

SCOPE AND SEQUENCE: SURF Maths (Prep to Year 4)

SURF board at the end of Year 4

Strategies (12+)	Understanding (11)	Reading (11)	Fast Facts (11)
Count On AND Hop On	Turnaround Facts (commutative property)	+	Counting forwards (20/100/1000/10000 and beyond)
Count Back AND Hop Back	More than one way	-	Counting backwards (20/100/1000/10000 and beyond)
Count Up AND Hop Up	Change the order (associative property)	=	Rainbow facts
Near Doubles	Partitioning [into 1000's (& beyond), 100's, 10's & 1's; tenths and hundredths]: What lives inside a number?	> And <	Doubles and Halves
Bridging through 10 (and multiples of 10)	Add. Undoes Sub.; Sub. Undoes Add.	1, 2 3 & 4-digit numbers (and beyond)	Subitising
Number splitting	Fractions	×	Friendly numbers
Double Double (Half Half)	Decimals	÷	Adding 10
Double Double Double (Half Half Half)	Estimation and checking	∞	Super Rainbow Facts
Compensate	Mult. Undoes Div.; Div. Undoes Mult.	Fraction notation (e.g. $\frac{1}{2}$)	Skip counting (all sequences)
Strategies produced through use of the distributive property (5's+2's=7's)	Adding and subtracting groups (distributive property)	Decimal notation (e.g., 0.34)	Timetables (2's to 10's)
	Factors and multiples		Multiplying by 10
	Prime and Composite numbers		

Scope and Sequence

Prep	Year 1	Year 2	Year 3 & 4
Strategies:	Strategies:	Strategies:	Strategies:
Count On (Introduced)	Count On (Consolidated)	Count Up (Consolidated)	Hop On (Consolidated)
Count Back (Introduced)	Count Back (Consolidated)	Near Doubles (Consolidated)	Hop Back (Consolidated)
	Count Up (Introduced)	Bridging through 10 (Consolidated)	Hop Up (Consolidated)
	Near Doubles (Introduced)	Number splitting (Consolidated)	Double Double (Consolidated)
	Bridging through 10 (Introduced)	Hop On (Introduced)	Compensate (Consolidated)
	Number splitting (Introduced)	Hop Back (Introduced)	Double double double (Introduced)
		Hop Up (Introduced)	Timetable strategies based on distributive property: $5's + 1's = 6's$; $5's + 2's = 7's$; $10's - 1's = 9's$; $10's + 2's = 12's$; $10's - 2's = 8's$ etc.
		Double Double (Introduced)	
		Compensate (Introduced)	

Prep	Year 1	Year 2	Year 3 & 4
Understanding:	Understanding:	Understanding:	Understanding:
More than one way (part-whole concept)	More than one way (Add. & Sub. – Introduced)	More than one way (Add. & Sub. – Consolidated)	More than one way (Mult. & Div.)
Partitioning: 1 group of 10 and some 1's	Partitioning: 10's, 1's	Partitioning: 100's, 10's, 1's	Partitioning: 1000's (and beyond), 100's, 10's, 1's, tenths and hundredths
Turnaround Facts (introduced)	Turnaround Facts (Consolidated)	Change the order (Consolidated)	Estimation and checking (Consolidated)
	Change the order (Introduced)	Add. Undoes Sub. (Consolidated)	Distributive Property (Consolidated)
	Add. Undoes Sub. (Introduced)	Fractions: region/set (Consolidated)	Fractions: region/set (Consolidated)
	Fractions: region/set (Introduced)	Estimation (Introduced)	Fractions: as number (Introduced)
		Distributive Property (Introduced)	Decimals (Introduced) – linked to partitioning and fractions
			Mult. Undoes Div. (Introduced)
			Factors and multiples (Introduced)
			Prime and composite numbers (Introduced)

Prep	Year 1	Year 2	Year 3 & 4
Reading:	Reading:	Reading:	Reading:
1-digit numbers (Consolidated)	2-digit numbers	3-digit numbers	4-digit numbers and beyond
2-digit numbers (Introduced)	+ (Consolidated)	> And < (Introduced)	> And < (Consolidated)
+ (Introduced)	- (Consolidated)	= (Consolidated)	= (Consolidated)
- (Introduced)	= (Consolidated)	× (Introduced)	× (Consolidated)
= (Introduced)		÷ Introduced)	÷ (Consolidated)
			∞ (Introduced)
			Fraction notation (e.g., $\frac{1}{4}$) (Introduced)
			Decimal notation (e.g., 0.36) (Introduced)

Prep	Year 1	Year 2	Year 3 & 4
Fast Facts:	Fast Facts:	Fast Facts:	Fast Facts:
Counting forwards (to 20)	Counting forwards (to 100)	Counting forwards (to 1000)	Counting forwards (to 10000 and beyond)
Counting backwards (to 20)	Counting backwards (to 100)	Counting backwards (to 1000)	Counting backwards (to 10000 and beyond)
Doubles and Halves (to 10)	Subitising (Consolidated)	Subitising (Consolidated)	Skip counting by 3's, 4's etc (Consolidated)
Subitising (Introduced)	Rainbow facts (Consolidated)	Rainbow Facts (Consolidated)	Super Rainbow Facts (Consolidated)
Rainbow facts (Introduced)	Doubles and Halves (to 20)	Doubles and Halves (to 30)	Timetables 2's, 3's, 5's, 10's (Year 3)
	Friendly numbers (Introduced)	Friendly numbers (Consolidated)	Timetables 4's, 6's, 7's, 8's, 9's (Year 4)
	Adding 10 (Introduced)	Adding 10 (Consolidated)	
	Skip counting by 2's, 5's, 10's (Introduced)	Skip counting by 2's, 5's, 10's (Consolidated)	
		Super Rainbow Facts (Introduced)	
		Skip counting by 3's, 4's etc (Introduced)	

Elaborations (Strategies, Understanding, Reading, Fast Facts)

Strategies

Teaching Point	Explanation	Example	When to use
Count On	When adding two numbers together, place the larger number in your head and count on.	$7 + 2 = 9$, because: 7, 8, 9.	When the smaller addend is 4 or less. When students do not have knowledge of more efficient strategies.
Hop On	Similar to count on, however begin by counting on in 10's before you count on in 1's. Students need to partition the smaller number (into 10's and 1's) before they use hop on.	$21 + 33 = 54$, because: 33, 43, 53 and 54.	When adding 2-digit (or larger) numbers. Should be represented as 'hops' on a number line.
Count Back	When subtracting two numbers, place the first (larger) number in your head and count back.	For example, $7 - 2 = 5$, because: 7, 6, 5.	When the number you are subtracting is 4 or less. When students do not have knowledge of more efficient strategies.
Hop Back	Similar to count back, however begin by counting back in 10's before you count back in 1's. Again, students need to partition the smaller number before they use hop back.	$54 - 21 = 33$, because: 54, 43, 33 and 32.	When subtracting 2-digit (or larger) numbers. When the teacher wants to demonstrate subtraction as 'take away'. Should be represented by 'hops' back on a number line.
Count Up	When subtracting two numbers, place the second (smaller) number in your head and count up. However far you have to count gives you the difference between the two numbers.	For example $16 - 14 = 2$, because: 14, 15, 16.	When the two numbers you are subtracting are relatively close together (i.e., when you anticipate an answer to your subtraction equation of 4 or less).
Hop Up	Similar to count up, however this is more accurately viewed as the difference between method for subtraction. Students begin by drawing a number line, marking the smallest and largest numbers. Students need to count up	For example $66 - 28 = 38$, because: $28 + 2 = 30$; $30 + 30 = 60$; $60 + 6 = 66$. The total amount the student has 'hopped up' is 38 ($2 + 30 + 6$). This is the	As an alternative to count back when subtracting two numbers. When the teacher wants to demonstrate that subtraction means 'difference between' or 'how many more'.

	from the smallest number to the next 'friendly number' (i.e., multiple of 10). Students then need to count up in multiples of 10 and then in multiples of 1 until they reach the larger number.	difference between the two numbers and the answer to the subtraction equation.	
Near Doubles	When adding two numbers, we can use our doubles facts (see Fast Facts below) to help if it is close to being a double.	For example, $6 + 5 = 11$, because: $5 + 5 = 10$ and $6 - 5 = 1$, and $10 + 1 = 11$.	When adding numbers with a difference of 1 or 2.
Bridging through 10 (and multiples of 10)	When adding two numbers together, 10 (or a multiple of 10) can be a good place to stop at on the way; if we know our rainbow facts and friendly numbers (see Fast Facts below).	For example, $9 + 7 = 16$, because: $9 + 1 = 10$, and $7 - 1 = 6$, and $10 + 6 = 16$.	When adding two single digit numbers whose total is going to be greater than 10. When adding a single-digit number to a multi-digit number, when the units column of the two numbers sum to be greater than 10.
Number splitting	When adding two multi-digit numbers together, it can help if we first partition the number into 10's and 1's. Ask students 'What lives inside this number' (see Understanding below). After both numbers are partitioned, students can add the 10's together, add the 1's together, and then recombine them into the answer.	For example, $35 + 27 = 62$ because: $35 = 30 + 5$ $27 = 20 + 7$ $30 + 20 = 50$ $5 + 7 = 12$ $50 + 12$ (or $50 + 10 + 2$) = 62	When adding 2-digit (or larger) numbers. Can be represented by students in different ways (e.g., by drawing lines to partition numbers).
Double Double (Half Half)	Students know that multiplying by 2 is the same as doubling a particular number, so multiplying by 4 is the same as doubling that particular number twice (and vice versa for division and halving).	4 groups of 13 = 52 because: Double 13 = 26 Double 26 = 52	When multiplying or dividing by 4. It means that students (at least up until grade 3) do not need to learn their 4 times tables by rote.
Double Double Double (Half Half Half)	Students know that multiplying by 2 is the same as doubling a particular number, so multiplying by 8 is the same as doubling that particular number three times (and vice versa for division and halving).	8 groups of 13 = 104 because: Double 13 = 26 Double 26 = 52 Double 52 = 104	When multiplying or dividing by 8. It means that students (at least up until grade 3) do not need to learn their 8 times tables by rote.

Compensating	<p>This is a flexible, sophisticated and multi-faceted strategy. It draws on the notion that numbers can be partitioned in different ways and that addition and subtraction are inverse operations. In this context, it can be used to aid with addition and subtraction (although the same principle can be clearly applied to multiplication and division). It involves changing both the numbers (or one of the numbers and the 'answer' – referred to as 'pay it back') to make an addition or subtraction easier.</p>	<p>For example, $298 + 76 = 374$ because:</p> <p>$298 + 2 = 300$ $76 - 2 = 74$ $300 + 74 = 374$</p> <p>$213 - 97 = 116$ because: $97 + 3 = 100$ $213 + 3 = 216$ $216 - 100 = 116$</p> <p>$58 - 19 = 39$ because: $58 + 1 = 59$ $59 - 19 = 40$ $40 - 1 = 39$</p>	<p>Can be used in a wide-variety of contexts, however particularly in circumstances where the student can identify that an addition or subtraction is <u>almost</u> something fairly straight forward. The idea is that the student can then manipulate the numbers to undertake the straightforward addition or subtraction and then compensate. I would recommend that students have mastered almost all other strategies, fast facts, symbols and ideas (with the exception of Adding and Subtracting Groups – see below) mentioned in this document before trying to explicitly teach compensating.</p>
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Understanding (Ideas)

Teaching Point	Explanation	Example	When to use
Turnaround Facts (commutative property)	$A + B = B + A$ I know that I can add (or multiply) two numbers in whatever order I like and still get the same result.	For example, because I know $6 + 3$ equals 9, $3 + 6$ must also equal 9.	Self-explanatory
More than one way	<p>There is often more than one strategy or approach for working out an answer. This is the most important idea in SURF: exploring different strategies and approaches, and deciding which are the most efficient.</p> <p>In Prep, it can be linked to the part-whole idea (e.g., $7 = 5+2 = 4+3 = 2+5$)</p> <p>In Year 1 and 2, it can be linked to addition and subtraction (see example)</p> <p>In Year 3 and 4, it can be linked to multiplication and division, and specifically with the distributive property (e.g., Double double double = 8's; $4's+4's = 8's$; $10's-2's = 8's$).</p>	For example, $6 + 5 = 11$ can be worked out using Near Doubles addition ($5 + 5 + 1$), Near Doubles subtraction ($6 + 6 - 1$), Count On (6,7, 8, 9, 10, 11) or bridge through 10 ($6 + 4 + 1$). Most students would choose the Near Doubles addition approach, however all the above strategies, possibly with the exception of Count On, would seem relatively efficient.	Constantly refer back to this idea.
Change the order (associative property)	$A + B + C = C + A + B = B + C + A$ etc Sometimes when adding three or more numbers together, the sum make the adding much easier if we can change the order in which we add.	For example, $8 + 5 + 2$ is easier if I work out the rainbow fact first ($8+2 = 10$, see Fast Facts below), and then add the 5 (using Friendly Numbers, see Fast Facts below).	When adding or subtracting more than two numbers, students should consider changing the order.
Partitioning (into 1000's, 100's, 10's and 1's, tenths,	Ask students: "What lives inside this number?". You are inviting the student to identify how many 10's and how many 1's make up a	What lives inside 36? 3 tens and 6 ones 30 and 6	When preparing to add or subtract multi-digit numbers.

hundredths): What lives inside a number?	particular number. Strong links to how we say 2-digit numbers (e.g., 7-ty-6 is comprised of 7 tens and 6 ones) and how we <u>should</u> say 2-digit numbers, if our mathematical language was more clearly structured (e.g., if 13 was pronounced 1-ty-3, it would be much clearer that it was comprised of 1 ten and 3 ones).		This idea can be extended to partitioning numbers in other ways, which links with the compensation strategy discussed above (e.g., seeing 307 as 290 and 17, which might be useful if faced with $307 - 289$).
Addition Undoes Subtraction; Subtraction Undoes Addition	This involves understanding the inverse relationship between addition and subtraction.	For example, because $8 + 3 = 11$, we know that $11 - 3 = 8$ and $11 - 8 = 3$.	Generally when tackling problems involving subtraction.
Fractions	Part of a whole. The numerator (top number) represents a number of equal parts, and the denominator (bottom number), which cannot be zero, indicates how many of those parts make up a unit or a whole. Region and Set models. Fractions as a number of a number line.	For example, in the fraction $\frac{3}{4}$, the numerator, 3, tells us that the fraction represents 3 equal parts, and the denominator, 4, tells us that 4 parts make up a whole. The whole can be a region (e.g., shape) or a set (e.g., 12 teddies). $\frac{3}{4}$ can also represent a number between 0 and 1 to be plotted on a number line.	Self-explanatory
Estimation and checking	Estimation – using your pre-existing knowledge so that you have some idea about the range of the answer before you attempt to solve a problem. Also encompasses approximation, when close enough is good enough. Checking – using a different strategy (or carefully repeating the same strategy) to check if your calculation is correct.	151 + 173 Estimation: It must be greater than 200 and less than 400 because $100 + 100 = 200$ and $200 + 200 = 400$, and both the addends are between these two numbers. Approximation: $150 + 150 = 300$, so the answer is a little over 300. Around 320. Checking: I used Hop Up to work out the exact answer to be 324. I will now use Number Splitting to check if I am right.	Encourage students to attempt to make an estimation whenever they are tackling a mathematical problem. Students should also be encouraged to check their work using a different strategy, particularly when solving problems involving multiple steps.

Multiplication Undoes Division; Division Undoes Multiplication	This involves understanding the inverse relationship between multiplication and division.	For example, because $4 \times 6 = 24$, we know $24 \div 6 = 4$	Generally when tackling problems involving division.
Adding and subtracting groups (Distributive Property)	$A \times (B + C) = A \times B + A \times C$ This refers to the notion that we can break up groups of numbers when multiplying or dividing to aid with calculation. It can take many different forms, links with the notions of partitioning and compensating, and is probably best explored through examples.	For example, $7 \times 7 = 49$, because $(5 + 2) \times 7 = 49$. More specifically: $7 \times 5 = 35$ $7 \times 2 = 14$ $35 + 14 = 49$ For example $13 \times 9 = 117$, because $13 \times (10 - 1) = 117$. More specifically: $13 \times 10 = 130$ $1 \times 13 = 13$ $130 - 13 = 117$	When multiplying or dividing numbers, and when the answer is not immediately apparent.
Factors and multiples	<p>Factors are numbers you can multiply together to get another number.</p> <p>A multiple is the <u>result</u> of multiplying a number by an integer (i.e., a whole number)</p>	<p>Example: 2 and 3 are factors of 6, because $2 \times 3 = 6$.</p> <p>Example 6 is a multiple of 3 and 2.</p>	Self-explanatory
Prime and composite numbers	A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself. A natural number greater than 1 that is not a prime number is called a composite number.	<p>7 has only two factors: 1 and 7 so it is prime.</p> <p>8 has more than two factors: 1,2,4,8 so it is composite.</p>	Self-explanatory
Decimals	Extension of place value system to represent a number smaller than one, or a number between two whole numbers.	3.36 contains 3 units (ones), 3 tenths and 6 hundreds or, using non-standard partitioning, 3 units and 36 hundredths or 336 hundredths or 33 tenths and 6 hundredths.	Self-explanatory

Reading

The Reading goal of SURF was designed to capture more than just mathematical symbols, it was also designed to address the relationship between these symbols and English language terms. Consequently, when teaching more applied topics across any year level, it is recommended that relevant terminology be included under the Reading goal on the SURF Board to draw attention to critical language for the duration of the topic. For example, a Year 2 class doing a unit of work on money may have language such as: dollars, cents, total, change, sale, discount, income etc included under the Reading goal, with appropriate linkages made with the relevant symbols (e.g., total links to =, discount to -, income to +). To avoid the SURF Board becoming cluttered and unmanageable, it is recommended that this key terminology be taken off the SURF Board once the topic is finished, and displayed on an anchor chart elsewhere in the classroom. As you move through the applied topics, and more anchor charts accumulate, this will encourage students to make connections between common language used across different topics (e.g., the word 'total').

Fast Facts

Teaching Point	Explanation	Example	When to use
Counting forwards	Counting forwards by 1's (initially to 20, then to 100 and 1000).	Self-explanatory	Self-explanatory
Counting backwards	Counting backwards by 1's (initially from 20, then from 100 and 1000).	Self-explanatory	Self-explanatory
Rainbow facts	All the different ways two numbers can be added together to equal 10.	For example: 3+7; 4+6; 5+5; 8+2 etc.	Self-explanatory
Doubles and Halves	Knowing how to double (half) numbers in your head, all the way up to 10 + 10 = 20 and beyond.	For example: Double 7 = 14 Half 14 = 7 Double 21 = 42 Half 42 = 21	Self-explanatory
Subitising	Knowing how many objects/ things there are without counting.	For example, knowing that \equiv represents three lines without counting.	Self-explanatory
Friendly numbers	Numbers ending with a zero can be easily added to single digit numbers. This is essentially the opposite process to partitioning into 10's and 1's, and establishing "What lives inside a number?" (see above) Friendly Numbers are useful for reinforcing how the place value system works in Year 1 and 2	For example, $30 + 6 = 36$, because 3 tens plus 6 ones equals 3 tens 6 ones, which is another way of saying 36.	Self-explanatory
Adding 10	Being able to add 10 to another number rapidly. Like friendly numbers, requires an understanding of place value and partitioning into 10's and 1's. In a similar manner to partitioning (see above), this fast fact would be more rapidly comprehended by students if our mathematical language was more clearly structured (e.g., if 13 was pronounced 1-ty-3, it would	For example, $23 + 10 = 33$.	Self-explanatory

	<p>be much clearer that it was comprised of 1 ten and 3 ones, and therefore, it would be much easier to add another 10 to make 2-ty-3).</p> <p>Adding 10 is useful for demonstrating an understanding of how place value system works in Year 1 and 2</p>		
Super Rainbow Facts (i.e., complements to 100)	<p>All the different ways two numbers can be added together to make 100.</p> <p>Can be useful for linking to compensate strategy when adding particular combinations of numbers (e.g., $58+43 = 58+42+1$).</p>	<p>For example:</p> <p>$56 + ? = 100$; $? = 44$ $89 + ? = 100$; $? = 11$</p>	Self-explanatory
Skip counting by 2's, 3's, 5's and 10's (and then more advanced sequences)	The different counting patterns evident when we count by 2's, 3's, 5's and 10's (initially starting from zero and then exploring other starting points).	<p>For example:</p> <p>2, 4, 6, 8, 10, 12, 14, 16 3, 6, 9, 12, 15, 18, 21, 24 5, 10, 15, 20, 25, 30, 35 10, 20, 30, 40, 50, 60, 70</p>	Useful back-up strategy for timestables
Timestables 2's, 3's, 5's, 10's (and then 4's, 6's, 7's, 8's & 9's)	The traditional timestables, and their division counterparts.	<p>For example:</p> <p>$7 \times 2 = 14$; $18 \div 2 = 9$ $5 \times 3 = 15$; $24 \div 3 = 8$ $8 \times 5 = 40$; $20 \div 5 = 4$ $6 \times 10 = 60$; $40 \div 10 = 4$</p>	Self-explanatory
Multiplying by 10 (and dividing by 10)	<p>When multiplying a number by 10, we can increase the number by an order of magnitude by placing a zero in the ones column, and making the number 1-digit longer. The 10's timestables introduces students to this pattern, however obviously it can be generalised to case where we are multiplying larger numbers by 10.</p> <p>Multiplying and dividing by 10 can be useful for exploring an understanding of place value system in Year 3 and 4</p>	<p>For example:</p> <p>$7 \times 10 = 70$ $19 \times 10 = 190$ $360 \div 10 = 36$ $0.7 \times 10 = 0.7$</p>	Self-explanatory