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Robust Coverage by a Mobile Robot of a Planar Workspace

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Abstract— **In this paper, we suggest a new way to plan coverage paths for a mobile robot whose position and velocity are subject to bounded error. Most prior approaches assume a probabilistic model of uncertainty and maximize the expected value of covered area. We assume a worst-case model of uncertainty and—for a particular choice of coverage path—are still able to guarantee complete coverage. We begin by considering the special case in which the region to be covered is a single point. The machinery we develop to express and solve this problem immediately extends to guarantee coverage of a small subset in the workspace. Finally, we use this subset as a sort of virtual coverage implement, achieving complete coverage of the entire workspace by tiling copies of the subset along boustrophedon paths.**

I. INTRODUCTION

In robotics, the coverage problem is to determine a motion strategy for a robot equipped with a coverage implement, such that the area swept out by the coverage implement includes the entire area to be covered [1]. Familiar applications include lawn mowing [2], de-mining [3], and painting [4].

In the ideal case, when there is no uncertainty in the robot's state, the coverage problem can be framed as a classical path planning problem, and complete and correct algorithms exist for computing coverage paths. Typically these paths comprise some variation of a raster scan (e.g., the boustrophedon algorithm [5]). In simply connected environments, such a path can be applied directly; in multiply connected environments, cell decomposition techniques can be used to partition the environment into simply connected cells, each of which can be covered using a raster scan path [6]. These methods apply equally well in unstructured environments, provided that sensors can robustly detect critical points in the environment [3].

For cases in which uncertainty cannot be neglected, most approaches to coverage resort to randomized algorithms, framing the problem as one of probabilistic coverage. Since such approaches cannot guarantee complete coverage in finite time, they typically attempt to maximize the expected value of the area covered [7], [8], or, in the case of probabilistic search (which can be framed as a coverage problem), to minimize the expected value of time required to cover some target area [9]–[12].

In this paper, we consider the problem of constructing guaranteed coverage paths when the robot is subject to uncertainty in both its position and its instantaneous velocity. For this problem, neither the complete algorithms for coverage without uncertainty nor the randomized algorithms for coverage under uncertainty are applicable.

More formally, the problem we confront is as follows. A robot is equipped with a circular coverage implement of radius γ. At any time t, the robot's position is known only to lie in an uncertainty ball of constant radius α, and its velocity is subject to an arbitrary error whose magnitude is bounded by a constant β. This model of uncertainty is applicable, for example, to robots equipped with GPS, and it has a long history in the robotics community, dating back to early work in motion planning under uncertainty [13]–[17]. Under these conditions, our problem is to construct a coverage path $p(t)$ such that every point in a workspace W is guaranteed to be covered in finite time.

We begin in Section II by considering the special case of covering a single point in the workspace. Using concepts from pursuit-evasion games, we formalize the problem in terms of minimum time to capture of an evader, who plays the role of nature in perturbing the position and velocity of the robot. We determine sufficient conditions for coverage in terms of α, β and γ, and provide a corresponding guaranteed coverage path. In Section III we extend this approach to the case of covering a small area W, by constructing a conservative bound on an area guaranteed to be covered by the path from Section II. We then treat W as a virtual coverage implement, and construct a guaranteed coverage path for the entire workspace by applying the boustrophedon algorithm (Section IV).

II. COVERAGE OF A POINT

A. γ-Coverage

Let us begin by considering the simple case in which we want a moving disk to cover a single point that is located at the origin of a planar workspace. As usual, what we mean by "cover" is that the origin is contained in the region swept out by the disk as it moves. Suppose that the center of the disk follows a trajectory $p\colon [0,\infty) \to \mathbb{R}^2$ that satisfies

$$\dot{p}(t) = u(t) \tag{1}$$

for some piecewise-continuous input $u\colon [0,\infty) \to \mathbb{R}^2$ and all $t \in [0,\infty)$. Suppose also that

$$\|u(t)\| \leq 1 \tag{2}$$

for all $t \in [0,\infty)$, so the disk moves at no more than unit speed. Finally, suppose that the disk has radius $\gamma > 0$. Then, we say that the origin is *γ-covered* by p if there exists some time $t \in [0,\infty)$ at which

$$\|p(t)\| < \gamma, \tag{3}$$

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where $\|\cdot\|$ is Euclidean distance. Equivalently, the origin is γ-covered by p if the minimum such time

$$\inf \{t \in [0, \infty) \colon \|p(t)\| < \gamma\}$$

is finite. Note our use of "γ-covered" rather than "covered" in order to emphasize dependence on the choice of radius γ.

B. (α, β, γ)-Coverage

Now, suppose that the center of the moving disk does not exactly follow the trajectory p. We would like to know if the origin remains γ-covered.

To answer this question, we begin by defining a class of admissible perturbations. Choose $\alpha, \beta \geq 0$ and denote by $\mathcal{E}$ the set of all absolutely continuous functions $e \colon [0, \infty) \to \mathbb{R}^2$ that satisfy

$$\dot{e}(t) = v(t), \qquad \|e(t)\| \leq \alpha, \qquad \|v(t)\| \leq \beta \tag{4}$$

for some piecewise-continuous function $v \colon [0, \infty) \to \mathbb{R}^2$ and all $t \in [0, \infty)$. For fixed p, we assume that it is possible for the center of the disk to follow any trajectory of the form $p - e$ for some $e \in \mathcal{E}$. Think of p as the desired trajectory and of e as the error that would result from tracking this trajectory with a mobile robot—i.e., the error due to model uncertainty, sensor and actuator noise, etc. The parameters α and β establish worst-case bounds on the "position error" $e(t)$ and "velocity error" $v(t)$, respectively.

For fixed $e \in \mathcal{E}$, the origin is γ-covered by $p - e$ if there exists some time $t \in [0, \infty)$ at which

$$\|p(t) - e(t)\| < \gamma. \tag{5}$$

Equivalently, the origin is γ-covered by $p - e$ if the minimum such time

$$\kappa(p, e) = \inf \{t \in [0, \infty) \colon \|p(t) - e(t)\| < \gamma\}$$

is finite. We say that the origin is (α, β, γ)-*covered* by p if

$$\bar{\kappa}(p) = \sup \{\kappa(p, e) \colon e \in \mathcal{E}\} \tag{6}$$

is finite, i.e., if the origin is γ-covered by $p - e$ even for the worst-case choice of error e. Again, our use of "(α, β, γ)-covered" rather than "covered" emphasizes dependence on the choice of bounds α and β and of radius γ.

It is clear that (α, β, γ)-coverage can be interpreted as the outcome of a pursuit-evasion game, in this case a so-called game against nature. Indeed, our use of the notation p and e is meant to evoke "pursuer" and "evader." The pursuer wins if $p(t)$ and $e(t)$ are ever closer than γ, i.e., if $\kappa(p, e) < \infty$. The evader wins if this event never occurs, i.e., if $\kappa(p, e) = \infty$. An optimal choice of strategy for the (omniscient) evader is a trajectory e that maximizes the time to capture, i.e. that achieves the supremum $\bar{\kappa}(p)$ in Equation (6). An optimal choice of strategy for the pursuer is evidently a trajectory p that minimizes the (worst-case) time to capture, i.e., that achieves the infimum

$$\bar{\kappa}_* = \inf \{\bar{\kappa}(p) \colon p \in \mathcal{P}\}, \tag{7}$$

where $\mathcal{P}$ is the set of all absolutely continuous functions $p \colon [0, \infty) \to \mathbb{R}^2$ that satisfy (1)-(2) for some piecewise-continuous function $u \colon [0, \infty) \to \mathbb{R}^2$ and all $t \in [0, \infty)$. Equation (7) is, in fact, a complete description of what we might call the "optimal (α, β, γ)-coverage planning problem"—computing $\bar{\kappa}_*$ is equivalent to finding the trajectory p that (α, β, γ)-covers the origin in minimum time.

For now, we are interested only in finding *some* trajectory p that (α, β, γ)-covers the origin, not in finding a trajectory that does so in minimum time. Before proceeding, we will establish an equivalent definition of (α, β, γ)-coverage that is more useful for this purpose. For any $t \in [0, \infty)$, let

$$E(t, p) = \left\{e(t) \in \mathbb{R}^2 \colon e \in \mathcal{E} \text{ and } \kappa(p, e) > t\right\}.$$

Notice that $E(t, p)$ is the forward reachable set of (4) at time t from any initial condition $e(0) \in \mathbb{R}^2$, subject to the constraint that $\|p(s) - e(s)\| \geq \gamma$ for all $s \in [0, t]$. It is clear that the origin is (α, β, γ)-covered by p if and only if there exists some time $t \in [0, \infty)$ at which $E(t, p) = \emptyset$. We will see in the following section that, for certain choices of p, it is easy to compute the reachable set $E(t, p)$, hence to verify that p achieves (α, β, γ)-coverage.

C. Verification of (α, β, γ)-Coverage for $\beta = 0$

In this section, we will show that the origin is (α, β, γ)-covered by a particular choice of trajectory p in the special case for which $\beta = 0$. This trajectory will be a sequence of switchbacks (i.e., it will be a Boustrophedon path). A similar trajectory will also suffice in the general case $\beta \geq 0$, as we will show in the sequel (Section II-D).

By assuming that $\beta = 0$, we are assuming that the only uncertainty in following p is in the initial error $e(0)$. To verify that the origin is (α, β, γ)-covered by p, we need only verify that p passes within a distance γ of all $e(0) \in \mathbb{R}^2$ satisfying $\|e(0)\| \leq \alpha$. This objective is exactly what we would encounter in a "standard" coverage problem (without uncertainty), in which the region to be covered is a disk of radius α. A common way to solve this problem is with a Boustrophedon path, in other words with a path that is a sequence of straight-line switchbacks [5]. Figure 1(a) shows a trajectory p of exactly this type. It is constructed starting from the initial condition

$$p(0) = \begin{bmatrix} -\alpha - \gamma \\ -\alpha + \delta \end{bmatrix}$$

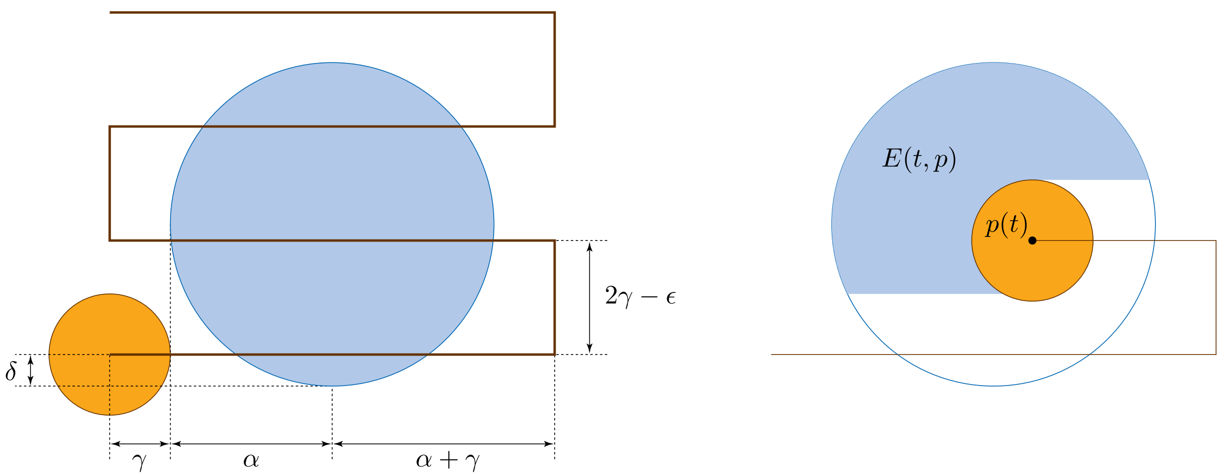
for some $\delta \in (-\gamma, \gamma)$ by repeated application of

$$u(t) = \begin{cases} (1, 0) & k\tau \leq t < k\tau + \ell \\ (0, 1) & \cdots \leq t < k\tau + \ell + h \\ (-1, 0) & \cdots \leq t < k\tau + 2\ell + h \\ (0, 1) & \cdots \leq t < k\tau + 2\ell + 2h \end{cases}$$

for $k \in \{0, 1, 2, \dots\}$ and $\tau = 2\ell + 2h$, where

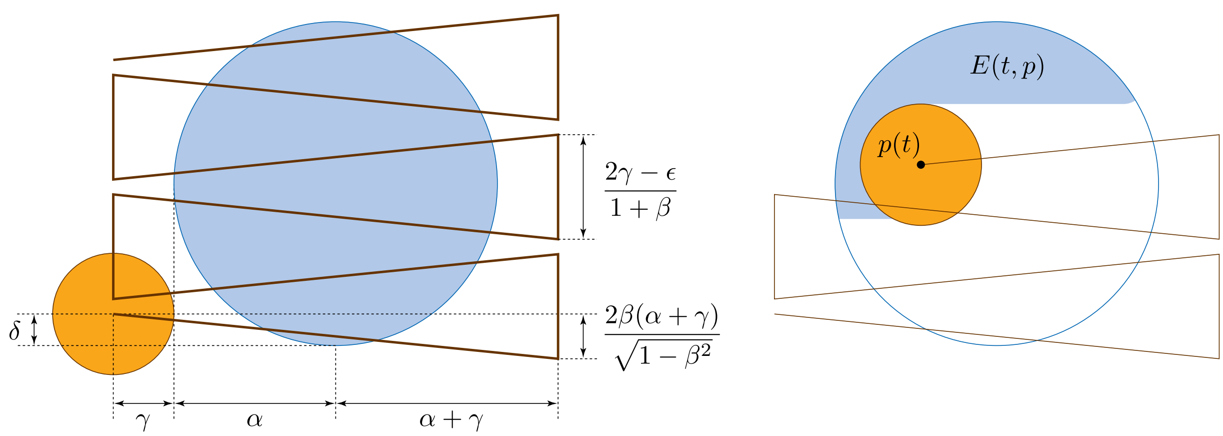
$$\ell = 2(\alpha + \gamma) \qquad h = 2\gamma - \epsilon$$

and the parameter $\epsilon \in (0, 2\gamma)$ is arbitrarily small. It is easy to compute the forward reachable set $E(t, p)$ for this choice



(a) (b)

Fig. 1. The switchback plan to cover a point when $(\alpha = 1, \beta = 0, \gamma = 3/8)$. The parameters $\delta \in (-\gamma, \gamma)$ and $\epsilon \in (0, 2\gamma)$ can be chosen arbitrarily.



(a) (b)

Fig. 2. The switchback plan to cover a point when $(\alpha = 1, \beta = 1/10, \gamma = 3/8)$. The parameters $\delta \in (-\gamma, \gamma)$ and $\epsilon \in (0, 2\gamma)$ can be chosen arbitrarily.

of p. In fact, we can describe this set explicitly as

$$E(t, p) = (L \cup R) \cap A,$$

where

$$\begin{aligned} L &= \left\{z \in \mathbb{R}^2 \colon z_1 \leq p_1(t) \text{ and } z_2 \geq p_2(t) + \gamma\right\} \\ R &= \left\{z \in \mathbb{R}^2 \colon z_1 \geq p_1(t) \text{ and } z_2 \geq p_2(t) - \gamma + \epsilon\right\} \end{aligned} \tag{8}$$

when $t \in [k\tau, k\tau + \ell + h)$,

$$\begin{aligned} L &= \left\{z \in \mathbb{R}^2 \colon z_1 \leq p_1(t) \text{ and } z_2 \geq p_2(t) - \gamma + \epsilon\right\} \\ R &= \left\{z \in \mathbb{R}^2 \colon z_1 \geq p_1(t) \text{ and } z_2 \geq p_2(t) + \gamma\right\} \end{aligned} \tag{9}$$

when $t \in [k\tau + \ell + h, (k+1)\tau)$, and

$$A = \left\{z \in \mathbb{R}^2 \colon \ \|z\| \leq \alpha \text{ and } \|p(t) - z\| \geq \gamma\right\} \tag{10}$$

in either case. Figure 1(b) shows a snapshot of $E(t, p)$ at one particular time $t \in [0\tau + \ell + h, (0+1)\tau]$. Notice that

$$\begin{aligned} \Delta y &\triangleq p_2((k+1)\tau) - p_2(k\tau) \\ &= 2h \\ &= 2(2\gamma - \epsilon), \end{aligned}$$

so after each interval $[k\tau, (k+1)\tau]$ the boundary of $E(t, p)$ moves "up" a distance $2(2\gamma - \epsilon)$. In particular, it is easy to verify that $E(t, p) = \emptyset$ for all

$$t > \lceil 2(\alpha + \gamma)/\Delta y \rceil \, \tau, \tag{11}$$

where $\lceil \cdot \rceil$ is the ceiling operator that rounds up to the nearest integer (note that (11) is a conservative bound). We conclude that the origin is (α, β, γ)-covered by p.

D. Verification of (α, β, γ)-Coverage for $\beta > 0$

We have seen that a Boustrophedon path suffices to achieve (α, β, γ)-coverage of the origin when $\beta = 0$. We might guess that a similar path would achieve (α, β, γ)-coverage when $\beta > 0$ is non-zero but still small. In particular, consider again the example shown in Figure 1(b). With $\beta = 0$, the two straight-line edges that form the lower boundary of $E(t, p)$ remain motionless. If instead $\beta > 0$, these two edges would move downward—in the outward normal direction—at speed β. Evidently, the trajectory p should also move downward at the same rate—in other words, p should be a sequence of switchbacks that slant slightly backward along each pass. Figure 2(a) shows a trajectory p of exactly this type. It is constructed starting from the initial condition

$$p(0) = \begin{bmatrix} -\alpha - \gamma \\ -\alpha + \delta \end{bmatrix}$$

for some $\delta \in (-\gamma, \gamma)$ by repeated application of

$$u(t) = \begin{cases} (\sqrt{1-\beta^2}, -\beta) & k\tau \leq t < k\tau + \ell \\ (0, 1) & \cdots \leq t < k\tau + \ell + h \\ (-\sqrt{1-\beta^2}, -\beta) & \cdots \leq t < k\tau + 2\ell + h \\ (0, 1) & \cdots \leq t < k\tau + 2(\ell + h) \end{cases}$$

for $k \in \{0, 1, 2, \dots\}$ and $\tau = 2(\ell + h)$, where

$$\ell = 2(\alpha + \gamma)/\sqrt{1-\beta^2} \qquad h = (2\gamma - \epsilon)/(1 + \beta)$$

and the parameter $\epsilon \in (0, 2\gamma)$ is arbitrarily small. It is easy to verify that, for this choice of trajectory, the forward reachable set evolves as shown in Figure 3. By extension, we can in fact verify that

$$E(t, p) \subseteq (L \cup R) \cap A,$$

where L, R, and A are exactly as given before in (8)-(10). Figure 2(b) shows a snapshot of $E(t, p)$ at one particular time $t \in [1\tau + \ell + h, (1 + 1)\tau]$. Notice that

$$\begin{aligned} \Delta y &\triangleq p_2((k+1)\tau) - p_2(k\tau) \\ &= 2\left(\frac{2\gamma - \epsilon}{1 + \beta} - \frac{2\beta(\alpha + \gamma)}{\sqrt{1-\beta^2}}\right), \end{aligned} \tag{12}$$

so after each interval $[k\tau, (k+1)\tau]$ the boundary of $E(t, p)$ moves "up" a distance Δy. So long as $\Delta y > 0$, it is easy to verify that $E(t, p) = \emptyset$ for all

$$t > \lceil 2(\alpha + \gamma)/\Delta y \rceil \tau. \tag{13}$$

Note that (13) is a conservative bound, as was (11). We conclude that the origin is (α, β, γ)-covered by p if $\Delta y > 0$.

E. Asymptotics of (α, β, γ)-Coverage

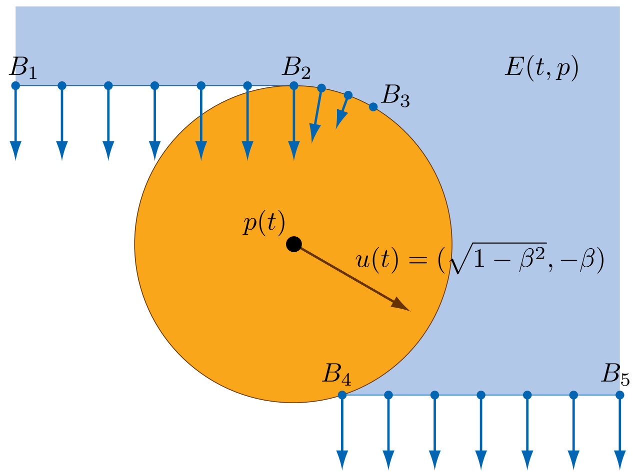
We have seen that a "backtracking" Boustrophedon path (Figure 2) suffices to achieve (α, β, γ)-coverage of the origin in the general case $\beta \geq 0$, so long as the quantity Δy in Equation (12) is positive. It is instructive to enumerate the values of α, β, and γ that would produce $\Delta y > 0$.

First, notice that $\Delta y > 0$ if and only if

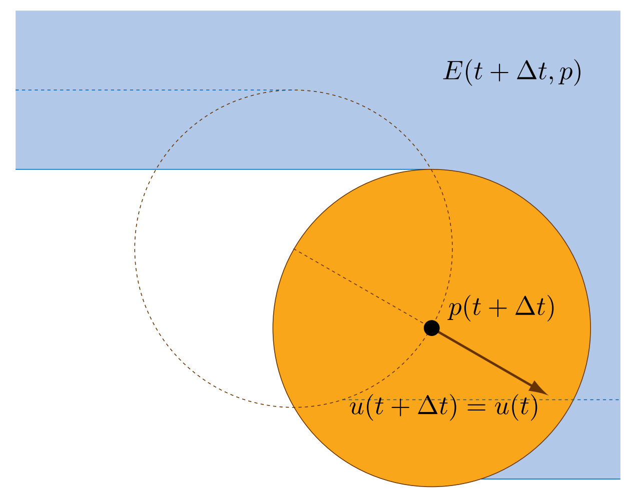
$$\gamma\left(\sqrt{1-\beta^2} - \beta(1+\beta)\right) - \left(\epsilon\sqrt{1-\beta^2} + \alpha\beta(1+\beta)\right) > 0,$$

which cannot possibly be true unless

$$f(\beta) \triangleq \sqrt{1-\beta^2} - \beta(1+\beta) > 0. \tag{14}$$



(a) Velocity of points on the free boundary of the forward reachable set $E(t, p)$. Each point moves in the outward normal direction. Points along B_1B_2 and B_4B_5 move at speed β. Points a distance s along the arc from B_3 to B_2 move at speed $\sin(s/\gamma)$ for $s \in [0, \gamma \sin^{-1}(\beta)]$.



(b) The new forward reachable set $E(t+\Delta t, p)$ after a time step $\Delta t > 0$.

Fig. 3. Evolution of $E(t, p)$ when $\dot{p}(t) = u(t) = (\sqrt{1-\beta^2}, -\beta)$.

The function f in Equation (14) is monotonic decreasing for $\beta \in [0, 1)$, and has a zero crossing at $\beta \approx 0.54$. As a consequence, a necessary condition for (α, β, γ)-coverage with our particular choice of p is that $\beta \in [0, 0.54)$.

Second, notice that we can explicitly compute the sensitivity of Δy to changes in the parameters α, β, and γ:

$$\begin{aligned} \frac{\partial \Delta y}{\partial \alpha} &= -\frac{4\beta}{\sqrt{1-\beta^2}} \\ \frac{\partial \Delta y}{\partial \beta} &= -2\left(\frac{2(\alpha+\gamma)}{(1-\beta^2)^{3/2}} + \frac{2\gamma + \epsilon}{(1+\beta)^2}\right) \\ \frac{\partial \Delta y}{\partial \gamma} &= 4\left(\frac{1}{1+\beta} - \frac{\beta}{\sqrt{1-\beta^2}}\right). \end{aligned}$$

Figure 4 plots these partial derivatives. These results tell us what changes in α, β, and γ produce the greatest increase in Δy (i.e., the greatest decrease in the time required to achieve (α, β, γ)-coverage of the origin with a switchback

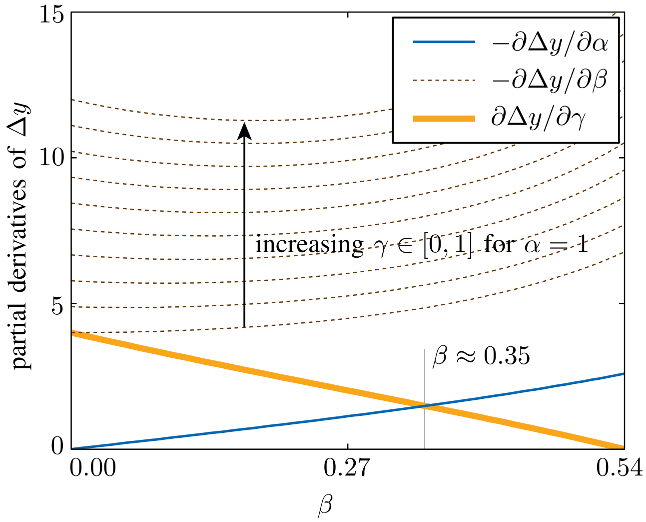


Fig. 4. Sensitivity of Δy—a measure of forward progress, as in Equation (12)—to differential changes in α, β, and γ for the switchback plan.

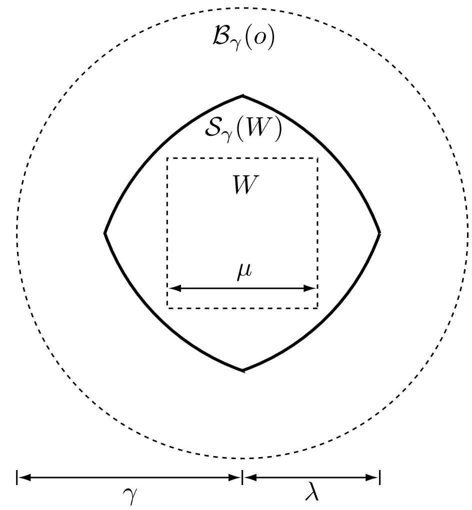


Fig. 5. Construction of $\mathcal{S}_\gamma(W)$ when W is a square with side-length μ that is centered at the origin.

plan). In particular, they tell us that we get the most payoff by decreasing β (i.e., by tightening our bound on worst-case velocity error), and that there is a critical value of $\beta \approx 0.35$ that determines if it is better to decrease α or increase γ.

These asymptotic results have real design implications, since the parameters α, β, and γ derive from the choice of sensors, actuators, and "implement size" (e.g., the size of the cutting blade on an automated residential lawn mower). One thing these results emphasize, for example, is the relative importance of a tight bound on worst-case velocity error (whereas we might *a priori* have assumed that worst-case position error is most important).

III. COVERAGE OF A SET

In the previous section, we saw how to cover a single point with a moving disk of radius γ, despite perturbations e that are characterized by worst-case bounds on position error ($\|e(t)\| \leq \alpha$) and on velocity error ($\|\dot{e}(t)\| \leq \beta$). In this section, we will see that exactly the same approach can be used to cover a small subset of $\mathbb{R}^2$ (not just a point). This result will be our foundation for achieving robust coverage of an entire workspace (Section IV).

A. From Points to Sets

As a direct extension of what appears in Section II-B, we say that a set $W \subset \mathbb{R}^2$ is (α, β, γ)-covered by a trajectory $p \in \mathcal{P}$ if every $w \in W$ is (α, β, γ)-covered by p. However, it is not immediately clear how to verify (α, β, γ)-coverage in this case, for arbitrary sets W. Our purpose in this section is to establish a sufficient condition for (α, β, γ)-coverage of a "small" set that is easy to verify, exactly as we did in Section II-D for coverage of a single point.

We begin by restricting our attention to compact sets $W \subset \mathbb{R}^2$ that are radially symmetric about the origin, so that $-w \in W$ for all $w \in W$. For any $r > 0$ and $q \in \mathbb{R}^2$, define

$$\mathcal{B}_r(q) = \left\{ z \in \mathbb{R}^2 \colon \ \|q - z\| < r \right\}$$

and

$$\mathcal{S}_r(W) = \left\{ z \in \mathbb{R}^2 \colon W \subset \mathcal{B}_\gamma(z) \right\}.$$

In particular, the set W is strictly contained in a disk of radius γ if this disk is centered at any point in $\mathcal{S}_\gamma(W)$. Henceforth, we will assume that $\mathcal{S}_\gamma(W) \neq \emptyset$. In this case, a sufficient condition for W to be (α, β, γ)-covered by p is clearly that there exists some time $t \in [0, \infty)$ at which

$$p(t) - e(t) \in \mathcal{S}_\gamma(W).$$

It is equivalent that

$$e(t) \in p(t) \oplus \mathcal{S}_\gamma(W), \tag{15}$$

where "$\oplus$" denotes Minkowski addition. Note that Equation (5) in Section II-B could similarly have been written

$$e(t) \in p(t) \oplus \mathcal{B}_\gamma(o),$$

where $o = (0, 0) \in \mathbb{R}^2$ denotes the origin. In particular, we see that a sufficient condition for (α, β, γ)-coverage of W by a disk $B_\gamma(o)$ is (α, β, γ)-coverage of the origin by $S_\gamma(W)$. All of the machinery we developed in Section II can immediately be applied to verify this condition.

B. Verifying (α, β, γ)-Coverage of a Square

Suppose the set W to be covered is a square of side-length μ that is centered at the origin:

$$W = \left\{ z \in \mathbb{R}^2 \colon \ |z_1| \leq \mu \text{ and } |z_2| \leq \mu \right\}.$$

This set is both compact and radially symmetric. It is easy to verify that $S_\gamma(W)$ is the "diamond" shape of radius

$$\lambda = \frac{-\mu + \sqrt{4\gamma^2 - \mu^2}}{2}$$

that is shown in Figure 5, and also that $S_\gamma(W) \neq \emptyset$ when $\mu < \gamma\sqrt{2}$. Hence, W satisfies the assumptions we made in Section III-A, and we are free to approach verification of (α, β, γ)-coverage as in Section II.

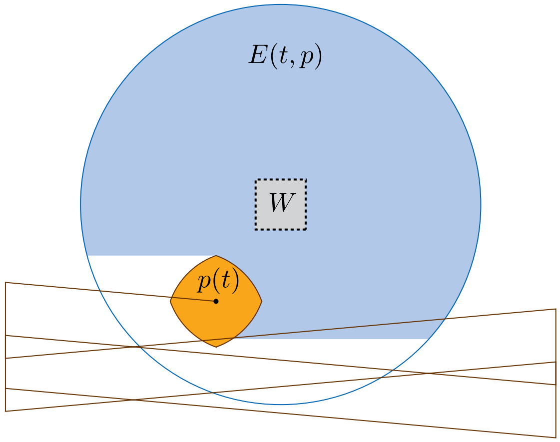


Fig. 6. Snapshot of the switchback plan to cover a square set W of side-length $\mu = 1/4$ when $(\alpha = 1, \beta = 1/10, \gamma = 3/8)$.

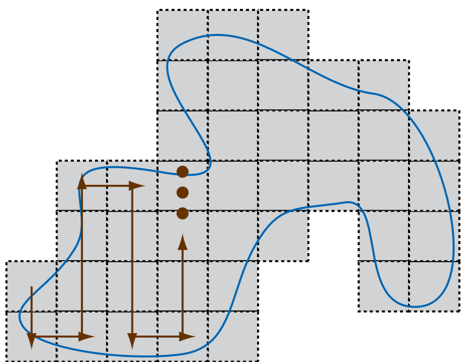


Fig. 7. Covering a workspace by tiling it with a sequence of squares, each one of which is (α, β, γ)-covered by repeating the trajectory in Figure 6.

In particular, we consider exactly the same switchback trajectory p that we defined in Section II-D, only replacing "γ" with "λ" wherever it appears. The forward reachable set $E(t,p)$ evolves exactly as before. Figure 6 shows a snapshot of $E(t,p)$ at one particular time. We may compute from Equations (12)-(13)—again, replacing "γ" with "λ"—an upper bound on the first time $t \in [0, \infty)$ at which $E(t,p) = \emptyset$. We conclude, as before, that the set W is (α, β, γ)-covered by p.

IV. COVERAGE OF A WORKSPACE

In the previous two sections, we saw how to achieve (α, β, γ)-coverage (i.e., "robust" coverage) of a point and of a square. We did so using a trajectory p that consists of a sequence of switchbacks, slanted slightly backward on each pass (see Figures 2 and 6). An important property of this trajectory—that we did not discuss before—is that p is *periodic*. An immediate consequence is that, by repeating this trajectory, we (α, β, γ)-cover not just one square but rather an entire sequence of squares. We now have an approach that allows us to cover a workspace of arbitrary shape—tile the workspace with squares of side-length μ, follow a Boustrophedon path (as in [5]) through the tiling, and cover each square by repeated application of our same basic trajectory p (see Figure 7).

V. CONCLUSION

In this paper we showed one way to plan guaranteed coverage paths for a mobile robot whose position and velocity are subject to bounded error. Our algorithm is conservative, in that we have made no attempt here to maximize the rate of coverage. There are several obvious modifications that would preserve the coverage guarantee, while improving the coverage rate (e.g., interlacing the "raster lines" for the paths in Section IV would likely lead to faster coverage in most circumstances, while still providing the same worst-case guarantee). And, of course, if α or β are decreased, or if γ is increased, the paths we present here will achieve a faster rate of coverage. To our knowledge, this work represents the first guaranteed coverage results for the case of bounded position and velocity error.

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