{2} \iff q_B(h_{k-1}) = \hat{\mu}(\gamma_B, h_{k-1}) + \frac{\lambda}{2}\]

and

\[-(1 - \pi) \gamma_G \delta^{k-1} f(h_{k-1} | \phi_L) [2q_G(h_{k-1}) - 2\mu_L] - (1 - \pi) (1 - \gamma_G) \delta^{k-1} f(h_{k-1} | \phi_H) [2q_G(h_{k-1}) - 2\mu_H] + \pi \lambda \delta^{k-1} \{\gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)\} = 0\]

\[\iff \gamma_G f(h_{k-1} | \phi_L) [q_G(h_{k-1}) - \mu_L] + (1 - \gamma_G) f(h_{k-1} | \phi_H) [q_G(h_{k-1}) - \mu_H] + \frac{\lambda}{1 - \pi} \frac{\lambda}{2} \{\gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)\} = 0\]

\[q_G(h_{k-1}) = \frac{\gamma_G f(h_{k-1} | \phi_L) \mu_L + (1 - \gamma_G) f(h_{k-1} | \phi_H) \mu_H}{\gamma_G f(h_{k-1} | \phi_L) + (1 - \gamma_G) f(h_{k-1} | \phi_H)} - \frac{\pi}{1 - \pi} \frac{\lambda}{2} \gamma_B f(h_{k-1} | \phi_L) + (1 - \gamma_B) f(h_{k-1} | \phi_H)\]

\[\iff q_G(h_{k-1}) = \hat{\mu}(\gamma_G, h_{k-1}) - \frac{\pi}{1 - \pi} \frac{\lambda}{2} \gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)\]
Define \( w \equiv \lambda/(2(1 - \pi)(\mu_H - \mu_L)) \). Hence, there exists some value \( w \) such that the optimal mechanism in the relaxed problem is given by:

\[
q_B(h_{k-1}) = \mu_G(h_{k-1}) + (1 - \pi)w(\mu_H - \mu_L) \\
q_G(h_{k-1}) = \mu_G(h_{k-1}) - \pi w \frac{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G)}(\mu_H - \mu_L),
\]

completing Step I.

Before moving to Step II, consider the difference \( q_G(h_{k-1}) - q_B(h_{k-1}) \) for a given value of \( w \):

\[
q_G(h_{k-1}) - q_B(h_{k-1}) = \mu_G(h_{k-1}) - \mu_B(h_{k-1}) - w(\mu_H - \mu_L)\{(1 - \pi) + \pi \frac{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G)}\} = \\
(\mu_H - \mu_L)\left(\frac{(\gamma_B - \gamma_G) \Lambda(h_{k-1})}{(\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B))(\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G))}\right) - w\{(1 - \pi) + \pi \frac{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G)}\}.
\]

Substituting \( \pi \equiv (\gamma - \gamma_G)/(\gamma_B - \gamma_G) \) yields:

\[q_G(h_{k-1}) - q_B(h_{k-1}) = (\mu_H - \mu_L)\frac{(\gamma_B - \gamma_G) \Lambda(h_{k-1}) - w(\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B))(\gamma \Lambda(h_{k-1}) + (1 - \gamma))}{(\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B))(\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G))}.
\]

Let \( J(l, w) \) be the following function:

\[J(l, w) \equiv (\mu_H - \mu_L)\frac{(\gamma_B - \gamma_G) l - w(\gamma_B l + (1 - \gamma_B))(\gamma l + (1 - \gamma))}{(\gamma_B l + (1 - \gamma_B))(\gamma_G l + (1 - \gamma_G))},\]

defined on \( R^2_+ \). Note from (2) that \( q_G(h_{k-1}) - q_B(h_{k-1}) = J(\Lambda(h_{k-1}), w) \).

**Step II.** We solve the relaxed problem by characterizing the optimal \( w \) and showing that it is strictly positive. Note first that the left hand side of (ICB) is a decreasing, linear function of \( w \). Indeed, its derivative with respect to \( w \) is negative and independent of \( w \):

\[
\frac{d}{dw} \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H)\} [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} = \\
- (\mu_H - \mu_L) \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H)\} \frac{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G)} dh_{k-1} < 0.
\]

Note second that for all \( l > 0 \),

\[J(l, 0) = (\mu_H - \mu_L)\frac{(\gamma_B - \gamma_G) l}{(\gamma_B l + (1 - \gamma_B))(\gamma_G l + (1 - \gamma_G))} > 0,
\]
and hence, for \( w = 0 \) the left hand side of (ICB) is positive:

\[
\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} = 0
\]

and equals zero for \( w > 0 \). By complementary slackness, (ICB) holds with equality in the solution to the relaxed problem. Because the left hand side of (ICB) is a decreasing, linear function of \( w \) that is positive for \( w = 0 \), a unique value of \( w \) exists for which equality is attained. Denote this value \( \omega_r \):

\[
w = \omega_r \iff \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} J(\Lambda(h_{k-1}), \omega_r) dh_{k-1} = 0
\]

Hence, if \( w = 0 \), or equivalently, \( \lambda = 0 \), then constraint (ICB) is violated. (ICB) is therefore binding in the relaxed problem, implying \( \lambda > 0 \), or equivalently \( w > 0 \). By complementary slackness, (ICB) holds with equality in the solution to the relaxed problem. Because the left hand side of (ICB) is a decreasing, linear function of \( w \) that is positive for \( w = 0 \), a unique value of \( w \) exists for which equality is attained. Denote this value \( \omega_r \):

\[
w = \omega_r \iff \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} = 0
\]

Hence, the optimal mechanism that solves the relaxed problem is

\[
q_B(h_{k-1}) = \hat{\mu}(\gamma_B, h_{k-1}) + (1 - \pi) \omega_r (\mu_H - \mu_L)
\]

\[
q_G(h_{k-1}) = \hat{\mu}(\gamma_G, h_{k-1}) - \pi \omega_r \frac{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G)} (\mu_H - \mu_L),
\]

completing Step II.

**Step III.** We show that \( \omega_r < \gamma_B - \gamma_G \). To do so, we establish that for \( w = \gamma_B - \gamma_G \):

(i) \[
\int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} = 0 \text{ for } k = 1,
\]

(ii) \[
\int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} < 0 \text{ for all } k \geq 2.
\]

If these are both established, then the left hand side of (ICB), which is the discounted sum of these terms, is strictly negative for \( w = \gamma_B - \gamma_G \). Because the left hand side of (ICB) is decreasing in \( w \) and equals zero for \( w = \omega_r \), the desired result, \( \omega_r < \gamma_B - \gamma_G \) follows. Therefore, suppose that \( w = \gamma_B - \gamma_G \) and consider the difference \( q_G(h_{k-1}) - q_B(h_{k-1}) \):

\[
q_G(h_{k-1}) - q_B(h_{k-1}) = J(\Lambda(h_{k-1}), \gamma_B - \gamma_G).
\]
Simplifying \( J(\Lambda(h_{k-1}), \gamma_B - \gamma_G) \) gives the following:

\[
q_G(h_{k-1}) - q_B(h_{k-1}) = \frac{(\mu_H - \mu_L)(\gamma_B - \gamma_G)(\gamma\Lambda(h_{k-1}) + (1 - \gamma))}{(\gamma_B\Lambda(h_{k-1}) + (1 - \gamma_B))(\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G))(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)})(\Lambda(h_{k-1}) - 1)}.
\]

Note that \( \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H) = f(h_{k-1}|\phi_H)(\gamma_B\Lambda(h_{k-1}) + (1 - \gamma_B)) \). Hence, the integrand in (i) and (ii) is

\[
-(\mu_H - \mu_L)(\gamma_B - \gamma_G)\gamma\Lambda(h_{k-1}) + (1 - \gamma)\frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H).
\]

Recall that \( \Lambda(h_0) = 1 \), and note that for \( k = 1 \) the integrand is zero. Hence, (i). Because \( (\mu_H - \mu_L) > 0 \) and \( (\gamma_B - \gamma_G) > 0 \), condition (ii) is equivalent to the following:

\[
(6) \quad \int_{X_{k-1}} \frac{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)})(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) > 0.
\]

Note that:

\[
\frac{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}) - (\gamma_B - (1 - \gamma)) = \frac{\Lambda(h_{k-1}) - 1}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}.
\]

and hence,

\[
\Lambda(h_{k-1}) > 1 \iff \frac{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}) > \gamma_B - (1 - \gamma)
\]

\[
\Lambda(h_{k-1}) < 1 \iff \frac{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}) < \gamma_B - (1 - \gamma)
\]

\[
\Lambda(h_{k-1}) = 1 \iff \frac{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}) = \gamma_B - (1 - \gamma).
\]

Because \( (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \geq 0 \iff \Lambda(h_{k-1}) \geq 1 \), multiplying the preceding inequalities by \( (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \) yields:

\[
(\Lambda(h_{k-1}) - 1) f(h_{k-1}|\phi_H) \frac{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G\Lambda(h_{k-1}) + (1 - \gamma_G)}(\gamma_B - \frac{1 - \gamma}{\gamma\Lambda(h_{k-1}) + (1 - \gamma)}) > (\Lambda(h_{k-1}) - 1) f(h_{k-1}|\phi_H)(\gamma_B - (1 - \gamma)),
\]
Step IV. We show that for the solution to the relaxed problem, \( q_G(h_0) - q_B(h_0) > 0 \). For the solution to the relaxed problem, \( q_G(h_0) - q_B(h_0) = J(\Lambda(h_0), \omega_r) \). Because \( \Lambda(h_0) = 1 \), it follows that \( q_G(h_0) - q_B(h_0) = J(1, \omega_r) \), and hence (3) implies:

\[
q_G(h_0) - q_B(h_0) = (\mu_H - \mu_L)(\gamma_B - \gamma_G - \omega_r) > 0,
\]

where the last inequality follows from Step III.

\[
\Lambda(h_{k-1}) < 1 \Rightarrow
\]
\[
(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \frac{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma)} (\gamma_B - \frac{1 - \gamma}{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}) >
\]
\[
(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H)(\gamma_B - (1 - \gamma)),
\]

\[
\Lambda(h_{k-1}) = 1 \Rightarrow
\]
\[
(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \frac{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma)} (\gamma_B - \frac{1 - \gamma}{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}) =
\]
\[
(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H)(\gamma_B - (1 - \gamma)).
\]

Hence,

\[
\forall h_{k-1} \in X_{k-1}, \quad (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \frac{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma)} (\gamma_B - \frac{1 - \gamma}{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}) \geq
\]
\[
(\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H)(\gamma_B - (1 - \gamma)),
\]

with strict inequality for histories satisfying \( \Lambda(h_{k-1}) \neq 1 \). Because \( \mu_H > \mu_L \) and the distributions have common support, for all \( k \) there must be a non-negligible set of histories satisfying \( \Lambda(h_{k-1}) \neq 1 \). Integrating implies:

\[
\int_{X_{k-1}} (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \frac{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma)} (\gamma_B - \frac{1 - \gamma}{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}) dh_{k-1} >
\]
\[
\int_{X_{k-1}} (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H)(\gamma_B - (1 - \gamma)) dh_{k-1}.
\]

Note that \( (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) = f(h_{k-1}|\phi_L) - f(h_{k-1}|\phi_H) \). Hence,

\[
\int_{X_{k-1}} (\Lambda(h_{k-1}) - 1)f(h_{k-1}|\phi_H) \frac{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma)} (\gamma_B - \frac{1 - \gamma}{\gamma \Lambda(h_{k-1}) + (1 - \gamma)}) dh_{k-1} >
\]
\[
\int_{X_{k-1}} (f(h_{k-1}|\phi_L) - f(h_{k-1}|\phi_H))(\gamma_B - (1 - \gamma)) dh_{k-1} = 0,
\]

where the last equality follows from common support of \( f(x|\phi_i) \) for \( i \in \{H, L\} \). Hence, (6), which is equivalent to (ii). This completes the proof of Step III.
**Step V.** We establish the existence of a finite $\Lambda^* > 1$, such that the solution to the relaxed problem satisfies the following:

$$\forall h_{k-1} \in X_{k-1}, \quad q_G(h_{k-1}) - q_B(h_{k-1}) \geq 0 \iff \Lambda(h_{k-1}) \leq \Lambda^*.$$  

First, consider function $J(l, \omega_r)$ for $l \in [0, \infty)$. Note the following simple properties:

1. $J(l, \omega_r)$ is a continuous function of $l$.
2. $J(0, \omega_r) = -\omega_r(1 - \gamma_B)(1 - \gamma) < 0$.
3. $\lim_{l \to \infty} J(l, \omega_r) = -\omega_r \frac{\gamma}{\gamma_G} < 0$.
4. The denominator of $J(l, \omega_r)$ is strictly positive for all $l \in [0, \infty)$.
5. The numerator of $J(l, \omega_r)$ is a quadratic function of $l$, with negative coefficient on the quadratic term, $-(\mu_H - \mu_L)\omega_r\gamma_B < 0$.

Properties 1, 4, and 5 imply that $J(l, \omega_r)$ has at most two sign changes for $l \in [0, \infty)$. Properties 1, 2, and 3 imply that the number of sign changes is either zero or two. Because $J(0, \omega_r) < 0$ and Step IV establishes that $J(1, \omega_r) > 0$, the number of sign changes is non-zero. Hence $J(l, \omega_r)$ has two sign changes for $l \in [0, \infty)$:

$$(7) \quad \exists l_1 \in (0, 1) \text{ and } l_2 \in (l_1, \infty) \text{ such that } J(l_1, \omega_r) = J(l_2, \omega_r) = 0,$$

$$J(l, \omega_r) > 0 \iff l \in (l_1, l_2),$$

$$J(l, \omega_r) < 0 \iff l \in [0, l_1) \cup (l_2, \infty).$$

Furthermore,

$$(8) \quad J(1, \omega_r) > 0 \Rightarrow l_2 > 1 > l_1,$$

$$(9) \quad J\left(\frac{(1 - \gamma)(1 - \gamma_B)}{\gamma_B}, \omega_r\right) = \frac{\gamma_B - \gamma_G - \omega_r}{\gamma_G(1 - \gamma) + \gamma_B(\gamma - \gamma_G)} > 0 \Rightarrow l_2 > \frac{(1 - \gamma)(1 - \gamma_B)}{\gamma B} > l_1.$$ 

Next, recall assumption (A):

$$\gamma_B \geq \frac{1 - \gamma}{\gamma L^{N-1} + (1 - \gamma)} \quad \text{where} \quad L \equiv \min_{x \in X} \Lambda(x).$$

Hence,

$$L^{N-1} \geq \frac{(1 - \gamma)(1 - \gamma_B)}{\gamma B}.$$ 

Because $L < 1$, for $k \leq N$,

$$L^{k-1} \geq L^{N-1} \geq \frac{(1 - \gamma)(1 - \gamma_B)}{\gamma B}.$$
From the definition of $L$, it follows that $\forall h_{k-1} \in X_{k-1}$, $\Lambda(h_{k-1}) \geq L^{k-1}$, and hence:

$$\forall h_{k-1} \in X_{k-1}, \quad \Lambda(h_{k-1}) \geq \frac{(1 - \gamma)(1 - \gamma_B)}{\gamma B}.$$  

Hence, (9) implies:

$$\forall h_{k-1} \in X_{k-1}, \quad \Lambda(h_{k-1}) > l_1.$$  

Hence, (7) implies:

$$\forall h_{k-1} \in X_{k-1},$$

\begin{align*}
J(\Lambda(h_{k-1}), \omega_r) = 0 & \iff \Lambda(h_{k-1}) = l_2, \\
J(\Lambda(h_{k-1}), \omega_r) > 0 & \iff \Lambda(h_{k-1}) < l_2, \\
J(\Lambda(h_{k-1}), \omega_r) < 0 & \iff \Lambda(h_{k-1}) > l_2.
\end{align*}

Hence, for the solution to the relaxed problem, there exists a finite value $\Lambda^* = l_2$ such that:

$$\forall h_{k-1} \in X_{k-1}, \quad q_G(h_{k-1}) - q_B(h_{k-1}) \geq 0 \iff \Lambda(h_{k-1}) \leq \Lambda^*.$$  

To complete Step V, note that (8) implies $l_2 > 1$, and hence, $\Lambda^* > 1$.

**Step VI.** We establish that the solution to the relaxed problem satisfies the following inequality for all $k \leq N$:

$$\int_{X_{k-1}} \{\gamma_G f(h_{k-1}|\phi_L) + (1 - \gamma_G)f(h_{k-1}|\phi_H)\}[q_G(h_{k-1}) - q_B(h_{k-1})]dh_{k-1} \geq \frac{\gamma_G \Lambda^* + (1 - \gamma_G)}{\gamma B \Lambda^* + (1 - \gamma_B)} \int_{X_{k-1}} \{\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)\}[q_G(h_{k-1}) - q_B(h_{k-1})]dh_{k-1}.$$  

Consider $k = 1$. Note that

$$1 - \frac{\gamma_G \Lambda^* + (1 - \gamma_G)}{\gamma B \Lambda^* + (1 - \gamma_B)} = (\gamma_B - \gamma_G)(\Lambda^* - 1) > 0,$$

where the last inequality follows from Step V, which establishes $\Lambda^* > 1$. From Step IV, $q_G(h_0) - q_B(h_0) > 0$, and hence:

$$q_G(h_0) - q_B(h_0) > \frac{\gamma_G \Lambda^* + (1 - \gamma_G)}{\gamma B \Lambda^* + (1 - \gamma_B)}[q_G(h_0) - q_B(h_0)],$$

establishing the result for $k = 1$.

Consider $k \geq 2$. Note that

$$\frac{\gamma_G \Lambda(h_{k-1}) + (1 - \gamma_G)}{\gamma B \Lambda(h_{k-1}) + (1 - \gamma_B)} - \frac{\gamma_G \Lambda^* + (1 - \gamma_G)}{\gamma B \Lambda^* + (1 - \gamma_B)} = \frac{\Lambda^* - \Lambda(h_{k-1})}{\gamma B \Lambda(h_{k-1}) + (1 - \gamma_B)(\gamma G \Lambda(h_{k-1}) + (1 - \gamma_G))},$$
and hence,

\[
\begin{align*}
\Lambda(h_{k-1}) & \leq \Lambda^* \Rightarrow \frac{\gamma G \Lambda(h_{k-1}) + (1 - \gamma G)}{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)} \geq \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} \\
\Lambda(h_{k-1}) & > \Lambda^* \Rightarrow \frac{\gamma G \Lambda(h_{k-1}) + (1 - \gamma G)}{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)} < \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)}.
\end{align*}
\]

From Step V,

\[
\forall h_{k-1} \in X_{k-1}, \quad \Lambda(h_{k-1}) \geq \Lambda^* \iff q_G(h_{k-1}) - q_B(h_{k-1}) \geq 0.
\]

Hence, multiplying (11) by \([q_G(h_{k-1}) - q_B(h_{k-1})]\) implies that \(\forall h_{k-1} \in X_{k-1}\),

\[
\begin{align*}
\Lambda(h_{k-1}) & \leq \Lambda^* \Rightarrow \frac{\gamma G \Lambda(h_{k-1}) + (1 - \gamma G)}{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)} [q_G(h_{k-1}) - q_B(h_{k-1})] \geq \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [q_G(h_{k-1}) - q_B(h_{k-1})] \\
\Lambda(h_{k-1}) & > \Lambda^* \Rightarrow \frac{\gamma G \Lambda(h_{k-1}) + (1 - \gamma G)}{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)} [q_G(h_{k-1}) - q_B(h_{k-1})] > \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [q_G(h_{k-1}) - q_B(h_{k-1})],
\end{align*}
\]

and hence, \(\forall h_{k-1} \in X_{k-1}\),

\[
\frac{\gamma G \Lambda(h_{k-1}) + (1 - \gamma G)}{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)} [q_G(h_{k-1}) - q_B(h_{k-1})] \geq \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [q_G(h_{k-1}) - q_B(h_{k-1})].
\]

Multiplying both sides by \(\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)\) gives:

\[
\frac{\gamma G \Lambda(h_{k-1}) + (1 - \gamma G)}{\gamma_B \Lambda(h_{k-1}) + (1 - \gamma_B)} [\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)] [q_G(h_{k-1}) - q_B(h_{k-1})] \geq \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)] [q_G(h_{k-1}) - q_B(h_{k-1})],
\]

and hence, \(\forall h_{k-1} \in X_{k-1}\),

\[
\frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)] [q_G(h_{k-1}) - q_B(h_{k-1})] \geq \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)] [q_G(h_{k-1}) - q_B(h_{k-1})].
\]

Integrating gives:

\[
\int_{X_{k-1}} \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} [\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)] [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} \geq \frac{\gamma G \Lambda^* + (1 - \gamma G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} \int_{X_{k-1}} [\gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B)f(h_{k-1}|\phi_H)] [q_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1},
\]

completing Step VI.
Step VII. To complete the proof, we show that the solution to the relaxed problem satisfies constraint (ICG). From Step VI, the solution to the relaxed problem satisfies the following inequality for all \( k \leq N \):

\[
\int_{X_{k-1}} \{ \gamma_G f(h_{k-1}|\phi_L) + (1 - \gamma_G) f(h_{k-1}|\phi_H) \} [g_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} \geq \frac{\gamma_G \Lambda^* + (1 - \gamma_G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} \int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} [g_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1}.
\]

Forming the discounted sum yields:

\[
\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_G f(h_{k-1}|\phi_L) + (1 - \gamma_G) f(h_{k-1}|\phi_H) \} [g_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} \geq \frac{\gamma_G \Lambda^* + (1 - \gamma_G)}{\gamma_B \Lambda^* + (1 - \gamma_B)} \sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_B f(h_{k-1}|\phi_L) + (1 - \gamma_B) f(h_{k-1}|\phi_H) \} [g_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1}.
\]

Note that from equation (4), the right hand side of this inequality is zero (it is actually the left hand side of (ICB), which is equal to zero in the solution to the relaxed problem). Hence,

\[
\sum_{k=1}^{N} \delta^{k-1} \int_{X_{k-1}} \{ \gamma_G f(h_{k-1}|\phi_L) + (1 - \gamma_G) f(h_{k-1}|\phi_H) \} [g_G(h_{k-1}) - q_B(h_{k-1})] dh_{k-1} \geq 0,
\]

and hence,

\[
\sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[g_G(h_{k-1}) - q_B(h_{k-1})|\phi_L] + (1 - \gamma_G) E[g_G(h_{k-1}) - q_B(h_{k-1})|\phi_H] \} \geq 0,
\]

which is (ICG), completing the proof of the proposition. ■
**Proof. Proposition 7.2.** We show that under the more informative information structure, the planner can offer an incentive compatible mechanism that dominates her payoff under the optimal mechanism associated with the less informative information structure. The planner’s payoff must therefore be higher with a more informative signal.

Suppose that the prior is \( \gamma \). Signal \( S' \) updates beliefs to \((\gamma'_B, \gamma'_G)\) and signal \( S \) updates beliefs to \((\gamma_B, \gamma_G)\). Signal \( S' \) is more informative in the Blackwell sense, i.e. \( \gamma'_G \leq \gamma < \gamma < \gamma_B \leq \gamma'_B \). Let \( m^* \equiv \{q^*_B(h_{k-1}), q^*_G(h_{k-1})\} \) denote the planner’s optimal evolving influence mechanism when the advocate observes signal \( S \). Because the first best mechanism for signal \( S \) violates incentive compatibility for type \( B \), the incentive constraint for type \( B \) must be binding, and hence:

\[
\begin{align*}
(12) \quad & \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[q^*_B(h_{k-1})|\phi_L] + (1 - \gamma_B)E[q^*_B(h_{k-1})|\phi_H] \} = \\
& \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[q^*_G(h_{k-1})|\phi_L] + (1 - \gamma_B)E[q^*_G(h_{k-1})|\phi_H] \}.
\end{align*}
\]

In addition, \( m^* \) satisfies incentive compatibility for type \( G \), and hence:

\[
\begin{align*}
(13) \quad & \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[q^*_G(h_{k-1})|\phi_L] + (1 - \gamma_G)E[q^*_G(h_{k-1})|\phi_H] \} \leq \\
& \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[q^*_G(h_{k-1})|\phi_L] + (1 - \gamma_G)E[q^*_G(h_{k-1})|\phi_H] \}.
\end{align*}
\]

Finally, let \( P(m^*, S) \) represent the planner’s maximized payoff when the advocate’s signal is \( S \):

\[
(14) \quad P(m^*, S) = -\pi \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[(q^*_B(h_{k-1}) - x)^2|\phi_L] + (1 - \gamma_B)E[(q^*_B(h_{k-1}) - x)^2|\phi_H] \} \\
- (1 - \pi) \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[(q^*_G(h_{k-1}) - x)^2|\phi_L] + (1 - \gamma_G)E[(q^*_G(h_{k-1}) - x)^2|\phi_H] \}.
\]

Consider an evolving influence mechanism \( \bar{m} \equiv \{\bar{q}_B(h_{k-1}), \bar{q}_G(h_{k-1})\}_{k=1}^{N} \) where:

\[
(15) \quad \begin{align*}
\bar{q}_B(h_{k-1}) &= bq^*_B(h_{k-1}) + (1 - b)q^*_G(h_{k-1}) \\
\bar{q}_G(h_{k-1}) &= gq^*_B(h_{k-1}) + (1 - g)q^*_G(h_{k-1}),
\end{align*}
\]

and \( b > g \).

**Step I.** We show that mechanism \( \bar{m} \) is incentive compatible when the advocate’s signal is \( S' \);
that is, we show that the following inequalities hold:

\[
\sum_{k=1}^{N} \delta^{k-1} \gamma_B' E[q_B(h_{k-1})|\phi_L] + (1 - \gamma_B') E[q_B(h_k)|\phi_H] \geq \\
\sum_{k=1}^{N} \delta^{k-1} \gamma_G' E[q_G(h_{k-1})|\phi_L] + (1 - \gamma_G') E[q_G(h_k)|\phi_H],
\]

\[
\sum_{k=1}^{N} \delta^{k-1} \gamma_B' E[q_B(h_{k-1})|\phi_L] + (1 - \gamma_B') E[q_B(h_k)|\phi_H] \geq \\
\sum_{k=1}^{N} \delta^{k-1} \gamma_G' E[q_G(h_{k-1})|\phi_L] + (1 - \gamma_G') E[q_G(h_k)|\phi_H].
\]

Inequality (16) is the incentive compatibility constraint for type B and (17) is the incentive compatibility constraint for type G when the advocate's signal is $S'$.

**Proof of (16).** Substituting $q_B(h_{k-1})$ and $q_G(h_{k-1})$ into (16) gives:

\[
\sum_{k=1}^{N} \delta^{k-1}\left(\gamma_B'\{bE[q_B^*(h_{k-1})|\phi_L] + (1 - b)E[q_B^*(h_k)|\phi_H]\} \\
+ (1 - \gamma_B')\{bE[q_B^*(h_{k-1})|\phi_H] + (1 - b)E[q_B^*(h_k)|\phi_H]\}\right) \geq \\
\sum_{k=1}^{N} \delta^{k-1}\left(\gamma_B'\{gE[q_B^*(h_{k-1})|\phi_L] + (1 - g)E[q_B^*(h_k)|\phi_L]\} \\
+ (1 - \gamma_B')\{gE[q_B^*(h_{k-1})|\phi_H] + (1 - g)E[q_B^*(h_k)|\phi_H]\}\right),
\]

which holds if and only if:

\[
(b - g) \sum_{k=1}^{N} \delta^{k-1}\{\gamma_B' E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma_B') E[q_B^*(h_k)|\phi_H]\} \geq \\
(b - g) \sum_{k=1}^{N} \delta^{k-1}\{\gamma_B' E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma_B') E[q_B^*(h_k)|\phi_H]\}
\]

Because $b > g$, dividing both sides of inequality (18) by $b - g > 0$ yields:

\[
\sum_{k=1}^{N} \delta^{k-1}\{\gamma_B' E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma_B') E[q_B^*(h_k)|\phi_H]\} \geq \\
\sum_{k=1}^{N} \delta^{k-1}\{\gamma_B' E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma_B') E[q_B^*(h_k)|\phi_H]\}.
\]
Subtracting (12) from (19) gives:

\[(\gamma_B' - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{ E[q_B^*(h_k - 1)|\phi_L] - E[q_B^*(h_k - 1)|\phi_H] \} \geq 0 \]  \tag{20}

\[(\gamma_B' - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{ E[q_G^*(h_k - 1)|\phi_L] - E[q_G^*(h_k - 1)|\phi_H] \}. \]

Multiplying both sides of (20) by \((\gamma_G - \gamma_B)/(\gamma_B' - \gamma_B) < 0\) gives:

\[(\gamma_B - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{ E[q_B^*(h_k - 1)|\phi_L] - E[q_B^*(h_k - 1)|\phi_H] \} \leq 0 \]

\[(\gamma_B - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{ E[q_G^*(h_k - 1)|\phi_L] - E[q_G^*(h_k - 1)|\phi_H] \}. \]

Adding (12) to (21) gives:

\[\gamma_G \sum_{k=1}^{N} \delta^{k-1} \{ E[q_B^*(h_k - 1)|\phi_L] - E[q_B^*(h_k - 1)|\phi_H] \} + \sum_{k=1}^{N} \delta^{k-1} E[q_B^*(h_k - 1)|\phi_H] \leq \]

\[\gamma_G \sum_{k=1}^{N} \delta^{k-1} \{ E[q_G^*(h_k - 1)|\phi_L] - E[q_G^*(h_k - 1)|\phi_H] \} + \sum_{k=1}^{N} \delta^{k-1} E[q_B^*(h_k - 1)|\phi_H]. \]

\[\gamma_G \sum_{k=1}^{N} \delta^{k-1} \{ E[q_B^*(h_k - 1)|\phi_L] - E[q_B^*(h_k - 1)|\phi_H] \} + \sum_{k=1}^{N} \delta^{k-1} E[q_B^*(h_k - 1)|\phi_H]. \]

Rearranging (22) gives:

\[\sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[q_B^*(h_k - 1)|\phi_L] + (1 - \gamma_G)E[q_B^*(h_k - 1)|\phi_H] \} \leq \]

\[\sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[q_G^*(h_k - 1)|\phi_L] + (1 - \gamma_G)E[q_G^*(h_k - 1)|\phi_H] \}. \]

Hence, (16) is true if and only if (23) is true. Note that (23) is identical to (13), and (13) is true because \(m^*\) is incentive compatible for type \(G\) with signal \(S\). Therefore, (16) is true.
Proof of (17). Substituting $q_B(h_{k-1})$ and $q_G(h_{k-1})$ into (17) gives:

\[
\sum_{k=1}^{N} \delta^{k-1} \left( \gamma_G' \{g E[q_B^*(h_{k-1})|\phi_L] + (1 - g) E[q_G^*(h_{k-1})|\phi_L]\} + (1 - \gamma'_G) \{g E[q_B^*(h_{k-1})|\phi_H] + (1 - g) E[q_G^*(h_{k-1})|\phi_H]\} \right) \geq \sum_{k=1}^{N} \delta^{k-1} \left( \gamma'_G \{b E[q_B^*(h_{k-1})|\phi_L] + (1 - b) E[q_G^*(h_{k-1})|\phi_L]\} + (1 - \gamma'_G) \{b E[q_B^*(h_{k-1})|\phi_H] + (1 - b) E[q_G^*(h_{k-1})|\phi_H]\} \right),
\]

which holds if and only if:

\[
(g - b) \sum_{k=1}^{N} \delta^{k-1} \left( \gamma'_G E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma'_G) E[q_B^*(h_{k-1})|\phi_H]\right) \geq (g - b) \sum_{k=1}^{N} \delta^{k-1} \left( \gamma'_G E[q_G^*(h_{k-1})|\phi_L] + (1 - \gamma'_G) E[q_G^*(h_{k-1})|\phi_H]\right).
\]

Because $b > g$, dividing both sides of inequality (24) by $g - b < 0$ yields:

\[
\sum_{k=1}^{N} \delta^{k-1} \left( \gamma'_G E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma'_G) E[q_B^*(h_{k-1})|\phi_H]\right) \leq \sum_{k=1}^{N} \delta^{k-1} \left( \gamma'_G E[q_G^*(h_{k-1})|\phi_L] + (1 - \gamma'_G) E[q_G^*(h_{k-1})|\phi_H]\right).
\]

Subtracting (12) from (25) gives:

\[
(g - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{E[q_B^*(h_{k-1})|\phi_L] - E[q_B^*(h_{k-1})|\phi_H]\} \leq (\gamma_G' - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{E[q_G^*(h_{k-1})|\phi_L] - E[q_G^*(h_{k-1})|\phi_H]\}.
\]

Multiplying both sides of (25) by $(\gamma_G - \gamma_B)/(\gamma_G' - \gamma_B) > 0$ gives:

\[
(\gamma_G - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{E[q_B^*(h_{k-1})|\phi_L] - E[q_B^*(h_{k-1})|\phi_H]\} \leq (\gamma_G - \gamma_B) \sum_{k=1}^{N} \delta^{k-1} \{E[q_G^*(h_{k-1})|\phi_L] - E[q_G^*(h_{k-1})|\phi_H]\}.
\]
Adding (12) to (27) gives:

\[
\gamma G \sum_{k=1}^{N} \delta^{k-1} \{ E[q_B^*(h_{k-1})|\phi_L] - E[q_B(h_{k-1})|\phi_H] \} + \sum_{k=1}^{N} \delta^{k-1} E[q_B^*(h_{k-1})|\phi_H] \leq \\
\gamma G \sum_{k=1}^{N} \delta^{k-1} \{ E[q_G^*(h_{k-1})|\phi_L] - E[q_G(h_{k-1})|\phi_H] \} + \sum_{k=1}^{N} \delta^{k-1} E[q_G^*(h_{k-1})|\phi_H].
\]

Rearranging (28) gives:

\[
\sum_{k=1}^{N} \delta^{k-1} \{ \gamma G E[q_B^*(h_{k-1})|\phi_L] + (1 - \gamma G) E[q_B^*(h_{k-1})|\phi_H] \} \leq \\
\sum_{k=1}^{N} \delta^{k-1} \{ \gamma G E[q_G^*(h_{k-1})|\phi_L] + (1 - \gamma G) E[q_G(h_{k-1})|\phi_H] \}.
\]

Hence, (17) is true if and only if (29) is true. Note that (29) is identical to (13), and (13) is true because \(m^*\) is incentive compatible for type \(G\) with signal \(S\). Therefore, (17) is true.

**Step II.** We show that for a particular combination of \((b, g)\), selecting mechanism \(\tilde{m}\) when the advocate’s signal is \(S'\) gives the planner a strictly larger expected payoff than selecting mechanism \(m^*\) when the advocate’s signal is \(S\).

Consider mechanism \(\tilde{m}\) as defined in (15), with the following values of \(b\) and \(g\):

\[
b = \left( \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G} \right) \left( \frac{\gamma_B - \gamma_G'}{\gamma - \gamma_G} \right) \quad g = \left( \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G} \right) \left( \frac{\gamma_B - \gamma_{G'}}{\gamma' - \gamma} \right).
\]

Clearly our assumptions on the Blackwell ranking of the signals \((\gamma_G' \leq \gamma < \gamma < \gamma_B \leq \gamma_{B'})\) imply that both \(b\) and \(g\) are positive. Furthermore,

\[
1 - b = \frac{(\gamma_B - \gamma)(\gamma_G - \gamma_G')}{(\gamma_B - \gamma_G)(\gamma - \gamma_G')} \geq 0 \quad 1 - g = \frac{(\gamma_B - \gamma)(\gamma_B' - \gamma_{G'})}{(\gamma_B' - \gamma_G)(\gamma_B - \gamma_G')} \geq 0,
\]

and hence, both \(b \in (0, 1)\) and \(g \in (0, 1)\). Note further that

\[
b - g = \left( \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G} \right) \left( \frac{\gamma_B - \gamma_G'}{\gamma - \gamma_G} \right) - \left( \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G} \right) \left( \frac{\gamma_B - \gamma_{B'}}{\gamma' - \gamma} \right) = \left( \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G} \right) \left( \frac{\gamma_B - \gamma}{\gamma_B - \gamma_G} \right) \left( \frac{\gamma_B - \gamma_{G'}}{\gamma' - \gamma} \right) > 0.
\]

The planner’s expected payoff of offering mechanism \(\tilde{m}\) when the advocate’s signal is \(S'\) is the
following:

\[
\begin{align*}
(32) \quad P(\bar{m}, S') & \equiv -\pi' \sum_{k=1}^{N} \delta^{k-1} \left\{ \gamma_B' E[(\bar{q}_B(h_{k-1}) - x)^2|\phi_L] + (1 - \gamma_B') E[(\bar{q}_B(h_{k-1}) - x)^2|\phi_H] \right\} \\
& -(1 - \pi') \sum_{k=1}^{N} \delta^{k-1} \left\{ \gamma_G' E[(\bar{q}_G(h_{k-1}) - x)^2|\phi_L] + (1 - \gamma_G') E[(\bar{q}_G(h_{k-1}) - x)^2|\phi_H] \right\}.
\end{align*}
\]

Observe that for \( i \in \{L, H\} \):

\[
\begin{align*}
E[(\bar{q}_B(h_{k-1}) - x)^2|\phi_i] &= E[(bq_B^*(h_{k-1}) + (1 - b)q_G^*(h_{k-1}) - x)^2|\phi_i] = \\
bE[(q_B(h_{k-1}) - x)^2|\phi_i] + (1 - b)E[(q_G^*(h_{k-1}) - x)^2|\phi_i] - b(1 - b)E[(q_B^*(h_{k-1}) - q_G^*(h_{k-1}))^2|\phi_i],
\end{align*}
\]

\[
\begin{align*}
E[(\bar{q}_G(h_{k-1}) - x)^2|\phi_i] &= E[(gq_B^*(h_{k-1}) + (1 - g)q_G^*(h_{k-1}) - x)^2|\phi_i] = \\
gE[(q_B(h_{k-1}) - x)^2|\phi_i] + (1 - g)E[(q_G^*(h_{k-1}) - x)^2|\phi_i] - g(1 - g)E[(q_B^*(h_{k-1}) - q_G^*(h_{k-1}))^2|\phi_i].
\end{align*}
\]

Hence, \( b \in (0, 1) \) and \( g \in (0, 1) \) imply:

\[
(33) \quad E[(\bar{q}_B(h_{k-1}) - x)^2|\phi_i] < bE[(q_B^*(h_{k-1}) - x)^2|\phi_i] + (1 - b)E[(q_G^*(h_{k-1}) - x)^2|\phi_i],
\]

\[
(34) \quad E[(\bar{q}_G(h_{k-1}) - x)^2|\phi_i] < gE[(q_B^*(h_{k-1}) - x)^2|\phi_i] + (1 - g)E[(q_G^*(h_{k-1}) - x)^2|\phi_i],
\]

for \( i \in \{L, H\} \). Using (33) and (34) in (32) yields:

\[
(35) \quad P(\bar{m}, S') > -\pi' \sum_{k=1}^{N} \delta^{k-1} \left\{ \gamma_B' \left\{ bE[(q_B^*(h_{k-1}) - x)^2|\phi_L] + (1 - b)E[(q_G^*(h_{k-1}) - x)^2|\phi_L] \right\} \\
+ (1 - \gamma_B') \left\{ bE[(q_B^*(h_{k-1}) - x)^2|\phi_H] + (1 - b)E[(q_G^*(h_{k-1}) - x)^2|\phi_H] \right\} \right\} \\
-(1 - \pi') \sum_{k=1}^{N} \delta^{k-1} \left\{ \gamma_G' \left\{ gE[(q_B^*(h_{k-1}) - x)^2|\phi_L] + (1 - g)E[(q_G^*(h_{k-1}) - x)^2|\phi_L] \right\} \\
+ (1 - \gamma_G') \left\{ gE[(q_B^*(h_{k-1}) - x)^2|\phi_H] + (1 - g)E[(q_G^*(h_{k-1}) - x)^2|\phi_H] \right\} \right\}.
\]
Collecting terms in the right hand side of (35) gives:

(36) \[ P(\tilde{m}, S') > - \sum_{k=1}^{N} \delta^{k-1} \left\{ (\pi' \gamma_B b + (1 - \pi') \gamma'_G g) E[(q_B^*(h_{k-1}) - x)^2 | \phi_L] + (\pi' (1 - \gamma_B) b + (1 - \pi') (1 - \gamma'_G) g) E[(q_B^*(h_{k-1}) - x)^2 | \phi_H] + (\pi' \gamma_B (1 - b) + (1 - \pi') \gamma'_G (1 - g)) E[(q_G^*(h_{k-1}) - x)^2 | \phi_L] + (\pi' (1 - \gamma_B) (1 - b) + (1 - \pi') (1 - \gamma'_G) (1 - g)) E[(q_B^*(h_{k-1}) - x)^2 | \phi_H] \right\}. \]

Next, recall

\[ \pi' = \frac{\gamma - \gamma'_G}{\gamma_B - \gamma'_G} \quad \text{and} \quad \pi = \frac{\gamma - \gamma_G}{\gamma_B - \gamma_G}, \]

and the definitions of \( b \) and \( g \) in (30). It follows that:

(37) \[ \pi' \gamma_B b + (1 - \pi') \gamma'_G g = \pi' \gamma_B \]

(38) \[ \pi' (1 - \gamma_B) b + (1 - \pi') (1 - \gamma'_G) g = \pi (1 - \gamma_B) \]

(39) \[ \pi' \gamma'_B (1 - b) + (1 - \pi') \gamma'_G (1 - g) = (1 - \pi) \gamma_G \]

(40) \[ \pi' (1 - \gamma'_B) (1 - b) + (1 - \pi') (1 - \gamma'_G) (1 - g) = (1 - \pi) (1 - \gamma_G). \]

Substituting (37-40) into (36) gives:

\[ P(\tilde{m}, S') > - \sum_{k=1}^{N} \delta^{k-1} \left\{ \pi \gamma_B E[(q_B^*(h_{k-1}) - x)^2 | \phi_L] + \pi (1 - \gamma_B) E[(q_B^*(h_{k-1}) - x)^2 | \phi_H] + (1 - \pi) \gamma_B E[(q_G^*(h_{k-1}) - x)^2 | \phi_L] + (1 - \pi) (1 - \gamma_B) E[(q_G^*(h_{k-1}) - x)^2 | \phi_H] \right\}, \]

and hence,

(41) \[ P(\tilde{m}, S') > -\pi \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_B E[(q_B^*(h_{k-1}) - x)^2 | \phi_L] + (1 - \gamma_B) E[(q_B^*(h_{k-1}) - x)^2 | \phi_H] \} + (1 - \pi) \sum_{k=1}^{N} \delta^{k-1} \{ \gamma_G E[(q_B^*(h_{k-1}) - x)^2 | \phi_L] + (1 - \gamma_G) E[(q_B^*(h_{k-1}) - x)^2 | \phi_H] \}. \]

Note that the right hand side of (41) is the planner’s expect payoff of selecting \( m^* \) when the advocate’s signal is \( S \), consult (14). Hence,

\[ P(\tilde{m}, S') > P(m^*, S). \]
Thus, offering mechanism $\bar{m}$ when the advocate’s signal is $S'$ yields a larger expected payoff than offering mechanism $m^*$ when the advocate’s signal is $S$.

**Step III.** Let $m'$ represent the planner’s optimal incentive compatible mechanism when the advocate observes signal $S'$, and let $P(m', S')$ represent the planner’s expected payoff from this mechanism. From Step I, offering mechanism $\bar{m}$ is incentive compatible for any values of $(b, g)$, and hence, $P(m', S') \geq P(\bar{m}, S')$ for all $(b, g)$. From Step II, offering mechanism $\bar{m}$ with $(b, g)$ as in (30) when the advocate’s signal is $S'$ yields a larger expected payoff than offering mechanism $m^*$ when the advocate’s signal is $S$; that is, $P(\bar{m}, S') > P(m^*, S)$. Hence, $P(m', S') \geq P(\bar{m}, S') > P(m^*, S)$, i.e. the planner’s payoff from offering the mechanism that is optimal when the advocate observes $S'$ is larger than her payoff from offering the mechanism that is optimal when the advocate observes $S$. 

■