Name : ______________________ **Class Number**: _____

Final Exam Practice Problems

Part II: Sequences and Series
Math 1C: Calculus III

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

What can I use on this exam?

- You may use one note card that is no larger then 11 inches by 8.5 inches. You may write on both sides of this note card. PLEASE SUBMIT YOUR NOTECARD WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of sequence and series notation including proper use of summation notation.

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false. After you've chosen your answer, mark the appropriate space on your Scantron form. Notice that letter A corresponds to true while letter B corresponds to false.

1. T F $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$

2. T F If $\sum_{n=0}^{\infty} a_n$ is divergent, then $\sum_{n=0}^{\infty} |a_n|$ is divergent.

3. T F If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

4. T F If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

5. T F If $0 \le a_n \le b_n$ for all n and $\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.

6. T F If $a_n > 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then $\lim_{n\to\infty} a_n = 0$.

7. T F The ratio test can be used to determine if the series $\sum_{n=0}^{\infty} \frac{1}{n^3}$ converges

8. T F The series $\sum_{n=1}^{\infty} 3ne^{-n^2}$ diverges

Multiple Choice

For the problems below, circle the correct response for each question. After you've chosen your answer, mark your answer on your Scantron form.

9. For which of the following values of k do both $\sum_{n=0}^{\infty}\left(\frac{3}{k}\right)^n$ and $\sum_{n=0}^{\infty}\frac{(3-k)^n}{\sqrt{n+3}}$ converge?

A. None B. 2 C. 3 D. 4 E. 5

10. The series $\sum_{n=1}^{\infty}\frac{1}{n^\alpha}$ converges if and only if

A. $1<\alpha$ B. $\alpha<1$ C. $-1<\alpha<1$ D. $\alpha\geq 1$ E. $-1<\alpha$

11. Which of the following series converge?

I. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\,(\ln(n))^4}$ II. $\displaystyle\sum_{n=1}^{\infty}\frac{3+\sin(n)}{n^4}$ III. $\displaystyle\sum_{n=1}^{\infty}\frac{7n^2-5}{e^n(n+3)^2}$

A. I only B. I and II only C. II and III only D. I and III only E. I, II, and III

12. Which of the following series converge?

I. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^2\cdot\ln(n)}$ II. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\cdot\ln(n)}$ III. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}\cdot\ln(n)}$

A. I only B. II only C. I and II only D. I and III only E. I, II, and III

13. Which of the following is the radius of convergence for the series $\sum_{n=0}^{\infty}\frac{(-1)^n n!\,x^n}{n^n}$?

A. 0 B. $\frac{1}{e}$ C. 1 D. e E. ∞

14. What are the values of x for which the series

$$(x+1) - \frac{(x+1)^2}{2} + \frac{(x+1)^3}{3} - \cdots + \frac{(-1)^{n+1}(x+1)^n}{n} + \cdots$$

converges?

A. $-2 < x \leq 0$ B. All real numbers C. $-2 < x < 0$ D. $-2 \leq x < 0$ E. $-2 \leq x \leq 0$

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15. For what real values of k could the series

$$\sum_{n=1}^{\infty} \frac{\ln(n) + n^3}{n^{5k} + 4}$$

converges?

A. $k \geq 1$ B. $k < 1$ C. all real k D. $k > 1/5$ E. $k > 4/5$

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16. Find the values of x for which the series $\sum_{n=1}^{\infty} (x-1)^n$:

A. $-2 < x < 0$ B. $0 < x \leq 2$ C. $0 < x < 2$ D. $0 \leq x \leq 2$ E. $-2 \leq x < 0$

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17. What are all values of x for which the series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{\sqrt[3]{n^2}}$$

converge?

A. $-1 \leq x \leq 1$ B. $0 < x < 1$ C. $0 \leq x < 1$ D. $0 < x \leq 1$ E. $-1 \leq x \leq 1$

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18. The series $\sum_{n=0}^{\infty} r^n$ converges if and only if:

A. $-1 < r < 1$ B. $-1 \leq r \leq 1$ C. $-1 \leq r < 1$ D. $-1 < r \leq 1$ E. $r < 1$

19. Consider the series

$$\frac{1}{2} + \frac{(x-25)}{4} + \frac{(x-25)^2}{8} + \cdots + \frac{(x-25)^n}{2^{n+1}} + \cdots$$

Which of the following statements are true about this series?

I. The interval where the series converges is $23 < x < 27$

II. The sum of the series where it converges is equal to $\dfrac{1}{27-x}$

III. If $x = 28$, the series converges to $\dfrac{1}{3}$

IV. The interval where the series converges is $21 < x < 29$

V. The series diverges

A. I only B. III only C. I and II only D. I and IV only E. V only

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20. How many terms of the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1} n^{-2}$$

must we add in order to be sure that the partial sum s_n is within 0.0001 of the sum s.

A. 10 B. 300 C. 30 D. 100 E. 1000

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21. The convergent power series in x that has the same equal to $\dfrac{x^3}{2-x^3}$ when $\|x\| < \sqrt[3]{2}$ is:

A. $\frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^{3n+3}}{2^n}\right)$ B. $\sum_{n=0}^{\infty} \frac{x^{3n+3}}{2^n}$ C. $\sum_{n=0}^{\infty} \frac{x^{3n-3}}{4^n}$ D. $\frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^{3n}}{2^n}\right)$ E. $\frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^{3n+3}}{4^n}\right)$

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22. Which of the following is the power series representation of $x\,e^{2x+1}$?

A. $x - 2 \cdot x^2 + 2^2 \cdot \frac{x^3}{2!} - 2^3 \cdot \frac{x^4}{3!} + \cdots$

B. $e \cdot x^2 + 2\,e \cdot x^4 + 2^2\,e \cdot \frac{x^6}{2!} + 2^3\,e \cdot \frac{x^8}{3!} + \cdots$

C. $e \cdot x + 2\,e \cdot x^2 + 2^2\,e \cdot \frac{x^3}{2!} + 2^3\,e \cdot \frac{x^4}{3!} + \cdots$

D. $x + (2x+1)\,x + \frac{(2x+1)^2}{2!} + \frac{(2x+1)^3}{3!} + \cdots$

E. $x^2 + x^4 + \frac{x^6}{2!} + \frac{x^8}{3!} + \cdots$

23. Which of the following series converge?

$$1)\sum_{n=1}^{\infty}\frac{1}{n} \qquad 2)\sum_{n=1}^{\infty}\frac{(-1)^n\cdot n}{\ln(n)} \qquad 3)\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$

A. 1 B. 2 C. 3 D. $2,3$ E. None

24. Which of the following is a power series representation for $f(x)=\dfrac{1}{4+6x}$ if $|x|<\dfrac{2}{3}$?

A. $\dfrac{1}{4}\left(\sum_{n=0}^{\infty}(-1)^n\left(\dfrac{3}{2}\right)^n x^n\right)$

B. $\dfrac{1}{6}\left(\sum_{n=0}^{\infty}(-1)^n\left(\dfrac{2}{3}\right)^n x^n\right)$

C. $\dfrac{1}{4}\left(\sum_{n=0}^{\infty}(-1)^{2n}\left(\dfrac{3}{4}\right)^n x^n\right)$

D. $\dfrac{1}{4}\left(\sum_{n=0}^{\infty}\left(\dfrac{3}{2}\right)^n x^n\right)$

E. $\dfrac{1}{6}\left(\sum_{n=0}^{\infty}(-1)^n\left(\dfrac{3}{2}\right)^n x^n\right)$

25. Which of the following is a power series representation for $f(x)=x\cdot(\arctan(x))$ if $|x|<1$?

A. $x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+\dfrac{x^7}{7}+\dfrac{x^9}{9}+\cdots$

B. $1-x+\dfrac{x^3}{3}-\dfrac{x^5}{5}+\dfrac{x^7}{7}-\dfrac{x^9}{9}+\cdots$

C. $1+x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+\dfrac{x^7}{7}+\dfrac{x^9}{9}+\dfrac{x^{11}}{11}+\cdots$

D. $x^2-\dfrac{x^4}{3}+\dfrac{x^6}{5}-\dfrac{x^8}{7}+\dfrac{x^{10}}{9}+\cdots$

E. $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+\dfrac{x^9}{9}+\cdots$

26. Which of the following is the power series representation of $\ln(1+2x)$?

A. $2x - \frac{4x^2}{2} + \frac{8x^3}{3} - \frac{16x^4}{4} + \frac{32x^5}{5} + \cdots$

B. $\frac{x^3}{3} - \frac{4x^4}{4} + \frac{8x^5}{5} - \frac{16x^6}{6} + \frac{32x^7}{7} + \cdots$

C. $2 - 4x + 8x^2 - 16x^3 + 32x^4 + \cdots$

D. $2x^2 - 4x^4 + 8x^6 - 16x^8 + 32x^{10} + \cdots$

E. $2x^2 - \frac{4x^4}{2} + \frac{8x^6}{3} - \frac{16x^8}{4} + \frac{32x^{10}}{5} + \cdots$

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27. Which of the following is a power series representation for $f(x) = \frac{3}{(1+x)^2}$ if $|x| < 1$?

A. $2x^2 - 3x^3 + 4x^4 - 5x^5 + \cdots$
B. $3 - 3x + 3x^2 - 3x^3 + \cdots$
C. $1 - x + x^2 - x^3 + \cdots$
D. $3 - 3\cdot(2x) + 3\cdot(3x^2) - 3\cdot(4x^3) + \cdots$
E. $1 - 2x + 3x^2 - 4x^3 + \cdots$

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28. Which of the following series converge?

1) $\sum_{n=1}^{\infty} \frac{3^{2n}}{2^{3n}}$ 2) $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$ 3) $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^3+2}}$

A. None B. 1 C. 2 D. 3 E. 2, 3

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29. The Taylor Series about $x = 2$ for a certain function f converges to $f(x)$ for all x in the radius of convergence. The nth derivative of f at $x = 2$ is given by

$$f^{(n)}(x) = \frac{2^n\, n!}{3^{n+1}(n+1)}$$

and $f(2) = 3$. Which of the following is the radius of convergence for the Taylor series for f about $x = 2$?

A. $-\frac{1}{2}$ B. $\frac{2}{3}$ C. 0 D. $\frac{3}{2}$ E. 3

30. Let f be a function having derivatives of all orders for all real numbers x. The third-order Taylor polynomial for f about $x = 4$ is given by

$$T(x) = \frac{1}{9} + 5(x-4)^2 - 8(x-4)^3.$$

If $\left|f^{(4)}(x)\right| \leq \frac{1}{4}$ for $3.5 \leq x \leq 4$, order the following from greatest to least?

I. The maximum value of $|T(x) - f(x)|$ for $3.5 \leq x \leq 4$

II. $|f''(4)|$

III. $|f(4)|$

A. I, II, III B. II, I, III C. III, I, II D. II, III, I E. III, II, I

31. Suppose the second-degree Taylor polynomial for $f(x)$ around $x = 0$ is given by

$$P(x) = 7x + \frac{5}{2}x^2$$

If $g(x)$ is the inverse of $f(x)$, what is $g'(0)$?

A. 1 B. $\frac{1}{5}$ C. $\frac{1}{7}$ D. 5 E. Undefined

32. Find the limit of the sequence $a_n = 2 + \left(-\frac{4}{5}\right)^n$:

A. 2 B. $\frac{6}{5}$ C. $-\frac{4}{5}$ D. -2 E. $\frac{4}{5}$

33. The coefficient of $(x-3)^2$ in the Taylor series for $f(x) = \arctan(x)$ about $x = 3$ is:

A. $\frac{3}{100}$ B. $-\frac{3}{100}$ C. $-\frac{3}{50}$ D. $\frac{3}{50}$ E. $\frac{1}{256}$

Free Response

34. (10 points) Find the value of $\sum_{n=2}^{\infty} \frac{3^n + 5^n}{15^n}$. Show your work.

35. (10 points) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln(n))^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

36. (10 points) Use a Taylor Polynomial to estimate the value of the integral

$$\int_0^{0.5} \ln(1+x^2)dx$$

with an absolute error of at most 1/1000. Justify your answer.

Challenge Problem

37. (Optional, Extra Credit, Challenge Problem)

Suppose that a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies

$$0 < a_n \leq a_{2n} + a_{2n+1}$$

for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

