Application of kurtosis to underwater sound\textsuperscript{a)}

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ABSTRACT:

Regulations for underwater anthropogenic noise are typically formulated in terms of peak sound pressure, root-mean-square sound pressure, and (weighted or unweighted) sound exposure. Sound effect studies on humans and other terrestrial mammals suggest that in addition to these metrics, the impulsiveness of sound (often quantified by its kurtosis $\beta$) is also related to the risk of hearing impairment. Kurtosis is often used to distinguish between ambient noise and transients, such as echolocation clicks and dolphin whistles. A lack of standardization of the integration interval leads to ambiguous kurtosis values, especially for transient signals. In the current research, kurtosis is applied to transient signals typical for high-power underwater noise. For integration time $(t_2 - t_1)$, the quantity $(t_2 - t_1)/\beta$ is shown to be a robust measure of signal duration, closely related to the effective signal duration, $T_{\text{eff}}$, for sounds from airguns, pile driving, and explosions. This research provides practical formulas for kurtosis of impulsive sounds and compares kurtosis between measurements of transient sounds from different sources.

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I. INTRODUCTION

There is concern about the risk of noise-induced effects on aquatic life exposed to high-intensity anthropogenic noise. Regulations are being developed for both impulsive and continuous underwater noise. Existing regulations and regulatory guidance are typically formulated in terms of peak sound pressure, root-mean-square (rms) sound pressure, and weighted or unweighted sound exposure (Andersson \textit{et al.}, 2017; BSH, 2013; Dekeling \textit{et al.}, 2014; NMFS, 2018). Studies of the effects of sound on humans and other terrestrial mammals suggest that in addition to these metrics, the consideration of the impulsiveness of fatiguing sound can improve the prediction of risk of hearing loss or other adverse effects. Impulsive sound has often been considered more damaging to the auditory system than a more stable sound of the same average intensity (e.g., Henderson and Hamernik, 1986; Melnick, 1991).

Following the impulsive sound definition by ISO 1996-1 (ISO, 2016, §3.4.8), impulsiveness implies a large variation in amplitude. The kurtosis, which can be interpreted as a measure for the variation in amplitude (Moors, 1986), is used to reflect the impulsiveness of sound and the resulting additional risk of physiological impact (Fuente \textit{et al.}, 2018; Goley \textit{et al.}, 2011; Hamernik and Qiu, 2001; Liu \textit{et al.}, 2015). For example, exposure to impulsive signals with a higher degree of kurtosis has been shown to induce more hair cell loss in chinchillas than exposure to a signal with a lower degree of kurtosis for a similar sound exposure (Hamernik and Qiu, 2001; Lei \textit{et al.}, 1994; Qiu \textit{et al.}, 2006). For factory workers as well, chronic exposure to sound with a higher value of kurtosis is correlated with a higher prevalence of measured hearing loss (Zhao \textit{et al.}, 2010). As a result of these findings, a correction factor taking kurtosis into account has recently been proposed (Xie \textit{et al.}, 2016) to refine the long-standing equal-energy rule for human noise exposure criteria (Roberto \textit{et al.}, 1985).

The concept of impulsiveness has a variety of definitions, often arbitrary or open to interpretation (Southall \textit{et al.}, 2007, p. 412). The ISO Online Browsing Platform lists many possible definitions for “impulsive noise” and “impulsive sound,” some of which use purely qualitative descriptions such as “brief bursts of sound.” Martin \textit{et al.} (2020) provide an overview of definitions of impulsivity currently used in the field of underwater sound assessment. The first steps made by Zhao \textit{et al.} (2010) and Goley \textit{et al.} (2011) to include kurtosis in a noise exposure criterion may facilitate defining a criterion which may not need distinguishing between impulsive and non-impulsive sources since the prevalence of periods with relatively high exposure can be captured in a more quantitative and less subjective manner. In addition to its potential application to estimate risk of hearing injury, kurtosis has been used in underwater acoustics to simulate underwater noise (Traverso \textit{et al.}, 2012; Webster, 1994) and for detection in noise of echolocation clicks (Gervaise \textit{et al.}, 2010), dolphin whistles (Millioz and Martin, 2010), and random signals (Dwyer, 1984).
In studies that measured kurtosis, continuous sound exposures were investigated, which had similar acoustic energy but different degrees of impulsiveness (Goley et al., 2011; Hamernik and Qiu, 2001; Lei et al., 1994; Qiu et al., 2006; Zhao et al., 2010). To understand the role of sound impulsiveness on the physiology of aquatic species, it is desirable to understand the degrees of kurtosis for anthropogenic activities that generate intermittent impulsive signals underwater, such as seismic surveys, underwater explosions, sonar exercises or impact pile driving, or continuous signals with different degrees of impulsiveness.

With some exceptions (Amaral et al., 2020; Kastelein et al., 2017; Martin et al., 2020), kurtosis has not been reported systematically for underwater sound sources, complicating interpretation of the calculation as well as its application in quantifying a risk of impact. The validity of the equal energy hypothesis for underwater sound exposure for different aquatic species has been a subject of ongoing debate (Finneran, 2015; Halvorsen et al., 2011; Mooney et al., 2009; Popper et al., 2014; Southall et al., 2019). The application of the term “impulsive signal” is ambiguous in the acoustic literature and requires clearer characterizations and delineations to allow comparability between studies (Henderson and Hamernik, 1986; Madsen, 2005; Martin et al., 2019; Southall et al., 2007).

This paper addresses how the concept of kurtosis can be used to improve the characterization of impulsive sounds. Fundamental properties of kurtosis are discussed, as well as the relation between the kurtosis of a series of pulses and the kurtosis of individual pulses. For intermittent sounds, the ways to measure and report kurtosis values are evaluated. From this, we derive recommendations on how kurtosis can unambiguously be evaluated, interpreted, and reported. In particular, for transient signals, it will be useful to report the derived metrics $\beta/(t_2 - t_1)$ and $\beta E/(t_2 - t_1)$ instead of kurtosis itself because these metrics rely less on the choice of integration time and the interpretation of their values is more straightforward. In this paper, “transient” refers to a signal of finite length (Ainslie et al., 2018) so a signal does not need to be impulse-like to qualify as a transient.

Examples of kurtosis values are provided for a variety of underwater sound sources. This paper also addresses some of the ambiguities surrounding kurtosis and “impulse” and proposes an approach to allow for comparability between measurements and studies. Last, it is recommended that the metrics resulting from this approach be investigated for their correlation to the risk of physiological impact on aquatic life.

II. DEFINITION

In its most basic form, the term and concept of kurtosis is familiar in statistics to quantify the degree of the heaviness of the tails (Fig. 1) of a statistical random variable or the distribution of values within a data set. In order to understand kurtosis, first the general statistical definition of kurtosis is considered and then the kurtosis of a sound pressure time series is described.

A. Coefficient of kurtosis

According to ISO 3534-1 (ISO, 2006), the coefficient of kurtosis is the moment of order 4 of the standardized probability distribution of a random variable. For the distribution of a random variable $X$ with mean $\mu$ and standard deviation $\sigma$, the corresponding standardized random variable is $(X - \mu)/\sigma$. The moment of order 4 is the expectation $E[(X - \mu)^4/\sigma^4]$. It is understood that the kurtosis is a statistical metric describing the probability distribution of a variable, not a specific realization or sample of this variable. As a result, the statistics of this variable are understood to be constant. Contrary to acoustic signals, these variables do not have qualities such as “impulsiveness,” and all samples are assumed to be independent of each other. A distribution with heavier tails has a higher coefficient of kurtosis. Since the kurtosis is a quality of standardized version of a
probability distribution, its value is insensitive to scaling (change of standard deviation \( \sigma \)) and shifting (change of mean value \( \mu \)) of the distribution. For example; any uniform distribution has a coefficient of kurtosis equal to 9/5; for any Gaussian distribution it is 3, and for any exponential distribution, it is 9 (Fig. 1). The Bernoulli ("coin toss") distribution has the lowest possible kurtosis of \( \beta = 1 \).

B. Sample coefficient of kurtosis

In the same ISO 3541-1 standard (ISO, 2006), the sample coefficient of kurtosis over \( n \) samples of the variable \( X \) is defined as

\[
\beta_X = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{X_i - \bar{X}}{S_X} \right)^4, \text{ where } S_X^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2. \tag{1}
\]

The ISO standard mentions that some statistical packages may compute the "excess kurtosis" \( \beta_X - 3 \), which is the kurtosis of the distribution at hand (\( \beta \)) minus the kurtosis of the Gaussian distribution (3). Excess kurtosis is sometimes referred to as "degree of kurtosis," or "Fisher kurtosis," as opposed to "Pearson kurtosis," which is simply \( \beta_X \) (Pearson, 1905; SciPy Community, 2019). Furthermore, the sample coefficient of kurtosis [Eq. (1)] is a biased estimator, meaning that it slightly underestimates the coefficient of kurtosis of the underlying distribution, thus statistical packages may implement a bias-free estimator \( \beta_X^* \). This difference is only significant for small numbers of samples and, therefore, not relevant in the context of sound pressure time series with many thousands of samples per second.

To give a few examples, the SciPy Community (2019) offers options to compute kurtosis or excess kurtosis, each with or without bias. The default behavior is biased excess kurtosis. MathWorks, Inc. (2019) computes kurtosis with or without bias, where the default is the biased version. Excel (Microsoft, 2019) only implements a bias-free estimator for the excess kurtosis.

C. Sound pressure kurtosis

In ISO 18405 (ISO, 2017) the sound pressure kurtosis is defined as \( \beta = \mu_4 / \mu_2^2 \), where \( \mu_n \) is the \( n \)th moment of the sound pressure. Specifically, \( \mu_4 = (1/t_2 - t_1) \int_{t_1}^{t_2} (p(t) - \bar{p})^4 \, dt \), the sound pressure variance \( \mu_2 = (1/t_2 - t_1) \int_{t_1}^{t_2} (p(t) - \bar{p})^2 \, dt \), where \( \bar{p} \) is the mean sound pressure at the same time interval (Erdreich, 1986). This expression written out is

\[
\beta = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} (p(t) - \bar{p})^4 \, dt \left( \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} (p(t) - \bar{p})^2 \, dt \right)^{-\frac{1}{2}}, \tag{2}
\]

which is the definition of kurtosis used in this paper. This definition is in line with the expressions for the sample coefficient of kurtosis (as described in Sec. II B) and for the coefficient of kurtosis (Sec. II A).

However, the coefficient of kurtosis in ISO 3534–1 (ISO, 2006) describes a random variable with constant statistics, which can be estimated by taking mutually independent samples from this variable. As a result, the sound pressure kurtosis does not capture any temporal structure of the signal. Effects such as recovery, the influence of the signal rise time, rate of pressure change (positive or negative), or adaptive reduction of hearing sensitivity is, therefore, not necessarily captured by evaluation of the sound pressure kurtosis (Dahl et al., 2020; Finneran, 2015; Nachtigall et al., 2018; Price, 2007).

In certain situations, where more is known about the temporal structure of the signal, kurtosis can be seen as a measure of duty cycle or proxy of impulsiveness. ISO 1996-1 (ISO, 2016), §3.4.8, for instance, defines impulsive sound as characterized by bursts of sound pressure, usually less than 1 s in duration. ISO 18405 (ISO, 2017) does not mention any requirement for the sound pressure to be statistically stationary (a time series with statistical properties that are constant over time).

Except for random white noise, sound pressure signals have a temporal structure and the integrals in Eq. (2) do not represent evaluation over mutually independent samples. For signals that are not statistically stationary, taking more samples does not necessarily lead to convergence of the value of \( \beta \).

D. Interpretation for statistically stationary signals

As noted before, ISO 18405 (ISO, 2017) mentions that the sound pressure kurtosis describes the distribution of values in a pressure signal in the time domain so that it can be used to distinguish between different kinds of white noise. Distributions with a higher value of kurtosis have heavier tails, and often a stronger peak near the mean (greater prevalence of values relatively close to the mean), leading to kurtosis being referred to as the "peakedness" of the signal. However, this need not be the case, as a small number of outliers will have a greater effect on the value of kurtosis than more values close to the mean (DeCarlo, 1997; Westfall, 2014) (Fig. 1).

E. Interpretation for transients

Kurtosis is the moment of order 4 in the standardized probability distribution of a random variable (Sec. II A). In this context, "standardized" means that the value of the kurtosis of a signal does not change when the signal is scaled in amplitude or shifted by an offset \( \bar{p} \). Also, the kurtosis does not change when the signal is scaled in time because scaling in time changes the time series, not the corresponding distribution (Fig. 2).

For nearly all relevant measurements, \( \bar{p} \) will be close enough to zero to be omitted from the expression for
kurtosis [Eq. (2)], so that the expression for kurtosis can be simplified to [Appendix A, Eq. (A1)]

$$\beta \approx \tilde{\beta} \equiv (t_2 - t_1) \frac{\int_{t_1}^{t_2} p(t)^4 \, dt}{\left( \int_{t_1}^{t_2} p(t)^2 \, dt \right)^2}. \quad (3)$$

In the following, a more compact notation is used for the averaging. The integration interval is made explicit by indices where relevant,

$$\langle x \rangle_{(t_1,t_2)} \equiv \frac{\int_{t_1}^{t_2} x \, dt}{t_2 - t_1}, \quad (4)$$

so that $\beta_{(t_1,t_2)} \approx \tilde{\beta}_{(t_1,t_2)} \equiv \langle p^4 \rangle_{(t_1,t_2)}/\langle p^2 \rangle_{(t_1,t_2)^2}$.

Since the difference between $\beta$ and $\tilde{\beta}$ is only significant for very short signals, no distinction has been made between the numerical values of both quantities in Secs. II E, II E 1, and III A, either in tabulated or graphical representation. For mathematical correctness, the distinction was made in derivations.

1. Characterization of an isolated pulse

The kurtosis of a single impulse (e.g., an underwater explosion or a single pile driving strike) depends on the integration interval duration $(t_2 - t_1)$. When more silence is added to the integration interval, the results of the integrals will not change, but the factor $t_2 - t_1$ will change. Thus, for an interval containing a single pulse, the ratio

$$\frac{\tilde{\beta}}{t_2 - t_1} = \frac{\int_{t_1}^{t_2} p(t)^4 \, dt}{\left( \int_{t_1}^{t_2} p(t)^2 \, dt \right)^2}$$

is independent of $(t_2 - t_1)$, which provides a more robust and unambiguous characterization for the kurtosis of an isolated pulse (Fig. 3).

2. Average of $p^2$ weighted by itself

The $w$-weighted average of $f(t)$ over the interval between $t_1$ and $t_2$ is given by

$$\frac{\int_{t_1}^{t_2} f(t) w(t) \, dt}{\int_{t_1}^{t_2} w(t) \, dt}. \quad (6)$$

ISO 18405 (ISO, 2017) defines the sound exposure as

$$E_{p,(t_2-t_1)} = \int_{t_1}^{t_2} p(t)^2 \, dt. \quad (7)$$

This can be combined with Eq. (5) to write

$$\frac{\beta_{E_{p,(t_2-t_1)}}}{(t_2 - t_1)} = \frac{\int_{t_1}^{t_2} p(t)^4 \, dt}{\int_{t_1}^{t_2} p(t)^2 \, dt}. \quad (8)$$

The right-hand side of Eq. (8) is identical to the expression in Eq. (6) when both $f(t)$ and the weighting function $w(t)$ are substituted by $p(t)^2$. Thus, $\beta_{E_{p,(t_2-t_1)}}/(t_2 - t_1)$ can be interpreted as the $p^2$-weighted average of $p^2$ (self-weighted mean of $p^2$).

Since this measure for signal amplitude can be used for transients and intermittent signals without prior interpretation of a “pulse,” $\beta_{E_{p,(t_2-t_1)}}/(t_2 - t_1)$ may allow for investigating both impulsive and non-impulsive sounds using the same metrics, and without the need for differentiation between both categories.

F. Interpretation for intermittent signals

For a transient signal containing only a small number of pulses, the kurtosis is not uniquely defined without stating...
the integration time. As the integration time increases, for a statistically stationary signal, the kurtosis converges to a unique value. For a signal consisting of multiple consecutive pulses (e.g., Fig. 4) of known kurtosis, variance, and duration, the kurtosis of the total signal can be computed as follows:

\[
\beta_{(t_0,t_n)} = (t_n - t_0) \frac{\int_{t_0}^{t_n} p^2 \, dt}{\left(\int_{t_0}^{t_n} p^2 \, dt\right)^{3/2}} \\
= (t_n - t_0) \frac{\sum_{i=0}^{n-1} \langle p^4 \rangle_{(t_0,t_{i+1})}(t_{i+1} - t_i)}{\left(\sum_{i=0}^{n-1} \langle p^2 \rangle_{(t_0,t_{i+1})}(t_{i+1} - t_i)\right)^{3/2}} \\
= (t_n - t_0) \frac{\sum_{i=0}^{n-1} E_{p,(t_0,t_{i+1})}^2 \beta_{(t_0,t_{i+1})}}{\left(\sum_{i=0}^{n-1} E_{p,(t_0,t_{i+1})}^2\right)^{3/2}}, \tag{9}
\]

\[
\beta = \frac{\beta_{t_2 - t_1}}{t_2 - t_1} = 36.6 \text{ ms}^{-1}
\]

\[
\beta = \frac{\beta_{t_4 - t_1}}{t_4 - t_1} = 18.3 \text{ ms}^{-1}
\]

For collections of segments that are described by \(p_{\text{rms}}^2 = \langle p^2 \rangle_{(t_0,t_{i+1})}/(t_{i+1} - t_i)\),

\[
\langle p^2 \rangle_{(t_0,t_{i+1})}(t_n - t_0) = \sum_{i=0}^{n-1} \langle p^2 \rangle_{(t_0,t_{i+1})}(t_{i+1} - t_i), \tag{11}
\]

and since \(\langle p^2 \rangle^2 \beta \equiv \langle p^4 \rangle\),

\[
\langle p^2 \rangle_{(t_0,t_{i+1})}^2 \hat{\beta}_{(t_0,t_{i+1})}(t_n - t_0) = \sum_{i=0}^{n-1} \langle p^2 \rangle_{(t_0,t_{i+1})}^2 \hat{\beta}_{(t_0,t_{i+1})}(t_{i+1} - t_i). \tag{12}
\]

Sections of the compound signal where the acoustic pressure is zero throughout can be left out of the summation. When all \(n\) pulses are identical, a single pulse is characterized by \(E_{p,\text{pulse}} = E_{p,(t_0,t_{i+1})}\) and \(\hat{\beta}_{(t_0,t_{i+1})}/(t_{i+1} - t_i) = \beta_{\text{pulse}}/\Delta t_{\text{int, pulse}}\). Equation (10) then simplifies to

\[
\frac{E_{p,(t_0,t_{i+1})}^2 \beta_{(t_0,t_{i+1})}}{t_{i+1} - t_i} = \sum_{i=0}^{n-1} \frac{E_{p,(t_0,t_{i+1})}^2 \hat{\beta}_{(t_0,t_{i+1})}}{t_{i+1} - t_i}. \tag{10}
\]

\[
\beta = 24.9 \text{ kPa/m} \text{s}^{2}
\]

\[
\beta = 49.8 \text{ kPa/m} \text{s}^{2}
\]

FIG. 3. (Color online) The effect of integration time on the kurtosis and kurtosis divided by integration time. For isolated pulses, kurtosis depends on the adopted integration time, as shown by the value of kurtosis listed under the left and middle panels. However, when divided by the integration time, it becomes a robust measure (left and middle panel). Multiple identical pulses after each other (right panel) have the same value for kurtosis as a single pulse (left panel), but kurtosis divided by integration time decreases as more pulses are added.

FIG. 4. (Color online) The kurtosis for a train of pulses, where for each pulse \((t_{i+1} - t_i)\), \(\langle p^2 \rangle_{(t_0,t_{i+1})}\), and \(\hat{\beta}_{(t_0,t_{i+1})}\) are known, can be computed according to Eq. (10). When all pulses are identical, the expression simplifies to Eq. (13).
\[
E^2_{\text{p},(t_0,t_n)} \frac{\bar{\beta}_{(t_0,t_n)}}{t_n - t_0} = \sum_{i=0}^{n-1} E^2_{\text{p},(t_i,t_{i+1})} \frac{\bar{\beta}_{(t_i,t_{i+1})}}{t_{i+1} - t_i}
\]
\[
(nE^2_{\text{p,pulse}}) \frac{\bar{\beta}_{(t_0,t_n)}}{t_n - t_0} = nE^2_{\text{p,pulse}} \frac{\bar{\beta}_{\text{pulse}}}{\Delta t_{\text{int,pulse}}}
\]
\[
t_n - t_0 \frac{\Delta t_{\text{int,pulse}}}{\bar{\beta}_{(t_0,t_n)}} = n \frac{\Delta t_{\text{int,pulse}}}{\bar{\beta}_{\text{pulse}}}. \tag{13}
\]

For a sequence of \(n\) pulses with a ratio \(\bar{\beta}_{\text{pulse}}/\Delta t_{\text{int,pulse}}\) per pulse, the kurtosis for the sequence is therefore inversely proportional to the pulse repetition rate \(n/(t_n - t_0)\).

**III. APPLICATION TO REALISTIC ANTHROPOGENIC SIGNALS**

Typical pressure time series of four different sounds were considered (Fig. 5) in order to compare kurtosis of different high-energy underwater sound sources:

- A hypothetical 1 s duration single-frequency (1 kHz) sinusoidal sonar transmission, used, e.g., in active search sonar (D’Amico and Pittenger, 2009).
- A pulse from marine pile driving of a monopile installed for the construction of a wind farm in the North Sea, without noise abatement measures, recorded at a range of 750 m in shallow (28.20 m) water (ITAP, 2015).
- An underwater explosion of 265 kg trinitrotoluene (TNT)-equivalent charge mass, measured at 982 m in shallow (between 26 and 28 m) water (von Benda-Beckmann et al., 2015).

For these four signals, the kurtosis was evaluated over an interval that was chosen large enough (2 s) to cover the longest of the example signals (Fig. 5 and Table I). For relatively small integration times (compared to the pulse repetition time), it was found that the kurtosis varied significantly and depended on the chosen start and end time (Fig. 6, left pane). However, the kurtosis converged with increasing integration time (Fig. 6, right pane). After every additional signal cycle, the value of the running evaluation of the kurtosis up to that time was 15 (the converged value). In between the full cycles, the deviations from this converged value (either up or down, depending on where the evaluation was started) declined in amplitude, inversely proportional to the integration time. The integration time required for the convergence of \(\beta\) depended on repetition rate and signal type and ranged between a period of approximately 20 s (pile driving) up to 100 s (Figs. 6 and 7).

![Graphs showing the kurtosis evaluation for different signals](https://doi.org/10.1121/10.0001631)

**FIG. 5.** (Color online) Four examples of anthropogenic underwater noise (left to right; top to bottom): airgun, impact pile driving (signal zero padded), sonar and airgun, and explosion (clearance of underwater unexploded ordnance, UXO; signal zero padded). The amplitudes (irrelevant to the analysis of kurtosis) are normalized by their maximum values.
between Appendix B). For a sinusoidal signal, constant in amplitude is constant, in the terminology of that paper, to characterize the time spread of the impulse response in forward scattering. In sonar signal processing, the temporal extent of a signal duration, closely related to the effective signal duration, (ISO, 2017; Burdic, 1984). In his Eqs. (2) and (3), Dahl (2001) used a measure corresponding to (s_\text{eff}/C_0). This is quantified by the effective signal duration (s_\text{eff}). In sonar signal processing, the temporal extent of a signal type, and extended between 20 s (pile driving) and approximately 100 s.

### 1. Comparison to \( \tau_{\text{eff}} \)

In sonar signal processing, the temporal extent of a signal is related to its frequency-spectral resolving capability. This is quantified by the effective signal duration (\( \tau_{\text{eff}} \)). For a sinusoidal signal, constant in amplitude between \( t_1 \) and \( t_2 \) and zero outside, the effective signal duration is \( \tau_{\text{eff}} = t_2 - t_1 \).

### A. Relationship between \( (t_2 - t_1)/\beta \), \( \tau_{\text{eff}} \), and \( \tau_{90\%} \)

Sound pressure kurtosis in the context of acoustics has been used in different forms. In Sec. II E 2 it is shown that \( \langle t_2 - t_1 \rangle/\beta \) can be interpreted as a weighted measure for the signal duration, closely related to the effective signal duration, \( \tau_{\text{eff}} \) (ISO, 2017; Burdic, 1984). In his Eqs. (2) and (3), Dahl (2001) used a measure corresponding to \( (t_2 - t_1)/\beta \), expressed in the terminology of that paper, to characterize the time spread of the impulse response in forward scattering. For a continuous harmonic signal such as \( p(t) = RA \exp (12\pi ft) = A \cos (2\pi ft) \) the ratio \( \tau_{\text{eff}}/\langle (t_2 - t_1)/\beta \rangle \) is constant,

\[
\frac{\tau_{\text{eff}}}{(t_2 - t_1)/\beta} = \left( \frac{\int_{-\infty}^{+\infty} |p_{\text{cont}}(t)|^2 dt}{\int_{-\infty}^{+\infty} |p_{\text{cont}}(t)|^4 dt} \right)^2 / \left( \frac{\int_{t_1}^{t_2} |p(t) - \bar{p}|^2 dt}{\int_{t_1}^{t_2} |p(t) - \bar{p}|^4 dt} \right)^2 ,
\]

where \( |p_{\text{cont}}| = A \) and \( \bar{p} = 0 \), \( t_1 = -\infty \) and \( t_2 = \infty \).

### TABLE I. Metrics describing four examples of anthropogenic impulsive sounds shown in Fig. 5.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Kurtosis ( \beta ) for 2 s interval</th>
<th>Repetition time/s</th>
<th>Converged kurtosis [Eq. (13)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sonar</td>
<td>3.0</td>
<td>10</td>
<td>150</td>
</tr>
<tr>
<td>Pile driving</td>
<td>72.8</td>
<td>1.5</td>
<td>54.6</td>
</tr>
<tr>
<td>Airgun</td>
<td>122.1</td>
<td>10</td>
<td>610.5</td>
</tr>
<tr>
<td>Explosion in shallow water</td>
<td>25.1</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

### FIG. 6. (Color online) Left two panels: 10% duty cycle of a continuous sine wave (“sonar” in Fig. 5) starting with either signal (top) or silence (bottom). Right panel: running evaluation of the sound pressure kurtosis from \( t = 0 \). Both signals converge to the same value of 1.5/(10%) = 15. Even after many cycles, the variation of \( \beta \) may still be significant. The variation of \( \beta_{(\text{sonar})} \) is inversely proportional to the integration time.

### FIG. 7. (Color online) Dependence of the kurtosis of a sequence of signals on the integration time for three different anthropogenic sound signals (sonar, pile driving, and airgun). The amount of integration time required to obtain convergence of \( \beta \) for these signals depended on signal type, and extended between 20 s (pile driving) and approximately 100 s.

\[
\frac{\tau_{\text{eff}}}{(t_2 - t_1)/\beta} = \frac{\int_{-\infty}^{+\infty} A^2 dt}{\int_{-\infty}^{+\infty} A^4 dt} \left( \frac{\int_{-\infty}^{+\infty} (A \cos 2\pi ft)^2 dt}{\int_{-\infty}^{+\infty} (A \cos 2\pi ft)^4 dt} \right)^2 = \frac{3}{2} .
\]

To investigate this relationship for other signals, the correlation between \( (t_2 - t_1)/\beta \) and \( \tau_{\text{eff}} \) was investigated for the impulsive signals discussed earlier in this section, as well as played back pile driving sounds for impact-assessment studies on fish (Halvorsen et al., 2011; labeled as “pile driving (playback)”) (Fig. 8).

It was found that there is a strong correlation between \( \tau_{\text{eff}} \) and \( (t_2 - t_1)/\beta \), showing a ratio consistently close to 3/2. The quantity \( \tau_{90\%} \), however, showed much less correlation with \( \tau_{\text{eff}} \) and \( (t_2 - t_1)/\beta \). This means that measurements of \( \tau_{90\%} \) cannot be used to estimate \( \tau_{\text{eff}} \) and \( (t_2 - t_1)/\beta \) and vice versa.
B. Other uses of the term “kurtosis”

Section II already mentioned alternative interpretations for the term “kurtosis.” Many more definitions found in acoustical and statistical literature are listed in the supplemental material.1 Not all papers reporting kurtosis clearly specify how the kurtosis was defined or computed. From this survey, it is clear that more consistent reporting and calculating of kurtosis would reduce confusion. For the purpose of this study, we investigated the kurtosis of different underwater noise sources, using the ISO 18405 (ISO, 2017) definition [Eq. (2)].

The definition given by ISO 18405 (ISO, 2017) [Eq. (2)] was used by some authors on the study of human noise exposure in the supplementary document (DeCarlo, 1997; Hamernik and Qiu, 2001; Webster, 1994). However, other investigators chose to report excess kurtosis \( \beta - 3 \) (DeCarlo, 1997; Erdreich, 1986; Plug et al., 2002). Few studies reported the bandwidth of the signal under consideration. Busson et al. (2010), Dwyer (1981, 1984), and Hamernik et al. (2003) computed the kurtosis over band-passed channels from a filter bank. Hamernik et al. (2003) used octave-band filters and referred to the result as “frequency-specific kurtosis (\( \beta - 3 \)).” Busson et al. (2010) used different (linearly-spaced) band filters. Most studies measured kurtosis over the time pressure series, in line with the definition of sound pressure kurtosis by ISO 18405. However, sometimes kurtosis was measured on the temporal variation of spectral bins from the Fourier-transformed sound pressure, which was then referred to as “frequency domain kurtosis” (Dwyer, 1981, 1984). Antoni (2006) and Randall and Antoni (2011) interpret this “frequency domain kurtosis” as the kurtosis over the complex envelope of the acoustic pressure within narrow frequency bands.

Finally, the intuitive interpretation of “kurtosis” can be explained in various ways, leading to further confusion, as mentioned by DeCarlo (1997, p. 300). Kurtosis may be interpreted solely in terms of peakedness, without mention of the importance of the tails of the pressure distribution (DeCarlo, 1997, p. 294). The relation between peak and tails of distribution with excess kurtosis (\( \beta > 3 \)) may be described or illustrated incorrectly. Variance may be

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FIG. 8. (Color online) Comparison between measures for signal duration for four datasets (left to right; top to bottom): clearance of underwater unexploded ordnance (UXO) (von Benda-Beckmann et al., 2015); played back pile driving sounds for impact-assessment studies on fish (Halvorsen et al., 2011); pulses from marine pile driving (ITAP, 2015); and pulses from various airguns (Lundsten, 2010). Yellow and cyan give a qualitative indication of the regions with a higher density of data points.
confused with kurtosis and the influence of tails may be described opposite of what they should be (DeCarlo, 1997, p. 295).

For a more in-depth overview of common uses of the term “kurtosis,” the reader is referred to the supplementary document. This paper relates the kurtosis of a sequence of multiple transient sounds to the kurtosis of individual transient sounds and describes some mathematical properties of the kurtosis of the individual transients. We have chosen to follow the definition of kurtosis from ISO 18405 (ISO, 2017), but many other definitions are used, often without specifying which definition is followed. To avoid unnecessary ambiguity in results and conclusions, when not following the ISO definition, a precise definition of the metrics used should always be given.

IV. DISCUSSION

Unlike the impact of noise on humans, understanding the possible effects of underwater noise on aquatic life is still in early development (Fields et al., 2019; McCauley et al., 2017; Popper et al., 2014; Richardson et al., 1995; Southall et al., 2007; Southall et al., 2019). Kurtosis has been proposed to improve the prediction of risk of auditory impairment due to impulsive sound, compared to considering only the sound exposure (Goley et al., 2011; Hamernik and Qiu, 2001; Southall et al., 2007; Zhao et al., 2010). In this context, kurtosis is used as a metric quantifying the presence of impulses. Another application of this interpretation is the detection of impulses by automated signal classification, such as identification of echolocation clicks (Gervaise et al., 2010). To assess the applicability of the kurtosis metric to underwater sound, studies need to measure and report kurtosis in a consistent manner.

A review of the use of kurtosis in acoustical and statistical literature showed that the term is used in a variety of ways, leading to ambiguity in interpretation when it is not defined for each use. In this paper, we followed the definition of ISO 18405 (ISO, 2017), and our results were general enough to also apply to most other definitions of kurtosis as well. In other papers, most of the variables referred to as “kurtosis” can easily be converted to comply with the ISO definition. When kurtosis is computed over a variable other than sound pressure \( p(t) \), the results of this paper will also apply to that variable.

The definition of ISO 18405 (ISO, 2017) is based on the expression \( \bar{p} \). For a measurement, the recorded representation of \( p \) and, therefore, also of \( \bar{p} \) could depend strongly on the high-pass filter implemented in the measurement system used. When \( \bar{p} \) is not negligible, it is worthwhile stating this in the measurement results. More generally, if the bandwidth of the signal is not fully supported by the measurement system, this should be stated. Furthermore, the integration interval should be stated. For transient signals, bounded in time, the integration time can be divided out by reporting \( \beta_1/(t_2 - t_1) \). For periodic signals, the period of the signal should be mentioned, and ideally, the integration interval should be a multiple thereof so that the resulting value of kurtosis is identical to the converged value over longer time.

This study shows that for non-stationary sounds the kurtosis itself is not a well-defined quantity, but requires additional information (measurement duration) to unambiguously describe the signal. A practical rule is provided [Eq. (9)] to apply kurtosis to intermittent impulse sounds, which are typical for exposures from underwater sound sources such as pile driving and air gun exposures.

To assess whether kurtosis provides improved description and understanding of the impact of underwater sound requires dedicated studies and systematic comparisons of effects studies with different sound types. Studies in marine mammals are limited to induced temporary hearing threshold shift (TTS) (below 30 dB) that can be investigated for application of kurtosis (Finneran, 2015; Kastelein et al., 2017). However, ethical considerations preclude investigation of the empirical relation between exposure metrics (sound exposure, peak sound pressure, kurtosis) and long-term hearing impairment, or loss of hair cells in aquatic mammals. On the other hand, experimental studies have demonstrated TTS (Halvorsen et al., 2013; Halvorsen et al., 2012; Popper et al., 2005a; Popper et al., 2005b), injury, and mortality (Casper et al., 2013; Halvorsen et al., 2011; Halvorsen et al., 2012) in fishes, and suggested injury and mortality in invertebrates (André et al., 2011; Day et al., 2019; McCauley et al., 2017; Solé et al., 2016), which can be used to investigate possible influence of kurtosis on risk of injury or long-term hearing impairment.

In Sec. II.E.2, it is shown that \( (t_2 - t_1)/\beta \) can be interpreted as a weighted measure for the signal duration, closely related to the effective signal duration \( \tau_{eff} \). Also, \( \beta \bar{p}_E(p(t_1), t_2 - t_1) \) can be interpreted as the \( p^2 \)-weighted average of \( p^2 \). Although kurtosis is closely related to a measure of the duration of a signal, we found that generally kurtosis cannot be derived from quantities such as the 90% exposure duration, which are commonly reported in underwater noise studies. If kurtosis is found to improve the prediction of risk of auditory impact in aquatic species, it is recommended to report kurtosis more systematically (Madsen, 2005). Evaluating the effective signal duration or \( (t_2 - t_1)/\beta \) may also have a use apart from estimating physiological effects. In measurements of marine impact pile driving using multiple acoustic metrics (Ainslie et al., 2019) it was found that the effective signal duration (or kurtosis) results in a tighter relationship between sound exposure level (SEL) and sound pressure level (SPL), when SPL is determined over the effective signal duration, compared to when it is determined over the 90% duration. SPL is typically used to characterize the risk of disturbance to marine mammals (e.g., Daly and Harrison, 2012), and when SEL is known, SPL can be predicted (Ainslie et al., 2019) based on an empirical relationship between SPL and SEL.
When reporting kurtosis in the context of underwater sound, we propose the following four guidelines to ensure harmonized and unambiguous interpretation:

1. Follow the ISO 18405 definition (ISO, 2017) of sound pressure kurtosis.
2. State and motivate integration interval clearly; preferably use the repetition time for periodic intermittent signals.
3. For isolated pulses, report the ratio \( \beta / (t_2 - t_1) \) instead of \( \beta \) itself.

Kurtosis is usually computed from the broadband sound pressure. However, to be relevant for understanding the possible physiological impact for a faunal group, it might need to be evaluated in a frequency range relevant to the hearing range of that group. One way of achieving this is to calculate the kurtosis of the auditory frequency-weighted sound pressure (Antoni, 2006; Lee and Seo, 2013; Martin et al., 2020; Southall et al., 2019). The influence of the bandwidth and phase response of filters used for this purpose on the value of kurtosis remains to be investigated.

A methodology to measure and apply alternative metrics (\( \bar{\beta} \), \( \tau_{\text{eff}} \)) is provided. However, investigation of the correlation between the metrics from this approach and the risk of physiological impact from impulsive noise will be required prior to the application of these terms in acoustic criteria for aquatic life.

**V. CONCLUSIONS**

This study demonstrates how the kurtosis (\( \beta \)) of underwater sounds can be measured unambiguously. For a transient signal, the kurtosis depends on the integration time. However, the ratio \( \bar{\beta} / (t_2 - t_1) \) of a transient is independent of integration time.

The effective signal duration \( \tau_{\text{eff}} \) is closely related to the kurtosis of a signal. For a wide range of observed signals, we find that the ratio \( \tau_{\text{eff}} / [(t_2 - t_1) / \beta] \) is close to a constant (3/2). The effective duration differs from the 90% exposure duration of the signal, although it could be estimated from \( \tau_{\text{eff}} \) if known.

It was found that a simple relationship can be used to scale the kurtosis of a single transient signal to that of repetitive signals, which we illustrated using different examples of underwater sound signals. Our results can be used to facilitate comparability between studies and harmonize reporting of characteristics of underwater sound.

**APPENDIX A: TREATMENT OF \( \bar{\beta} \)**

Most measurement systems include a high-pass filter so that \( \bar{\beta} \) will converge to zero for any measurement of sufficient length. Even when the sound pressure impulse \( J_p = \int p(t) \, dt \) is not equal to zero, for increasing integration time, \( \bar{\beta} \) of a transient will tend asymptotically to zero. For example, for a Gaussian pulse \( p(t) = p_0 e^{-t^2/t_0^2} \), the total pressure impulse \( J_p = \int p(t) \, dt \) is

\[
J_p = \int_{-\infty}^{\infty} p(t) \, dt = p_0 \sqrt{\pi}, \quad \text{and} \quad \bar{\beta} = J_p / \infty = 0.
\]

Using this observation, the expression for kurtosis for such measurements can be simplified to

\[
\beta \approx \bar{\beta} = \left( \frac{t_2 - t_1}{t_0} \right) \left( \frac{\int_0^{t_0} p(t)^4 \, dt}{\left( \int_0^{t_0} p(t)^2 \, dt \right)^2} \right),
\]

or using the more compact notation from Sec. IIE,

\[
\beta_{(t_1,t_2)} \approx \bar{\beta}_{(t_1,t_2)} \equiv \frac{(\langle p^4 \rangle_{(t_1,t_2)})}{(\langle p^2 \rangle_{(t_1,t_2)})^2}.
\]

For very short integration times, relative to the period of the pressure fluctuations in the signal, there can be a significant discrepancy between kurtosis computed by its formal definition [Eq. (2)] and the approximation from Eq. (A1) (illustrated for a sine wave in Figs. 9 and 10). However, as the integration time is increased, the discrepancy converges to zero, inversely proportional to the number of cycles in the signal (Fig. 10).

**FIG. 9.** (Color online) Left pane: running evaluation of kurtosis \( \beta_{(t_0,0)} \) and \( \bar{\beta}_{(t_0,0)} \) = \( (\langle p^4 \rangle_{(t_0,0)}) / (\langle p^2 \rangle_{(t_0,0)}^2) \), which both converge to 1.5 for a continuous sine wave. Right pane: \( \rho \), running evaluation of \( \rho \) and \( \rho - \sigma \) for the same signal.
APPENDIX B: RELATION BETWEEN $\tau_{\text{eff}}$ AND $\beta/(t_2 - t_1)$

We compared kurtosis to $\tau_{\text{eff}}$ because the mathematical definition of $\tau_{\text{eff}}$ looks very similar to $1/\beta$ [Eq. (A1)], and because both can be used to define a signal-weighted average [see Eq. (6)] of the signal amplitude. The full definition of the effective signal duration $\tau_{\text{eff}}$ listed in ISO 18405 (ISO, 2017) is

$$\tau_{\text{eff}} = \frac{\left( \int_{-\infty}^{+\infty} |p_{an}(t)|^2 \, dt \right)^2}{\int_{-\infty}^{+\infty} |p_{an}(t)|^4 \, dt}, \quad \text{(B1)}$$

where $p_{an}(t)$ is the analytic representation of the sound pressure signal (Burdic, 1984),

$$p_{an}(t) = p(t) + i\Delta p(t). \quad \text{(B2)}$$

The main difference with $1/\beta$ is the use of the complex envelope $|p_{an}(t)|$ instead of pressure $p(t) = \Re p_{an}(t)$.

When $\beta = 0$, then $\int_{-\infty}^{+\infty} |p_{an}(t)|^2 \, dt = 2 \int_{0}^{t} |p(t)|^2 \, dt$, it then follows that $E_p/(t_1, t_2) = \int_{0}^{t} |p_{an}(t)|^2 / 2 \, dt$. Therefore, $E_p/\tau_{\text{eff}}$ can be rewritten in the form of the expression in Eq. (8) as

$$\frac{E_p}{\tau_{\text{eff}}} = \frac{\left( \int_{-\infty}^{+\infty} |p_{an}(t)|^2 / 2 \right)^2}{\int_{-\infty}^{+\infty} |p_{an}(t)|^2 / 2 \, dt}, \quad \text{(B3)}$$

which can be interpreted to be the $|p_{an}(t)|^2 / 2$-weighted average of $|p_{an}(t)|^2 / 2$.

The definitions of $\tau_{\text{eff}}$ and $(t_2 - t_1)/\beta$ look similar [Eqs. (2) and (B1)]. Both can be used to define a (whether in analytic or real) signal-weighted average of the signal amplitude [Eqs. (3) and (B3)]. Similar to $\tau_{\text{eff}}$, the term $(t_2 - t_1)/\beta$ can be interpreted as a measure for signal duration. For a continuous harmonic signal \(p(t) = \Re A \exp(i2\pi ft) = A \cos(2\pi ft)\) the ratio $\tau_{\text{eff}}/(t_2 - t_1)/\beta$ has a fixed value, $\left(\int_{-\infty}^{+\infty} |p_{an}(t)|^2 \, dt \right)^2 / \int_{-\infty}^{+\infty} |p_{an}(t)|^4 \, dt$.

where $|p_{an}| = A$ and $\beta = 0$, $t_1 = -\infty$ and $t_2 = \infty$,

$$\frac{\tau_{\text{eff}}}{(t_2 - t_1)/\beta} = \frac{\left( \int_{-\infty}^{+\infty} |p_{an}(t)|^2 \, dt \right)^2}{\int_{-\infty}^{+\infty} |p_{an}(t)|^4 \, dt} / \frac{\left( \int_{-\infty}^{+\infty} |p(t)|^2 \, dt \right)^2}{\int_{-\infty}^{+\infty} |p(t)|^4 \, dt},$$

$\text{1See supplementary material at https://doi.org/10.1121/10.0001631 for an overview of the use of the term “kurtosis” found in statistical and acoustical literature.}$


