The Role of Moral Sentiments and Audit Perceptions in Tax Compliance*

Brian Erard
Department of Economics
Carleton University

and

Jonathan S. Feinstein
School of Organization and Management
Yale University

Abstract

The standard expected utility model of tax compliance substantially overpredicts both the frequency and level of noncompliance among U.S. individual income taxpayers. In order to improve the realism of this model we extend it to address two important issues: the role of moral sentiments, specifically guilt and shame, in taxpayer decision-making; and the relationship between taxpayer perceptions of the probability of audit and the true audit function. We estimate the parameters of our model using a dataset containing detailed tax and audit information from the state of Oregon and the U.S. Internal Revenue Service. Our empirical results provide an indication of the roles of audit misperceptions and moral sentiments in shaping compliance behavior. In particular, we find evidence of considerable heterogeneity and a general upward bias in audit perceptions amongst filers, and we find that the importance of the sentiment of shame in explaining reporting behavior is much more sensitive than the sentiment of guilt to the degree of bias in audit perceptions.
1. Introduction

In the conventional economic model of tax compliance, the taxpayer's choice of how much income to report on his return is determined by applying the expected utility theory of decision-making under uncertainty. Although its mathematical rigor and its foundation in a well-established theory of decision-making are appealing, the conventional model substantially overpredicts both the frequency and the amount of tax evasion in the United States and other developed nations. Many different elements probably play a role in the failure of the standard model to explain compliance behavior, but we believe that the two most important explanations relate to taxpayer motivations and taxpayer perceptions of the probability of an audit. In the conventional expected utility model it is assumed that taxpayer reporting decisions are motivated solely by financial considerations, embodied in the risk of audit and the threat of subsequent fines. In contrast, intuition, survey evidence, and certain psychological theories suggest that moral sentiments, especially guilt and shame, may strongly influence reporting behavior, reducing the perceived benefits of cheating. In the conventional model, taxpayers are also assumed to correctly perceive the probability of audit, whereas survey evidence and psychological theories of judgment under uncertainty and social cognition suggest that individuals may find it difficult to form realistic perceptions of the audit function they face, and may tend to overestimate the probability of audit.

In this paper we extend the conventional economic model to address these two issues. We then estimate our model using a sample of high income U.S. taxpayers, and show that, under certain conditions, our model is capable of explaining observed reporting behavior. We incorporate the sentiments of guilt and shame directly into our specification of taxpayer utility, hypothesizing that a taxpayer will feel guilty when he under-reports and escapes detection and will feel ashamed when he under-reports and gets caught. We address the issue of misperceptions by positing a model of taxpayer perceptions of the audit function that includes one bias parameter measuring misperceptions of the average probability of audit and a second bias parameter measuring misperceptions of how changes in reported income relate to changes in the probability of audit. In addition, we allow for heterogeneity across taxpayers in both the intensity of moral sentiments and the extent of misperceptions.

The data includes both detailed line-item information from the federal tax returns filed by the individuals in our sample, an indicator for which returns were subjected to a federal audit, and the audit results (Aim, et. al. [1993] is the only other academic study we are aware of that links micro-level tax return information to audit records for the same tax year; however, see Erard [1992] for a study that links tax return information with prior year audit records). The econometric procedure we use involves first constructing an estimate of the
actual audit function faced by the filers in the sample, and then embedding this estimated audit function in our structural model and estimating the model’s remaining parameters.

Our results indicate that accounting for both moral sentiments and audit misperceptions is necessary in order to provide a reasonable explanation of actual compliance behavior. In particular, when the perception parameters are left unconstrained, the results imply unrealistic overassessments of audit risk and no role for guilt or shame. However, when we constrain the perceptions parameters to more realistic values, guilt and shame become important. Furthermore, as we tighten the constraints on the perceptions parameters the estimated level of guilt increases relatively slowly, while the estimated level of shame increases relatively rapidly. Finally, the results also indicate considerable variation across taxpayers in both audit perceptions and the intensity of feelings of guilt and shame.

Although the model and results presented in this paper can be read independently of our other work, we feel that it is appropriate to mention that this paper is part of a larger project on tax compliance (see Erard and Feinstein [1991, 1994]).

2. Models

This section is divided into four parts. First, we review the conventional expected utility model of evasion, emphasizing the difficulties this model has in explaining observed compliance behavior. Second, we extend the standard model to incorporate sentiments of guilt and shame, drawing on dictionary definitions and psychological descriptions to support our claim that these sentiments can play an important role in compliance decisions. Next, extending the standard model in a different direction, we develop a model of taxpayer perceptions that allows for the possibility that taxpayers may misperceive the audit function they face. We argue that our approach is consistent both with survey evidence and with psychological theories of judgment under uncertainty and social cognition; however, we also claim that a correct formulation must go beyond previous theories of probability assessment to study how individuals form perceptions of entire probability functions. Finally, we conclude the section with a mathematical overview of our complete model.

The conventional expected utility model

Consider a particular taxpayer with true income \( y \), on which he owes taxes \( t(y) \), where \( t(\cdot) \) represents the tax function. The taxpayer chooses to report income \( x \), which may be equal to or less than \( y \), and pays taxes according to how much income he reports. Thus, if he reports truthfully the taxpayer pays his full tax liability; however, if he reports \( x < y \), he pays only \( t(x) < t(y) \).

In choosing how much income to report, the taxpayer takes
into account the probability that his return will be audited as well as the penalties that will be assessed if he is audited and found to have underreported. We define \( p(x) \) to be the probability that the taxpayer's return will be subjected to an audit if he reports \( x \); our notation emphasizes that the probability of audit is a function of the taxpayer's report, a fact that will be important throughout our analysis, especially during our discussion of audit perceptions. If the taxpayer is not audited his net after-tax income is \( y-t(x) \). If he is audited we assume that his true income \( y \) is determined with certainty and that he must pay both the difference between his true tax liability and his initial tax payment, \( t(y)-t(x) \), and a penalty (which may include a substantial interest charge) of \( \theta (t(y)-t(x)) \). We assume that the taxpayer's utility depends only on his net income and that his utility function belongs to the constant relative risk aversion family. In this case if the taxpayer has coefficient of relative risk aversion \( \alpha \), he chooses \( x \) to maximize

\[
p(x)[y-t(y)-\theta(t(y)-t(x))]^\alpha + (1-p(x))[y-t(x)]^\alpha \quad (1)
\]

The expected utility model, like all theoretical constructs, may be evaluated both in terms of the realism of its assumptions and the validity of its predictions. The unrealistic nature of the model's assumptions is explored in some depth in the next two subsections. In the remainder of this subsection, we briefly comment on the model's failure to correctly predict observed compliance behavior.

The expected utility model predicts that, at the audit and penalty rates prevalent in the U.S. and most other developed countries, many taxpayers should choose to evade a substantial amount of their taxes. As an illustration of this claim, consider the case of a risk neutral taxpayer (\( \alpha = 1 \)) who faces an audit schedule for which the probability of audit does not vary with the taxpayer's report. The conventional model, as formalized in equation (1), predicts that in this example the taxpayer will report zero income whenever the audit probability he faces is less than \( 1/(1+\theta) \). Data from federal audits in the U.S. indicate that the penalty charges associated with an audit are such that \( \theta \) is between .25 and .5. When \( \theta \) equals .5, \( 1/(1+\theta) \) equals .67, far above the average audit rate in the U.S., which is less than 1%, and indeed far above the rate faced by any class of filers. Even if the effective penalty rate were much larger than .5 as a consequence of the financial costs, time costs, and stress associated with an audit, the conventional model continues to predict complete evasion; for example, even if \( \theta \) were equal to 9, the taxpayer would report zero income as long as the probability of audit was below .10.

In cases in which taxpayers are risk averse and the probability of audit does vary with reported income, analogous calculations to those above demonstrate that the conventional expected utility model continues to predict very large amounts
of cheating over a wide range of audit schedules and penalty rates. For example, consider a filer whose true income is $100,000 and whose coefficient of relative risk aversion is .5. Let $\theta$ equal .5, assume a constant proportional tax rate of .3, and suppose that $p(x)$ is of the form $.5 - x/200,000$ (so that the probability of audit is zero if the filer reports truthfully, and rises linearly to 1/2 as reported income approaches zero). Such a filer is predicted to report only $38,350 as income, understating true income by 62%. Finally, the main thrust of the example remains valid when the example is extended to take into account a number of important practical issues, such as the fact that even strategic taxpayers will tend to fully report income items subject to informational reporting (for which the detection rate is likely to be quite high).1

In striking contrast to the predictions of the standard expected utility model, there is considerable evidence both that many taxpayers choose to report truthfully, and that even among those who do cheat, most do so by far less than predicted by the standard model. The evidence in Alexander and Feinstein [1986] and Feinstein [1991] indicates that between one-third and one-half of all taxpayers attempt to report their taxes correctly. A survey by Harris and Associates [1987] found that nearly 60% of taxpayers poled claimed to fill out their taxes honestly; although it is likely that some of these respondents were answering falsely, this survey finding is at least consistent with the notion that many taxpayers report truthfully. Erard and Feinstein [1994] provide a more extensive discussion of honesty in tax reporting.

Econometric evidence presented in Alexander and Feinstein [1986] and Feinstein [1991], both based on the Internal Revenue Service's Taxpayer Compliance Measurement Program (TCMP) data files, indicates that amongst those filers who do cheat the typical level of cheating is only a few hundred dollars, in most cases far below their tax liability. In the dataset we employ later in this paper, which consists of high income filers (average income in our sample is above $180,000), the average amount of detected understatement amongst those who do underreport and are later subjected to an audit is approximately $10,000, or 6% of true income. We refer the reader to Alexander and Feinstein [1986], Skinner and Slemrod [1985], and Alm, McClelland, and Schulze [1992] for additional discussions of the failure of the conventional model of tax compliance to predict observed compliance outcomes.

**Moral sentiments**

One important reason why the conventional expected utility model of tax compliance overpredicts the prevalence and extent of tax evasion is that compliance behavior is assumed to be motivated solely by financial considerations, whereas in reality many taxpayers are influenced by a variety of other feelings, which we will call *moral sentiments*. In this subsection we extend the conventional model to account for the influence of two of these sentiments, guilt and shame. Our approach is
motivated by the same facts that previously lead us to emphasize the role of honest taxpayers in tax compliance systems (see Erard and Feinstein [1994]). However, whereas in the earlier study we simply assumed that some taxpayers are inherently honest, while others follow the conventional model and respond only to financial considerations, in the present analysis we develop a somewhat deeper and more flexible model in which taxpayer honesty itself emerges from the internalization of feelings of guilt and shame.

The feeling of guilt is defined by The Random House Unabridged Dictionary of the English Language (Second Edition) to be "a feeling of responsibility or remorse for some offence, crime, wrong, etc..., whether real or imagined" (second definition listed). Conventional usage suggests that a person feels guilty when he voluntarily acts against the dictates of his conscience, violating his internal code of responsibility. Psychoanalysts link guilt to censure by the "superego." Thus, Lewis [1971] argues that guilt arises when an individual realizes that he has acted irresponsibly and in violation of a rule or social norm that he has previously internalized, either by committing a specific wrongful act or by failing to fulfill a recognized ethical duty; her discussion emphasizes that the experience of guilt is harsh, is limited in its target to the specific transgression, and is linked to impersonal rules and obligations. Since the obligation to pay one's fair share of taxes to the government is a widely accepted social norm, it makes sense that individuals who choose not to pay all of their taxes may feel guilty; it also seems reasonable that the amount of guilt a taxpayer feels is likely to be positively related to the extent of evasion. In our opinion, the feeling of guilt is most likely to arise when a taxpayer who evades some or all of his tax obligation is not subsequently audited, since he then feels that he has unfairly "gotten away with something." We doubt that the taxpayer will feel much guilt if it turns out that he is subsequently audited; he is then far more likely to feel ashamed, as we discuss below.

Survey evidence supports the notion that taxpayers feel guilty when they intentionally evade taxes. In a 1980 Taxpayer Opinion Survey by Aitken and Bonneville [1980], more than 50% of respondents claimed that their consciences would be bothered "a lot" as a result of engaging in any of the following activities: (i) padding business activities, (ii) overstating medical expenses, (iii) understating income, (iv) not filing a return, or (v) claiming an extra dependent. In a separate survey Grasmick and Bursick [1990] interviewed 355 individuals regarding their future inclination to perform various legal offences, including tax evasion. Their analysis of the survey results indicated that the anticipated guilt associated with committing tax evasion served as a much greater deterrent than the perceived threat of legal sanctions. Recent empirical work by Scholz and Pinney [1993] also emphasizes the importance of civic duty and guilt in compliance decisions.

As the dictionary definition cited above states, guilt may arise from the contemplation of either real or imagined actions.
We would suggest that, while filling out his return, the taxpayer will experience guilt while contemplating a sequence of events in which he underreports his taxes and is not subsequently audited. In turn the feeling of guilt will lead him to attribute lower utility to this sequence of events than the conventional expected utility model predicts; further, his guilt will be larger and his utility reduced further below the conventional model's prediction the smaller the report he considers. In keeping with our earlier remarks, we believe that the taxpayer is unlikely to feel guilty when contemplating the alternative sequence of events in which he is audited; hence, utility in this state will be unaffected.\(^2\)

To formalize the impact of guilt, we modify our previous expression for taxpayer utility in the not audited state. We introduce a new parameter \(g\), which is assumed to be non-negative. Utility in the non-audited state is redefined as

\[
\frac{[y-t(x)]^a}{1+g \left( \frac{t(y)-t(x)}{y} \right)} \tag{2}
\]

The denominator of this expression becomes larger than one when the taxpayer underpays and feels guilty, which causes his overall utility level to be diminished; furthermore, the magnitude of the taxpayer's guilt is proportional to the ratio of his underpayment \((t(y)-t(x))\) to his true income \((y)\).

Although our derivation of (2) has emphasized the role that guilt plays in reducing utility when a taxpayer cheats and is not audited, one who prefers not to speculate about "deeper" psychological justifications for this expression may simply be view it as an adjustment to utility.

The amount of guilt experienced may well vary across taxpayers, since some have internalized this (and other) social norms more completely or possess "harsher" superegos than others. To capture such variations in guilt, we develop and estimate an extension to our basic model in which the parameter \(g\) in expression (2) is replaced by a stochastic disturbance \(\varepsilon_g\), distributed independently across filers according to the uniform distribution with support \([0,G]\), where \(G\) is a parameter to be estimated. This extended model is computationally more burdensome than the main model and, as it turns out, generates relatively similar results. Consequently, we confine our formal model development in this and the subsequent section to the simpler case in which \(g\) is the same for all filers; however, we do discuss estimates of the extended model in section 4.

A second moral sentiment that may affect reporting behavior is shame. Shame is defined by The Random House Dictionary to be "the painful feeling arising from the consciousness of something dishonorable, improper, ridiculous, etc..., done by oneself or another" (first definition listed). Psychoanalysts such as Lewis [1971] and Wurmser [1981] make two useful distinctions between shame and guilt. First, whereas guilt is associated with the transgression of an impersonal rule or norm (or with
the failure to perform a specific, impersonal duty), shame wears a "human face" and arises in response to the imagination of a human "other" who judges one's self or behavior to be inappropriate or immoral. Second, guilt is typically focused on a specific action or intention, but shame is a more global feeling in which a person's whole self-image is implicated; thus, a person who commits a wrongful act may feel guilty but continue to experience a global sense of self-esteem, while an individual who becomes ashamed due to some public mishap is likely to feel a global sense of self-effacement or loss of self-esteem. Reflecting these distinctions, Lewis [1971, page 39] writes that shame is an experience in which an internalized other:

"seems to scorn, despise, or ridicule the self.... Shame may be experienced in private; or it may be evoked by an actual encounter with a specific or ill-defined 'other'. During this encounter, the self is acutely self-conscious, whether the encounter occurs in fantasy or in reality."

These definitions are consistent with the notion that a taxpayer is likely to feel shame if he has underpaid his taxes and is subsequently subjected to an audit, both because he may be physically present when the tax examiner uncovers his evasion and because family members and friends are quite likely to learn of his evasion.

At the time when the taxpayer is filling out his tax return he has not yet committed any wrongdoing or undergone the shameful experience of an audit. Nonetheless, as Lewis makes clear shame refers to the ridiculing or scornful gaze of either a real or imagined other. Hence, we expect that the filer will experience shame when filling out his return and contemplating scenarios in which he underpays his taxes and is subsequently subjected to the embarrassment of an audit. This sense of shame will reduce the filer's assessment of the utility he will experience in this state below the level predicted by the standard compliance model. We also hypothesize, although the psychology literature provides no evidence on this point, that the filer will feel more shame the larger the amount by which he contemplates underreporting. We do not think he will feel any shame when contemplating scenarios in which he is not audited and his evasion remains undiscovered by others.

We incorporate shame into the taxpayer's utility function in a manner analogous to the way in which we incorporated guilt. Specifically, we redefine utility in the state in which the taxpayer is audited to be

\[
\frac{[y-t(y)-\theta(t(y)-t(x))]^\alpha}{1+s[t(y)-t(x)]/\gamma}
\]

where \(s\) is a parameter that again is assumed to be non-negative.
Just as for expression (2), expression (3) can be interpreted in a more pragmatic way as a simple adjustment to utility, rather than as a specific expression for shame.

As with guilt an extension of our basic formulation allows shame to vary across filers by redefining shame to be $\xi_g$, where $\xi_g$ is a stochastic disturbance distributed independently across filers according to the uniform distribution with support $[0, S]$. Again, we do not discuss this extended model in the remainder of this section or in section 3, but we do discuss empirical estimates of it in section 4.

To obtain a sense of the impact of the guilt and shame parameters on taxpayer utility, consider the case where $g(s)$ is .33 and $t(y) = .3y$. In this case, a taxpayer who underreports his income by 10% will experience, as a result of his sense of guilt (shame), a reduction in the total utility he would otherwise obtain in the non-audited state (audited state) of about 1%.

Although we believe that our formulation of moral sentiments is a useful step in the direction of more realistic models of tax compliance, we readily acknowledge that there are a host of other moral, ethical, and sociologic issues that we have not addressed, including fairness and satisfaction with government (see the recent papers by Frey [1992] and Pommerehne, Hart, and Frey [1993]).

Perceptions of the audit function

In this subsection we first review several reasons why taxpayers may have difficulty forming accurate perceptions of the audit function they face and then introduce a model that allows for biases and variations in taxpayer perceptions.

Taxpayer surveys offer one important kind of evidence about taxpayer perceptions of the probability of audit. All of the taxpayer surveys that we have reviewed indicate that taxpayers tend to overestimate the probability of an audit. For example, results from a survey of U.S. taxpayers conducted by Harris and Associates [1988] indicate that on average taxpayers believe that 5% of all tax returns were subjected to a federal audit in 1986, whereas in fact only slightly more than 1% were audited; this survey's results also reveal substantial variation in perceptions amongst taxpayers: 36% of those surveyed believe that the actual audit rate was between 0 and 5%, 12% of those surveyed believe that it was between 5% and 10%, and 12% believe that it was between 10% and 20% (40% did not respond).

The survey finding that taxpayers typically overestimate the probability of an audit is consistent with evidence from psychological experiments (see the articles collected in Kahneman, Slovic, and Tversky [1982]) that individuals tend to overestimate the probabilities of unlikely events (which a tax audit surely is for most taxpayers) in a wide variety of contexts (Alm, McClelland, and Schulze [1992] also make this point).

Drawing on principles from cognitive psychology, Kahneman and Tversky have developed the theory of "representativeness" to
explain why perceptions tend to be biased in this way (see Kahneman and Tversky [1979] as well as their contributions to Kahneman, Slovic, and Tversky). They suggest that individuals form a rough assessment of the likelihood of an event by constructing scenarios of the future that either lead or do not lead to the event occurring; then, hoping that these few scenarios are "representative" of the future, they use them to compute an estimate of the probability of the event occurring. In fact, however, some scenarios may be more "available" than others, coming to mind disproportionaterly often, leading to biases in probability assessments; for example, rare but dramatic events, such as a nuclear power plant accident, will tend to come to mind far more readily than relatively mundane, though actually far more likely, outcomes.

There are several reasons why scenarios in which an audit occurs will tend to come into a taxpayer's mind disproportionaterly often, as compared with scenarios in which an audit does not occur. First, an audit is a highly salient and dramatic event. Second, consideration of the possibility of a future audit will create assosiational links to memories of audits that a taxpayer has either personally experienced or of which he has knowledge, memories which offer fertile material for the imaginative construction of scenarios in which a future audit occurs. Finally, emotional associations may lead a taxpayer to imagine or recall audit situations disproportionaterly often while filling out his return. Gilligan and Bower [1984] have experimentally demonstrated that the experience of a particular emotion tends to call forth memories or thoughts laden with similar emotions; in the context of tax compliance this effect implies that if a taxpayer feels anxious while filling out his tax return, he is likely either to recall or imagine other anxious experiences, of which an audit is a prime example, whose emotional hue matches his current mood.

Although the taxpayer survey evidence and psychological arguments we have reviewed above all provide strong support for the hypothesis that taxpayers often misperceive the probability of audit, a further consideration of exactly what taxpayers must know before deciding how much income to report on their tax returns reveals one subtlety that we do not believe has been adequately addressed. This subtlety arises from the fact that, whereas the survey and psychology literatures commonly focus on the assessment of a single probability $p$, a sophisticated taxpayer will recognize that her probability of being audited depends in part on her report $x$, so that in order to maximize her expected utility she must assess an entire audit schedule, $p(x)$. We now present a simple model of perceptions of such a function.\(^3\)

Our model of perceptions focuses on linking subjective taxpayer assessments of the probability of audit to the true (objective) audit function employed by the tax authority. Let the true audit function be

\[ p(t) = \beta_0 + \beta_1 x + \beta_2 x^2 + \gamma \]  

(4).
where $\beta_0$, $\beta_1$, $\beta_2$, and $\gamma$ are parameters, $x$ is reported income, and $z$ represents fixed and exogenous filer characteristics that affect the probability of an audit. We impose as a constraint on expression (4) that the probability of audit falls as the report $x$ rises.

We link perceptions to this true audit function by assuming that the taxpayer subjectively assesses the audit function to be of the form

$$p(s) = \beta_0 + \eta_0 + x(\beta_1 + \eta_1) + x^2\beta_2 + z\gamma$$

(5)

where $p(\bullet)$ is of the same functional form as in expression (4), and $\eta_0$ and $\eta_1$ are stochastic disturbances assumed to be drawn from a bivariate distribution $f_\eta(\bullet, \bullet)$, which is common across all filers and is characterized by means $\mu_0$ and $\mu_1$, standard deviations $\sigma_0$ and $\sigma_1$, and correlation coefficient $\rho$. The disturbances $\eta_0$ and $\eta_1$ are assumed to be independent across filers; together, they measure the errors in a taxpayer's assessments of the parameters $\beta_0$ and $\beta_1$ which contribute to the argument of the audit probability function. To ensure that the perceived audit probability schedule is decreasing in reported income, the distribution of $\eta_1$ is restricted to values below $-(\beta_1 + 2\beta_2 y_i)$, where $y_i$ is taxpayer $i$'s true taxable income.

In our formulation of audit perceptions we assume that taxpayers know the functional form that characterizes $p(\bullet)$, the variables that enter into the audit function, and the values of the coefficients $\beta_2$ and $\gamma$. Nonetheless, the presence of the disturbances $\eta_0$ and $\eta_1$ allows for considerable flexibility in the relationship between taxpayer perceptions and the true audit function. Three types of configurations of $\eta_0$ and $\eta_1$ are of particular interest. First, as discussed above, we expect that the mean values of these disturbances will be consistent with a tendency for taxpayers to overestimate the probability of an audit. Second, we suspect that the average values of $\eta_0$ and $\eta_1$ also will imply that taxpayers tend to overestimate the responsiveness of the audit function to changes in reported income levels. Third, we wonder whether some taxpayers suffer from what we call "self-other confusion," a state of mind described by social cognitive theorists (e.g., Flavell [1975]) in which an individual (often a child, but sometimes an adult) tends to ascribe his own knowledge or beliefs to another person without recognizing that the other person may not possess his knowledge or beliefs. In the context of tax compliance this type of self-other attribution error would lead a taxpayer into two kinds of mistakes. On the one hand, he would imagine that should he report honestly, the tax authority will somehow "know" this fact and consequently will choose not to audit him; on the other hand, he would believe that to the extent he does cheat, the tax authority will be able to "see" this misbehavior and consequently will be likely to audit him. The phenomenon of self-other confusion would be consistent with configurations of $\eta_0$ and $\eta_1$ such that the taxpayer underestimates the probability
of audit in the case in which he reports truthfully \((x=y)\), but overestimates the slope relating a reduction in \(x\) to an increase in \(p\).

Finally, the argument in Scholz and Pinney suggests that the bias terms \(\eta_0\) and \(\eta_1\) may be correlated with a taxpayer's sense of guilt and shame; we do not investigate this possibility in our empirical work.

**Full Model**

The theoretical development of this section can be summarized in a single model, which links our discussions of moral sentiments and audit perceptions together in an expanded expected utility framework. For convenience, we present this integrated model in the form in which we specify it for our econometric estimation, a form that makes a distinction between true taxable income, which we continue to denote by \(y\), and a taxpayer's total positive income (a gross income measure defined as the sum of the positive amounts of a taxpayer's various income sources), which we take to be a better measure of available resources, and which we denote \(w\).

With this modification, the taxpayer's expected utility when he reports \(x\) can be written as

\[
p(\beta_0 + \eta_0 + x(\beta_1 + \eta_1) + x^2 \beta_2 + zy) \frac{w - (1 + \theta)\mu(y) + \theta \mu(x)}{1 + s \left[ \frac{\mu(y) - \mu(x)}{w} \right]} \]

\[
+ [1 - p(\beta_0 + \eta_0 + x(\beta_1 + \eta_1) + x^2 \beta_2 + zy)] \frac{w - \mu(x)}{1 + s \left[ \frac{\mu(y) - \mu(x)}{w} \right]} \]

The taxpayer chooses \(x\) to maximize expression (6). The appendix presents the first-order condition associated with this equation. For a valid solution, the second-order condition (which we do not present but do check in our estimation) must also be satisfied.

**3. Data and econometric specifications**

The data used in this study is a small subsample drawn from a much larger dataset of tax returns and audit information we have obtained from the state of Oregon and the U.S. Internal Revenue Service. The data used in this study is restricted to filers in a single audit class, consisting of those returns with total positive income in excess of \$100,000 and no business (schedule C) or farm (schedule F) income. The sample consists of 716 filers, of whom 158 were subjected to a federal audit. The dataset includes substantial line item detail from the 1987 federal tax return. The data also includes, for those filers who were subjected to an audit, the assessed tax change and penalty.
resulting from the audit. We use the entire sample of 716 filers to estimate the true audit function in this class and use the 158 audited filers, for whom we have a good measure of true taxable income, to estimate our structural model.\textsuperscript{6}

Median true total positive income is $175,846 amongst the 158 filers in the audited group. The median reported tax liability is $29,162 for this group, while the median post-audit tax liability is $31,933. We have constructed a measure of "residual income" (income not subject to information reporting); the median value for this variable is $10,866 in the audited group. Most audited returns (63\%) are associated with a positive tax change (meaning that the filer owed additional taxes). Moreover, the median tax change is over twice as large for taxpayers with a positive change ($2371) as for those with a negative change ($-975; 7.9\% of the sample). We treat a negative tax change as identical to no change in our empirical estimation. For estimation, we have normalized our measures of income and taxes by dividing each measure by $100,000.

Estimation of our model proceeds in two stages. In the first stage we estimate the true audit function, expression (4), by applying a standard logit specification to our full sample of audited and non-audited taxpayers; the dependent variable is whether or not the filer was selected for an audit, and the explanatory variables include taxpayer and tax return characteristics. Unfortunately, our contractual agreement with the state of Oregon prevents us from divulging these results. In the second stage our analysis is restricted to audited taxpayers, for whom we know both the reported and true (post audit results) levels of income. We maximize the likelihood function associated with the taxpayer's first-order condition, conditional on the estimated parameters of the audit function from the first stage. The likelihood function is derived in the appendix.

4. Empirical results

Since we are unable to report results of the first step in our analysis, the estimation of an audit function, we move immediately to a discussion of our second stage results. In discussing the second stage analysis, we begin with and focus on the basic model developed in sections two and three, for which guilt and shame are identical fixed parameters for all filers; however, later we present results for the extended model in which guilt and shame vary in the population.

Preliminary findings from the second stage structural estimation process revealed several difficulties. First, we found it difficult to estimate \( \theta \) jointly with the other parameter of the model. To resolve this problem we fixed \( \theta \) at the value .25; we have also estimated the model with \( \theta \) fixed at .5, and we discuss these results where they differ from the results for \( \theta = .25 \). We also found it difficult to estimate \( \alpha \). To resolve this difficulty we fixed \( \alpha \) at the value one (risk neutrality); again, we have experimented with other values for
α and mention results for these cases where appropriate below. In all of the results we present below the correlation ρ between η₀ and η₁ has been set to zero. However, we have estimated ρ as a free parameter in a limited number of additional specifications and have found qualitatively similar results to those we present.

Table 1 presents results for an "unconstrained specification" that treats all of the audit perceptions terms (μ₀, μ₁, σ₀, and σ₁) as well as the guilt and shame terms (g and s) as free parameters. The results indicate substantial misperceptions of the overall shape of the audit function, substantial variability among taxpayers in the nature of this misperception, and no role for either guilt or shame. The estimated values for μ₀ and μ₁, 5.069 and -7.001, indicate that on average taxpayers greatly overestimate both the level and slope of the audit function; the estimated values for the standard deviations σ₀ and σ₁ associated with these mean biases are also quite large, 6.422 and 3.878.

**TABLE 1**

Results for Unrestricted Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
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<tbody>
<tr>
<td>μ₀</td>
<td>5.069 (7.53)</td>
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<tr>
<td>μ₁</td>
<td>-7.001 (3.02)</td>
</tr>
<tr>
<td>σ₀</td>
<td>6.422 (3.38)</td>
</tr>
<tr>
<td>σ₁</td>
<td>3.878 (2.38)</td>
</tr>
<tr>
<td>g</td>
<td>1.0E-10 (1.0E-6)</td>
</tr>
<tr>
<td>s</td>
<td>1.0E-10 (1.0E-6)</td>
</tr>
<tr>
<td>Likelihood Value</td>
<td>-139.53</td>
</tr>
</tbody>
</table>

(*t*-statistics in parentheses)

Although our contract with the state of Oregon prohibits us from presenting the exact audit perception function implied by our estimates, we are able to discuss certain qualitative features of this function. Most importantly, whereas the true average audit rate in the audit class represented by our sample is approximately 2.7%, the estimates from the unconstrained specification imply a much higher perceived average probability, approximately 72%, which is close to but slightly less than 1/(1 + 0), which equals 80%. We believe one key to understanding why the best statistical fit of the unconstrained model involves such an unrealistically high average perceived probability is the result, related to the discussion in section 2, that a risk neutral taxpayer who believes that he faces a constant audit probability of 1/(1 + 0) is indifferent about what income level to report. Regardless of the precise report a filer in this situation chooses to make, his behavior will be consistent with the expected utility model and will generate a high likelihood
value. The likelihood maximization algorithm chooses high average bias terms so as to generate an average perceived probability of audit close to this indifference point. Recall now that the true audit rate varies across filers due to variations in the exogenous $z$ variables and note that the overall perceived audit probability is the sum of the true rate plus the bias term. Together, these two facts imply that, for fixed parameters, the range of values of the disturbances $\eta_0$ and $\eta_1$ for which the perceived probability of audit is close to $1/(1 + \theta)$ varies across filers. To ensure that many such filers simultaneously have a significant probability mass associated with the perceived risk of audit $1/(1 + \theta)$, the search algorithm chooses large values for $\sigma_0$ and $\sigma_1$, thereby generating a high likelihood value.

A second interesting feature of the unconstrained estimates is that, even assuming both $\beta_1$ and $\beta_2$ (the estimates of which we are prohibited from reporting) to be zero, the magnitude of the estimated bias parameters $\mu_0$ and $\mu_1$ is so large as to imply that a $1,000 reduction in the report $x$ would increase the perceived probability of audit by more than 1% for nearly all relevant values of $x$.

In an attempt to obtain more realistic results than those generated by the unconstrained model, we have estimated a series of constrained specifications in which the mean values of $\eta_0$ and $\eta_1$ are fixed at smaller absolute values than their unconstrained estimates and the model's remaining parameters are estimated. For the disturbance $\eta_0$ we have restricted the mean to the values 3, 2, 1, and 0; and for the disturbance $\eta_1$ we have restricted the (truncated) mean to the values -6, -5, -2, and -1, leading to 16 different combinations. In the following discussion, we will refer to cases in which the mean of $\eta_0$ is relatively large (3 or 2) and the absolute value of the mean of $\eta_1$ is also relatively large (6 or 5) as weakly binding cases; the remaining cases will be referred to as strongly binding cases.

Table 2 provides estimates for the constrained specifications.

The results exhibit several broad features. First, as the constraints become more binding the roles of both guilt and shame tend to become more important, as expected. Second, guilt and shame differ markedly in the way in which they are affected by the constraints. Guilt is substantial when the constraints are weakly binding but grows only slightly as the constraints become more strongly binding, while shame is very small when the constraints are weakly binding and grows sharply as the constraints become more strongly binding. Thus, when the mean of $\eta_0$ is 3 and the mean of $\eta_1$ is -6, the weakest constraints, $g$ is estimated to be .635 and $s$ is estimated to be zero; in contrast, when the mean of $\eta_0$ is 0 and the mean of $\eta_1$ is -1, the strongest constraints, the estimated value of $g$ has risen only to .988, while that of $s$ has risen to 1.21. Intuitively, we do not find it surprising that shame is more sensitive to changes in audit perceptions at relatively low perceived probabilities than guilt. Shame enters into the
<table>
<thead>
<tr>
<th>$E(\eta_0)$</th>
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<th>3</th>
<th>3</th>
<th>3</th>
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<tbody>
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<td>-2</td>
<td>-1</td>
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<td>0.744</td>
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<td>(1.55)</td>
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<td>(3.51)</td>
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<td>0.915</td>
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<td>0.968</td>
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<td>(3.14)</td>
<td>(11.23)</td>
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<tr>
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<td>0.783</td>
<td>0.913</td>
<td>1.135</td>
<td>0.988</td>
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<td>(0.58)</td>
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<td>Likelihood</td>
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<td>$E(\eta_1)$</td>
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<td>Estimated Parameters</td>
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<td>$\sigma_0$</td>
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<td>(3.45)</td>
<td>(3.49)</td>
<td>(3.61)</td>
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<tr>
<td>g</td>
<td>0.947</td>
<td>0.954</td>
<td>0.976</td>
<td>0.988</td>
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<td>(4.48)</td>
<td>(6.02)</td>
<td>(23.61)</td>
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<tr>
<td>Likelihood</td>
<td>-164.62</td>
<td>-167.56</td>
<td>-199.16</td>
<td>-244.05</td>
</tr>
</tbody>
</table>

$t$-statistics in parentheses; $E(\eta_1)$ is the truncated mean of $\eta_1$. 

15
taxpayer's objective function as a term multiplying the probability of audit \((p)\), whereas guilt enters as a term multiplying the probability of no audit \((1-p)\). As the constraints on perceptions become tighter, the perceived probability of audit falls relatively quickly, while the perceived probability of no audit rises relatively slowly; hence the relative importance of shame grows rapidly, while the relative importance of guilt declines.

A second broad feature of the results in table 3 is that as tighter constraints are imposed the fit of the model, as measured by the likelihood statistic, becomes dramatically worse. Thus, whereas the value of this statistic is -139.5 in the unconstrained specification, it falls to -244.05 in the most constrained case. A third feature of the results is that the standard deviations \(\sigma_\theta\) and \(\sigma_\gamma\), which measure the heterogeneity in perceptions across taxpayers, become smaller as the constraints become more binding.

In order to quantify the link between the perceived and actual audit functions, we have performed the following simulation experiment. For the unconstrained as well as each of the constrained specifications, we have simulated the impact of an increase of 1% in the true probability of audit on reported taxes at the median income level in our sample. For the unconstrained model, the results indicate that reported taxes would increase by $873 as a result of this shift in the true audit schedule. For the weakly binding cases, the corresponding increase ranges from $600 to $1200; while for the strongly binding cases, the increase ranges from $1,800 to $2,400.

We have compared our results for the case \(\theta = .25\), presented above in tables 1 and 2, to the alternative case in which \(\theta = .5\), for both the unconstrained and constrained specifications. We find that all parameter estimates are similar except those for \(s\), which is nearly twice as large as the values reported in the previous case (though still zero in the unconstrained specification). This finding supports the notion that the penalty rate \(\theta\) and the shame parameter \(s\) are substitutes.

Table 3 presents results from two specifications of the extended model, in which guilt and shame as well as audit perceptions are allowed to vary amongst filers. For these models, we have set \(\theta = .5\), have constrained \( \mathbb{E}(\eta_0) \) to be 1 and \( \mathbb{E}(\eta_1) \) to be -2, and have considered both the case in which \( \alpha = 1 \) and the case in which \( \alpha = .5 \). In this extended model we estimate the parameters \(G\) and \(S\), the upper bounds of the supports of guilt and shame, respectively, instead of \(g\) and \(s\).
TABLE 3

Results for Extended Model

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Guilt and shame nonstochastic; risk neutral taxpayers</th>
<th>Guilt and shame stochastic; risk neutral taxpayers</th>
<th>Guilt and shame stochastic; risk averse taxpayers</th>
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<tr>
<td>$\sigma_0$</td>
<td>2.699</td>
<td>2.272</td>
<td>2.079</td>
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<td>(9.40)</td>
<td>(9.06)</td>
<td>(9.39)</td>
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<tr>
<td>$\sigma_1$</td>
<td>1.076</td>
<td>0.753</td>
<td>0.750</td>
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<td>(3.35)</td>
<td>(5.83)</td>
<td>(6.52)</td>
</tr>
<tr>
<td>$g(G)$</td>
<td>0.948</td>
<td>1.496</td>
<td>.727</td>
</tr>
<tr>
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<td>(11.09)</td>
<td>(19.64)</td>
<td>(15.90)</td>
</tr>
<tr>
<td>$s(S)$</td>
<td>0.817</td>
<td>1.754</td>
<td>1.940</td>
</tr>
<tr>
<td></td>
<td>(1.14)</td>
<td>(7.74)</td>
<td>(2.02)</td>
</tr>
</tbody>
</table>

*t*-statistics in parentheses.

Two comments may help clarify why we chose to estimate the particular versions of the extended model depicted in table 3. We chose not to estimate the unconstrained or loosely constrained (in terms of perceptions) versions of this model because in these models we expect guilt and shame to be relatively unimportant, in the sense that $G$ or $S$ will be nearly zero. We considered the case of risk aversion ($\alpha = .5$) because this case is estimable in the extended model but was not estimable in the base model.

The results in table 3 reveal that moving to the extended model does not greatly alter the qualitative conclusions we reached in analyzing the base model. As compared with the base model, the standard deviations $\sigma_0$ and $\sigma_1$ decrease slightly. When $\alpha = 1$ the upper bounds $G$ and $S$ are estimated to be 1.496 and 1.754, implying average levels of $g$ and $s$ of .748 and .877, whereas in the base model with the same perceptions constraints $g$ and $s$ were estimated to be .948 and .817. Thus guilt has fallen in average value, while shame has risen. When $\alpha = .5$ $G$ falls to about one-half its value for $\alpha = 1$ and $S$ rises slightly, to 1.94.

5. Conclusion

Our results suggest that taxpayers have substantial and varied misperceptions about the probability of audit, overestimating both its level and the rate at which it rises as the reported income falls. Moreover, when we constrain our estimates of taxpayer perceptions to realistic levels, we find that guilt and shame become important determinants of reporting behavior.
Endnotes

*We thank Pierre Pestieau, Werner Pommerehne and the other participants of the 49th IIPF Congress, seminar participants at the University of Colorado at Boulder Economics Workshop, and Rcn Shachar for helpful comments. We are also grateful to the Oregon Department of Revenue, particularly to Brian Reeder, and to the Internal Revenue Service, particularly to Ellie Convery, Dennis Cox, Chih-Chin Ho, and Alan Plumley, for assisting us in obtaining the dataset used in this paper. The first author thanks the Social Sciences and Humanities Research Council of Canada and the second author thanks the John M. Olin Foundation and the Yale SOM Summer Faculty Research Support Program for financial support.

1In fact, in the audit class we use to estimate our model nearly all individuals possess either some "residual" income (income not subject to information reporting) or report itemized deductions, so this issue is not directly relevant to our empirical analysis.

2Unfortunately, we are not aware of any surveys that inquire about the relationship between the event of an audit (or the probability of an audit) and the level of guilt.

3In reality, we doubt whether all taxpayers recognize the need to assess an entire probability function; when taxpayers do not recognize this need, they may form a single numerical assessment of the probability of audit, implicitly assuming that they are facing a "random audit" threat.

4We make this distinction primarily because in our data sample a number of filers have a high level of total positive income but a very low (or zero) level of true taxable income (based on their audit results); in this situation, we feel that total positive income is a better measure of their overall attainable utility.

5The complete dataset contains information on approximately 45,000 Oregon filers, including a large proportion of those filers who were subjected to either a federal or state audit for tax year 1987.

6We are aware of the fact that restricting our structural estimation to audited filers raises issues of selection bias. However, we feel that this difficulty is more than outweighed by the fact that we possess no reasonable measure of true income, and therefore no measure of evasion, for those filers who were not audited.
Appendix

In this appendix, we provide a derivation of the likelihood function and a discussion of related issues. To facilitate the analysis we make use of the following definitions:

\[ r_i = \frac{t(y_i) - t(x_i)}{w_i} \]
\[ u_i = w_i - (1+\theta)t(y_i) + \theta t(x_i) \]
\[ v_i = w_i - t(x_i) \]
\[ A_i = -\frac{\alpha v_i^{\alpha-1}t'(x_i)[1+gr_i] - \frac{gr_i v_i^n}{w_i}}{[1+gr_i]^2} \]
\[ B_i = \frac{\alpha u_i^{\alpha-1}t'(x_i)[1+sr_i] - \frac{sr_i u_i^n}{w_i}}{[1+sr_i]^2} - A_i \]
\[ C_i = (\beta_1 + \eta_{i1} + 2x_i\beta_2)[\frac{w_i^n}{1+sr_i} - \frac{v_i^n}{1+gr_i}] \]
\[ K_i = \exp\left[-(\beta_0 + \eta_{i0} + x(\beta_1 + \eta_{i1}) + x_i^2\beta_2 + z_i\gamma)\right] \]

where the subscript \( i \) refers to the \( i \)th taxpayer in the sample. Recalling that the perceived audit probability function is logistic, the term \( K_i \) is recognized as the odds that taxpayer \( i \) will escape audit; i.e., \((1-p_i(\bullet)/p_i(\bullet))\).

Using the above definitions the first-order condition for taxpayer \( i \) may be expressed as

\[ A_i + \frac{B_i}{1+K_i} + \frac{C_i K_i}{(1+K_i)^2} \geq 0 \]

where the first-order condition is satisfied as an equality if the taxpayer reports an amount \( x_i \) less than true income \( y_i \) and as an inequality (evaluated at \( x_i = y_i \)) if the taxpayer reports his true income. In the latter case, the term \( C_i \) reduces to zero and the likelihood function is defined in terms of the probability that \( K_i \) is less than \(-\frac{(A_i+B_i)/A_i\} \) where \( A_i \) and \( B_i \) are evaluated at the point where \( x_i = y_i \). Making use of the fact that \( \eta_{0i} \) and \( \eta_{1i} \) are (truncated) bivariate normal random variables, the likelihood function for this case may be expressed as
\[
L_i = \frac{D_i}{E_i}
\]

\[
f_i = \frac{\eta_{i0} - \mu_1}{\sigma_1}
\]

\[
h_i = \frac{\beta_0 + x_i(\beta_1 + \eta_{i0}) + x_i^2 \beta_2 + z_i \gamma + \ln\left(-\frac{A_i + B_i}{\theta_i}\right) + \mu_0}{\sigma_0}
\]

\[
D_i \text{ is the integral over } \eta_{i0} \text{ from } -\infty \text{ to } -(\beta_1 + 2\beta_2 y_i) \text{ of the expression}
\]

\[
\frac{1}{\sigma_1} \phi(f_i) \Phi\left(\frac{h_i + \eta_{i0}}{\sqrt{1-\rho^2}}\right)
\]

\[
E_i = \Phi\left(\frac{-\beta_1 + 2\beta_2 y_i + \mu_1}{\sigma_1}\right)
\]

where \( \phi(\bullet) \) and \( \Phi(\bullet) \) are the standard normal probability density and cumulative density functions, respectively.

For the case in which taxpayer \( i \) chooses to report an amount \( x_i \) less than \( y_i \), the first-order condition may be rewritten as

\[(A_i + B_i) + (2A_i + B_i + C_i)K_i + A_iK_i^2 = 0\]

which may be recognized as a quadratic equation in \( K_i \). It can be shown that one of the two roots of this equation can be ruled out, allowing us to express \( K_i \) as a unique function of \( A_i, B_i, \) and \( C_i \). Calling this solution \( K_i^* \) and noticing that the random disturbance \( \eta_{0i} \) appears only in the definition of the term \( K_i \), which was presented at the beginning of this appendix, we may express \( \eta_{0i} \) in terms of \( \eta_{1i}, x_i, K_i^*, z_i \), and the parameters of the model as follows:

\[
\eta_{0i} = -\left[\beta_0 + x(\beta_1 + \eta_{i0}) + x_i^2 \beta_2 + z_i \gamma + \ln(K_i^*)\right]
\]

The likelihood function \( L_i \) for this case may therefore be expressed as

\[
\frac{1}{\sigma_0 \sigma_1} \J_i
\]

\[
\J_i \text{ is the integral over } \eta_{i0} \text{ from } -\infty \text{ to } -(\beta_1 + 2\beta_2 y_i) \text{ of}
\]

\[
\text{pdfbvn}\left[\frac{\eta_{0i} - \mu_0}{\sigma_0}, \frac{\eta_{i0} - \mu_1}{\sigma_1}, \rho\right] \frac{d\eta_{0i}}{dx_i}
\]

where \( \text{pdfbvn}(\bullet, \bullet, \bullet) \) represents the bivariate standard normal probability density function, and \( d\eta_{0i} / dx_i \) is the Jacobian term that relates changes in \( \eta_{0i} \) to changes in \( x_i \).

There are two important considerations in maximizing the likelihood function. First, it is necessary to check that the taxpayer's second-order is satisfied at the solution to his
first-order condition. We have determined that for certain configurations of the disturbances \( \eta_{01} \) and \( \eta_{11} \), the second-order condition will be violated for some taxpayers. For these configurations the contribution to the likelihood function must be set to zero. Second, there is an upper bound for the guilt parameter \( g \) for taxpayers who choose to underreport their income; namely, the value \( aw/[w-t(x)-a(t(y)-t(x))] \). If \( g \) exceeds this bound, the marginal change in expected utility from underreporting the last dollar of income is negative, even if the probability of audit is zero, which implies that the taxpayer would want to report an amount greater than the observed amount \( x \).
References


