Gait & Posture 40 (2014) 134–139

Contents lists available at ScienceDirect

Gait & Posture

journal homepage: www.elsevier.com/locate/gaitpost

Interpreting lateral dynamic weight shifts using a simple inverted pendulum model

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ARTICLE INFO

Article history:
Received 15 October 2013
Received in revised form 28 January 2014
Accepted 7 March 2014

Keywords:
Balance
Lateral shifting
Inverted pendulum model
Standing
Non-minimum phase behavior

ABSTRACT

Seventy-five young, healthy adults completed a lateral weight-shifting activity in which each shifted his/her center of pressure (CoP) to visually displayed target locations with the aid of visual CoP feedback. Each subject's CoP data were modeled using a single-link inverted pendulum system with a spring-damper at the joint. This extends the simple inverted pendulum model of static balance in the sagittal plane to lateral weight-shifting balance. The model controlled pendulum angle using PD control and a ramp setpoint trajectory, and weight-shifting was characterized by both shift speed and a non-minimum phase (NMP) behavior metric. This NMP behavior metric examines the force magnitude at shift initiation and provides weight-shifting balance performance information that parallels the examination of peak ground reaction forces in gait analysis. Control parameters were optimized on a subject-by-subject basis to match balance metrics for modeled results to metric values calculated from experimental data. Overall, the model matches experimental data well (average percent error of 0.35% for shifting speed and 0.05% for NMP behavior). These results suggest that the single-link inverted pendulum model can be used effectively to capture lateral weight-shifting balance, as it has been shown to model static balance.

1. Introduction

Balance assessment is important in rehabilitation after neurotrauma such as stroke, allowing clinicians to identify balance problems (functional approach) and determine underlying causes (systems approach) [1]. Quiet standing postural control, in which subjects attempt to center their center of mass (CoM) and reduce sway, has been a major focus of research. Static balance assessment examines center of gravity (CoG), the vertical projection of the CoM onto the ground, and center of pressure (CoP), the location of the resultant ground reaction force (GRF). CoP location affects CoG motion as CoG acceleration is proportional to the difference between the two [2]. Several metrics have been shown to distinguish between static balance data for young, elderly, and balance-impaired subjects [3,4]. Many clinical studies take a functional approach [5–9], while others examine human postural control with a systems approach [2,10]. In either case, ceiling effects limit the utility of balance assessments based purely on static balance as patients grow increasingly adept at standing upright [8]. Analyzing dynamic gait is another approach [2] characterized by floor effects due to the higher difficulty of the walking task.

Lateral weight shifting is a balance task more difficult than quiet standing and less difficult than walking in which subjects laterally translate their CoM, often using visual targets and CoP feedback. Easily applied clinically, lateral weight shifting is a robust task for evaluating stroke patient balance [7] that provides information beyond that available through static balance assessment [6]. Additionally, weight-shifting balance training may reduce fall risk in hemiplegic patients [5]. While previous studies have examined its relationship with functional balance [5,6,11–14], a systems approach to weight-shifting assessment that could shed light on underlying control mechanisms and fundamental differences between healthy and pathologic weight-shifting is currently lacking. Systems approaches can enable analysis of standing balance within a dynamic controls context, generally using PID or PD control. For example, a PID static balance control model postulates that increased control stiffness and damping compensate for added noise in elderly static balance [10]. The stiffness/damping ratio from PD control in other work demonstrated the significance of body velocity in balance control [15].

This study is the first to take a systems approach to establish the validity of a simple inverted pendulum model for lateral weight shifting and to build the foundation for clinical applications.

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http://dx.doi.org/10.1016/j.gaitpost.2014.03.011
0966-6362/

Inverted pendulum models of anterior–posterior [10,16], medio-lateral [17], and bilateral [18,19] quiet standing are common, and some more complex models, such as the parallelogram [2] and multi-link inverted pendulum models [20], have been proposed to better mimic human physiology. No such models, though, have been applied to lateral weight shifting with the focus on balance control herein. Furthermore, existing quantitative metrics for lateral weight shifting are limited to weight-shifting speed, precision of weight shifting, temporal symmetry, and force symmetry [4–6,12]. This study examines a new metric based on non-minimum phase (NMP) behavior to focus on the control of shift initiation.

2. Methods

2.1. Non-minimum phase behavior

In control theory, a non-minimum phase system is one in which the output initially moves in the direction opposite that of a new reference position [21]. For weight shifting with visual feedback, the output is the CoP, and the reference is the target CoP position to which the shift occurs. From a mechanics viewpoint, the leg opposite the shift direction generates an increased GRF with a lateral component that accelerates the CoM toward the target (assuming no other contacts and no foot adhesion to the floor). This GRF increase causes the CoP to briefly move in the direction opposite the weight shift until the CoG has shifted far enough to cancel the effects of the initial GRF increase. This NMP behavior is readily observed in the CoP trajectories of visually guided weight shifting (Fig. 1). Similar behavior has been observed during gait initiation, and GRF peaks have been used to characterize dynamic gait [9,22]. As vertical CoM movement is typically small during weight-shifting, the total vertical GRF is nearly constant. More meaningful is the difference in vertical GRF between the feet, which peaks during weight shift initiation, causing NMP behavior.

In this study, NMP behavior magnitude, the distance the CoP travels in the direction opposite the weight shift, is introduced as a characteristic property of lateral weight shifting to establish a metric for quantitative analysis. This metric parallels use of peak GRF in gait analysis since the timings of the peak NMP behavior and the peak GRF difference between the feet coincide and their magnitudes are directly related. Therefore, this NMP metric measures the strength of shift initiation as a subject prepares to move his/her CoP toward the target.

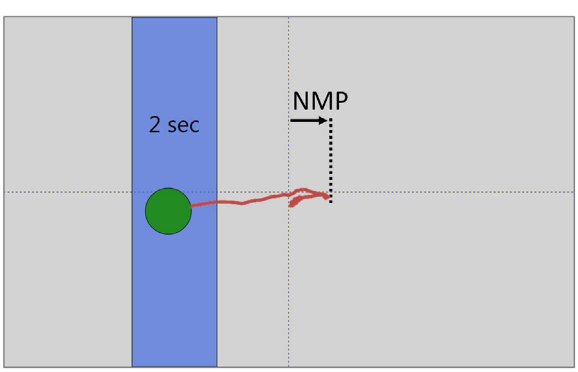

Fig. 1. Non-minimum phase behavior demonstrated by a lateral weight shift. The magnitude of the NMP behavior along the *x*-axis is illustrated by the arrow terminating at the thick dotted line. In shifting from a laterally symmetrical position to the blue target region on the left, the red CoP trace travels to the right before moving toward the target region. The subject's CoP is shown as a green circle, the target CoP region is shown as a blue rectangle, and the time that the subject's CoP has remained within the target CoP region is shown by the text inside the target region. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

2.2. Weight-shifting task

Subjects were led through a lateral weight-shifting task using the WeHab system [23]. Each subject stood with one foot on either of two Nintendo Wii balance boards and shifted his/her CoP, presented as a green circle on a rectangular field, from one target region to another (Fig. 1). While both sagittal and lateral CoP information were presented, the task was based on lateral CoP information alone. Target regions were presented as blue rectangles twice the width of the CoP marker. Subjects were instructed to hit as many targets as possible within the time provided. A new target appeared once the center of the CoP marker entered and remained within the target region for 3 s. Targets alternated between central (symmetrical stance) and offset locations positioned at a 70–30% weight distribution randomly located to the left/right. To account for anticipation effects, only offset target shifts were examined.

2.3. Metrics

The reaction time (t_R) is the time required for a subject to recognize the new target location and begin shifting his/her weight. Due to the natural sway in static balance, it is difficult to determine the start of a purposeful shift. Therefore, t_R was estimated using a three-sample ($\approx$0.05 s; see Section 2.4) moving window that iterated backward in time from the point of maximum NMP displacement ($\mathrm{NMP}_{\max}$). When this window no longer contained a point closer to the target location than any point previously examined, the point with the last minimum distance was taken to mark the reaction instant (Fig. 2a). The time between the target shift and the reaction instant was t_R, and the subject's CoP position at the reaction instant was the initial CoP position.

The initial time-to-target (t_S), the time required to shift the CoP to a target region, quantifies the speed of weight shifting [4,6]. This metric was calculated by subtracting t_R from the time between the target shift and the CoP entering the target region (Fig. 2a).

The NMP shift ratio (r_{NMP}) was measured as $d_{\mathrm{NMP}}/d_{shift}$, where the NMP magnitude d_{NMP} is the distance from the initial CoP position to the CoP trajectory's farthest point from the target and the shift distance d_{shift} is from the initial CoP position to the target region's center. This ratio supplements the t_S metric by characterizing shifting force at the onset of a weight shift. Fig. 2a shows d_{NMP} and d_{shift} in the context of experimental shift data, both calculated based on lateral CoP balance alone.

2.4. Data

Lateral weight-shifting data were obtained from 79 healthy subjects participating in a visual feedback study [24]. All subjects gave informed consent, and the study received approval from the appropriate Institutional Review Board. Four subjects' data were discarded, two due to missing height data and two due to an error in selecting single board instead of dual-board configuration in the software. The remaining subjects included 35 males and 40 females, 17–22 years old (body mass 66.7 $\pm$ 11.7 kg; height 173.6 $\pm$ 9.4 cm; mean $\pm$ standard deviation). Data were collected at 63.9 $\pm$ 2.0 Hz (mean $\pm$ standard deviation).

Each weight shift consisted of the CoP trajectory starting from a target location shift and ending when the CoP first enters the target region. Considering all shifts from all subjects, weight shifts with t_S or t_R values outside of three standard deviations of the mean were excluded. For r_{NMP}, shifts with values greater than 1 were excluded to account for false starts in the wrong direction. Metric values were averaged across each subject (10.7 $\pm$ 1.5 shifts per subject; mean $\pm$ standard deviation) and used to determine subject-specific control parameters.

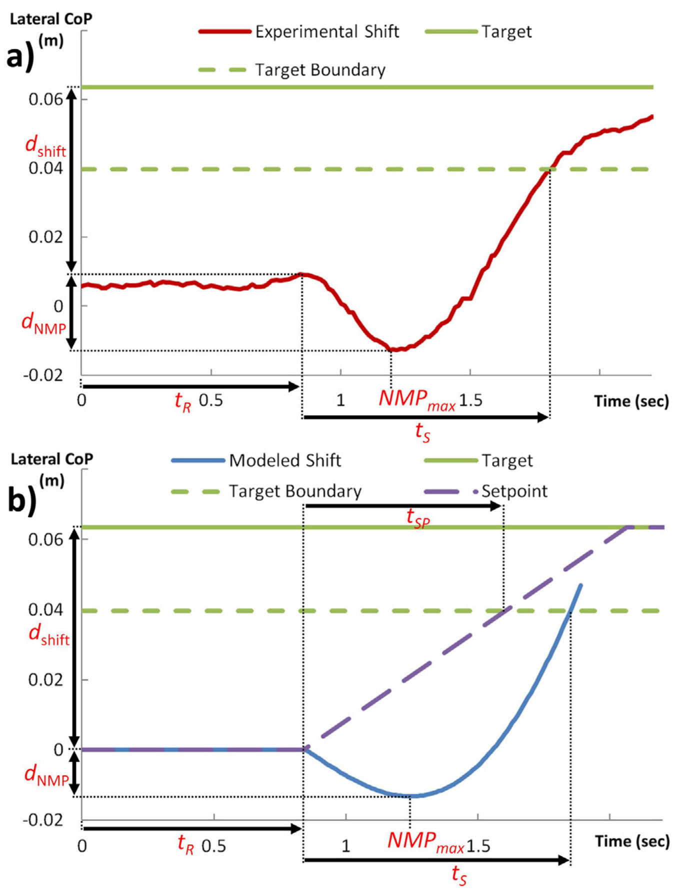

Fig. 2. Measurable properties applied to a representative example of experimental weight-shift data (a) and a simulated weight-shift trajectory (b). These properties are non-minimum phase distance (d_{NMP}), shift distance (d_{shift}), non-minimum phase time (t_{NMP}), and shift time (t_S). Reaction time (t_R) and the control setpoint shift time (t_{SP}; based on a CoP representation of the setpoint trajectory) are also shown.

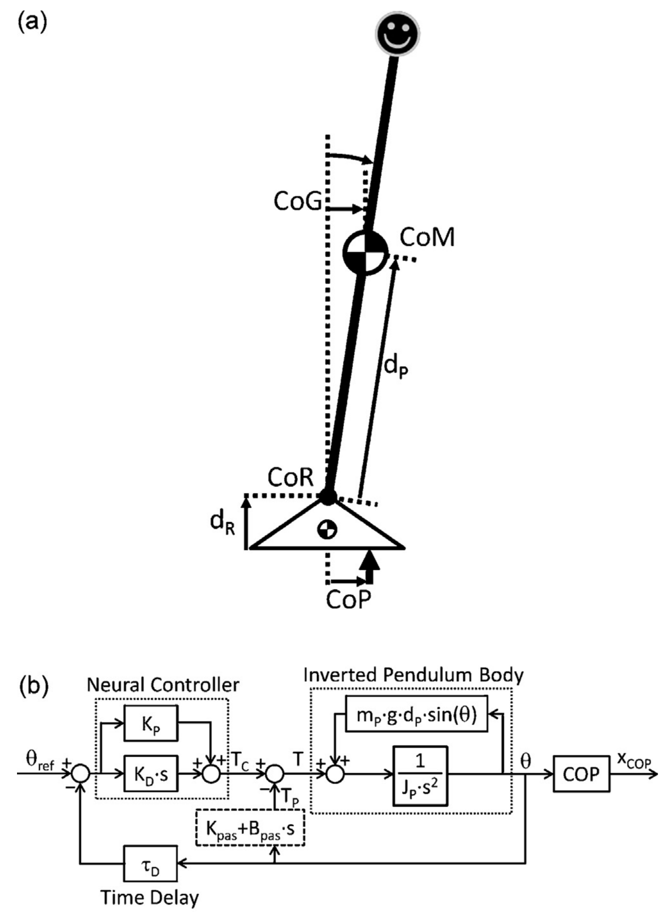

Fig. 3. Model and control used in simulation of weight-shifting balance. Inverted pendulum model for lateral balance (a) and block diagram of weight-shifting control (b).

2.5. Model

This study examines lateral weight shifting with the inverted pendulum model (Fig. 3a) in [10]. The novel application of this single-link inverted pendulum model to lateral weight shifting parallels previous static balance work taking a systems approach [10,16] and provides an intuitive extension to bilateral weight-shifting. The present model, however, does not incorporate random noise commonly used to simulate the passive sway in static balance [10]. It also parallels the simple inverted pendulum's use in examining dynamic gait [25] and a dynamic balance task [26]. Like the gait model, it represents an abstraction of balance control that groups the effects of multiple joint actuators together to achieve a CoM trajectory. As such, actuation of the model's pendulum joint is not meant to represent ankle control but rather whole body CoM control, and the pendulum's center of rotation (CoR) height is chosen to be the ankle height only for consistency with sagittal plane models. Physical parameters were estimated anthropometrically based on each subject's height (H) and mass (M) [27]. The CoR height d_R = 0.039H, the distance from the CoR to the CoM d_P = 0.579H – d_R, the pendulum mass m_P = 0.971M, and the static mass of the base m_B = 0.029M. Subject height was measured by hand, while mass was obtained from force data. Pendulum moment of inertia about the anterior–posterior axis through the CoR was calculated as

$$J_P = 0.352MH^2, \tag{1}$$

which was derived from anthropometric data [27] and yielded values consistent with previous work [10,28].

2.6. Control

Previous work has shown that closed-loop PD control accurately models static balance [15]. Fig. 3b is a block diagram of the inverted pendulum model and neural controller in this study. Like previous standing balance models [10,16–19], this model assumes that subjects control CoG, not CoP, so the system exhibits NMP behavior but is not an NMP system since the output and controlled variable differ. The input is the torque at the base consisting of the control torque (T_C) generated by the PD neural controller based on the error between the angle setpoint and actual angle and the resistance torque due to passive stiffness and damping. The CoP position is the output calculated from the CoG position and the horizontal, vertical, and angular accelerations [10]:

$$x_{CoP} = \frac{(m_P d_P^2 - J_P)\ddot{\theta}_P + m_P x_P (g + \ddot{y}_P) - m_P y_P \ddot{x}_P - m_P d_R \ddot{x}_P}{m_P (g + \ddot{y}_P) + m_B g}. \tag{2}$$

Unlike some work [10], this study does not include integral control because a static control strategy is assumed dominant only after a weight shift. The PD-control angle setpoint (Fig. 2b) is a ramp function from the neutral CoP position to the target position over a setpoint shift time (t_{SP}). While the target location changes as a step function, a ramp function, representing constant speed control, better corresponds to the desired pendulum angle change since an instantaneous shift is infeasible. Thus, t_{SP} represents a planned shift time and the ramp a planned CoP trajectory. Time delay for the feedback loop was 1 ms. Passive stiffness and damping terms were set to 0, as consistent with static balance results in [10].

2.7. Analysis

Proportional gain, derivative gain, and t_{SP} were optimized to match metric values of the model's lateral weight shifts to those from experimental data. Modeled values for both metrics ($t_{S,mod}$ and $r_{\mathrm{NMP,mod}}$) were compared to experimental values ($t_{S,exp}$ and $r_{\mathrm{NMP,exp}}$) using percent error. The cost function was:

$$\mathrm{cost} = \left(\frac{t_{S,\mathrm{exp}} - t_{S,\mathrm{mod}}}{t_{S,\mathrm{exp}}}\right)^2 + \left(\frac{r_{\mathrm{NMP,exp}} - r_{\mathrm{NMP,mod}}}{r_{\mathrm{NMP,exp}}}\right)^2. \quad (3)$$

Using MATLAB's pattern search function, control parameter values were optimized for each subject. The MADS Positive Basis 2N pattern was used for both polling and search, with both complete polling and complete searching enabled. All three control parameters were varied with a maximum of 300 iterations. The set of control parameters resulting in the lowest cost function value was taken to be the representative control for each subject. Lower and upper bounds were: K_P: [0, 2500], K_D: [0, 1000], and t_{SP}: [0.1, 5]. Nine initial guesses were used for each subject, using combinations of K_P = 500, 1000, and 1500, K_D = 100, 300, and 500, and $t_{SP} = t_{S,exp}$.

A sensitivity analysis was performed on a subject-by-subject basis for each model parameter to evaluate the effects on both $t_{S,mod}$ and $r_{\mathrm{NMP},mod}$. This analysis incorporated a range from 70% to 130% of those parameter values, centered around the optimized values. Pearson correlation coefficients were calculated using the SAS software's CORR procedure.

3. Results

3.1. Data-matched balance measures

Optimized control parameter values resulted in an average percent error of 0.35% for t_S and 0.05% for r_{NMP} across all 75 subjects, and Table 1 shows all balance metric values.

Correlation analysis showed that $t_{S,mod}$ was negatively correlated with K_P (-0.2669, $p = 0.0206$) and positively correlated with t_{SP} (0.4880, $p < 0.0001$) but not significantly correlated with K_D ($p = 0.5637$), while $r_{\mathrm{NMP},mod}$ was positively correlated with K_D (0.6042, $p < 0.0001$) and negatively correlated with t_{SP} (-0.4566, $p < 0.0001$) but not significantly correlated with K_P ($p = 0.9610$). Among the three control parameters, K_D and K_{SP} were significantly correlated (0.2327, $p = 0.0446$), and t_{SP} was significantly correlated with both K_P (0.3791, $p = 0.0008$) and K_D (0.2792, $p = 0.0153$).

3.2. Sensitivity analysis

Both K_P and K_D were positively correlated with $r_{\mathrm{NMP},mod}$ (0.7862 and 0.7871, respectively), while K_P was negatively correlated with $t_{S,mod}$ (-0.6626) and K_D was positively correlated with $t_{S,mod}$ (0.4125). The t_{SP} metric was positively correlated with $t_{S,mod}$ (0.8964) and negatively correlated with $r_{\mathrm{NMP},mod}$ (-0.9835). All p-values were less than 0.0001. Figs. 4a and b contain results for $t_{S,mod}$ and $r_{\mathrm{NMP},mod}$, respectively. In terms of magnitude, $t_{S,mod}$ and $r_{\mathrm{NMP},mod}$ were most sensitive to t_{SP}, while $t_{S,mod}$ was least sensitive to K_D and $r_{\mathrm{NMP,mod}}$ was least sensitive to K_P.

Table 1
Balance metric and control parameter results from optimized inverted pendulum model.

	Mean	Std. Dev.
Metrics		
$t_{S,mod}$	1.01	0.13
$r_{\mathrm{NMP},mod}$	0.31	0.11
t_S error (%)	0.35	1.06
r_{NMP} error (%)	0.05	0.38
Control parameters		
K_P	1090	610
K_D	278	219
t_{SP}	1.19	0.37

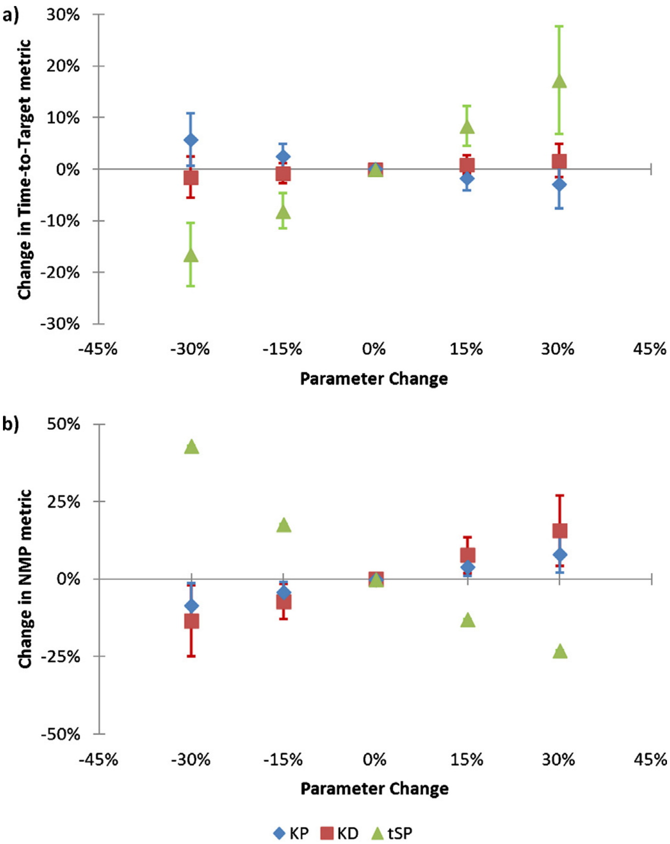

Fig. 4. Results from a sensitivity analysis for the effects of control parameters on modeled shift time (t_S; a) and modeled non-minimum phase ratio (r_{NMP}; b). Error bars indicate standard deviation.

4. Discussion

4.1. Error analysis

The model matched t_S within 1% for 74 subjects. Considering this subgroup alone, the mean matching errors for t_S and r_{NMP} were 0.2% and 0.001%, respectively.

4.2. Setpoint shift time and weight-shifting balance performance

The setpoint shift time t_{SP} is significantly correlated with both weight-shifting balance metrics. A faster planned shift results in a more quickly executed shift (positive correlation with t_S) and a greater shift initiation force (negative correlation with r_{NMP}).

4.3. Non-minimum phase behavior as a weight-shifting balance metric

NMP behavior is an important metric for weight-shifting due to its focus on the initial push-off phase. Unlike existing shift precision metrics, NMP behavior is appropriate for analysis of lateral-only tasks and directly captures dynamic behavior. In comparison, the directional control metric of shifting precision [5] is based on balance perpendicular to the shift direction and presupposes that a straight-line path is optimal. Similarly, the imprecision metric [6] incorporates static balance performance into characterization of weight-shifting balance. Also, unlike existing symmetry metrics that quantify the equivalence of performance to either side, the NMP metric describes absolute balance performance. Furthermore, NMP behavior is easily captured using simple, inexpensive instruments, requiring only CoP information obtained through vertical GRF data. While two balance boards were used in this study, CoP information can readily be obtained using just one.

NMP behavior provides additional information about GRF in weight-shifting balance control, similar to previous characterization of dynamic gait using GRF in gait initiation [9,22]. While the cost function equally weights t_S and r_{NMP}, percent r_{NMP} error was lower. This may be because NMP focuses specifically on the initial phase, while shift time is a property of the entire CoP trajectory and thus has the potential for greater variability. The negative correlation between r_{NMP} and t_S supports use of NMP behavior as a measure of weight-shifting push-off, being that a greater push-off force results in a quicker shift. At the same time, the correlation's intermediate magnitude (-0.4543) shows that r_{NMP} and t_S are not simply interdependent metrics conveying the same information.

4.4. Inverted pendulum model

This novel application of the inverted pendulum model (Fig. 3) was able to match weight-shifting balance metrics from experimental data, with a mean percent error of less than 0.4% for t_S and 0.1% for r_{NMP}. While accurate matching with such a simple model is desirable, the simplicity is also the primary limitation of the model. Since it does not capture individual joint control, it represents a level of abstraction higher than most previous implementations of the model. Greater understanding of dynamic lateral balance control could be implemented through a more complex, multi-link model, perhaps even including upper-body dynamics; however, this would require data beyond the scope of that collected through use of force plates alone.

In terms of extension to clinical applications, previous work has shown that elderly subjects demonstrated increased stiffness and damping parameters compared to younger subjects [10]. Also, stiffness and damping have been shown to be reduced in children with cerebral palsy [19]. Similar to these studies, this model could allow comparison of healthy lateral weight-shifting control parameter values with those obtained from pathological balance, albeit parameters describing abstracted whole body CoM control rather than individual joint control. Future work could also combine the weight-shifting and static balance control schemes to capture the entire weight shift. Measurement of weight-shifting balance performance using two metrics represents a lower-dimensional comparison than prior work matching 15 sway metrics [10]. This is due in part to the greater number of established metrics for static sway, and highlights the need for additional weight-shifting metrics such as NMP behavior.

In summary, the single-link inverted pendulum model can be used to evaluate lateral weight-shifting balance, which parallels the inverted pendulum model's effectiveness in modelling static balance [16].

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant no. IIS-1117706.

Many thanks to Charles R. Crowell, Michael Villano, and Aaron D. Striegel for useful discussion that informed this study. Thanks also to Leilani Aldridge, Austin Atherton, Kathryn Elliot, Angela Ferreira, Allison Jeter, Brittany Jones, Mitchell Kajzer, Francis Kamen, Kalyn McGinnis, Brendan O'Shea, Jamie Segerson, Stephanie Sonnick, Gerald Wilson, and Julaine Zenk for their assistance with data collection.

Conflicts of interest statement The authors have no conflict of interests to report related to this work.

Appendix A. Supplementary Data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.gaitpost.2014.03.011.

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