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Making culturally responsive mathematics teaching explicit: a lesson analysis tool

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In the United States, there is a need for pedagogical tools that help teachers develop essential pedagogical content knowledge and practices to meet the mathematical education needs of a growing culturally and linguistically diverse student population. In this article, we introduce an innovative lesson analysis tool that focuses on integrating mathematical thinking, language, culture, and social justice. We argue that these are essential components to culturally responsive mathematics teaching. Using data collected from a 3-year professional development initiative focused on culturally responsive mathematics teaching including teacher discussions, lesson artefacts, and interviews, the study yields two main results about the tool and its impact on teacher thinking: (1) the tool enabled teachers to systematically analyse and critique mathematics lessons with multiple dimensions including mathematical thinking, language, culture, and social justice; (2) the tool fostered purposeful pedagogical dialogue to improve a mathematics lesson along specific culturally responsive dimensions. The findings illustrate the potential of this tool to engage teachers in strategic lesson planning and purposeful discussions to make mathematics lessons culturally responsive and improve the quality of mathematics teaching in the classroom.

Keywords: mathematics education; culturally responsive; mathematics teaching

Introduction

There is a sense of urgency to foster mathematics teacher education that can positively respond to the growing cultural and linguistic diversity of US public schools and the continued emphasis on improving students' mathematics performance (Grossman, Schoenfeld, & Lee, 2005; Sowder, 2007). Yet, tensions persist about just how to reach the higher equity and excellence standards in mathematics teaching and learning currently in demand in our nation and across the globe (Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000; OECD Programme for International Student Assessment [PISA], 2009). One avenue of research and professional development that has made some traction in this area focuses on culturally responsive mathematics teaching (CRMT) – a set of specific pedagogical knowledge, dispositions, and practices that privilege mathematical thinking, cultural and linguistic funds of knowledge, and issues of power and social justice in mathematics education (Aguirre, 2009; Gay, 2009; Gutiérrez, 2009; Kitchen, 2005; Leonard, Napp, & Adeleke, 2009; Turner et al.,

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2012). We argue that CRMT is essential to advance student learning. However, few tools are available to explicitly support teachers' development of CRMT competencies within their daily practices of planning and implementing mathematics lessons.

Our purpose is to introduce a specific professional development tool designed to support teachers to assess and integrate multiple resources into their mathematics lessons to make them culturally responsive. The CRMT lesson tool helps teachers evaluate their mathematics lessons through an integrated focus on mathematical thinking, language, culture, and social justice. Developed as part of a culturally responsive mathematics professional development initiative for new teachers, the CRMT tool enabled participating teachers to engage in systematic analysis and critique of mathematics lessons. The CRMT tool fostered purposeful pedagogical dialogue among teachers as they learned to utilize this tool to assess and modify their mathematics lessons to become culturally responsive. We think the insights gained from the tool in this professional development context have the potential to enhance mathematical pedagogies that will benefit mathematics learning for culturally and linguistically diverse students.

Constructing culturally responsive mathematics teaching

Two primary literature bases inform our definition of CRMT and the development of the CRMT lesson analysis tool: pedagogical content knowledge (PCK) and culturally responsive pedagogy (CRP). Independently, important elements from both literature bases contribute to what we think of as "good mathematics teaching." However, neither alone offers a comprehensive focus on mathematics teaching and learning that privileges equity and mathematics (Aguirre, 2009). CRMT integrates important elements of both literatures to create a more comprehensive context to support mathematics instruction. Dimensions of CRP and concepts from research into PCK in mathematics contribute to our understanding of productive avenues for developing culturally responsive mathematics teachers. Our conceptual framework illustrating these relationships is represented in Figure 1.

In the 1980s and 1990s, scholars in mathematics education started using cognition as a lens on children's mathematics thinking, turning the instructional focus of teaching onto how children construct mathematical knowledge (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999). New research in how children construct mathematical ideas, combined with sociocultural learning theories grounded in the

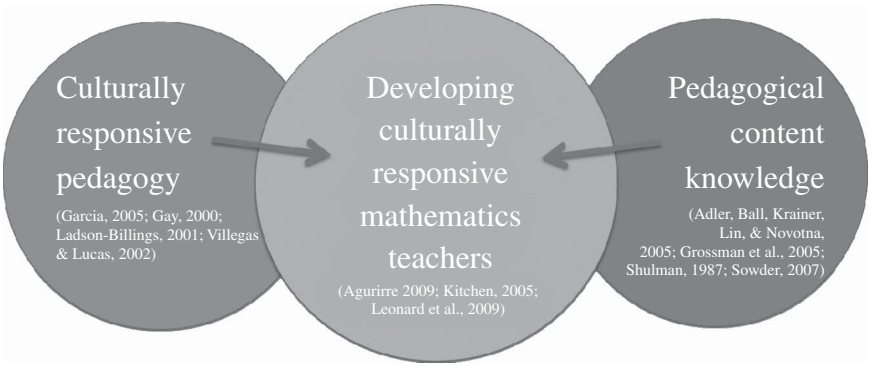

Figure 1. A conceptual framework for developing culturally responsive mathematics teachers.

work of Vygotsky, influenced a new direction in understanding mathematics learning and teaching (Cobb & Bowers, 1999; Yackel & Cobb, 1996). In teacher education, scholars turned their attention to what specialized teaching knowledge, apart from content knowledge, teachers needed to successfully teach children mathematics: PCK. Shulman (1987) defined PCK as

> The most regularly taught topics in one's subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations – in a word, ways of representing and formulating the subject that make it comprehensible to others. Pedagogical content knowledge also includes an understanding of what makes learning specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those frequently taught topics and lessons. (pp. 9–10)

Grossman (1990) further articulated four general PCK categories[1]: (1) an overarching knowledge and belief about the purpose for teaching; (2) knowledge of students' understandings, conceptions, and potential misunderstandings; (3) knowledge of curriculum and curricular materials; and (4) knowledge of the instructional strategies and representations for teaching particular topics. In relation to mathematics teaching, Sowder (2007) argued that Grossman's delineation of PCK into four components "are helpful for those developing teacher education programs and professional development opportunities for mathematics teachers" (p. 164). The attention to PCK in mathematics education contributes to a growing consensus for teachers to pay close attention to how students construct mathematical ideas, to learn explicitly how to support student engagement in meaningful mathematics, and to explicitly attend to mathematical discourses in the classroom (Sherin, Jacobs, & Philipp, 2011; Sowder, 2007; Turner et al., 2012).

In the last two decades, scholars have brought attention to other variables that impact mathematics learning such as power relationships, culture, race, and language (Diversity in Mathematics Education (DiME) Center for Learning and Teaching, 2007; Gutiérrez, 2002, 2009; Gutstein, 2006; Martin, 2009; Moschkovich, 2007, 2010; Secada, Fennema, & Adajian, 1995). Attending to the mathematics needs of second language learners has gained traction as an equity issue focusing on access to and participation in mathematics learning. Where English is the dominant language, scholars have sought ways to address best practices in scaffolding the participation of English language learners (ELL) that include drawing on their first language as a resource, activating prior knowledge, using realia, manipulatives, and other visual and tactile resources, as well as grouping strategies in the classroom for students to be resources for each other (Barwell, 2009; Khisty & Chval, 2002; Moschkovich, 2007, 2010). However, work on pedagogical content knowledge, with only few exceptions, lacks explicit attention to the relationships among power, race, culture, and language in the classroom (Aguirre, 2009; Gay, 2009). Furthermore, while Grossman et al. (2005) acknowledge that PCK and instructional practices for specific groups of students "might include culturally responsive pedagogies," CRP is considered an optional, rather than a required, knowledge base for new mathematics teachers.

We argue that PCK literature, while critically important to the development of mathematics teaching, is not enough to facilitate access and advancement of mathematics learning for all students. What is missing is a way to help teachers attend to the multiple dimensions of teaching mathematics that are based in research on children's mathematical thinking as well as language, culture, and power. Thus, we also draw from the CRP literature to address these essential dimensions needed for CRMT.

Culturally responsive educators have developed specific frameworks for instructional practices that promote excellence and equity for historically marginalized youth in the United States (see, for example, Gay, 2000; Ladson Billings, 1995). Gay (2000) provides an all-encompassing description of CRP that privileges academic excellence and equity:

> Culturally responsive pedagogy simultaneously develops, along with academic achievement, social consciousness and critique, cultural affirmation, competence, and exchange; community-building and personal connections; individual self-worth and abilities; and an ethic of caring . . . Culturally responsive teachers have unequivocal faith in the human dignity and intellectual capabilities of their students. They view learning as having intellectual, academic, personal, social, ethical, and political dimensions, all of which are developed in concert with one another. (pp. 43–44)

The importance of CRP cannot be overstated. However, a new teacher's introduction to CRP is usually reserved for multicultural education classes, not subject-specific methods courses (Cochran-Smith, Davis, & Fries, 2004). And while new teachers may gain inspiration from this work, finding ways to integrate these essential strands of CRP in a mathematics context has been elusive until recently (Nasir, Hand, & Taylor, 2008). Therefore, it is useful to visit two current areas in the literature that extends CRP into mathematics education and provides the foundation for CRMT: community funds of knowledge and teaching mathematics for social justice.

Teaching mathematics by attending to students' funds of knowledge

Moll and Gonzalez (2004) use the term funds of knowledge (FoK) to refer to "the knowledge base that underlies the productive and exchange activities of households" (p. 700). They argue that a FoK approach to education recontextualizes students in broader social contexts, and in doing this, raises teacher awareness of the resources and opportunities for teaching. Research studies of FoK approaches to teaching mathematics demonstrate how family activities such as gardening, sewing, scheduling, cooking, and playing games are mathematical resources available to students and teachers to support mathematics learning of children (Civil, 2007; Civil & Kahn, 2001; Turner et al., 2012). In addition to specific mathematical contexts embedded in household activities, a second conceptualization of FoK refers to specific mathematical resources immigrant parents and students learn in the schools of their home countries. As scholars have argued, symbolic notation is not universal and immigrant parents and older students may have unacknowledged mathematical skills that could serve as resources for mathematical learning (Perkins & Flores, 2002; Staats, 2009). A third area of research focuses on drawing upon community practices occurring locally to contextualize and extend mathematical understanding. These contexts might include exploring the mathematical practices of businesses near a school, such as bakeries, auto shops, and food markets, as well as community services like parks, libraries, cultural centres, and fire stations (Díez-Palomar, Simic-Muller, & Varley, 2007; Turner, Varley Gutiérrez, Simic-Muller, & Díez-Palomar, 2009).

The research on teaching mathematics and drawing on students' FoK emphasizes connections to students' broader lives in authentic ways, but may not explicitly address issues of power and justice. Turner and Strawhun (2007) argue that the problem context must be *authentic* for students. This includes constructing mathematics lessons based on **known** (not perceived, contrived, or assumed) student experiences as well as problems that stem from a local issue or situation that students find "genuinely problematic" such as

school overcrowding (Turner & Strawhun, 2007). Research on teaching that explicitly connects social issues, especially ones students find problematic, with mathematical tools for understanding and acting on the issue is discussed later.

Teaching mathematics for social justice

Mathematics education scholars with a specific focus on social justice see mathematics as an analytical tool to understand power relationships and structures of social, economic, and civic issues within a local, national, or global context (Aguirre, 2009; Christiansen, 2008; Gutiérrez, 2009; Gutstein, 2006; Mukhopadhyay & Greer, 2008; Tate, 1994; Turner & Strawhun, 2007). There is often a social action/civic engagement component that position students to challenge and change these inequitable situations/structures. Examples within the U.S context include secondary students investigating the role of racism in mortgage lending practices and police actions (Gutstein, 2006), as well as investigations that develop "statistical empathy" through critical examination of statistics linked to atrocities of war and violence (Mukhopadhyay & Greer, 2008). At the elementary level, examples include exploring large numbers through budgetary expenditures contexts (Peterson, 2005), using multiplication, measurement, and data analysis to challenge district decisions to close a neighbourhood school (Varley-Gutierrez, 2011), utilizing measurement concepts to challenge societal messages about body image (Kitchen & Lear, 2000), and engaging in data analysis and representation to address race and racism in primary grades (Tenorio, 2004). Teachers play an important role in guiding the students' mathematical investigations to understand, challenge, and change power relationships and structures.

The literature on CFoK and CM/SJ suggests that (1) powerful mathematics teaching builds on knowledge students bring from outside of school and leverages contexts in which their community members are experts and (2) broadens students' understanding of the utility of mathematics to interpret the world around them and act on social issues relevant to their lives. These components closely align with the purposes of CRP within a mathematics context. However, *defining* powerful mathematics teaching and *enacting* it are distinct processes. In the next section, we propose a framework for what CRMT means in terms of teacher development.

Developing culturally responsive mathematics teachers

As previously stated, CRMT involves a set of specific pedagogical knowledge, dispositions, and practices that privilege mathematical thinking, cultural and linguistic funds of knowledge, and issues of power and social justice in mathematics education. Recently some mathematics teaching frameworks have integrated aspects of PCK and CRP to help educate new teachers in mathematics understanding and equity (Aguirre, 2009; Gutiérrez, 2009; Leonard, 2007; Rodríguez & Kitchen, 2005; Turner et al., 2012). We will describe three key dimensions of CRMT that have linkages to both PCK and CRP, drawing on the culturally responsive teacher preparation framework offered by Villegas and Lucas (2002).

First, culturally responsive mathematics teachers must develop a socio-cultural–political consciousness. This means teachers must understand teaching and learning as part of a broader socio-political context and see their job as preparing students to engage in these contexts. Teachers must see mathematics teaching as a political activity rather than neutral activity, develop an awareness of the role power plays in school policies and curriculum practices, and actively seek to dismantle structures and practices that perpetuate inequities in mathematics education. Teachers must also be able to understand how

mathematics is used as a primary academic performance indicator of students to evaluate state, national, and global competitiveness. In addition, teachers must identify and challenge how mathematics is a primary gatekeeper to advanced courses and career trajectories with serious economic and social consequences for students if placed in lower mathematics tracks (Boaler, 2002; Oakes, 2005).

Second, teachers must understand and embrace social constructivist and socio-cultural theories of learning. Both CRP and PCK connect to these perspectives on knowledge, learning and teaching. Students construct their knowledge through social interaction that is mediated by language and culture (Cobb & Bowers, 1999). Learning is a complex, non-linear, and meaning-making process based on prior knowledge and experience. Because of the complexities of learning, teaching involves bridge building between previous knowledge and new knowledge (Lampert, 2001). PCK and CRP are required to make constant adjustments to instruction that can build on what students know and experience, which includes how students are mathematically socialized through discourse practices (Khisty & Chval, 2002; Moschkovich, 2002, 2007; Setati & Adler, 2001; Turner & Celedón-Pattichis, 2011).

Finally, teachers must focus on getting to know and leveraging the mathematical resources of students, their families, and their communities. Families and communities are assumed to have knowledge, experiences, and skills that can be intellectual resources (rather than deficits) for mathematics teaching and learning. Teachers discover ways mathematics is used at home and in communities to bridge school mathematical competencies and skills (Civil, 2007; Jackson & Remillard, 2005; Moll & González, 2004).

To summarize, culturally responsive mathematics teachers leverage mathematical learning by expanding children's mathematical thinking, building bridges between previous knowledge and new knowledge, supporting bilingualism and academic language development, fostering connections with cultural funds of knowledge and experiences, and cultivating critical mathematical knowledge that enables students to analyse and address authentic problems. The framework for CRMT takes powerful elements from both PCK and CRP to help teachers embrace a multifaceted approach to their practice aimed to advance mathematics excellence and equity.

A culturally responsive mathematics lesson analysis tool for teaching mathematics

Teachers need tools to help make CRMT explicit in the daily practices of teaching. We designed the CRMT tool for use in lesson planning and analysis contexts. The main goal is to make the elements of CRMT concrete for teachers.

The CRMT tool is made up of eight dimensions that approximate the categories of mathematical thinking, language, culture, and social justice. Each dimension may be examined within a single mathematics lesson or across lessons. Each category utilizes a rubric scale of 1–5 with descriptions of what evidence constitutes a specific rating. In addition, each category has a guiding question crafted to help the user attend to the essential elements of the dimension (see Table 1 for abbreviated version of CRMT Lesson Analysis Tool and Appendix 1 for full version of tool).

These dimensions come from a variety of sources. Categories 1–5 were drawn from a classroom observation protocol developed at the Wisconsin Center for Educational Research and extensively used by Richard Kitchen and his colleagues (Kitchen, 2005; Kitchen, DePree, Celedón-Pattichis, & Brinkerhoff, 2007; WCER, 1992). These first five categories align well with the PCK research which privileges attention to children's

Table 1. The eight dimensions of the CRMT lesson analysis tool, with guiding questions.

	Lesson analysis tool dimension	Guiding question
Mathematical thinking	1) Intellectual support	*How does my lesson make intellectual support visible?*
	2) Depth of student knowledge and understanding	*How does my lesson make student thinking/understanding visible and deep?*
	3) Mathematical analysis	*How does my lesson enable students to examine mathematical concepts and/or procedures?*
	4) Mathematical discourse and communication	*How does my lesson create opportunities to discuss mathematics in meaningful and rigorous ways (e.g. debate ideas, use math terminology, develop explanations, communicate reasoning, and/or make generalizations)?*
	5) Student engagement	*How does my lesson foster sustained and widespread student involvement in mathematical activity?*
Language	6a) Academic Language Support for ELL – Use of L1	*How does my lesson utilize L1 (home language) to support academic language development for English Learners?*
	6b) Use of ESL scaffolding strategies	*How does my lesson provide ESL scaffolding strategies to support academic language development for English Learners?*
Culture	7) Funds of knowledge/Culture/Community Support	*How does my lesson help students connect mathematics with relevant/authentic situations in their lives?*
Social justice	8) Use of critical knowledge/social justice.	*How does my lesson support students' use of mathematics to understand, critique, and change an important equity or social justice issue in their lives?*

mathematical thinking and engagement. Kitchen et al. (2007) detailed and utilized this five-dimension protocol in mathematics lesson observations in secondary schools (grades 6–12) that effectively serve poor and working class communities in the United States.

Categories 6–8 were added to explicitly address different dimensions of CRMT outlined in the literature that focus on language, culture, and social justice. The language categories (6a and 6b) were modified from an all-encompassing academic language support for ELL category that was developed by the Center for Mathematics Education of Latinos/as (CEMELA, 2006). The two language subcategories focus on the use of L1 (home language) and ESL (English as a second language) scaffolding supports for English learners. This adaption focuses instructional attention on the use of home languages as a resource for mathematics learning during lessons and specific instructional practices that scaffold academic language in the second language, in this case English. The rubric descriptors for category 6a (use of L1) provide specificity of how the use of home language may or may not be utilized in the math lesson. A score of 1 suggests complete intolerance for speaking any language other than the dominant language, in this case English. As the rubric increases, the frequency and affirmation of the use of L1 during the lesson by students is prioritized. Admittedly, there is some overlap between categories 6a and 6b because use of L1 is also considered a scaffolding strategy for students learning mathematics in a second language. However, category 6b also points to additional strategies that help students learning in a second language to access and participate in mathematics lessons such as use of graphic organizers, direct modelling of vocabulary in the dominant language, revoicing student comments, use of realia (physical models), and attention to non-verbal communication such as gestures. The use and frequency of these scaffolding strategies among students and teacher increase with corresponding rubric scores.

The seventh dimension focuses on community/cultural funds of knowledge (CFoK). As discussed previously, this dimension builds on research demonstrating the richness of mathematical knowledge and practices embedded in daily household and out-of-school activities of families and communities. The rubric descriptors attend to the *authenticity* of problem contexts for students. The characteristic of authenticity overlaps extensively with the eighth dimension of the tool: critical knowledge/social justice. However, within the CFoK dimension, higher rubric scores correlate with intentional, sustained connections to students' lives to engage in rigorous mathematics (i.e., sustained attention to mathematics with or through family and community knowledge). A score of 1 means that the lesson connects to a generic context. For example, in the United States, a common context for developing fractions sense is the use of fair-sharing situations involving popular food such as pizza or brownies. The rationale for such problem contexts is that "all" students can relate to popular food and therefore will be more engaged with such fraction tasks involving this context. While mathematically useful, these generic contexts are culturally "neutral" which can explain their common occurrence as "application" examples in mathematics textbooks. Instead, lessons that explicitly connect to known family activities and student interests or local community events/issues provide a different and more authentic context for students to explore mathematics.

The eighth dimension of the CRMT tool relates to critical knowledge/social justice. The rubric for this dimension is designed to get teachers to pay attention to whether and how they connect math lessons to authentic problems that demand social/civic action. The lower rubric scores include no evidence (score 1) as well as a missed opportunity to connect to such a social justice context (score 2). The higher rubric scores describe an intentional connection with and throughout the lesson(s) (scores 4 and 5). Most lessons

taken directly from elementary school curricula would rate low, as attending to social justice is not a regular part of mathematics textbooks. However, as teachers look for opportunities to customize lessons to include authentic problems or social issues similar to those described previously, explicit connections would increase the rubric score.

The CRMT tool is designed to provide guideposts for teachers to plan and analyse their mathematics lessons among multiple dimensions that include children's mathematical thinking, language, culture, and social justice. While the CRMT tool delineates separate categories, there are clear overlaps across the dimensions. We feel the distinctions and overlaps are strengths that add comprehensiveness and specificity to analysing lessons from a culturally responsive mathematics standpoint.

Methods

Study design

This analysis is part of a larger teacher professional development study called the CRMT project. The focus of CRMT project was to document and describe how beginning K-8 mathematics teachers developed and implemented culturally responsive mathematics pedagogy in their early years of teaching.

This study utilizes a qualitative research design with multiple data sources (Miles & Huberman, 1994). A major goal was to analyse the development and impact on pedagogical decisions, visions, and mathematics instruction over time. In service of this goal, all teacher participants were recent alumni of the lead author's university mathematics methods course taught in the 2007–2008 academic year. This course contained specific activities and assignments designed to develop teacher knowledge and competencies related to mathematics and equity (Aguirre, 2009). Six beginning teachers agreed to be a part of this research study. Participants included five White female teachers and one White male teacher. The following year, three participants acquired full-time teaching jobs in elementary schools in three regional school districts (Melissa, Natalie, and Vanessa) and two participants acquired full-time teaching jobs at the middle school level (Alex and Elise). One teacher (Diane) continued to look for full-time employment while working as a substitute teacher her first year and a part-time Spanish teacher for a private school the second year. All five full-time public school teachers worked in different school districts and at schools with significant populations of minority (42–71%) and economically disadvantaged youth (32–71%). The participants agreed to a series of professional development workshops and teacher interviews aimed at developing their CRMT competencies. A limitation of the study is varied teacher participation in project activities overtime. While all six teachers participated in the first year project activities, due to logistical constraints (e.g., family emergencies, time conflicts, no longer teaching mathematics) three elementary teachers remained active with the project between 2008 and 2010.

Data sources

The data sources for this analysis include teacher interviews, three professional development sessions, and written artifacts (i.e., teacher-written reflections, lesson plans). Although initially part of the research design, there were no class observations of participating teachers' classrooms due to complex logistical issues with districts meeting university human subjects requirements. This analysis primarily draws on professional

development sessions occurring in May and July of 2009, and the first two teacher interviews conducted in summer 2008 and fall 2009.

Teacher interviews. Three semi-structured audio-taped interviews (average length 60 min) were conducted during the project. All teachers participated in the first interview (summer 2008), five teachers in the second interview (fall 2009), and three teachers participated in a third interview in fall 2010 (see Appendix 2 for sample interview questions related to this analysis). The first interview was conducted before or early in their first year of teaching before the first professional development session. We asked participants to describe their math teaching vision, define CRMT, discuss their views of the roles language and culture play in mathematics teaching and learning, and identify supports and challenges in learning to teach. The second interview was completed after one full year of teaching and two professional development sessions occurring in May and July 2009. The interview questions again focused on the role of language and culture in their lesson planning and implementation as well as for comment on the professional development session they experienced with the CRMT lesson planning analysis tool, and supports and challenges for mathematics instruction.

Professional development sessions. These sessions occurred in November 2008, May 2009, and July 2009. Each session was approximately 3 hours. These sessions were responsive in nature and provided a supportive environment for participating teachers to ask questions and troubleshoot teaching issues. Though session attendance fluctuated with participants' schedules, four out of six teachers attended all the sessions, and all teachers attended the first session. During these professional development sessions, observations of instructional practice centred on the practice of lesson planning and analysis, namely, how teachers conceptualize, design, reflect upon, and refine a mathematics lesson (self-constructed or from a specific curriculum program). This part of a teacher's practice happens outside of the immediate physical presence of students, but can reflect the understandings that influence what happens in the mathematics classroom (Little & Horn, 2007). The first session focused on reconnecting with the participating teachers early into their first year of teaching and providing a forum for them to share their challenges and discuss supports. In the second session, we introduced the lesson analysis tool through two activities. The participants were acquainted with the tool by observing a commercially available math lesson. The teachers discussed ratings for different categories in pairs. Next, teachers rated their own math lesson plan according to each dimension. Teachers were asked to locate evidence in the math lessons to support their ratings and discuss their findings with the group. The third session built on the previous session using the tool to analyze math lessons and develop teaching goals for the year. We transcribed all professional development sessions and teacher interviews.

Data analysis

Data analysis began with a systematic review of participant interview data to begin forming in-depth case studies (Miles & Huberman, 1994). Our analysis included preliminary coding to condense the interview data by summarizing specific data linked to interview questions. For example, specific codes included teacher's definitions of culturally responsive mathematic teaching and their views of the roles of families, culture, and language in mathematics lessons and learning. Then we analysed the video transcripts of the summer professional development sessions using a similar iterative coding procedure. Data

transcripts were analysed using a constant comparative method (Strauss & Corbin, 1997). Using this method for triangulation purposes, central themes were identified across the data. The themes focused on multiple ways teachers defined CRMT, as well as struggles around what counts as evidence for the different categories aligned with the goals of CRMT. We discuss findings in relation to these themes later.

Results

A promising finding from this research is the development of a lesson analysis tool to help support pedagogical design and mathematics instruction that is culturally responsive. Two prominent themes emerged from this analysis. First, the CRMT tool enabled teachers to engage in systematic analysis and critique of mathematics lessons with multiple dimensions including attention to mathematical thinking, language, culture, and social justice. In particular, we found teachers grappling with multiple interpretations of cultural/community funds of knowledge (CFoK). Second, the tool fostered purposeful pedagogical dialogue in designing their mathematics lessons. Participating teachers began to self-evaluate strengths and limitations of their mathematics lessons using rubric dimensions and apply these insights to adapt lessons and improve their practice in the future.

Negotiating meaning of CFoK in mathematics lessons

In the first professional development session, teachers discussed sources of funds of knowledge, raising questions about the location of community that counts as drawing on "community funds of knowledge." Teachers distinguished two different sources: whether the community of reference was the classroom community or the home communities of students. This distinction was also found in interviews. For example, when defining CRMT in her first interview, Vanessa included a hybrid stance of CFoK that included both inside and outside resources, saying that it is not just about what kids bring into class, but also "being able to tap into . . . the classroom environment that we create together. And, drawing from that to build math into that so they can see that math happens in their world in school and beyond."

The CRMT tool raised opportunities to push conceptions of CFoK as reflected in mathematics lessons towards an explicit anti-stereotype dimension that was not captured on the protocol. For example, in the third professional development session, teachers debated what constituted a low score on the rubric for CFoK – culturally neutral, culturally absent, and culturally stereotypical. When viewing a video that focused on generating factors of 48 using arrays (Cameron, Dolk, Fosnot, Hersch, & Tieg, 2005), the following discussion took place among partners Melissa and Vanessa:

Melissa: I saw a huge draw on classical math, community knowledge but it is school-based, not real cultural at all.

Vanessa: Oh this is hard to answer. We don't know what their community is about. Maybe she was bringing in something from the community, but -

Melissa: But it is very much numbers. The numbers were not used in the context of anything -

Vanessa: Other than mathematical concepts like arrays.

Melissa: Maybe this is a part of a string of lessons where now they got this solidified they will apply it in a way that is more responsive culturally. This was more the nuts and bolts.

Vanessa: Culturally neutral. I like that term.
Melissa: Yeah. But one thing I might say about it is that maybe this, the kind of scale might go into negative numbers. Because culturally neutral is one thing, but there was not anything in the lesson that I could see that would leave people behind. They had developed a base of knowledge that was thorough enough that the class could have access too.
Vanessa: True.

Both teachers agreed that while the lesson supported the development of mathematical ideas – school-based mathematical knowledge – there was limited evidence of cultural/community knowledge from outside the school. They identified the context as being "culturally neutral" or more "nuts and bolts." Melissa's critique related to CFoK was buffeted by her observation that the lesson promoted mathematical access that would not "leave kids behind."

At this point the other participants and both researchers joined the conversation. Melissa posed the question about whether there were "negative" scores for the rubric:

Melissa: Do they ever have a scale with negative numbers?
Researcher 1: What would that be like?
Melissa: Well, if they ever did a lesson that she assumed that they all knew something or did a lesson that was disrespectful to the knowledge a kid brought to it.
Researcher 1: I think that is a good thing to think about what the rubric represents. Super low expectations, negative responses to children based on behavior, issues of respect? [*Pause*] What would justify a rating of 1?

As the conversation continued, Vanessa and Melissa reported a rating of one on this dimension because there was no evidence of tapping into cultural funds of knowledge. The researcher offered an anecdotal story about a pre-service teacher watching a veteran teacher try to make a cultural connection to students during a math lesson. The veteran teacher announced to the class of students, many of whom were Vietnamese descent, "your parents all own nail salons, right?" The significance of this incident in an American context is that owning nails salons is a labour stereotype of Vietnamese immigrants. When recounting this story to the teachers, they responded negatively about this action by shaking their heads. The researcher then asked where would something like that lesson situation fit onto the rubric. In response, Melissa contemplated whether culturally neutral should be bumped up to a three, and the negative stereotype context rated a one. The following exchange took place, with researcher 2 beginning by offering her reaction to what such a rating system would mean:

Researcher 2: I would hate people to think that neutral is okay. It is an interesting dilemma. If you make that (referring to culturally neutral) a three, then you can always think that you are doing okay.
Melissa: Yeah.
Researcher 2: This is designed to make you think about how to actually do it, so if you are attempting to engage in this.
Melissa: It made me think about [*pause*] the lesson has to fit into a spectrum. I think this is a strong lesson yet it scored a 1 on this. Does that make it a bad lesson? Or where does it go from there? Is it one of a string, where the kids if they solidify the math knowledge and engage with it in a day or two? Then it all works and it's good.

> Natalie: But I like what you are saying though (to Researcher 2). Because if you have to do a negative on here, it means that you did not meet it at all, you are not even on the board. So do something to change.

In this exchange, the teachers wrestle with what constitutes evidence for drawing upon cultural/community funds of knowledge and how to evaluate that connection while at the same time recognizing mathematical strengths of the lesson. The discussion reflects two inherent tensions. The first tension relates to the degree to which CFoK are reflected in math lessons. The group examined the rubric descriptor of culturally neutral and posed alternative descriptors to more accurately evaluate mathematical lessons in terms of this dimension. Lessons that might reflect cultural stereotypes were offered as potential situations that were not reflected within the current rubric, whereas potential situations that might be deemed "culturally neutral" were problematized as to whether they were really the lowest level on a rubric. Clearly, the discussion raised instructional planning scenarios that connected to challenging cultural stereotypes of students and their families. Culturally neutral contexts, stereotypical contexts, and the complete absence of contexts that connect to CFoK were contenders for a low rubric score on this dimension. This information could help teachers plan lessons in different ways or in the words of Natalie, "do something to change."

The second tension focuses on what constitutes a good mathematics lesson. By not including a community context, does that make it a bad lesson? The teachers considered other evidence they would look for to rethink their rating. For example, Melissa reflected on the instruction beyond this lesson to where this lesson fits within a "spectrum," again evoking her attention to thinking of lessons as part of a series of activities teachers plan over time. The use of the CRMT tool in conjunction with a concrete lesson made her contemplate the possibility of CFoK being addressed later in another lesson and thus the sequence of lessons be considered "good."

Self-evaluation of mathematics lessons: a purposeful dialogue

The CRMT tool also fostered purposeful pedagogical dialogue in assessing mathematics lessons that highlight strengths and areas for improvement. To illustrate this, we focus on the case of Melissa's self-analysis of her curriculum textbook lesson using estimation and rounding numbers strategies for multi-digit addition using the CRMT tool. We follow her responses during a small group discussion of her ratings on the language subcategory use of L1 (home language) (6a) and critical knowledge in mathematics (8). Melissa, a third-grade teacher with Spanish- and Russian-speaking students, and "a sprinkling of other languages," rated herself between a 3 and 4 in category 6a. She explained her decision:

> So this year one of my goals is to label things in different languages because we are learning, our school and our district, are going to have a focus on explicit academic language instruction this year, and so I want to expand that to include the other languages that are represented in our classroom next year. And so in this, you know, just be part of what we do teaching when we talk about vocabulary, we'll be doing it in all the languages. So, I gave myself a three, possibly into a four, because when we're labeling the numbers on the number lines I would be asking the kids, okay, how do you say half? Teach me your language, and the other kids in the class. How do you say fifty, five hundred, all those half numbers?

Melissa justified her rating and voiced her plan to incorporate her students' home languages to develop their academic language in mathematics. In this way, she located how she would

adapt her lesson to incorporate a student's home language (L1) as a language support on the rubric in conjunction with the increasing demand of supporting academic language of students. Melissa also noticed, "So I didn't think that it reached the level of extensive and sustained attention to students' linguistic funds of knowledge," using the language of the rubric rating of five to support where her lesson fell short. She concluded, "I think I need more training in ELL strategies," a conclusion that logically followed from her own assessment using the rubric.

During the same activity, Melissa rated her lesson a 1 in the category of critical knowledge/social justice. She discussed this rating and how she could make this connection more explicit with Vanessa and one of the researchers, using the higher scores on the rubric scale to anchor the discussion. She started by sharing her rating of 1, explaining she had "no evidence of connection to critical knowledge," at which point the researcher reminded her about an idea she brought up earlier, thinking about how estimating the number of people in a whole family or how many hours are in a work week could start that connection. Vanessa articulated particular challenges she had with this category, which lead to a brief discussion of challenges but also prompted Melissa to think about a possible connection to a topic in social studies:

Vanessa:	Yeah, this is the one I have the most trouble with, I'm usually down in this range [*gestures towards lower end of rubric on paper*] because I don't know how to make it socially justice – socially (Melissa: Socially just.) Socially just AND age appropriate. You know, something that is on their level that they can relate to.
Melissa:	Yeah, and that it doesn't open up troubling problems that we're not able to really resolve in any way.
Vanessa:	Or that maybe they're not ready to attack just yet, you know?
Researcher 2:	Yeah, it's an interesting dilemma.
Melissa:	Well, one of the things in our social studies is talking about immigration and the great migration, you know? I think there are ways you can bring up social justice issues that are related historically, but are also still present in day-to-day lives. So maybe you can have it in the context of what we're doing in social studies.
Vanessa:	Mathematizing.

The protocol enabled both Melissa and Vanessa to engage in substantive reflections around Melissa's lesson. In this case, both acknowledged that addressing issues of social justice/critical knowledge was not evident in this lesson, or in Vanessa's case most of her lessons. The discussion opened up the opportunity to think about linkages to other subjects in which social justice is discussed such as history/social studies. In addition, both Vanessa and Melissa identified potential tensions related to addressing social justice in mathematics lessons (e.g., age appropriate and readiness) that are not uncommon in the culturally responsive mathematics literature (Aguirre, 2009; Christiensen, 2008; Leonard et al., 2009).

The resulting interaction with the CRMT tool enabled teachers to see lesson planning as "an overarching" process – not just one lesson but a set of lessons – with the rubric categories as guideposts. During our group debrief, Melissa reflected on the activities we did together and revisited the idea of thinking of lessons over time as a unit of instruction:

> I really liked that format, I mean to practice the exercise on a video, so it's something that I'm not, you know, it's not in my gut that I feel really personal about, that I get to practice on, and then to apply it to my own lesson plan was really useful. And I think it would be neat over

> time to see things moving up a little bit or, you know, this was an isolated lesson plan and I'm down here at a one on this one, but if it was an over-arching unit, you know what I mean? To see changes.

Melissa articulated the main intention of this tool: to provide teachers opportunities to critically reflect on their practice and make substantive changes to improve their mathematics lesson. These changes can be specific to the particular lesson or more broadly what to include in a specific mathematics unit. In addition, self-analysis using the CRMT tool offered these teachers specific milestones to gauge and improve their practice over time, using the detailed categories in the rubric. Melissa could envision what progress might look like and set pedagogical goals to improve her own ratings of her lessons.

Discussion with implications

We believe the CRMT tool has potential widespread uses beyond formal professional development contexts that can promote both strategic and comprehensive professional growth and reflection. At the classroom level, teachers can use this tool to reflect on a single lesson or unit. An individual can target a few dimensions or all dimensions. The rubric might provide a visual representation that aids lesson modification from a culturally responsive standpoint. At a collegial level, teachers engaged in professional learning communities such as elementary grade level planning teams or secondary mathematics departments may find the comprehensiveness and specificity of the CRMT tool a useful instrument to promote deeper pedagogical discussions about instructional practice.

However, there are some cautionary points to the use of the CRMT tool for discussion purposes only. For example, *all* dimensions of the CRMT contribute to CRMT. Attention to only children's mathematical thinking or academic language or community funds of knowledge over time would diminish its power as a comprehensive analytical tool for lesson design and implementation. Furthermore, the dimensions corresponding to children's mathematical thinking have a widespread history as a classroom observation tool (Kitchen et al., 2007) unlike the latter dimensions that focus on academic language for second language learners, cultural/community funds of knowledge, and social justice. Thus the recommendation to leverage the CRMT tool as a productive discussion instrument would be to insist that selection span both the historical and more current dimensions of the tool. For instance, a group of teachers could select 1–2 dimension(s) from the children's mathematical thinking category and 1–2 dimensions from among the CFoK, language, and critical knowledge/social justice categories to analyse lesson design and implementation. This will broaden mainstream attention to a more comprehensive understanding of CRMT.

Beyond a discussion-only instrument, another possible usage of the CRMT tool could be for teacher evaluation purposes. The CRMT tool is designed for teachers to self-reflect on their lesson design and implementation, however with one important caveat: *The teacher being observed will get the opportunity to select which criteria they want instructional feedback to improve practice*. Lesson observations are an important mechanism for teachers to get constructive professional support. Since teaching is a complex practice, the process should be transparent and helpful – not punitive. As an example, Aguirre has used the CRMT tool as a lesson observation tool in university mathematics methods courses for practicing teachers. In this context, teachers selected 3–4 dimensions for specific feedback when they modelled a lesson. Then multiple observers completed the specific dimensions with evidence from the lesson. The scored and annotated CRMT rubrics were returned to the teacher for reflection and lesson improvement purposes. In addition, the teacher was asked to use the tool to reflect on the lesson. With these multiple sources of feedback,

teachers discussed the lesson's strengths and areas for improvement. This feedback and reflection process took time. The tool was not designed nor should it ever be used as a high stakes teaching evaluation tool.

Conclusion

As mathematics teachers work towards meeting the increased standards for excellence and equity, they need tools to help them effectively engage in this work. The analysis presented here suggests teachers can engage in purposeful pedagogical discussions that focus on mathematical thinking, language, culture, and social justice when given tools with an integrated focus. CRMT expands and integrates the kind of pedagogical content knowledge teachers need to effectively teach mathematics to culturally, linguistically, and socio-economically diverse students. The CRMT tool provides some specific guideposts to meet the diverse mathematics learning needs of their students. The tool offers a way to expand instructional vision to improve practice within a lesson or a unit over time. In addition, it provides specificity to evaluate a teacher's own practice in multiple dimensions representing culturally responsive mathematics instruction. As calls continue for teacher education to effectively address the mathematics education needs and advancement of our increasingly culturally, ethnically, and linguistically diverse student population, the use of this tool demonstrates a positive step towards this goal.

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Note

1. It is important to understand that Grossman's work on PCK was developed through an in-depth study of secondary English/Language Arts teachers in the United States. However, this concept is generalizable to other domains of teaching and teacher preparation (Grossman et al., 2005; Sowder, 2007).

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Appendix 1. Mathematics lesson analysis protocol (condensed version)

		1	2	3	4	5
	Guiding question: How does my lesson make intellectual support visible?					
1) Intellectual support	Description of rating	Intellectual support is negative. Action/comments by teacher or students result in put-downs of students' academic efforts. Students interfere with one another's efforts to learn. Classroom atmosphere for learning is negative.	Intellectual support is mixed. Both negative and positive actions/comments by teacher or students concerning students' academic efforts are observed. Teacher fails to call upon students who want to participate repeatedly.	Intellectual support is neutral or mildly positive. Evidence may be mainly in the form of verbal approval for student effort and work. Support tends to be given to students who are already taking initiative in the class, and it tends not to be given to those who are reluctant participants or less articulate or skilled in the subject.	Intellectual support from the teacher is clearly positive. There is some evidence of intellectual support among students for their peers. Evidence of special efforts by the teacher take the form of expressions that convey high academic expectations for all, mutual respect, and a need to try hard and risk initial failure.	Intellectual support is strong. The class is characterized by high academic expectations, challenging work, strong effort, mutual respect, and assistance in achievement for all students. Both teacher and students demonstrate a number of these attitudes by soliciting and welcoming contributions from all students who are expected to put forth their best efforts. Broad participation may be an indication that low achieving students receive intellectual support for learning.

Guiding question: How does my lesson make student thinking/understanding visible and deep?

2) Depth of knowledge and student understanding	Description of rating	Knowledge is very thin because concepts are treated trivially or presented as non-problematic. Students are not involved in the coverage of information they are to remember.	Knowledge remains superficial and fragmented. Underlying or related concepts and ideas might be mentioned or covered, but only a superficial acquaintance or trivialized understanding of these ideas is evident.	Knowledge is treated unevenly during instruction. Deep understanding of some mathematical concepts is countered by superficial understanding of some other ideas. At least one idea may be presented in depth and its significance grasped by some (10%–20%) students, but in general the focus is not sustained.	Knowledge is relatively deep because the students provide information, arguments, or reasoning that demonstrates the complexity of one or more ideas. The teacher structures the lesson so that many students (20%–50%) do at least one of the following: • sustain a focus on a significant topic for a period of time; • demonstrate their understanding of the problematic nature of information and/or ideas; • demonstrate understanding by arriving at a reasoned, supported conclusion; • explain how they solved a relatively complex problem.	Knowledge is very deep because the teacher successfully structures the lesson so that most students (50%–90%) do at least one of the following: • sustain a focus on a significant topic; • demonstrate their understanding of the problematic nature of information or ideas; • demonstrate complex understanding by arriving at a reasoned, supported conclusion; • explain how they solved a complex problem. In general, students' reasoning, explanations, and arguments demonstrate fullness and complexity of understanding.

(*Continued*)

Table A1. (*Continued*).

	1	2	3	4	5
	Guiding question: How does my lesson enable students to examine mathematical concepts and/or procedures?				
3) Mathematical analysis	Students receive, recite, or perform routine procedures. In no activities during the lesson do students engage in mathematical analysis.	Students primarily receive, recite, or perform routine procedures.	There is at least one significant activity involving mathematical analysis in which some students (10%–20%) engage. Or, mathematical analysis that is primarily diversionary in nature occurs throughout the lesson.	There is at least one major activity in which students engage in mathematical analysis. This activity occupies a substantial portion of the lesson; and many students (20%–50%) are engaged in it.	Most students (50%–90%), for most of the time (50%–90%), are engaged in mathematical analysis.
	Guiding question: How does my lesson create opportunities to discuss mathematics in meaningful and rigorous ways (e.g., debate ideas, use math terminology, develop explanations, communicate reasoning, and/or make generalizations)?				
4) Mathematics discourse and communication	Virtually no features of mathematical discourse and communication occur, or what occurs is of a fill-in-the-blank nature.	Sharing and the development of collective understanding among a few students (or between a single student and the teacher) occur briefly.	There is at least one sustained episode of sharing and developing collective understanding about mathematics that involves: (a) a small group of students or (b) a small group of students and the teacher.	There are many sustained episodes of sharing and developing collective understandings about mathematics in which many students (20%–50%) participate.	The creation and maintenance of collective understandings permeates the entire lesson. This could include the use of a common terminology and the careful negotiation of meanings. Most students (50%–90%) participate.

			Or, brief episodes of sharing and developing collective understandings occur sporadically throughout the lesson.			
Guiding question: How does my lesson foster sustained and widespread student involvement in mathematical activity?						
5) Student engagement	Disruptive disengagement; students are frequently off-task as evidenced by gross inattention or serious disruptions by many students (20%–50%); this is the central characteristic during much of the class.	Passive disengagement; students appear lethargic and are only occasionally on-task; for most of time, many students (20%–50%) are either clearly off-task or nominally on-task but not trying very hard.	Sporadic or episodic engagement; most students (50%–90%), some of the time (20%–50%), are engaged in class activities, but this engagement is uneven; mildly enthusiastic or dependent on frequent prodding from the teacher.	Engagement is widespread; most students (50%–90%), most of the time (50%–90%), are on-task pursuing the substance of the lesson; most students seem to be taking the work seriously and trying hard.	Serious engagement; almost all students (90% or more) are deeply involved, almost all of the time (90% or more), in pursuing the substance of the lesson.	

(*Continued*)

Table A1. (*Continued*).

	1	2	3	4	5
	Guiding question: How does my lesson use L1 to support academic language development for ELLS?				
6A) Academic language support for ELLs	An explicit intolerance towards students' use of L1 is evident such as translation or code-switching (e.g. "We only use English in this classroom."). Students who are not yet fully proficient in English are ignored and/or seated apart from their classmates.	No acknowledgement of ELL students' needs or presence is evident. Although there is no explicit use of ESL strategies, or attention to L1 (such as explicit attention to cognates), students' use of L1 is tolerated.	There is at least one instance of support for L1. Even if teacher does not use L1, it is evident that students' linguistic repertoires are valued and that they are encouraged to build on them (e.g. students can present in L1, students work in groups in L1, students work in groups in L1.	Sustained encouragement of L1 usage is observed at least between teacher and one, or small group, of students. Focus is on mathematical discourse.	Extensive and sustained attention to students' linguistic funds of knowledge. Sustained encouragement of L1 usage, or hybrid language (ex. code-switching) is observed among teacher and students, in a variety of interactions (Teacher-students, small group, and whole class). The main focus is the development of mathematical discourse and meaning making, not students' production of "correct" English.

Guiding question: How does my lesson utilize scaffolding strategies to provide academic language development for ELLS?

6B) Use of ESL scaffolding strategy	No evidence of an ESL scaffolding strategy. Students who are not yet fully proficient in English are ignored and/or seated apart from their classmates.	Although there is no explicit use of ESL strategies, or attention to L1 (such as explicit attention to cognates), students' use of L1 is tolerated. Focus on correct usage of English vocabulary.	There is at least one instance in which an ESL scaffolding strategy is used to develop academic language (i.e. revoicing, use of graphic organizers).	Sustained use of at least a couple of ESL strategies, such as the use of revoicing and attention to cognates, direct modelling of vocabulary, or encouragement of L1 usage is observed at least between teacher and one, or small group, of students.	Deliberate and continuous use of ESL strategies, such as gesturing, use of objects (realia), use of cognates, revoicing, graphic organizers and manipulatives are observed during whole class and/or small group instruction and discussions. The main focus is the development of mathematical discourse and meaning making, not students' production of "correct" English.

(*Continued*)

Table A1. (*Continued*).

	1	2	3	4	5
	Guiding question: How does my lesson help students connect mathematics with relevant/authentic situations in their lives?				
7) Funds of knowledge/ culture/ community support	No evidence of connecting to students' cultural funds of knowledge (parental/community knowledge, student interest). Lesson incorporates culturally neutral contexts that "all students" will be interested in.	There is at least one instance in connecting math lesson to community/cultural knowledge and experience. Lesson draws on student knowledge and experience. Focus is with one student or a small group of students.	There is at least one sustained episode of sharing and developing collective understanding about mathematics that involves connecting to community/cultural knowledge. Or, brief episodes of sharing and developing collective understandings occur sporadically throughout the lesson.	There are many sustained episodes of sharing and developing collective understandings about mathematics that involves connecting to cultural/community knowledge (e.g. student experiences are mathematized, student/parent connections with math work; math examples are embedded in local community/cultural contexts and activities – i.e. games).	The creation and maintenance of collective understandings about mathematics that involves intricate connections to community/cultural knowledge and permeates the entire lesson. This would include hook/intro, main activities, assessment, closure and homework. Students are asked to analyze the mathematics within the community context and how the mathematics helps them understand that context.

	Guiding question: How does my lesson support students' use of mathematics to understand, critique, and change an important equity or social justice issue in their lives?				
8) Use of critical knowledge/ social justice	No evidence of connection to critical knowledge (socio-political contexts, issues that concern students)	Opportunity to critically mathematize a situation went unacknowledged or unaddressed when present.	There is at least one instance of connecting mathematics to analyze a sociopolitical/cultural context.	There is at least one major activity in which students collectively engage in mathematical analysis within a sociopolitical/authentic or problem-posing context. Mathematical arguments are provided to solve the problems. Pathways to change/transform the situation are briefly addressed.	Deliberate and continuous used of mathematics as an analytical tool to understand an issue/context, formulate mathematically based arguments to address the issues and provide substantive pathways to change/transform the issue.

Appendix 2. Sample interview questions

Interview 1

- How do you think you will teach mathematics in this next coming year, and how does this compare to the way you were taught mathematics as a student?
- Why do you think it is important for kids to learn math?
- When you are planning a math lesson, what kinds of things do you consider, and what roles do those things play in the planning of your math lessons?
- How would you describe mathematics teaching that is culturally responsive?
- When you plan a mathematics lesson, what are considerations you make that are culturally responsive?
- What role do you think language (home and math); culture and family/community play in learning and teaching mathematics?

Interview 2

- Talk me through how you plan a typical math lesson. Talk aloud about the considerations you make and the materials you use in this process. Tell me more about how, if at all, these considerations are culturally responsive.
- How have your views about the role(s) of language (math, home); culture; family/community play in math instruction changed, if at all, since you started teaching math this year?*
- In thinking about the pd session and the activities we did related to analyzing lessons we are interested in how those activities/discussions resonated and/or challenged your thinking about mathematics teaching in culturally responsive ways. Please share some of your insights or experiences. *[Be sure to probe for things that resonated and things that challenged thinking]*
- How have you been supported to teach mathematics in culturally responsive ways?*
- What challenges have you faced in teaching mathematics this year? When you think about teaching mathematics in culturally responsive ways, are these challenges similar? Different? *[if probe needed – Rephrase as, "Are there unique challenges to teaching math in culturally responsive ways?"]**

*Questions repeated in Interview #3.