Make It 'Til You Fake It*

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Abstract

We study a dynamic game of fraud and trust between an entrepreneur and an investor. The investor wishes to fund a real project and reject a fake. The entrepreneur is either an ethical type that produces only a real project or a strategic type that has the ability to produce a fake. A real innovation takes a positive and uncertain amount of time, while a fake innovation can be manufactured instantaneously at some cost. We characterize the equilibrium of the game and explore two institutional remedies that can improve the investor's welfare: opaque standards, and a dynamic liquidity constraint.

JEL Classifications: C73, D21, D82, L15, M42.

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Suspect each moment, for it is a thief, tiptoeing away with more than it brings.
—John Updike, *A Month Of Sundays*

1 Introduction

> [In March 2018] Elizabeth Holmes, founder and chief executive of the blood-testing company Theranos, was charged by the Securities and Exchange Commission with an 'elaborate, years-long fraud' in which she and former company president Ramesh 'Sunny' Balwani allegedly 'deceived investors into believing that its key product – a portable blood analyzer – could conduct comprehensive blood tests from finger drops of blood,' the SEC said.
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> 'The Theranos story is an important lesson for Silicon Valley,' Jina Choi, director of the SEC's San Francisco regional office, said in a statement. 'Innovators who seek to revolutionize and disrupt an industry must tell investors the truth about what their technology can do today, not just what they hope it might do someday.'[1]

In this paper we present and analyze a novel dynamic model of fraud and trust in the context of an entrepreneur (he) seeking an injection of capital from an investor (she). The investor wishes to approve funding for a *real* innovation and reject funding for a *fake*. Real innovations and fakes have different arrival processes: real project development takes a positive and uncertain amount of time (i.e., a research breakthrough), while a fake project can be manufactured instantaneously at some cost. The entrepreneur dislikes waiting for the capital injection: he may get scooped by another innovator; the investor may decide to use her capital to fund a different project; he is naturally impatient. Whatever the reason, the entrepreneur faces pressure to generate a project quickly, and he may therefore be tempted to produce a fake. The entrepreneur is of two possible types. Both types have the same ability to develop a real project, but they vary in their opportunity or willingness to commit fraud. An ethical type is unwilling or unable to produce a fake, while a strategic type can generate a costly fake at will. Furthermore, real innovations and fakes are indistinguishable for the investor at the moment they are submitted for funding approval. The investor only observes the time at which a project is proffered when deciding whether to *approve* or *reject* it. Thus, the time of innovation plays a critical role in the investor's funding decision, allowing her to learn both about the entrepreneur's ethics and the project's authenticity.[2]

[1] Carolyn Y. Johnson, "Theranos chief executive Elizabeth Holmes charged with massive fraud," *The Washington Post,* March 14, 2018.

[2] We have also investigated extending the baseline model by endowing the investor with a costly auditing

In the unique equilibrium of the game, both the investor and strategic type of entrepreneur play mixed strategies on a finite time interval at the beginning of the interaction, which we refer to as the *phase of doubt*. Specifically, during this interval the strategic entrepreneur searches for a real innovation, but also randomly creates a fake. Thus, when the investor receives a submission, she is uncertain whether it constitutes a real breakthrough by an ethical entrepreneur, a real breakthrough by a strategic entrepreneur, or a fake. In equilibrium, the investor employs a random approval policy over the phase of doubt. To offset the strategic entrepreneur's desire to commit fraud as early as possible, the investor's equilibrium approval probability increases over time. Furthermore, as time passes without receiving a submission, the investor's confidence that the entrepreneur is ethical grows. Indeed, the strategic type submits a fake with probability 1 by some finite date, at which point the phase of doubt ends. After that, the investor is fully confident that the entrepreneur is ethical and she approves funding for any subsequent submission for sure.

Because the investor is indifferent between approving and rejecting projects that are submitted during the phase of doubt, her expected payoff is equal to her payoff from rejecting. In other words, she benefits from the arrival of a project if and only if the arrival occurs after the phase of doubt is over, when her trust in the entrepreneur is fully established. Since a submission is made after the phase of doubt only if (i) the entrepreneur actually is ethical and (ii) he does not make a discovery during the phase of doubt, the investor's equilibrium payoff is relatively low in the baseline model. The rest of the paper, therefore, is dedicated to investigating two institutional remedies that the investor can use to improve her situation.

The first remedy that we consider concerns opacity of the investor's standards. Specifically, we consider a setting in which the entrepreneur interacts with one of two types of investor with different tolerances for funding fake projects. We show that both the high-standard and low-standard types of investor are better off (weakly or strictly) when the entrepreneur is uncertain about which type of investor he faces. In particular, when the investor is sufficiently likely to have high standards for approval, in equilibrium the entrepreneur behaves as if he *only* faces this type. Thus, the high standards investor obtains the same equilibrium payoff under opacity as under transparency. However, we show that the low-standards investor is strictly better off. In the remaining case, when the probability of the high standards investor is low, the phase of doubt is divided into two stages: an initial stage with aggressive (stochastic) cheating, followed by a second stage with mild cheating. In this case, opacity strictly benefits both types of investor.

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technology which she can deploy before deciding whether to approve or reject funding for the project. If the auditing technology is not too costly, then access to it increases the investor's welfare. We do not present these results for the sake of brevity, but the details are available on request.

The second remedy that we analyze limits the investor's ability to fund projects. In particular, we consider an investor who is liquidity constrained initially, and cannot fund the project until she acquires additional capital via a Poisson arrival.[3] For example, the investor might need to wait for another project to mature in order to recover sufficient funds to finance the current project. The liquidity constraint effectively imposes a time-varying upper bound on the ex ante probability that a submission arriving at a given time is approved, which becomes less restrictive over time but is never completely relaxed. In this case, the equilibrium structure is characterized by *two* phases of credibility surrounding a single phase of doubt. As in the main model, the phase of doubt is an interval (with duration identical to that of the main model), but unlike the main model, the phase of doubt begins at a strictly positive time, rather than at the beginning of the game. Intuitively, the entrepreneur expects the investor to be constrained with high probability early on. Expecting that it is unlikely that the investor can fund his project immediately, the entrepreneur has less incentive to pay the cost of submitting a fake innovation. Because an unconstrained investor funds early submissions and these are always real, the first credibility phase generates an expected benefit relative to the main model, where early arrivals generate no expected surplus. However, the second phase of credibility imposes a relative cost. Though the investor is confident that arrivals in the second credibility phase are also real, the investor cannot fund them with probability 1, while in the main model she approves any submission with probability 1 once the phase of doubt is over. We show that when the probability of a strategic entrepreneur is large (small), the dynamic liquidity constraint benefits (harms) the investor.

2 Literature

At a broad level, our paper is related to a recent stream of work concerned with cheating, gaming, and subterfuge in principal-agent relationships. Barron, Georgiadis, and Swinkels (2019) consider the design of compensation contracts for agents who can "game the system" by gambling with intermediate output, thereby adding mean-preserving noise. In such an environment, the agent's wage must be a concave function of his output, necessitating linear ironing on intervals where the standard contract is convex. A different perspective on gaming is presented in Frankel and Kartik (2019), who study a signaling model in which agents differ both in their "natural actions" and in their "gaming ability." The authors show that actions convey muddled information about both dimensions and derive conditions under which an

[3] In order to make the underlying forces as clear as possible, we focus on the case where the arrival of additional capital is privately observed by the investor. We have also analyzed the case in which capital arrives publicly and have verified that the investor also can benefit in this case. Details available on request.

increase in the stakes tilts information provision toward gaming ability. Glazer, Herrera, and Perry (2019) study the informativeness of a product review when the evaluator may be a dishonest type, who can submit a fake review in order to make the product appear good. In equilibrium, the informativeness of reviews is compressed: past a cutoff, all positive reviews have the same effect on beliefs. Perez-Richet and Skreta (2018) consider the design of an optimal test, when the agent has the ability to manipulate the process by which the test determines his type, showing that the possibility of manipulation induces the principal to select a more-informative test.

The evolution of the investor's belief about the entrepreneur's integrity plays a key role in our analysis. In this sense, our work is connected to the literature on reputation in long term relationships. To our knowledge, ours is the first paper in this area that explores the link between the maturation of projects and the growth of reputational capital. Sobel (1985) considers a repeated cheap talk game in which the agent may be either a "friend" of the principal, with aligned preferences, or an "enemy," with opposing preferences. The enemy cultivates his reputation by sometimes issuing honest advice in periods with moderate stakes. When the stakes become sufficiently high, the enemy exploits his reputation by issuing a self-serving recommendation, thereby revealing his type. Bar-Isaac (2003) studies how reputation affects a monopolist's decision to abandon a market. In equilibrium, the good type of seller signals that his product is likely to be of high quality by staying in the market, despite an unlucky run in which realized product quality is low. Ely and Välimäki (2003) study a model of advice in which a long-lived expert advises a sequence of short-lived principals, who observe past recommendations, but not past states. The authors highlight a perverse incentive, whereby the "good" advisor is disinclined to make recommendations that might make him appear to be the "bad" type, even if such recommendations are actually warranted. Deb, Mitchell, and Pai (2019) also explore a dynamic model of expertise. In each period, the agent privately observes the arrival of information before choosing whether to act on it. Only a good agent can acquire information, which can be either high or low quality. To maintain his reputation, a good agent is sometimes tempted to act on low quality information. Kolb and Madsen (2020) develop a dynamic principal agent model in which a principal runs a project, which may be implemented by a disloyal agent. The principal controls the evolution of the project stakes, which increase both the principal's flow benefit from honest performance and a disloyal agent's flow benefit from undermining. The principal detects undermining stochastically, and fires the agent as soon as undermining is detected. Thus, the evolution of stakes affects both the principal's flow payoff and her ability to root out disloyalty.

While our paper focuses on an agent's ability to generate an artificial arrival, another strand of literature focuses on an agent's ability to suppress or delay an arrival, particularly in the

context of information or news. Gratton, Holden, and Kolotilin (2018) study a dynamic persuasion model in which a stochastic arrival privately informs the sender of his type. Once the sender discloses that he has learned his type (without disclosing what it is), the receiver begins to draw informative signals about it. Early disclosure provides the receiver with more opportunities to learn about the sender and therefore signals good news. Shadmehr and Bernhardt (2015) analyze a ruler's incentive to suppress media reports, showing that the ruler can benefit from a commitment to censor less than he does in equilibrium. Sun (2018) considers a dynamic model of censorship, demonstrating that when the arrival of bad news is inconclusive, it is censored aggressively by the good type of ruler, which can improve information quality and lead to a Pareto improvement. In a different vein, Li, Matouschek, and Powell (2017) study power dynamics in a relational contract. In each period, the principal approves or vetoes an agent's recommended project, without observing whether her own preferred project is available. Thus, the agent can suppress the arrival of the principal's preferred project, hoping to implement his own.

Our analysis is also tangentially related to the literature on dynamic moral hazard contracts in which the agent's effort accelerates a project's arrival (Bergemann and Hege, 1998, 2005; Mason and Välimäki, 2015; Sun and Tian, 2018). In these papers, the agent's effort is costly but increases the arrival rate of a success.[4] In contrast, in our analysis, cheating *increases* the arrival rate of a "success" while *decreasing* its quality to the point that the investor would prefer to deny funding. We are aware of only two dynamic contracting papers — Klein (2016) and Varas (2018) — that allow the agent to act in a similar manner.

In Klein (2016), the principal hires an agent to experiment by generating public information in the form of a state-contingent Poisson process. In addition to the experimentation technology, the agent has access to a specious technology which produces Poisson successes (that appear identical to the ones generated by the experimentation technology) at a rate that is independent of the state. Thus, a specious success is uninformative and worthless to the principal. The author shows that the optimal compensation contract backloads payments. By contrast, the entrepreneur in our model possesses a technology for generating a single fake project rather than a stream of false data. In this context, we find that an early arrival has no value to the investor, whereas a late (enough) arrival must be authentic.

In the contracting environment investigated by Varas (2018), the agent chooses in each instant whether to *work, shirk*, or *gamble*. Working generates high quality output after an

[4]In Bergemann and Hege (1998, 2005), the type of the project is also uncertain, and only a good project can deliver a success. Thus when the agent shirks, he becomes more optimistic than the principal about the project's type. In other words, shirking not only allows the agent to save on cost, but also generates private information.

uncertain amount of time and effort, while gambling generates an output of random quality that is difficult for the principal to verify. The optimal contract derived by Varas (2018) exhibits two phases: an initial phase of diminishing payments followed by a stationary phase in which the agent is not punished for production delays. The principal in Varas (2018) learns about project quality post–submission, while the principal in our setting learns about the integrity of the agent pre–submission. More generally, Varas (2018) underscores the limits of high–powered incentive contracts, whereas our findings point to the crucial role played by reputation and trust in a setting marked by limited commitment.

Finally, the institutional remedies we consider are related in some degree to a body of prior work. The first remedy that we study allows the principal to maintain private information about her preferences. Ederer, Holden, and Meyer (2018) study a related form of opacity in a multi-task moral hazard model, where the principal deters gaming by committing to randomize between different performance measures. The final remedy we consider forces the principal to postpone her decision until she has a private Poisson arrival, which incentivizes the entrepreneur to delay faking and improves the investor's welfare. This result is reminiscent of Fuchs and Skrzypacz (2015, 2019), who show that shutting down a market at certain times changes the dynamic incentives to trade and enhances efficiency.

3 The Model

An investor (she) interacts with an entrepreneur (he) over an indefinite horizon. Time is continuous and both parties discount future payoffs at rate $\rho > 0$.[5] The entrepreneur develops a project over time that he submits to the investor for funding approval. The project can be developed using a technology that is either *authentic* or *fraudulent*. If the entrepreneur uses the authentic technology at time t, then a *real* innovation arrives at Poisson rate λ. The fraudulent technology allows the entrepreneur to instantly develop a *fake* innovation, at fixed cost $\phi \in [0, 1)$. Thus, the authentic technology is free but slow, while the fraudulent technology is costly (if $\phi > 0$) but fast.[6]

[5]For most of our results, whether ρ is the rate of time preference or the rate at which the game ends does not matter. However, when we investigate the possibility of a liquidity constraint in Section 6, the latter interpretation is most natural.

[6]If the authentic technology imposed a positive flow cost on the enrepreneur, then he could engage in fraud in order to shirk, thereby avoiding the cost of authentic project development. Our goal is to explore the dynamic incentives to commit fraud, which have received relatively little formal analysis, as opposed to incentives to shirk, which have been researched extensively. Furthermore, incorporating a small positive flow cost for the authentic technology would have no impact on our results. Normalizing the expected flow cost under the real technology back to zero would manifest technically as a reduction in ϕ (calculations in

The entrepreneur is one of two types: *strategic* with probability $\sigma \in (0,1)$ or *ethical* with probability $1-\sigma$. The strategic entrepreneur can use either technology while the ethical entrepreneur can only use the authentic one. The entrepreneur's type is private information, but the value σ is common knowledge. Below whenever we write of the entrepreneur choosing whether to generate a fake innovation, it should be understood that we are referring to the strategic type.

Once a project has been developed, it is instantly submitted to the investor for approval.[7] The state of the project — real or fake — is not directly observable upon submission. The investor only observes the time at which the project was submitted when deciding whether to approve it for funding or reject it.

The investor would like to approve real projects and reject fakes. Her gross expected return from funding a real project is 1 and from funding a fake is 0.[8] The capital injection required for bringing a project to maturity is $k \in (0,1)$. If the investor denies funding, then she receives 0 regardless of the project's state. The strategic entrepreneur would like his project to be funded regardless of its state, obtaining a gross benefit of 1 from approval and 0 from rejection.[9]

4 Equilibrium Characterization

In this section, we characterize the weak Perfect Bayesian equilibrium (henceforth equilibrium) of the game and show that it is generically unique.[10] An equilibrium consists of strategies for the strategic entrepreneur and the investor and a belief function for the investor regarding the state of a submitted project, such that (i) the entrepreneur's strategy is opti-

Appendix).

[7]This assumption rules out equilibria in which both types of entrepreneur are compelled to delay by the investor's off–path belief that submissions arriving at certain times must be fake. A refinement in the spirit of D1 would eliminate such off–path beliefs, ruling out equilibria involving delays. In addition, in all model variants, the equilibrium that we derive is robust to allowing the entrepreneur to delay the submission of the project.

[8]A real project could still fail with positive probability in which case it would be indistinguishable from a fake.

[9]The analysis and results are similar if the entrepreneur's payoff from having a fake project approved is smaller than his payoff from having a real project approved (i.e., if there is a small expected penelty for submitting a project that fails). Details available upon request.

[10]Multiple equilibria do exist iff $\phi = \frac{\rho}{\rho+\lambda}$. To streamline presentation, we abstract from this and other non-generic cases.

mal given the investor's approval strategy, (ii) the investor's approval strategy is sequentially rational given her beliefs, (iii) the investor's belief about a submitted project is derived from Bayes' rule.[11]

Strategies. A pure strategy for the strategic entrepreneur is a choice of a "cheating time" $t \in \{\mathbb{R}_+ \cup \infty\}$ at which he will submit a fake project for funding if a real one has not yet arrived. A mixed strategy for a strategic entrepreneur is a probability measure over finite cheating times represented by cumulative distribution function $F(\cdot)$.[12] A strategy for the investor is an approval function $a(\cdot)$ on the domain $\mathbb{R}_+$, which specifies the probability with which a submission at time t is approved for funding.

Beliefs. If a project is submitted at time t, the investor's belief that it is real must be derived by Bayes' rule as the probability of a real arrival at t given an arrival at t.

Lemma 4.1 (Beliefs)**.** *If the strategic entrepreneur submits a fake project according to the cumulative distribution function $F(\cdot)$ with density $f(\cdot)$, then, the probability that a submission at time t is real is*

$$g(t) = \frac{\lambda}{\lambda + \mu(t)}, \tag{1}$$

where

$$\mu(t) \equiv \frac{\sigma f(t)}{1 - \sigma F(t)}.$$

The function $\mu(\cdot)$ is the hazard rate of a fake arrival: it is the likelihood that a fake arrival is generated at time t, given that one was not generated earlier. It is important to point out that $\mu(\cdot)$ is the hazard rate of a fake *from the investor's perspective*, because it accounts for her uncertainty about the entrepreneur's type, reflected in the parameter σ. Note that λ is the hazard rate of a real arrival, (which can be generated by either type of entrepreneur) and thus the investor's belief that an arrival at time t is real is simply the ratio of the hazard rate of a real arrival to the sum of the hazard rates of a real and fake arrival.

[11] Because a real innovation arrives at a positive rate and is assumed to be submitted immediately, all times $t \geq 0$ are on the equilibrium path. Thus, our characterization does not exploit the freedom to specify off-path beliefs granted by weak PBE.

[12] Strictly speaking, the strategic entrepreneur can choose never to cheat with positive probability, thereby allocating some probability mass to $t = \infty$. However, given Assumption 1 below, the strategic entrepreneur cheats with probability 1 in finite time in equilibrium.

Investor's Decision. If the investor believes that a project that arrives at time t is real with probability $g(t)$, then her expected payoff from funding it is

$$g(t)(1-k) - (1-g(t))k = g(t) - k,$$

and therefore, the investor's sequentially rational approval strategy must satisfy

$$a(t) = \begin{cases} 1 & \text{if} \quad g(t) > k \\ [0,1] & \text{if} \quad g(t) = k \\ 0 & \text{if} \quad g(t) < k. \end{cases} \tag{2}$$

Entrepreneur's Decision. If the strategic entrepreneur adopts a pure strategy in which he will cheat at time t given that no real arrival has been generated by that point, then – holding fixed the investor's strategy – the entrepreneur's expected payoff is

$$u(t) = \int_0^t \lambda \exp(-(\rho+\lambda)s)a(s)\,\mathrm{d}s + \exp(-(\rho+\lambda)t)(a(t)-\phi). \tag{3}$$

The integral represents the discounted expected payoff to the entrepreneur from real arrivals that occur at all times $s < t$. If no real arrival occurs before t, then the entrepreneur submits a fake project which costs ϕ and is approved for funding with probability $a(t)$. In the limit where the entrepreneur *never* submits a fake project, he can secure a non-negative payoff of $u(\infty)$. Indeed, when the cost of submitting a fake project is sufficiently high, then the entrepreneur never submits one in equilibrium. This is formalized in the following lemma.

Lemma 4.2 (No Fakes)**.** *If $\phi > \widehat{\phi} \equiv \frac{\rho}{\rho+\lambda}$, then there is a unique equilibrium of the game and it involves the entrepreneur never submitting a fake project and the investor approving any submission she receives with probability 1. If $\phi < \widehat{\phi}$, the entrepreneur fakes with strictly positive probability in equilibrium.*

The intuition is straightforward. Suppose for the moment that the investor approves any submission she receives with probability 1. Given the stationarity of the environment, the strategic entrepreneur effectively faces two alternatives: he can submit a fake immediately and earn payoff $1-\phi$, or he can wait for a real arrival, earning payoff $\lambda/(\rho+\lambda) = 1-\widehat{\phi}$. Obviously, when $\phi > \widehat{\phi}$, he prefers the second alternative. In this case, the entrepreneur never submits a fake, and it is sequentially rational for the investor to approve any submission she receives with probability 1. Motivated by this observation, we maintain the following assumption (without restatement) below.

Assumption 1. *The cost of faking is sufficiently low that the equilibrium probability of a fake submission is strictly positive: $\phi < \widehat{\phi}$.*

When $\phi < \widehat{\phi}$, the strategic entrepreneur must submit a fake project with positive probability in equilibrium. He cannot, however, submit a fake with positive probability at any specific point in time t because the probability of a real arrival at t is 0, implying that the investor's best response would be to reject with probability 1. This suggests that in equilibrium, the strategic type of entrepreneur must partially pool with the ethical type by submitting a fake project according to some probability density function, $f(\cdot)$. In order for randomization to be optimal for the strategic enrepreneur, he must be indifferent between all cheating times that he might select. Given the entrepreneur's impatience, he will be tempted to fake early, because an early approval is more valuable. To maintain indifference, it must be that early submissions are approved less often than later ones. This intuition is formalized in the following lemma.

Lemma 4.3 (Equilibrium Structure)**.** *In any equilibrium of the game*

(i) the time at which the entrepreneur submits a fake is drawn from a continuous mixed strategy with no mass points or gaps supported on an interval $[0, \bar{t}]$*, where* $\bar{t} \in (0, \infty)$*.*

(iii) for $t \in [\bar{t}, \infty)$*, the investor always approves the project,* $a(t) = 1$*.*

(ii) for $t \in [0, \bar{t})$*, the investor's strategy* $a(\cdot)$ *is strictly increasing, continuous, and differentiable almost everywhere, with* $\lim_{t \to \bar{t}} a(t) = 1$*.*

In equilibrium, the interval of arrival times is divided into two phases: an early *phase of doubt* $[0, \bar{t})$, in which the entrepreneur's submission is treated with skepticism, inducing the investor to deny funding with positive probability, and a late *phase of credibility*, $[\bar{t}, \infty)$ in which a submission originates only from the ethical type and is funded with certainty. The strategic entrepreneur's mixed strategy is supported continuously on the entire phase of doubt. It can have no mass points because this would induce rejection, and it can have no gaps because this would induce approval thereby creating a profitable deviation. Building on these observations, it is also possible to show that the phase of doubt must be finite (for $\sigma < 1$). Mathematically, it is simple to show that the cheating rate, $\mu(t)$, approaches zero as time passes, regardless of the entrepreneur's strategy. It follows that at some finite time, the cheating rate becomes small enough that the investor strictly prefers to approve an innovation, and the entrepreneur never waits past this time to submit a fake, resulting in a finite phase of doubt.

Because the strategic entrepreneur mixes over the phase of doubt in equilibrium, his payoff $u(t)$ must be constant. Using this observation, we show that the approval probability $a(\cdot)$ is continuous and differentiable. Furthermore, because the entrepreneur is impatient, the approval probability must rise over time so as to maintain indifference between early fake

submissions and later ones. Moreover, the approval probability approaches one at the end of the phase of doubt. Indeed, once the phase of doubt ends, the investor knows that the entrepreneur is ethical. By implication, for times near the end of the phase of doubt, the investor must approve with probability approaching one; otherwise, the entrepreneur could benefit by delaying his fake until the phase of doubt is over.

Lemma 4.3 implies that during the phase of doubt the mixed strategies for the investor and entrepreneur must obey a pair of first-order linear differential equations,

$$g(t) = k \;\Rightarrow\; \frac{\sigma f(t)}{1-\sigma F(t)} = \frac{\lambda(1-k)}{k}, \tag{4}$$

$$u'(t) = 0 \;\Rightarrow\; a'(t) - \rho a(t) + \phi(\rho+\lambda) = 0. \tag{5}$$

The first equation requires the investor to be indifferent between approving and rejecting an innovation for funding, while the second requires the entrepreneur to be indifferent about submitting a fake over all times inside the phase of doubt. Solving the first equation with boundary condition $F(0) = 0$ (which comes from the absence of a mass point at $t = 0$) yields the entrepreneur's equilibrium mixed strategy. Using the equilibrium mixed strategy, we find $\bar{t}$ by solving $F(\bar{t}) = 1$. Finally, solving the second differential equation with boundary condition $a(\bar{t}) = 1$ yields the investor's approval strategy. To characterize the equilibrium most succinctly, define

$$\mu \equiv \frac{\lambda(1-k)}{k}.$$

Note that the constant μ defined above is the *equilibrium* cheating rate and should not be confused with the function $\mu(t) = \sigma f(t)/(1-\sigma F(t))$, the cheating rate in general.

Proposition 4.1 (Equilibrium Fakes and Approvals.)**.** *The unique equilibrium of the game is characterized as follows.*

Strategies. *The entrepreneur's cheating time is drawn from the distribution function*

$$F(t) = \frac{1}{\sigma}(1 - \exp(-\mu t)) \tag{6}$$

supported on interval $[0, \bar{t})$*, where*

$$\bar{t} = -\frac{\ln(1-\sigma)}{\mu}. \tag{7}$$

If $t \in [0, \bar{t})$*, then the investor approves with probability*

$$a(t) = \frac{\phi}{\widehat{\phi}} + \left(1 - \frac{\phi}{\widehat{\phi}}\right) \exp\{-\rho(\bar{t} - t)\}, \tag{8}$$

and with probability 1 otherwise.

Beliefs. *The investor's belief that she faces the ethical entrepreneur given no submission by* t *is* $\frac{1-\sigma}{1-\sigma F(t)}$*. On the other hand, if she receives a submission at* $t \in (0, \bar{t})$*, then she believes it is real with probability* $g(t) = k$*, and* $g(t) = 1$ *otherwise.*

Payoffs. *The strategic entrepreneur's equilibrium payoff is* $U^S = a(0) - \phi$*, and the ethical entrepreneur's payoff is* $U^E = U^S - (\widehat{\phi} - \phi)\exp(-(\rho+\lambda)\bar{t})$*. The investor's payoff is*

$$V = (1-\sigma)(1-k)\int_{\bar{t}}^{\infty} \lambda \exp\{-(\rho+\lambda)s\}\,\mathrm{d}s = (1-\sigma)(1-k)\frac{\lambda}{\rho+\lambda}\exp(-(\rho+\lambda)\bar{t}) \quad (9)$$

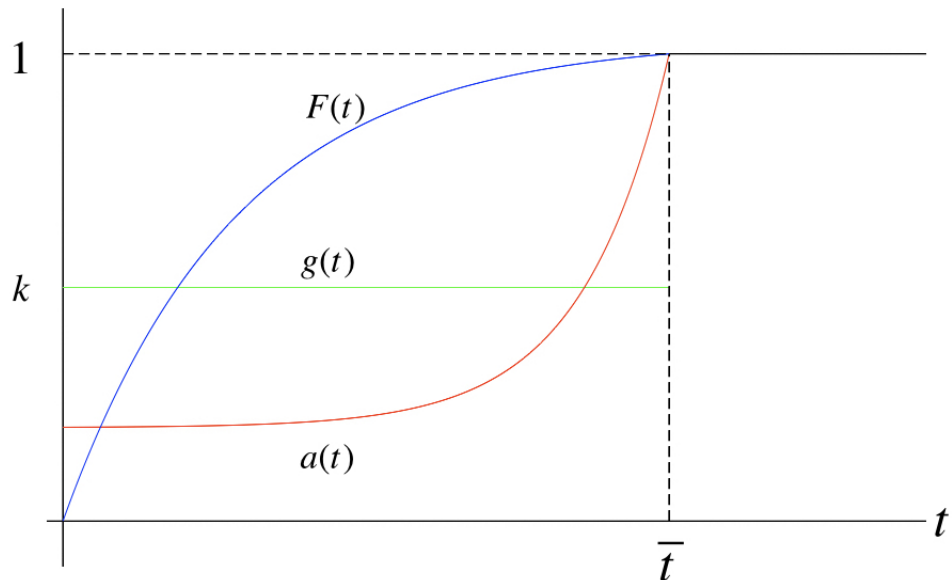

Figure 1: *Proposition 4.1*

We discuss each aspect of Proposition 4.1, beginning with the strategies. Formally, the entrepreneur's indifference condition, coupled with the finite phase of doubt implies that the investor must approve early submissions—that are more tempting to fake—with lower probability and late submissions with higher probability. The differential equation and boundary condition for the entrepreneur's indifference deliver an approval function of a particular functional form: a constant, plus an exponential function with growth rate ρ, the discount rate.

To understand the shape of the approval strategy it is helpful to consider the differential equation for $a(\cdot)$, given in (4). This equation admits a particular solution with a constant approval function $a(t) = \phi/\widehat{\phi}$ and a complementary solution $a(t) = \exp(\rho t)$.[13] To see where the particular solution comes from, note that by delaying, the strategic entrepreneur loses

[13] A constant particular solution satisfies $-\rho a + \phi(\rho+\lambda) = 0$; dividing by ρ and substituting yields the claimed solution. The complementary solution satisfies $a'(t) - \rho a(t) = 0$ and is evidently exponential with growth rate ρ.

the opportunity to generate an instant approval with probability $a(t)$, losing $\rho a(t)$ at the margin. However, by delaying, the entrepreneur may save on the faking cost ϕ, either if the game ends due to an exogenous shock (at rate ρ) or a real innovation occurs (at rate λ), resulting in a marginal gain of $\phi(\rho + \lambda)$. Thus, the constant term equates the marginal loss and gain from delay, leaving the entrepreneur indifferent. To see where the exponential term appears, note that (4) also accounts for the increases in the approval probability over time, reflected in the term $a'(t)$. Thus, by marginally delaying, the entrepreneur also increases the probability that his fake will be approved. To maintain the entrepreneur's indifference, the increase in the approval probability must be exactly offset by discounting, resulting in an approval strategy that grows at rate ρ.

What about the entrepreneur's strategy? If the investor mixes over the phase of doubt, then her belief that an innovation is real must be constant, $g(t) = k$. Thus, from the investor's perspective, the arrival rate of a fake must also be constant, as in (5). Because the authentic technology has a constant arrival rate, the fraudulent technology must appear to the investor to be deployed with a constant rate so that she does not learn about the state of a project from observing the time at which it was submitted. Solving (5), we find that the only distributions with constant arrival rate μ are truncated exponentials of the form $\frac{1}{\sigma}(1 - \kappa \exp(-\mu t))$, where κ is an integration constant. Finally, ruling out a mass point at zero yields, $\kappa = 1$, delivering the stated distribution.

We consider payoffs next. In equilibrium, the strategic entrepreneur is indifferent over submitting a fake project at any time within the phase of doubt. Thus, his payoff must equal the expected return to faking at $t = 0$, where it is approved with the lowest probability, $a(0)$. Unlike the strategic enrepreneur, the ethical entrepreneur has no opportunity to fake. In other words, he must "wait forever" to fake, which yields payoff $u(\infty)$. By implication, the ethical entrepreneur's equilibrium payoff is strictly lower than the strategic entrepreneur's, who could always mimic the ethical type's strategic but strictly prefers not to (when $\phi < \widehat{\phi}$). Furthermore, compared to the case when $\sigma \approx 0$, both the ethical and strategic entrepreneur s are worse off. If the investor believes the entrepreneur is very likely ethical, the phase of doubt collapses to zero, and both types' submissions are almost certain to be approved.

To understand the investor's payoff and the normative implications of faking, note first that for an arrival during the phase of doubt ($t \leq \bar{t}$), the investor mixes between approval and rejection, and therefore, she expects zero surplus from any arrival during this phase. Past the phase of doubt ($t > \bar{t}$), only the ethical entrepreneur is still active: in equilibrium, the strategic entrepreneur submits a fake before $\bar{t}$ with probability 1. Consequently, when a project is submitted past time $\bar{t}$, the investor is confident that it is real, and she approves it. Thus, an arrival after $\bar{t}$ generates expected surplus $1 - k$ for the investor.

Together, these observations imply that the investor expects a positive surplus from the project only when two conditions are met. First, the entrepreneur must be ethical: if the entrepreneur is strategic, then he will submit during the phase of doubt with probability 1 in equilibrium, and his arrival, whether real or fake, generates zero *expected* surplus for the investor. Second, the real technology must produce an arrival relatively late, after the phase of doubt is over. If the ethical entrepreneur is "lucky" and produces a real arrival quickly, it also generates no expected surplus for the investor. This normative implication is particularly pernicious: absent fraud, it is the early arrivals that are most valuable to her. In the next two sections we explore possible institutional remedies to this problem, opaque standards, and a dynamic liquidity constraint.[14]

5 Opaque Standards

In this section we analyze an environment in which the investor can be one of two types. With probability ν she has high standards, k_H, and with probability $1-\nu$, low standards, k_L where $1 > k_H > k_L > 0$. One interpretation is that the investor, herself, may need to raise capital or otherwise exert effort once she decides to fund the project, and this is easier for one type (low standards) than for the other (high standards).[15] Under "transparent standards" the investor's type is observed by the entrepreneur at the beginning of the game. For each realization of the investor's type, the equilibrium is identical to the main model. With "opaque standards" the entrepreneur cannot observe the investor's type. Such a situation would arise, for example, if several investors operate as a group, and the entrepreneur is uncertain which individual investor within the group will evaluate and fund his project.

The goal is to characterize the equilibrium with opaque standards, and analyze its normative properties. Denote the approval strategy of each type of investor $a_i(\cdot)$, where $i \in \{H, L\}$, and let $a_U(t) \equiv \nu a_H(t) + (1-\nu)a_L(t)$ denote the expected probability of approval at time t, accounting for the entrepreneur's uncertainty about the investor's type.[16] The entrepreneur's expected payoff of selecting cheating time t is identical to his payoff in the main model,

[14] As noted previously, we have also explored auditing a submission with an imperfect costly test. We do not include this analysis because the normative results are unsurprising, although the positive ones are intriguing.

[15] According to Gompers, Gornall, Kaplan, and Strebulaev (2021) investors regularly interact with the companies they fund and provide a large number of services including "strategic guidance... connections to other investors... connections to customers... operational guidance... help hiring board members... and help hiring employees."

[16] Because the investor makes a decision only *after* the entrepreneur makes a submission and her type concerns her preferences, the entrepreneur does not update beliefs about the investor over time.

substituting the *expected* approval probability $a_U(\cdot)$ for the approval probability $a(\cdot)$ of the main model. Similar arguments to those in Lemma 4.3 establish that the entrepreneur mixes continuously on an interval from time zero to some finite threshold $\bar{t}_U$, defining a finite "phase of doubt." Furthermore, over this interval, the *expected* approval probability inherits the features of the appproval probability in the main model: $a_U(\cdot)$ is strictly greater than ϕ, increasing, continuous, differentiable, and approaches one at the end of the phase of doubt. After the phase of doubt, the approval probability is one, $a_U(t) = 1$ for $t > \bar{t}_U$.

With opaque standards, the entrepreneur's mixing distribution is the same, regardless of what standard he actually faces. In other words, both types of investor face the same mixed strategy. Because the investor's type orders her payoff according to single-crossing, it cannot be that both types of investor are simultaneously indifferent between approving and rejecting. Consequently, if one type of investor mixes in equilibrium, then the other strictly prefers approving or rejecting. Furthermore, the low standards investor has a stronger incentive to approve: thus, whenever the high type mixes, the low type approves, and whenever the low type mixes, the high type rejects. As we show in the following lemma, this ordering of the investor's incentives implies that under opaque standards, the phase of doubt is divided into two sub-phases. In the first (possibly degenerate) sub-phase the low standards investor mixes and the high standards investor rejects. In the second sub-phase, the low standards investor approves, and the high standards investor mixes.

Lemma 5.1 (Opaque standards equilibrium structure.)**.** *In equilibrium with uncertain standards, there exists $\bar{t}_U \in (0, \infty)$ and $\widetilde{t}_U \in [0, \bar{t}_U)$ such that*

(i) the entrepreneur's cheating time is drawn from a continuous mixed strategy with no mass points or gaps supported on an interval $[0, \bar{t}_U]$.

(ii) for $t \in [\bar{t}_U, \infty)$, both types of investor approve the project, $a_L(t) = a_H(t) = 1$.

(iii) for $t \in [\widetilde{t}_U, \bar{t}_U)$ the low standards investor always approves, $a_L(t) = 1$, and the high standards investor's approval strategy is strictly increasing, continuous, and differentiable almost everywhere, with $\lim_{t \to \bar{t}_U} a_H(t) = 1$.

(iv) if $\widetilde{t}_U > 0$, then for $t \in [0, \widetilde{t}_U)$ the high standards investor always rejects $a_H(t) = 0$, and the low standards investor's approval strategy $a_L(\cdot)$ is strictly increasing, continuous, and differentiable, with $\lim_{t \to \widetilde{t}_U} a_L(t) = 1$. Furthermore, $\lim_{t \to \widetilde{t}_U} a_H(t) = 0$.

To complete the characterization, we separately consider equilibria with a "one stage" structure, corresponding to the case $\widetilde{t}_U = 0$, and a "two stage" structure, corresponding to

$\widetilde{t}_U > 0$.[17] First, we introduce some additional notation that simplifies the exposition. For $i \in \{H, L\}$, let

$$\mu_i \equiv \lambda \frac{1-k_i}{k_i} \qquad \bar{t}_i \equiv -\frac{\ln(1-\sigma)}{\mu_i} \qquad \delta_U \equiv \frac{-\ln(1-\frac{\nu\widehat{\phi}}{\widehat{\phi}-\phi})}{\rho} \qquad \nu^* \equiv (1-\frac{\phi}{\widehat{\phi}})(1-\exp(-\rho\bar{t}_H))$$

Note that μ_i is the equilibrium cheating rate when the entrepreneur observes that he faces a type $i \in \{H, L\}$ investor. In other words, it is the equilibrium cheating rate under transparency for the type i investor. Similarly, $\bar{t}_i$ is the duration of the phase of doubt under transparency with a type i investor. By implication, $\mu_L > \mu_H$ and $\bar{t}_L < \bar{t}_H$. As we will see, δ_U is the duration of the second stage in the two-stage equilibrium. Note that δ_U is well-defined whenever $\nu < 1 - \phi/\widehat{\phi}$. Finally, we will also see that the relationship between ν and ν^* determines whether the equilibrium has one or two stages of faking.

In a one stage equilibrium, the low standards investor approves all arrivals, while the high standards investor mixes for arrivals before $\bar{t}_U$ and approves thereafter. Because only the high type investor mixes in the phase of doubt, in such equilibria, the entrepreneur behaves as if he faces *only* the high type investor. Thus, the entrepreneur mixes over the same phase of doubt as in the main model, $[0, \bar{t}_H]$. By implication, the low standards investor strictly prefers approval in both phases ($k_L < k_H < 1$). Furthermore, from the entrepreneur's perspective, the expected approval probability is the same as in the main model. However, because the low standards investor always approves, the high standards investor's approval strategy must be adjusted to maintain the same expected approval probability as in the main model, $\nu a_H(t) + (1-\nu) = a(t)$. Therefore, a one stage equilibrium exists only if ν is relatively large: if ν is small, then $1 - \nu > a(0)$, which would imply $a_H(0) < 0$. Intuitively, if the probability of a low standards investor is high, then the probability that an early arrival is approved is also high. Consequently, the strategic entrepreneur will be tempted to cheat early, even if he is rejected by the high type. Given the low probability of a high-standards investor, no adjustment in the high type's approval probability can offset this.

Proposition 5.1 (Opaque standards, One Stage.)**.** *With opaque standards, a one phase equilibrium exists if and only if $\nu > \nu^*$, and it is characterized below. Furthermore, with opaque standards, no other one phase equilibrium exists.*

Strategies. *The entrepreneur's cheating time is drawn from distribution function*

$$F(t) = \frac{1}{\sigma}(1 - \exp(-\mu_H t))$$

[17] It is worth mentioning that, from Lemma 5.1, every equilibrium of the game has either a "one stage" structure or a "two stage" structure. Below, we characterize each type of equilibrium and derive conditions under which it exists. We show that these conditions are mutually eclusive, and thus, the equilibrium is unique.

supported on interval $[0, \bar{t}_H]$. *If* $t \in [0, \bar{t}_H]$, *then the high type investor approves with probability*

$$a_H(t) = \frac{1}{\nu}\Big(\frac{\phi}{\hat{\phi}} + (1 - \frac{\phi}{\hat{\phi}})\exp\{-\rho(\bar{t}_H - t)\} - (1 - \nu)\Big),$$

and with probability 1 otherwise. The low type investor always approves, $a_L(t) = 1$. *The expected approval probability* $a_U(\cdot)$ *is identical to the approval probability in the main model, with investor's standard known to be* k_H.

Beliefs. *If* $t \in (0, \bar{t}_H)$, *then* $g(t) = k_H$, *and* $g(t) = 1$ *otherwise.*

Payoffs. *The strategic entrepreneur's equilibrium payoff is* $a_U(0) - \phi$, *identical to the main model with investor's standard known to be* k_H. *The high standards investor's payoff is*

$$V_H = (1 - k_H)(1 - \sigma)\int_{\bar{t}_H}^{\infty} \lambda \exp\{-(\rho + \lambda)t\}dt,$$

and the low type investor's payoff is

$$V_L = \lambda(1 - \frac{k_L}{k_H})\int_0^{\bar{t}_H} \exp\{-(\rho + \frac{\lambda}{k_H})t\}dt + (1 - k)(1 - \sigma)\int_{\bar{t}_H}^{\infty} \lambda \exp\{-(\rho + \lambda)t\}dt.$$

Normative Ranking. *In the one phase equilibrium with opaque standards, (i) the high type investor's payoff is the same as in the unique equilibrium with transparent standards. (ii) The low type investor's payoff is strictly higher than in the unique equilibrium with transparent standards.*

When the investor has high standards with high probability, opacity (weakly) increases the payoffs of both types of investor. Because the entrepreneur is most likely to interact with the high standards investor, it is the high type's incentive to approve funding that restrains the strategic entrepreneur's incentive to cheat. In equilibrium, the strategic type effectively targets only the high type investor, completely ignoring the low type. In particular, the entrepreneur's strategy is identical to the baseline model, assuming that the investor's standard is known to be k_H. Thus, the high type investor obtains the same equilibrium payoff with opacity as with transparency. In contrast, when the investor's realized standard is low, she approves every submission, obtaining a positive payoff in both the doubt and credibility phases. However, the entrepreneur cheats more slowly with opacity than transparency if he faces the low type investor; thus, opacity also delays the onset of the credibility phase for the low type. The low type investor faces a tradeoff with opacity: a higher belief during the phase of doubt (and approval of all arrivals), but a longer phase of doubt. It turns out that the benefit generated by a higher belief during the phase of doubt outweighs the delay in restoring credibility; i.e., opacity strictly benefits the low type investor.

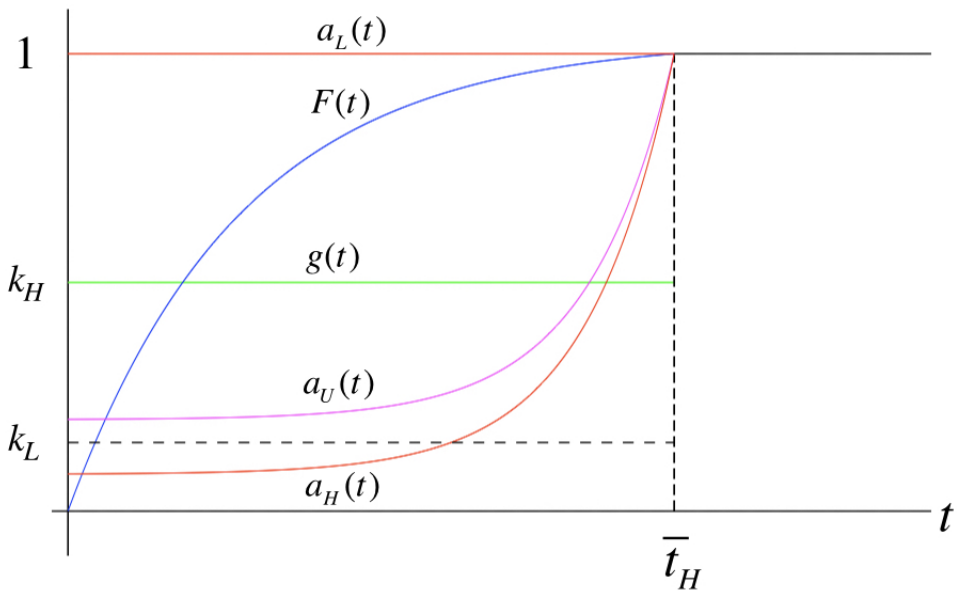

Figure 2: *Proposition 5.1*

We turn next to the two stage equilibrium, in which $\widetilde{t}_U > 0$. The second stage resembles the one stage equilibrium—the low type investor always approves and high type mixes. In contrast to the one stage equilibrium, however, the high type's approval probability begins at zero, $a(\widetilde{t}_U) = 0$, finishing at one, $a(\bar{t}_U) = 1$. Furthermore, in the first phase, the high type investor rejects, while the low type investor mixes. The low type's approval probability is positive at time zero, and increases during the first stage hitting one at the transition time $\widetilde{t}_U$. The entrepreneur's mixed strategy, while continuous, also takes a different form in the two sub-phases. In the first sub-phase, the entrepreneur cheats at a faster rate, inducing mixing by the low type investor; in the second, the entrepreneur cheats more slowly, inducing mixing by the high type investor. The entrepreneur and investor indifference conditions over the two stages, combined with the appropriate continuity boundary conditions define a system of differential equations that characterize the equilibrium.

Proposition 5.2 (Opaque standards, Two Stages.)**.** *With opaque standards, a two stage equilibrium exists if and only if $\nu < \nu^*$, and it is characterized below. Furthermore, with opaque standards, no other two stage equilibrium exists.*

Stage Transitions. *The transition times $\widetilde{t}_U, \bar{t}_U$ in the two stage equilibrium are*

$$\widetilde{t}_U = \bar{t}_L - \frac{\mu_H}{\mu_L}\delta_U \qquad \bar{t}_U = \widetilde{t}_L + \delta_U.$$

Furthermore, $0 < \widetilde{t}_U < \bar{t}_U < \bar{t}_H$.

Strategies. *The entrepreneur's cheating time is drawn from continuous distribution function*

$$F(t) = \begin{cases} \frac{1}{\sigma}(1 - \exp(-\mu_L t)) & \text{for } t \in [0, \widetilde{t}_U) \\ \frac{1}{\sigma}(1 - \exp(-\mu_H t - (\mu_L - \mu_H)\widetilde{t}_U)) & \text{for } t \in [\widetilde{t}_U, \bar{t}_U] \end{cases}$$

supported on $[0, \bar{t}_U]$. *If* $t \in [0, \widetilde{t}_U]$, *then the high type investor always rejects,* $a_H(t) = 0$, *and the low type investor approves with probability*

$$a_L(t) = (\frac{1}{1-\nu})(\frac{\phi}{\widehat{\phi}} + (1 - \frac{\phi}{\widehat{\phi}} - \nu)\exp\{-\rho(\widetilde{t}_U - t)\}).$$

If $t \in [\widetilde{t}_U, \bar{t}_U]$, *then the high type investor approves with probability*

$$a_H(t) = \frac{1}{\nu}\Big(\frac{\phi}{\widehat{\phi}} + (1 - \frac{\phi}{\widehat{\phi}})\exp\{-\rho(\bar{t}_U - t)\} - (1-\nu)\Big),$$

and the low type investor always approves, $a_L(t) = 1$. *If* $t \geq \bar{t}_U$, *then both types of investor always approve funding,* $a_L(t) = a_H(t) = 1$.

Beliefs. *If* $t \in (0, \widetilde{t}_U)$, *then* $g(t) = k_L$. *If* $t \in (\widetilde{t}_U, \bar{t}_U)$, *then* $g(t) = k_H$. *Otherwise* $g(t) = 1$.

Payoffs. *The entrepreneur's equilibrium payoff is* $a_U(0) - \phi$. *The high standards investor's payoff is*

$$V_H = (1 - k_H)(1 - \sigma)\int_{\bar{t}_U}^{\infty} \lambda \exp\{-(\rho + \lambda)s\}ds.$$

The low type investor's payoff is

$$V_L = \exp(-(\mu_L - \mu_H)\widetilde{t}_U)(1 - \frac{k_L}{k_H})\int_{\widetilde{t}_U}^{\bar{t}_U} \lambda \exp(-(\rho + \frac{\lambda}{k_H})t)dt + (1 - k_L)(1 - \sigma)\int_{\bar{t}_U}^{\infty} \lambda \exp(-(\rho + \lambda)t)dt.$$

Normative Ranking. *In the two stage equilibrium with opaque standards, (i) the high type investor's payoff is strictly higher than in the unique equilibrium with transparent standards. (ii) The low type investor's payoff is strictly higher than in the unique equilibrium with transparent standards.*

When the probability of the low type investor is sufficiently high, then the entrepreneur does not ignore her—as he did in the one phase equilibrium. Instead, in the initial stage, $t \in [0, \widetilde{t}_U)$, the entrepreneur cheats aggressively, gambling that the investor is the low type, who approves even early arrivals with positive probability. As a consequence of this increase in the cheating rate, the high type investor is strictly better off than under transparency. Recall that under transparency, the entrepreneur cheats with rate μ_H throughout the phase of doubt. However, in the two stage equilibrium with opacity, the entrepreneur starts off cheating at rate $\mu_L > \mu_H$, and switches to rate μ_H at some interior time. Thus, the entrepreneur's

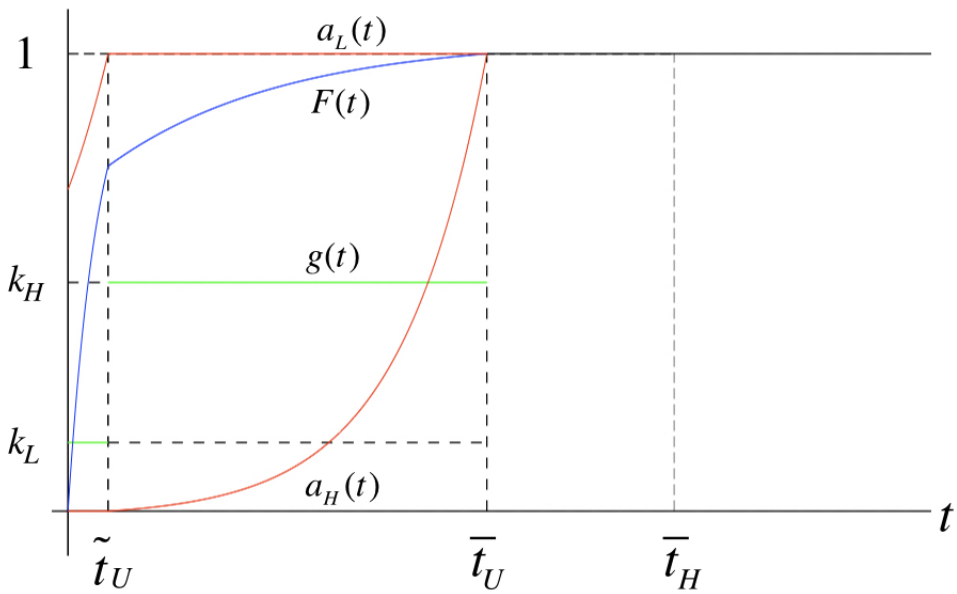

Figure 3: *Proposition 5.2. Note that $g(\cdot)$ jumps from k_L to k_H at $\widetilde{t}_U$.*

overall credibility is restored more quickly, at time $\bar{t}_U < \bar{t}_H$. Because the high type investor's payoff is determined exclusively by the duration of the phase of doubt, she strictly benefits from opacity. The low type faces an initial phase with a high cheating rate and no surplus, a second phase with a lower cheating rate and positive surplus, and finally the restoration of full credibility. Though it takes longer for the entrepreneur's credibility to be restored fully, the low type expects positive surplus in the second phase. On net, she also benefits from opacity.

It is worth pointing out that endowing the investor with a disclosure technology by which she can verify her type to the entrepreneur at the outset does not undermine this equilibrium. That is, even if the investor has such a technology, an equilibrium exists in which neither type of investor uses it.[18]

6 Dynamic Liquidity Constraint

In this section, we consider a liquidity constraint that may prevent the investor from funding the project immediately. In particular, we suppose that the investor initially lacks sufficient capital to fund the project. The investor acquires these funds privately, at known Poisson

[18]Because both types of investor prefer opacity to transparency (at least weakly), if the type i investor is expected not to disclose her type, then it is a best response for the type j investor not to disclose. In some cases, equilibria with partial disclosure also exist.

rate γ.[19] If the entrepreneur submits a project before the investor has acquired the necessary capital to fund it, then the investor observes the arrival time, but she must delay her approval decision until she has the necessary funds. This situation might arise, for example, if the investor is obliged to wait for another investment to produce a return in order to recover seed capital for the current project. As in the main model, the project's payoffs are realized when the investor makes her funding decision.[20] In this setting, it is convenient to interpret ρ as the rate at which the game ends (e.g., because another entrepreneur innovates first). Then, by delaying the investor's decision, the liquidity constraint admits the possibility that the game ends before a submission can be approved for funding, even if the investor would like to do so.

There are multiple equilibria in this version of the game, but they are all expected payoff equivalent. Note that at any given time, the investor can be of two possible types: the unconstrained type has acquired capital already, while the constrained type is still waiting. Multiplicity arises because both types of the investor have the same beliefs about the project, and thus, the same approval incentives. In other words, given an arrival at time t, both types either strictly prefer to approve, strictly prefer to reject, or are indifferent. When both types are indifferent they may mix with different approval strategies in equilibrium—nevertheless the *expected probability of approval* is pinned down uniquely by the entrepreneur's indifference condition, which ensures that he is willing to mix.

With these qualifications in mind, we economize on notation and solve for a pooling equilibrium in which the constrained and unconstrained types of investor use the same approval strategy, $a_C(t)$. In other words, if the investor has acquired capital already, then she approves immediately with probability $a_C(t)$, and if she has not acquired capital, then she approves with this same probability as soon as she is able, provided the game does not end first.[21]

From the entrepreneur's perspective, the possibility that the investor is still liquidity constrained imposes an upper bound on the probability that a project is approved. In particular, if the project is submitted at time t and the investor would like to fund it, the probability

[19] It is not essential for the capital to arrive privately in order for a liquidity constraint to be beneficial to the investor, though the forces at play are more transparent when the arrival is private. Details for the case where capital arrives publicly are available upon request.

[20] It is worth clarifying that cost ϕ is paid by the entrepreneur at the time the fake is generated.

[21] The passage of additional time while waiting for capital does not change the constrained investor's belief about whether a project that was submitted earlier is real or fake.

that she is eventually able to do so is

$$\bar{a}(t) \equiv \underbrace{1 - \exp(-\gamma t)}_{\text{unconstrained type}} + \underbrace{\frac{\gamma}{\gamma + \rho} \exp(-\gamma t)}_{\text{constrained type}}$$
$$= 1 - \frac{\rho}{\rho + \gamma} \exp(-\gamma t).$$

The first term is the probability that capital is acquired before t, in which case the investor can fund the project at the time it is submitted; the second term is the probability that capital has not been acquired by time t, multiplied by the probability that it is acquired before the game terminates.

To highlight our main case of interest, we focus on capital that arrives slowly.

Assumption 2. *The rate at which capital arrives is sufficiently low that $\bar{a}(0)$ is less than the equilibrium approval of the main model at time zero for all $\sigma \in (0, 1)$,*

$$\frac{\gamma}{\gamma + \rho} < \frac{\phi}{\widehat{\phi}}.$$

To develop intuition for the effect of the liquidity constraint, consider a possible equilibrium in which the entrepreneur never fakes. In such a putative equilibrium, the investor always approves and hence, the expected approval probability is $\bar{a}(\cdot)$. The entrepreneur's best response is determined by the condition

$$u'(t) = \exp(-(\lambda + \rho)t)\{\bar{a}'(t) - \rho\bar{a}(t) + \phi(\rho + \lambda)\}$$
$$= \exp(-(\lambda + \rho)t)\rho\{\exp(-\gamma t) - (1 - \frac{\phi}{\widehat{\phi}})\}.$$

It follows that the entrepreneur would submit a fake at time $t^* \in (0, \infty)$, where

$$\exp(-\gamma t^*) = 1 - \frac{\phi}{\widehat{\phi}}.$$

Of course, if the entrepreneur did so, the investor would infer that a submission at this time is fake, which implies that such an equilibrium does not exist.[22] Unlike the main model, where the entrepreneur would like to deviate by faking *immediately*, in the current model the entrepreneur would like to delay faking until $t^* > 0$. Intuitively, the entrepreneur expects that the investor is unlikely to have the required capital early, so there is less incentive for the entrepreneur to pay the cost of faking in order to rush his project out for approval.

[22] The corresponding calculation implies that the ethical type's payoff of submitting a project at time t is strictly decreasing when the approval probability is $\bar{a}(\cdot)$. Thus, the ethical type would like to submit a real project as soon as it arrives.

This observation has implications for the equilibrium structure. When selecting his optimal cheating time, the entrepreneur can deviate from t^* both to later times ($t > t^*$) and to earlier times ($t < t^*$), which suggests that the support of the entrepreneur's equilibrium mixed strategy is an interval around t^*. Furthermore, if capital arrives slowly, then t^* is large, which suggests that the bottom of the support is strictly positive.

Lemma 6.1. *(Liquidity Constraint Equilibrium Structure). Let $a(t) \equiv a_C(t)\bar{a}(t)$ be the expected probability of approval. In an equilibrium with a liquidity constraint, there exist $\bar{t}_C \in (t^*, \infty)$ and $\widetilde{t}_C \in (0, t^*)$ such that*

(i) the time at which the entrepreneur submits a fake is drawn from a continuous mixed strategy with no mass points or gaps supported on interval $[\widetilde{t}_C, \bar{t}_C]$.

(ii) the investor's expected approval strategy $a(t)$ is continuous and increasing for all $t \geq 0$.

(iii) for $t \in (\widetilde{t}_C, \bar{t}_C)$, $a_C(t) < 1$ so that $a(t) < \bar{a}(t)$.

(iv) for $t \notin (\widetilde{t}_C, \bar{t}_C)$ $a_C(t) = 1$ so that $a(t) = \bar{a}(t)$.

This lemma highlights two main differences between the equilibrium with a liquidity constraint and the main model. First, with a liquidity constraint, there is an early phase ($t < \widetilde{t}_C$), in which the entrepreneur does not fake and the investor approves with the maximum expected probability $\bar{a}(\cdot)$—there is no such phase in the main model. Because the investor is likely to be constrained at early times, there is less incentive for the entrepreneur to accelerate the arrival of his project by faking. Second, once the entrepreneur's credibility is fully restored ($t > \bar{t}_C$) the investor is still constrained to approve with expected probability $\bar{a}(\cdot) < 1$, from an ex ante perspective. There is a strictly positive probability that the capital needed to fund the project will take longer than $\bar{t}_C$ to arrive, and thus, the investor cannot fund such projects with probability 1, even though she is sure that they are real. From an ex ante perspective, the first effect is beneficial to the investor, because she is confident that early arrivals are real and she is able to approve them some of the time, while the second effect is harmful, because it prevents her from approving late real arrivals as often as she would like.

We complete the characterization with the following proposition.

Proposition 6.1. *(Liquidity Constraint Equilibrium.) With a liquidity constraint, the pooling equilibrium of the game is characterized as follows.*

Strategies. *The entrepreneur's cheating time is drawn from continuous distribution function*

$$F(t) = \frac{1}{\sigma}(1 - \exp(-\mu(t - \widetilde{t}_C))$$

supported on interval $[\widetilde{t}_C, \bar{t}_C]$*, where* $\widetilde{t}_C$ *is such that*

$$\exp(-\gamma\widetilde{t}_C) = (1 - \frac{\phi}{\widehat{\phi}})(1 + \frac{\gamma}{\rho})\frac{1 - \exp(-\rho\bar{t})}{1 - \exp(-(\rho + \gamma)\bar{t})}$$

and $\bar{t}_C = \widetilde{t}_C + \bar{t}$*. If* $t \in (\widetilde{t}_C, \bar{t}_C)$*, then*

$$a(t) = \frac{\phi}{\widehat{\phi}} + (\overline{a}(\bar{t}_C) - \frac{\phi}{\widehat{\phi}})\exp(-\rho(\bar{t}_C - t))$$

and $a(t) = \overline{a}(t)$ *otherwise.*

Beliefs. *If* $t \in (\widetilde{t}_C, \bar{t}_C)$*, then* $g(t) = k$*, and* $g(t) = 1$ *otherwise.*

Payoffs. *The strategic entrepreneur's equilibrium payoff is*

$$U_C = \int_0^{\widetilde{t}_C} \lambda \exp(-(\rho + \lambda)t)\overline{a}(t)dt + \exp(-(\rho + \lambda)\widetilde{t}_C)(\overline{a}(\widetilde{t}_C) - \phi).$$

The investor's ex ante equilibrium payoff is

$$V_C = (1 - k)(\int_0^{\widetilde{t}_C} \lambda \exp(-(\rho + \lambda)t)\overline{a}(t)dt + (1 - \sigma)\int_{\bar{t}_C}^{\infty} \lambda \exp(-(\rho + \lambda)t)\overline{a}(t)dt).$$

Normative Ranking. *The investor is strictly better off in the liquidity constraint equilibrium than in the equilibrium of the main model if* σ *is sufficiently large, and she is strictly worse off if* σ *is sufficiently small.*

With a liquidity constraint, the equilibrium has *two* phases of credibility, an early one $(0, \widetilde{t}_C)$, and a late one $(\bar{t}_C, \infty)$, with a single phase of doubt $[\widetilde{t}_C, \bar{t}_C]$, sandwiched between them. During the initial phase of credibility, the investor approves all submissions with maximum probability, but because the strategic type does not fake, the investor's belief does not evolve. When the phase of doubt is reached, the entrepreneur begins to fake at rate μ, as in the main model, which leaves the investor indifferent between approving and rejecting. Because the investor's belief about the entrepreneur at time $\widetilde{t}_C$ is the same as at the beginning of the game and the entrepreneur fakes at the same rate during the phase of doubt, it takes the same amount of time for his credibility to be fully restored. That is, the duration of the phase of doubt is the same as in the main model, $\bar{t}_C - \widetilde{t}_C = \bar{t}$. Finally, we reach the second phase of credibility, in which the investor is confident that the entrepreneur is ethical and approves all submissions with maximum probability.

The initial phase of credibility generates a normative gain for the investor: an innovation in this phase is real and is approved with expected probability $\bar{a}(\cdot)$. At the same time, the second phase of credibility generates a normative loss for the investor. Although the investor

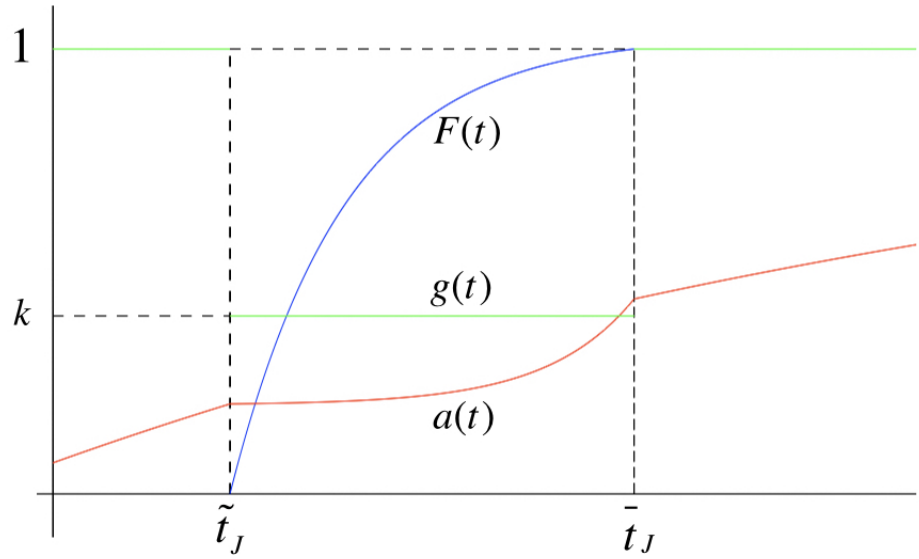

Figure 4: *Proposition 6.1. Note that $g(\cdot)$ jumps from 1 to k at $\widetilde{t}_C$ and back up to 1 at $\bar{t}_C$.*

is confident that such an arrival is real, the possibility of still being constrained prevents her from approving it with probability 1. In addition, the second phase of credibility is reached later than in the main model, at time $\bar{t}_C = \widetilde{t}_C + \bar{t}$, rather than at time $\bar{t}$. Thus, the positive surplus generated by such an arrival is discounted more heavily. The normative impact of the liquidity constraint depends on which of these effects dominates. When σ is large, the phase of doubt is very long in the main model and the investor's surplus approaches zero. We show that with the liquidity constraint, the duration of the *initial* phase of credibility approaches a strictly positive limit as σ gets large. Thus, the investor's expected surplus with the liquidity constraint is bounded away from zero. By implication, when she faces an entrepreneur who is likely to be strategic, the liquidity constraint benefits the investor. When σ becomes small, the duration of the phase of doubt collapses to zero. In the main model, the investor approves all submissions, which she is sure are real, and her payoff converges to the first best. With the liquidity constraint, she approves all projects with maximum probability, and she is sure that they are real. However, her payoff is strictly smaller because she cannot approve all projects with probability 1. Consequently, the liquidity constraint is harmful if the entrepreneur is most likely ethical.

7 Conclusion

In this paper we have investigated a novel dynamic model of fraud and trust in the context of an entrepreneur seeking a capital injection from an investor. The investor wishes to

approve funding for a *real* project and reject funding for a *fake*. The entrepreneur's type — ethical or strategic — is private information. Both types face pressure to perform, but an ethical entrepreneur does not have an opportunity to fake, or is unable to rationalize doing so. Therefore, he only produces a real innovation, while a strategic entrepreneur chooses between producing a real innovation and a fake one.

In the unique equilibrium of the baseline model, the strategic entrepreneur randomizes about when to commit fraud and the investor randomizes about whether to approve funding for a project as a function of its time of submission. As time passes without an innovation, the investor's belief that the entrepreneur is ethical rises until the point when she completely trusts the entrepreneur and approves any subsequent submission with certainty. Indeed, the investor receives a positive expected payoff from funding an innovation if and only if it arrives after her trust in the entrepreneur is fully restored. This generates a relatively low present expected payoff for the investor in the equilibrium of the baseline model. We explore two institutional remedies capable of improving the investor's welfare: opaque standards, and a dynamic liquidity constraint, deriving conditions under which these raise the investor's welfare.

In our analysis, the "pressure" to commit fraud is rooted in the entrepreneur's time-preference.[23] The entrepreneur commits fraud in order to accelerate the arrival of his utility from project approval. Other forms of "pressure" can also lead to fraud. For example, if there is a positive probability that the real technology may *never* deliver an arrival, then the entrepreneur's pessimism about the viability of the real technology grows over time. If enough time passes, the entrepreneur might resort to faking, because he doubts that a real project can ever be produced. A related form of pressure is generated by a deadline. As the deadline approaches, the probability of meeting it by honest means decreases, and the entrepreneur may resort to faking in order to do so. In a somewhat different vein, an entrepreneur may be tempted to fake in order to improve or maintain a reputation. For example, if there is uncertainty about the arrival rate of the entrepreneur's real technology, then an entrepreneur who would like to be perceived as "fast" might want to fake an arrival in order to affect the belief about his ability. Many of these alternate forms of pressure can be analyzed within the framework we present here.

We believe that the analysis can also be extended to study fraud and trust building in a number of related settings. For instance, it would be edifying to know how the potential for repeated interactions influences the entrepreneur's equilibrium incentives to commit fraud and the investor's incentives to approve his project submissions. Another possibility is to

[23] Though we model time-preference as discounting, similar results hold if the entrepreneur's time preference is modelled as a flow cost.

investigate the impact on incentives of competition between several entrepreneur s engaged in an innovation race. We look to extend the analysis presented above to address these and other related questions in future work.

A Proofs

Calculations for Footnote 6. Suppose that the real technology imposes flow cost c on the entrepreneur and the entrepreneur pays this flow cost until either (i) he has a real arrival, at rate λ, (ii) the game exogenously ends, at rate ρ, (iii) he submits a fake. Equation 3 becomes,

$$\begin{aligned} u_c(t) &= \int_0^t \lambda \exp(-(\rho+\lambda)s)(a(s)-cs) - \rho \exp(-(\rho+\lambda)s)cs \,\mathrm{d}s + \exp(-(\rho+\lambda)t)(a(t)-ct-\phi) \\ &= u(t) - c(\int_0^t (\lambda+\rho)\exp(-(\rho+\lambda)s)s \,\mathrm{d}s + \exp(-(\rho+\lambda)t)t), \end{aligned}$$

where $u(t)$ (as in (3)) is the entrepreneur's payoff function when $c = 0$. Note that

$$\int_0^t (\lambda+\rho)\exp(-(\rho+\lambda)s)s \,\mathrm{d}s + \exp(-(\rho+\lambda)t)t = K - \frac{\exp(-(\rho+\lambda)t)}{\lambda+\rho},$$

where K does not depend on t. Thus,

$$u_c(t) = \int_0^t \lambda \exp(-(\rho+\lambda)s)a(s)ds + \exp(-(\rho+\lambda)t)(a(t) - (\phi - \frac{c}{\lambda+\rho})) - cK.$$

Thus, the model with flow cost is strategically equivalent to the model without flow cost, with a smaller value of ϕ.

A.1 Proofs for Baseline Model

Proof of Lemma 4.1. We derive the probability distributions for the time of submission and the time of submission for a real project. If only the authentic technology is used, then the submission time for a *real* project is $T_R \sim H(t) \equiv 1 - \exp(-\lambda t)$. The waiting time for a fake project is $T_F \sim F(t)$. The overall waiting time for a submission is

$$T = (1-\sigma)T_R + \sigma \min\{T_R, T_F\}.$$

An ethical entrepreneur only uses the real technology, while a strategic entrepreneur uses the minimum of the real and fake waiting time. Therefore, the CDF of the arrival time is

$$\begin{aligned} W(t) = (1-\sigma)H(t) + \sigma(H(t) + F(t) - F(t)H(t)) &= H(t) + \sigma F(t)(1-H(t)) \\ &= 1 - \exp(-\lambda t)(1-\sigma F(t)), \end{aligned}$$

with associated density $w(t) = \exp(-\lambda t)[\lambda(1-\sigma F(t)) + \sigma f(t)]$. Using similar reasoning, the waiting time for a real arrival is distributed according to

$$\begin{aligned} W_A(t) &= (1-\sigma)H(t) + \sigma \int_0^t h(s)(1-F(s))\,\mathrm{d}s \\ &= (1-\sigma)(1-\exp(-\lambda t)) + \sigma \int_0^t \lambda \exp(-\lambda s)(1-F(s))\,\mathrm{d}s. \end{aligned}$$

If entrepreneur is ethical, a real project arrives before time t with probability $H(t)$. If entrepreneur is strategic, then a real project arrives before time t if a real arrival occurs at any $s \leq t$ and the fake arrival takes longer than s. Integrating over $s \leq t$ yields the expression. The density of the waiting time for a real arrival is therefore,

$$w_A(t) = \lambda \exp(-\lambda t)(1-\sigma F(t)).$$

By Bayes' Rule, the probability that a submission at time t is real is

$$g(t) = \frac{w_A(t)}{w(t)} = \frac{\lambda(1-\sigma F(t))}{\lambda(1-\sigma F(t)) + \sigma f(t)}.$$

Dividing numerator and denominator by $1-\sigma F(t)$ yields the desired result. □

Proof of Lemma 4.2. We verify that the prescribed behavior is an equilibrium when $\phi > \widehat{\phi}$. Suppose $a(t) = 1$ for all $t \geq 0$. Substituting into (3) and evaluating gives $u'(t) > 0 \iff \phi > \widehat{\phi}$. Thus, when $\phi > \widehat{\phi}$ the entrepreneur's best response to $a(\cdot) = 1$ is never to submit a fake; furthermore, because the entrepreneur never fakes, we have $g(t) = 1$ for all $t \geq 0$, and from (2), we have $a(\cdot) = 1$.

Next, we show that when $\phi < \widehat{\phi}$, the entrepreneur submits a fake with positive probability in equilibrium. Consider a possible equilibrium in which the entrepreneur does not submit a fake. By implication, $g(\cdot) = 1$, and hence, $a(\cdot) = 1$. Substituting into (3) and evaluating gives $u'(t) < 0 \iff \phi < \widehat{\phi}$. Therefore, the entrepreneur's best response is to fake at time 0, contradicting the assumption that the entrepreneur fakes with probability 0.

Assuming $\phi > \widehat{\phi}$, we verify uniqueness with the following steps.

Step 1. We show that the entrepreneur's mixed strategy cannot have a mass point. If such mass point exists at t, then $g(t) = a(t) = 0$. An elementary calculation implies that $u(t) < u(\infty)$, and thus faking at t is not a best response.

Step 2. We show that there exists some finite t^* such that $a(t) = 1$ and $f(t) = 0$ for $t \geq t^*$. Integrability implies $\lim_{t\to\infty} f(t) = 0$. Furthermore, $1-\sigma F(t) \geq 1-\sigma$, and hence, $\lim_{t\to\infty} \mu(t) = 0$. It follows that there exists t^* such that, $\mu(t) < k$ for $t > t^*$. From (2), we have $a(t) = 1$ for $t > t^*$. Substituting into (3) and differentiating at $t > t^*$, we find that $u'(t) > 0 \iff \phi > \widehat{\phi}$. Hence, $u(t) < u(\infty)$ for $t > t^*$. By implication, $f(t) = 0$ for such t.

Let $\bar{t}$ be $\sup\{t : f(t) > 0\}$. From Step 2, we have $\bar{t} < \infty$.

Step 3. We show that no equilibrium with faking exists. If $\bar{t} = 0$ and faking occurs in equilibrium, then $f(0) > 0$, contradicting Step 1. Therefore if such an equilibrium exists, then it must have $\bar{t} > 0$. Suppose this is the case. By definition of $\bar{t}$, for any small ϵ, there exists $t \in (\bar{t} - \epsilon, \bar{t}]$ such that $f(t) > 0$. For any such t we have that the entrepreneur's equilibrium payoff must be

$$u(t) = \int_0^t \lambda \exp(-(\rho + \lambda)s)a(s)\,\mathrm{d}s + \exp(-(\rho + \lambda)t)(a(t) - \phi).$$

This must be at least as large as

$$u(\infty) = \int_0^t \lambda \exp(-(\rho+\lambda)s)a(s)\,\mathrm{d}s + \int_t^{\bar{t}} \lambda \exp(-(\rho+\lambda)s)a(s)\,\mathrm{d}s + \int_{\bar{t}}^{\infty} \lambda \exp(-(\rho+\lambda)s)\,\mathrm{d}s.$$

Thus, for any choice of ϵ and a corresponding t, we have

$$\exp(-(\rho + \lambda)t)(a(t) - \phi) \geq \int_t^{\bar{t}} \lambda \exp(-(\rho + \lambda)s)a(s)\,\mathrm{d}s + \int_{\bar{t}}^{\infty} \lambda \exp(-(\rho + \lambda)s)\,\mathrm{d}s.$$

Since $\exp(-(\rho + \lambda)t)$ is decreasing, $a(\cdot) \in [0, 1]$, we have

$$\exp(-(\rho + \lambda)(\bar{t} - \epsilon))(1 - \phi) \geq \int_{\bar{t}}^{\infty} \lambda \exp(-(\rho + \lambda)s)\,\mathrm{d}s.$$

Integrating and simplifying, we have

$$\exp((\rho + \lambda)\epsilon))(1 - \phi) \geq (1 - \widehat{\phi}),$$

for any $\epsilon > 0$. Choosing $\epsilon \approx 0$ implies that $\phi < \widehat{\phi}$, contradicting the maintained hypothesis. □

Proof of Lemma 4.3. We prove several steps which we then combine.

Step 1. We show that $a(t) \geq \phi$ for all $t \geq 0$. By way of contradiction, consider some $t \geq 0$ and suppose that $a(t) < \phi$. Because $a(t) - \phi < 0$, and $\int_t^{\infty} \lambda \exp\{-(\rho + \lambda)s\}a(s)ds \geq 0$, submitting a fake at time t is worse for the strategic entrepreneur than never submitting a fake, that is $u(t) < u(\infty)$. It follows that, $\mu(t) = 0$, which implies $g(t) = 1$, and hence $a(t) = 1$ by (2). Since we assumed $a(t) < \phi$, we find that $\phi > 1$, a contradiction.

Step 2. We show $f(t) = 0 \implies a(t) = 1$. This follows from $f(t) = 0 \implies \mu(t) = 0 \implies g(t) = 1 \implies a(t) = 1$, by (2).

Step 3. We show that if $a(t) = 1$, then $f(t') = 0$ and $a(t') = 1$ for all $t' > t$. Suppose $a(t) = 1$. Then for all $t' > t$ we have,

$$\begin{aligned} u(t') - u(t) &= \int_t^{t'} \lambda \exp(-(\rho+\lambda)s) a(s)\,\mathrm{d}s + \exp(-(\rho+\lambda)t')(a(t') - \phi) - \exp(-(\rho+\lambda)t)(1-\phi) \\ &\leq \int_t^{t'} \lambda \exp(-(\rho+\lambda)s)\,\mathrm{d}s + \exp(-(\rho+\lambda)t')(1-\phi) - \exp(-(\rho+\lambda)t)(1-\phi) \\ &= \left(\phi - \frac{\rho}{\rho+\lambda}\right)(\exp(-(\rho+\lambda)t) - \exp(-(\rho+\lambda)t') < 0, \end{aligned}$$

where the last inequality follows because $\phi < \widehat{\phi}$ and $t' > t$. Therefore, the entrepreneur receives a strictly higher payoff from submitting a fake at t than at any $t' > t$. This implies $f(t') = 0$ which implies $\mu(t') = 0$ which implies $g(t') = 1$ and by (2) $a(t') = 1$.

For the rest of the proof, let $\bar{t} \equiv \inf\{t : a(t) = 1\}$ or $\bar{t} = \infty$ if $a(t) < 1$ for all $t \geq 0$.

Step 4. We show that if $t < \bar{t}$, then $a(t) \in [\phi, 1)$ and $f(t) > 0$. That $a(t) < 1$ follows from Step 3 and the definition of $\bar{t}$. That $a(t) \geq \phi$ follows from Step 1. From the investor's indifference condition (2), we get $\mu(t) = \lambda(1-k)/k > 0$. Hence $f(t) > 0$.

Step 5. We show that $F(\cdot)$ has no mass point for any $t < \infty$. If $F(\cdot)$ has a mass point at t, then $\mu(t) = \infty$, and hence, $a(t) = 0$, which contradicts Step 1.

Step 6. We show that $\bar{t} \in (0, \infty)$. first suppose $\bar{t} = 0$. Then $a(t) = 1$ for all $t \geq 0$ by Step 3. Substituting into (3) and simplifying yields $u'(t) < 0 \iff \phi < \widehat{\phi}$. Hence, it is optimal for the entrepreneur to submit a fake with probability 1 at $t = 0$. From (2), $a(0) = 0$ is sequentially rational, a contradiction. Next, suppose that $\bar{t} = \infty$, i.e. for all t we have $f(t) > 0$. From Step 4, we have $a(t) \in (0, 1)$ for all t. Hence, $\mu(t) = \lambda(1-k)/k$ for all t. By implication, $f(t) = (1 - \sigma F(t))\lambda(1-k)/(\sigma k) \geq (1-\sigma)\lambda(1-k)/(\sigma k) > 0$. Thus the integral of $f(\cdot)$ is unbounded, a contradiction.

Proof of (i). This follows from Steps 3, 4, 5, and 6.
Proof of (ii). This follows from Steps 3 and 6.

Step 7. We show that $a(\cdot)$ is continuous at $\bar{t}$. Note that for $t > \bar{t}$ we have $\lim_{t\to\bar{t}} a(t) = a(\bar{t}) = 1$. We seek to show that for $t < \bar{t}$, we have $\lim_{t\to\bar{t}} a(t) = 1$. Consider $t < \bar{t}$. Because $f(t) > 0$, we must have $u(t) \geq u(\bar{t})$. Hence,

$$u(\bar{t}) - u(t) = \int_t^{\bar{t}} \lambda \exp(-(\rho+\lambda)s) a(s)\,\mathrm{d}s + \exp(-(\rho+\lambda)\bar{t})(1-\phi) - \exp(-(\rho+\lambda)t)(a(t) - \phi) \leq 0$$

Because $a(\cdot)$ is bounded, in the limit as $t \to \bar{t}$, we have $\lim_{t\to\bar{t}}\{u(\bar{t}) - u(t)\} = \exp(-(\rho + \lambda)\bar{t})(1 - \lim_{t\to\bar{t}} a(t)) \leq 0$. Because $a(t) \leq 1$ for all t, we have $\lim_{t\to\bar{t}} a(t) = 1$.

Step 8. We show that for $t < \bar{t}$, $a(t)$ is continuous, differentiable, and strictly increasing. Let $t, t' < \bar{t}$. Because $t, t' < \bar{t}$, from Claim (i) we have $f(t)\,f(t') > 0$. Hence, $u(t') = u(t)$.

Therefore,

$$\begin{aligned}
0 &= u(t') - u(t) \\
&= \int_t^{t'} \lambda \exp(-(\rho+\lambda)s) a(s)\,\mathrm{d}s + \exp(-(\rho+\lambda)t')(a(t')-\phi) - \exp(-(\rho+\lambda)t)(a(t)-\phi) \\
&= \int_t^{t'} \lambda \exp(-(\rho+\lambda)s) a(s)\,\mathrm{d}t + [\exp(-(\rho+\lambda)t') - \exp(-(\rho+\lambda)t)](a(t')-\phi) \\
&\qquad\qquad + \exp(-(\rho+\lambda)t)(a(t')-a(t)).
\end{aligned}$$

Because the integrand above is bounded, taking the limit as $t' \to t$ yields $a(t') \to a(t)$. Hence, $a(t)$ is continuous.

To show that $a(t)$ is differentiable, divide the preceding equation by $t'-t$ to obtain,

$$\begin{aligned}
\frac{\int_t^{t'} \lambda \exp(-(\rho+\lambda)s) a(s)\,\mathrm{d}s}{t'-t} + \frac{\exp(-(\rho+\lambda)t') - \exp(-(\rho+\lambda)t)}{t'-t}(a(t')-\phi) \\
+ \exp(-(\rho+\lambda)t)\frac{a(t')-a(t)}{t'-t} = 0.
\end{aligned}$$

Because $a(\cdot)$ is continuous, the limit as $t' \to t$ gives

$$\lambda \exp(-(\rho+\lambda)t)a(t) - (\rho+\lambda)\exp(-(\rho+\lambda)t)(a(t)-\phi) + \exp(-(\rho+\lambda)t)\lim_{t'\to t}\frac{a(t')-a(t)}{t'-t} = 0.$$

Hence the derivative of $a(\cdot)$ exists at t. Furthermore, from this equation we have,

$$a'(t) = (\rho+\lambda)(a(t)-\phi) - \lambda a(t) = (\rho+\lambda)[a(t)\frac{\rho}{\rho+\lambda} - \phi] = (\rho+\lambda)[a(t)\widehat{\phi} - \phi].$$

It follows that $a(t)$ does not change monotonicity for any $t \in (0, \bar{t})$. It is either constant, strictly decreasing, or strictly increasing. Suppose $a(t)$ is constant or strictly decreasing for all $t < \bar{t}$. It follows that $a(t) \le \phi/\widehat{\phi}$ for all $t < \bar{t}$. Because $\phi/\widehat{\phi} < 1$, we have $\lim_{t\to\bar{t}} a(t) < 1$, contradicting Step 7.

Proof of (iii). This follows from Steps 7 and 8. □

Proof of Proposition 4.1. Strategies. The entrepreneur must be indifferent about submitting at all times $t \in (0, \bar{t})$ and $a(\cdot)$ is differentiable. Therefore, for such t,

$$\begin{aligned}
u'(t) &= \exp(-(\rho+\lambda)t)\{\lambda a(t) - (\rho+\lambda)(a(t)-\phi) + a'(t)\} \\
&= \exp(-(\rho+\lambda)t)\{a'(t) - \rho a(t) + \phi(\rho+\lambda)\} = 0,
\end{aligned}$$

and hence, for $t \in [0, \bar{t}]$, we have $a(t) = \phi(1+\frac{\lambda}{\rho}) + \kappa_1 \exp(\rho t)$ for some integration constant $\kappa_1 > 0$. Because $a(t) \in (0,1)$ for $t \in (0, \bar{t})$, we also have

$$g(t) = k \implies \mu(t) = \mu \implies F(t) = \frac{1}{\sigma}(1 - \kappa_2 \exp(-\mu t)),$$

where κ_2 is another integration constant. Note that the entrepreneur's mixed strategy cannot have a mass point, and hence $F(0) = 0$, which implies $\kappa_2 = 1$. It follows that

$$\bar{t} = -\ln(1-\sigma)\frac{k}{\lambda(1-k)}.$$

From the boundary condition $a(\bar{t}) = 1$ we find

$$\kappa_1 = (1 - \phi(1 + \frac{\lambda}{\rho}))\exp(-\rho\bar{t}) = (1 - \phi(1 + \frac{\lambda}{\rho}))(1-\sigma)^{\frac{\rho k}{\lambda(1-k)}}.$$

Observing that $\phi(1 + \frac{\lambda}{\rho}) = \frac{\phi}{\widehat{\phi}}$ completes the characterization of strategies.

Beliefs. Obvious.

Payoffs. **Strategic Entrepreneur .** The strategic entrepreneur's payoff is identical for all cheating times $t \in [0, \bar{t})$, and hence, $U^S = a(0) - \phi$. Simplifying, we have

$$U^S = \frac{\phi(\rho+\lambda)}{\rho} + (1 - \frac{\phi(\rho+\lambda)}{\rho})\exp\{-\rho\bar{t}\} - \phi = \frac{\phi\lambda}{\rho} + (1 - \phi - \frac{\phi\lambda}{\rho})(1-\sigma)^{\frac{\rho}{\mu}}.$$

Ethical Entrepreneur . Payoff of the ethical entrepreneur is

$$U^E = \int_0^\infty \lambda\exp(-(\rho+\lambda)t)a(t)\,\mathrm{d}t = \lim_{t\to\infty} u(t).$$

Recall that $u(t) = U^S$ on $[0, \bar{t}]$ and in particular $u(\bar{t}) = U^S$. Furthermore, since $a(t) = 1$ on $[\bar{t}, \infty)$, differentiation reveals that for $t \in (\bar{t}, \infty)$ we have

$$u'(t) = -(\widehat{\phi} - \phi)(\rho+\lambda)\exp(-(\rho+\lambda)t).$$

It follows that

$$U^E = \lim_{t\to\infty} u(t) = u(\bar{t}) + \int_{\bar{t}}^\infty u'(t)\,\mathrm{d}t = U^S - (\widehat{\phi} - \phi)\exp(-(\rho+\lambda)\bar{t}).$$

Investor. The investor is indifferent between approving and rejecting for all $t < \bar{t}$, and therefore her expected payoff is 0 if an arrival occurs before time $\bar{t}$. Furthermore, if the entrepreneur is strategic, then an arrival will certainly occur before time $\bar{t}$. If the arrival occurs after $\bar{t}$, then it is real and will be approved with probability 1. Hence,

$$V = (1-k)(1-\sigma)\int_{\bar{t}}^\infty \lambda\exp\{-(\rho+\lambda)t\}\,\mathrm{d}t = (1-k)(1-\sigma)\frac{\lambda}{\rho+\lambda}\exp\{-(\rho+\lambda)\bar{t}\}$$

□

A.2 Proofs for Opaque Standards

Proof of Lemma 5.1. Proof of (i). This point follows exactly from the arguments in the proof of Lemma 4.3, replacing $a(\cdot)$ with $a_U(\cdot)$.

Step 0. We claim that for $t \in [0, \bar{t}_U)$, the expected approval probability $a_U(\cdot)$ is strictly greater than ϕ, strictly increasing, continuous, and differentiable almost everywhere, with $\lim_{t \to \bar{t}_U} a_U(t) = 1$. Furthermore, for $t \in [\bar{t}_U, \infty)$, the expected approval probability is 1, $a_U(t) = 1$. These points follow exactly from the arguments in the proof of Lemma 4.3, replacing $a(\cdot)$ with $a_U(\cdot)$.

By analogy with the proof of Lemma 4.3, let $\bar{t}_U \equiv \inf\{t : a_U(t) = 1\}$. Existence of $\bar{t}_U \in (0, \infty)$ is established by analogy with Lemma 4.3.

Proof of (ii). From Step 0, if $t \geq \bar{t}_U$ then $a_U(t) = 1$, and hence, $\nu a_H(t) + (1-\nu) a_L(t) = 1$. Because $a_L(t) \leq 1$ and $a_H(t) \leq 1$, it follows that $a_H(t) = a_L(t) = 1$.

Step 1. We show that for any $t \geq 0$, (a) if $a_H(t) > 0$, then $a_L(t) = 1$ and (b) if $a_L(t) < 1$, then $a_H(t) = 0$. Both (a) and (b) follow immediately from each type of investor's sequentially rational approval strategy, coupled with $k_H > k_L$.

Step 2. We show that for any $t \in [0, \bar{t}_U)$, exactly one of the following three conditions (A,B,C) must hold: (A) $a_H(t) \in (0,1)$ and $a_L(t) = 1$, (B) $a_L(t) \in (0,1)$ and $a_H(t) = 0$, (C) $a_L(t) = 1$ and $a_H(t) = 0$. From the definition of $\bar{t}_U$, we know that $a_U(t) < 1$ for $t < \bar{t}_U$. Hence, for such t, at least one of $a_i(t) < 1$ for $i \in \{H, L\}$.

If $a_L(t) < 1$, then $a_H(t) = 0$ from Step 1 (b). Furthermore, from Step 0, we know that $a_U(t) > \phi$. Coupled with $a_H(t) = 0$, this implies $a_L(t) > 0$. Hence, we have (B).

If $a_H(t) < 1$, then there are two possibilities. If $a_H(t) > 0$, then from Step 1 (a), we have $a_L(t) = 1$, case (A). If $a_H(t) = 0$, then we must have $a_L(t) > 0$ (otherwise $a_U(t) = 0$, contradicting Step 0). If $a_L(t) < 1$, then case (B). If $a_L(t) = 1$ then case (C).

Step 3. Consider $0 \leq t < t' < \bar{t}_U$. We show that (a) If $a_H(t) \in (0,1)$ then $a_H(t) < a_H(t') < 1$ and $a_L(t') = 1$, (b) If $a_L(t') \in (0,1)$ then $a_L(t) < a_L(t') < 1$ and $a_H(t) = 0$. From Step 0, we have $a_U(t') > a_U(t)$. Thus,

$$\nu a_H(t') + (1-\nu) a_L(t') > \nu a_H(t) + (1-\nu) a_L(t). \tag{A.1}$$

To prove claim (a), suppose that $a_H(t) \in (0,1)$. By Step 1 (a), we have $a_L(t) = 1$. Hence, (A.1) implies

$$\begin{aligned} \nu a_H(t') + (1-\nu) a_L(t') &> \nu a_H(t) + (1-\nu) \\ \nu(a_H(t') - a_H(t)) &> (1-\nu)(1 - a_L(t')) \geq 0, \end{aligned}$$

and hence, $a_H(t') > a_H(t) > 0$. From Step 2, we have $a_H(t') < 1$ and $a_L(t') = 0$ (Case A).

To prove claim (b), suppose that $a_L(t') \in (0,1)$. By Step 1 (b) we have $a_H(t') = 0$. Hence, (A.1) implies

$$(1-\nu)a_L(t') > \nu a_H(t) + (1-\nu)a_L(t)$$
$$(1-\nu)(a_L(t') - a_L(t)) > \nu a_H(t) \geq 0$$

and hence, $a_L(t) < a_L(t') < 1$. From Step 2, we must have $a_L(t) > 0$ and $a_H(t) = 0$.

Step 4. We show that there exists some $t < \bar{t}_U$ such that $a_H(t) \in (0,1)$. Suppose not. From Step 2, we have that $a_H(t)$ for all $t < \bar{t}_U$. Thus, for all such t, we have $a_U(t) = (1-\nu)a_L(t) \leq (1-\nu) < 1$. By implication $\lim_{t\to\bar{t}_U} a_U(t) \leq 1-\nu < 1$, contradicting Part (ii).

Let $\widetilde{t}_U \equiv \inf\{t : a_H(t) > 0\}$.

Proof of (iii). From Step 4, we know that $\widetilde{t}_U < \bar{t}_U$. Thus, for any $t \in (\widetilde{t}_U, \bar{t}_U)$, there exists $t' = t - \epsilon$ such that $a_H(t') > 0$. Applying Step 3, we know that for any $t \in (\widetilde{t}_U, \bar{t}_U)$ we have $a_H(t) \in (0,1)$ and $a_L(t) = 1$. Thus, for such t, we have $a_U(t) = \nu a_H(t) + (1-\nu)$. Because $a_U(\cdot)$ is continuous, increasing, and differentiable on $[0, \bar{t}_U)$ and $\widetilde{t}_U < \bar{t}_U$, we have that $a_H(\cdot)$ is continuous, increasing, and differentiable for such t. Finally, from $\lim_{t\to\bar{t}_U} a_U(t) = 1$, we have $\lim_{t\to\bar{t}_U} a_H(t) = 1$.

Step 5. Suppose $\widetilde{t}_U > 0$. We show that $a_L(t) \in (0,1)$ and $a_H(t) = 0$ for $t < \widetilde{t}_U$. From the definition of $\widetilde{t}_U$, we have $a_H(t) = 0$ for $t < \widetilde{t}_U$. From Step 2, for all such t, we have either $a_L(t) \in (0,1)$ for $a_L(t) = 1$. Consider $t, t' < \widetilde{t}_U$ with $t' > t$. From Step 0, we have $a_U(t') > a(t)$, and hence, $a_L(t') > a_L(t)$. By implication $a_L(t) < 1$. Hence, $a_L(t) \in (0,1)$.

Step 6. Suppose $\widetilde{t}_U > 0$. We show that $a_L(t)$ is continuous, increasing, differentiable for $t < \widetilde{t}_U$. From Step 5, we have $a_H(t) = 0$ for $t < \widetilde{t}_U$. Therefore $a_U(t) = (1-\nu)a_L(t)$. Because $a_U(\cdot)$ is continuous, increasing, and differentiable, the conclusion follows.

Step 7. Suppose $\widetilde{t}_U > 0$. We show that $\lim_{t\to\widetilde{t}_U} a_L(t) = 1$ and $\lim_{t\to\widetilde{t}_U} a_H(t) = 0$. Let $t^- \equiv \widetilde{t}_U - \epsilon$ and $t+- \equiv \widetilde{t}_U + \epsilon$ for $\epsilon > 0$. Note that for $t > \widetilde{t}_U$ we have $a_L(t) = 1$. It is therefore obvious that $\lim_{\epsilon\to 0} a_L(t^+) = 1$. Similarly, for $t < \widetilde{t}_U$ we have $a_H(t) = 0$. Therefore it is obvious that $\lim_{\epsilon\to 0} a_H(t^-) = 0$. What remains to establish is

$$\lim_{\epsilon\to 0} a_L(t^-) = 1 \qquad \lim_{\epsilon\to 0} a_H(t^+) = 0.$$

Note that continuity of $a_U(\cdot)$ at $\widetilde{t}_U$ implies

$$\lim_{\epsilon\to 0}[a_U(t^-) - a_U(t^+)] = 0.$$

Substituting $a_L(t^+) = 1$ and $a_H(t^-) = 0$ we have,

$$\lim_{\epsilon\to 0}[(1-\nu)a_L(t^-) - \nu a_H(t^+) - (1-\nu)] = 0 \Rightarrow \lim_{\epsilon\to 0}[(1-\nu)(a_L(t^-) - 1) - \nu a_H(t^+)] = 0.$$

Because $a_L(\cdot) \leq 1$ and $a_H(\cdot) \geq 0$ the result follows.

Proof of (iv). Follows from Steps 5-7. □

Proof of Proposition 5.1. We construct a one phase equilibrium, consistent with Lemma 5.1, showing that such an equilibrium exists if and only if $\nu > (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp(-\rho \bar{t}_H))$, and that the only such equilibrium is the one characterized in the statement of the proposition. To this end, consider the one phase structure, characterized by Lemma 5.1 with $\widetilde{t}_U = 0$.

Strategies. If an equilibrium with the one phase structure exists, then for all $t \in [0, \bar{t}_U)$ we have $a_H(t) \in (0, 1)$. By implication,

$$g(t) = k_H \Rightarrow \mu(t) = \lambda \frac{1 - k_H}{k_H} \Rightarrow F(t) = \frac{1}{\sigma}(1 - \exp(-\lambda \frac{1 - k_H}{k_H} t)),$$

where the last step uses point (i) of Lemma 5.1 to rule out a mass point in the entrepreneur's mixed strategy, thereby identifying an integration constant. Using point (i) of Lemma 5.1, we have $F(\bar{t}_U) = 1$, implying $\bar{t}_U = \bar{t}_H$ as stated in the proposition.

For the approval probability, note that the entrepreneur's expected payoff of waiting to cheat until time t is

$$u(t) = \int_0^t \exp(-(\lambda + \rho)s)(\nu a_H(s) + (1 - \nu))ds + \exp(-(\lambda + \rho)t)(\nu a_H(t) + (1 - \nu) - \phi),$$

where we have used Lemma 5.1 (iii) to establish $a_H(t) \in (0, 1)$ and $a_L(t) = 1$. Furthermore, since $a_H(\cdot)$ is differentiable for $t < \bar{t}_U$. It follows that

$$u'(t) = 0 \iff \nu a_H'(t) - \rho(\nu a_H(t) + (1 - \nu)) + \phi(\rho + \lambda) = 0.$$

Solving, we have

$$a_H(t) = \frac{1}{\nu}\Big(\frac{\phi}{\widehat{\phi}} - (1 - \nu)\Big) + \kappa \exp\{\rho t\},$$

where κ is an integration constant. Using the boundary condition $a_H(\bar{t}_H) = 1$ we find

$$\begin{aligned} a_H(t) &= \frac{1}{\nu}(\frac{\phi}{\widehat{\phi}} - (1 - \nu)) + \Big(1 - \frac{1}{\nu}(\frac{\phi}{\widehat{\phi}} - (1 - \nu))\Big) \exp\{-\rho(\bar{t}_H - t)\} \\ &= \frac{1}{\nu}(\frac{\phi}{\widehat{\phi}} + \Big(1 - \frac{\phi}{\widehat{\phi}}\Big) \exp\{-\rho(\bar{t}_H - t)\} - (1 - \nu)). \end{aligned}$$

Therefore, such an equilibrium exists provided two additional conditions,

$$a_H'(t) > 0 \iff \frac{1}{\nu}(\frac{\phi}{\widehat{\phi}} - (1 - \nu)) < 1 \iff \phi < \widehat{\phi}$$

and

$$a_H(0) \geq 0 \iff \nu \geq (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp\{-\rho \bar{t}_H\}).$$

The first of these is Assumption 1, the second is stated in the proposition.

Payoffs. The strategic entrepreneur's payoff is equal to his payoff of submitting at time 0, which is

$$a_U(0) - \phi = \nu a_H(t) + (1 - \nu) - \phi = \frac{\phi}{\widehat{\phi}} + (1 - \frac{\phi}{\widehat{\phi}}) \exp(-\rho \bar{t}_H) - \phi,$$

which is identical to the strategic entrepreneur's payoff in the main model, when facing an investor with known standard k_H. The high type investor gets payoff zero from any arrival inside the phase of doubt, and a payoff of one if the phase of credibility is reached. Thus, the high type investor's payoff is

$$(1 - k_H)(1 - \sigma) \int_{\bar{t}_H}^{\infty} \lambda \exp(-(\lambda + \rho)s) ds,$$

exactly as in the main model where the investor is known to be type H. The low type investor's payoff, in this equilibrium is

$$\int_0^{\bar{t}_H} \exp(-(\rho+\lambda)t)(\lambda(1-\sigma F(t))(1-k_L) - \sigma f(t) k_L) dt + (1-k_L)(1-\sigma) \int_{\bar{t}_H}^{\infty} \lambda \exp(-(\rho+\lambda)t) dt.$$

In this equilibrium, $1 - \sigma F(t) = \exp(-r_H t)$ and $\sigma f(t) = r_H \exp(-r_H t)$, where $r_H \equiv \lambda \frac{1-k_H}{k_H}$. Therefore, the low type investor's payoff is

$$V_L = \int_0^{\bar{t}_H} \exp(-(\rho + \lambda + r_H)t)(\lambda(1 - k_L) - r_H k) dt + (1 - k_L)(1 - \sigma) \int_{\bar{t}_H}^{\infty} \lambda \exp(-(\rho + \lambda)t) dt.$$

Substituting for r_H, we have

$$\begin{aligned} V_L =& \int_0^{\bar{t}_H} \exp(-(\rho + \frac{\lambda}{k_H})t)(\lambda(1 - k_L) - k_L \lambda \frac{1 - k_H}{k_H}) \, \mathrm{d}t + (1 - \sigma)(1 - k_L) \int_{\bar{t}_H}^{\infty} \lambda \exp(-(\rho + \lambda)t) \, \mathrm{d}t \\ =& \lambda(1 - \frac{k_L}{k_H}) \int_0^{\bar{t}_H} \exp(-(\rho + \frac{\lambda}{k_H})t) \, \mathrm{d}t + (1 - \sigma)(1 - k_L) \int_{\bar{t}_H}^{\infty} \lambda \exp(-(\rho + \lambda)t) \, \mathrm{d}t. \end{aligned}$$

Normative Analysis. Point (i) is obvious, since the high type investor's payoff is the same as under transparency, where she is known to be the high type.

(ii) We seek to show that this is larger than the payoff in the main model, V, given in Proposition 4.1. Note first that for $k_H = k_L$, the two expressions are equal, i.e. $V_L = V$. Differentiating with respect to k_H, we have

$$\begin{aligned} \frac{dV_L}{dk_H} =& \frac{\lambda k_L}{k_H^2} \int_0^{\bar{t}_H} \exp(-(\rho + \frac{\lambda}{k_H})t) \, \mathrm{d}t \\ &+ \lambda(1 - \frac{k_L}{k_H})[\exp(-(\rho + \frac{\lambda}{k_H})\bar{t}_H) \frac{d\bar{t}_H}{dk_H} + \int_0^{\bar{t}_H} \exp(-(\rho + \frac{\lambda}{k_H})t) \frac{\lambda}{k_H^2} t dt] - (1 - \sigma)(1 - k_L) \lambda \exp(-(\rho + \lambda)\bar{t}_H) \frac{d\bar{t}_H}{dk_H}. \end{aligned}$$

Note that

$$(1-\sigma)\exp(-(\rho+\lambda)\bar{t}_H) = \exp(\{1+(\rho+\lambda)\frac{k_H}{\lambda(1-k_H)}\}\ln(1-\sigma)) = \exp(-(\rho+\frac{\lambda}{k_H})\bar{t}_H),$$

and

$$\frac{d\bar{t}_H}{dk_H} = -\frac{\lambda(1-k_H)+\lambda k_H}{\lambda^2(1-k_H)^2}\ln(1-\sigma) = -\frac{1}{\lambda(1-k_H)^2}\ln(1-\sigma) = \frac{\bar{t}_H}{k_H(1-k_H)}.$$

Substituting and simplifying, we have

$$\frac{dV_L}{dk_H} = \frac{\lambda k_L}{k_H^2}\int_0^{\bar{t}_H}\exp(-(\rho+\frac{\lambda}{k_H})t)\,\mathrm{d}t - \frac{\lambda k_L}{k_H^2}\exp(-(\rho+\frac{\lambda}{k_H})\bar{t}_H)\bar{t}_H + \lambda(\frac{k_H-k_L}{k_H})\int_0^{\bar{t}_H}\exp(-(\rho+\frac{\lambda}{k_H})t)\frac{\lambda}{k_H^2}t dt.$$

Noting that the last integral is strictly positive, we have

$$\frac{dV_L}{dk_H} > \frac{\lambda k}{k_H^2}[\int_0^{\bar{t}_H}\exp(-(\rho+\frac{\lambda}{k_H})t)\,\mathrm{d}t - \exp(-(\rho+\frac{\lambda}{k_H})\bar{t}_H)\bar{t}_H] > 0,$$

where the last inequality follows because $\exp(-(\rho+\lambda/k_H)t)$ is a decreasing function, and thus, its average value on interval $[0,\bar{t}_H]$ is larger than its value at the right endpoint. Because (i) V_L is increasing in k_H, (ii) $k_H = k \Rightarrow V_L = V$, and (iii) $k_H > k_L$, we have that in the one stage auditing equilibrium $V_L > V$. □

Proof of Proposition 5.2. We construct a two stage equilibrium, consistent with Lemma 5.1, showing that such an equilibrium exists if and only if $\nu < (1-\frac{\phi}{\widehat{\phi}})(1-\exp(-\rho\bar{t}_H))$, and that the only such equilibrium is the one characterized in the statement of the proposition. To this end, consider the two phase structure, characterized by Lemma 5.1 with $\widetilde{t}_U > 0$.

Strategies and Phase Transitions. From Lemma 5.1, in phase 1, we have $a_L(t) \in (0,1)$, and hence

$$\mu(t) = \mu_L \Rightarrow F(t) = \frac{1}{\sigma}(1-\exp\{-\mu_L t\}),$$

where use has been made of the fact that $F(0) = 0$ (i.e., $F(\cdot)$ has no mass point) which allows us to determine that the integration constant in the solution is 1.

For the approval strategy in phase 1, we substitute $a_U(t) = (1-\nu)a_L(t)$ into the entrepreneur's indifference condition to obtain

$$(1-\nu)a_L'(t) - \rho(1-\nu)a_L(t) + \phi(\rho+\lambda) = 0.$$

Solving, with boundary condition $a_L(\widetilde{t}_U) = 1$, we have

$$\begin{aligned} a_L(t) &= (\frac{1}{1-\nu})\frac{\phi}{\widehat{\phi}} + (1-(\frac{1}{1-\nu})\frac{\phi}{\widehat{\phi}})\exp\{-\rho(\widetilde{t}_U - t)\} \\ &= (\frac{1}{1-\nu})[\frac{\phi}{\widehat{\phi}} + (1-\frac{\phi}{\widehat{\phi}}-\nu)\exp\{-\rho(\widetilde{t}_U - t)\}]. \end{aligned}$$

From Lemma 5.1, in phase 2, we have $a_H(t) \in (0,1)$, and hence,

$$\mu(t) = \mu_H \Rightarrow F(t) = \frac{1}{\sigma}(1 - \kappa_2 \exp\{-\mu_H t\}),$$

where κ_2 is an integration constant. From the boundary condition $F(\bar{t}_U) = 1$ we have

$$\frac{1}{\sigma}(1 - \kappa_2 \exp\{-\mu_H \bar{t}_U\}) = 1 \Rightarrow \kappa_2 = (1-\sigma)\exp\{\mu_H \bar{t}_U\}.$$

Note that $\kappa_2 > 0$. Thus, $F(\cdot)$ is increasing in the second phase.

The differential equation for the entrepreneur's indifference condition in phase 2 is identical to the differential equation for the indifference condition in the one phase equilibrium, characterized in Proposition 5.1. Following the same argument with boundary condition $a_H(\bar{t}_U) = 1$, we have

$$a_H(t) = \frac{1}{\nu}\Big(\frac{\phi}{\widehat{\phi}} + (1 - \frac{\phi}{\widehat{\phi}})\exp\{-\rho(\bar{t}_U - t)\} - (1-\nu)\Big).$$

To determine the phase transitions, $\widetilde{t}_U, \bar{t}_U$, we use continuity of $F(\cdot)$ at $\widetilde{t}_U$, and the boundary condition $a_H(\widetilde{t}_U) = 0$, both of which come from Lemma 5.1.

$$\frac{1}{\sigma}(1 - \exp\{-\mu_L \widetilde{t}_U\}) = \frac{1}{\sigma}(1 - (1-\sigma)\exp\{\mu_H(\bar{t}_U - \widetilde{t}_U)\})$$
$$\frac{1}{\nu}\Big(\frac{\phi}{\widehat{\phi}} + (1 - \frac{\phi}{\widehat{\phi}})\exp\{-\rho(\bar{t}_U - \widetilde{t}_U)\} - (1-\nu)\Big) = 0$$

Solving, we have

$$\widetilde{t}_U = \bar{t}_L - \frac{\mu_H}{\mu_L}\delta_U \qquad \bar{t}_U = \bar{t}_L + (1 - \frac{\mu_H}{\mu_L})\delta_U,$$

where $\delta_U = -\ln(1 - \frac{\nu\widehat{\phi}}{\widehat{\phi}-\phi})/\rho$. Note that for this system to have any solution, we must have $\nu < 1 - \phi/\widehat{\phi}$, so that δ_U is well-defined. Additional details are available in the Online Supplement.

Thus, we have a unique candidate for the two stage equilibrium. This candidate is indeed an equilibrium if and only if $\widetilde{t}_U > 0$.

Claim 1. We show that $\widetilde{t}_U > 0 \iff \nu < (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp\{-\rho\bar{t}_H\})$. Note that

$$\widetilde{t}_U > 0 \iff \bar{t}_L > \frac{\mu_H}{\mu_L}\delta_U \iff \bar{t}_H > \delta_U \iff -\rho\bar{t}_H < \ln(1 - \frac{\nu\widehat{\phi}}{\widehat{\phi}-\phi}) \iff$$
$$\exp\{-\rho\bar{t}_H\} < 1 - \frac{\widehat{\phi}\nu}{\widehat{\phi}-\phi} \iff \nu < (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp\{-\rho\bar{t}_H\}).$$

It follows that the two stage equilibrium exists when $\nu < (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp\{-\rho\bar{t}_H\})$, and in this equilibrium the strategies and phase transitions are the ones given in the proposition. One additional claim is made in the phase transitions part of the proposition, which we verify.

Claim 2. We show that if $\nu < (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp\{-\rho\bar{t}_H\})$, then $\bar{t}_U < \bar{t}_H$. From claim 1, for such ν we have

$$\widetilde{t}_U > 0 \Rightarrow \bar{t}_L > \frac{\mu_H}{\mu_L}\delta_U \Rightarrow \bar{t}_H > \delta_U \Rightarrow \bar{t}_H(1 - \frac{\mu_H}{\mu_L}) > (1 - \frac{\mu_H}{\mu_L})\delta_U \Rightarrow$$
$$-\ln(1-\sigma)\frac{\mu_L - \mu_H}{\mu_H\mu_L} > (1 - \frac{\mu_H}{\mu_L})\delta_U \Rightarrow -\ln(1-\sigma)(\frac{1}{\mu_H} - \frac{1}{\mu_L}) > (1 - \frac{\mu_H}{\mu_L})\delta_U \Rightarrow$$
$$\bar{t}_H - \bar{t}_L > (1 - \frac{\mu_H}{\mu_L})\delta_U \Rightarrow \bar{t}_H > \bar{t}_L + (1 - \frac{\mu_H}{\mu_L})\delta_U = \bar{t}_U$$

Claim 3. We show that $\kappa_2 = (1-\sigma)\exp\{\mu_H\bar{t}_U\} = \exp\{-(\mu_L - \mu_H)\widetilde{t}_U\}$. Consequently, the entrepreneur's strategy can be written as stated in the proposition. Note that

$$\mu_H\bar{t}_U + \ln(1-\sigma) = \mu_H(-\frac{\ln(1-\sigma)}{\mu_L} + (1 - \frac{\mu_H}{\mu_L})\delta_U) + \ln(1-\sigma) =$$
$$(1 - \frac{\mu_H}{\mu_L})\ln(1-\sigma) + \mu_H(1 - \frac{\mu_H}{\mu_L})\delta_U =$$
$$(\mu_L - \mu_H)(\frac{\ln(1-\sigma)}{\mu_L} + \frac{\mu_H}{\mu_L}\delta_U) = -(\mu_L - \mu_H)\widetilde{t}_U.$$

The claim follows by applying $\exp(\cdot)$ to both sides of the equation.

Beliefs. Follows immediately from Lemma 5.1 and the investor's sequentially rational approval decision.

Payoffs. The strategic entrepreneur is indifferent about submitting a fake at all times inside the phase of doubt, and thus, the entrepreneur's payoff is the payoff of submitting a fake at time zero. Thus, the strategic entrepreneur's payoff is

$$a_U(0) - \phi = (1-\nu)a_L(0) - \phi = \frac{\phi}{\widehat{\phi}} + (1 - \frac{\phi}{\widehat{\phi}} - \nu)\exp(-\rho\widetilde{t}_U) - \phi.$$

The high type investor either rejects or mixes at all times $t \in [0, \bar{t}_U)$. Thus, the high type investor's payoff is

$$V_H = (1-k_H)(1-\sigma)\int_{\bar{t}_U}^{\infty} \lambda\exp\{-(\rho+\lambda)t\}dt.$$

To calculate the low type investor's payoff, note that the low type mixes in phase 1, and approves in phase 2. Thus, the low type's payoff is

$$V_L = \int_{\widetilde{t}_U}^{\bar{t}_U} \exp(-(\rho+\lambda)t)(\lambda(1-\sigma F(t))(1-k_L) - \sigma f(t)k_L)dt + (1-k_L)(1-\sigma)\int_{\bar{t}_U}^{\infty} \lambda\exp(-(\rho+\lambda)t)dt$$

where $F(\cdot)$ and $f(\cdot)$ are the entrepreneur's CDF and PDF of the entrepreneur's mixed strategy. From the equilibrium characterization, we have $(1-\sigma F(t)) = (1-\sigma)\exp(\mu_H(\bar{t}_U - t))$

and $\sigma f(t) = (1-\sigma)\mu_H \exp(\mu_H(\bar{t}_U - t))$, which implies that the low type investor's payoff is

$$(1-\sigma)\exp(\mu_H \bar{t}_U)\int_{\widetilde{t}_U}^{\bar{t}_U} \exp(-(\rho + \frac{\lambda}{k_H})t)(\lambda(1-k_L) - \mu_H k_L)dt + (1-k_L)(1-\sigma)\int_{\bar{t}_U}^{\infty} \lambda \exp(-(\rho+\lambda)t)dt =$$

$$(1-\sigma)\exp(\mu_H \bar{t}_U)(1-\frac{k_L}{k_H})\int_{\widetilde{t}_U}^{\bar{t}_U} \lambda \exp(-(\rho + \frac{\lambda}{k_H})t)dt + (1-k_L)(1-\sigma)\int_{\bar{t}_U}^{\infty} \lambda \exp(-(\rho+\lambda)t)dt$$

Using Claim 3 to simplify the leading term, we have

$$\exp(-(\mu_L - \mu_H)\widetilde{t}_U)(1-\frac{k_L}{k_H})\int_{\widetilde{t}_U}^{\bar{t}_U} \exp(-(\rho + \frac{\lambda}{k_H})t)dt + (1-k_L)(1-\sigma)\int_{\bar{t}_U}^{\infty} \lambda \exp(-(\rho+\lambda)t)dt.$$

Normative Comparison. That the high type investor strictly benefits from opaque standards in this equilibrium follows immediately from $\bar{t}_U < \bar{t}_H$, proved in Claim 2.

We prove that the low type investor's payoff is higher in the two stage equilibrium than in the baseline model in three steps.

In Step 1 we show that there exists $\widetilde{\sigma}$ such that the two stage equilibrium obtains iff $\sigma > \widetilde{\sigma}$ and that the low type investor's payoff in the two stage equilibrium approaches her payoff in the 1-phase equilibrium as $\sigma \downarrow \widetilde{\sigma}$. Because we showed above that the investor's payoff is strictly higher in the one stage opaque standards equilibrium than in the baseline model, we conclude that there exists $\epsilon > 0$ such that her payoff in the two stage auditing equilibrium is also higher than in the baseline model for all $\sigma \in (\widetilde{\sigma}, \widetilde{\sigma} + \epsilon)$.

In Step 2 we show that if the low type investor's payoff is higher in the 2-stage equilibrium than in the baseline model for any value of σ, then it is higher for all larger values as well.

In Step 3, we combine Steps 1 and 2 to show that the investor's payoff is higher in the two stage equilibrium than in the baseline model.

Step 1: We show that for any σ such that the two stage equilibrium exists, there exists $\sigma' < \sigma$ such that (i) the two stage equilibrium exists at σ', and (ii) at σ' the investor's payoff in the two stage equilibrium is higher than her payoff in the baseline model.

Consider parameters at which the two stage equilibrium exists; by Proposition 5.2, we have $\nu < \nu^*$. Note that

$$\nu < \nu^* = (1 - \frac{\phi}{\widehat{\phi}})(1 - \exp(-\rho \bar{t}_H)) \iff \nu < 1 - \frac{\phi}{\widehat{\phi}} \quad \text{and} \quad \bar{t}_H \text{ is sufficiently large.}$$

Recalling that $\bar{t}_H$ is monotone increasing in σ, we have that the two stage equilibrium exists whenever $\nu < 1 - \frac{\phi}{\widehat{\phi}}$ and $\sigma > \widetilde{\sigma}$, for some $\widetilde{\sigma} \in (0,1)$. By implication, if $\sigma > \widetilde{\sigma}$, then the two stage equilibrium exists for all $\sigma \in (\widetilde{\sigma}, \sigma)$.

Next, we argue that as $\sigma \downarrow \widetilde{\sigma}$, the low type investor's payoff in the two stage equilibrium approaches her payoff in the one stage equilibrium. Note that in the two stage equilibrium,

the low type investor's payoff is

$$V_L = \exp(-(\mu_L - \mu_H)\widetilde{t}_U)(1 - \frac{k_L}{k_H}) \int_{\widetilde{t}_U}^{\bar{t}_U} \exp(-(\rho + \frac{\lambda}{k_H})t)dt + (1 - k_L)(1 - \sigma) \int_{\bar{t}_U}^{\infty} \lambda \exp(-(\rho + \lambda)t)dt.$$

A straightforward modification of Claim 1 shows that as $\sigma \downarrow \widetilde{\sigma}$, we have $\widetilde{t}_U \downarrow 0$. By implication, $\bar{t}_L \to \frac{\mu_H}{\mu_L}\delta_U$, and hence $\bar{t}_H \to \delta_U$. Furthermore $\bar{t}_U = \bar{t}_L + (1 - \frac{\mu_H}{\mu_L})\delta_U$. Substituting, we have $\bar{t}_U \to \delta_U$. Combining, we have $\bar{t}_U \to \bar{t}_H$. By routine simplification, as $\sigma \downarrow \widetilde{\sigma}$, the low type investor's payoff in the two stage equilibrium approaches

$$(1 - \frac{k_L}{k_H}) \int_0^{\bar{t}_H} \lambda \exp(-(\rho + \frac{\lambda}{k_H})t)dt + (1 - k_L)(1 - \widetilde{\sigma}) \int_{\bar{t}_H}^{\infty} \lambda \exp(-(\rho + \lambda)t)dt,$$

which is the low type investor's payoff in the one phase equilibrium that obtains at $\widetilde{\sigma}$. From Proposition 5.1, this payoff strictly exceeds the low type investor's payoff in the baseline model. By implication, there exists $\epsilon > 0$ such that for an $\sigma' \in (\widetilde{\sigma}, \widetilde{\sigma} + \epsilon)$, the low type investor's payoff in the two stage equilibrium at σ' strictly exceeds her payoff in the baseline model. Hence, for any $\sigma > \widetilde{\sigma}$, there exists $\sigma' \in (\widetilde{\sigma}, \sigma)$ at which the low type investor's payoff in the two stage equilibrium is higher than her payoff in the baseline model.

Step 2: Consider $\sigma > \widetilde{\sigma}$. We show that if the investor's payoff is higher in the two stage equilibrium with auditing than in the baseline model at σ, then the same is true for all $\sigma'' > \sigma$.

Consider the payoff difference between the two stage equilibrium and the baseline model,

$$\begin{aligned}
&\exp(-(\mu_L - \mu_H)\widetilde{t}_U)(1 - \frac{k_L}{k_H}) \int_{\widetilde{t}_U}^{\bar{t}_U} \exp(-(\rho + \frac{\lambda}{k_H})t)\, \mathrm{d}t \\
&+ (1 - k_L)(1 - \sigma) \left[\int_{\bar{t}_U}^{\infty} \lambda \exp(-(\rho + \lambda)t)\, \mathrm{d}t - \int_{\bar{t}_L}^{\infty} \lambda \exp(-(\rho + \lambda)t)\, \mathrm{d}t \right].
\end{aligned}$$

We simplify the previous expression in order to isolate σ. To keep the exposition organized, we proceed line-by-line.

We simplify the first line.

$$\exp(-(\mu_L - \mu_H)\widetilde{t}_U)(1 - \frac{k_L}{k_H})[\exp(-(\rho + \frac{\lambda}{k_H})\widetilde{t}_U) - \exp(-(\rho + \frac{\lambda}{k_H})\bar{t}_U)].$$

Note that

$$-(\mu_L - \mu_H) = -\lambda \frac{k_H(1 - k_L) - k_L(1 - k_H)}{k_H k_L} = -\lambda \frac{k_H - k_L}{k_H k_L} = \frac{\lambda}{k_H} - \frac{\lambda}{k_L}.$$

Substituting, and using $\bar{t}_U = \widetilde{t}_U + \delta_U$, we have

$$\exp(-(\rho + \frac{\lambda}{k_L})\widetilde{t}_U)(1 - \frac{k_L}{k_H})[1 - \exp(-(\rho + \frac{\lambda}{k_H})\delta_U)].$$

Using $\widetilde{t}_U = \bar{t}_L - \frac{\mu_H}{\mu_L}\delta_U$, we have

$$\exp(-(\rho + \frac{\lambda}{k_L})\bar{t}_L)\exp((\rho + \frac{\lambda}{k_L})\frac{\mu_H}{\mu_L}\delta_U)(1 - \frac{k_L}{k_H})[1 - \exp(-(\rho + \frac{\lambda}{k_H})\delta_U)].$$

Substituting the definition of $\bar{t}_L$, the first line is

$$(1-\sigma)^{(\rho+\frac{\lambda}{k_L})\frac{k_L}{\lambda(1-k_L)}}\exp((\rho + \frac{\lambda}{k_L})\frac{\mu_H}{\mu_L}\delta_U)(1 - \frac{k_L}{k_H})[1 - \exp(-(\rho + \frac{\lambda}{k_H})\delta_U)] =$$
$$(1-\sigma)^{\frac{\rho k_L+\lambda}{\lambda(1-k_L)}}\exp((\rho + \frac{\lambda}{k_L})\frac{\mu_H}{\mu_L}\delta_U)(1 - \frac{k}{k_H})[1 - \exp(-(\rho + \frac{\lambda}{k_H})\delta_U)] =$$
$$\kappa_1(1-\sigma)^{\frac{\rho k_L+\lambda}{\lambda(1-k_L)}},$$

where $\kappa_1 \equiv \exp((\rho + \frac{\lambda}{k_L})\frac{\mu_H}{\mu_L}\delta_U)(1 - \frac{k_L}{k_H})[1 - \exp(-(\rho + \frac{\lambda}{k_H})\delta_U)]$ is independent of σ.

Next, we simplify the second line.

$$(1-k_L)(1-\sigma)[\int_{\bar{t}_U}^{\infty}\lambda\exp(-(\rho+\lambda)t)\,\mathrm{d}t - \int_{\bar{t}_L}^{\infty}\lambda\exp(-(\rho+\lambda)t)\,\mathrm{d}t] =$$
$$(1-k_L)(1-\sigma)\int_{\bar{t}_U}^{\bar{t}_L}\lambda\exp(-(\rho+\lambda)t)\,\mathrm{d}t = (1-k_L)(1-\sigma)\frac{\lambda}{\lambda+\rho}[\exp(-(\rho+\lambda)\bar{t}_U) - \exp(-(\rho+\lambda)\bar{t}_L)].$$

Substituting $\bar{t}_U = \bar{t}_L + (1 - \frac{\mu_H}{\mu_L})\delta_U$, we have

$$(1-k_L)(1-\sigma)\frac{\lambda}{\lambda+\rho}\exp(-(\rho+\lambda)\bar{t}_L)[\exp(-(\rho+\lambda)(1 - \frac{\mu_H}{\mu_L})\delta_U) - 1].$$

Note that

$$(1-\sigma)\exp(-(\rho+\lambda)\bar{t}_L) = \exp(1 - (\rho+\lambda)\frac{k_L}{\lambda(1-k_L)})\ln(1-\sigma)) =$$
$$\exp(-(\rho + \frac{\lambda}{k_L})(-\frac{k_L}{\lambda(1-k_L)})\ln(1-\sigma)) = \exp(-(\rho + \frac{\lambda}{k_L})\bar{t}_L).$$

Continuing the simplification,

$$(1-k_L)\frac{\lambda}{\lambda+\rho}\exp(-(\rho + \frac{\lambda}{k_L})\bar{t}_L)[\exp(-(\rho+\lambda)(1 - \frac{\mu_H}{\mu_L})\delta_U) - 1]$$
$$(1-k_L)\frac{\lambda}{\lambda+\rho}[\exp(-(\rho+\lambda)(1 - \frac{\mu_H}{\mu_L})\delta) - 1](1-\sigma)^{\frac{\rho k_L+\lambda}{\lambda(1-k_L)}} =$$
$$\kappa_2(1-\sigma)^{\frac{\rho k_L+\lambda}{\lambda(1-k_L)}},$$

where $\kappa_2 \equiv (1-k_L)\frac{\lambda}{\lambda+\rho}[\exp(-(\rho+\lambda)(1 - \frac{\mu_H}{\mu_L})\delta_U) - 1]$ is independent of σ. Combining terms, the payoff difference as a function of σ is simply

$$(\kappa_1 + \kappa_2)(1-\sigma)^{\frac{\rho k_L+\lambda}{\lambda(1-k_L)}}.$$

Therefore, the payoff difference is positive if and only if $\kappa_1 + \kappa_2 > 0$. Thus, if the payoff difference is positive for some value of $\sigma > \widetilde{\sigma}$, then it is also positive for $\sigma'' \in (\sigma, 1]$.

Step 3. We show that the low type investor's payoff is higher in the two stage equilibrium than in the baseline model. Consider $\sigma > \widetilde{\sigma}$. From Step 1, there exists $\sigma' \in (\widetilde{\sigma}, \sigma)$ such that the investor's payoff in the two stage equilibrium at σ' is higher than in the baseline model. Applying Step 2, the low type investor's payoff in the two stage auditing equilibrium at $\sigma > \sigma'$ is also higher than in the baseline model. □

A.3 Proofs for Liquidity Constraint

Proof of Lemma 6.1. Step 1. Let $S \equiv \{t : a(t) < \overline{a}(t)\}$. We show that S is an open set, i.e. S is a countable union of open intervals $S = \cup\,(t_i, t_{i+1})$.

Consider some t such that $a(t) < \overline{a}(t)$, i.e. the constraint is slack. By implication, we must have $g(t) = k$, and hence $f(t) > 0$.

If there exists a small $\delta > 0$ such that $a(t') < \overline{a}(t')$ for all $t' \in (t - \delta, t + \delta)$, then the set S is open, and the claim is established. To derive a contradiction, suppose that for all $\delta > 0$, there exists some $t' \in [t - \delta, t + \delta]$ such that $a(t') = \overline{a}(t')$. Let $\overline{\epsilon} \equiv \exp(-(\rho + \lambda)t)(\overline{a}(t) - a(t)) > 0$ and let $\epsilon \equiv \overline{\epsilon}/4$. From $a(\cdot) \in [\phi, 1)$ and continuity of exponential, there exists some $\delta' > 0$ such that the following conditions hold for $t' \in (t - \delta', t + \delta')$,

$$-\epsilon \leq \int_{t'}^{t} \lambda \exp(-(\rho + \lambda)s) a(s) ds \leq \epsilon$$

$$-\epsilon \leq (\exp(-(\rho + \lambda)t) - \exp(-(\rho + \lambda)t'))(a(t') - \phi) \leq \epsilon$$

$$-\epsilon \leq \exp(-(\rho + \lambda)t)(\overline{a}(t) - \overline{a}(t')) \leq \epsilon.$$

Furthermore, by assumption, there exists some $t' \in [t - \delta', t + \delta']$ such that $a(t') = \overline{a}(t')$.

$$u(t) - u(t') = \int_{t'}^{t} \lambda \exp(-(\rho + \lambda)s) a(s) ds + (\exp(-(\rho + \lambda)t) - \exp(-(\rho + \lambda)t'))(a(t') - \phi) + \exp(-(\rho + \lambda)t)[\overline{a}(t) - \overline{a}(t') - (\overline{a}(t) - a(t)].$$

It follows that $u(t) - u(t') \leq 3\epsilon - \overline{\epsilon} = -\frac{\overline{\epsilon}}{4}$, which contradicts $f(t) > 0$.

Step 2. We show that $a(\cdot)$ is continuous at all $t \geq 0$.

First (1), consider $t \in S$, i.e. $a(t) < \overline{a}(t)$. From Step A we have that $(t - \delta, t + \delta) \in S$ for some $\delta > 0$, and hence, $f(t') > 0$ for all $t' \in (t - \delta, t + \delta)$. By implication, $u(t) - u(t') = 0$

for all $t' \in (t-\delta, t+\delta)$. It follows that for all such t',

$$\int_{t'}^{t} \lambda \exp(-(\rho+\lambda)s)a(s)ds + (\exp(-(\rho+\lambda)t) - \exp(-(\rho+\lambda)t')(a(t)-\phi) + \\ \exp(-(\rho+\lambda)t')(a(t)-a(t')) = 0.$$

Taking the limit as $t' \to t$, we find $a(t) - \lim_{t'\to t} a(t') = 0$, establishing continuity at t.

Second (2), consider t such that $a(t) = \overline{a}(t)$, and assume that t is in the interior of S^C, i.e. there exists a small δ such that for all $t' \in (t-\delta, t+\delta)$, we have $a(t') = \overline{a}(t')$. In this case continuity of $a(\cdot)$ at t follows immediately from continuity of $\overline{a}(\cdot)$ on interval $(t-\delta, t+\delta)$.

Third (3), consider t such that $a(t) = \overline{a}(t)$, and assume that t is on the boundary of S, so that in any interval $(t-\delta, t+\delta)$ there exists some t'_δ such that $a(t'_\delta) < \overline{a}(t'_\delta)$ and some t''_δ such that $a(t''_\delta) = \overline{a}(t''_\delta)$. We will show that for a sufficiently small δ, we have $|\overline{a}(t) - a(t')| < \epsilon$, whenever $t' \in (t-\delta, t+\delta)$, regardless of whether $t' \in S$ or $t' \in S^C$.

(i) Consider $t' \in S^C$. From continuity of $\overline{a}(\cdot)$ at t, we know that for any $\epsilon > 0$ there exists δ_C such that $|\overline{a}(t) - \overline{a}(t')| < \epsilon$ for any $t' \in (t-\delta_C, t+\delta_C)$.

(ii) Consider $t' \in S$, i.e. $a(t') < \overline{a}(t')$. We have

$$u(t') - u(t) = \int_{t}^{t'} \lambda \exp(-(\rho+\lambda s))a(s)ds + (\exp(-(\rho+\lambda)t') - \exp(-(\rho+\lambda)t))(a(t')-\phi) \\ + \exp(-(\rho+\lambda)t)(a(t') - \overline{a}(t)).$$

Because $a(\cdot) \in (\phi, 1)$ and the exponential is continuous and $\overline{a}(\cdot)$ is continuous, some $\delta_S > 0$ exists such that the following inequalities hold for all $t' \in (t-\delta_S, t+\delta_S)$:

$$\int_{t}^{t'} \lambda \exp(-(\rho+\lambda s))a(s)ds \le \frac{\epsilon}{2}\exp(-(\rho+\lambda)t),$$
$$(\exp(-(\rho+\lambda)t') - \exp(-(\rho+\lambda)t))(a(t')-\phi) \le \frac{\epsilon}{2}\exp(-(\rho+\lambda)t),$$

Recall that $t' \in S$, and hence $f(t') > 0$. For such t', we have

$$0 \le u(t') - u(t) \le \exp(-(\rho+\lambda)t)(\epsilon + a(t') - \overline{a}(t)),$$

and hence $\overline{a}(t) - a(t') \le \epsilon$.

(iii) Next, note that $\overline{a}(t) - a(t') \ge \overline{a}(t) - \overline{a}(t') \ge -\epsilon$ for $t' \in (t-\delta_C, t+\delta_C)$.

Let $\delta^* = \min\{\delta_C, \delta_S\}$, and consider $t' \in (t-\delta^*, t+\delta^*)$. If $t' \in S^C$, then $|\overline{a}(t) - a(t')| = |\overline{a}(t) - \overline{a}(t')| < \epsilon$, from (i). If $t' \in S$, then (ii) implies that $\overline{a}(t) - a(t') \le \epsilon$ and (iii) implies $\overline{a}(t) - a(t') \ge -\epsilon$, and hence, $|\overline{a}(t) - a(t')| < \epsilon$. It follows that $a(\cdot)$ is continuos at t.

Points (1)-(3) establish continuity of $a(\cdot)$ at all t in the interior of S, the interior of S^C, and the boundary of S, thereby proving the result.

Step 3. We show that $u(\cdot)$ is continuous. For any t, t' we have

$$u(t') - u(t) = \int_t^{t'} \lambda \exp(-(\rho + \lambda s))a(s)ds + (\exp(-(\rho+\lambda)t') - \exp(-(\rho+\lambda)t))(a(t') - \phi) + \exp(-(\rho+\lambda)t)(a(t') - \overline{a}(t)).$$

Taking the limit as $t \to t'$ and using the continuity of $a(\cdot)$ yields the result.

Step 4. Suppose that for $t \in (t_L, t_H)$ we have $a(t) = \overline{a}(t)$, where $t_H < t^*$. We show that $u(\cdot)$ is strictly increasing on (t_L, t_H). After substituting $a(\cdot) = \overline{a}(\cdot)$ to calculate $u(\cdot)$, the result follows from straightforward differentiation.

Definition. Let $u^* \equiv \max_t u(t)$.

Step 5. We show that if $t' < t < t^*$ and $u(t) < u^*$, then $u(t') < u^*$. Consider $X = \{t'' \in [0, t] : u(t'') = u^*\}$. If X is empty, then the result holds. By way of contradiction, suppose X is nonempty. Set X has an upper bound, and therefore it has a least upper bound, denoted T. Continuity of $u(\cdot)$ (Step 3) implies (1) $u(T) = u^*$, and (2), that $T < t$. It follows that for all $t' \in (T, t)$, we have $u(t') < u^*$ and thus, $f(t') = 0$. Therefore, for all such t', we have $a(t') = \overline{a}(t')$. Using Step 4, we have that $u(\cdot)$ is strictly increasing on (T, t), and hence $u(T) < u(t) < u^*$, resulting in a contradiction.

Step 6. Suppose that for $t \in (t_L, t_H)$ we have $a(t) = \overline{a}(t)$, where $t_L > t^*$. We show that $u(\cdot)$ is strictly decreasing on (t_L, t_H). Similar to Step 4.

Step 7. We show that if $t^* < t < t'$ and $u(t) < u^*$, then $u(t') < u^*$. Similar to Step 5.

Step 8. There exist $\widetilde{t}_C, \overline{t}_C$ with $0 \le \widetilde{t}_C \le t^* \le \overline{t}_C \le \infty$ such that (1) $u(t) < u^*$ for $t \in [0, \widetilde{t}_C)$, (2) $u(t) = u^*$ for $t \in [\widetilde{t}_C, \overline{t}_C]$, (3) $u(t) < u^*$ for $t \in (\overline{t}_C, \infty)$. Follows from Steps 5 and 7.

Step 9. We show that if $t \in (\widetilde{t}_C, \overline{t}_C)$ then $a(t) < \overline{a}(t)$ and $f(t) > 0$. From Step 8, we know that $u(t) = u^*$ for $t \in (\widetilde{t}_C, \overline{t}_C)$. Following a similar argument to the one in Lemma 4.1 it is possible to show that $a(\cdot)$ is differentiable on this interval, and that

$$a'(t) = \rho(a(t) - \frac{\phi}{\widehat{\phi}}) \Rightarrow a(t) = \frac{\phi}{\widehat{\phi}} + \kappa \exp(\rho t).$$

Suppose $t' \in (\widetilde{t}_C, \overline{t}_C)$ and $a(t') = \overline{a}(t')$. We consider two cases. (1) If $\kappa \le 0$, then $a(t)$ is weakly decreasing. Thus, $a(t') = \overline{a}(t')$ implies $a(t'') \ge a(t')$ for $t'' \in (\widetilde{t}_C, t')$. By assumption $a(t') = \overline{a}(t') > \overline{a}(t'')$, where the last inequality follows because $\overline{a}(t)$ is a strictly increasing function. Thus, we have shown $a(t'') > \overline{a}(t'')$, violating the constraint. (2) If $\kappa > 0$, then $a(\cdot)$ is a strictly increasing convex function on $(\widetilde{t}_C, \overline{t}_C)$. Recall that $\overline{a}(\cdot)$ is a strictly increasing concave function. If $a'(t') < \overline{a}'(t')$, then the constraint $a(\cdot) \le \overline{a}(\cdot)$ is violated in a small interval $(t' - \delta, t')$. Similarly if $a'(t') > \overline{a}'(t')$ then the constraint is violated in a small interval $(t', t' + \delta)$. Finally if $a'(t') = \overline{a}'(t')$, then $a(\cdot)$ and $\overline{a}(\cdot)$, can be separated by a tangent

line at t', and hence $a(\cdot) > \overline{a}(\cdot)$ around t', violating the constraint. Thus, we have shown that $t' \in (\widetilde{t}_C, \overline{t}_C) \Rightarrow a(t') < \overline{a}(t')$. By implication $g(t') = k$, and hence $f(t') > 0$.

Step 10. We show (1) if $\widetilde{t}_C > 0$, then $a(t) = \overline{a}(t)$ for $t \in [0, \widetilde{t}_C]$, and (2) if $\overline{t}_C < \infty$ then $a(t) = \overline{a}(t)$ for $t \in [\overline{t}_C, \infty)$. From Step 8, we know that $u(t) < u^*$ on intervals $[0, \widetilde{t}_C)$ and $(\overline{t}_C, \infty)$. Thus, $f(t) = 0$ on such intervals, and consequently, $a(\cdot) = \overline{a}(\cdot)$. By continuity of $a(\cdot)$ at $\widetilde{t}_C$, $\overline{t}_C$, we have $a(\overline{t}_C) = \overline{a}(\overline{t}_C)$ and $a(\widetilde{t}_C) = \overline{a}(\widetilde{t}_C)$.

Step 11. We show that $\overline{t}_C < \infty$. Suppose that $\overline{t}_C = \infty$. From Step 9, we have $a(t) < \overline{a}(t)$ for $t \in (\widetilde{t}_C, \overline{t}_C)$. Thus, for $t \to \infty$ we have $g(t) = k$, and hence $f(t) = \mu(1-\sigma F(t)) \geq \mu(1-\sigma) > 0$. It follows that $F(t) \to \infty$ as $t \to \infty$.

Step 12. We show that $\widetilde{t}_C < \overline{t}_C$. Suppose $\widetilde{t}_C = \overline{t}_C$. Combined with the conditions $\widetilde{t}_C \leq t^* \leq \overline{t}_C$, we have $\widetilde{t}_C = t^* = \overline{t}_C$. Because $\phi/\hat{\phi} < 1$, we have $t^* > 0$. From Step 8, $u(t) < u^*$ for $t \neq t^*$. Thus, for $t \neq t^*$ we have $a(t) = \overline{a}(t)$, and by continuity of $a(\cdot)$, the same is true at t^*. A straightforward calculation reveals that the entrepreneur's optimal cheating time t^*, and thus $g(t^*) = 0$. By implication $a(t^*) = 0$, contradicting continuity of $a(\cdot)$.

Step 13. We show that $a(\cdot)$ is increasing. From Step 12, we know that $\widetilde{t}_C < \overline{t}_C$. From Step 8 we know that $u(t) = u^*$ on $[\widetilde{t}_C, \overline{t}_H]$. Using the entrepreneur's indifference condition, we have $a(t) = \phi/\hat{\phi} + \kappa \exp(\rho t)$. Thus, $a(\cdot)$ is either strictly increasing, strictly decreasing, or constant on this interval. Next, note that whether $\widetilde{t}_C > 0$ or $\widetilde{t}_C = 0$, we have $a(\widetilde{t}_C) \leq \overline{a}(\widetilde{t}_C) < \overline{a}(\overline{t}_C)$, where the last inequality follows from Step 12 combined with the fact that $\overline{a}(\cdot)$ is strictly increasing. Furthermore, from Steps 10 and 11 we have $\overline{a}(\overline{t}_C) = a(\overline{t}_C)$ for some $\overline{t}_C \in (\widetilde{t}_C, \infty)$. Thus, we have shown that $a(\widetilde{t}_C) < a(\overline{t}_C)$. Since the monotonicity of $a(\cdot)$ does not change on $(\widetilde{t}_C, \overline{t}_C)$, it must be increasing on this interval. Applying Step 10, it follows that outside this interval $a(t) = \overline{a}(t)$, which is itself an increasing function.

Step 14. We show that $\widetilde{t}_C > 0$. Suppose $\widetilde{t}_C = 0$. Using Step 9, we have $a(0) < \overline{a}(0) = \gamma/(\gamma + \rho) < \frac{\phi}{\hat{\phi}}$. The entrepreneur's indifference condition on $[\widetilde{t}_C, \overline{t}_C]$ implies $a(t) = \phi/\hat{\phi} + \kappa \exp(\rho t)$. Thus, $a(0) < \phi/\hat{\phi}$ implies $\kappa < 0$. By implication, $a(\cdot)$ is decreasing on $(\widetilde{t}_C, \overline{t}_C)$, contradicting Step 13.

Step 15. We show that $\widetilde{t}_C < t^*$; that $\overline{t}_C > t^*$ is proved in a similar way. Suppose that $\widetilde{t}_C = t^*$. We know that $\overline{t}_C > \widetilde{t}_C$. From the entrepreneur's indifference condition on $[\widetilde{t}_C, \overline{t}_C]$, we have $a(t) = \phi/\widehat{\phi} + \kappa \exp(\rho t)$. Combined with Step 10, we have

$$\frac{\phi}{\widehat{\phi}} + \kappa \exp(\rho t^*) = 1 - \frac{\rho}{\rho + \gamma} \exp(-\gamma t^*).$$

From the definition of t^*, we have

$$\frac{\phi}{\widehat{\phi}} + \kappa \exp(\rho t^*) = 1 - \frac{\rho}{\rho + \gamma}(1 - \frac{\phi}{\widehat{\phi}}) \Rightarrow \kappa \exp(\rho t^*) = \frac{\gamma}{\gamma + \rho}(1 - \frac{\phi}{\widehat{\phi}}).$$

Next, consider the derivative of $a(\cdot)$ and $\overline{a}(\cdot)$ at t^*. We have

$$a'(t^*) = \rho\kappa\exp(\rho t^*), \qquad \overline{a}(t^*) = \frac{\rho\gamma}{\rho+\gamma}\exp(-\gamma t^*) \Rightarrow$$
$$a'(t^*) = \frac{\rho\gamma}{\gamma+\rho}(1-\frac{\phi}{\widehat{\phi}}), \qquad \overline{a}(t^*) = \frac{\rho\gamma}{\rho+\gamma}(1-\frac{\phi}{\widehat{\phi}}) \Rightarrow$$
$$a'(t^*) = \overline{a}(t^*).$$

Because (1) $a(\cdot)$ is increasing and convex (2) $\overline{a}(\cdot)$ is increasing and concave, (3) $a(t^*) = \overline{a}(t^*)$ and (4) $a'(t^*) = \overline{a}'(t^*)$, we have $a(\cdot) > \overline{a}(\cdot)$ for $t \in (t^*, \overline{t}_C)$, contradicting $a(\cdot) \leq \overline{a}(\cdot)$.

Proof of (i). That the entrepreneur's mixing distribution has no mass points follows from continuity of $a(\cdot)$ (Step 2). Combined, Steps 8, 11, 12, and 14 establish the rest of the claim.

Proof of (ii). Proved in Steps 2 and 13.

Proof of (iii). Follows from Steps 10-15.

Proof of (iv). Follows from Step 9 and 11-15. □

Proof of Proposition 6.1. Strategies. Based on Lemma 6.1, there exist $\{\widetilde{t}_C, \overline{t}_C\}$ with $0 < \widetilde{t}_C < t^* < \overline{t}_C < \infty$ such that for $t \in (\widetilde{t}_U, \overline{t}_U)$, equilibrium strategies are characterized by the differential equations $\mu(t) = \mu$ and $a'(t) - \rho a(t) + \phi(\rho+\lambda) = 0$ with boundary conditions, $F(\widetilde{t}_C) = 0$, $F(\overline{t}_C) = 1$, $a(\widetilde{t}_C) = \overline{a}(\widetilde{t}_C)$,$a(\overline{t}_C) = \overline{a}(\overline{t}_C)$.. Solving the first differential equation and boundary conditions, we have

$$F(t) = \frac{1}{\sigma}(1-\exp(-\mu(t-\widetilde{t}_C)), \qquad \overline{t}_C = \widetilde{t}_C + \overline{t}.$$

Solving the second differential equation, we have

$$a(t) = \frac{\phi}{\hat{\phi}} + \kappa\exp(\rho t),$$

where κ is an integration constant. Thus, the remaining boundary conditions become

$$\frac{\phi}{\hat{\phi}} + \kappa\exp(\rho\widetilde{t}_C) = 1 - \frac{\rho}{\rho+\gamma}\exp(-\gamma\widetilde{t}_C)$$
$$\frac{\phi}{\hat{\phi}} + \kappa\exp(\rho(\widetilde{t}_C+\overline{t})) = 1 - \frac{\rho}{\rho+\gamma}\exp(-\gamma(\widetilde{t}_C+\overline{t})).$$

Solving, we have

$$\kappa = \exp(-\rho\widetilde{t}_C)(\overline{a}(\widetilde{t}_C) - \frac{\phi}{\widehat{\phi}}),$$
$$\exp(-\gamma\widetilde{t}_C) = (1-\frac{\phi}{\widehat{\phi}})(1+\frac{\gamma}{\rho})\frac{\exp(\rho\overline{t})-1}{\exp(\rho\overline{t})-\exp(-\gamma\overline{t})}.$$

Obviously, $\widetilde{t}_C > 0$ and $\bar{t}_C < \infty$. That $\widetilde{t}_C < t^* < \bar{t}_C$ follows from a straightforward application of L'Hopital's rule. Note that, if we solve the second equation for κ, we find $\kappa = \exp(-\rho\bar{t}_C)(\overline{a}(\bar{t}_C) - \frac{\phi}{\widehat{\phi}})$, which corresponds to the expression of the approval strategy presented in the proposition.

Beliefs. Obvious.

Payoffs. The entrepreneur is indifferent between faking at all times inside $[\widetilde{t}_C, \bar{t}_C]$, and therefore his equilibrium payoff is $u(\widetilde{t}_C)$ as stated in the proposition. In the next part of the proof, we give a simpler expression for the entrepreneur's equilibrium payoff. For the investor's payoff, note that an arrival inside $(\widetilde{t}_C, \bar{t}_C)$ delivers no surplus, while an arrival outside of this time interval is known to be real, and is approved with probability $\overline{a}(\cdot)$, delivering payoff $(1-k)$. Furthermore, an arrival comes at $t > \bar{t}_C$ only if the entrepreneur is ethical.

Normative Implications. Consider the investor's payoff. Note that as $\sigma \to 1$, we have $\bar{t} \to \infty$, and hence,

$$\widetilde{t}_C \to -\frac{1}{\gamma}\ln[(1-\frac{\phi}{\widehat{\phi}})(1+\frac{\gamma}{\rho})] = -\frac{1}{\gamma}\ln(\frac{1-\frac{\phi}{\widehat{\phi}}}{1-\frac{\gamma}{\gamma+\rho}}).$$

Under the maintained assumption that $\gamma/(\gamma+\rho) < \phi/\widehat{\phi}$, this limit is strictly positive. Therefore, with a logjam, the investor's payoff is bounded away from zero as $\sigma \to 1$, while it converges to zero in the main model. Now consider $\sigma \to 0$. We have $\bar{t} \to 0$. Using L'Hopital's rule,

$$\frac{\exp(\rho\bar{t}) - 1}{\exp(\rho\bar{t}) - \exp(-\gamma\bar{t})} \to \frac{\rho}{\rho+\gamma},$$

which in turn implies that $\widetilde{t}_C \to t^*$, $\bar{t}_C \to t^*$. It follows that the investor's payoff approaches

$$(1-k)\int_0^\infty \lambda\exp(-(\lambda+\rho)t)\overline{a}(t)dt < (1-k)\int_0^\infty \lambda\exp(-(\lambda+\rho)t)dt,$$

which is the limit as $\sigma \to 0$ in the main model. □

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