“Sequential Majoritarian Blotto Games”
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Authors consider *Sequential Majoritarian Blotto Games*.

- Two players \((A, B)\) have fixed resource budgets \((a, b)\).
- In each period, players fight a “battle,” simultaneously choosing how much of their current resource budget to allocate.
- If \(A\) allocates \(x\) and \(B\) allocates \(y\), then \(A\) wins the battle with probability \(p(x, y)\).
- The battle winner and the resource levels are revealed.
- Play continues until one player wins a certain number of battles, \(n\), the majority of the max number of battles \(N\).
- This player wins the overall game.
- Resources have no scrap value.
Summary

- Authors focus on equilibria in Markovian strategies: the current period allocation depends on players’ remaining budgets, and on each side’s number of wins.
- Present conditions under which the equilibrium has a simple and elegant structure.
- Each player calculates the length of the “longest route” to game termination, and divides his current budget by this number. Allocates this fraction to the current battle.
- For example, if the remaining battle determines winner (max. length 1), then both players allocate all remaining resources to it.
- If today one player needs to win 2 battles and the other needs to win 1 (max length 2), then each player allocates 1/2 of his budget to today’s contest.
Iterating forward from the initial game state, each player simply divides his budget by $N$ and allocates this share to each battle that he fights.

Probability of winning a battle does not depend on number of wins or losses for each player, only on initial resources and total number of possible battles $N$.

No “momentum” effects.

Conditions are not that easy to interpret but are satisfied by a number of standard contest success functions.

Very nice comparative statics with respect to $N$ and extension to initial costly fundraising.
Incentives.

▶ Early battles less “decisive” of overall winner. Need to save resources for important battles later in game.

▶ More uncertainty about whether a later battle will be fought—game could end before a particular late battle is reached. Later battles less-likely, so investing in earlier battles better use of resources.

▶ In authors’ setting, two forces offset exactly.

▶ No adjustment in resource allocation as game is played.
Questions and Suggestions

Highly tractable model for studying a complicated setting. Some questions and possible next steps...

- Authors consider one round of fundraising at beginning of the game. Possibility of fundraising in each period? Is equilibrium fundraising lumpy? Momentum effects?
- If one player could commit to his resource allocation strategy first, (how) would he like to deviate from the equilibrium strategy?
- If a player not only wants to win, but also wants to win early, (the payoff of winning is discounted) would a suitable extension of the equal-split equilibrium arise? Perhaps a “proportional” split rule whereby the ratio between investments is constant over time?
Questions and Suggestions

Highly tractable model for studying a complicated setting. Some questions and possible next steps...

▶ Is the sequential timing important? Given that the game state does not matter in equilibrium, one may wonder whether the same “equal-split” strategies constitute an equilibrium if all battles are fought at once.

▶ Finally, to help the reader appreciate the elegance of the results, may be helpful to illustrate what can happen without (3).
Example

- Legislature consists of three voting blocks led by politicians $i = 1, 2, 3$.
- Two lobbyists with initial budget 1 support different policies, $SQ, R$.
- In period $i = 1, 2, 3$, politician $i$ draws a private preference parameter $v_i \sim U[-1, 1]$, representing politician’s private benefit from voting for policy $R$. Private benefit of voting for SQ normalized to zero.
- Lobbyists simultaneously offer campaign contributions $(x_S, x_R)$ to politician $i$.
- Politician publicly commits to vote $SQ$ iff $x_S \geq v_i + x_R \Leftrightarrow v_i \leq x_S - x_R$.
- Probability of voting $SQ$ is therefore $p(x_S, x_R) = \frac{x_S - x_R + 1}{2}$.
Example

centaje $p \in [0, 1]$ for all $(x_S, x_R)$ satisfying budget restriction. Also, does not satisfy (3).

- Obviously, if first two politicians split, expend all remaining budget on third.
Let winner of first battle be *Leader* (L) with budget $B_L$ and loser of first battle be *Laggard* (G), with budget $B_G$.

Let $p_i(x_L, x_G)$ be the prob. that $i = L, G$ wins given resources $(x_L, x_G)$.

In second battle,

$$u_L(x_L, x_G) = p_L(x_L, x_G) + p_G(x_L, x_G)p_L(B_L - x_L, B_G - x_G)$$

$$u_G(x_L, x_G) = p_G(x_L, x_G)p_G(B_L - x_L, B_G - x_G)$$

In this example, leader’s payoff function is *convex* in $x_L$. Leader either expends remaining budget (go for broke) or saves entire remaining budget (play safe).

Optimal for Leader to go for broke if Laggard expends less than half of his resources, and play it safe if Laggard expends more than half of his resources. If Laggard spends half of remaining resources, leader willing to mix.
Example

- Laggard payoff function concave in $x_G$.
- If Laggard expects Leader to go for broke, responds by expending more than half of remaining resources. Leader prefers to play safe.
- If Laggard expects Leader to play it safe, responds by expending less than half of remaining resources. Leader prefers to go for broke.
- In equilibrium, Laggard spends half of remaining resources, Leader mixes between go for broke and play it safe with probability $1/2$.
- Backing up to first stage only symmetric pure strategy equilibrium is for both sides to expend *no resources* on first battle.
Example

- Notable momentum effects in equilibrium.
- The probability that Leader wins the second vote is 1/2. Thus, winning the first battle does not increase the probability of winning the second.
- However, the probability that the Laggard wins the third battle given that he wins the second battle is $3/8 < 1/2$. 
In equilibrium, the first win is equally likely for each side.

Regardless of which side got the first win, each side is equally likely to get the second win.

The side that got the first win is more likely to get the third win, if it loses the second battle.

Thus, the “momentum effect” of winning the first battle manifests in the third battle (if it is reached) but not in the second.

Example shows potential for interesting results if (3) is relaxed.
Conclusion

Nice paper! Thanks for listening!
If Leader goes for broke, wins w/ prob. \( p_L(1, 1/2) = 3/4 \)
If Leader plays safe, then wins w/ prob. \( p_L(0, 1/2) = 1/4 \).
Leader mixes between GFB and PS with probability 1/2.

Thus, the probability that leader wins second vote is

\[
\frac{1}{2} \left( \frac{3}{4} \right) + \frac{1}{2} \left( \frac{1}{4} \right) = \frac{1}{2}.
\]
Given that Laggard wins the second vote, what is the prob. that Laggard wins the third vote?

\[
\Pr(G \text{ wins } 3 | G \text{ wins } 2) = \frac{\Pr(G \text{ wins 3 and 2})}{\Pr(G \text{ wins 2})} = \frac{1}{2} p_G(0, 1/2)p_G(1, 1/2) + \frac{1}{2} p_G(1, 1/2)p_G(0, 1/2)
\]

\[
= \frac{1}{2} p_G(1, 1/2)p_G(0, 1/2)
\]

\[
2p_G(0, 1/2)p_G(1, 1/2) = \frac{3}{8} < \frac{1}{2}.
\]