20th ICCRTS

Title: Applying Multi-sensor Information Fusion Technology to Problems of National Defense in the Complicated Social, Information, And Communication Environment

Paper Id: 004

Suggested Primary topic:
Topic 12: ISR for decision making

Alternate topics:
Topic 1: Concepts Theory & Practice
Topic 3: Data, Information & Knowledge

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Applying Multi-sensor Information Fusion Technology to Problems of National Defense in the Complicated Social, Information, and Communication Environment

Draft (Version 1.0)

BY

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Abstract

In 2014, commercial flight MH17 was downed in Ukraine with 293 passengers aboard, US fighter Jets began striking ISIS terrorists in IRU, Japan and China confront each other over rights to an island in the South China Sea, and North Korea and Iran continue to threaten the world with nuclear weapon development. All these events Indicated that we are still living in a world of conflict. There is a need to make the correct national defense decision under the complicated social, information and communication environment.

This paper proposes a best possible decision making model by comparing the Bayesian Fusion Model and the Dempster-Schafer Fusion Model. In complicated situations, decisions are not unique, both Bayesian and Dempster-Schafer can provide the final resolution.

In this comparison study, two examples are used to show the differences and similarities of the two fusion models. The first example is a real life problem in defense involving discrimination of friendly aircraft from enemy aircraft with radar, electronic warfare, and CNI sensors. The second example is from medicine. The two methods are used by a team of medical experts to positively discriminate between patients with a brain tumor, concussion, or meningitis.

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**Introduction**

The main objective of this comparison study is to explore multi-sensor fusion technology and its applications. The multi-sensor Bayesian fusion model and the Dempster-Shafer fusion model are the most popular and wildly applied fusion models. Both Bayesian fusion model and Dempster-Shafer fusion model have been widely applied in defense, medicine, and meteorology, for automatic classification and positive identification problems. In this comparison study, we try to find the mathematical expression for both the Bayesian fusion model and the Dempster-Shafer fusion model. We will also explain the differences and similarities between the Bayesian fusion model and the Dempster-Shafer fusion model.

The Bayesian fusion model is based on the conditional probability model. It was developed by the British mathematician Thomas Bayes in the seventeen century. The Dempster-Shafer fusion model was developed by Arthur P. Dempster, a professor from Harvard University, and Glenn Shafer, a professor from Princeton University.

The Bayesian fusion model is a probability model whereas the Dempster-Shafer fusion model is a logical and mathematical model. The Bayesian fusion model requires a multivariate normal distribution assumption, whereas the Dempster-Shafer fusion model does not require a probability distribution assumption.

In the information age, as electronics and computer technology advance day-by-day, more sensor products are used, meaning more information needs to be extracted and properly applied. This kind of problem is a great challenge for people in the field of command and control.

In this comparison study, we provide two examples. The first example from defense, applies the models to the problem of positively identifying multiple targets in a multi-sensor environment. The second example from medicine, applies the models to the emergency room “triage problem”. In both these examples, we apply the Bayesian fusion model and the Dempster-Shafer fusion model to prove that both fusion models generate an identical decision from the same data set or numerical problem. This application proves that the Bayesian fusion model and the Dempster-Shafer fusion model are mathematically equivalent.

**Bayesian Fusion Model**

The fused target probability in the multi-sensor and multiple target environment can be expressed as [Hall, 1992]:

\[
P\{ Tk / (S1,S2,S3, ... Sn) \} = \frac{\{P(S1/Tk) \times P(S2/Tk) \times P(S3/Tk) \times ... \times P(Sn/Tk) \}}{\sum_{i=1}^{n} \{P(S1/Ti) \times P(S2/Ti) \times P(S3/Ti) \times ... \times P(Sn/Ti)\}}\]

**equation A [Hall, 1992]**

Where: \( P\{ Tk / (S1,S2,S3, ... Sn) \} \) is the probability of target Tk based on \( n \) sensors.

For the single sensor with multiple targets, the fused target probability reduces to the following:
\[ P(Tk/S) = \left\{ P(S/Tk) \right\} / \left\{ \sum_{i=1,n} P(S/Ti) \right\} \]  

[Jeun, 1979]

This expression is the regular Bayesian probability model.

**Dempster-Shafer Fusion Model**  
[Glenn Shafer, 1976]

The Dempster-Shafer theory was first developed by Professor Arthur P. Dempster, at Harvard University in the 1960’s. It was later extended by Professor Glenn Shafer at Princeton University. Later on, the new theory was known as the theory of evidence. The Dempster-Shafer theory, in general is a collection of evidence from multiple sources used to form logical decisions to interpret uncertainty.

The most important part of Dempster-Shafer theory is Dempster’s rule of combinations. A combination (called a **joint mass**) is calculated from two sets of masses \( m_1 \) and \( m_2 \) in the following manner.

The two dimensional matrix of probability mass for two sensors (with mass \( m_1 \) and \( m_2 \)), and three targets (A, B and C), can be constructed as following:

<table>
<thead>
<tr>
<th></th>
<th>m2</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>m1 A</td>
<td>m1(A) x m2(A)</td>
<td>m1(A) x m2(B)</td>
<td>m1(A) x m2(C)</td>
<td></td>
</tr>
<tr>
<td>m1 B</td>
<td>m1(B) x m2(A)</td>
<td>m1(B) x m2(B)</td>
<td>m1(B) x m2(C)</td>
<td></td>
</tr>
<tr>
<td>m1 C</td>
<td>m1(C) x m2(A)</td>
<td>m1(C) x m2(B)</td>
<td>m1(C) x m2(C)</td>
<td></td>
</tr>
</tbody>
</table>

The probability mass for targets A, B, and C in the two sensor environment can be defined as following:

\[
Pr-m(A) = \left\{ \frac{1.0}{1.0 - K} \right\} \{ m1(A)*m2(A) \}\]

\[
Pr-m(B) = \left\{ \frac{1.0}{1.0 - K} \right\} \{ m1(B)*m2(B) \}\]

\[
Pr-m(C) = \left\{ \frac{1.0}{1.0 - K} \right\} \{ m1(C)*m2(C) \}\]

where \( K = \) the measure of conflict between the two sensors

\[
= \{ m1(A)*m2(B) + m1(A)*m2(C) \}
\]

\[
+ \{ m1(B)*m2(A) + m1(B)*m2(C) \}
\]

\[
+ \{ m1(C)*m2(A) + m1(C)*m2(B) \}
\]

and \( (1.0 - K) = \) the normalization factor

The two dimensional matrix of probability mass for two sensors (with mass \( m_1 \) and \( m_2 \)) and multiple targets (\( T_1, T_2, T_3, \ldots T_n \)) can be constructed as following:
Table 2 Probability Mass for 2 sensors and multiple targets

<table>
<thead>
<tr>
<th>m2</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>...</th>
<th>Tn</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>m1(T1) x m2(T1)</td>
<td>m1(T1) x m2(T2)</td>
<td>m1(T1) x m2(T3)</td>
<td>...</td>
<td>m1(T1) x m2(Tn)</td>
</tr>
<tr>
<td>T2</td>
<td>m1(T2) x m2(T1)</td>
<td>m1(T2) x m2(T2)</td>
<td>m1(T2) x m2(T3)</td>
<td>...</td>
<td>m1(T2) x m2(Tn)</td>
</tr>
<tr>
<td>T3</td>
<td>m1(T3) x m2(T1)</td>
<td>m1(T3) x m2(T2)</td>
<td>m1(T3) x m2(T3)</td>
<td>...</td>
<td>m1(T3) x m2(Tn)</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>Tn</td>
<td>m1(Tn) x m2(T1)</td>
<td>m1(Tn) x m2(T2)</td>
<td>m1(Tn) x m2(T3)</td>
<td>...</td>
<td>m1(Tn) x m2(Tn)</td>
</tr>
</tbody>
</table>

The probability mass for multiple targets T1, T2, T3, ... Tn in the two sensor environment can be defined as following:

\[
\begin{align*}
\text{Pr-m}(T1) &= \frac{1.0}{(1.0 - K)} \times \{m(T1) \times m2(T1)\} \\
\text{Pr-m}(T2) &= \frac{1.0}{(1.0 - K)} \times \{m(T2) \times m2(T2)\} \\
\text{Pr-m}(T3) &= \frac{1.0}{(1.0 - K)} \times \{m(T3) \times m2(T3)\} \\
\vdots & \quad \vdots \quad \vdots \quad \vdots \\
\text{Pr-m}(Tn) &= \frac{1.0}{(1.0 - K)} \times \{m(Tn) \times m2(Tn)\}
\end{align*}
\]

where \( K \) = the measure of conflict between the two sensors

\[
K = \{m(T1) \times m2(T2) + m(T1) \times m2(T3) + \cdots m(T1) \times m2(Tn)\} \\
+ \{m(T2) \times m2(T1) + m(T2) \times m2(T3) + \cdots m(T2) \times m2(Tn)\} \\
\vdots \quad \vdots \quad \vdots \\
+ \{m(Tn) \times m2(T1) + m(Tn) \times m2(T2) + \cdots m(Tn-1) \times m2(Tn-1)\}
\]

and \( (1.0 - K) \) = the normalization factor

For multi-sensor and multiple target problems, the Dempster-Shafer fusion model can provide accurate solutions. The complete procedure for solving with multi-sensor and multiple targets is as follows.

Suppose there are four sensors (S1, S2, S3 and S4) and “n” targets (T1,T2,T3 ,T4, ... Tn).

Step 1, solve using the first two sensors (S1 and S2). Construct a two dimensional matrix of probability mass for the two sensors and all targets as shown in Table 2 Probability Mass for 2 sensors and multiple targets Obtain a completion solution for Pr-m(T1), Pr-m(T2), Pr-m(T3), ... Pr-m(Tn).

Step 2, next solve using the probability mass for targets T1, T2, ... Tn obtained from Step 1. Combine the new information with sensor S3 information as a new two sensor and multiple target problem.

Repeat Step 2, until the all sensors have been included.
Similarities between the Bayesian Fusion Model and the Dempster-Shafer Fusion Model
Both the Bayesian fusion model and the Dempster-Shafer fusion model can resolve conflicting decisions. They are powerful research tools for the scientist and engineer in different fields of application.

The Bayesian fusion model and the Dempster-Shafer fusion model can both solve multi-sensor and multiple target problems in defense, medicine, and many other scientific areas.

Both fusion models can be applied to classify and positively identification problems in the area of statistical pattern recognition and image analysis. They are also known as accurate classifiers.

Differences between the Bayesian Fusion Model and the Dempster-Shafer Fusion Model
The Bayesian fusion model is a probability model whereas the Dempster-Shafer fusion model is a logical and mathematical model. The Bayesian fusion model requires a multivariate probability distribution assumption to solve multi-sensor and multiple target problems. The Dempster-Shafer fusion model does not require multivariate probability distribution to solve multi-sensor and multiple target problems.

The Bayesian fusion model can solve multi-sensor and multiple target problems, with just one single substitution to reach a solution. The Dempster-Shafer fusion model requires multiple steps and repeated procedures to reach a final solution when solving multi-sensor and multiple target problems. Refer to example #1 in this comparison study to see the solution of a three sensor and three target problem. The Dempster-Shafer fusion model requires two steps to reach the final solution.

Application in Defense (Example #1)

Figure 1 Target1 (F22 fighter aircraft)
Detection, tracking, and positive identification of aircraft in the multi-sensor environment is a very difficult and challenging defense task. Each individual sensor, such as radar, EW and CNI may not give identical information. In that case, a decision conflict may occur. Conflict resolution is needed. Usually multi-sensor information fusion technology is used to resolve such conflicts and provide a decision. For example, in a three sensor (radar, EW, and CNI) and three target (T1—F22, T2—P3, and T3—MH370) problem, probabilities of detection associated to each target are indicated in the following table.

Table 3 Example Probability of Detection Data

<table>
<thead>
<tr>
<th></th>
<th>T1 - F22</th>
<th>T2 - P3</th>
<th>T3 – MH370</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar (S1)</td>
<td>0.80</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>EW (S2)</td>
<td>0.30</td>
<td>0.60</td>
<td>0.05</td>
</tr>
<tr>
<td>CNI (S3)</td>
<td>0.20</td>
<td>0.70</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The above table strongly indicates that the three sensors, Radar, EW and CNI, do not have an identical decision on what target has been detected.

Solution by Bayesian Fusion Model

Now applying the Bayesian fusion model to the data set of example #1 gives the equation for three targets and three sensors as following:

\[
\Pr(T_k|S_1,S_2,S_3) = \left\{ \Pr(S_1/T_k) \times \Pr(S_2/T_k) \times \Pr(S_3/T_k) \right\} / \left\{ \sum_{i=1,2,3} \Pr(S_1/T_i) \times \Pr(S_2/T_i) \times \Pr(S_3/T_i) \right\}
\]

From the sensor information giving in example #1, we have:
\[ \begin{align*}
\Pr(S_1/T_1) &= 0.80 \\
\Pr(S_2/T_1) &= 0.30 \\
\Pr(S_3/T_1) &= 0.20 \\
\Pr(S_1/T_2) &= 0.05 \\
\Pr(S_2/T_2) &= 0.60 \\
\Pr(S_3/T_2) &= 0.70 \\
\Pr(S_1/T_3) &= 0.05 \\
\Pr(S_2/T_3) &= 0.05 \\
\Pr(S_3/T_3) &= 0.05
\end{align*} \]

By substituting equation \(<B>\) into equation \(<A>\), we have:

\[ \Pr(T_1/S_1, S_2, S_3) = \frac{(0.80 \times 0.30 \times 0.20)}{(0.80 \times 0.30 \times 0.20) + (0.05 \times 0.60 \times 0.70) + (0.05 \times 0.05 \times 0.05)} = \frac{0.048}{0.048 + 0.021 + 0.000125} = \frac{0.048}{0.0691} = 0.6944 \text{ or } 69.44\% \]

Therefore, the fused probability for target T1 (F22 fighter jet) is \( \Pr(T_1/S_1, S_2, S_3) = 69.44\% \) in the three sensor (Rader, EW and CNI) environment.

Similarly, for target 2 substituting the information from equation \(<B>\), into equation \(<A>\), we get:

\[ \Pr(T_2/S_1, S_2, S_3) = \frac{(0.05 \times 0.60 \times 0.70)}{(0.80 \times 0.30 \times 0.20) + (0.05 \times 0.60 \times 0.70) + (0.05 \times 0.05 \times 0.05)} = \frac{0.021}{0.069125} = 0.3038 \text{ or } 30.38\% \]

and for target 3 we get:

\[ \Pr(T_3/S_1, S_2, S_3) = \frac{(0.05 \times 0.05 \times 0.05)}{(0.80 \times 0.30 \times 0.20) + (0.05 \times 0.60 \times 0.70) + (0.05 \times 0.05 \times 0.05)} \]
= 0.000125/0.069125
= 0.0018 or 0.18%

Therefore, the solution of example #1 by the Bayesian fusion model is:

\[ \text{Pr}(\ T1/\ S1, S2, S3) = 69.44\% \]
\[ \text{Pr}(\ T2/\ S1, S2, S3) = 30.38\% \]
\[ \text{Pr}(\ T3/\ S1, S2, S3) = 0.18\% \]

That is, T1 (the F22) is detected with probability of 69.44%,

T2 (the P3) is detected with probability of 30.38%, and

T3 (MH370) is detected with probability of 0.18%

in the three sensor (Radar, EW, CNI) environment.

Solution by Dempster-Shafer Fusion Model

Now, we will apply the Dempster-Shafer fusion model to example #1. Since it is a three-sensor and three-target problem, it requires two steps for a complete solution. Step one - construct a two dimensional matrix of a probability mass for Radar and EW and find the probability mass for each target.

Step two - using the results from step one, construct a new two dimensional matrix of probability mass, and find the probability mass for all targets from the remaining CNI sensor.

Step 1:
### Table 4 Probability Mass of Detection of 3 targets from 2 sensors

<table>
<thead>
<tr>
<th>S2(EW)</th>
<th>S1 (radar)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T1(0.80)</td>
<td>T2(0.05)</td>
<td>T3(0.05)</td>
</tr>
<tr>
<td>T1(0.30)</td>
<td>0.240</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>T2(0.60)</td>
<td>0.480</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>T3(0.05)</td>
<td>0.040</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

From the above two dimensional matrix of probability mass, we have:

\[ K = 0.015 + 0.015 + 0.480 + 0.030 + 0.040 + 0.003 = 0.583 \] as the conflict.

\[ 1.0 - K = 0.417 \] as the normalization factor

and the required probability mass for target 1, target 2, and target 3 can be obtained as:

\[ \text{Pr}_m(T1) = (1.0/0.417) \times 0.240 = 0.5755 \text{ or } 57.55\% \]

\[ \text{Pr}_m(T2) = (1.0/0.417) \times 0.030 = 0.0719 \text{ or } 7.19\% \]

\[ \text{Pr}_m(T3) = (1.0/0.417) \times 0.003 = 0.0047 \text{ or } 0.47\% \]

\( \text{Pr}_m(T1) \) is the probability mass for Target T1, the F22

\( \text{Pr}_m(T2) \) is the probability mass for Target T2, the P3

\( \text{Pr}_m(T3) \) is the probability mass for Target T3, the airliner

Step #2: now use the results obtained from Radar and EW, to construct probability mass matrix for Sensor CNI:

### Table 5 Probability Mass after adding 3rd sensor

<table>
<thead>
<tr>
<th>S3 (CNI)</th>
<th>S1 &amp; S2 (Radar &amp; EW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T1(0.576)</td>
</tr>
<tr>
<td>T1(0.20)</td>
<td>0.114</td>
</tr>
<tr>
<td>T2(0.70)</td>
<td>0.399</td>
</tr>
<tr>
<td>T3(0.05)</td>
<td>0.285</td>
</tr>
</tbody>
</table>

from the above probability mass matrix for Radar and EW, we find

\[ K = 0.014 + 0.001 + 0.399 + 0.003 + 0.285 + 0.003 = 0.705 \]

\[ 1.0 - k = 0.295 \]

where \( K \) is the conflict and normalization factor for targets T1, T2, and T3 in the three sensor S1, S2, and S3 environment.

\[ \text{Pr}_m(T1) = 0.114/0.295 = 0.3864 \text{ or } 38.64\% \]

\[ \text{Pr}_m(T2) = 0.049/0.295 = 0.1661 \text{ or } 16.61\% \]
Pr_m (T3) = 0.003/0.295 = 0.0101 or 1.01%

Therefore, the probability mass for Target T1 (the F22) is 38.64 %, the probability mass for Target T2 (the P3) is 16.61 %, and the probability mass for Target T3 (MH370) is 1.01% in the three sensor environment.

The results are identical to the solution provided by the Bayesian fusion model.

Application in Medicine (Example #2)
In example #2, a patient is diagnosed by two neurologists - doctor A and doctor B. The doctors express their diagnosis as a probability of three possible diseases:

Table 6 Probability of Disease

<table>
<thead>
<tr>
<th></th>
<th>T1 (brain tumor)</th>
<th>T2 (concussion)</th>
<th>T3 (meningitis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctor A</td>
<td>0.75</td>
<td>0.20</td>
<td>0.05</td>
</tr>
<tr>
<td>Doctor B</td>
<td>0.80</td>
<td>0.12</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The purpose of this example is to demonstrate how to discriminate between brain tumor, concussion, and meningitis when two brain doctors are involved. Just looking at the data, one can conclude that the patient is most likely suffering from a brain tumor. Now we will apply the Bayesian fusion model and Dempster-Shafer fusion model to the data of the two neurologists. We will compare the answers obtained by the two models and check whether the results agree or disagree.
First, we determine that example #2 is a two sensor and three targets problem. S1 denotes doctor A and S2 denotes doctor B. Target T1 denotes the perceived probability of the disease being a brain tumor, target T2 denotes concussion, and target T3 denotes Meningitis.

**Solution by the Bayesian Fusion Model:**
To apply the Bayesian fusion Model, we obtain a formula that can provide a method to estimate fused probabilities for T1, T2, and T3 from Sensor S1, and S2 as following:

\[
Pr(T1/S1,S2) = \frac{Pr(S1/T1)*Pr(S2/T1)}{\{ [Pr(S1/T1) * Pr(S2/T1)]
+ [Pr(S1/T2) * Pr(S2/T2)]
+ [Pr(S1/T3) * Pr(S2/T3)] \}}
\]

\[
Pr(T2/S1,S2) = \frac{Pr(S1/T2)*Pr(S2/T2)}{\{ [Pr(S1/T1) * Pr(S2/T1)]
+ [Pr(S1/T2) * Pr(S2/T2)] \}}
\]
\[
Pr(T3/S1,S2) = \frac{Pr(S1/T3) \cdot Pr(S2/T3)}{\left\{ \left[ \frac{Pr(S1/T1) \cdot Pr(S2/T1)}{Pr(S1/T1)} \right] + \left[ \frac{Pr(S1/T2) \cdot Pr(S2/T2)}{Pr(S1/T2)} \right] + \left[ \frac{Pr(S1/T3) \cdot Pr(S2/T3)}{Pr(S1/T3)} \right] \right\}} \text{ equation } A
\]

and from the example #2 given information, we have:

\[
Pr(S1/T1) = 0.75 \\
Pr(S1/T2) = 0.20 \\
Pr(S1/T3) = 0.05 \\
Pr(S2/T1) = 0.80 \\
Pr(S2/T2) = 0.12 \\
Pr(S2/T3) = 0.08 \text{ equation } B
\]

Now, substituting equation B into equation A, we have:

\[
Pr(T1/S1,S2) = \frac{(0.75) \cdot (0.80)}{\left\{ \left[ (0.75) \cdot (0.80) \right] + \left[ (0.20) \cdot (0.12) \right] + \left[ (0.05) \cdot (0.08) \right] \right\} = \frac{0.60}{0.6 + 0.024 + 0.004} = \frac{0.6}{0.628} = 0.9554
\]

That is, the doctors are 95.54% certain that the patient is suffering from a brain tumor.

Similarly, we have:

\[
Pr(T2/S1,S2) = \frac{0.024}{0.628} = 0.0382 \text{ or } 3.82\% \text{ probability of concussion} \\
Pr(T3/S1,S2) = \frac{0.004}{0.628} = 0.0063 \text{ or } 0.6\% \text{ probability of meningitis}
\]
One can conclude that both Doctors A and B agree the patient is suffering from a brain tumor with 95.54% certainty, suffering from a concussion with 3.82% certainty, and suffering from meningitis with 0.6% certainty.

**Solution by Dempster-Shafer Fusion Model:**

Now, build a two dimensional matrix of sensors and targets to estimate the required probability mass for example #2 as following:

**Table 10 Probability Mass**

<table>
<thead>
<tr>
<th></th>
<th>S1 (doctor A)</th>
<th>S2 (doctor B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T1(0.75) T2(0.20) T3(0.05)</td>
<td>T1(0.80) T2(0.12) T3(0.08)</td>
</tr>
<tr>
<td>T1</td>
<td>0.60 0.09 0.06</td>
<td>0.60 0.09 0.06</td>
</tr>
<tr>
<td>T2</td>
<td>0.16 0.024 0.016</td>
<td>0.16 0.024 0.016</td>
</tr>
<tr>
<td>T3</td>
<td>0.04 0.006 0.004</td>
<td>0.04 0.006 0.004</td>
</tr>
</tbody>
</table>

From the above two dimensional matrix of probability mass, we have:

\[ K = 0.16 + 0.04 + 0.09 + 0.006 + 0.06 + 0.016 = 0.372 \]

as the measure of conflict.

\[ 1.0 - K = 0.628 \]

as the normalization factor

and the required probability mass for target 1, target 2, and target 3 are obtained as follows:

\[ Pr_m(T1) = 0.60 / 0.628 = 0.9554 \quad \text{or} \quad 95.54\% \quad \text{probability of brain tumor} \]

\[ Pr_m(T2) = 0.02 / 0.628 = 0.0382 \quad \text{or} \quad 3.82\% \quad \text{probability of concussion} \]

\[ Pr_m(T3) = 0.01 / 0.628 = 0.006 \quad \text{or} \quad 0.6\% \quad \text{probability of meningitis} \]
One can conclude that both doctors A and B agree that the patient is suffering from a brain tumor with 95.54% certainty, suffering from a concussion with 3.82% certainty, and suffering from meningitis with 0.6% certainty.

**Conclusion**

Applying the Bayesian fusion model and Dempster-Shafer fusion model to example #1 and example #2, both provide an identical solution. Therefore the Bayesian fusion model and the Dempster-Shafer fusion model are equivalent.

We can demonstrate mathematically that the Bayesian fusion model probability is a function of the Dempster-Shafer fusion model probability mass and measurement of confusion, K.

\[ \Pr(T) = f\{ \Pr_m(T), K \} \]

proof upon request

Both Bayesian fusion model and Dempster-Shafer fusion model can solve multi-sensor and multiple target problems from the field of defense, medical, and other fields.

The Bayesian fusion model is a conditional probability model which requires multivariate normal distribution assumptions whereas the Dempster-Shafer fusion model is a mathematical and logical model which does not require multivariate normal distribution assumptions.

Both Bayesian fusion model and Dempster-Shafer fusion model have been used to verify numerical multi-sensor and multiple target problems with examples published in well known academic journals.[Dempster, 1968] and [Sandia.gov, 2002]

When compared to the Dempster-Shafer fusion model, the Bayesian fusion model is easier to use in solving multi-sensor and multiple target problems. The Bayesian fusion model requires simple
substitution. Whereas the Dempster-Shafer fusion model requires multiple steps to reach a solution in the multi-sensor and multiple target environment [Example #1 and Example #2].

References

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