I have two conflicting interests. One is computational design and digital fabrication—commonly assumed to be rapid, fashionable, and surface-based. The other is volume—thick, heavy, stone, compression-only permanent structures. Much of my previous research\(^1\) dealt with the problem of creating volumetric occupation from economically friendly sheet material. More recently this desire has formalized into stereotomic research with such projects as *Periscope: Foam Tower\(^2\)* and my Princeton thesis *Temporal Tenancy*. These projects mined the past knowledge of stereotomy—the technique of precisely cutting solids to specific forms and dimensions—as a way to robotically carve foam for temporary installations. The irony of these projects is they apply knowledge from heavy stone construction to light temporary projects which required tensile cables to stand. These exercises in carving solids could also be applied to materials with significant mass as a way to re-engage the thick, heavy, and permanent compression-only architecture of the past.

The majority of contemporary research in compression-only structures focuses on the ideal—super-thin, form-found, catenary or funicular shaped geometry. This approach considers just one variable: structure. Architecture, on the other hand, has to navigate a number of concerns, from acoustics, to formal concerns, and, yes, structure. How is a gothic cathedral able to stand without the constraint of this ideal catenary geometry? The answer is volume. With the aid of variable thickness, the gothic stonemason was able to re-direct the thrust vector back inside the thickness of the column to ensure the cathedral would stand.

The following inquiry is intended to marry my two interests of digital fabrication and volume. It proposes a method of automating and calculating the most efficient variable-depth required to ensure any geometry will result in a compression-only structure. This method is dedicated to addressing architectural concerns with structural results. I am not advocating for the reversion to a past architecture. I am promoting the insertion of lost knowledge into our current means and methods of making.

Abstract
Particle-spring systems are commonly used to develop compression-only form finding systems. We propose to use a particle-spring system to respond to a desired form in order to generate a variable depth, compression-only structure. As a variable depth system, loads can be re-directed through the material in order to result in a desired form as opposed to a structurally optimal form which assumes a uniform-thickness approach. This paper proposes to generate, build, and test, compression-only structures in response to a desired architectural geometry. This research will allow for integration with external programs to input a desired form, and result in a constructible compression-only structure.

1 Introduction
Thin-shell compression-only structural systems are a relatively new phenomenon in the built environment. Compression-only structures on the other hand are an ancient one. Thin-shell structures assume a minimal and consistent cross-section. This assumption is driven by material efficiency. The results of this assumption are forms based exclusively on structure. Architecture does have to navigate structural concerns, but it also has to address a variety of other issues—acoustic, formal, program, etc. It is not necessary that form be driven strictly by structural requirements. For example, Gothic Cathedrals contain the thrust-vector within the variable depth of the stone's cross-section. These Cathedrals are not simply determined by idealized catenary form, but through a confluence of architectural formal desires, with compression only structural principles. With this approach as inspiration, this inquiry addresses the potentials of compression only systems to be resolved through a variable depth system in order to obtain a desired form.

Much research has been done in analyzing existing variable depth structures to determine if a thrust vector falls inside the depth of
the material [Block et al., 2006]. Other methods assume a fixed depth of material in order to generate a design. The method proposed here assumes a fixed geometry and allows for a variable thickness to re-direct the thrust vector as a means to produce a viable design that concerns both structure and other formal concerns. If typically one would assume a uniformly thin structure, this paper assumes a dynamically variable volume.

2 Particle-Spring Systems

Particle-spring systems are based on lumped masses, called particles, which are connected by linear elastic springs. The solver used for this research is part of a particle-spring system implemented by Simon Greenwold [Fry and Reas]. Each particle in the system has a position, a velocity, and a variable mass, as well as a summarized vector for all of the forces acting on it [Kilian and Ochsendorf, 2005]. This Runge-Kutta solver is not necessary to generate a catenary (even load distribution), but it is necessary when evaluating an irregular load case. The method applied in this research will always be an irregular load case because if the desired geometry fit a catenary, it would not have a variable depth.

Other virtual form-finding methods have been explored such as Kilian’s CADenary tool [Kilian and Ochsendorf, 2005].

3 Compression-Only Structures

A compression-only structure will stand if the thrust vector of the system falls within the middle third of its cross-section. It is possible for a system to stand if the thrust vector lands between the middle third and the outer surface; however, it is likely to develop a hinge. For the purposes of this paper, we will maintain a thrust vector inside the middle third of the structure, assuring a result with zero tension.

It is important to note it is not always predictable that a structure will fail. Though it is possible to know if it will stand. “The Stone Skeleton” [Heyman, 1966] introduced the safe theorem for masonry structures. This theorem states that a compression only structure can stand so long as one network of compression forces can be found in equilibrium within the section of the structure. This solution is a possible lower-bound solution. When evaluating existing structures, it is not always possible to understand where this force network is exactly [Block and Ochsendorf, 2008]. The method applied in this paper can calculate and assure a thrust vector falls within the thickness of material; however, it cannot guarantee a thrust vector will not. Because of this uncertainty, a number of assumed failures did not fail (see Section 7).

For further reading on lower-bound analysis for unreinforced masonry structures see [Heyman, 1995] and [Huerta, 2001, 2004].
4 Form-Responding

Form-finding analog models by such researchers as Otto and Gaudi, or even the virtual versions like Kilian’s CADenary [Kilian and Ochsendorf, 2005] have proved it is difficult to control and predict the results of the final form. If that form does not correspond with a force that is external to the form-finding model, it is difficult to resolve the two into a solution. A form-responding approach takes a desired form as an input and produces a variable depth to allow for the interaction with the solver-based model.

5 Methodology

The method applied communicates the dynamic relaxation of the particle-spring system with a desired geometry. This system uses a
solver-based approach to inform each particle in the particle system with a distance from the desired geometry. The model can be broken into the following categories:

- Base Geometry
- Particle Spring System
- Vertical Distance
- Expansion / Contraction
- Sub-Division

5.1. Base Geometry

This research has been developed with the principle that the base geometry is fixed. The assumption is that this base geometry is determined by a force that is external to structure, be it acoustic, formal, building-code, etc. Future research could allow for a more fluid and reciprocal relationship between the structural requirements and the other formal drivers. For the purposes of this research, this geometry has been predominantly free-form as a way of testing variability; however, it has been determined that not all geometries are currently possible. For instance, undercut is not yet a possible geometry to solve with this method. The vertical intersection rule (to be explained in 5.3) cannot yet compensate for undercut. Resolution can also be a factor with the system. If extremely tight curves are skipped by the minimal number of particles applied, the system does not understand those nuances in the geometry. It is possible to solve for this problem by increasing the number of particles to increase the resolution of the understood lower geometry. For this reason, smooth curves with a minimal change in radius tend to work best as they can compensate for the discrepancies between two nodes easily. Sharp changes can sometimes be skipped.

A base geometry is required to inform the system. This base geometry is the datum with which the particle system compares...
itself against informing each node of its new vertical thrust. Without this measure, the system would produce a catenary as each particle would have a consistent weight compared to its neighbors—even distribution. Experiments have been conducted where a 2D curve is created in Processing to serve as this base geometry. Experiments have also been conducted where a 3D surface is created in Rhinoceros/Grasshopper and Processing communicates with this surface through a vertical intersection calculation via User Datagram Protocol (UDP) (see Figure 2).

5.1.1. 2D Geometry
The majority of this research has been dedicated to 2D geometries. In Processing, a simple five-node b-spline serves as the base geometry. The middle node of this b-spline is interactive. By hovering with the curser, then clicking and dragging, it is possible for the user to manipulate the curve dynamically to interact with the system, and better understand how that geometry informs the particle-spring system. While this was a helpful demonstrative device, the dynamism is confusing. This paper proposes to create a hard geometry in which the particle system responds against. The interactivity the user has with that base geometry is intended only for demonstration, not as method (see Figure 5).

5.1.2. 2.5D Geometry
While the previous 2D research appears to be a single sectional example, by incorporating Processing with Rhinoceros / Grasshopper via a UDP language, the sectional approach is capable of operating on a 3D Surface as well. This operation is a simple planar intersection with the surface to extract a curve that influences the particle-spring system. These 2D arches can then be aggregated to produce a network of arches approximating the desired surface (see Figure 7).

5.1.3. 3D Geometry
Future work needs to be completed to expand the system to a network of particle-springs. The working model of this system would be similar as each particle would project down to intersect with the surface. To examine relevant examples of particle-spring network systems see Kilian and Ochsendorf, 2005.

5.2. Particle-Spring System
The particle-spring system setup is consistent no matter what the base geometry input is. The system is informed by the number of particles, the length of the spring that connects each particle, and the resultant force on each particle (to be informed by 5.3 Vertical Distance).
5.3. Vertical Distance

This particle-spring system does not have a uniform load. In order to determine the irregular load that will determine the final rest lower-bound geometry, a vertical distance between the particle and the base geometry needs to be determined. This is achieved by drawing a line vertically from each particle, and intersecting that line with the base geometry. The distance between the particle and the intersection is the new vertical thrust vector for that particular particle. For example, the spire of a Gothic Cathedral has a deeper cross section, and therefore weighs more, re-directing the thrust vector down into the column.

When analyzing masonry arches, it is common practice to use static block analysis to break down an arch into a few polygons. The area of each polygon will determine the vertical thrust vector. [Block, et al, 2006 and Block and Ochsendorf, 2007] In this particle-spring system, vertical distance is more applicable as the resolution is higher and manual calculation is not necessary. Each particle is influenced by its own distance.

If the distance was a negative number, the vertical line was drawn in red to indicate a solution had not yet been found (see Figure 5).

5.4. Expansion / Contraction

In order to determine a solution where the thrust-vector falls above the desired geometry (base geometry), the length of each spring needs to expand evenly throughout the system. It is possible to produce a solution with too large of a mass by expanding this length too long, so a balance needs to be determined.

Before entering into the solution stage, the designer can determine the thinnest possible cross section of the result. The system then continually looks for the shortest vertical dimension and if that dimension is shorter than the required minimum, the entire system expands. If the shortest distance is longer than the minimum, the system contracts. The result of this approach is an automated expansion and contraction of the system, creating an arch that will have a variable depth cross section. The thinnest point of that cross section will be the depth the designer determined as the minimum (see Figure 9).

5.5. Sub-Division

Once a desired solution is obtained, it is necessary to break the system down into construable blocks. In order to resolve an equilibrium of shear forces (and not have to rely on friction) the break lines are created perpendicular to the funicular (or thrust vector). These lines have been evenly spaced, but the number of breaks, and the regularity of these breaks does not alter the solution. In future work, a 3D version of this sub-division system would be normal (3D version of perpendicular) to the thrust-vector network (surface). In this approach, as long as a break is normal to this

Figure 11: The breaks here are determined based on a 3D Voronoi operation. The points are located on the thrust vector surface. The resultant breaks ensure the thrust vector is perpendicular to the break surface.
surface, the resulting blocks would press against each other creating a zero shear solution.

6 Prototyping
The first method of prototyping was to extract the curves from Processing via a dxf export (or in the UDP version, simply bake the geometry from Grasshopper) as 2D geometry. This geometry was then laser cut from 1/8” Medium Density Fibreboard (MDF) sheet material and stacked 6 deep to compensate for lateral buckling. A lower shoring was also laser cut. The blocks are stacked on top of the shoring. The shoring is lowered down to test the stability of the arch.

All of these prototypes have been simple extrusions of a section. This method does have the potential to calculate the vertical thrust vector based on surface area and not simply vertical distance. In this case, other cross-sections are possible, though other prototyping methods would be required to produce these forms. Future experiments intend to test via rapid prototype 3D prints.

6.1. Material
The only requirement for this prototyping is to ensure that the material density is uniform throughout the cross section. 1/8” thick MDF sheet was readily available and a quick method for prototyping; however, the manual nature of aggregating the pieces to make a block reduced the precision of the fit. The kerf of the laser also reduced the precision.

This research is embedded in a method of making that has roots in stereotomy (the art of carving stone). For this reason, future research could pursue carving from materials such as Expanded Polystyrene (EPS) foam block, Autoclave Aerated Concrete (AAC), Solid Wood, Stone, etc. As long as the material is of consistent density, it does
not matter. The small scale of the previous MDF prototype speaks to the potential to work at a larger scale to resolve this problem.

6.2. Drop Mechanism
A number of problems arose with the drop mechanism. Figure 15 captures the dropping of one side of the drop mechanism faster than the other, the shoring pushed the blocks to one side, resulting in a failure. This failure is the result of a poorly designed drop mechanism. Had the mechanism dropped the form straight down, it is possible the prototype would stand. In working with this prototype, it was determined the drop mechanism was too unreliable, and wasteful. Later experiments incorporated a raceway for a string to post tension and erect the system.

7 Analysis
Not all of the arches created would stand. Various theories were developed to explain why. These theories ranged from blaming the drop mechanism, to human error in the gluing of blocks, to lack of buckling bracing in the z-direction (meaning six plies of MDF was not enough to account for the buckling). Video taping the failures was helpful to play back in slow motion to verify the theories. In the case of the drop-mechanism, the theory was true.

Additionally, some arches created specifically to demonstrate failure, did not fail. As mentioned previously, as long as there is a lower bound solution, the arch will stand. This method
currently guarantees a thrust vector can fall within the thickness of material, but cannot guarantee one does not. In prototypes such as demonstrated in Figure 16, material is removed from the cross-section in the hopes there would be a failure. Unfortunately, the arch did not fail, and the demonstration confused the issue.

7.1. Isolated External Load Case

In order to verify the variable depth was contributing to ensuring the thrust vector remained inside the middle third of the cross-section, it was necessary to produce a calculated failure. For this prototype, a pair of isolated loads were placed on a few particles on each side of the arch. These particles were overwritten in the code to understand they were to be two or sometimes four times more impacted by the vertical thrust. This was then realized by extending the vertical line the factor expressed in the code. The result can be seen in Figure 12. These masses are reminiscent of the spires used in gothic cathedrals to re-direct the thrust vector down inside the columns.

The arch was then video-recorded as the mass was removed and the arch exposed the hinge point where the new thrust vector left the arch cross-section. Future research will create an interactive ability to place such isolated load cases on the system.

Conclusion

This research has successfully demonstrated the potential to use a particle-spring system to respond to a given geometry and result in a constructable compression-only structure. Currently the system produces a single solution, though there are an infnant number of ways to vary the depth of an arch to produce a desired bottom geometry. These ways include isolated load cases, expansion / contraction of the system, etc. The current approach uses the minimal vertical dimension rule; however, future research could produce an array of solutions and select the one with the least cross-section (using the least amount of material).

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