SECTION 7.4: Absolute-Value Equations and Inequalities (p. 547 – 558)

- Absolute value (p. 547)
- The absolute-value principle for equations (p. 548)
- Algebraic and graphical approaches to solve absolute value equations (p. 549)
- The absolute-value principle for equations with TWO absolute values (p. 551)
- Principles for solving absolute-value problems (p. 552)

For problems 1 through 7 below, answer the following questions in groups. You do not need to show your work. Think about it and write down your thoughts. Write down anything you think might help another student understand how you came up with your answers.

1. Nico takes the absolute value of a number, and gets 5. What number(s) did he use?

2. Nanci takes the absolute value of a number, and gets 0. What number(s) did she use?

   Nanci’s number must be zero since zero is 0 units away from itself.

3. Jayshawn takes the absolute value of a number to get -8. What number(s) did he use?

   No solution

4. Mavic doubles the absolute value of a number to get 18. What number(s) did he use?

   q or -q

5. Sam takes the absolute value of a number then subtracts 5 from the result to get -1. What number(s) did she use?

   4 or -4

6. Jaclyn adds 1 to a number then takes the absolute value of the result and gets 3. What number(s) did she use?

   2 or -4

7. Char says that she can multiply the absolute value of a number by -4 and the result will be 20. What number(s) could she be thinking of?

   No solution
SECTION 7.4: WRITING EQUATIONS INVOLVING ABSOLUTE VALUES

The descriptions from the previous problems are below. Use algebraic symbols to translate the problems you just solved into equations. If you had to think through more than one step to arrive at your answers, try to write the steps using algebraic symbols as well.

8. The absolute value of Nico’s number is 5.
   Equation: \( |n| = 5 \)
   Solution(s): \( n = 5 \) or \( n = -5 \)

9. The absolute value of Nanci’s number is 0.
   Equation: \( |n| = 0 \)
   Solution(s): \( n = 0 \)

10. The absolute value of Jayshawn’s number is -8.
    Equation: \( |j| = -8 \)
    Solution(s): No solution

11. Mavic doubles the absolute value of the number to get 18.
    Equation: \( 2 \cdot |m| = 18 \)
    Solution(s): \( m = 9 \) or \( m = -9 \)

12. Sam takes the absolute value, then subtracts 5 from the result to get -1.
    Equation: \( |s| - 5 = -1 \)
    Solution(s): \( s = 4 \) or \( s = -4 \)

13. When Jacyn adds 1 to a number, then takes the absolute value of the result, she gets 3.
    Equation: \( |j + 1| = 3 \)
    Solution(s): \( j = 2 \) or \( j = -4 \)

14. Char multiplies the absolute value of a number by -4 and the result will be 20. What number(s) could she be thinking of?
    Equation: \( -4 \cdot |c| = 20 \)
    Solution(s): No solution
SECTION 7.4: Determine the next step to solve each absolute value equation, and then solve the equation.

15. \(|2b + 1| + 2 = 7\)
   \[ \Rightarrow \quad |2b + 1| + 2 - 2 = 7 - 2 \]
   \[ \Rightarrow \quad |2b + 1| + 0 = 5 \]
   \[ \Rightarrow \quad |2b + 1| = 5 \]
   \[ \Rightarrow \quad 2b + 1 = 5 \quad \text{or} \quad 2b + 1 = -5 \]
   \[ \Rightarrow \quad 2b = 4 \quad \text{or} \quad 2b = -6 \]
   \[ \Rightarrow \quad b = 2 \quad \text{or} \quad b = -3 \]

16. \(3|8 + w| = 42\)
   \[ \Rightarrow \quad \frac{3}{3} \cdot \frac{|w + 8|}{1} = \frac{42}{3} \]
   \[ \Rightarrow \quad |w + 8| = 14 \]
   \[ \Rightarrow \quad w + 8 = 14 \quad \text{or} \quad w + 8 = -14 \]
   \[ \Rightarrow \quad w = 6 \quad \text{or} \quad w = -22 \]

17. \(-3|x + 8| = 12\)
   \[ \Rightarrow \quad \frac{-3 \cdot |x + 8|}{-3} = \frac{12}{-3} \]
   \[ \Rightarrow \quad |x + 8| = -4 \]
   No solution: recall that the absolute value function \(f(x) = |x|\) must have non-negative output values. In other words, for any input value \(x\), we know \(|x| \geq 0\). Thus, there is no value for \(x\) such that \(|x + 8| < 0\), and we conclude there is no solution.

18. \(-4|2m - 15| + 9 = -11\)
   \[ \Rightarrow \quad -4 \cdot |2m - 15| = -20 \]
   \[ \Rightarrow \quad \frac{-4}{-4} \cdot \frac{|2m - 15|}{1} = \frac{-20}{-4} \]
   \[ \Rightarrow \quad |2m - 15| = 5 \]
   \[ \Rightarrow \quad 2m - 15 = 5 \quad \text{or} \quad 2m - 15 = -5 \]
   \[ \Rightarrow \quad 2m = 20 \quad \text{or} \quad 2m = 10 \]
   \[ \Rightarrow \quad m = 10 \quad \text{or} \quad m = 5 \]
15. Let’s solve this problem using our graphical method. To do so, we graph the expressions on either side of the equals sign separately by setting

\[ y_1 = |2b + 1| + 2, \quad y_2 = 7 \]

We remember that the solution of our equation are the “x-coordinates” of the points of intersection.

Using the graph below (our on our graphing calculator), we see the two points of intersection are

\((-3, 7)\) corresponding to solution \[ b = -3 \]

\((2, 7)\) corresponding to solution \[ b = 2 \]
Determinate the number of solutions to the give equation.

19. \(|3x + 6| = |6x - 12|
\[
\begin{align*}
\Rightarrow & \quad 3x + 6 = 6x - 12 \\
\Rightarrow & \quad 3x + 18 = 6x \\
\Rightarrow & \quad 18 = 3x \\
\Rightarrow & \quad \frac{18}{3} = \frac{3x}{3} \\
\Rightarrow & \quad x = 6
\end{align*}
\]

\[\text{or} \quad 3x + 6 = -(6x - 12)\]
\[
\begin{align*}
\Rightarrow & \quad 3x + 6 = -6x + 12 \\
\Rightarrow & \quad 3x = -6x + 6 \\
\Rightarrow & \quad 9x = 6 \\
\Rightarrow & \quad \frac{9x}{9} = \frac{6}{9} \\
\Rightarrow & \quad x = \frac{2}{3}
\end{align*}
\]

20. \(-2|4y - 12| - 5 = 4|y + 1| + 15\)
\[
\begin{align*}
\Rightarrow & \quad -2|4y - 12| = 4|y + 1| + 20 \\
\Rightarrow & \quad -2 \cdot |4y - 12| = 4 \cdot |y + 1| + 20 \\
\Rightarrow & \quad \frac{-2}{-2} \cdot |4y - 12| = \frac{4 \cdot |y + 1| + 20}{-2} \\
\Rightarrow & \quad |4y - 12| = -2 \cdot |y + 1| -10
\end{align*}
\]

No solution.

Consider the LHS of this equation given by \(|4y - 12|\). The output to the LHS will always be greater than or equal to zero. Compare this to the RHS \(-2 \cdot |y + 1| -10\). The output to the right hand side will always be less than or equal to \(-10\). There is no way the two outputs can be the same. Thus we conclude there is no solution to this equation.
19. Let’s solve this problem using our graphical method. To do so, we graph the expressions on either side of the equals sign separately by setting

\[ y_1 = |3x + 6| \quad \quad y_2 = |6x - 12| \]

We remember that the solution of our equation are the “x-coordinates” of the points of intersection.

Using the graph below (our on our graphing calculator), we see the two points of intersection are

\( \left( \frac{2}{3}, 8 \right) \) corresponding to solution \( x = \frac{2}{3} \)

\( (6, 24) \) corresponding to solution \( x = 6 \)