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Why Panel Tests of Purchasing Power Parity Should Allow for Heterogeneous Mean Reversion

Kees G. Koedijk
Tilburg University and CEPR
e-mail: c.koedijk@uvt.nl

Ben Tims
Rotterdam School of Management, Erasmus University
e-mail: btims@rsm.nl

Mathijs A. van Dijk
Rotterdam School of Management, Erasmus University
e-mail: madijk@rsm.nl

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Abstract
Recent studies of purchasing power parity (PPP) use panel tests that fail to take into account heterogeneity in the speed of mean reversion across real exchange rates. In contrast to several other severe restrictions of panel models and tests of PPP, the assumption of homogeneous mean reversion is still widely used and its consequences are virtually unexplored. This paper analyzes the properties of homogeneous and heterogeneous panel unit root testing methodologies. Using Monte Carlo simulation, we uncover important adverse properties of the panel approach that relies on homogeneous estimation and testing. More specifically, power functions are low and assume irregular shapes. Furthermore, homogeneous estimates of the mean reversion parameters exhibit potentially large biases. These properties can lead to misleading inferences on the validity of PPP. Our findings highlight the importance of allowing for heterogeneous estimation when testing for a unit root in panels of real exchange rates.

Keywords: PPP, real exchange rates, panel models, unit root tests, heterogeneity, panel tests

JEL subject codes: F31, F33, F47

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Appendix B (internet appendix)

This appendix presents the proof that the homogeneous mean reversion estimate of a mixed panel converges to unity when the sample length T approaches infinity. Furthermore, we show that in heterogeneous panels, the estimated common mean reversion coefficient converges to a weighted average of the mean reversion parameters of the individual series with the second moments of these series as weights. We examine a panel consisting of two series:

$$y_{1,t} = \beta_1 y_{1,t-1} + \varepsilon_{1,t} \qquad \varepsilon_{1,t} \sim N(0,\sigma_1^2)$$

$$y_{2,t} = \beta_2 y_{2,t-1} + \varepsilon_{2,t} \qquad \varepsilon_{2,t} \sim N(0,\sigma_2^2)$$

Let $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $y_{-1} = \begin{pmatrix} y_{1,-1} \\ y_{2,-1} \end{pmatrix}$, $\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$.

Define $A_i \equiv y_{i,-1}' y_{i,-1} = \sum_{t=1}^{T} y_{i,t-1}^2$ and $B_i \equiv y_{i,-1}'\varepsilon_i = \sum_{t=1}^{T} y_{i,t-1}\varepsilon_{i,t}$ $\quad$ (i=1,2).

Then the estimate of the homogeneous mean reversion parameter β can be expressed as follows:

$$\beta = (y_{-1}' y_{-1})^{-1} y_{-1}' y = \frac{\beta_1 A_1 + B_1 + \beta_2 A_2 + B_2}{A_1 + A_2}.$$

We consider the following cases:

(i) $\beta_1 = 1 \wedge \beta_2 < 1$ (mixed panel)

(ii) $\beta_1 < 1 \wedge \beta_2 < 1 \wedge \beta_1 \neq \beta_2$ (heterogeneous panel)

(i) Mixed panel

Hamilton (1994, p. 486) shows the following two properties for the non-stationary series:

$$\frac{1}{T^2} A_1 \xrightarrow{L} \sigma_1^2 \int_0^1 [W(r)]^2 dr,$$

$$\frac{1}{T} B_1 \xrightarrow{L} \frac{1}{2}\sigma_1^2 \left([W(1)]^2 - 1\right).$$

For the stationary series, Hamilton (1994, p. 216) demonstrates that:

$$\frac{1}{T} A_2 \xrightarrow{p} \frac{\sigma_2^2}{1-\beta_2^2},$$

$$\frac{1}{\sqrt{T}} B_2 \xrightarrow{L} \mathrm{N}(0, \frac{\sigma_2^4}{1-\beta_2^2}).$$

Therefore, for $T \to \infty$, we find that:

$$\beta = \frac{A_1 + B_1 + \beta_2 A_2 + B_2}{A_1 + A_2} = \frac{\frac{1}{T^2} A_1 + \frac{1}{T}\left(\frac{1}{T} B_1\right) + \beta_2 \frac{1}{T}\left(\frac{1}{T} A_2\right) + \frac{1}{T^{3/2}}\left(\frac{1}{\sqrt{T}} B_2\right)}{\frac{1}{T^2} A_1 + \frac{1}{T}\left(\frac{1}{T} A_2\right)} \xrightarrow{p} 1.$$

(ii) Heterogeneous panel

In this case, both series are stationary, so that

$$\frac{1}{T} A_i \xrightarrow{p} \frac{\sigma_i^2}{1-\beta_i^2} \text{ for } i = 1,2$$

$$\frac{1}{\sqrt{T}} B_i \xrightarrow{L} \mathrm{N}(0, \frac{\sigma_i^4}{1-\beta_i^2}) \text{ for } i = 1,2 .$$

which implies for the common mean reversion parameter that

$$\beta = \frac{\beta_1 A_1 + B_1 + \beta_2 A_2 + B_2}{A_1 + A_2} =$$

$$= \frac{\beta_1\left(\frac{1}{T} A_1\right) + \frac{1}{\sqrt{T}}\left(\frac{1}{\sqrt{T}} B_1\right) + \beta_2\left(\frac{1}{T} A_2\right) + \frac{1}{\sqrt{T}}\left(\frac{1}{\sqrt{T}} B_2\right)}{\frac{1}{T} A_1 + \frac{1}{T} A_2} \xrightarrow{p} \frac{\beta_1 \frac{\sigma_1^2}{1-\beta_1^2} + \beta_2 \frac{\sigma_2^2}{1-\beta_2^2}}{\frac{\sigma_1^2}{1-\beta_1^2} + \frac{\sigma_2^2}{1-\beta_2^2}} .$$