

ANSWERS AND SOLUTIONS

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Student Notes and Problems

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PHYSICS 12
British Columbia

Credits

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MEASUREMENT OF MOTION

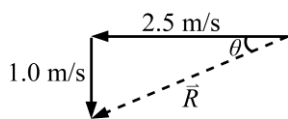
Lesson 1—Frames of Reference

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $2.5 \text{ m/s south} + 1.0 \text{ m/s south} = 3.5 \text{ m/s south}$

2. $2.5 \text{ m/s north} + 1.0 \text{ m/s south} = 1.5 \text{ m/s north}$

3.



$$R = \sqrt{(v_1)^2 + (v_2)^2}$$

$$= \sqrt{(2.5 \text{ m/s})^2 + (1.0 \text{ m/s})^2}$$

$$= 2.7 \text{ m/s}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{1.0 \text{ m/s}}{2.5 \text{ m/s}}$$

$$\theta = \tan^{-1} \left(\frac{1.0 \text{ m/s}}{2.5 \text{ m/s}} \right)$$

$$= 22^\circ$$

$$\vec{R} = 2.7 \text{ m/s } 22^\circ \text{ S of W}$$

4. The spacecraft accelerates relative to the stars because of the force acting on it. However, there is no force acting on the floating object. By Newton's first law, the object remains in its state of motion with the same velocity relative to the stars. To the astronaut, who is fixed to the spacecraft, it seems that the object is accelerating in the opposite direction to the spacecraft, i.e., toward him.

5. From the point of view (reference frame) of the shark, it appears that the fish is approaching it at (64–15) km/h. Therefore, the motion of the fish appears to be 49 km/h west.

Lesson 2—Special Theory of Relativity

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$1. \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.25 \text{ years}}{\sqrt{1 - \left(\frac{2.40 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2}}$$

$$= 2.08 \text{ years}$$

$$2. \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{3.00 \text{ years}}{\sqrt{1 - \left(\frac{7.50 \times 10^7 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2}}$$

$$= 3.10 \text{ years}$$

$$3. \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.50 \text{ h}}{\sqrt{1 - \left(\frac{44.4 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2}}$$

$$= 1.50 \text{ h}$$

$$4. \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_0 = t \left(\sqrt{1 - \frac{v^2}{c^2}} \right)$$

$$= (50.0 \text{ years}) \left(\sqrt{1 - \left(\frac{2.80 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2} \right)$$

$$= 18.0 \text{ years}$$

$$\begin{aligned}
 5. \quad t &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 t_0 &= t \left(\sqrt{1 - \frac{v^2}{c^2}} \right) \\
 &= (35.0 \text{ years}) \left(\sqrt{1 - \left(\frac{2.90 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2} \right) \\
 &= 8.96 \text{ years}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad t &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 18.0 \text{ years} &= \frac{10.0 \text{ years}}{\sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}}} \\
 \sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}} &= \frac{10.0 \text{ years}}{18.0 \text{ years}} \\
 1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2} &= \left(\frac{10.0}{18.0} \right)^2 \\
 \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2} &= 1 - \left(\frac{10.0}{18.0} \right)^2 \\
 &= 0.691 \\
 v &= \sqrt{(0.691)(3.00 \times 10^8 \text{ m/s})^2} \\
 &= 2.49 \times 10^8 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad t &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 60.0 \text{ years} &= \frac{15.0 \text{ years}}{\sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}}} \\
 \sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}} &= \frac{15.0 \text{ years}}{60.0 \text{ years}} \\
 1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2} &= \left(\frac{15.0 \text{ years}}{60.0 \text{ years}} \right)^2 \\
 \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2} &= 1 - \left(\frac{15.0 \text{ years}}{60.0 \text{ years}} \right)^2 \\
 &= 0.938 \\
 v &= \sqrt{(0.938)(3.00 \times 10^8 \text{ m/s})^2} \\
 &= 2.90 \times 10^8 \text{ m/s}
 \end{aligned}$$

8. The teacher should give 2.5 h according to the space bus clock. Not only would the space bus clock slow down as the space bus travels, but the biological clocks of all the passengers would slow down at the same rate. If you travel at a high speed through space, you are not aware that there is anything different. Writing an exam for 2.5 h on the space bus will seem the same to you as writing the same exam in your class room.

Lesson 3—Length Contraction

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= (35.0 \text{ m}) \sqrt{1 - \frac{(2.55 \times 10^8 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}} \\
 &= 18.4 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 L_0 &= \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 &= \frac{80.0 \text{ m}}{\sqrt{1 - (0.900)^2}} \\
 &= 184 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 2.85 \text{ light years} &= (6.25 \text{ light years}) \sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}}
 \end{aligned}$$

Note: You do not have to convert light years.

$$\begin{aligned}
 \frac{2.85 \text{ light years}}{6.25 \text{ light years}} &= \sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}} \\
 (0.456)^2 &= 1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2} \\
 \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2} &= 1 - (0.456)^2 \\
 v &= \sqrt{(0.792)(3.00 \times 10^8 \text{ m/s})^2} \\
 &= 2.67 \times 10^8 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= (90.0 \text{ m}) \sqrt{1 - (0.800)^2} \\
 &= 54.0 \text{ m}
 \end{aligned}$$

NOTE: Height does not change. It is only the dimension parallel to the direction of travel that changes.

$$\begin{aligned}
 5. \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= (1.20 \text{ km}) \sqrt{1 - \frac{(3.10 \times 10^8 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}}
 \end{aligned}$$

You will soon discover that the mathematics is impossible to do as one cannot find the square root of a negative value. According to Einstein's special theory of relativity, nothing can travel faster than light ($3.00 \times 10^8 \text{ m/s}$). The salesman was not telling the truth.

$$\begin{aligned}
 6. \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= (1.00 \text{ m}) \sqrt{1 - (0.600)^2} \\
 &= 0.800 \text{ m}
 \end{aligned}$$

Lesson 4—Mass Increase

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad m &= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 &= \frac{1.67 \times 10^{-27} \text{ kg}}{\sqrt{1 - (0.95)^2}} \\
 &= 5.3 \times 10^{-27} \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad m &= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 m_0 &= m \left(\sqrt{1 - \frac{v^2}{c^2}} \right) \\
 &= (7.20 \times 10^{-20} \text{ kg}) \left(\sqrt{1 - \left(\frac{2.50 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2} \right) \\
 &= 3.98 \times 10^{-20} \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad m &= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 m_0 &= \sqrt{1 - \frac{v^2}{c^2}} \\
 &= (5.50 \times 10^{-10} \text{ kg}) \left(\sqrt{1 - (0.800)^2} \right) \\
 &= 3.30 \times 10^{-10} \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \text{let } x &= \frac{v}{c} \\
 m &= \frac{m_0}{\sqrt{1 - x^2}} \\
 x &= \sqrt{1 - \left(\frac{m_0}{m} \right)^2} \\
 &= \sqrt{1 - \left(\frac{6.05 \times 10^{-27} \text{ kg}}{8.00 \times 10^{-27} \text{ kg}} \right)^2} \\
 &= 0.654 \\
 v &= xc \\
 &= (0.654)(3.00 \times 10^8 \text{ m/s}) \\
 &= 1.96 \times 10^8 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \text{let } x &= \frac{v}{c} \\
 m &= \frac{m_0}{\sqrt{1 - x^2}} \\
 x &= \sqrt{1 - \left(\frac{m_0}{m} \right)^2} \\
 &= \sqrt{1 - \left(\frac{1}{3} \right)^2} \\
 &= 0.943 \\
 v &= xc \\
 &= (0.943)(3.00 \times 10^8 \text{ m/s}) \\
 &= 2.83 \times 10^8 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \text{let } x &= \frac{v}{c} \\
 m &= \frac{m_0}{\sqrt{1-x^2}} \\
 x &= \sqrt{1-\left(\frac{m_0}{m}\right)^2} \\
 &= \sqrt{1-\left(\frac{1}{4}\right)^2} \\
 &= 0.968 \\
 v &= xc \\
 &= (0.968)(3.00 \times 10^8 \text{ m/s}) \\
 &= 2.90 \times 10^8 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad E &= mc^2 \\
 &= (10.0 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \\
 &= 9.00 \times 10^{17} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad E &= mc^2 \\
 m &= \frac{E}{c^2} \\
 &= \frac{7.25 \times 10^{16} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} \\
 &= 0.806 \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad E &= mc^2 \\
 m &= \frac{E}{c^2} \\
 &= \frac{5.3 \times 10^{10} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} \\
 &= 5.9 \times 10^{-7} \text{ kg}
 \end{aligned}$$

NOTE: A grain of sand may have a mass approximately 2.00×10^{-6} kg. $\therefore 5.9 \times 10^{-7}$ kg is very small amount of gasoline, approximately one-third of the mass of a grain of sand.

NOTE: in order to obtain 5.3×10^{10} J of energy by chemical means, you would burn approximately 1500 L of gasoline in your car.

$$\frac{16\,000 \text{ km}}{10.6 \text{ km/L}} = 1500 \text{ L}$$

Lesson 5—Relativistic Addition of Velocities

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{0.80c + 0.75c}{1 + \frac{(0.80c)(0.75c)}{c^2}} \\
 &= 0.97c \text{ away}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{-0.80c + 0.75c}{1 + \frac{(-0.80c)(0.75c)}{c^2}} \\
 &= -0.13c \text{ or } 0.13c \text{ toward}
 \end{aligned}$$

NOTE: The negative sign indicates that the object is moving toward the stationary observer.

$$\begin{aligned}
 3. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{0.75c + 0.60c}{1 + \frac{(0.75c)(0.60c)}{c^2}} \\
 &= 0.93c \text{ toward}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{0.80c + (-c)}{1 + \frac{(0.80c)(-c)}{c^2}} \\
 &= -c \text{ or } c \text{ toward the observer}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{0.80c + c}{1 + \frac{(0.80c)(c)}{c^2}} \\
 &= c \text{ toward the observer}
 \end{aligned}$$

NOTE: It does not matter if the vehicle is travelling toward or away from the stationary observer. The velocity of light is the same.

$$\begin{aligned}
 6. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{2.5 \times 10^8 \text{ m/s} + 2.0 \times 10^8 \text{ m/s}}{1 + \frac{(2.5 \times 10^8 \text{ m/s})(2.0 \times 10^8 \text{ m/s})}{(3.00 \times 10^8 \text{ m/s})^2}} \\
 &= 2.9 \times 10^8 \text{ m/s away}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \vec{u} &= \frac{\vec{v} + \vec{u}'}{1 + \frac{\vec{v}\vec{u}'}{c^2}} \\
 &= \frac{2.5 \text{ m/s} + 6.0 \text{ m/s}}{1 + \frac{(2.5 \text{ m/s})(6.0 \text{ m/s})}{(3.00 \times 10^8 \text{ m/s})^2}} \\
 &= 8.5 \text{ m/s away}
 \end{aligned}$$

NOTE: This is the same answer that you would get if you just added the two velocity vectors together as you added vectors earlier in this course. Relativistic effects are only observable at very high velocities.

Practice Test

ANSWERS AND SOLUTIONS

1. Michelson and Morley designed an experiment to measure the speed of light relative to ether. Their result is known as the null result, because they did not detect any evidence of the ether. This made many scientists question its existence.
2. An inertial frame of reference is any frame of reference that is at rest or is moving at a constant velocity.

3. a) Observer 1 will observe the illumination of light 1 before light 2 because for observer 1, light 1 travels less distance than light 2.
b) Observer 2 will observe light from light 1 and 2 at precisely the same time because the light from both sources travels precisely the same distance.
4. a) Proper time is measured by the observer who is at rest with respect to the event. José is at rest with respect to the event.

$$\begin{aligned}
 \text{b)} \quad t &= \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \\
 18.0 \text{ years} &= \frac{3.00 \text{ years}}{\sqrt{1 - \frac{v^2}{(3.00 \times 10^8 \text{ m/s})^2}}} \\
 v &= 2.96 \times 10^8 \text{ m/s}
 \end{aligned}$$

5. Proper time is the time measured by an observer who is at rest with respect to the event. Proper time is also less than the time measured by an observer who is not at rest with respect to the event. Note that for non-relativistic speeds, the discrepancy from the proper time is extremely small.

$$t = \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

6. Proper length is defined as the length as determined by the observer who is at rest with respect to the length determined. But, unlike proper time, proper length is greater than the length determined by the observer that is moving with respect to the length determined.

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

where L_0 is the proper length.

7. Einstein's special theory of relativity has two postulates:
 1. the laws of physics in all inertial frames of reference are the same
 2. the speed of light in space is the same for all observers, regardless of the motion of the source or observer.

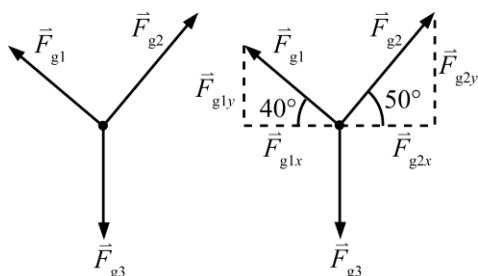
8. If Einstein's postulates are correct, then space and time are not absolute. It also follows that mass and energy are equivalent.
9. The length contraction will only be in the direction of the motion. Therefore, side Y will remain at 50 m. There is length contraction of side Z so it will be shorter.
10. An observer will have the same experiences in any inertial frame of reference. Pavel will not observe any change to his watch other than the time changing as he would normally notice.

FORCES CAUSE MOTION

Lesson 1—The First Condition of Equilibrium

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1.



$$F_{g1x} = F_{g1} \cos \theta_1 = F_{g1} \cos 40^\circ = 0.766 F_{g1}$$

$$F_{g1y} = F_{g1} \sin \theta_1 = F_{g1} \sin 40^\circ = 0.643 F_{g1}$$

$$F_{g2x} = F_{g2} \cos \theta_2 = F_{g2} \cos 50^\circ = 0.643 F_{g2}$$

$$F_{g2y} = F_{g2} \sin \theta_2 = F_{g2} \sin 50^\circ = 0.766 F_{g2}$$

$$\sum \vec{F}_x = 0$$

$$F_{g2x} - F_{g1x} = 0$$

$$0.643 F_{g2} - 0.766 F_{g1} = 0$$

$$F_g = \frac{0.766 F_{g1}}{0.643}$$

$$F_{g2} = 1.19 F_{g1}$$

$$\sum \vec{F}_y = 0$$

$$F_{g1y} + F_{g2y} - F_{g3} = 0$$

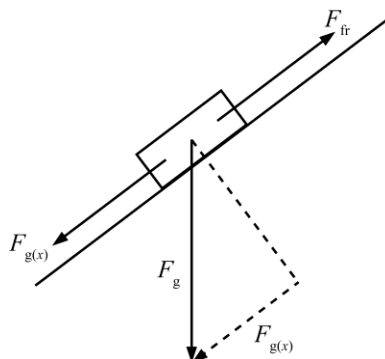
$$0.643 F_{g1} + 0.766 F_{g2} - 12 \text{ N} = 0$$

$$1.55 F_{g1} = 12 \text{ N}$$

$$F_{g1} = 7.74 \text{ N}$$

Again $F_{g2} = 1.19 F_{g1}$
 $= (1.19)(7.74 \text{ N})$
 $= 9.21 \text{ N} \approx 9.2 \text{ N}$

2.



$$\sum F_x = 0$$

$$F_T - F_{gx} = 0$$

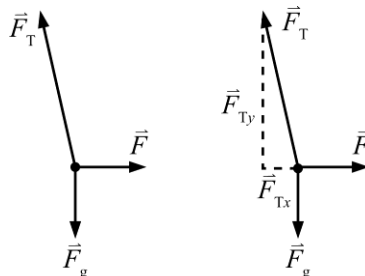
$$F_{gx} = F_T = 5000 \text{ N}$$

$$F_{gx} = F_g \sin \theta$$

$$F_g = \frac{F_{gx}}{\sin \theta} = \frac{5000 \text{ N}}{\sin 25.0^\circ}$$

$$= 1.18 \times 10^4 \text{ N}$$

3.



$$F_{Tx} = F_T \cos \theta = F_T \cos 63.0^\circ = 0.454 F_T$$

$$F_{Ty} = F_T \sin \theta = F_T \sin 63.0^\circ = 0.891 F_T$$

$$\sum \vec{F}_y = 0$$

$$F_{Ty} - F_g = 0$$

$$F_{Ty} = F_g = 20.0 \text{ N}$$

$$F_T = \frac{F_{Ty}}{0.891} = \frac{20.0 \text{ N}}{0.891} = 22.4 \text{ N}$$

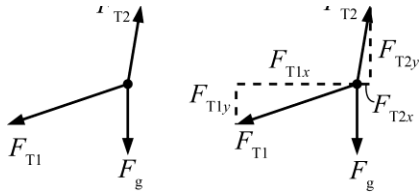
$$\sum \vec{F}_x = 0$$

$$F_{app} - F_{Tx} = 0$$

$$F_{app} - 0.454 F_T = 0$$

$$F_{app} = (0.454)(22.4 \text{ N}) = 10.2 \text{ N}$$

4.

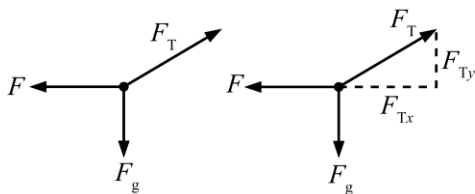


$$\begin{aligned} F_{T1x} &= F_{T1} \cos \theta_1 & F_{T1y} &= F_{T1} \sin \theta_1 \\ &= F_{T1} \cos 22.0^\circ & &= F_{T1} \sin 22.0^\circ \\ &= 0.927 F_{T1} & &= 0.375 F_{T1} \end{aligned}$$

$$\begin{aligned} F_{T2x} &= F_{T2} \cos \theta_2 & F_{T2y} &= F_{T2} \sin \theta_2 \\ &= F_{T2} \cos 80.0^\circ & &= F_{T2} \sin 80.0^\circ \\ &= 0.174 F_{T2} & &= 0.985 F_{T2} \end{aligned}$$

$$\begin{aligned} \sum \vec{F}_x &= 0 \\ F_{T2x} - F_{T1x} &= 0 \\ 0.174 F_{T2} - 0.927 F_{T1} &= 0 \\ F_{T2} &= \frac{0.927 F_{T1}}{0.174} \\ &= 5.33 F_{T1} \\ \sum \vec{F}_y &= 0 \\ F_{T2y} - F_{T1y} - F_g &= 0 \\ 0.985 F_{T2} - 0.375 F_{T1} &= 75.0 \text{ N} \\ (0.985)(5.33 F_{T1}) - 0.375 F_{T1} &= 75.0 \text{ N} \\ 4.88 F_{T1} &= 75 \text{ N} \\ F_{T1} &= 15.4 \text{ N} \\ \therefore F_{T2} &= 5.33 (15.4 \text{ N}) = 82.1 \text{ N} \end{aligned}$$

5.



$$\begin{aligned} F_{Tx} &= F_T \cos \theta & F_{Ty} &= F_T \sin \theta \\ &= F_T \cos 30^\circ & &= F_T \sin 30^\circ \\ &= 0.866 F_T & &= 0.500 F_T \end{aligned}$$

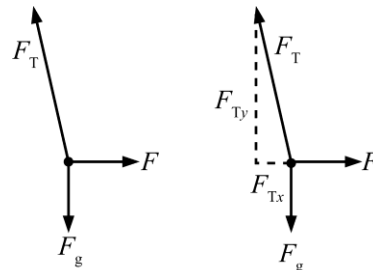
$$\begin{aligned} F &= F_f = \mu mg \cos \theta \\ &= (0.27)(15 \text{ kg})(9.81 \text{ m/s}^2)(\cos 0) \\ &= 39.7 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum \vec{F}_x &= 0 \\ F_{Tx} - F &= 0 \\ F_{Tx} &= 39.7 \text{ N} \\ F_{Tx} &= 0.866 F_T \\ F_T &= \frac{F_{Tx}}{0.866} = \frac{39.7 \text{ N}}{0.866} \\ &= 45.9 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum \vec{F}_y &= 0 \\ F_{Ty} - F_g &= 0 \\ 0.500 F_T &= F_g \\ \Rightarrow F_g &= (0.500)(45.9 \text{ N}) \\ &= 22.9 \text{ N} \end{aligned}$$

$$\begin{aligned} F_g &= mg \\ m &= \frac{F_g}{g} = \frac{22.9 \text{ N}}{9.81 \text{ m/s}^2} \\ &= 2.3 \text{ kg} \end{aligned}$$

6.

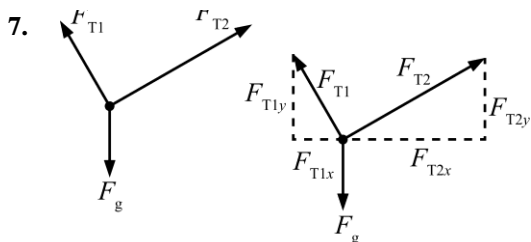


$$\begin{aligned} F_{Tx} &= F_T \cos \theta \\ &= F_T \cos 78^\circ \\ &= 0.208 F_T \end{aligned}$$

$$\begin{aligned} F_{Ty} &= F_T \sin \theta \\ &= F_T \sin 78^\circ \\ &= 0.978 F_T \end{aligned}$$

$$\begin{aligned} \sum \vec{F}_x &= 0 \\ F_{Tx} - F &= 0 \\ F &= F_{Tx} \\ &= 0.208 F_T \end{aligned}$$

$$\begin{aligned} \sum \vec{F}_y &= 0 \\ F_{Ty} - F_g &= 0 \\ F_{Ty} &= F_g \\ 0.978 F_T &= 735 \text{ N} \\ F_T &= \frac{735 \text{ N}}{0.978} \\ &= 751 \text{ N} \\ &= 7.5 \times 10^2 \text{ N} \end{aligned}$$



$$F_{T1x} = F_{T1} \cos \theta_1 \quad F_{T1y} = F_{T1} \sin \theta_1$$

$$= (96 \text{ N}) \cos 60^\circ \quad = (96 \text{ N}) \sin 60^\circ$$

$$= 48 \text{ N} \quad = 83 \text{ N}$$

$$F_{T2x} = F_{T2} \cos \theta_2 \quad F_{T2y} = F_{T2} \sin \theta_2$$

$$= F_{T2} \cos 30^\circ \quad = F_{T2} \sin 30^\circ$$

$$= 0.866 F_{T2} \quad = 0.500 F_{T2}$$

$$\sum \vec{F}_x = 0$$

$$F_{T2x} - F_{T1x} = 0$$

$$48 \text{ N} = 0.866 F_{T2}$$

$$F_{T2} = \frac{48 \text{ N}}{0.866}$$

$$= 55.43 \text{ N} \approx 55 \text{ N}$$

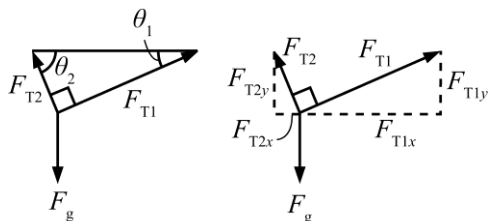
$$\sum \vec{F}_y = 0$$

$$F_{T1y} + F_{T2y} - F_g = 0$$

$$83 \text{ N} + 0.500 F_{T2} = F_g$$

$$83 \text{ N} + (0.500)(55 \text{ N}) = F_g$$

8.



Find θ_1

$$\sin \theta_1 = \frac{\text{op}}{\text{hyp}} = \frac{3.0 \text{ m}}{6.7 \text{ m}}$$

$$\theta_1 = 26.5^\circ$$

Find θ_2

$$\sin \theta_2 = \frac{\text{op}}{\text{hyp}} = \frac{6.0 \text{ m}}{6.7 \text{ m}}$$

$$\theta_2 = 63.5^\circ$$

$$F_{T1x} = F_{T1} \cos \theta_1 \quad F_{T1y} = F_{T1} \sin \theta_1$$

$$= F_{T1} \cos 26.5^\circ \quad = F_{T1} \sin 26.5^\circ$$

$$= 0.895 F_{T1} \quad = 0.446 F_{T1}$$

$$F_{T2x} = F_{T2} \cos \theta_2$$

$$= F_{T2} \cos 63.5^\circ$$

$$= 0.446 F_{T2}$$

$$F_{T2y} = F_{T2} \sin \theta_2$$

$$= F_{T2} \sin 63.5^\circ$$

$$= 0.895 F_{T2}$$

$$\sum \vec{F}_x = 0$$

$$F_{T1x} - F_{T2x} = 0$$

$$0.895 F_{T1} + 0.446 F_{T2} = 0$$

$$F_{T1} = \frac{0.446 F_{T2}}{0.895}$$

$$= 0.498 F_{T2}$$

$$0.498 F_{T2} = F_{T1}$$

$$F_{T2} = \frac{55 \text{ N}}{0.498}$$

$$F_{T2} = 110.4 \text{ N or } 1.1 \times 10^2 \text{ N}$$

$$\sum \vec{F}_y = 0$$

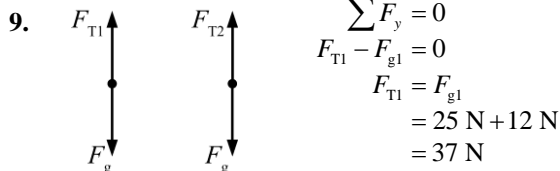
$$F_{T1y} + F_{T2y} - F_g = 0$$

$$.446 F_{T1} + 0.895 F_{T2} = F_g$$

$$0.446(55 \text{ N}) + 0.895(110 \text{ N}) = F_g$$

$$\therefore F_g = 123 \text{ N or } 1.2 \times 10^2 \text{ N}$$

For the whole system,



$$\sum \vec{F}_y = 0$$

$$F_{T1} - F_{g1} = 0$$

$$F_{T1} = F_{g1}$$

$$= 25 \text{ N} + 12 \text{ N}$$

$$= 37 \text{ N}$$

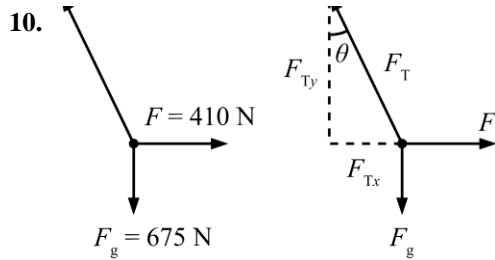
For the lower block,

$$\sum \vec{F}_y = 0$$

$$F_{T2} - F_g = 0$$

$$F_{T2} = F_g$$

$$= 12 \text{ N}$$



$$\begin{aligned} \sum \vec{F}_x &= 0 & \sum \vec{F}_y &= 0 \\ F_{Tx} - F &= 0 & F_{Ty} - F_g &= 0 \\ F_{Tx} &= F & F_{Ty} &= F_g \\ F &= 410 \text{ N} & &= 675 \text{ N} \end{aligned}$$

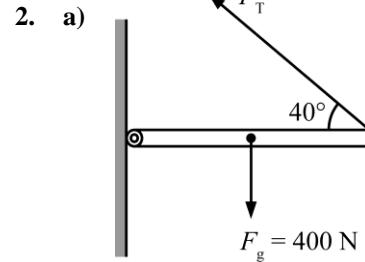
$$\begin{aligned} \tan \theta &= \frac{\text{op}}{\text{adj}} \\ &= \frac{410 \text{ N}}{675 \text{ N}} \\ \therefore \theta &= 31.3^\circ \end{aligned}$$

Lesson 2—The Second Condition of Equilibrium

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

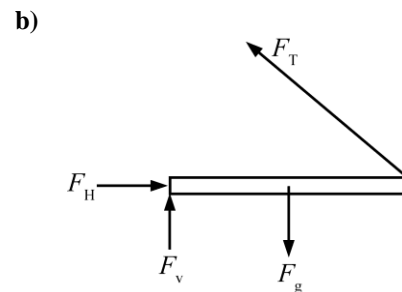
1. $\tau = rF \sin \theta$

$$\begin{aligned} F &= \frac{\tau}{r \sin \theta} \\ &= \frac{45 \text{ N} \cdot \text{m}}{(0.35 \text{ m})(\sin 90^\circ)} \\ &= 129 \text{ N or } 1.3 \times 10^2 \text{ N} \end{aligned}$$



$$\begin{aligned} \sum \tau &= 0 \\ \tau_1 - \tau_2 &= 0 \\ r_1 F_1 \sin \theta_1 - r_2 F_2 \sin \theta_2 &= 0 \\ (L)(F_T) \sin 40^\circ - \left(\frac{1}{2}L\right)(F_g) \sin 90^\circ &= 0 \\ 0.643 L F_T - (200 \text{ N})L &= 0 \\ 0.643 F_T &= 200 \text{ N} \end{aligned}$$

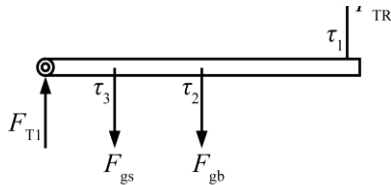
$$\begin{aligned} F_T &= \frac{200 \text{ N}}{0.643} \\ &= 311 \text{ N} \\ &= 3.1 \times 10^2 \text{ N} \end{aligned}$$



$$\begin{aligned} F_{Tx} &= F_T \cos \theta \\ &= (311 \text{ N})(\cos 40^\circ) \\ &= 238 \text{ N} \\ F_{Ty} &= F_T \sin \theta \\ &= (311 \text{ N})(\sin 40^\circ) \\ &= 200 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_x &= 0 \\ F_H - F_{Tx} &= 0 \\ F_H &= F_{Tx} \\ F_H &= 238 \text{ N} \\ \sum F_y &= 0 \\ F_{Ty} - F_g + F_v &= 0 \\ 200 \text{ N} - 400 \text{ N} + F_v &= 0 \\ F_v &= 238 \text{ N} \\ F_v &= 200 \text{ N} \end{aligned}$$

3.



Point of rotation is arbitrary

a)
$$\sum \tau = 0$$

$$\tau_1 - \tau_2 - \tau_3 = 0$$

$$r_1 F_1 \sin \theta_1 - r_2 F_2 \sin \theta_2 - r_3 F_3 \sin \theta_3 = 0$$

$$(5.00 \text{ m})(F_{TR}) \sin 90^\circ - (2.50 \text{ m})(250 \text{ N}) \sin 90^\circ - (1.50 \text{ m})(650 \text{ N}) \sin 90^\circ = 0$$

$$(5.00 \text{ m})(F_{TR}) - 625 \text{ N} - 975 \text{ N} = 0$$

$$\Rightarrow F_{TR} = \frac{(625 \text{ N} \cdot \text{m} + 975 \text{ N} \cdot \text{m})}{5.00 \text{ m}}$$

$$\therefore F_{TR} = 320 \text{ N}$$

b)
$$\sum F_x = 0$$

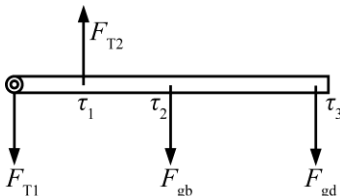
$$F_{TR} + F_{TL} - F_{gs} - F_{gb} = 0$$

$$320 \text{ N} + F_{TL} - 650 \text{ N} - 250 \text{ N} = 0$$

$$F_{TL} = 900 \text{ N} - 320 \text{ N}$$

$$\therefore F_{TL} = 580 \text{ N}$$

4.



point of rotation is arbitrary

$$\sum \tau = 0$$

$$\tau_1 - \tau_2 - \tau_3 = 0$$

$$r_1 F_1 \sin \theta_1 - r_2 F_2 \sin \theta_2 - r_3 F_3 \sin \theta_3 = 0$$

$$r_1 F_{T2} - r_2 F_{gb} - r_3 F_{gd} = 0$$

$$\left(\frac{1}{4}L\right)F_{T2} - \left(\frac{1}{2}L\right)(400 \text{ N}) - (L)(75.0 \text{ kg} \times 9.81 \text{ m/s}^2) = 0$$

$$\frac{1}{4}F_{T2} - 200 \text{ N} - 736 \text{ N} = 0$$

$$\Rightarrow F_{T2} = 4(936 \text{ N}) = 3744 \text{ N or } 3.74 \times 10^3 \text{ N}$$

$$\sum F_x = 0$$

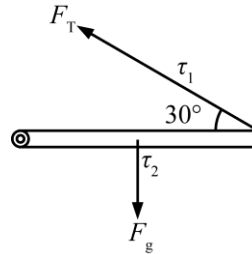
$$F_{T2} - F_{gb} - F_{gd} + F_{T1} = 0$$

$$3744 \text{ N} - 400 \text{ N} - 735 \text{ N} + F_{T1} = 0$$

$$F_{T1} = -3744 \text{ N} + 400 \text{ N} + 736 \text{ N}$$

$$\therefore F_{T1} = -2605 \text{ N or } -2.61 \times 10^3 \text{ N}$$

5.



$$\sum \tau = 0$$

$$\tau_1 - \tau_2 = 0$$

$$r_1 F_1 \sin \theta_1 - r_2 F_2 \sin \theta_2 = 0$$

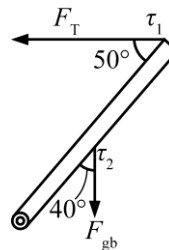
$$(L)(F_T) \sin 30^\circ - \left(\frac{1}{2}L\right)(F_{gb}) \sin 90^\circ = 0$$

$$0.500F_T - 100 \text{ N} = 0$$

$$0.500F_T = 100 \text{ N}$$

$$F_T = \frac{100 \text{ N}}{0.500} = 200 \text{ N} = 2.0 \times 10^2 \text{ N}$$

6.



$$\sum \tau = 0$$

$$\tau_1 - \tau_2 = 0$$

$$r_1 F_1 \sin \theta_1 - r_2 F_2 \sin \theta_2 = 0$$

$$(L)(F_T) \sin 50^\circ - \left(\frac{1}{2}L\right)(F_{gb}) \sin 40^\circ = 0$$

$$0.766F_T - 64.3 \text{ N} = 0$$

$$0.766F_T = 64.3 \text{ N}$$

$$F_T = \frac{64.3 \text{ N}}{0.766} = 83.9 \text{ N} = 84 \text{ N}$$

Lesson 3—Uniform Circular Motion

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad F_c &= \frac{mv^2}{r} \\
 &= \frac{(925 \text{ kg})(22 \text{ m/s})^2}{75 \text{ m}} \\
 &= 6.0 \times 10^3 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad C &= 2\pi r \\
 &= (2\pi)(3282 \text{ m}) \\
 &= 2.06 \times 10^4 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 v &= \frac{d}{t} \\
 &= \frac{2.06 \times 10^4 \text{ m}}{(2.0 \text{ min})(60 \text{ s/min})} \\
 &= 172 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 a_c &= \frac{v^2}{r} \\
 &= \frac{(172 \text{ m/s})^2}{3282 \text{ m}} \\
 &= 9.0 \text{ m/s}^2
 \end{aligned}$$

$$\begin{aligned}
 3. \quad F_c &= \frac{mv^2}{r} \\
 &= \frac{(822 \text{ kg})(28.0 \text{ m/s})^2}{105 \text{ m}} \\
 &= 6.14 \times 10^3 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad v &= \frac{2\pi r}{T} \\
 &= \frac{2\pi(2.8 \text{ m})}{(0.98 \text{ s})} \\
 &= 18 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad F_c &= \frac{mv^2}{r} \\
 v^2 &= \frac{(2.00 \times 10^{-2} \text{ m})(4.60 \times 10^{-14} \text{ N})}{(9.11 \times 10^{-31} \text{ kg})} \\
 v &= \sqrt{\frac{(4.60 \times 10^{-14} \text{ N})(2.00 \times 10^{-2} \text{ m})}{9.11 \times 10^{-31} \text{ kg}}} \\
 &= 3.18 \times 10^7 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad F_c &= \frac{mv^2}{r} \\
 &= \frac{(925 \text{ kg})(25 \text{ m/s})^2}{72 \text{ m}} \\
 &= 8.03 \times 10^3 \text{ N}
 \end{aligned}$$

Here the frictional force is equal to the centripetal force.

Again

$$\begin{aligned}
 F_N &= F_g = mg \\
 &= (925 \text{ kg})(9.81 \text{ m/s}^2) \\
 &= 9.07 \times 10^3 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_{fr} &= \mu F_N \\
 \mu &= \frac{F_{fr}}{F_N} \\
 &= \frac{8.03 \times 10^3 \text{ N}}{9.07 \times 10^3 \text{ N}} \\
 &= 0.89
 \end{aligned}$$

$$\begin{aligned}
 7. \quad F_c &= \frac{mv^2}{r} \\
 &= \frac{(2.7 \times 10^3 \text{ kg})(4.7 \times 10^3 \text{ m/s})^2}{1.8 \times 10^7 \text{ m}} \\
 &= 3.3 \times 10^3 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \text{a) } v &= \frac{2\pi r}{T} \\
 &= \frac{2\pi(0.90 \text{ m})}{0.30 \text{ s}} \\
 &= 19 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } F_c &= \frac{mv^2}{r} \\
 &= \frac{(3.7 \text{ kg})(19 \text{ m/s})^2}{0.90 \text{ m}} \\
 &= 1.5 \times 10^3 \text{ N}
 \end{aligned}$$

c) Find time to reach the ground using the vertical component

v_0	v	a	d	t
0	×	-9.81 m/s^2	-1.2 m	×

$$d_y = v_0 t + \frac{1}{2} a t^2$$

$$-1.2 \text{ m} = \frac{1}{2} (-9.81 \text{ m/s}^2) (t^2)$$

$$t = \sqrt{\frac{2(1.2 \text{ m})}{9.81 \text{ m/s}^2}}$$

$$= 0.495 \text{ s}$$

Horizontally

$$v_x = \frac{d_x}{t}$$

$$d_x = v_x t$$

$$= (19 \text{ m/s})(0.495 \text{ s})$$

$$= 9.3 \text{ m}$$

9. Speed $2r = \text{Diameter}$

$$r = \frac{30.0 \times 10^{-2} \text{ m}}{2}$$

$$= 15.0 \times 10^{-2} \text{ m}$$

$$T = \frac{1}{f}$$

$$f = 33 \frac{1}{3} \text{ rev/minute} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= \frac{33 \frac{1}{3}}{60} \text{ Hz}$$

$$T = \frac{60}{33 \frac{1}{3}} \text{ Hz}$$

$$= 1.80 \text{ s}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi(15.0 \times 10^{-2} \text{ m})}{1.80 \text{ s}}$$

$$= 0.524 \text{ m/s}$$

Acceleration

$$a_c = \frac{v^2}{r}$$

$$= \frac{(0.524 \text{ m/s})^2}{15.0 \times 10^{-2} \text{ m}}$$

$$= 1.83 \text{ m/s}^2$$

$$10. \quad F_c = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{F_c r}{m}}$$

$$= \sqrt{\frac{(135 \text{ N})(1.10 \text{ m})}{2.00 \text{ kg}}}$$

$$= 8.62 \text{ m/s}$$

$$11. \text{ a) } F_N = mg$$

$$= (932 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 9143 \text{ N}$$

$$F_{fr} = \mu F_N$$

$$= (0.95)(9143 \text{ N})$$

$$= 8.69 \times 10^3 \text{ N}$$

$$F_c = \frac{mv^2}{r}$$

$$8.69 \times 10^3 \text{ N} = \frac{(932 \text{ kg})(v^2)}{82 \text{ m}}$$

$$v = \sqrt{\frac{(8.69 \times 10^3 \text{ N})(82 \text{ m})}{932 \text{ kg}}}$$

$$= 28 \text{ m/s}$$

$$\text{b) } F_{fr} = \mu F_N$$

$$= (0.40)(9143 \text{ N})$$

$$= 3.66 \times 10^3 \text{ N}$$

$$F_c = \frac{mv^2}{r}$$

$$3.66 \times 10^3 \text{ N} = \frac{(932 \text{ kg})(v^2)}{82 \text{ m}}$$

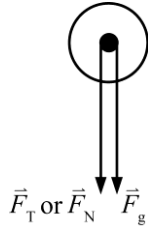
$$v = \sqrt{\frac{(3.66 \times 10^3 \text{ N})(82 \text{ m})}{932 \text{ kg}}}$$

$$= 18 \text{ m/s}$$

Lesson 4—Vertical Circular Motion

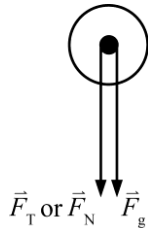
PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1.



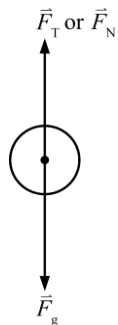
$$\begin{aligned}
 F_T &= 0 \\
 \therefore F_c &= F_g \\
 \frac{mv^2}{r} &= mg \\
 v &= \sqrt{rg} \\
 &= \sqrt{(5.00 \text{ m})(9.81 \text{ m/s}^2)} \\
 &= 7.00 \text{ m/s}
 \end{aligned}$$

2.



$$\begin{aligned}
 F_T &= 0 \\
 \therefore F_c &= F_g \\
 \frac{mv^2}{r} &= mg \\
 v &= \sqrt{rg} \\
 &= \sqrt{(0.95 \text{ m})(9.81 \text{ m/s}^2)} \\
 &= 3.1 \text{ m/s}
 \end{aligned}$$

3.

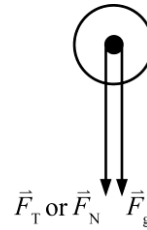


$$\begin{aligned}
 F_c &= F_N - F_g \\
 &= 1.96 \times 10^3 \text{ N} - 655 \text{ N} \\
 &= 1.305 \times 10^3 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_g &= mg \\
 m &= \frac{F_g}{g} \\
 &= \frac{655 \text{ N}}{9.81 \text{ m/s}^2} \\
 &= 66.8 \text{ kg}
 \end{aligned}$$

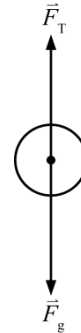
$$\begin{aligned}
 F_c &= \frac{mv^2}{r} \\
 v &= \sqrt{\frac{F_c r}{m}} \\
 &= \sqrt{\frac{(1.305 \times 10^3 \text{ N})(18.0 \text{ m})}{66.8 \text{ kg}}} \\
 &= 18.7 \text{ m/s}
 \end{aligned}$$

4.



$$\begin{aligned}
 F_T &= 0 \\
 \therefore F_c &= F_g \\
 \frac{mv^2}{r} &= mg \\
 v &= \sqrt{rg} \\
 &= \sqrt{(2.90 \text{ m})(9.81 \text{ m/s}^2)} \\
 &= 5.33 \text{ m/s}
 \end{aligned}$$

5.



$$\begin{aligned}
 F_g &= mg \\
 &= (1.50 \text{ kg})(9.81 \text{ m/s}^2) \\
 &= 14.7 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_c &= F_T - F_g \\
 &= 186 \text{ N} - 14.7 \text{ N} \\
 &= 171 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_c &= \frac{mv^2}{r} \\
 v &= \sqrt{\frac{F_c r}{m}} \\
 &= \sqrt{\frac{(171 \text{ N})(1.90 \text{ m})}{1.50 \text{ kg}}} \\
 &= 14.7 \text{ m/s}
 \end{aligned}$$

6. a)

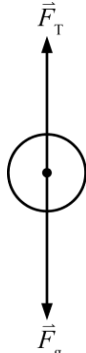


$$\begin{aligned}
 F_c &= \frac{4\pi^2 rm}{T^2} \\
 &= \frac{(4\pi^2)(1.0 \text{ m})(2.2 \text{ kg})}{(0.97 \text{ s})^2} \\
 &= 92.3 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_g &= mg \\
 &= (2.2 \text{ kg})(9.81 \text{ m/s}^2) \\
 &= 21.6 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_c &= F_T + F_g \\
 F_T &= F_c - F_g \\
 &= 92.3 \text{ N} - 21.6 \text{ N} \\
 &= 71 \text{ N}
 \end{aligned}$$

b)




$$F_c = F_T - F_g$$

$$F_T = F_c + F_g$$

$$= 92.3 \text{ N} + 21.6 \text{ N}$$

$$= 114 \text{ N}$$

7.



$$F_T = 0$$

$$\therefore F_c = F_g$$

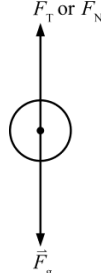
$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{(48 \text{ m})(9.81 \text{ m/s}^2)}$$

$$= 22.7 \text{ m/s or } 22 \text{ m/s}$$

8.



$$F_T = 0$$

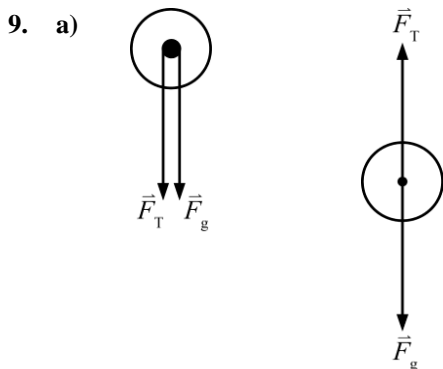
$$\therefore F_c = F_g$$

$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{(43 \text{ m})(9.81 \text{ m/s}^2)}$$

$$= 20.5 \text{ m/s or } 21 \text{ m/s}$$



High	Low
$F_c = F_T + F_g$	$F_c = F_T - F_g$
$F_T = F_c - F_g$	$F_T = F_c + F_g$

It is clear from the above formulas that the tension in the string greater at its low point than at its high point

b)

$$F_g = mg$$

$$= (2.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 24.5 \text{ N}$$

$$F_c = \frac{mv^2}{r}$$

$$= \frac{(2.5 \text{ kg})(12 \text{ m/s})^2}{0.75 \text{ m}}$$

$$= 4.8 \times 10^2 \text{ N}$$

i) The tension of the string at its low point is

$$F_T = F_c + F_g$$

$$= 480 \text{ N} + 24.5 \text{ N}$$

$$= 5.0 \times 10^2 \text{ N}$$

ii) The tension of the string at its high point is

$$F_T = F_c - F_g$$

$$= 480 \text{ N} - 24.5 \text{ N}$$

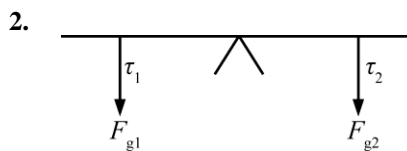
$$= 4.6 \times 10^2 \text{ N}$$

Practice Test

ANSWERS AND SOLUTIONS

1. Force of friction depends on the surfaces (coefficient of friction, μ) and the normal force ($F_N = mg \cos\theta$).

$$\therefore F_{fr} = \mu mg \cos\theta$$



$$\tau_1 = \tau_2$$

$$m_1 g L_1 = m_2 g L_2$$

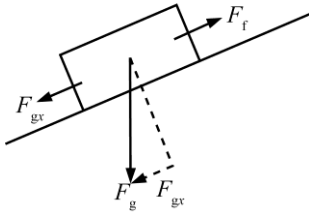
$$(32 \text{ kg})(1.5 \text{ m}) = (42 \text{ kg})(d)$$

$$d = 1.14 \text{ m}$$

$$\approx 1.1 \text{ m}$$

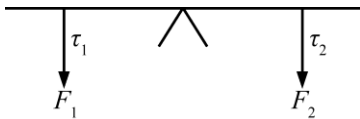
3. Rotational motion is related to torque. Therefore, if a wooden beam has no rotational motion, then the torques acting on the beam must be balanced.

4.



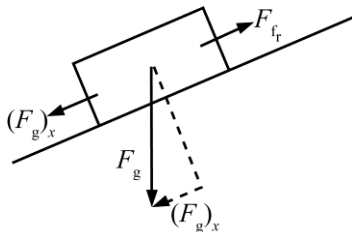
$$\begin{aligned}
 F_{\text{net}} &= 0 \\
 F_{\text{fr}} &= F_{g(x)} \\
 F_{g(x)} &= F_g \sin \theta \\
 &= (2.55 \text{ N})(9.81 \text{ m/s}^2)(\sin 30^\circ) \\
 &= 12.5 \text{ N}
 \end{aligned}$$

5.



$$\begin{aligned}
 \sum \tau &= 0 \\
 \tau_1 - \tau_2 &= 0 \\
 m_1 g d - m_2 g (1 \text{ m} - d) &= 0 \\
 (3.00 \text{ kg})(d) - (5.00 \text{ kg})(1 \text{ m} - d) &= 0 \\
 (3.00 \text{ kg})d - 5.00 \text{ kg} \cdot \text{m} + (5.00 \text{ kg})d &= 0 \\
 (8.00 \text{ kg})d &= 5.00 \text{ kg} \cdot \text{m} \\
 d &= 0.625 \text{ m} \\
 &\text{or } 62.5 \text{ cm}
 \end{aligned}$$

6.



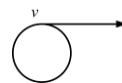
$$\begin{aligned}
 F_{\text{net}} &= 0 \\
 F_{\text{net}} &= F_{g(x)} - F_{\text{fr}} \\
 F_{\text{fr}} &= F_{g(x)} \\
 F_{g(x)} &= F_g \sin \theta \\
 &= (3.0 \text{ N})(9.81 \text{ m/s}^2)(\sin 37^\circ) \\
 &= 17.7 \text{ N} \\
 F_{\text{fr}} &= \mu m g \sin \theta \\
 \mu &= \frac{F_{\text{fr}}}{m g \cos \theta} \\
 &= \frac{(17.7 \text{ N})}{(3.0 \text{ kg})(9.81 \text{ m/s}^2)(\cos 37^\circ)} \\
 &= 0.75
 \end{aligned}$$

7.



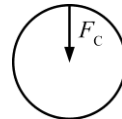
$$\begin{aligned}
 \sum \vec{F} &= 0 \\
 F_{T1} - F_g &= 0 \\
 F_{T1} &= F_g = mg \\
 &= (18 \text{ kg})(9.81 \text{ m/s}^2) \\
 &= 177 \text{ N or } 1.8 \times 10^2 \text{ N}
 \end{aligned}$$

8.



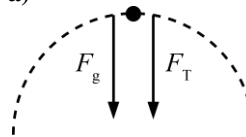
In circular motion, the velocity of a moving object is tangent to the path.

9.



In circular motion, the net force (F_c) is toward the centre of the path, and the acceleration of an object is always in the direction of the net force.

10. a)



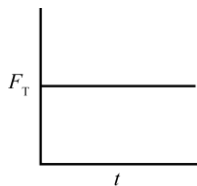
The tension in the cord will be directed toward the centre of the circle wherever the mass is in its path in the vertical circle. The force due to gravity will be directed straight down to Earth's surface wherever the mass is in its path in the vertical circle. Therefore, when the mass is precisely at the top of the vertical circle, both the force due to gravity and the tension are directed down through the centre of the circle.

10. b)

At the top of the vertical circle, both the tension and the force of gravity act toward the centre of the circle. Therefore, the magnitude of the centripetal force at that moment is equal the sum of both forces.

$$F_c = F_g + F_T$$

11. The key to drawing this graph is recognizing that the object is twirled at a constant speed. If the speed remains constant, the tension in the cord will remain constant. This is represented in a graph that looks like this:



12. Centripetal force is the net force that causes an object to travel in a circular path.

13. $F_c = F_{fr}$

$$\frac{mv^2}{r} = \mu mg \cos \theta$$

$$v = \sqrt{r \mu g \cos \theta}$$

$$= \sqrt{(40.0 \text{ m})(0.50)(9.81 \text{ m/s}^2)}$$

$$= 14 \text{ m/s}$$

FORCES CAUSE MOTION

Lesson 1—Newton's Law of Universal Gravitation

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1.
$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(70.0 \text{ kg})(52.0 \text{ kg})}{(1.50 \text{ m})^2}$$

$$= 1.08 \times 10^{-7} \text{ N}$$

2.
$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{(7.35 \times 10^{22} \text{ kg})}$$

$$= \frac{(7.35 \times 10^{22} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2}$$

$$= 1.99 \times 10^{20} \text{ N}$$

3.
$$F_g = \frac{Gm_1m_2}{r^2}$$

As both the objects have equal mass,

$$m^2 = \frac{F_g r^2}{G}$$

$$= \frac{(2.30 \times 10^{-8} \text{ N})(10.0 \text{ m})^2}{6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}}$$

$$= 3.45 \times 10^4 \text{ kg}^2$$

$$m = 1.86 \times 10^2 \text{ kg}$$

4.
$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{\times (6.50 \times 10^2 \text{ kg})}$$

$$= \frac{(4.15 \times 10^6 \text{ m} + 6.37 \times 10^6 \text{ m})^2}{2.34 \times 10^3 \text{ N}}$$

5.
$$F_g = \frac{Gm_1m_2}{r^2}$$

$$m_2 = \frac{F_g r^2}{Gm_1}$$

$$= \frac{(3.2 \times 10^{-6} \text{ N})(2.1 \times 10^{-1} \text{ m})^2}{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.5 \times 10^1 \text{ kg})}$$

$$= 3.8 \times 10^1 \text{ N}$$

6.
$$F_g = \frac{Gm_1m_2}{r^2}$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(2.0 \times 10^2 \text{ kg})}{\times (2.0 \times 10^2 \text{ kg})}}$$

$$= \sqrt{\frac{3.7 \times 10^{-6} \text{ N}}{3.7 \times 10^{-6} \text{ N}}}$$

$$= 8.5 \times 10^{-1} \text{ m}$$

$$\begin{aligned}
 7. \quad F_g &= \frac{Gm_1m_2}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{(70.0 \text{ kg})} \\
 &= \frac{\quad}{(6.38 \times 10^6 \text{ m})^2} \\
 &= 6.86 \times 10^2 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad F_g &= \frac{Gm_1m_2}{r^2} \\
 F_g &\propto m
 \end{aligned}$$

Mass changes by a factor of:

$$\begin{aligned}
 \frac{\text{To}}{\text{From}} &= \frac{35.0 \text{ kg}}{70.0 \text{ kg}} \\
 &= 0.500
 \end{aligned}$$

Mass changes by a factor of 0.500

$\therefore F_g$ will change by a factor of 0.500

$$\begin{aligned}
 F_g &= (6.86 \times 10^2 \text{ N})(0.500) \\
 &= 3.43 \times 10^2 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad F_g &= \frac{Gm_1m_2}{r^2} \\
 F_g &\propto \frac{1}{r^2}
 \end{aligned}$$

Distance changes by a factor of:

$$\begin{aligned}
 \frac{\text{to}}{\text{from}} &= \frac{1.276 \times 10^7 \text{ m}}{6.38 \times 10^6 \text{ m}} \\
 &= 2.00
 \end{aligned}$$

Distance changes by a factor of 2.00

$$\therefore F_g \text{ changes by a factor of } 0.250 = \frac{1}{(2.00)^2}$$

$$\begin{aligned}
 F_g &= (6.86 \times 10^2 \text{ N})(0.250) \\
 &= 1.72 \times 10^2 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad F_g &= \frac{Gm_1m_2}{r^2} \\
 F_g &\propto \frac{1}{r^2}
 \end{aligned}$$

Distance changes by a factor of:

$$\begin{aligned}
 \frac{\text{to}}{\text{from}} &= \frac{9.57 \times 10^6 \text{ m}}{6.38 \times 10^6 \text{ m}} \\
 &= 1.50
 \end{aligned}$$

Distance changes by a factor of 1.50

$$\therefore F_g \text{ changes by a factor of } \frac{1}{(1.50)^2} = 0.444$$

$$\begin{aligned}
 F_g &= (6.86 \times 10^2 \text{ N})(0.444) \\
 &= 3.05 \times 10^2 \text{ N}
 \end{aligned}$$

11.



$F_{g1} = F_{g2}$, and since these forces are in opposite directions, the net force will be zero.

12.



$$\begin{aligned}
 F_{g1} &= \frac{Gm_1m_2}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(10.0 \text{ kg})(10.0 \text{ kg})}{(5.00 \times 10^{-1} \text{ m})^2} \\
 &= 2.67 \times 10^{-8} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_{g2} &= \frac{Gm_1m_2}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(10.0 \text{ kg})(15.0 \text{ kg})}{(5.00 \times 10^{-1} \text{ m})^2} \\
 &= 4.00 \times 10^{-8} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_{\text{net}} &= F_{g2} - F_{g1} \\
 &= 4.00 \times 10^{-8} \text{ N} - 2.67 \times 10^{-8} \text{ N} \\
 &= 1.33 \times 10^{-8} \text{ N}
 \end{aligned}$$

$$13. \quad F_g = \frac{Gm_1m_2}{r^2}$$

$$\begin{aligned}
 F_g &\propto \frac{m_1m_2}{r^2} \\
 &\propto \frac{(2)(2)}{(3)^2} \\
 &\propto 0.444
 \end{aligned}$$

$$\begin{aligned}
 F_{g(\text{new})} &= (3.24 \times 10^{-7} \text{ N})(0.444) \\
 &= 1.44 \times 10^{-7} \text{ N}
 \end{aligned}$$

Lesson 2—Field Explanation

ANSWERS AND SOLUTIONS PRACTICE EXERCISES

$$\begin{aligned}
 1. \quad g &= \frac{GM}{r^2} \\
 &= \frac{Gm_M}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) (6.37 \times 10^{23} \text{ kg})}{(3.43 \times 10^6 \text{ m})^2} \\
 &= 3.61 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad g &= \frac{GM}{r^2} \\
 &= \frac{Gm_E}{r^2} \\
 r &= \sqrt{\frac{Gm_E}{g}} \\
 &= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) (5.98 \times 10^{24} \text{ kg})}{7.33 \text{ N/kg}}} \\
 &= 7.377 \times 10^6 \text{ m}
 \end{aligned}$$

From surface:

$$\begin{aligned}
 d &= 7.377 \times 10^6 \text{ m} - 6.38 \times 10^6 \text{ m} \\
 &= 1.0 \times 10^6 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad g &= \frac{F_g}{m} \\
 &= \frac{63.5 \text{ N}}{22.5 \text{ kg}} \\
 &= 2.82 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad g &= \frac{F_g}{m} \\
 &= \frac{50.0 \text{ N}}{30.0 \text{ kg}} \\
 &= 1.67 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 g &= \frac{GM}{r^2} \\
 &= \frac{Gm_Y}{r^2} \\
 r &= \sqrt{\frac{Gm_Y}{g}} \\
 &= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) (4.83 \times 10^{24} \text{ kg})}{1.67 \text{ N/kg}}} \\
 &= 1.39 \times 10^7 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad g &= \frac{GM}{r^2} \\
 g &\propto \frac{1}{r^2} \\
 &\propto \frac{1}{(2)^2} \\
 &\propto 0.25
 \end{aligned}$$

$$\begin{aligned}
 g_{\text{new}} &= (9.81 \text{ N/kg})(0.25) \\
 &= 2.45 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad g &= \frac{GM}{r^2} \\
 &= \frac{Gm_B}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) (4.00 \times 10^{22} \text{ kg})}{(6.0 \times 10^5 \text{ m})^2} \\
 &= 7.4 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad g_A &= \frac{GM}{r_A^2} \\
 g_B &= \frac{GM}{r_B^2} \\
 \frac{g_A}{g_B} &= \frac{r_B^2}{r_A^2} \\
 r_A^2 &= r_B^2 \frac{g_B}{g_A} \\
 r_A &= r_B \sqrt{\frac{1}{1.20}} \quad (\because g_A = 1.20 g_B) \\
 r_A &= (0.913) r_B
 \end{aligned}$$

$$\begin{aligned}
 8. \quad g &= \frac{GM}{r^2} \\
 &= \frac{Gm_m}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(7.35 \times 10^{22} \text{ kg})}{(1.74 \times 10^6 \text{ m})^2} \\
 &= 1.62 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 g &= \frac{F_g}{m} \\
 F_g &= mg \\
 &= (20.0 \text{ kg})(1.62 \text{ N/kg}) \\
 &= 32.4 \text{ N}
 \end{aligned}$$

Lesson 3—Gravitational Potential Energy

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad E_p &= -\frac{Gm_E m_s}{r} \\
 &= -\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})(5.00 \times 10^3 \text{ kg})}{9.90 \times 10^6 \text{ m}} \\
 &= -2.01 \times 10^{11} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad W &= \Delta E_p \\
 \Delta E_p &= Gm_E m_s \left(\frac{1}{r_i} - \frac{1}{r_f} \right) \\
 &= \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) (5.98 \times 10^{24} \text{ kg})(5.00 \times 10^3 \text{ kg}) \\
 &\quad \times \left(\frac{1}{6.38 \times 10^6 \text{ m}} - \frac{1}{9.90 \times 10^6 \text{ m}} \right) \\
 &= 1.11 \times 10^{11} \text{ J}
 \end{aligned}$$

$$3. \quad W = \Delta E_p$$

The work done to lift the satellite into orbit was $1.11 \times 10^{11} \text{ J}$, so the change in gravitational potential energy was $1.11 \times 10^{11} \text{ J}$.

$$\begin{aligned}
 4. \quad \Delta E_p &= Gm_{\text{moon}} m_{\text{meteor}} \left(\frac{1}{r_i} - \frac{1}{r_f} \right) \\
 &= \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) (7.35 \times 10^{22} \text{ kg})(1750 \text{ kg}) \\
 &\quad \times \left(\frac{1}{15000 \text{ m} + 1.74 \times 10^6 \text{ m}} - \frac{1}{1.74 \times 10^6 \text{ m}} \right) \\
 &= -4.21 \times 10^7 \text{ J}
 \end{aligned}$$

$$\Delta E_k = -\Delta E_p = 4.21 \times 10^7 \text{ J}$$

$$\begin{aligned}
 \Delta E_k &= (E_k)_f - (E_k)_i \\
 &= \frac{1}{2} m v^2 - \frac{1}{2} m v_o^2 \\
 &= \frac{1}{2} m (v^2 - v_o^2)
 \end{aligned}$$

$$4.21 \times 10^7 \text{ J} = \frac{1}{2} (1750 \text{ kg}) (v^2 - (1.00 \times 10^3 \text{ m/s})^2)$$

$$v^2 - (1.00 \times 10^3 \text{ m/s})^2 = \frac{2(4.21 \times 10^7 \text{ J})}{1750 \text{ kg}}$$

$$v^2 = \frac{2(4.21 \times 10^7 \text{ J})}{1750 \text{ kg}} + (1.00 \times 10^3 \text{ m/s})^2$$

$$\Rightarrow v = 1.02 \times 10^3 \text{ m/s}$$

$$\begin{aligned}
 5. \quad E_p &= -\frac{Gm_E m_o}{r} \\
 &= -\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})(10.0 \text{ kg})}{6.38 \times 10^6 \text{ m}} \\
 &= -6.25 \times 10^8 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \Delta E_p &= Gm_E m_s \left(\frac{1}{r_i} - \frac{1}{r_f} \right) \\
 &= \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) (5.98 \times 10^{24} \text{ kg})(2.50 \times 10^3 \text{ kg}) \\
 &\quad \times \left(\frac{1}{6.38 \times 10^6 \text{ m}} - \frac{1}{6.90 \times 10^6 \text{ m}} \right) \\
 &= 1.18 \times 10^{10} \text{ J}
 \end{aligned}$$

Lesson 4—Satellites: Natural & Artificial

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad v &= \sqrt{\frac{2GM}{r}} \\
 &= \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})}{(1.74 \times 10^6 \text{ m})}} \\
 &= 2.37 \times 10^3 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad v &= \sqrt{\frac{2GM}{r}} \\
 &= \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.9 \times 10^{27} \text{ kg})}{(7.18 \times 10^7 \text{ m})}} \\
 &= 5.94 \times 10^4 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad v &= \sqrt{\frac{2GM}{r}} \\
 M &= \frac{v^2 r}{2G} \\
 &= \frac{(9.0 \times 10^3 \text{ m/s})^2 (7.2 \times 10^6 \text{ m})}{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} \\
 &= 4.37 \times 10^{24} \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad v &= \sqrt{\frac{2GM}{r}} \\
 M &= \frac{v^2 r}{2G} \\
 &= \frac{(2.92 \times 10^3 \text{ m/s})^2 (2.57 \times 10^6 \text{ m})}{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} \\
 &= 1.64 \times 10^{23} \text{ kg}
 \end{aligned}$$

Lesson 5—Satellites in Orbit

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad v &= \sqrt{\frac{Gm_E}{r}} \\
 &= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{3.84 \times 10^8 \text{ m}}} \\
 &= 1.02 \times 10^3 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad v &= \sqrt{\frac{Gm_E}{r}} \\
 &= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{(4.4 \times 10^5 \text{ m} + 6.38 \times 10^6 \text{ m})}} \\
 &= 7.65 \times 10^3 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad v &= \sqrt{\frac{Gm_1}{r}} \\
 &= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(1.90 \times 10^{27} \text{ kg})}{(5.60 \times 10^6 \text{ m} + 7.18 \times 10^7 \text{ m})}} \\
 &= 4.06 \times 10^4 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad v &= \sqrt{\frac{Gm_s}{r}} \\
 &= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(1.98 \times 10^{30} \text{ kg})}{1.50 \times 10^{11} \text{ m}}} \\
 &= 2.97 \times 10^4 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad T &= \frac{2\pi r^{\frac{3}{2}}}{\sqrt{Gm}} \\
 &= \frac{(2\pi)(2.3 \times 10^{11} \text{ m})^{\frac{3}{2}}}{\sqrt{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(1.98 \times 10^{30} \text{ kg})}} \\
 &= 6.0 \times 10^7 \text{ s or 1.9 Earth years}
 \end{aligned}$$

Lesson 6—Electric Forces

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad F_g &= \frac{Gm_1 m_2}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(70.0 \text{ kg})(52.0 \text{ kg})}{(1.50 \text{ m})^2} \\
 &= 1.08 \times 10^{-7} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad F_g &= \frac{Gm_1m_2}{r^2} \\
 &= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} \\
 &= 1.99 \times 10^{20} \text{ N}
 \end{aligned}$$

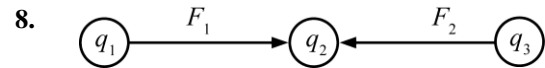
$$\begin{aligned}
 3. \quad F_e &= \frac{kq_1q_2}{r^2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.00 \times 10^{-6} \text{ C})}{(2.00 \times 10^{-2} \text{ m})^2} \\
 &= 2.70 \times 10^2 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad F_e &= \frac{kq_1q_2}{r^2} \\
 q^2 &= \frac{F_e r^2}{k} \\
 &= \frac{(3.40 \times 10^{-2} \text{ N})(1.00 \times 10^{-1} \text{ m})^2}{9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}} \\
 &= 3.78 \times 10^{-14} \text{ C}^2 \\
 q &= 1.94 \times 10^{-7} \text{ C}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad F_e &= \frac{kq_1q_2}{r^2} \\
 r^2 &= \frac{kq_1q_2}{F_e} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.0 \times 10^{-6} \text{ C})}{(5.6 \times 10^{-1} \text{ N})^2} \\
 &= 1.28 \times 10^{-1} \text{ m}^2 \\
 r &= 3.6 \times 10^{-1} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad F_e &= \frac{kq_1q_2}{r^2} \\
 F_e &\propto \frac{1}{r^2} \\
 &\propto \frac{1}{(3)^2} \\
 &\propto 0.111 \\
 F_e &= (6.20 \times 10^{-2} \text{ N})(0.111) \\
 &= 6.89 \times 10^{-3} \text{ N}
 \end{aligned}$$

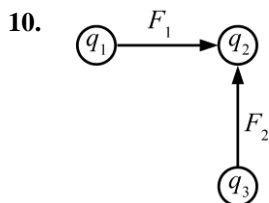
$$\begin{aligned}
 7. \quad F_e &= \frac{kq_1q_2}{r^2} \\
 F_e &\propto \frac{q_1q_2}{r^2} \\
 &\propto \frac{(3)(3)}{(2)^2} \\
 &\propto 2.25 \\
 F_e &= (4.5 \times 10^{-3} \text{ N})(2 \cdot 25) \\
 &= 1.0 \times 10^{-2} \text{ N}
 \end{aligned}$$



$$\begin{aligned}
 F_1 &= \frac{kq_1q_2}{r_1^2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.0 \times 10^{-6} \text{ C})(2.0 \times 10^{-6} \text{ C})}{(0.40 \text{ m})^2} \\
 &= 2.25 \times 10^{-1} \text{ N} \\
 F_2 &= \frac{kq_2q_3}{r_2^2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})}{(0.40 \text{ m})^2} \\
 &= 3.37 \times 10^{-1} \text{ N} \\
 \vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 \\
 &= 2.25 \times 10^{-1} \text{ N} - 3.37 \times 10^{-1} \text{ N} \\
 &= -1.1 \times 10^{-1} \text{ N} \\
 &= 1.1 \times 10^{-1} \text{ N left}
 \end{aligned}$$

Therefore the magnitude of the net electric force is $1.1 \times 10^{-1} \text{ N}$

$$\begin{aligned}
 9. \quad F_e &= \frac{kq_1q_2}{r^2} \\
 F_e &\propto \frac{1}{r^2} \\
 \text{Distance changes by a factor of:} \\
 \frac{\text{to}}{\text{from}} &= \frac{4.04 \times 10^{-1} \text{ m}}{3.11 \times 10^{-1} \text{ m}} \\
 &= 1.30 \\
 \text{Distance increases by a factor of 1.30.} \\
 F_e &\propto \frac{1}{(1.30)^2} \\
 &\propto 0.592 \\
 F_e' &= (5.2 \times 10^{-4} \text{ N})(0.592) \\
 &= 3.1 \times 10^{-4} \text{ N}
 \end{aligned}$$



$$F_1 = \frac{kq_1q_2}{r_1^2}$$

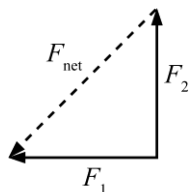
$$= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(3.0 \times 10^{-6} \text{ C})(4.0 \times 10^{-6} \text{ C})}{(0.60 \text{ m})^2}$$

$$= 3.00 \times 10^{-1} \text{ N}$$

$$F_2 = \frac{kq_2q_3}{r_2^2}$$

$$= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.0 \times 10^{-6} \text{ C})^2}{(0.60 \text{ m})^2}$$

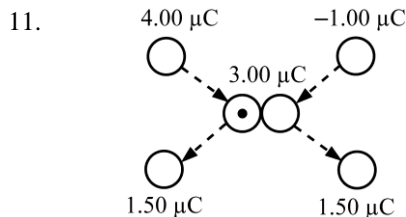
$$= 4.00 \times 10^{-1} \text{ N}$$



$$F_{\text{net}} = \sqrt{(F_1)^2 + (F_2)^2}$$

$$= \sqrt{(3.00 \times 10^{-1} \text{ N})^2 + (4.00 \times 10^{-1} \text{ N})^2}$$

$$= 5.0 \times 10^{-1} \text{ N}$$



$$F_2 = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.50 \times 10^{-6} \text{ C})^2}{(2.00 \times 10^{-1} \text{ m})^2}$$

$$= 5.06 \times 10^{-1} \text{ N}$$

12. $F_e = \frac{kq_1q_2}{r^2}$

$$= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.00 \times 10^{-6} \text{ C})(3.00 \times 10^{-6} \text{ C})}{(3.50 \times 10^{-1} \text{ m})^2}$$

$$= 4.40 \times 10^{-1} \text{ N}$$

$$F_{\text{net}} = ma$$

$$a = \frac{F_{\text{net}}}{m}$$

$$= \frac{4.40 \times 10^{-1} \text{ N}}{2.00 \times 10^{-5} \text{ kg}}$$

$$= 2.20 \times 10^4 \text{ m/s}^2$$

13. $F_{\text{A on B}} = \frac{q_2}{r_1^2}$

$$F_{\text{C on B}} = \frac{q_2}{r_2^2}$$

$$\frac{F_{\text{C on B}}}{F_{\text{A on B}}} = \frac{r_1^2}{r_2^2}$$

$$= \frac{(1.5 \text{ cm})^2}{(4.5 \text{ cm})^2}$$

$$= 0.11$$

Lesson 7—Electric Fields

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $|\vec{E}| = \frac{kq_1}{r^2}$

$$= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(8.00 \times 10^{-6} \text{ C})}{(7.50 \times 10^{-1} \text{ m})^2}$$

$$= 1.28 \times 10^5 \text{ N/C}$$

2. $g = \frac{GM}{r^2}$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(6.37 \times 10^{23} \text{ kg})}{(3.43 \times 10^6 \text{ m})^2}$$

$$= 3.61 \text{ N/kg}$$

$$\begin{aligned}
 3. \quad |\vec{E}| &= \frac{kq_1}{r^2} \\
 r &= \sqrt{\frac{kq}{|\vec{E}|}} \\
 &= \sqrt{\frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.60 \times 10^{-6} \text{ C})}{2.75 \times 10^5 \text{ N/C}}} \\
 &= 3.88 \times 10^{-1} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad g &= \frac{F_g}{m} \\
 &= \frac{63.5 \text{ N}}{22.5 \text{ kg}} \\
 &= 2.82 \text{ N/kg}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad F_e &\propto q \\
 \frac{\text{to}}{\text{from}} &= \frac{1.60 \times 10^{-19} \text{ C}}{3.20 \times 10^{-19} \text{ C}} \\
 &= 0.500 \\
 F_e &= (0.500)(0.250 \text{ N}) \\
 &= 0.125 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad |\vec{E}| &= \frac{F_e}{q} \\
 &= \frac{7.11 \times 10^{-3} \text{ N}}{5.20 \times 10^{-6} \text{ C}} \\
 &= 1.37 \times 10^3 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad |\vec{E}| &= \frac{F_e}{q} \\
 &= \frac{(3.20 \times 10^{-19} \text{ C})(7.60 \times 10^4 \text{ N/C})}{2.43 \times 10^{-14} \text{ N}}
 \end{aligned}$$

$$\begin{aligned}
 F_e &= ma \\
 a &= \frac{F_e}{m} \\
 &= \frac{2.43 \times 10^{-14} \text{ N}}{6.65 \times 10^{-27} \text{ kg}} \\
 &= 3.65 \times 10^{12} \text{ m/s}^2
 \end{aligned}$$

$$8. \quad \begin{array}{c} \oplus \xrightarrow{|\vec{E}_1|} \bullet \xleftarrow{|\vec{E}_2|} \ominus \end{array}$$

$$\begin{aligned}
 |\vec{E}_1| &= |\vec{E}_2| = \frac{kq}{r^2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.50 \times 10^{-6} \text{ C})}{(2.50 \times 10^{-1} \text{ m})^2} \\
 &= 6.48 \times 10^5 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 \vec{E}_{\text{net}} &= \vec{E}_1 + \vec{E}_2 \\
 &= 6.48 \times 10^5 \text{ N/C} + 6.48 \times 10^5 \text{ N/C} \\
 &= 1.296 \times 10^6 \text{ N/C towards right}
 \end{aligned}$$

$$|\vec{E}_{\text{net}}| = 1.296 \times 10^6 \text{ N/C}$$

$$9. \quad \begin{array}{c} \oplus \xrightarrow{|\vec{E}_1|} \bullet \xleftarrow{|\vec{E}_2|} \oplus \end{array}$$

$$\begin{aligned}
 |\vec{E}_1| &= \frac{kq_1}{r_1^2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(3.0 \times 10^{-6} \text{ C})}{(4.0 \times 10^{-1} \text{ m})^2} \\
 &= 1.688 \times 10^5 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 |\vec{E}_2| &= \frac{kq_2}{r_2^2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(6.0 \times 10^{-6} \text{ C})}{(4.0 \times 10^{-1} \text{ m})^2} \\
 &= 3.375 \times 10^5 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 \vec{E}_{\text{net}} &= \vec{E}_1 + \vec{E}_2 \\
 &= 1.688 \times 10^5 \text{ N/C} - 3.375 \times 10^5 \text{ N/C} \\
 &= -1.7 \times 10^5 \text{ N/C towards left}
 \end{aligned}$$

$$|\vec{E}_{\text{net}}| = 1.7 \times 10^5 \text{ N/C}$$

$$10. \quad \begin{array}{c} \oplus \xrightarrow{|\vec{E}_1|} \bullet \xleftarrow{|\vec{E}_2|} \oplus \end{array}$$

Since both the point charges have equal value and their distance are same on both side of the test charge

$$\begin{aligned}
 |\vec{E}_1| &= |\vec{E}_2| \\
 \therefore |\vec{E}_{\text{net}}| &= 0
 \end{aligned}$$

$$\begin{aligned}
 11. \quad F_e &= ma \\
 &= (9.11 \times 10^{-31} \text{ kg})(7.50 \times 10^{12} \text{ m/s}^2) \\
 &= 6.83 \times 10^{-18} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 |\vec{E}| &= \frac{F_e}{q} \\
 &= \frac{6.83 \times 10^{-18} \text{ N}}{1.60 \times 10^{-19} \text{ C}} \\
 &= 4.27 \times 10^1 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad |\vec{E}| &= \frac{kq}{r^2} \\
 |\vec{E}| &\propto \frac{1}{r^2}
 \end{aligned}$$

Distance changes by a factor of:

$$\begin{aligned}
 \frac{\text{To}}{\text{From}} &= \frac{4.50 \times 10^{-1} \text{ m}}{3.00 \times 10^{-1} \text{ m}} \\
 &= 1.50
 \end{aligned}$$

Distance changes by a factor of 1.50.

$$\therefore |\vec{E}| \text{ will change by a factor of } \left(\frac{1}{1.50}\right)^2 = 0.444$$

$$\begin{aligned}
 |\vec{E}'| &= (3.60 \times 10^5 \text{ N/C})(0.444) \\
 &= 1.60 \times 10^5 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad |\vec{E}| &= \frac{kq}{r^2} \\
 |\vec{E}| &\propto \frac{1}{r^2} \\
 r^2 &\propto \frac{1}{|\vec{E}|} \Rightarrow r = \sqrt{\frac{1}{|\vec{E}|}}
 \end{aligned}$$

Field is changed by a factor of:

$$\begin{aligned}
 \frac{\text{To}}{\text{From}} &= \frac{4.20 \times 10^4 \text{ N/C}}{2.10 \times 10^5 \text{ N/C}} \\
 &= 2.00
 \end{aligned}$$

$|\vec{E}|$ changes by a factor of 2.00

$$\therefore r \text{ changes by a factor of } \sqrt{\frac{1}{2.00}} = 0.707$$

$$\begin{aligned}
 r' &= (7.50 \times 10^{-1} \text{ m})(0.707) \\
 &= 5.30 \times 10^{-1} \text{ m}
 \end{aligned}$$

Lesson 8—Electric Potential due to a Point

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$\begin{aligned}
 1. \quad V &= \frac{kq_1}{r} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.5 \times 10^{-6} \text{ C})}{6.0 \times 10^{-2} \text{ m}} \\
 &= 3.8 \times 10^5 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad V &= \frac{kq_1}{r} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(-2.5 \times 10^{-6} \text{ C})}{25 \times 10^{-2} \text{ m}} \\
 &= -9.0 \times 10^4 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad V_1 &= \frac{kq_1}{r_1} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(5.0 \times 10^{-6} \text{ C})}{45 \times 10^{-2} \text{ m}} \\
 &= 9.99 \times 10^4 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_2 &= \frac{kq_1}{r_2} \\
 &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(5.0 \times 10^{-6} \text{ C})}{15 \times 10^{-2} \text{ m}} \\
 &= 3.00 \times 10^5 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \Delta V &= V_2 - V_1 \\
 &= 3.00 \times 10^5 \text{ V} - 9.99 \times 10^4 \text{ V} \\
 &= 2.00 \times 10^5 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 W &= \Delta E = qV \\
 &= (0.030 \times 10^{-6} \text{ C})(2.00 \times 10^5 \text{ V}) \\
 &= 6.0 \times 10^{-3} \text{ J}
 \end{aligned}$$

$$4. \quad V_1 = \frac{kq_1}{r_1} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(6.4 \times 10^{-18} \text{ C})}{2.0 \times 10^{-11} \text{ m}} \\ = 2.88 \times 10^3 \text{ V}$$

$$V_2 = \frac{kq_1}{r_2} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(6.4 \times 10^{-18} \text{ C})}{0.50 \text{ m}} \\ = 1.15 \times 10^{-7} \text{ V}$$

$$W = \Delta E = qV \\ = (1.60 \times 10^{-19} \text{ C})(2.88 \times 10^3 \text{ J}) \\ = 4.60 \times 10^{-16} \text{ J}$$

$$\Delta E = E_k \\ E_k = \frac{1}{2}mv^2 \\ v = \sqrt{\frac{2E_k}{m}} \\ = \sqrt{\frac{2(4.60 \times 10^{-16} \text{ J})}{1.67 \times 10^{-27} \text{ kg}}} \\ = 7.4 \times 10^5 \text{ m/s}$$

$$5. \quad V_1 = \frac{kq_1}{r_1} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.0 \times 10^{-6} \text{ C})}{7.5 \times 10^{-2} \text{ m}} \\ = 1.20 \times 10^5 \text{ V}$$

$$V_2 = \frac{kq_2}{r_2} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(-3.0 \times 10^{-6} \text{ C})}{17.5 \times 10^{-2} \text{ m}} \\ = -1.54 \times 10^5 \text{ V}$$

$$V_3 = \frac{kq_3}{r_3} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(-5.0 \times 10^{-6} \text{ C})}{27.5 \times 10^{-2} \text{ m}} \\ = -1.63 \times 10^5 \text{ V}$$

$$V_T = V_1 + V_2 + V_3 \\ = 1.20 \times 10^5 \text{ V} + (-1.54 \times 10^5 \text{ V}) + (-1.63 \times 10^5 \text{ V}) \\ = -2.0 \times 10^5 \text{ V}$$

$$6. \quad V_1 = \frac{kq_1}{r_1} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.0 \times 10^{-6} \text{ C})}{8.0 \times 10^{-2} \text{ m}} \\ = 1.2 \times 10^5 \text{ V}$$

$$r_2 = \sqrt{r_1^2 + r_3^2} \\ = \sqrt{(8.0 \text{ cm})^2 + (6.0 \text{ cm})^2} \\ = 1.0 \times 10^1 \text{ cm}$$

$$V_2 = \frac{kq_2}{r_2} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(-3.0 \times 10^{-6} \text{ C})}{1.0 \times 10^{-1} \text{ m}} \\ = -2.70 \times 10^5 \text{ V}$$

$$V_3 = \frac{kq_3}{r_3} \\ = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.0 \times 10^{-6} \text{ C})}{6.0 \times 10^{-2} \text{ m}} \\ = 5.99 \times 10^5 \text{ V}$$

$$V_T = V_1 + V_2 + V_3 \\ = 1.12 \times 10^5 \text{ V} + (-2.70 \times 10^5 \text{ V}) + 5.99 \times 10^5 \text{ V} \\ = 4.4 \times 10^5 \text{ V}$$

$$7. \quad E_k + E_p = E_k' + E_p' \\ 0 + E_p = E_k' + E_p'$$

$$\frac{kq_1q_2}{r} = \frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2 + \frac{kq_1q_2}{r'}$$

Both masses m_1 and m_2 are alpha particles, so they are equal in size. Therefore, the speeds v_1 and v_2 are also equal. Therefore, $m_1(v_1')^2$ and $m_2(v_2')^2$ are like terms.

$$m_1 = m_2 = m \\ (v_1')^2 = (v_2')^2 = (v')^2$$

$$\frac{kq_1q_2}{r} = \frac{1}{2}m(v')^2 + \frac{1}{2}m(v')^2 + \frac{kq_1q_2}{r'}$$

$$\frac{kq_1q_2}{r} = m(v')^2 + \frac{kq_1q_2}{r'}$$

$$m(v')^2 = \frac{kq_1q_2}{r} - \frac{kq_1q_2}{r'}$$

$$v' = \sqrt{\frac{\frac{kq_1q_2}{r} - \frac{kq_1q_2}{r'}}{m}}$$

$$\frac{kq_1q_2}{r} = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(3.20 \times 10^{-19} \text{ C})^2}{2.5 \times 10^{-12} \text{ m}}$$

$$= 3.69 \times 10^{-16} \text{ N} \cdot \text{m}$$

$$\frac{kq_1q_2}{r'} = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(3.20 \times 10^{-19} \text{ C})^2}{0.75 \text{ m}}$$

$$= 1.23 \times 10^{-27} \text{ N} \cdot \text{m}$$

Inserting the values into equation 1:

$$v' = \sqrt{\frac{3.69 \times 10^{-16} \text{ N} \cdot \text{m} - 1.23 \times 10^{-27} \text{ N} \cdot \text{m}}{6.65 \times 10^{-27} \text{ kg}}}$$

$$= 2.4 \times 10^5 \text{ m/s}$$

8. $W = \Delta E = qV$

$$V = \frac{\Delta E}{q}$$

$$= \frac{4.40 \times 10^{-5} \text{ J}}{3.00 \times 10^{-6} \text{ C}}$$

$$= 14.7 \text{ V}$$

Lesson 9—Electric Potential in a Uniform Electric Field

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $|\vec{E}| = \frac{V}{d}$

$$= \frac{12.0 \text{ V}}{9.00 \times 10^{-2} \text{ m}}$$

$$= 1.33 \times 10^2 \text{ V/m}$$

2. $|\vec{E}| = \frac{V}{d}$

$$d = \frac{V}{|E|}$$

$$= \frac{2.0 \times 10^2 \text{ V}}{5.0 \times 10^3 \text{ V/m}}$$

$$= 4.0 \times 10^{-2} \text{ m}$$

3. $|\vec{E}| = \frac{V}{d}$

$$V = |\vec{E}|d$$

$$\therefore V = (2.0 \times 10^3 \text{ V/m})(7.3 \times 10^{-2} \text{ m})$$

$$= 1.5 \times 10^2 \text{ V}$$

4. $V = \frac{\Delta E_k}{q}$

$$= \frac{1.50 \times 10^{-15} \text{ J}}{3.20 \times 10^{-14} \text{ C}}$$

$$= 4.69 \times 10^3 \text{ V}$$

5. $V = \frac{\Delta E_k}{q}$

$$\Delta E_k = qV$$

$$= (1.60 \times 10^{-19} \text{ C})(7.20 \times 10^2 \text{ V})$$

$$= 1.15 \times 10^{-16} \text{ J}$$

6. $V = \frac{\Delta E_{p(e)}}{q}$

$$\Delta E_{e(p)} = qV$$

$$= (3.20 \times 10^{-19} \text{ C})(7.50 \times 10^3 \text{ V})$$

$$= 2.40 \times 10^{-15} \text{ J}$$

kinetic energy gained by the alpha particle

$$\Delta E_k = \Delta E_{p(e)}$$

$$\therefore E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2(2.40 \times 10^{-15} \text{ J})}{6.65 \times 10^{-27} \text{ kg}}}$$

$$= 8.50 \times 10^5 \text{ m/s}$$

7. Because there is no change in the potential energy of the proton, there is no work done.

$$\begin{aligned}
 8. \quad |\bar{E}| &= \frac{V}{d} \\
 &= \frac{7.5 \times 10^1 \text{ V}}{6.0 \times 10^{-2} \text{ m}} \\
 &= 1.25 \times 10^3 \text{ V/m}
 \end{aligned}$$

$$\begin{aligned}
 |\bar{E}| &= \frac{F_e}{q} \\
 F_e &= q|\bar{E}| \\
 &= (1.60 \times 10^{-14} \text{ C})(1.25 \times 10^3 \text{ V/m}) \\
 &= 2.00 \times 10^{-16} \text{ N} \\
 \text{work} &= Fd \\
 &= (2.00 \times 10^{-16} \text{ N})(3.0 \times 10^{-2} \text{ m}) \\
 &= 6.0 \times 10^{-18} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad V &= \frac{\Delta E_k}{q} \\
 V &\propto \Delta E
 \end{aligned}$$

Energy is changed by a factor of:

$$\begin{aligned}
 \frac{\text{to}}{\text{from}} &= \frac{9.00 \times 10^{-17} \text{ J}}{3.00 \times 10^{-17} \text{ J}} \\
 &= 3.00
 \end{aligned}$$

The energy changes by a factor of 3.00. Therefore, the potential difference changes by a factor of 3.00.

$$\begin{aligned}
 V' &= (4.20 \times 10^2 \text{ V})(3.00) \\
 &= 1.26 \times 10^3 \text{ V}
 \end{aligned}$$

10. Find initial kinetic energy first:

$$\begin{aligned}
 E_k &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2}(6.65 \times 10^{-27} \text{ kg})(7.15 \times 10^4 \text{ m/s})^2 \\
 &= 1.70 \times 10^{-17} \text{ J}
 \end{aligned}$$

Now find the change in energy using following steps:

$$\begin{aligned}
 |\bar{E}| &= \frac{V}{d} \\
 V &= |\bar{E}|d \\
 &= (1.70 \times 10^2 \text{ V/m})(9.00 \times 10^{-2} \text{ m}) \\
 &= 1.53 \times 10^1 \text{ V} \\
 V &= \frac{\Delta E}{q} \\
 &= \Delta E = qV \\
 &= (3.20 \times 10^{-19} \text{ C})(1.53 \times 10^1 \text{ V}) \\
 &= 4.90 \times 10^{-18} \text{ J}
 \end{aligned}$$

\therefore energy (kinetic) of the alpha particle when it hit the plate is:

$$\begin{aligned}
 E'_k &= 4.90 \times 10^{-18} \text{ J} + 1.70 \times 10^{-17} \text{ J} \\
 &= 2.19 \times 10^{-17} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 E'_k &= \frac{1}{2}m(v')^2 \\
 (v') &= \sqrt{\frac{2E_k}{m}} \\
 &= \sqrt{\frac{2(2.19 \times 10^{-17} \text{ J})}{6.65 \times 10^{-27} \text{ kg}}} \\
 &= 8.11 \times 10^4 \text{ m/s}
 \end{aligned}$$

11. Find initial energy:

$$\begin{aligned}
 E_{k1} &= \frac{1}{2}mv_1^2 \\
 &= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(5.0 \times 10^5 \text{ m/s})^2 \\
 &= 1.14 \times 10^{-19} \text{ J}
 \end{aligned}$$

Find energy of electron at plate:

$$\begin{aligned}
 E_{k2} &= \frac{1}{2}mv_2^2 \\
 &= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(1.0 \times 10^5 \text{ m/s})^2 \\
 &= 4.56 \times 10^{-21} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \Delta E &= 1.14 \times 10^{-19} \text{ J} - 4.56 \times 10^{-21} \text{ J} \\
 &= 1.09 \times 10^{-19} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{\Delta E}{q} \\
 &= \frac{1.09 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ C}} \\
 &= 6.8 \times 10^{-1} \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad |\bar{E}| &= \frac{V}{d} \\
 |\bar{E}| &\propto \frac{1}{d}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\text{To}}{\text{From}} &= \frac{5.0 \text{ cm}}{7.0 \text{ cm}} \\
 &= 0.714
 \end{aligned}$$

Distance changes by a factor of 0.714.

\therefore the electric field will change by a factor of

$$\frac{1}{0.714} = 1.40$$

$$\begin{aligned}
 |\bar{E}'| &= (9.3 \times 10^2 \text{ V/m})(1.40) \\
 &= 1.3 \times 10^3 \text{ V/m}
 \end{aligned}$$

$$\begin{aligned}
 |\vec{E}| &= \frac{V}{d} \\
 &= \frac{3.00 \times 10^2 \text{ V}}{5.00 \times 10^{-2} \text{ m}} \\
 13. \quad &= 6.00 \times 10^3 \text{ V/m}
 \end{aligned}$$

$$\begin{aligned}
 E_k &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(6.00 \times 10^6 \text{ m/s})^2 \\
 14. \quad &= 1.64 \times 10^{-17} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \Delta E &= 1.64 \times 10^{-17} \text{ J} - 0 \\
 &= 1.64 \times 10^{-17} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{\Delta E}{q} \\
 &= \frac{1.64 \times 10^{-17} \text{ J}}{1.60 \times 10^{-19} \text{ C}} \\
 &= 1.02 \times 10^2 \text{ V}
 \end{aligned}$$

15. Uniform accelerated motion:

$$\begin{aligned}
 v_i &= 0 \\
 t &= 2.50 \times 10^{-5} \text{ s} \\
 d &= 4.00 \times 10^{-2} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 d &= v_i t + \frac{1}{2}at^2 \\
 a &= \frac{2d}{t^2} \\
 &= \frac{2(4.00 \times 10^{-2} \text{ m})}{(2.50 \times 10^{-5} \text{ s})^2} \\
 &= 1.28 \times 10^8 \text{ m/s}^2
 \end{aligned}$$

$$\begin{aligned}
 F_{\text{net}} &= F_e = ma \\
 &= (6.65 \times 10^{-27} \text{ kg})(1.28 \times 10^8 \text{ m/s}^2) \\
 &= 8.51 \times 10^{-19} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 |\vec{E}| &= \frac{F_e}{q} \\
 &= \frac{8.51 \times 10^{-19} \text{ N}}{3.20 \times 10^{-19} \text{ C}} \\
 &= 2.66 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad |\vec{E}| &= \frac{F_e}{q} \\
 &= \frac{5.30 \times 10^{-14} \text{ N}}{3.20 \times 10^{-19} \text{ C}} \\
 &= 1.66 \times 10^5 \text{ N/C}
 \end{aligned}$$

$$\begin{aligned}
 |\vec{E}| &= \frac{V}{d} \\
 V &= |\vec{E}|d \\
 &= (1.66 \times 10^5 \text{ N/C})(7.50 \times 10^{-2} \text{ m}) \\
 &= 1.24 \times 10^4 \text{ V}
 \end{aligned}$$

17. When a charged object is moved perpendicular to an electric field, no work is done.

$$\begin{aligned}
 18. \text{ a) } \quad \Delta E_k + \Delta E_p &= 0 \\
 \frac{1}{2}mv^2 + \frac{1}{2}mv_0^2 + qV &= 0 \\
 \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(v^2) \\
 + (1.60 \times 10^{-19} \text{ C})(60.0 \text{ V}) &= 0 \\
 \Rightarrow v &= 1.07 \times 10^5 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \quad \Delta E_k + \Delta E_p &= 0 \\
 \frac{1}{2}mv^2 + \frac{1}{2}mv_0^2 + qV &= 0 \\
 \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(v^2) \\
 + (-1.60 \times 10^{-19} \text{ C})(60.0 \text{ V}) &= 0 \\
 \Rightarrow v &= 4.59 \times 10^6 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad E_k &= \frac{1}{2}mv^2 \\
 \Delta E &= qV \\
 qV &= \frac{1}{2}mv^2 \\
 v^2 \propto V \text{ or } V &\propto v^2
 \end{aligned}$$

speed changes by a factor of:

$$\begin{aligned}
 \frac{\text{To}}{\text{From}} &= \frac{7.25 \times 10^4 \text{ m/s}}{2.90 \times 10^4 \text{ m/s}} \\
 &= 2.50
 \end{aligned}$$

$\therefore V$ changes by a factor of $(2.50)^2 = 6.25$

$$\begin{aligned}
 V' &= (6.25)(2.50 \times 10^5 \text{ V}) \\
 &= 1.56 \times 10^6 \text{ V}
 \end{aligned}$$

$$\begin{aligned}\Delta E &= qV \\ &= (1.60 \times 10^{-19} \text{ C})(5.00 \times 10^3 \text{ V}) \\ \mathbf{20.} \quad &= 8.00 \times 10^{-16} \text{ J}\end{aligned}$$

$$\begin{aligned}\Delta E &= E_k - 0 \\ E_k &= \Delta E \\ E_k &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} \\ &= \sqrt{\frac{2(8.00 \times 10^{-16} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} \\ &= 4.19 \times 10^7 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\mathbf{21. a)} \quad |\vec{E}| &= \frac{V}{d} \\ &= \frac{12.0 \text{ V}}{2.00 \times 10^{-2} \text{ m}} \\ &= 6.00 \times 10^2 \text{ V/m}\end{aligned}$$

$$\begin{aligned}F_e &= q|E| \\ &= (3.20 \times 10^{-19} \text{ C})(6.00 \times 10^2 \text{ V/m}) \\ &= 1.92 \times 10^{-16} \text{ N}\end{aligned}$$

$$\begin{aligned}\mathbf{b)} \quad F_g &= mg \\ &= (6.65 \times 10^{-27} \text{ kg})(9.81 \text{ N/kg}) \\ &= 6.52 \times 10^{-26} \text{ N}\end{aligned}$$

$$\begin{aligned}\mathbf{c)} \quad F_{\text{net}} &= F_e - F_g \\ &= 1.92 \times 10^{-16} \text{ N} - 6.52 \times 10^{-26} \text{ N} \\ &= 1.92 \times 10^{-16} \text{ N}\end{aligned}$$

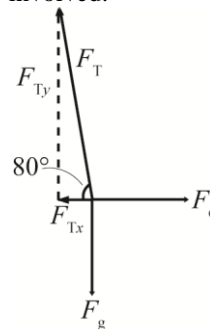
$$\begin{aligned}\mathbf{d)} \quad F_{\text{net}} &= ma \\ a &= \frac{F_{\text{net}}}{m} \\ &= \frac{1.92 \times 10^{-16} \text{ N}}{6.65 \times 10^{-27} \text{ kg}} \\ &= 2.89 \times 10^{10} \text{ m/s}^2\end{aligned}$$

$$\begin{aligned}\mathbf{e)} \quad F_e &= F_g \\ F_e &= 6.52 \times 10^{-26} \text{ N}\end{aligned}$$

$$\begin{aligned}|\vec{E}| &= \frac{F_e}{q} \\ &= \frac{6.52 \times 10^{-26} \text{ N}}{3.20 \times 10^{-19} \text{ C}} \\ &= 2.04 \times 10^{-7} \text{ N/C}\end{aligned}$$

$$\begin{aligned}V &= |\vec{E}|d \\ &= (2.04 \times 10^{-7} \text{ N/C})(2.00 \times 10^{-2} \text{ m}) \\ &= 4.07 \times 10^{-9} \text{ V}\end{aligned}$$

- 22.** Draw a free-body diagram to identify the forces involved.



$$\begin{aligned}F_g &= mg \\ &= (0.500 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 4.90 \text{ N}\end{aligned}$$

Considering equilibrium condition:

$$\begin{aligned}\sum \vec{F}_y &= 0 \\ F_{Ty} - F_g &= 0 \\ F_T \sin \theta - F_g &= 0 \\ F_T \sin 80.0^\circ - 4.90 \text{ N} &= 0 \\ 0.985 F_T &= 4.90 \text{ N} \\ F_T &= \frac{4.90 \text{ N}}{0.985} \\ &= 4.975 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum \vec{F}_x &= 0 \\ F_c - F_{Tx} &= 0 \\ q|\vec{E}| - F_T \cos \theta &= 0 \\ (1500 \text{ N/C})q &= F_T \cos 80.0^\circ \\ (1500 \text{ N/C})q &= 0.174 F_T \\ (1500 \text{ N/C})q &= (0.174)(4.975 \text{ N}) \\ q &= 5.77 \times 10^{-4} \text{ C}\end{aligned}$$

- 23.** Find how long the electron will be in the electric field:

$$\begin{aligned}v &= \frac{d}{t} \\ t &= \frac{d}{v} \\ &= \frac{14.0 \times 10^{-2} \text{ m}}{8.70 \times 10^6 \text{ m/s}} \\ &= 1.61 \times 10^{-8} \text{ s}\end{aligned}$$

Find the electric force on the electron:

$$\begin{aligned}F_e &= q|\vec{E}| \\ &= (1.60 \times 10^{-19} \text{ C})(1.32 \times 10^3 \text{ N/C}) \\ &= 2.11 \times 10^{-16} \text{ N}\end{aligned}$$

$$\begin{aligned}
 F_{\text{net}} &= ma \\
 a &= \frac{F_{\text{net}}}{m} \\
 &= \frac{2.11 \times 10^{-16} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} \\
 &= 2.32 \times 10^{14} \text{ m/s}^2
 \end{aligned}$$

Now find the vertical component of displacement of the electron:

$$\begin{aligned}
 v_i &= 0 \\
 t &= 1.61 \times 10^{-8} \text{ s} \\
 a &= 2.32 \times 10^{14} \text{ m/s}^2
 \end{aligned}$$

Magnitude of displacement

$$\begin{aligned}
 d &= v_i t + \frac{1}{2} a t^2 \\
 &= \frac{1}{2} (2.32 \times 10^{14} \text{ m/s}^2) (1.61 \times 10^{-8} \text{ s})^2 \\
 &= 3.00 \times 10^{-2} \text{ m}
 \end{aligned}$$

Lesson 10—Magnetic Forces and Fields

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

- $$\begin{aligned}
 B &= \mu_0 I n \\
 &= \frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right) (1.25 \text{ A})(1800)}{25.0 \times 10^{-2} \text{ m}} \\
 &= 1.13 \times 10^{-2} \text{ T}
 \end{aligned}$$
- $$\begin{aligned}
 B &= \mu_0 I n \\
 n &= \frac{B}{\mu_0 I} \\
 &= \frac{2.1 \times 10^{-3} \text{ T}}{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right) (0.72 \text{ A})} \\
 &= 2.32 \times 10^3 / \text{m} \\
 \frac{2.32 \times 10^3}{100 \text{ cm}} &= \frac{x}{25 \text{ cm}} \\
 x &= 580
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{775 \text{ turns}}{30.0 \text{ cm}} &= \frac{n}{100 \text{ cm}} \\
 n &= 2583 \\
 B &= \mu_0 I n \\
 I &= \frac{B}{\mu_0 n} \\
 &= \frac{0.100 \text{ T}}{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right) (2583)} \\
 &= 31 \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad B &= \mu_0 I n \\
 &= (4\pi \times 10^{-7} \text{ N/A}^2) (2.0 \text{ A}) \left(\frac{800}{0.30 \text{ m}}\right) \\
 &= 6.7 \times 10^{-3} \text{ T}
 \end{aligned}$$

Lesson 11—Magnetic Forces on Current-Carrying Conductors

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

- $$\begin{aligned}
 F_m &= qvB_{\perp} \\
 B_{\perp} &= \frac{F_m}{qv} \\
 &= \frac{9.50 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(2.10 \times 10^5 \text{ m/s})} \\
 &= 2.83 \text{ T}
 \end{aligned}$$
- $$\begin{aligned}
 F_m &= B_{\perp} I l \\
 &= (7.50 \times 10^{-4} \text{ T})(0.960 \text{ A})(0.222 \text{ m}) \\
 &= 1.60 \times 10^{-4} \text{ N} \\
 \vec{F}_m &= 1.60 \times 10^{-4} \text{ N east}
 \end{aligned}$$
- $$\begin{aligned}
 F_m &= qvB_{\perp} \\
 &= (1.60 \times 10^{-19} \text{ C})(3.52 \times 10^5 \text{ m/s}) \\
 &\quad \times (2.80 \times 10^{-1} \text{ T}) \\
 &= 1.58 \times 10^{-14} \text{ N} \\
 \vec{F}_m &= 1.58 \times 10^{-14} \text{ N west}
 \end{aligned}$$
- $$\begin{aligned}
 F_m &= qvB_{\perp} \\
 &= (3.20 \times 10^{-19} \text{ C})(7.50 \times 10^4 \text{ m/s})(5.50 \text{ T}) \\
 &= 1.30 \times 10^{-13} \text{ N} \\
 \vec{F}_m &= 1.30 \times 10^{-13} \text{ N west}
 \end{aligned}$$

5. $F_m = qvB_{\perp}$
 $B_{\perp} = \frac{F_m}{qv}$

$$= \frac{1.70 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(1.40 \times 10^4 \text{ m/s})}$$

$$= 5.59 \text{ T}$$

$$\vec{B}_{\perp} = 5.59 \text{ T up}$$
6. $F_m = qvB_{\perp}$
 $B_{\perp} = \frac{F_m}{qv}$

$$= \frac{7.1 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(2.7 \times 10^5 \text{ m/s})}$$

$$= 1.6 \text{ T}$$

$$\vec{B}_{\perp} = 1.6 \text{ T west}$$
7. The alpha particle is moving parallel to the magnetic field.
 \therefore there is no force.
 $F_m = qvB \sin \theta$

$$= 0$$
8. $F_m = B_{\perp} Il$

$$= (5.00 \times 10^{-1} \text{ T})(3.60 \text{ A})(2.50 \times 10^{-1} \text{ m})$$

$$= 4.50 \times 10^{-1} \text{ N}$$
9. $\Delta E = qV$

$$= (1.60 \times 10^{-19} \text{ C})(1.70 \times 10^3 \text{ V})$$

$$= 2.72 \times 10^{-16} \text{ J}$$
- $$E_k = \frac{1}{2}mv^2$$
- $$v = \sqrt{\frac{2E_k}{m}}$$
- $$= \sqrt{\frac{2(2.72 \times 10^{-16})}{9.11 \times 10^{-31} \text{ kg}}}$$
- $$= 2.44 \times 10^7 \text{ m/s}$$
- $$F_m = qvB_{\perp}$$
- $$= (1.60 \times 10^{-19} \text{ C})(2.44 \times 10^7 \text{ m/s})$$
- $$\quad \times (2.50 \times 10^{-1} \text{ T})$$
- $$= 9.77 \times 10^{-13} \text{ N}$$

10. $F_m = qvB_{\perp}$
 $v = \frac{F_m}{qB_{\perp}}$

$$= \frac{4.1 \times 10^{-13} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(7.20 \times 10^{-1} \text{ T})}$$

$$= 3.56 \times 10^6 \text{ m/s}$$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(3.56 \times 10^6 \text{ m/s})^2$$

$$= 5.77 \times 10^{-18} \text{ J}$$

$$\Delta E = qV$$

$$V = \frac{\Delta E}{q}$$

$$= \frac{5.77 \times 10^{-18} \text{ J}}{1.60 \times 10^{-19} \text{ C}}$$

$$= 3.6 \times 10^1 \text{ V}$$
11. $F_m = qvB_{\perp}$

$$= (1.60 \times 10^{-19} \text{ C})(6.20 \times 10^5 \text{ m/s})$$

$$\quad \times (2.30 \times 10^{-1} \text{ T})$$

$$= 2.28 \times 10^{-14} \text{ N}$$

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{F_m}{m}$$

$$= \frac{2.28 \times 10^{-14} \text{ N}}{9.11 \times 10^{-31} \text{ kg}}$$

$$= 2.50 \times 10^{16} \text{ m/s}^2$$
12. $F_m = F_g$
 $F_g = mg$

$$= (1.76 \times 10^{-2} \text{ kg})(9.81 \text{ N/kg})$$

$$= 1.72 \times 10^{-1} \text{ N}$$

$$\therefore F_m = 1.72 \times 10^{-1} \text{ N}$$

$$F_m = B_{\perp} Il$$

$$B_{\perp} = \frac{F_m}{Il}$$

$$= \frac{1.72 \times 10^{-1} \text{ N}}{(6.00 \text{ A})(1.90 \times 10^{-2} \text{ m})}$$

$$= 1.51 \text{ T}$$

Lesson 12—Cathode Rays

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$v = \frac{qB_{\perp}r}{m}$$

$$= \frac{(3.20 \times 10^{-19} \text{ C})(4.22 \times 10^{-1} \text{ T})(1.50 \times 10^{-3} \text{ m})}{6.65 \times 10^{-27} \text{ kg}}$$

$$= 3.05 \times 10^4 \text{ m/s}$$

2. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$B_{\perp} = \frac{mv}{qr}$$

$$= \frac{(1.67 \times 10^{-27} \text{ kg})(5.40 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(7.20 \times 10^{-3} \text{ m})}$$

$$= 7.83 \times 10^{-1} \text{ T}$$

3. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$\frac{q}{m} = \frac{v}{B_{\perp}r}$$

$$= \frac{(3.60 \times 10^5 \text{ m/s})}{(6.10 \times 10^{-1} \text{ T})(7.40 \times 10^{-2} \text{ m})}$$

$$= 7.98 \times 10^6 \text{ C/kg}$$

4. $F_e = F_m$

$$q|\vec{E}| = qvB_{\perp}$$

$$|\vec{E}| = vB_{\perp}$$

$$= (7.80 \times 10^5 \text{ m/s})(2.20 \times 10^{-1} \text{ T})$$

$$= 1.72 \times 10^5 \text{ N/C}$$

5. $F_e = F_m$

$$q|\vec{E}| = qvB_{\perp}$$

$$v = \frac{|\vec{E}|}{B_{\perp}}$$

$$= \frac{2.10 \times 10^5 \text{ N/C}}{6.50 \times 10^{-1} \text{ T}}$$

$$= 3.23 \times 10^5 \text{ m/s}$$

6. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$\frac{qB_{\perp}r}{m} = \frac{v^2}{v}$$

$$v = \frac{qB_{\perp}r}{m}$$

$$= \frac{(3.20 \times 10^{-19} \text{ C})(3.60 \times 10^{-1} \text{ T})(8.20 \times 10^{-2} \text{ m})}{6.65 \times 10^{-27} \text{ kg}}$$

$$= 1.42 \times 10^6 \text{ m/s}$$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(6.65 \times 10^{-27} \text{ kg})(1.42 \times 10^6 \text{ m/s})^2$$

$$= 6.71 \times 10^{-15} \text{ J}$$

7. $V = \frac{\Delta E}{q}$

$$\Delta E = qV$$

$$= (1.60 \times 10^{-19} \text{ C})(2.50 \times 10^3 \text{ V})$$

$$= 4.00 \times 10^{-16} \text{ J}$$

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2(4.00 \times 10^{-16} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}}$$

$$= 2.96 \times 10^7 \text{ m/s}$$

8. $E_k = \frac{1}{2}mv^2$

$$= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(4.75 \times 10^7 \text{ m/s})^2$$

$$= 1.03 \times 10^{-15} \text{ J}$$

$$V = \frac{\Delta E}{q}$$

$$= \frac{1.03 \times 10^{-15} \text{ J}}{1.60 \times 10^{-19} \text{ C}}$$

$$= 6.42 \times 10^3 \text{ V}$$

9. $V = \frac{\Delta E}{q}$

$$\Delta E = qV$$

$$= (1.00 \times 10^{-14} \text{ C})(1.40 \times 10^3 \text{ V})$$

$$= 2.24 \times 10^{-16} \text{ J}$$

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2(2.24 \times 10^{-16} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}}$$

$$= 2.23 \times 10^7 \text{ m/s}$$

$$F_m = F_c$$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB_{\perp}}$$

$$= \frac{(9.11 \times 10^{-31} \text{ kg})(2.23 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(2.20 \times 10^{-2} \text{ T})}$$

$$= 5.74 \times 10^{-3} \text{ m}$$

10. $F_e = F_m$

$$q|\vec{E}| = qvB_{\perp}$$

$$v = \frac{|\vec{E}|}{B_{\perp}}$$

$$= \frac{4.20 \times 10^5 \text{ N/C}}{1.40 \times 10^{-2} \text{ T}}$$

$$= 3.00 \times 10^7 \text{ m/s}$$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^7 \text{ m/s})^2$$

$$= 4.10 \times 10^{-16} \text{ J}$$

$$V = \frac{\Delta E}{q}$$

$$= \frac{4.10 \times 10^{-16} \text{ J}}{1.6 \times 10^{-14} \text{ C}}$$

$$= 2.56 \times 10^3 \text{ V}$$

11. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$q = \frac{mv}{rB_{\perp}}$$

$$= \frac{(8.4 \times 10^{-27} \text{ kg})(5.6 \times 10^5 \text{ m/s})}{(3.5 \times 10^{-2} \text{ m})(2.8 \times 10^{-1} \text{ T})}$$

$$= 4.8 \times 10^{-19} \text{ C}$$

$$\# e^- = \frac{4.8 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$$

$$= 3$$

The particle carries an extra 3 electrons.

12. When charged particles travel parallel to a magnetic field there is no force. If there is no force, there is no deflection.

13. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$v = \frac{qB_{\perp}r}{m}$$

$$= \frac{(1.60 \times 10^{-19} \text{ C})(0.75 \text{ T})(0.30 \text{ m})}{1.67 \times 10^{-27} \text{ kg}}$$

$$= 2.16 \times 10^7 \text{ m/s}$$

14. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$v = \frac{qB_{\perp}r}{m}$$

$$= \frac{(1.60 \times 10^{-19} \text{ C})(2.2 \times 10^{-4} \text{ T})(0.77 \text{ m})}{9.11 \times 10^{-31} \text{ kg}}$$

$$= 2.98 \times 10^7 \text{ m/s}$$

$$\Delta E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(2.98 \times 10^7 \text{ m/s})^2$$

$$= 4.03 \times 10^{-16} \text{ J}$$

$$V = \frac{\Delta E}{q}$$

$$= \frac{4.03 \times 10^{-16} \text{ J}}{1.60 \times 10^{-19} \text{ C}}$$

$$= 2.5 \times 10^3 \text{ V}$$

15. $F_m = F_c$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$v = \frac{qB_{\perp}r}{m}$$

$$= (1.0 \times 10^4 \text{ C/kg})(9.10 \times 10^{-1} \text{ T})(0.240 \text{ m})$$

$$= 2.4 \times 10^3 \text{ m/s}$$

$$d = C = 2\pi r$$

$$= 2\pi(0.240 \text{ m})$$

$$= 1.51 \text{ m}$$

$$v = \frac{d}{t}$$

$$t = \frac{d}{v}$$

$$= \frac{1.51 \text{ m}}{2.40 \times 10^3 \text{ m/s}}$$

$$= 6.28 \times 10^{-4} \text{ s}$$

Lesson 13—Electromagnetic Induction

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\text{area} = \pi r^2$
 $= \pi(5.0 \times 10^{-2} \text{ m})^2$
 $= 7.85 \times 10^{-3} \text{ m}^2$
 $\Phi = BA \cos \theta$
 $= (3.2 \times 10^{-3} \text{ T})(7.85 \times 10^{-3} \text{ m}^2)(\cos 0^\circ)$
 $= 2.5 \times 10^{-5} \text{ Wb}$

2. $\text{area} = \pi r^2$
 $= \pi(6.0 \times 10^{-2} \text{ m})^2$
 $= 1.13 \times 10^{-2} \text{ m}^2$
 $\Phi_f = BA \cos \theta$
 $= (3.6 \times 10^{-4} \text{ T})(1.13 \times 10^{-2} \text{ m}^2)(\cos 90^\circ)$
 $= 0$

$\Phi_0 = BA \cos \theta$
 $= (3.6 \times 10^{-4} \text{ T})(1.13 \times 10^{-2} \text{ m}^2)(\cos 0^\circ)$
 $= 4.07 \times 10^{-6} \text{ Wb}$

$\Delta\Phi = \Phi_f - \Phi_0$
 $= 0 - 4.07 \times 10^{-6} \text{ Wb}$
 $= -4.07 \times 10^{-6} \text{ Wb}$

$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$
 $= (-200) \left(\frac{-4.07 \times 10^{-6} \text{ Wb}}{0.015 \text{ s}} \right)$
 $= 5.4 \times 10^{-2} \text{ V}$

3. $\Phi_f = BA \cos \theta$
 $= (2.2 \times 10^{-2} \text{ T})(0)(\cos 90^\circ)$
 $= 0$

$\Phi_0 = BA \cos \theta$
 $= (2.2 \times 10^{-2} \text{ T})(2.5 \times 10^{-3} \text{ m}^2)(\cos 0^\circ)$
 $= 5.5 \times 10^{-5} \text{ Wb}$

$\Delta\Phi = \Phi_f - \Phi_0$
 $= 0 - 5.5 \times 10^{-5} \text{ Wb}$
 $= -5.5 \times 10^{-5} \text{ Wb}$

$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$
 $= - \left(\frac{-5.5 \times 10^{-5} \text{ Wb}}{0.10 \text{ s}} \right)$
 $= 5.5 \times 10^{-4} \text{ V}$

Direction: clockwise

4. $\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$
 $= -50(15 \text{ Wb/s})$
 $= -7.5 \times 10^2 \text{ V}$

5. $\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$
 $\Delta\Phi = - \frac{\mathcal{E}(\Delta t)}{N}$
 $= - \frac{(0.92 \text{ V})(0.12 \text{ s})}{100}$
 $= -1.10 \times 10^{-3} \text{ Wb}$

$\Phi = BA \cos \theta$
 $B = \frac{\Phi}{A \cos \theta}$
 $= \frac{-1.10 \times 10^{-3} \text{ Wb}}{4.0 \times 10^{-3} \text{ m}^2}$
 $= 2.8 \times 10^{-1} \text{ T}$

6. $\text{area} = \pi r^2$
 $= \pi(12.5 \times 10^{-2} \text{ m})^2$
 $= 4.91 \times 10^{-2} \text{ m}^2$
 $\Phi_0 = BA \cos \theta$
 $= (2.7 \times 10^{-3} \text{ T})(4.91 \times 10^{-2} \text{ m}^2)(\cos 0^\circ)$
 $= 1.33 \times 10^{-4} \text{ Wb}$

$\Phi_f = BA \cos \theta$
 $= (2.7 \times 10^{-3} \text{ T})(4.91 \times 10^{-2} \text{ m}^2)(\cos 180^\circ)$
 $= -1.33 \times 10^{-4} \text{ Wb}$

$\Delta\Phi = \Phi_f - \Phi_0$
 $= -1.33 \times 10^{-4} \text{ Wb} - 1.33 \times 10^{-4} \text{ Wb}$
 $= -2.66 \times 10^{-4} \text{ Wb}$

$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$
 $= -(10) \left(\frac{-2.66 \times 10^{-4} \text{ Wb}}{0.30 \text{ s}} \right)$
 $= 8.8 \times 10^{-3} \text{ V}$

7. $\Phi_0 = BA \cos \theta$
 $= (0.40 \text{ T})(5.0 \times 10^{-3} \text{ m}^2)(\cos 0^\circ)$
 $= 2.0 \times 10^{-3} \text{ Wb}$

$\Phi_f = BA \cos \theta$
 $= (0)(5.0 \times 10^{-3} \text{ m}^2)(\cos 0^\circ)$
 $= 0$

$\Delta\Phi = \Phi_f - \Phi_0$
 $= 0 - 2.0 \times 10^{-3} \text{ Wb}$
 $= -2.0 \times 10^{-3} \text{ Wb}$

$$\begin{aligned}\varepsilon &= -N \frac{\Delta\Phi}{\Delta t} \\ &= -(25) \left(\frac{-2.0 \times 10^{-3} \text{ Wb}}{1.0 \text{ s}} \right) \\ &= 5.0 \times 10^{-2} \text{ V}\end{aligned}$$

8. a) $\Phi_0 = BA \cos \theta$
 $= (1.6 \text{ T})(7.2 \times 10^{-3} \text{ m}^2)(\cos 0^\circ)$
 $= 1.15 \times 10^{-2} \text{ Wb}$

$$\begin{aligned}\Phi_f &= BA \cos \theta \\ &= (0)(7.2 \times 10^{-3} \text{ m}^2)(\cos 0^\circ) \\ &= 0\end{aligned}$$

$$\begin{aligned}\Delta\Phi &= \Phi_f - \Phi_0 \\ &= 0 - 1.15 \times 10^{-2} \text{ Wb} \\ &= -1.15 \times 10^{-2} \text{ Wb}\end{aligned}$$

$$\begin{aligned}\varepsilon &= -N \frac{\Delta\Phi}{\Delta t} \\ &= - \frac{(-1.15 \times 10^{-2} \text{ Wb})}{0.050 \text{ s}} \\ &= 2.3 \times 10^{-1} \text{ V}\end{aligned}$$

b) $\varepsilon = IR$

$$\begin{aligned}I &= \frac{\varepsilon}{R} \\ &= \frac{2.3 \times 10^{-1} \text{ V}}{12.0 \Omega} \\ &= 1.9 \times 10^{-2} \text{ A}\end{aligned}$$

c) clockwise

9. a) area = l^2
 $= (4.0 \times 10^{-2} \text{ m})^2$
 $= 1.6 \times 10^{-3} \text{ m}^2$

$$\begin{aligned}\Phi_0 &= BA \cos \theta \\ &= (0.20 \text{ T})(1.6 \times 10^{-3} \text{ m}^2)(\cos 0^\circ) \\ &= 3.2 \times 10^{-4} \text{ Wb}\end{aligned}$$

$$\begin{aligned}\Phi_f &= BA \cos \theta \\ &= (0.50 \text{ T})(1.6 \times 10^{-3} \text{ m}^2)(\cos 0^\circ) \\ &= 8.0 \times 10^{-4} \text{ Wb}\end{aligned}$$

$$\begin{aligned}\Delta\Phi &= \Phi_f - \Phi_0 \\ &= 8.0 \times 10^{-4} \text{ Wb} - 3.2 \times 10^{-4} \text{ Wb} \\ &= 4.8 \times 10^{-4} \text{ Wb}\end{aligned}$$

$$\begin{aligned}\varepsilon &= -N \frac{\Delta\Phi}{\Delta t} \\ &= - \frac{4.8 \times 10^{-4} \text{ Wb}}{0.30 \text{ s}} \\ &= -1.6 \times 10^{-3} \text{ V}\end{aligned}$$

$$\varepsilon = IR$$

$$\begin{aligned}I &= \frac{\varepsilon}{R} \\ &= \frac{1.6 \times 10^{-3} \text{ V}}{2.0 \Omega} \\ &= 8.0 \times 10^{-4} \text{ A}\end{aligned}$$

b) counter-clockwise

Lesson 14—Electric Generator

PRACTICE QUESTIONS— ANSWERS AND SOLUTIONS

1. $V_{\text{back}} = \varepsilon - Ir$
 $= 120 \text{ V} - (12.0 \text{ A})(6.0 \Omega)$
 $= 48 \text{ V}$

2. a) initial

$$\begin{aligned}V_{\text{back}} &= \varepsilon - Ir \\ r &= \frac{\varepsilon - V_{\text{back}}}{I} \\ r &= \frac{120 \text{ V} - 0}{40.0 \text{ A}} \\ &= \frac{120 \text{ V}}{40.0 \text{ A}} \\ &= 3.00 \Omega\end{aligned}$$

b) operating full

$$\begin{aligned}V_{\text{back}} &= \varepsilon - Ir \\ &= 120 \text{ V} - (15.0 \text{ A})(3.00 \Omega) \\ &= 75.0 \text{ V}\end{aligned}$$

3. a) $V_{\text{back}} = \varepsilon - Ir$
 $= 120 \text{ V} - (9.0 \text{ A})(5.0 \Omega)$
 $= 75 \text{ V}$

b) no back emf

c) initial

$$V_{\text{back}} = 0$$

$$V_{\text{back}} = \varepsilon - Ir$$

$$0 = 120 \text{ V} - I(5.0 \Omega)$$

$$I = \frac{120 \text{ V}}{5.0 \Omega} = 24 \text{ A}$$

4. a) Greater current—more heat
(Remember: $P = I^2 r$)

b) Back emf has decreased

c) i) $V_{\text{back}} = \varepsilon - Ir$
 $= 120 \text{ V} - (3.6 \text{ A})(6.0 \Omega)$
 $= 98 \text{ V}$

ii) $V_{\text{back}} = \varepsilon - Ir$
 $= 120 \text{ V} - (8.4 \text{ A})(6.0 \Omega)$
 $= 70 \text{ V}$

5. 45.0 V

We can use $\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$, which tells us that the induced emf varies directly with the rate of change in the magnetic flux. The back emf is the emf induced by the motor. Therefore, if the rate of rotation is halved, the back emf is halved.

6. a) initial $V_{\text{back}} = 0$

$$V_{\text{back}} = \varepsilon - Ir$$

$$r = \frac{\varepsilon - V_{\text{back}}}{I}$$

$$r = \frac{120 \text{ V} - 0}{10.0 \text{ A}} = \frac{120 \text{ V}}{10.0 \text{ A}} = 12.0 \Omega$$

b) operating full speed

$$V_{\text{back}} = \varepsilon - Ir$$

$$= 120 \text{ V} - (3.0 \text{ A})(12.0 \Omega)$$

$$= 84 \text{ V}$$

Lesson 15—EMF Continued

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\varepsilon = B_{\perp} lv$
 $= (0.75 \text{ T})(0.35 \text{ m})(1.5 \text{ m/s})$
 $= 0.39 \text{ V}$

2. $\varepsilon = B_{\perp} lv$
 $B_{\perp} = \frac{\varepsilon}{lv}$
 $= \frac{0.075 \text{ V}}{(0.28 \text{ m})(0.80 \text{ m/s})}$
 $= 0.33 \text{ T}$

3. $\varepsilon = B_{\perp} lv$
 $= (0.150 \text{ T})(0.220 \text{ m})(1.25 \text{ m/s})$
 $= 0.0413 \text{ V}$
 $\varepsilon = IR$
 $I = \frac{\varepsilon}{R}$
 $= \frac{0.0413 \text{ V}}{2.25 \Omega}$
 $= 1.83 \times 10^{-2} \text{ A}$
 counter-clockwise

4. a) $\varepsilon = IR$
 $= (5.6 \times 10^{-2} \text{ A})(1.5 \Omega)$
 $= 8.4 \times 10^{-2} \text{ V}$

Again

$$\varepsilon = B_{\perp} lv$$

$$B_{\perp} = \frac{\varepsilon}{lv}$$

$$= \frac{8.4 \times 10^{-2} \text{ V}}{(15 \times 10^{-2} \text{ m})(0.95 \text{ m/s})}$$

$$= 0.59 \text{ T}$$

b) counter-clockwise

5. $\varepsilon = B_{\perp}lv$
 $= (0.950 \text{ T})(0.300 \text{ m})(1.50 \text{ m/s})$
 $= 0.428 \text{ V}$
 $\varepsilon = IR$
 $I = \frac{\varepsilon}{R}$
 $= \frac{0.428 \text{ V}}{3.25 \Omega}$
 $= 0.132 \text{ A}$
 $F_m = BIl$
 $= (0.950 \text{ T})(0.132 \text{ A})(0.300 \text{ m})$
 $= 3.75 \times 10^{-2} \text{ N}$
6. $\varepsilon = B_{\perp}lv$
 $= (4.70 \times 10^{-6} \text{ T})(6.25 \text{ m})(95.0 \text{ m/s})$
 $= 2.79 \times 10^{-3} \text{ V}$
7. $\varepsilon = B_{\perp}lv$
 $= (0.600 \text{ T})(0.300 \text{ m})(3.00 \text{ m/s})$
 $= 0.540 \text{ V}$
 $P = \frac{\varepsilon^2}{R}$
 $= \frac{(0.540 \text{ V})^2}{2.25 \Omega}$
 $= 0.130 \text{ W}$
 $P = \frac{W}{t}$
 $W = Pt$
 $= (0.130 \text{ W})(15.0 \text{ s})$
 $= 1.94 \text{ J}$
8. $\varepsilon = B_{\perp}lv$
 $= (0.75 \text{ T})(1.2 \text{ m})(2.5 \text{ m/s})$
 $= 2.25 \text{ V}$
 $\varepsilon = IR$
 $R = \frac{\varepsilon}{I}$
 $= \frac{2.25 \text{ V}}{0.45 \text{ A}}$
 $= 5.0 \Omega$
9. $\varepsilon = B_{\perp}lv$
 $= (0.95 \text{ T})(1.0 \text{ m})(3.0 \text{ m/s})$
 $= 2.85 \text{ V}$
 $P = \frac{\varepsilon^2}{R}$
 $= \frac{(2.85 \text{ V})^2}{45.0 \Omega}$
 $= 0.181 \text{ W}$

$$P = \frac{W}{t}$$

$$W = Pt$$

$$= (0.181 \text{ W})(1.0 \times 10^{-2} \text{ s})$$

$$= 1.8 \times 10^{-3} \text{ J}$$

10. $\varepsilon = IR$
 $= (5.2 \times 10^{-2} \text{ A})(2.9 \Omega)$
 $= 0.151 \text{ V}$
 $\varepsilon = B_{\perp}lv$
 $v = \frac{\varepsilon}{B_{\perp}l}$
 $= \frac{0.151 \text{ V}}{(0.65 \text{ T})(0.50 \text{ m})}$
 $= 0.46 \text{ m/s}$

11. a) $\varepsilon = B_{\perp}lv$
 $= (1.30 \text{ T})(0.625 \text{ m})(1.80 \text{ m/s})$
 $= 1.463 \text{ V}$
 $\varepsilon = IR$
 $I = \frac{\varepsilon}{R}$
 $= \frac{1.463 \text{ V}}{1.50 \Omega}$
 $= 0.975 \text{ A}$

b) clockwise

12. a) $\varepsilon = (B_{\perp}lv)N$
 $= (1.1 \text{ T})(0.18 \text{ m})(2.7 \text{ m/s})(5)$
 $= 2.67 \text{ V}$
 $\varepsilon = IR$
 $I = \frac{\varepsilon}{R}$
 $= \frac{2.67 \text{ V}}{3.5 \Omega}$
 $= 0.76 \text{ A}$

b) clockwise

Lesson 16—Transformers (Induction Coil)

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

$$1. \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$\frac{0.95 \text{ A}}{0.25 \text{ A}} = \frac{1.0 \times 10^3}{N_s}$$

$$N_s = 2.6 \times 10^2$$

$$2. \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$\frac{5.00 \text{ A}}{I_p} = \frac{1.20 \times 10^3}{1.50 \times 10^2}$$

$$I_p = 0.625 \text{ A}$$

$$3. \text{ a) } P_p = I_p V_p$$

$$I_p = \frac{P_p}{V_p}$$

$$= \frac{2.4 \times 10^3 \text{ W}}{3.6 \times 10^3 \text{ V}}$$

$$= 0.67 \text{ A}$$

$$\text{b) } P_s = I_s V_s$$

$$I_s = \frac{P_s}{V_s}$$

$$= \frac{2.4 \times 10^3 \text{ W}}{1.20 \times 10^2 \text{ V}}$$

$$= 2.0 \times 10^1 \text{ A}$$

$$4. \quad \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$\frac{1.20 \times 10^2 \text{ V}}{V_s} = \frac{1.50 \times 10^2}{25}$$

$$V_s = 20 \text{ V}$$

$$V_s = I_s R_s$$

$$I_s = \frac{V_s}{R_s}$$

$$= \frac{20 \text{ V}}{75 \Omega}$$

$$= 0.27 \text{ A}$$

$$\frac{I_s}{I_p} = \frac{V_p}{V_s}$$

$$\frac{0.267 \text{ A}}{I_p} = \frac{1.20 \times 10^2 \text{ V}}{20 \text{ V}}$$

$$I_p = 4.4 \times 10^{-2} \text{ A}$$

$$5. \quad P_p = I_p V_p$$

$$= (3.0 \text{ A})(1.20 \times 10^2 \text{ V})$$

$$= 3.6 \times 10^2 \text{ W}$$

Power delivered to the secondary coil is also $3.6 \times 10^2 \text{ W}$
(same in both cells)

$$6. \quad P_p = I_p V_p$$

$$I_p = \frac{P_p}{V_p}$$

$$= \frac{5.0 \times 10^1 \text{ W}}{1.20 \times 10^2 \text{ V}}$$

$$= 0.42 \text{ A}$$

$$7. \quad P_p = I_p V_p$$

$$= (1.0 \text{ A})(1.20 \times 10^2 \text{ V})$$

$$= 1.2 \times 10^2 \text{ W}$$

The power in the secondary coil is also $1.2 \times 10^2 \text{ W}$
(same in both cells)

$$8. \quad P_p = I_p V_p$$

$$I_p = \frac{P_p}{V_p}$$

$$= \frac{1.0 \times 10^2 \text{ W}}{9.0 \text{ V}}$$

$$= 11.1 \text{ A}$$

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$\frac{0.65 \text{ A}}{11.1 \text{ A}} = \frac{N_p}{1.5 \times 10^3}$$

$$N_p = 88$$

Practice Test

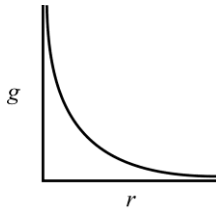
ANSWERS AND SOLUTIONS

1. $F_g = \frac{Gm_1m_2}{r^2}$ or $F_g \propto \frac{1}{r^2}$ (r is doubled, because the final distance from Earth's centre is $2r$)

$$\begin{aligned}\therefore F_g &\propto \frac{1}{4} \\ F'_g &= \frac{2.0 \text{ N}}{4} \\ &= 0.50 \text{ N}\end{aligned}$$

2. $g = \frac{GM}{r^2}$
 $\therefore g \propto \frac{1}{r^2}$

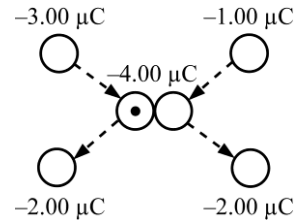
This is an inverse square relationship. An inverse square relationship is graphed as:



3. The equation $E_p = mgh$ cannot be used in the case of an object separated from Earth's surface by a large distance because the magnitude of g is not constant— g decreases as the distance from the earth increases.

4. $E_p = -\frac{Gm_1m_2}{r}$
- $$\begin{aligned}&= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})(1.25 \times 10^3 \text{ kg})}{4.00 \times 10^6 \text{ m} + 6.38 \times 10^6 \text{ m}} \\ &= -4.80 \times 10^{10} \text{ J}\end{aligned}$$

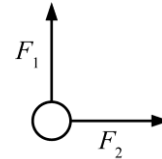
5.



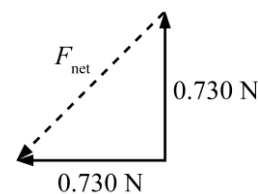
$$\begin{aligned}F_e &= \frac{kq_1q_2}{r^2} \\ &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.00 \times 10^{-6} \text{ C})}{(2.00 \times 10^{-2} \text{ m})^2} \\ &= 90.0 \text{ N}\end{aligned}$$

The force between the pith balls is 90.0 N.

6.



$$\begin{aligned}F_2 = F_1 &= \frac{kq_1q_2}{r^2} \\ &= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(3.00 \times 10^{-6} \text{ C})^2}{(0.333 \text{ m})^2} \\ &= 0.730 \text{ N}\end{aligned}$$



$$\begin{aligned}F_{\text{net}} &= \sqrt{F_1^2 + F_2^2} \\ &= \sqrt{(0.730 \text{ N})^2 + (0.730 \text{ N})^2} \\ &= 1.03 \text{ N}\end{aligned}$$

The net electric force on sphere B is 1.03 N.

7. $F_e = \frac{kq_1q_2}{r^2}$ or $F_e \propto \frac{1}{r^2}$

r decreases by $\frac{1}{3}$

$\therefore r^2$ changes by 0.111

F_e changes by 9 times

New force = 9.0F

The new force between the two point charges is 9.0F.

8. $F_{\text{net}} = ma$
 $a \propto F_{\text{net}}$

In this case the net force is the electric force.

$$F_{\text{net}} = F_e = \frac{kq_1q_2}{r^2}$$

$$F_{\text{net}} \propto \frac{1}{r^2}$$

This expression tells that when the distance increases 3 times, the force becomes $\frac{1}{9}$.

\therefore if the force decreases to $\frac{1}{9}$, then according to

the expression, $a \propto F_{\text{net}}$ the acceleration is also $\frac{1}{9}$.

New acceleration is

$$\left(1.00 \times 10^{10} \text{ m/s}^2\right) \frac{1}{9}$$

$$= 1.11 \times 10^9 \text{ m/s}^2$$

The initial acceleration of sphere A was $1.11 \times 10^9 \text{ m/s}^2$.

9. $F_{\text{net}} = ma$
 $a \propto F_{\text{net}}$

In this case the net force is the electric force.

$$F_{\text{net}} = F_e = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) (3.00 \times 10^{-6} \text{ C})^2}{(2.00 \times 10^{-2} \text{ m})^2}$$

$$= 2.03 \times 10^2 \text{ N}$$

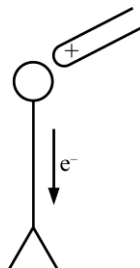
$$a = \frac{F_{\text{net}}}{m}$$

$$= \frac{2.02 \times 10^2 \text{ N}}{5.00 \times 10^{-4} \text{ kg}}$$

$$= 4.05 \times 10^5 \text{ m/s}^2$$

The initial acceleration of sphere A was $4.05 \times 10^5 \text{ m/s}^2$.

10.



The negatively charged rod will induce the electrons in the electroscope to move away from the rod, i.e., move from the head to the leaves. Therefore, the head of the electroscope will be positive.

11.

$$\Delta E_k + \Delta E_p = 0$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + qV = 0$$

$$\frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = -qV$$

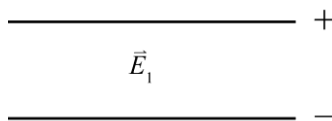
$$\frac{1}{2}(6.65 \times 10^{-27} \text{ kg})v^2 = -(3.20 \times 10^{-19} \text{ C})(-2.00 \times 10^3 \text{ V})$$

$$v^2 = \frac{-2(3.20 \times 10^{-19} \text{ C})(-2.00 \times 10^3 \text{ V})}{(6.65 \times 10^{-27} \text{ kg})}$$

$$v = 4.39 \times 10^5 \text{ m/s}$$

The maximum speed of the alpha particle is $4.39 \times 10^5 \text{ m/s}$.

12.



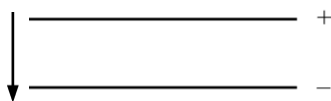
The electron will accelerate in the direction of the electric force which is away from the negative plate (opposite direction to the electric field).

13. Electric potential is the potential electric energy per unit of charge.

$$14. F_e = \frac{kq_1q_2}{r^2}$$

According to this equation, as the distance between the charges decreases, the electric force increases. If the electric force increases, the electric potential energy increases.

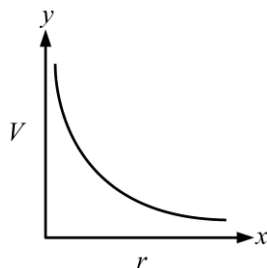
15. The direction of an electric field is defined in terms of the direction of the electric force on a positive test charge.
16. The electric field is away from a positive object toward a negative object. Therefore, the sketch should show that the field is directed down toward the negative plate.



$$17. V = \frac{kq_1}{r}$$

$$V \propto \frac{1}{r}$$

The magnitude of the electric potential (V) as a function of the distance (r) from a point charge is an inverse relationship. This can be shown as a curve approaching the x -axis as it moves away from the y -axis.



18. Lenz's law states that the induced field will oppose the original change of flux of the magnet. In other words, the induced magnetic field inside the solenoid must oppose the motion of the magnet. That means that the right side of the solenoid is the induced magnetic north. Use the right-hand rule to determine the direction of the current within the circuit. Point your thumb of your right hand to the right and curl your fingers in the direction of the electric current. For this to happen, the current flows from A to B through G.

$$19. \frac{N_p}{N_s} = \frac{I_s}{I_p}$$

$$\frac{1}{10} = \frac{I_s}{I_p}$$

$$I_p = 10I_s$$

This shows that the current in the primary coil is 10 times greater than the current in the secondary coil.

20. A "step-up" transformer is used to increase (hence *step up*) the voltage. But, if the transformer increases the voltage, the current must decrease. Remember the law of conservation of energy. The power in each coil must be the same. Therefore, a step-up transformer is used to increase voltage and decrease current.
21. A transformer makes use of the principle of electromagnetic induction. This means that there must be a change in the magnetic field inside the secondary coil. In order to change this magnetic field, a switch is placed in the primary circuit.
22. Lenz's law is an example of conservation of energy.
23. Use the right-hand rule. The opposing magnetic force must be in the opposite direction to its motion (i.e., to the left). Place your right palm to face the left along the conductor, and direct your fingers into the page (in the direction of the magnetic field). Your thumb will now point along the conductor from end B toward end A. This is the direction of the induced electric current.

$$\mathcal{E} = B_{\perp}lv \quad \text{and} \quad I = \frac{\mathcal{E}}{R}$$

From these mathematical relationships, if the velocity decreases, the current also decreases.

Therefore, the current within rod AB will be directed toward end A, and it will decrease as the rod passes through the magnetic field.

24. Note that the alpha particle is moving parallel to the magnetic field. Therefore, there is no magnetic force acting upon the alpha particle.
- $$F_m = qvB\sin\theta$$

25. Use the right-hand rule. Point your thumb east. Point your fingers north. Your palm should be directed up. The magnetic force on the proton is directed upward.
26. Lenz's law states that the induced current flows in such a way that the magnetic field inside the coil opposes the magnetic field of the magnet. Using the right-hand rule, curl the fingers on your right hand in a counterclockwise direction. Your right thumb will thus point upward. The magnetic field inside of the loop is directed upward.
27. Use the left-hand rule (electrons). Point your thumb north. When your left-hand fingers curl above your thumb, they should be pointing west. Therefore, the magnetic field above the conductor will be pointed west.
28. $\varepsilon = B_{\perp}lv$
If the period (time) of the rotation decreases, the loop will be rotating at a higher speed. From the above equation, as the speed increases, so does the emf. Therefore, if the period of rotation was cut in half, the induced voltage would increase as well.
29. Generators convert mechanical energy into electric energy.
30. Motors convert electric energy into mechanical energy.
31. Both transformers and generators operate on the principle of electromagnetic induction.

32.

$$F_m = qvB_{\perp}$$

$$F_m \propto v$$

and F_m does not depend on the mass.

\therefore if the speed is halved, you will halve the magnetic force.

$$\therefore F_m = \frac{F}{2}$$

33. Apply Lenz's law here. Use the right-hand rule for conventional current.

In this scenario, as the north pole of the magnet moves down into the loop, the magnetic field within the loop will point upward out of the loop. Point your right thumb in this direction and curl your fingers to identify the direction of the current. Your fingers should be pointing in a counterclockwise direction.

However, when the magnet is pulled out of the loop, the motion of the magnet, the direction of the magnetic field, and the direction of the current all change direction. Therefore, the direction of the current as the magnet is pulled out of the loop is in the clockwise direction.

Therefore, as viewed from above the loop, the direction of the induced current in the loop would first be counterclockwise and then clockwise.

34. J.J. Thomson determined the charge-to-mass ratio of the electron by balancing the electric and magnetic forces.
35. A cathode ray is a beam of electrons.
36. Statements **i)** and **ii)** are true. Cathode rays are electrons, and charged particles can be deflected by electric and magnetic fields. Therefore, cathode rays can be deflected by both electric and magnetic fields.

Statement **iii)** is incorrect. Only electromagnetic radiation travels at 3.00×10^8 m/s.

Electromagnetic radiation does not consist of charged particles as cathode rays do.

37. The greater the deflection, the smaller the radius. Therefore, in order to increase the deflection, determine which variables in the following equation will result in a smaller radius.

$$F_m = F_c$$

$$qvB_{\perp} = \frac{mv^2}{R}$$

$$R = \frac{mv}{qB_{\perp}}$$

The radius depends on the mass, speed, charge, and magnetic field strength. In order to decrease the radius, increase the charge of the particle, increase the strength of the magnetic field, or decrease either the mass or speed of the particle.

38. If electrons pass through superimposed electric and magnetic fields without deflection, that means the electric and magnetic forces acting on the electrons must be equal but in opposite directions.

$$F_e = F_m$$

$$q|\vec{E}| = qvB_{\perp}$$

$$v = \frac{|\vec{E}|}{B_{\perp}}$$

Therefore, the speed of the electrons can be determined from the strengths of the superimposed fields.

MOMENTUM

Lesson 1—Momentum

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\vec{p} = m\vec{v}$
 $= (4.0 \text{ kg})(12.0 \text{ m/s east})$
 $= 48 \text{ kg} \cdot \text{m/s east}$

2. $\vec{p} = m\vec{v}$
 $\vec{v} = \frac{\vec{p}}{m}$
 $= \frac{25.0 \text{ kg} \cdot \text{m/s west}}{5.0 \text{ kg}}$
 $= 5.0 \text{ m/s west}$

3. $\vec{p} = m\vec{v}$
 $m = \frac{\vec{p}}{\vec{v}}$
 $= \frac{36.0 \text{ kg} \cdot \text{m/s south}}{8.0 \text{ m/s south}}$
 $= 4.5 \text{ kg}$

4. $\vec{p} = m\vec{v}$
 $m = \frac{\vec{p}}{\vec{v}}$
 $= \frac{29 \text{ kg} \cdot \text{m/s east}}{2.0 \text{ m/s east}}$
 $= 14.5 \text{ kg}$

$$F_g = mg$$

$$= (14.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 1.4 \times 10^2 \text{ N}$$

5. $F_g = mg$
 $m = \frac{F_g}{g}$
 $= \frac{6.6 \text{ N}}{9.81 \text{ m/s}^2}$
 $= 0.673 \text{ kg}$

$$\vec{p} = m\vec{v}$$

$$= (0.673 \text{ kg})(3.0 \text{ m/s north})$$

$$= 2.0 \text{ kg} \cdot \text{m/s north}$$

6. $\vec{v} = \frac{\vec{d}}{t}$
 $= \frac{2.6 \text{ m west}}{1.1 \text{ s}}$
 $= 2.36 \text{ m/s west}$

$$\vec{p} = m\vec{v}$$

$$= (7.0 \text{ kg})(2.36 \text{ m/s west})$$

$$= 17 \text{ kg} \cdot \text{m/s west}$$

7. $\vec{a} = \frac{\vec{v} - \vec{v}_0}{t}$
 $-9.81 \text{ m/s}^2 = \frac{\vec{v} - 0}{0.25 \text{ s}}$
 $\vec{v} = -2.45 \text{ m/s}$

$$\vec{p} = m\vec{v}$$

$$= (5.0 \text{ kg})(-2.45 \text{ m/s})$$

$$= -12 \text{ kg} \cdot \text{m/s}$$

$$= 12 \text{ kg} \cdot \text{m/s down}$$

8. Impulse = $\vec{F}_{\text{net}} t$
 $= (17.0 \text{ N east})(2.5 \times 10^2 \text{ s})$
 $= 4.3 \times 10^3 \text{ N} \cdot \text{s east}$

9. Impulse = $\vec{F}_{\text{net}} t$
 $t = \frac{\text{Impulse}}{\vec{F}_{\text{net}}}$
 $= \frac{7.00 \text{ N} \cdot \text{s west}}{11.2 \text{ N west}}$
 $= 0.625 \text{ s}$

10. Consider north direction as positive

$$\vec{F}_{\text{net}} t = m\Delta\vec{v}$$

$$\vec{F}_{\text{net}} (2.60 \text{ s}) = (26.3 \text{ kg})(-21.0 \text{ m/s})$$

$$\vec{F}_{\text{net}} = -212 \text{ N or } 212 \text{ N south}$$

11. Magnitude of impulse

$$\begin{aligned}\vec{F}_{\text{net}} t &= m\Delta\vec{v} \\ (31.6 \text{ N})t &= (15.0 \text{ kg})(10.0 \text{ m/s}) \\ t &= 4.75 \text{ s}\end{aligned}$$

- 12.
- $\vec{F}_{\text{net}} t = m\Delta\vec{v}$

$$\begin{aligned}\text{Therefore, } m\Delta\vec{v} &= (25.0 \text{ N})(7.20 \times 10^{-1} \text{ s}) \\ &= 18.0 \text{ N} \cdot \text{s north}\end{aligned}$$

13. Impulse =
- $m\Delta\vec{v}$

$$\begin{aligned}&= (5.00 \text{ kg})(15.0 \text{ m/s east}) \\ &= 75.0 \text{ kg} \cdot \text{m/s east}\end{aligned}$$

- 14.

$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\vec{d}}{t} \\ &= \frac{26.3 \text{ m west}}{3.2 \text{ s}} \\ &= 8.22 \text{ m/s west}\end{aligned}$$

$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\vec{v} - \vec{v}_0}{2} \\ 8.22 \text{ m/s west} &= \frac{\vec{v} - 0}{2} \\ \vec{v} &= 16.4 \text{ m/s west}\end{aligned}$$

$$\begin{aligned}\Delta\vec{p} &= m\Delta\vec{v} \\ &= (11.0 \text{ kg})(16.4 \text{ m/s west}) \\ &= 181 \text{ kg} \cdot \text{m/s west}\end{aligned}$$

15. The magnitude of momentum

$$\begin{aligned}p &= mv \\ v &= \frac{p}{m} \\ &= \frac{6.0 \text{ kg} \cdot \text{m/s}}{3.0 \text{ kg}} \\ &= 2.0 \text{ m/s} \\ v^2 &= v_0^2 + 2ad \\ (2.0 \text{ m/s})^2 &= 2(9.81 \text{ m/s}^2)d \\ d &= 0.20 \text{ m}\end{aligned}$$

The object has fallen 0.20 m when its momentum is 6.0 kg m/s down.

$$\begin{aligned}16. \quad \vec{p}_1 &= m\vec{v}_1 \\ &= (1.0 \text{ kg})(-2.0 \text{ m/s}) \\ &= -2.0 \text{ kg} \cdot \text{m/s}\end{aligned}$$

$$\begin{aligned}\vec{p}_2 &= m\vec{v}_2 \\ &= (1.0 \text{ kg})(1.6 \text{ m/s}) \\ &= 1.6 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\Delta\vec{p} &= \vec{p}_2 - \vec{p}_1 \\ &= 1.6 \text{ kg} \cdot \text{m/s} - (-2.0 \text{ kg} \cdot \text{m/s}) \\ &= 3.6 \text{ kg} \cdot \text{m/s upward}\end{aligned}$$

17. Consider up as positive,

$$\begin{aligned}\vec{F}_{\text{net}} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_{\text{net}}}{m} \\ &= \frac{1.5 \times 10^5 \text{ N}}{9.5 \times 10^3 \text{ kg}} \\ &= 15.8 \text{ m/s}^2 \\ \vec{a} &= \frac{\vec{v} - \vec{v}_0}{t} \\ 15.8 \text{ m/s}^2 &= \frac{\vec{v} - 0}{15 \text{ s}} \\ \vec{v} &= 2.4 \times 10^2 \text{ m/s}\end{aligned}$$

or

$$\begin{aligned}\vec{F}_{\text{net}} t &= m\Delta\vec{v} \\ (1.5 \times 10^5 \text{ N})(15 \text{ s}) &= (9.5 \times 10^3 \text{ kg})\vec{v} \\ \vec{v} &= 2.4 \times 10^2 \text{ m/s}\end{aligned}$$

$$\begin{aligned}18. \quad \vec{F}_{\text{net}} t &= m\Delta\vec{v} \\ \vec{F}_{\text{net}} (0.75 \text{ s}) &= (5.4 \text{ kg})(3.0 \text{ m/s east}) \\ \vec{F}_{\text{net}} &= 22 \text{ Neast}\end{aligned}$$

19. The magnitude of impulse

$$\begin{aligned}F_{\text{net}} t &= m\Delta v \\ (225 \text{ N})t &= (1.0 \times 10^3 \text{ kg})(5.0 \text{ m/s} - 2.0 \text{ m/s}) \\ t &= 13 \text{ s}\end{aligned}$$

$$\begin{aligned}20. \quad \vec{F}_{\text{net}} t &= m\Delta\vec{v} \\ (95 \text{ N north})(1.65 \text{ s}) &= (15 \text{ kg})(\Delta v) \\ \Delta v &= 1.0 \times 10^1 \text{ m/s north}\end{aligned}$$

Lesson 2—Collisions

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. before collision



Consider right as positive and left as negative

$$\begin{aligned}m_1 &= 30.0 \text{ kg} & m_2 &= 20.0 \text{ kg} \\ \vec{v}_1 &= 1.00 \text{ m/s} & \vec{v}_2 &= -5.00 \text{ m/s} \\ \vec{p}_1 &= 30.0 \text{ kg} \cdot \text{m/s} & \vec{p}_2 &= -100 \text{ kg} \cdot \text{m/s}\end{aligned}$$

$$\therefore \vec{p}_{\text{sys(before)}} = \vec{p}_1 + \vec{p}_2 = -70.0 \text{ kg} \cdot \text{m/s}$$

after collision



$$\begin{aligned}
 m_1 &= 30.0 \text{ kg} & m_2 &= 20.0 \text{ kg} \\
 \vec{v}'_1 &=? & \vec{v}'_2 &= -1.25 \text{ m/s} \\
 \vec{p}'_1 &=? & \vec{p}'_2 &= -25.0 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

$$\vec{p}_{\text{sys(after)}} = \vec{p}_{\text{sys(before)}} = -70.0 \text{ kg} \cdot \text{m/s}$$

$$\begin{aligned}
 \vec{p}_{\text{sys(after)}} &= \vec{p}'_1 + \vec{p}'_2 = -70.0 \text{ kg} \cdot \text{m/s} \\
 \Rightarrow \vec{p}'_1 &= -70.0 \text{ kg} \cdot \text{m/s} - (-25.0 \text{ kg} \cdot \text{m/s}) \\
 \therefore \vec{p}'_1 &= -45.0 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

$$\begin{aligned}
 \vec{p}'_1 &= m_1 \vec{v}'_1 \\
 \vec{v}'_1 &= \frac{\vec{p}'_1}{m_1} \\
 &= \frac{-45.0 \text{ kg} \cdot \text{m/s}}{30.0 \text{ kg}} \\
 &= -1.50 \text{ m/s}
 \end{aligned}$$

The velocity of the 30.0 kg ball after the collision is 1.50 m/s left

2. before collision



Consider east as positive

$$\begin{aligned}
 m_1 &= 4.50 \times 10^3 \text{ kg} & m_2 &= 6.50 \times 10^3 \text{ kg} \\
 \vec{v}_1 &= 5.0 \text{ m/s} & \vec{v}_2 &= 0 \text{ m/s} \\
 \vec{p}_1 &= 2.25 \times 10^4 \text{ kg} \cdot \text{m/s} & \vec{p}_2 &= 0 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

$$\therefore \vec{p}_{\text{before}} = \vec{p}_1 + \vec{p}_2 = 2.25 \times 10^4 \text{ kg} \cdot \text{m/s}$$

after collision



$$\begin{aligned}
 m &= m_1 + m_2 = 1.10 \times 10^4 \text{ kg} \\
 \vec{v} &=? \\
 \vec{p} &= \vec{p}_{\text{after}} = 2.25 \times 10^4 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

$$\begin{aligned}
 \vec{p} &= m\vec{v} \\
 \vec{v} &= \frac{\vec{p}}{m} \\
 &= \frac{2.25 \times 10^4 \text{ kg} \cdot \text{m/s}}{1.10 \times 10^4 \text{ kg}} \\
 &= 2.1 \text{ m/s east}
 \end{aligned}$$

3. before collision



$$\begin{aligned}
 m_1 &= 925 \text{ kg} & m_2 &=? \\
 \vec{v}_1 &= 18.0 \text{ m/s} & \vec{v}_2 &= 0 \text{ m/s} \\
 \vec{p}_1 &= 1.67 \times 10^4 \text{ kg} \cdot \text{m/s} & \vec{p}_2 &= 0 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

$$\therefore \vec{p}_{\text{before}} = \vec{p}_1 + \vec{p}_2 = 1.67 \times 10^4 \text{ kg} \cdot \text{m/s}$$

after collision



$$\begin{aligned}
 m &=? \\
 \vec{v} &= 6.50 \text{ m/s} \\
 \vec{p} &= \vec{p}_{\text{after}} \\
 &= 1.67 \times 10^4 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

$$\begin{aligned}
 m &= \frac{\vec{p}}{\vec{v}} \\
 &= \frac{1.67 \times 10^4 \text{ kg} \cdot \text{m/s}}{6.50 \text{ m/s}} \\
 &= 2.57 \times 10^3 \text{ kg} \\
 m_2 &= 2.57 \times 10^3 \text{ kg} = 925 \text{ kg} \\
 &= 1.65 \times 10^3 \text{ kg}
 \end{aligned}$$

4. before collision



$$\begin{aligned}
 m_1 &= 0.050 \text{ kg} & m_2 &= 7.00 \text{ kg} \\
 \vec{v}_1 &=? & \vec{v}_2 &= 0 \text{ m/s} \\
 \vec{p}_1 &=? & \vec{p}_2 &= 0 \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

after collision



bullet and block

$$\begin{aligned}
 m &= m_1 + m_2 = 7.05 \text{ kg} \\
 \vec{v} &= 5.00 \text{ m/s} \\
 \vec{p} &= \vec{p}_{\text{after}} = 35.25 \text{ kg} \cdot \text{m/s} \\
 \vec{p}_{\text{before}} &= \vec{p}_{\text{after}} = 35.25 \text{ kg} \cdot \text{m/s} \\
 \vec{p}_1 &= \vec{p}_{\text{before}} = 35.25 \text{ kg} \cdot \text{m/s} \\
 \vec{v}_1 &= \frac{\vec{p}_1}{m_1} \\
 &= \frac{35.25 \text{ kg} \cdot \text{m/s}}{0.050 \text{ kg}} \\
 &= 705 \text{ m/s}
 \end{aligned}$$

5. before collision



Consider right as positive and left as negative

$$m_1 = 40.0 \text{ g}$$

$$\vec{v}_1 = 9.00 \text{ m/s}$$

$$\vec{p}_1 = 3.60 \times 10^2 \text{ g} \cdot \text{m/s}$$

$$m_2 = 55.0 \text{ g}$$

$$\vec{v}_2 = -6.00 \text{ m/s}$$

$$\vec{p}_2 = -3.30 \times 10^2 \text{ g} \cdot \text{m/s}$$

$$\therefore \vec{p}_{\text{before}} = \vec{p}_1 + \vec{p}_2 = 3.00 \times 10^1 \text{ g} \cdot \text{m/s}$$

after collision



$$m = m_1 + m_2 = 95.0 \text{ kg}$$

$$\vec{v} = ?$$

$$\vec{p} = \vec{p}_{\text{after}} = \vec{p}_{\text{before}} = 3.00 \times 10^1 \text{ g} \cdot \text{m/s}$$

$$\begin{aligned} \vec{v} &= \frac{\vec{p}}{m} \\ &= \frac{3.00 \times 10^1 \text{ g} \cdot \text{m/s}}{95.0 \text{ g}} \\ &= 0.316 \text{ m/s right} \end{aligned}$$

6. before collision



$$m = 76 \text{ kg}$$

$$v = 0$$

$$p = 0$$

$$\vec{p}_{\text{before}} = 0$$

after collision



Consider left as negative

$$m_o = 0.20 \text{ kg}$$

$$\vec{v}_o = -22 \text{ m/s}$$

$$\vec{p}_o = -4.4 \text{ kg} \cdot \text{m/s}$$

$$m_s = 76 \text{ kg}$$

$$\vec{v}_s = ?$$

$$\vec{p}_s = ?$$

$$\vec{p}_{\text{after}} = \vec{p}_{\text{before}} = 0$$

$$\begin{aligned} \vec{p}_{\text{after}} &= \vec{p}_o + \vec{p}_s = 0 \\ \vec{p}_s &= 4.4 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$\begin{aligned} \vec{v}_s &= \frac{\vec{p}_s}{m} \\ &= \frac{4.4 \text{ kg} \cdot \text{m/s}}{76 \text{ kg}} \\ &= 0.058 \text{ m/s right} \end{aligned}$$

7. before collision



$$m = 1.13 \times 10^3 \text{ kg}$$

$$v = 0$$

$$p = 0$$

$$\vec{p}_{\text{before}} = 0$$

after collision



Consider east as positive

$$m_L = 1.1 \times 10^3 \text{ kg}$$

$$\vec{v}_L = ?$$

$$\vec{p}_L = ?$$

$$m_P = 25 \text{ kg}$$

$$\vec{v}_P = 325 \text{ m/s}$$

$$\vec{p}_P = 8.13 \times 10^3 \text{ kg} \cdot \text{m/s}$$

$$\vec{p}_{\text{after}} = \vec{p}_{\text{before}} = 0$$

$$\vec{p}_{\text{after}} = \vec{p}_L + \vec{p}_P = 0$$

$$\vec{p}_L = -8.13 \times 10^3 \text{ kg} \cdot \text{m/s}$$

$$\begin{aligned} \vec{v}_L &= \frac{p_L}{m_L} \\ &= \frac{-8.13 \times 10^3 \text{ kg} \cdot \text{m/s}}{1.1 \times 10^3 \text{ kg}} \\ &= -7.4 \text{ m/s} \\ &= 7.4 \text{ m/s west} \end{aligned}$$

8. before collision



$$m = ?$$

$$v = 0$$

$$p = 0$$

$$\vec{p}_{\text{before}} = 0$$

after collision



Consider right as positive

$$\begin{aligned} m_V &= ? & m_G &= 4.5 \times 10^2 \text{ kg} \\ \vec{v}_V &= -45 \text{ m/s} & \vec{v}_G &= 1.4 \times 10^3 \text{ m/s} \\ \vec{p}_V &= ? & \vec{p}_G &= 6.3 \times 10^5 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$\vec{p}_{\text{after}} = \vec{p}_{\text{before}} = 0$$

$$\vec{p}_{\text{after}} = \vec{p}_V + \vec{p}_G = 0$$

$$\vec{p}_V = -6.3 \times 10^5 \text{ kg} \cdot \text{m/s}$$

$$\begin{aligned} m_V &= \frac{\vec{p}_V}{\vec{v}_V} \\ &= \frac{-6.3 \times 10^5 \text{ kg} \cdot \text{m/s}}{-45 \text{ m/s}} \\ &= 1.4 \times 10^4 \text{ kg} \end{aligned}$$

9. before collision



$$m = 7.0 \text{ kg}$$

$$v = 0$$

$$p = 0$$

$$\vec{p}_{\text{before}} = 0$$

after collision



Consider right as positive

$$\begin{aligned} m_B &= 5.0 \text{ kg} & m_A &= 2.0 \text{ kg} \\ \vec{v}_B &= ? & \vec{v}_A &= 10.0 \text{ m/s} \\ \vec{p}_B &= ? & \vec{p}_A &= 20.0 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$\vec{p}_{\text{after}} = \vec{p}_{\text{before}} = 0$$

$$\vec{p}_{\text{after}} = \vec{p}_A + \vec{p}_B = 0$$

$$\vec{p}_B = -20.0 \text{ kg} \cdot \text{m/s}$$

$$\begin{aligned} \vec{v}_B &= \frac{\vec{p}_B}{m_B} \\ &= \frac{-20.0 \text{ kg} \cdot \text{m/s}}{5.0 \text{ kg}} \\ &= 4.0 \text{ m/s left} \end{aligned}$$

10. before collision



Consider north as positive

$$m_T = 1.02 \times 10^4 \text{ kg}$$

$$\vec{v}_T = 15 \text{ m/s}$$

$$\vec{p}_T = 1.53 \times 10^5 \text{ kg} \cdot \text{m/s}$$

$$m_C = 1.02 \times 10^3 \text{ kg}$$

$$\vec{v}_C = -25 \text{ m/s}$$

$$\vec{p}_C = -2.55 \times 10^4 \text{ kg} \cdot \text{m/s}$$

$$\therefore \vec{p}_{\text{before}} = \vec{p}_T + \vec{p}_C = 1.28 \times 10^5 \text{ kg} \cdot \text{m/s}$$

after collision



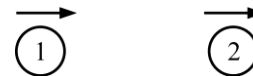
$$m = 1.12 \times 10^4 \text{ kg}$$

$$\vec{v} = ?$$

$$\vec{p} = \vec{p}_{\text{after}} = 1.28 \times 10^5 \text{ kg} \cdot \text{m/s}$$

$$\begin{aligned} \vec{v} &= \frac{\vec{p}}{m} \\ &= \frac{1.28 \times 10^5 \text{ kg} \cdot \text{m/s}}{1.12 \times 10^4 \text{ kg}} \\ &= 11 \text{ m/s north} \end{aligned}$$

11. a) before collision



Consider right as positive

$$m_1 = 225 \text{ g}$$

$$\vec{v}_1 = 30.0 \text{ cm/s}$$

$$\vec{p}_1 = 6.75 \times 10^3 \text{ g} \cdot \text{cm/s}$$

$$m_2 = 125 \text{ g}$$

$$\vec{v}_2 = 10.0 \text{ cm/s}$$

$$\vec{p}_2 = 1.25 \times 10^3 \text{ g} \cdot \text{cm/s}$$

$$\therefore \vec{p}_{\text{before}} = \vec{p}_1 + \vec{p}_2 = 8.00 \times 10^3 \text{ g} \cdot \text{cm/s}$$

after collision



$$m_1 = 225 \text{ g}$$

$$m_2 = 125 \text{ g}$$

$$\vec{v}'_1 = ?$$

$$\vec{v}'_2 = 24 \text{ cm/s}$$

$$\vec{p}'_1 = ?$$

$$\vec{p}'_2 = 3.00 \times 10^3 \text{ g} \cdot \text{cm/s}$$

$$\vec{p}_{\text{sys(after)}} = \vec{p}_{\text{sys(before)}} = 8.00 \times 10^3 \text{ g} \cdot \text{cm/s}$$

$$\vec{p}'_1 = 5.00 \times 10^3 \text{ g} \cdot \text{cm/s}$$

$$\vec{v}'_1 = \frac{\vec{p}'_1}{m}$$

$$= \frac{5.00 \times 10^3 \text{ g} \cdot \text{cm/s}}{225 \text{ g}}$$

$$= 22.2 \text{ cm/s right}$$

- b) Kinetic energy calculations, unlike momentum, require converting units to 'before collision' and consider speed.

$$\textcircled{1} \qquad \textcircled{2}$$

$$m_1 = 0.225 \text{ kg} \quad m_2 = 0.125 \text{ kg}$$

$$v_1 = 0.300 \text{ m/s} \quad v_2 = 0.100 \text{ m/s}$$

after collision

$$\textcircled{1} \qquad \textcircled{2}$$

$$m_1 = 0.225 \text{ kg} \quad m_2 = 0.125 \text{ kg}$$

$$v_1' = 0.222 \text{ m/s} \quad v_2' = 0.240 \text{ m/s}$$

Kinetic energy of object 1 before

$$E_{k1} = \frac{1}{2} m_1 v_1^2$$

$$= \frac{1}{2} (0.225 \text{ kg})(0.300 \text{ m/s})^2$$

$$= 1.0125 \times 10^{-2} \text{ J}$$

Kinetic energy of object 2, before

$$E_{k2} = \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} (0.125 \text{ kg})(0.100 \text{ m/s})^2$$

$$= 6.25 \times 10^{-4} \text{ J}$$

Kinetic energy of object 1, after

$$E_{k1}' = \frac{1}{2} m_1 (v_1')^2$$

$$= \frac{1}{2} (0.225 \text{ kg})(0.222 \text{ m/s})^2$$

$$= 5.5445 \times 10^{-3} \text{ J}$$

Kinetic energy of object 2, after

$$E_{k2}' = \frac{1}{2} m_2 (v_2')^2$$

$$= \frac{1}{2} (0.125 \text{ kg})(0.240 \text{ m/s})^2$$

$$= 3.60 \times 10^{-3} \text{ J}$$

Total kinetic energy before collision

$$= 1.0125 \times 10^{-2} \text{ J} + 6.25 \times 10^{-4} \text{ J}$$

$$= 1.075 \times 10^{-2} \text{ J}$$

Total kinetic energy after collision

$$= 5.5445 \times 10^{-3} \text{ J} + 3.60 \times 10^{-3} \text{ J}$$

$$= 9.1445 \times 10^{-3} \text{ J}$$

Mechanical (kinetic) energy lost in collision

$$= 1.075 \times 10^{-2} \text{ J} - 9.1445 \times 10^{-3} \text{ J}$$

$$= 1.61 \times 10^{-3} \text{ J}$$

\therefore collision is inelastic.

12. a) before collision

$$\begin{array}{cc} \longrightarrow & \\ \textcircled{1} & \textcircled{2} \end{array}$$

Consider right as positive

$$m_1 = 10.0 \text{ g} \quad m_2 = 30.0 \text{ g}$$

$$\vec{v}_1 = 20.0 \text{ cm/s} \quad \vec{v}_2 = 0$$

$$\vec{p}_1 = 200 \text{ g} \cdot \text{cm/s} \quad \vec{p}_2 = 0$$

$$\therefore \vec{p}_{\text{before}} = \vec{p}_1 + \vec{p}_2 = 200 \text{ g} \cdot \text{cm/s}$$

after collision

$$\begin{array}{cc} \longleftarrow & \longrightarrow \\ \textcircled{1} & \textcircled{2} \end{array}$$

$$m_1 = 10.0 \text{ g} \quad m_2 = 30 \text{ g}$$

$$\vec{v}_1' = -6.0 \text{ cm/s} \quad \vec{v}_2' = ?$$

$$\vec{p}_1' = -60 \text{ g} \cdot \text{cm/s} \quad \vec{p}_2' = ?$$

$$\vec{p}_{\text{sys(after)}} = \vec{p}_{\text{sys(before)}} = 200 \text{ g} \cdot \text{cm/s}$$

$$\vec{p}_2' = 200 \text{ g} \cdot \text{cm/s} - (-60 \text{ g} \cdot \text{cm/s})$$

$$= 260 \text{ g} \cdot \text{cm/s}$$

$$\vec{v}_2' = \frac{\vec{p}_2'}{m}$$

$$= \frac{260 \text{ g} \cdot \text{cm/s}}{30.0 \text{ g}}$$

$$= 8.67 \text{ cm/s right}$$

- b) Kinetic energy calculations, unlike momentum, require converting units to standard units.

before collision

$$\begin{array}{cc} \textcircled{1} & \textcircled{2} \\ m_1 = 0.0100 \text{ kg} & m_2 = 0.0300 \text{ kg} \\ v_1 = 0.200 \text{ m/s} & v_2 = 0 \end{array}$$

after collision

$$\begin{array}{cc} \textcircled{1} & \textcircled{2} \\ m_1 = 0.0100 \text{ kg} & m_2 = 0.0300 \text{ kg} \\ = -0.060 \text{ m/s} & v_2' = 0.0867 \text{ m/s} \end{array}$$

Kinetic energy of object 1 before

$$\begin{aligned} E_{k1} &= \frac{1}{2} m_1 v_1^2 \\ &= \frac{1}{2} (0.0100 \text{ kg})(0.200 \text{ m/s})^2 \\ &= 2.00 \times 10^{-4} \text{ J} \end{aligned}$$

Kinetic energy of object 1 after

$$\begin{aligned} E'_{k1} &= \frac{1}{2} m_1 (v'_1)^2 \\ &= \frac{1}{2} (0.0100 \text{ kg})(-0.060 \text{ m/s})^2 \\ &= 1.8 \times 10^{-5} \text{ J} \end{aligned}$$

Kinetic energy of object 2 before

$$\begin{aligned} E_{k2} &= \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2} (0.0300 \text{ kg})(0)^2 \\ &= 0 \end{aligned}$$

Kinetic energy of object 2 after

$$\begin{aligned} E'_{k2} &= \frac{1}{2} m_2 (v'_2)^2 \\ &= \frac{1}{2} (0.0300 \text{ kg})(0.0867 \text{ m/s})^2 \\ &= 1.13 \times 10^{-4} \text{ J} \end{aligned}$$

Total kinetic energy before collision

$$\begin{aligned} &= 2.00 \times 10^{-4} \text{ J} + 0 \\ &= 2.00 \times 10^{-4} \text{ J} \end{aligned}$$

Total kinetic energy after collision is

$$\begin{aligned} &= 1.8 \times 10^{-5} \text{ J} + 1.13 \times 10^{-4} \text{ J} \\ &= 1.31 \times 10^{-4} \text{ J} \end{aligned}$$

Mechanical (kinetic) energy lost in collision

$$\begin{aligned} &= 2.00 \times 10^{-4} \text{ J} - 1.31 \times 10^{-4} \text{ J} \\ &= 6.9 \times 10^{-5} \text{ J} \end{aligned}$$

\therefore collision is inelastic.

- c) Most of this energy was converted to thermal energy—some to sound, etc.

Lesson 3—Two-Dimensional Interactions

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. before collision



$$\begin{aligned} m_1 &= 2.0 \times 10^3 \text{ kg} \\ \vec{v}_1 &= 35 \text{ km/h north} \\ \vec{p}_1 &= 7.00 \times 10^4 \text{ kg} \cdot \text{km/h north} \\ m_2 &= 1.4 \times 10^3 \text{ kg} \\ \vec{v}_2 &= 37.0 \text{ m/s west} \\ \vec{p}_2 &= 28.0 \text{ kg} \cdot \text{m/s west} \end{aligned}$$

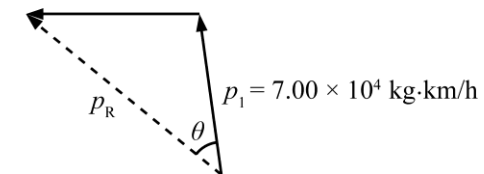
after collision



$$\begin{aligned} m &= m_1 + m_2 = 3.4 \times 10^3 \text{ kg} \\ \vec{v} &= ? \\ \vec{p} &= ? \end{aligned}$$

First find out the momentum of the system before collision ($\vec{p}_{\text{sys(before)}}$)

$$p_2 = 5.18 \times 10^4 \text{ kg} \cdot \text{km/h}$$



The magnitude of the resultant momentum is

$$\begin{aligned} p_R &= \sqrt{p_1^2 + p_2^2} \\ &= \sqrt{(7.00 \times 10^4 \text{ kg} \cdot \text{km/h})^2 + (5.18 \times 10^4 \text{ kg} \cdot \text{km/h})^2} \\ &= 8.71 \times 10^4 \text{ kg} \cdot \text{km/h} \end{aligned}$$

Now, find the direction

$$\begin{aligned} \tan \theta &= \frac{p_2}{p_1} \\ &= \frac{5.18 \times 10^4 \text{ kg} \cdot \text{km/h}}{7.00 \times 10^4 \text{ kg} \cdot \text{km/h}} \\ &= 0.750 \\ \theta &= 36.5^\circ \end{aligned}$$

Therefore

$$\begin{aligned}\vec{p}_{\text{sys(before)}} &= \vec{p}_R \\ &= 8.71 \times 10^4 \text{ kg} \cdot \text{km/h } 36.5^\circ \text{ W of N}\end{aligned}$$

Again $\vec{p} = \vec{p}_{\text{sys(after)}} = \vec{p}_{\text{sys(before)}} = \vec{p}_R$

$$\begin{aligned}\vec{v} &= \frac{\vec{p}}{m} \\ &= \frac{8.71 \times 10^4 \text{ kg} \cdot \text{km/h } 36.5^\circ \text{ W of N}}{3.4 \times 10^3 \text{ kg}} \\ &= 26 \text{ km/h } 37^\circ \text{ W of N}\end{aligned}$$

2. before collision

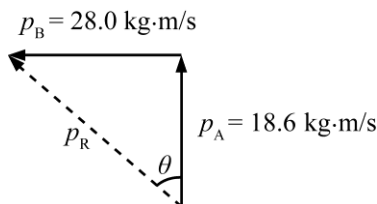


$$\begin{aligned}m_A &= 6.2 \text{ kg} \\ \vec{v}_A &= 3.0 \text{ m/s north} \\ \vec{p}_A &= 18.6 \text{ kg} \cdot \text{m/s north} \\ m_B &= 8.0 \text{ kg} \\ \vec{v}_B &= 3.5 \text{ k} \cdot \text{m/s west} \\ \vec{p}_B &= 28.0 \text{ kg} \cdot \text{m/s west}\end{aligned}$$

after collision



$$\begin{aligned}m &= m_A + m_B = 14.2 \text{ kg} \\ \vec{v} &= ? \\ \vec{p} &= ?\end{aligned}$$



$$\begin{aligned}p_R &= \sqrt{p_A^2 + p_B^2} \\ &= \sqrt{(18.6 \text{ kg} \cdot \text{m/s})^2 + (28.0 \text{ kg} \cdot \text{m/s})^2} \\ &= 33.6 \text{ kg} \cdot \text{m/s}\end{aligned}$$

$$\begin{aligned}\tan \theta &= \frac{p_B}{p_A} \\ &= \frac{28.0 \text{ kg} \cdot \text{m/s}}{18.6 \text{ kg} \cdot \text{m/s}} \\ &= 1.51 \\ \theta &= 56.4^\circ \\ &\doteq 56^\circ\end{aligned}$$

$$\vec{p} = \vec{p}_R = 33.6 \text{ kg} \cdot \text{m/s } 56^\circ \text{ W of N}$$

$$\begin{aligned}\vec{v} &= \frac{\vec{p}}{m} \\ &= \frac{33.6 \text{ kg} \cdot \text{m/s } 56^\circ \text{ W of N}}{14.2 \text{ kg}} \\ &= 24 \text{ m/s } 65^\circ \text{ W of N}\end{aligned}$$

3. before collision

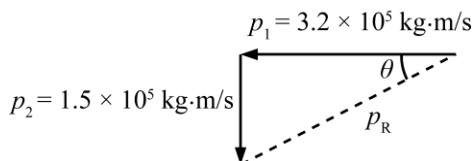


$$\begin{aligned}m_1 &= 4.0 \times 10^4 \text{ kg} \\ \vec{v}_1 &= 8.0 \text{ m/s west} \\ \vec{p}_1 &= 3.2 \times 10^5 \text{ kg} \cdot \text{m/s west} \\ m_2 &= 3.0 \times 10^4 \text{ kg} \\ \vec{v}_2 &= 5.0 \text{ m/s south} \\ \vec{p}_2 &= 1.5 \times 10^5 \text{ kg} \cdot \text{m/s south}\end{aligned}$$

after collision



$$\begin{aligned}m &= m_1 + m_2 = 7.0 \times 10^4 \text{ kg} \\ \vec{v} &= ? \\ \vec{p} &= ?\end{aligned}$$



$$\begin{aligned}p_R &= \sqrt{p_1^2 + p_2^2} \\ &= \sqrt{(3.2 \times 10^5 \text{ kg} \cdot \text{m/s})^2 + (1.5 \times 10^5 \text{ kg} \cdot \text{m/s})^2} \\ &= 3.53 \times 10^5 \text{ kg} \cdot \text{m/s}\end{aligned}$$

$$\begin{aligned}\tan \theta &= \frac{p_2}{p_1} \\ &= \frac{1.5 \times 10^5 \text{ kg} \cdot \text{m/s}}{3.2 \times 10^5 \text{ kg} \cdot \text{m/s}} \\ &= 0.469 \\ \theta &= 25^\circ\end{aligned}$$

$$\vec{p} = \vec{p}_R = 3.53 \times 10^5 \text{ kg} \cdot \text{m/s } 26^\circ \text{ S of W}$$

$$\begin{aligned}\vec{v} &= \frac{\vec{p}}{m} \\ &= \frac{3.53 \times 10^5 \text{ kg} \cdot \text{m/s } 26^\circ \text{ S of W}}{7.0 \times 10^4 \text{ kg}} \\ &= 5.0 \times 10^1 \text{ m/s } 25^\circ \text{ S of W}\end{aligned}$$

4. a) before collision



$$m_1 = 50.0 \text{ kg} \quad m_2 = 60.0 \text{ kg}$$

$$\vec{v}_1 = ? \quad \vec{v}_2 = 0$$

$$\vec{p}_1 = ? \quad \vec{p}_2 = 0$$

before collision



$$m_1 = 50.0 \text{ kg}$$

$$\vec{v}'_1 = 6.00 \text{ m/s } 50.0^\circ \text{ N of E}$$

$$\vec{p}'_1 = 300 \text{ kg} \cdot \text{m/s}$$

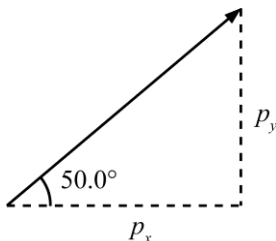
$$50.0^\circ \text{ N of E}$$

$$m_2 = 60.0 \text{ kg}$$

$$\vec{v}'_2 = 6.30 \text{ m/s } 38.0^\circ \text{ S of E}$$

$$\vec{p}'_2 = 378 \text{ kg} \cdot \text{m/s } 38.0^\circ \text{ S of E}$$

Find horizontal and vertical components of $300 \text{ kg} \cdot \text{m/s } 50.0^\circ \text{ N of E}$.



To solve this problem consider east and north direction as positive.

$$\vec{p}'_{1x} = p'_1 \cos \theta$$

$$= (300 \text{ kg} \cdot \text{m/s})(\cos 50.0^\circ)$$

$$= 193 \text{ kg} \cdot \text{m/s}$$

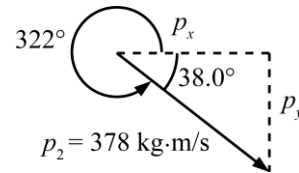
$$\vec{p}'_{1y} = p'_1 \sin \theta$$

$$= (300 \text{ kg} \cdot \text{m/s})(\sin 50.0^\circ)$$

$$= 230 \text{ kg} \cdot \text{m/s}$$

Find horizontal and vertical components of $378 \text{ kg} \cdot \text{m/s } 38.0^\circ \text{ S of E}$.

$38.0^\circ \text{ S of E} = 322^\circ$ heading counter clockwise from positive x -axis



$$\vec{p}'_{2x} = p'_2 \cos \theta$$

$$= (378 \text{ kg} \cdot \text{m/s})(\cos 322^\circ)$$

$$= 298 \text{ kg} \cdot \text{m/s}$$

$$\vec{p}'_{2y} = p'_2 \sin \theta$$

$$= (378 \text{ kg} \cdot \text{m/s})(\sin 322^\circ)$$

$$= -233 \text{ kg} \cdot \text{m/s}$$

Do the vector addition for the components of the momentum

$$\sum \vec{p}'_x = \vec{p}'_{1x} + \vec{p}'_{2x}$$

$$= 193 \text{ kg} \cdot \text{m/s} + 298 \text{ kg} \cdot \text{m/s}$$

$$= 491 \text{ kg} \cdot \text{m/s}$$

$$\sum \vec{p}'_y = \vec{p}'_{1y} + \vec{p}'_{2y}$$

$$= 230 \text{ kg} \cdot \text{m/s} + (-233 \text{ kg} \cdot \text{m/s})$$

$$= -3.00 \text{ kg} \cdot \text{m/s}$$

Now add $\sum p_x$ and $\sum p_y$ using Pythagoras theorem, to find out the magnitude of the resultant momentum.

$$p_R = \sqrt{(p_x)^2 + (p_y)^2}$$

$$= \sqrt{(491 \text{ kg} \cdot \text{m/s})^2 + (3.00 \text{ kg} \cdot \text{m/s})^2}$$

$$= 491 \text{ kg} \cdot \text{m/s}$$

$$\tan \theta = \frac{p_y}{p_x}$$

$$= \frac{3.00 \text{ kg} \cdot \text{m/s}}{491 \text{ kg} \cdot \text{m/s}}$$

$$\theta = 0.35^\circ$$

$$\doteq 0^\circ$$

$$\vec{p}_{\text{sys(after)}} = \vec{p}_R = 491 \text{ kg} \cdot \text{m/s east}$$

$$\vec{p}_{\text{sys(after)}} = \vec{p}_{\text{sys(before)}} = \vec{p}_1$$

$$\vec{p}_1 = m_1 \vec{v}_1$$

$$\vec{v}_1 = \frac{\vec{p}}{m}$$

$$= \frac{491 \text{ kg} \cdot \text{m/s east}}{50.0 \text{ kg}}$$

$$= 9.82 \text{ m/s east}$$

b) before collision



$$m_1 = 50.0 \text{ kg} \quad m_2 = 60.0 \text{ kg}$$

$$v_1 = 9.82 \text{ m/s} \quad v_2 = 0$$



$$m_1 = 50.0 \text{ kg} \quad m_2 = 60.0 \text{ kg}$$

$$v'_1 = 6.00 \text{ m/s} \quad v'_2 = 6.30 \text{ m/s}$$

NOTE: Do not include direction with the magnitude of velocity because direction is not necessary in calculating kinetic energy.

Kinetic energy of object 1 before

$$E_{k1} = \frac{1}{2} m_1 v_1^2$$

$$= \frac{1}{2} (50.0 \text{ kg})(9.82 \text{ m/s})^2$$

$$= 2.41 \times 10^3 \text{ J}$$

Kinetic energy of object 1 after

$$E'_{k1} = \frac{1}{2} m_1 (v'_1)^2$$

$$= \frac{1}{2} (50.0 \text{ kg})(6.00 \text{ m/s})^2$$

$$= 9.00 \times 10^2 \text{ J}$$

Kinetic energy of object 2 before

$$E_{k2} = \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} (60.0 \text{ kg})(0)^2$$

$$= 0$$

Kinetic energy of object 2 after

$$E'_{k2} = \frac{1}{2} m_2 (v'_2)^2$$

$$= \frac{1}{2} (60.0 \text{ kg})(6.30 \text{ m/s})^2$$

$$= 1.20 \times 10^3 \text{ J}$$

Total kinetic energy before collision

$$= 2.41 \times 10^3 \text{ J} + 0$$

$$= 2.41 \times 10^3 \text{ J}$$

Total kinetic energy after collision

$$= 9.0 \times 10^2 \text{ J} + 1.20 \times 10^3 \text{ J}$$

$$= 2.10 \times 10^3 \text{ J}$$

Mechanical (kinetic) energy lost in collision is $2.41 \times 10^3 \text{ J} - 2.10 \times 10^3 \text{ J} = 3.10 \times 10^2 \text{ J}$,
 \therefore collision is inelastic. Kinetic energy was not conserved.

c) Most of the loss in mechanical energy was converted to thermal energy—some to sound, etc.

5. a) before collision



$$m_1 = 15.0 \text{ kg} \quad m_2 = 10.0 \text{ kg}$$

$$\vec{v}_1 = 7.0 \text{ m/s east} \quad \vec{v}_2 = 0$$

$$\vec{p}_1 = 105 \text{ kg} \cdot \text{m/s east} \quad \vec{p}_2 = 0$$

before collision



$$m_1 = 15.0 \text{ kg}$$

$$\vec{v}'_1 = 4.2 \text{ m/s } 20.0^\circ \text{ S of E}$$

$$\vec{p}'_1 = 63 \text{ kg} \cdot \text{m/s } 20.0^\circ \text{ S of E}$$

$$m_2 = 10.0 \text{ kg}$$

$$\vec{v}'_2 = ?$$

$$\vec{p}'_2 = ?$$

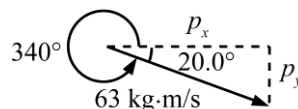
$$p_{\text{after}} = p_{\text{before}}$$

$$p_{\text{before}} = 105 \text{ kg} \cdot \text{m/s east}$$

$$\therefore p_{\text{after}} = 105 \text{ kg} \cdot \text{m/s east}$$

Find horizontal and vertical components of $63 \text{ kg} \cdot \text{m/s } 20.0^\circ \text{ S of E}$.

$20.0^\circ \text{ S of E} = 340^\circ$ heading counter clockwise from positive x -axis



$$\vec{p}'_{1x} = p'_{1x} \cos \theta$$

$$= (63 \text{ kg} \cdot \text{m/s})(\cos 340^\circ)$$

$$= 59.2 \text{ kg} \cdot \text{m/s}$$

$$\vec{p}'_{1y} = p'_{1y} \sin \theta$$

$$= (63 \text{ kg} \cdot \text{m/s})(\sin 340^\circ)$$

$$= -21.5 \text{ kg} \cdot \text{m/s}$$

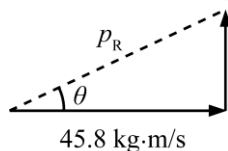
After collision, the 10.0 kg object must have a horizontal component of

$$105 \text{ kg}\cdot\text{m/s} - 59.2 \text{ kg}\cdot\text{m/s} = 45.8 \text{ kg}\cdot\text{m/s east}$$

After collision, the 10.0 kg object must have a vertical component of

$$0 - (-21.5 \text{ kg}\cdot\text{m/s}) = 21.5 \text{ kg}\cdot\text{m/s north}$$

Now, add 45.8 kg·m/s east and 21.5 kg·m/s north using Pythagoras theorem.



$$\begin{aligned} p_R &= \sqrt{(p_x)^2 + (p_y)^2} \\ &= \sqrt{(45.8 \text{ kg}\cdot\text{m/s})^2 + (21.5 \text{ kg}\cdot\text{m/s})^2} \\ &= 50.6 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Magnitude of velocity

$$\begin{aligned} v_2 &= \frac{p_2}{m} \\ &= \frac{50.6 \text{ kg}\cdot\text{m/s}}{10.0 \text{ kg}} \\ &= 5.1 \text{ m/s} \end{aligned}$$

Now find out the direction using

$$\begin{aligned} \tan \theta &= \frac{p_y}{p_x} \\ &= \frac{21.5 \text{ kg}\cdot\text{m/s}}{45.8 \text{ kg}\cdot\text{m/s}} \\ \theta &= 25^\circ \text{ N of E} \\ \vec{v}_2 &= 5.1 \text{ m/s } 25^\circ \text{ N of E} \end{aligned}$$

b)

before collision

$$\textcircled{1} \qquad \textcircled{2}$$

$$\begin{aligned} m_1 &= 15.0 \text{ kg} & m_2 &= 10.0 \text{ kg} \\ v_1 &= 7.0 \text{ m/s} & v_2 &= 0 \end{aligned}$$

after collision

$$\textcircled{1} \qquad \textcircled{2}$$

$$\begin{aligned} m'_1 &= 15.0 \text{ kg} & m'_2 &= 10.0 \text{ kg} \\ v'_1 &= 4.2 \text{ m/s} & v'_2 &= 5.1 \text{ m/s} \end{aligned}$$

Kinetic energy of object 1, before

$$\begin{aligned} E_{k1} &= \frac{1}{2} m_1 v_1^2 \\ &= \frac{1}{2} (15.0 \text{ kg})(7.0 \text{ m/s})^2 \\ &= 3.7 \times 10^2 \text{ J} \end{aligned}$$

Kinetic energy of object 1, after

$$\begin{aligned} E'_{k1} &= \frac{1}{2} m_1 (v'_1)^2 \\ &= \frac{1}{2} (15.0 \text{ kg})(4.2 \text{ m/s})^2 \\ &= 1.3 \times 10^2 \text{ J} \end{aligned}$$

Kinetic energy of object 2, before

$$\begin{aligned} E_{k2} &= \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2} (10.0 \text{ kg})(0)^2 \\ &= 0 \end{aligned}$$

Kinetic energy of object 2, after

$$\begin{aligned} E'_{k2} &= \frac{1}{2} m'_2 (v'_2)^2 \\ &= \frac{1}{2} (10.0 \text{ kg})(5.1 \text{ m/s})^2 \\ &= 1.30 \times 10^2 \text{ J} \end{aligned}$$

Total kinetic energy before collision

$$\begin{aligned} &= 3.7 \times 10^2 \text{ J} + 0 \\ &= 3.7 \times 10^2 \text{ J} \end{aligned}$$

Total kinetic energy after collision

$$\begin{aligned} &= 1.3 \times 10^2 \text{ J} + 1.3 \times 10^2 \text{ J} \\ &= 2.6 \times 10^2 \text{ J} \end{aligned}$$

Mechanical (kinetic) energy lost in collision is

$$3.7 \times 10^2 \text{ J} - 2.6 \times 10^2 \text{ J} = 1.1 \times 10^2 \text{ J},$$

\therefore collision is inelastic. Kinetic energy was not conserved.

c) Most of the loss in mechanical energy was converted to thermal energy—some to sound, etc.

6. before explosion



$$\begin{aligned} m &= 3m \\ \vec{v} &= 0 \\ \vec{p} &= 0 \end{aligned}$$

after explosion

①

$$m_1 = m$$

$$\vec{v}_1 = 15 \text{ m/s east}$$

$$\vec{p}_1 = (15m) \text{ m/s east}$$

②

$$m_2 = m$$

$$\vec{v}_2 = 10 \text{ m/s}$$

$$45^\circ \text{ S of E}$$

$$\vec{p}_2 = (10m) \text{ m/s}$$

$$45^\circ \text{ S of E}$$

③

$$m_3 = m$$

$$\vec{v}_3 = ?$$

$$\vec{p}_3 = ?$$

$$\vec{p}_{\text{sys(after)}} = \vec{p}_{\text{sys(before)}}$$

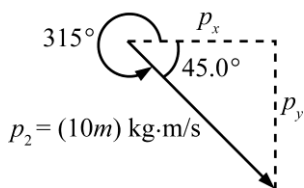
$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

Find the horizontal and vertical components of \vec{p}_1 .

$$\vec{p}_{1x} = (15m) \text{ m/s east}$$

$$\vec{p}_{1y} = 0$$

Find the horizontal and vertical components of \vec{p}_2 .
 $45.0^\circ \text{ S of E} = 315^\circ$ heading counter clockwise from positive x -axis



$$\vec{p}_{2x} = p_2 \cos \theta$$

$$= ((10m) \text{ m/s})(\cos 315^\circ)$$

$$= (7.07m) \text{ m/s}$$

$$\vec{p}_{2y} = p_2 \sin \theta$$

$$= ((10m) \text{ m/s})(\sin 315^\circ)$$

$$= -(7.07m) \text{ m/s}$$

$$\sum \vec{p}_x = (15m) \text{ m/s} + (7.07m) \text{ m/s}$$

$$= (22.1m) \text{ m/s}$$

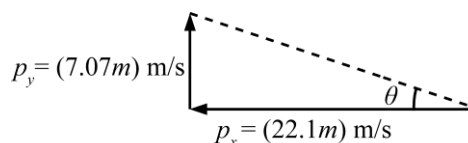
$$\sum \vec{p}_y = 0 + ((-7.07m) \text{ m/s})$$

$$= (-7.07m) \text{ m/s}$$

$\sum p_x$ should be zero,
 x -component of $\vec{p}_3 = (-22.1 \text{ m}) \text{ m/s}$

$\sum p_y$ should be zero,
 $\therefore y$ -component of $\vec{p}_3 = (7.07 \text{ m}) \text{ m/s}$

Now add the x - and y -components of \vec{p}_3 .



$$p_{3(R)} = \sqrt{(p_{3x})^2 + (p_{3y})^2}$$

$$= \sqrt{((22.1m) \text{ m/s})^2 + ((7.07m) \text{ m/s})^2}$$

$$= (23.2m) \text{ m/s}$$

$$\tan \theta = \frac{p_{3y}}{p_{3x}}$$

$$= \frac{(7.07m) \text{ m/s}}{(22.1m) \text{ m/s}}$$

$$\theta = 17.8^\circ \text{ N of W}$$

$$\vec{p}_3 = \vec{p}_{3R} = (23.2m) \text{ m/s } 17.8^\circ \text{ N of W}$$

$$\vec{v}_3 = \frac{\vec{p}_3}{m}$$

$$= \frac{(23.2m) \text{ m/s } 17.8^\circ \text{ N of W}}{m}$$

$$= 23.2 \text{ m/s } 17.8^\circ \text{ N of W}$$

Practice Test

ANSWERS AND SOLUTIONS

1. $E_k = \frac{1}{2}mv^2$

Since both the mass and the velocity remain constant, the kinetic energy remains constant.

$$E_p = mgh$$

Since the height (h) increases, the gravitational potential energy increases.

$$p = mv$$

Like kinetic energy, since both the mass and the velocity remain constant, the momentum remains constant.

2. Momentum is conserved in all collisions and explosions.

3. $Ft = m\Delta v$

Units of $Ft = \text{N}\cdot\text{s}$

Units of $m\Delta v = \text{kg}\cdot\text{m/s}$

\therefore units of impulse can be: $\text{N}\cdot\text{s}$ or $\text{kg}\cdot\text{m/s}$.

4. If the object has momentum, it has mass and velocity; and if it has mass and velocity, it has kinetic energy also.

Mechanical energy is the sum of the gravitational potential energy and the kinetic energy. Therefore, if it has kinetic energy, it must also have mechanical energy.

Note: The object does not need to have potential energy.

5. An elastic collision is defined as a collision in which both the momentum and kinetic energy are conserved.

$$6. \quad E_k = \frac{1}{2}mv^2$$

$$\therefore v = \sqrt{\frac{2E_k}{m}}$$

Again magnitude of the momentum

$$p = mv$$

$$\therefore p = m \left(\sqrt{\frac{2E_k}{m}} \right)$$

$$p^2 = \frac{m^2 2E_k}{m}$$

$$p = \sqrt{m2E_k} \text{ or } \sqrt{2mE_k}$$

7. $p = mv$
If you triple the velocity, you triple the momentum.
- $$p' = m(3v)$$
- $$p' = 3mv$$
- $$p' = 3p$$

Inertia is a measure of mass only.

8. If the object slides along a horizontal frictionless surface, there will be no acceleration (i.e., constant velocity). If the velocity remains constant, then the object's momentum, potential energy, and kinetic energy all remain constant.
9. Impulse can be defined as the change in momentum. Therefore, when an object experiences an impulse, it is experiencing a change in its momentum.

10. Find the velocity of the spacecraft before collision



$$m = 1.30 \times 10^2 \text{ kg} + 2.80 \times 10^3 \text{ kg}$$

$$= 2.93 \times 10^3 \text{ kg}$$

$$\vec{v} = 0$$

$$\vec{p} = m\vec{v}$$

$$= 0$$

after collision



$$m_1 = 1.30 \times 10^2 \text{ kg} \quad m_2 = 2.80 \times 10^3 \text{ kg}$$

$$\vec{v}_1 = 9.00 \text{ m/s} \quad \vec{v}_2 = ?$$

$$\vec{p}_1 = 1.17 \times 10^3 \text{ kg} \cdot \text{m/s} \quad \vec{p}_2 = -1.17 \times 10^3 \text{ kg} \cdot \text{m/s}$$

$$\vec{v}_2 = \frac{\vec{p}_2}{m_2}$$

$$= \frac{-1.17 \times 10^3 \text{ kg} \cdot \text{m/s}}{2.80 \times 10^3 \text{ kg}}$$

$$= -0.418 \text{ m/s}$$

\therefore relative velocity of the astronaut with respect to the spacecraft is

$$9.00 \text{ m/s} - (-0.418 \text{ m/s}) = 9.42 \text{ m/s}$$

$$v = \frac{d}{t}$$

$$t = \frac{d}{v}$$

$$= \frac{22.0 \text{ m}}{9.42 \text{ m/s}}$$

$$= 2.34 \text{ s}$$

It will take 2.34 s for the safety line to reach its full extension.

11. $\Delta p = m(v_f - v_0)$

Note that the final velocity of the subatomic particle has a magnitude approximately equal to the initial velocity; but the direction is opposite to its original direction.

i.e., if $v_0 = v$, then $v_f = -v$, or if $v_0 = -v$, then $v_f = v$.

$$\therefore \Delta p = m(v - (-v))$$

$$= m(2v)$$

$$= 2mv$$

The change in momentum for the subatomic particle is $2mv$.

12. A perfect elastic collision is defined as a collision in which both kinetic energy and momentum are conserved.

$$\begin{aligned}
 13. \text{ impulse} &= m\Delta v \\
 &= (0.25 \text{ kg})(v_f - v_0) \\
 &= (0.25 \text{ kg})(-2.5 \text{ m/s} - 3.0 \text{ m/s}) \\
 &= 1.4 \text{ N}\cdot\text{s}
 \end{aligned}$$

The impulse on the ball was 1.4 N·s.

14. before collision



Consider east as positive

$$\begin{aligned}
 m_1 &= 1.1 \times 10^3 \text{ kg} \\
 \vec{v}_1 &= 25 \text{ km/h} \\
 \vec{p}_1 &= 2.75 \times 10^4 \text{ kg}\cdot\text{km/h}
 \end{aligned}$$

$$\begin{aligned}
 m_2 &= 2.3 \times 10^3 \text{ kg} \\
 \vec{v}_2 &= -15 \text{ km/h} \\
 \vec{p}_2 &= -3.45 \times 10^4 \text{ kg}\cdot\text{km/h}
 \end{aligned}$$

$$\vec{p}_{\text{before}} = \vec{p}_1 + \vec{p}_2 = -7.0 \times 10^3 \text{ kg}\cdot\text{km/h}$$

after collision



$$\begin{aligned}
 m &= 3.4 \times 10^3 \text{ kg} \\
 \vec{v} &= ? \\
 \vec{p} &= -7.0 \times 10^3 \text{ kg}\cdot\text{km/h}
 \end{aligned}$$

$$\begin{aligned}
 \vec{v} &= \frac{\vec{p}}{m} \\
 &= \frac{-7.0 \times 10^3 \text{ kg}\cdot\text{km/h}}{3.4 \times 10^3 \text{ kg}} \\
 &= -2.1 \text{ km/h or } 2.1 \text{ km/h west}
 \end{aligned}$$

The velocity of the locked cars after the collision was 2.1 km/h west.