Radial Error Feedback Geometric Adaptive Control for Bar Turning in CNC Turning Centers

by

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Abstract

In-process measurement and control of CNC machines can improve machining accuracy while increasing productivity and eliminating waste in a cost effective way. This paper describes an in-process measurement and control system called radial error feedback geometric adaptive control (REFGAC) system for bar turning in CNC turning centers. REFGAC system was designed to compensate for the radial error caused by deflection under cutting force. In a dry cutting environment, a non-contact laser sensor was used to directly measure the radial error of the workpiece while it is being cut. To compensate for the radial error, Kalman filters for prediction together with PID control laws were designed, based on models of the deflection of the bar. Two modeling techniques, namely, analytical modeling (using classical beam vibration theory) and finite element modeling (FEM), were employed to develop simple models for the bar bending. The analytical modeling is suitable for uniform bars while the FEM can be used for both uniform and non-uniform bars. Experimental results show that the dimensional and geometric accuracy of the workpiece is substantially improved by this radial error feedback geometric adaptive control technique.

Keywords: CNC turning center, in-process measurement and control, Kalman filter, finite element modeling
1. Introduction

Bar turning is a popular function of computer numerically controlled (CNC) turning machines. Bars can be used as shafts of all kinds of machinery, including automobiles, aircraft, submarines, satellites, and so forth. The dimensional and geometric accuracy of these bars directly influences the performance of these machines. Because of increasing global market competition, more stringent tolerances have been imposed on machined parts. In-process measurement and control of CNC machines can improve machining accuracy while increasing productivity and eliminating waste in a cost effective way.

The major strategy for in-process measurement and control of CNC machines is geometric adaptive control (GAC), which improves part dimensional precision by applying geometric error compensation techniques to correct imprecisions caused by varying machine temperature, tool wear, part deflection, and so forth. Shiraishi and Sato\textsuperscript{12} used an optical device to measure the surface roughness of a workpiece on a lathe. Based on the algebraic relationship between the surface roughness and diameter, the workpiece diameter was monitored indirectly. A unity gain feedback GAC was used to control the radial position of the tool using the drive unit of the axis. Due to the indirect measurement of the diameter, the tool was forced to stop the cut after cutting 2\textit{mm} to 3\textit{mm} to determine the reference diameter level on the sensor. Gao, et al.\textsuperscript{5} developed a contact stylus sensor for the plunge grinding process. Again, part diameter could be measured only indirectly. A Z80A microprocessor was used to implement the size controller. Uda, et al.\textsuperscript{13} developed an optical surface sensor and micro-tool servo which were attached rigidly on the same tool holder. If the sensor detected that the tool holder moved away from or toward the workpiece surface, the piezoelectric micro-tool servo would compensate for the movement. With the improvement of the openness of the CNC controller, there is no difficulty in realizing error compensation via software. Hence, hardware compensation is not preferable.

In another work,\textsuperscript{10,11} an on-line system was used for the evaluation of workpiece size and geometrical tolerances in bar turning. The measurement was performed using three ultrasonic non-contact proximity sensors, which operate in a wet cutting environment. A state space mechanistic
model was used to overcome the delay in the feedback loop. A Kalman Filter was used in a predictor-corrector fashion to update model predictions using on-line measurements. A full state feedback controller with integral control action was designed and used to manipulate the tool position in real-time for machining error compensation. It was reported that the system was capable of improving the diameter accuracy of the workpiece by more than 90 percent. Shawky, et al.\textsuperscript{10,11} proposed a successful control scheme which can solve the sensing delay problem. However, the three ultrasonic sensors are difficult to align, and the part diameter was not measured directly. The time between the transmission and reception of the sound wave was used to calculate the part diameter. For different cutting fluids, the sensor system had to be recalibrated. On the other hand, due to environmental concerns, dry cutting processes are becoming more and more popular in current industries. In this case ultrasonic sensing cannot be used. Another deficiency of this approach is that the determination of the coefficients in the mechanistic model requires various time consuming experiments to be performed.

The Quality in Automation (QIA) program is a comprehensive instance of machining process control. It was conducted at the Manufacturing Engineering Laboratory of the National Institute of Standards and Technology (NIST). There were three loops in the QIA architecture:\textsuperscript{14} (1) the real-time control loop, (2) the process-intermittent control loop, and (3) the post-process control loop. Real-time error compensations in QIA was achieved by implementing tool-path modifications via a real-time error corrector (RTEC), using errors predicted by kinematics and geometric-thermal (GT) models of the machine-tool errors. The models were derived off-line from the structure of the machine and the pre-process characterization of the machine measurements. In order to use the models, machine tool temperatures were measured by thermal couples while the workpiece was in cut. After the cut, the workpiece was measured on-machine by a touch-trigger probe. A least-squares curve fitting of the measurement data was utilized to determine the adjusted tool path curve, which was called the process-intermittent loop. The post-process control loop was used primarily to detect and correct long-term system errors and was still under development. The major difficulty in implementing NIST QIA in practice is the development of the GT model, which requires a prohibitively long time and numerous thermocouples. Process intermittent measurements using
a touch trigger probe cannot achieve high measuring speed, and require an extra probing process which greatly increases machining cost.

Based on this review, it can be concluded that problems in in-process measurement and control have not yet been fully resolved; therefore, more research is required. Radial error feedback geometric adaptive control (REFGAC) is proposed and demonstrated in this paper to overcome the limitations of the current GAC methods.

Due to the high speed of cutting processes, non-contact sensors have been proven to be more suitable than contact sensors. Instead of using indirect measurements of part diameters, REFGAC uses a non-contact laser sensor to measure the part diameter directly in a dry cutting environment. The dry cutting environment was created by replacing cutting fluid with pressured cold air generated by an air gun. In comparison to ultrasonic sensors, optical sensors are more compact and easier to calibrate and install, and are thus more practical and efficient for industrial applications.

For turning processes, error sources in machined workpieces can be classified into four categories:\(^8\) (1) machine tool geometric errors; (2) deflections of the machining system which is composed of the machine, fixture, tool and workpiece (MFTW); (3) thermal deformations of the MFTW system; and (4) wear errors of cutting tool and machine components. McMurty says that the deflection of the machining system due to cutting forces dominates the error budget.\(^9\) Hence, instead of building a complicated GT model or mechanistic model, REFGAC employed simple vibration models for the deflection of the machining system. In addition to developing the vibration model analytically, this research also developed a model using finite element modeling. The advantage of finite element modeling is that it can be used to develop models for non-uniform bars.

Section 2 describes control design and implementation based on analytical modeling. First, the modeling of the workpiece deflection in the turning process using classical beam vibration theory is described. A PID control law and a Kalman filter are then designed based on the model to compensate for the deflection error. Simulation and experimental results are also given. Section 3 describes control design and implementation based on finite element modeling. Section 4 presents conclusions.
2. Control Design and Implementation Based on Analytical Modeling

As previously mentioned, deflection of the machining system due to the cutting force dominates the error budget in bar turning. Hence, in this section, an analytical model of the deflection is derived based on vibration theory. A Kalman filter and a PID control law are then designed and implemented to compensate for the radial error based on the analytical model.

2.1. Model Derivation

The part deflection in bar turning can be characterized by modeling the bar as a cantilever beam subject to a force of constant magnitude that moves across the beam with constant speed (Figure 1). A continuous-time state-space (SS) model is derived from the equation of motion of the vibration problem.

\[ \delta = L - vt \]

\[ x = L - vt \]

\[ L \]

\[ F \]

\[ v \]

\[ \delta \]

**Figure 1**: Vibration Model of the Turning Process

The equation of motion of the beam vibration problem just described is given by

\[ EI \frac{\partial^4 \delta}{\partial x^4} + c \frac{\partial \delta}{\partial t} + \gamma \frac{\partial^2 \delta}{\partial t^2} = F \nabla(x - (L - vt)). \]  \tag{1} \]

For a transverse vibration beam, the bending moment and shear force must vanish at the free end while the deflection and slope must be zero at the fixed end. Hence, the above PDE is subjected
to following boundary conditions when \( t > 0 \):

\[
EI \frac{\partial^2 \delta}{\partial x^2}(L, t) = 0, \tag{2}
\]

\[
EI \frac{\partial^3 \delta}{\partial x^3}(L, t) = 0, \tag{3}
\]

\[
EI \frac{\partial \delta}{\partial x}(0, t) = 0, \tag{4}
\]

\[
\delta(0, t) = 0. \tag{5}
\]

For the problem considered in this research the parameters given in (1) have the following definitions and values:

- \( E \) modulus of elasticity, \( 1.58e5 N/mm^2 \),
- \( I \) moment of inertia, \( \pi D^4 \) \( 64 \), unit is \( mm^4 \),
- \( D \) diameter of the workpiece, \( 49 mm \),
- \( \delta \) deflection of beam, unit is \( mm \),
- \( x \) distance between force and fixed end, unit is \( mm \),
- \( c \) damping factor,
- \( \gamma \) mass per unit length = \( \rho A \), unit is \( kg/mm \),
- \( \rho \) mass density, \( 7.4e-6 kg/mm^3 \),
- \( A \) area of cross section of the beam, \( \pi D^2 / 4 \),
- \( t \) machining time, unit is \( sec \),
- \( F \) radial cutting force, unit is \( N \),
- \( \nabla(x) \) Dirac delta function, \( \begin{cases} 1, & x = 0, \\ 0, & x \neq 0. \end{cases} \)
- \( L \) length of workpiece, \( 350 mm \)
- \( v \) cutting speed, \( 1 mm/sec \).

Next, (1) through (5) are used to develop a state-space (SS) model that characterizes the workpiece deflection. First, define \( Y(x) \) to be an eigenfunction of the eigenvalue problem defined by the undamped, unforced system,

\[
EI \frac{\partial^4 \delta(x, t)}{\partial x^4} + \gamma \frac{\partial^2 \delta(x, t)}{\partial t^2} = 0. \tag{6}
\]

We assume the product solution for (6),

\[
\delta(x, t) = Y(x)f(t). \tag{7}
\]

Differentiating (7), substituting the results into (6), and separating variables yields,

\[
\frac{1}{\gamma Y(x)} EI \frac{d^4 Y(x)}{dx^4} = - \frac{1}{f(t)} \frac{d^2 f(t)}{dt^2}. \tag{8}
\]
Equating each side of (8) to the constant $\omega^2$ so that the solution in time is harmonic then gives

$$\frac{d^2 f(t)}{dt^2} + \omega^2 f(t) = 0,$$

(9)

and

$$EI \frac{d^4 Y(x)}{dx^4} = \omega^2 \gamma Y(x).$$

(10)

Since all functions can be expanded in terms of the eigenfunctions, $Y_j(x) \ (j = 1, 2, \ldots)$, assume

$$F\nabla(x - (L - vt)) = \sum_{j=1}^{\infty} p_j(t)Y_j(x),$$

(11)

where $p_j(t)$ are time functions. Multiplying both sides of (11) by $\gamma Y_k(x)$ and integrating over the beam span gives

$$\int_0^L \gamma F \nabla(x - (L - vt))Y_k(x)dx = \sum_{j=1}^{\infty} p_j(t) \int_0^L \gamma Y_j(x)Y_k(x)dx.$$  

(12)

Note that, by the orthogonality properties of the eigenfunctions (or normal modes), the integral on the right-hand side of (12) equals zero for $k \neq j$, i.e.,

$$\int_0^L \gamma Y_j(x)Y_k(x)dx = 0 \quad k \neq j.$$  

(13)

Furthermore, if the eigenfunctions are normalized, then

$$\int_0^L \gamma Y_j(x)Y_j(x)dx = 1.$$  

(14)

Therefore, (12) may be expressed as

$$\int_0^L \gamma F \nabla(x - (L - vt))Y_j(x)dx = p_j(t).$$  

(15)

Since a property of the Dirac delta function is

$$\int_0^\infty \nabla(x - a)f(x)dx = f(a),$$  

(16)

(15) may be more simply expressed as

$$\gamma FY_j(L - vt) = p_j(t).$$  

(17)
Next, express the deflection as

$$\delta(x, t) = \sum_{j=1}^{\infty} y_j(t)Y_j(x). \hspace{1cm} (18)$$

Then multiplying and integrating as above yields

$$\int_0^L \gamma \delta(x, t)Y_j(x)dx = y_j(t). \hspace{1cm} (19)$$

Equation (18) can be differentiated with respect to $x$ and $t$ so that all the terms in the equation of motion (1) can be put into the modal expansion form,

$$\sum_{j=1}^{\infty} \left[ EIy_j(t)\frac{d^4Y_j(x)}{dx^4} + c \frac{dy_j(t)}{dt}Y_j(x) + \gamma \frac{d^2y_j(t)}{dt^2}Y_j(x) \right] = \sum_{j=1}^{\infty} p_j(t)Y_j(x). \hspace{1cm} (20)$$

Substituting (10) into (20) then yields

$$\sum_{j=1}^{\infty} \left[ \omega_j^2 \gamma y_j + c \dot{y}_j + \gamma \ddot{y}_j - p_j(t) \right] Y_j(x) = 0. \hspace{1cm} (21)$$

Since the eigenfunctions $Y_j(x)$ are nonzero, the expression in the square brackets must vanish for every $j$, i.e.,

$$\ddot{y}_j + \frac{c}{\gamma} \dot{y}_j + \omega_j^2 y_j = \frac{1}{\gamma} p_j(t), \hspace{1cm} j = 1, 2, \ldots \hspace{1cm} (22)$$

Substituting (17) into (22), then yields

$$\ddot{y}_j + \frac{c}{\gamma} \dot{y}_j + \omega_j^2 y_j = FY_j(L - vt). \hspace{1cm} (23)$$

Assume that only the first mode is considered in (18), then

$$y_1(t) = \frac{\delta(x, t)}{Y_1(x)}. \hspace{1cm} (24)$$

Substituting (24) into (23), we have

$$\ddot{\delta}(x, t) + \frac{c}{\gamma} \dot{\delta}(x, t) + \omega_1^2 \delta(x, t) = Y_1(x)Y_1(L - vt)F. \hspace{1cm} (25)$$

To ensure the boundary conditions (2)-(5) and the normal condition (14), we choose

$$Y_1(x) = c_1[\cosh \beta_1 x - \cos \beta_1 x - \sigma_1(\sinh \beta_1 x - \sin \beta_1 x)], \hspace{1cm} (26)$$
where

\[ c_1 = 0.10263, \quad \sigma_1 = 0.7341, \quad \beta_1 L = 1.87510407. \] (27)

The cutting force is proportional to the actual depth of cut, i.e.,

\[ F = k_f (d_c + \Delta d_d - \delta), \] (28)

where \( d_c \) is the commanded depth of cut, and \( \Delta d_d \) is the actual change of the depth of cut. Since

\[ \frac{c}{\gamma} = 2 \xi_1 \omega_1, \]

where \( \xi_1 \) is the damping ratio, and \( x = L - vt \), (25) becomes

\[ \ddot{\delta}(L - vt, t) + 2 \xi_1 \omega_1 \dot{\delta}(L - vt, t) + \omega_1^2 \delta = Y_1^2 (L - vt) k_f (d_c + \Delta d_d - \delta). \] (29)

Rearranging (29) and for simplicity omitting the explicit dependence of \( \delta \) and \( Y_1 \) on \( t \) gives

\[ \ddot{\delta} = -2 \xi_1 \omega_1 \dot{\delta} - (\omega_1^2 + Y_1^2 k_f) \delta + Y_1^2 k_f d_c + Y_1^2 k_f \Delta d_d, \] (30)

where \( \omega_1 = \beta_1 \sqrt{\frac{EI}{\gamma}} \), \( \epsilon \) and \( \xi_1 \) and \( k_f \) are obtained by matching the response of the model with the radial error curve and their values are given in Table 1.

For a commanded change of depth of cut \( \Delta d_c \), the actual change of the depth of cut \( \Delta d_d \) satisfies

\[ \ddot{\Delta d_d} + 2 \xi_d \omega_d \dot{\Delta d_d} + \omega_d^2 \Delta d_d = k_d \omega_d^2 \Delta d_c, \] (31)

where \( \xi_d \) is the damping ratio and \( \omega_d \) is natural frequency of the CNC’s servo response. The parameters \( k_d, \omega_d, \) and \( \xi_d \) are obtained from the step response of the CNC’s servo system (Figure 2) and their values are given in Table 1.

Choose the state vector of the state space model as \([\delta \dot{\delta} \Delta d_d \dot{\Delta d_d}]^T\), the input vector as \([\Delta d_c \quad d_c]^T\), and the output as the radial error \( e \). Then the SS model for the deflection is
\[
\begin{bmatrix}
\dot{\delta} \\
\dot{\delta} \\
\dot{\Delta d_d} \\
\dot{\Delta d_d}
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
-\omega_d^2 & -Y^2 k_f & -2\xi_1 \omega_1 & Y^2 k_f & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -\omega_d^2 & -2\xi_3 \omega_d
\end{bmatrix}
\begin{bmatrix}
\delta \\
\delta \\
\Delta d_d \\
\Delta d_d
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & Y^2 k_f & 0 & Y^2 k_f & 0 \\
0 & 0 & 0 & 0 & 0 \\
k_d \omega_d^2 & 0 & k_d \omega_d^2 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\triangle d_c \\
d_c \\
w_1 \\
w_2 \\
v
\end{bmatrix}, \quad (32)
\]

\[
\begin{bmatrix}
y_m \\
y_p
\end{bmatrix} = 
\begin{bmatrix}
-1 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \\
\delta \\
\Delta d_d \\
\Delta d_d
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\triangle d_c \\
d_c \\
w_1 \\
w_2 \\
v
\end{bmatrix}. \quad (33)
\]

where \(w_1\) and \(w_2\) are process noises, \(v\) is sensor measurement noise, \(y_m\) is the measured radial error, and \(y_p\) is the actual radial error of the plant. Since the linear accuracy of the CNC machine is \(\pm 40 \mu m\), the process noises are simulated as a zero mean random number between \(\pm 40 \mu m\).

The sensor noise is obtained by detrending the machine shop measurement data. The mean and variance of \(v\) are 0 and 9.45e−7 respectively. Since \(Y_1 = Y_1 (L - vt)\), the system matrices in (32) are time-dependent. Hence, the SS model (32)-(33) describes a time-varying, linear system.

\[\text{Figure 2: Step Response of the CNC Servo System}\]
Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Variables</th>
<th>( \omega_1 ) (rad/sec)</th>
<th>( \xi_1 )</th>
<th>( k_f ) (GN/m)</th>
<th>( k_d )</th>
<th>( \omega_d ) (rad/sec)</th>
<th>( \xi_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1647.88</td>
<td>0.047</td>
<td>5.0</td>
<td>1.0</td>
<td>2.0</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Figure 3 compares the radial error obtained from the model (simulated with measurement noise) with that obtained experimentally. It is seen that the model can predict the radial error in bar turning.

![Figure 3: Comparison of the Simulated and Experimentally Obtained Radial Error](image)

**Figure 3: Comparison of the Simulated and Experimentally Obtained Radial Error**

### 2.2. Simulated Response of Radial Error Feedback with PID Compensation

Figure 4 shows the block-diagram of the closed-loop system with radial error feedback. The low pass filter (using the moving average method) was used to filter the high frequency noise. The parameters of the PID control law \( K(1 + \frac{1}{T_I}s + T_Ds) \), where \( K = 2.000 \), \( T_I = 1.667 \text{sec} \), \( T_D = 0.000 \text{sec} \) were found by applying the Ziegler-Nichols PID tuning method\(^3\) to the model (32)-(33).

Figure 5 shows the simulated radial error without any sensor delay. Comparing Figure 5 with Figure 3, it is seen that the radial error is reduced from about 70\( \mu \text{m} \) to 5\( \mu \text{m} \).

The sensor cannot measure the diameter of the bar at the cutting point in practice, which results in a 2-second sensor delay. Hence, it is necessary to add this delay to the simulated feedback system.
Figure 4: Closed-Loop Radial Error Feedback

Figure 5: Simulated Radial Error for PID Control without Sensor Delay

Figure 6: Design PID for the State Space Model with Noise and the Sensor Delay

2.3. Experimental Response of Radial Error Feedback with PID Compensation

The experiments were performed on a Cinturn universal turning center. A Keyence laser sensor is used to measure the diameter of the workpiece in real-time. A Pentium 200 PC reads the measurements using a data acquisition card. The measurements are filtered first using the moving
average method, and then compared to the desired diameter of the workpiece to arrive at the radial error. The control algorithm is programmed in the PC. The control signal is calculated by the algorithm based on the radial error. The original X axis feedback loop is intercepted using resolver to digital (R/D) and digital to resolver (D/R) PC cards. The control signal is transformed to resolver signals using a D/R card. The transformed resolver signals (instead of the original resolver feedback signals) are sent back to the original controller of the CNC machine. As a result, the cutting tool’s X position can be changed in real-time to compensate for the radial error.

**Figure 8**: Interception of the Original Feedback Loop with R/D and D/R cards

Figure 8 shows how the original feedback loop is intercepted and demonstrates how the personal computer is added into the loop. In original feedback loop, the resolver was connected to the servo
amplitude (SAM) board and data management (DUMP) board via an 18-pin connector. The only place that control signals from the PC can be inserted into the original feedback loop is between the resolver and the connector. It is important to note the non-deterministic characteristics of the PC with Windows NT OS observed during experiments. While the PC is in the loop, the loop cannot be guaranteed to be closed all the time, especially when there are other Windows applications running. Once the loop is open, the servo board cannot get an updated control signal, and the axes’ movement will be out of control. Thanks to the efforts of the OMAC users’ group, some companies have developed Hard Real Time Extensions (HRTE) for Windows NT. According to the evaluation report published by the OMAC users’ group, the deterministic capabilities of Windows NT are greatly improved by HRTE.

The Windows NT HRTE software RTX from VenturCom is used in our research. RTX achieves hard real time by replacing NT’s hardware abstraction layer (HAL). The HAL of RTX provides a clock with much higher resolution, and changes the first-in-first-out (FIFO) scheduling system of NT. Applications can be assigned to 128 threads which have different proprietary values, whereas normal NT applications are automatically assigned the lowest proprietary value. Users can assign high proprietary values to real-time applications. By examining the process of REFGAC, two threads can be programmed (Figure 9). Thread 1 (we call it resolver thread) is reading from a R/D card and writing to a D/R card, and is assigned the highest proprietary value to ensure that the loop is closed all the time. Thread 2 (it is called measurement and control thread) includes data acquisition from the laser sensor and computation of the compensation data using the control algorithm, which is assigned the second highest proprietary value. These two threads were programmed using RTX API (application programming interface). With the help of RTX, the non-deterministic characteristics of the PC were no longer observed during experiments.

The solid line in Figure 7 and the third column of Table 2 show the results of actually implementing the PID controller on the experimental apparatus. Like the simulation result, the experimental result shows the damped oscillations caused by sensing delay. The 2nd row of Table 2 shows the average radial error of the workpiece (averaged along the workpiece length), machined with and without radial error feedback PID control. It is seen that the radial error with PID control is 91%
smaller. The 3rd row shows the difference between the maximum and minimum radial error. It is seen that the difference is reduced by 56% with PID control. The 4th row shows the standard deviation of the radial error from mean which was reduced by 75% using the PID compensator. The standard deviation from the trendline, shown in the 5th row of Table 2, measures the surface finish of the workpiece. The surface finish with the PID control law is 2.86 times worse than that without the PID control law. This is due to the oscillations caused by the measurement delay.

Table 2: Comparison of the Experimental Results With and Without REFGAC for Straight Bars

<table>
<thead>
<tr>
<th></th>
<th>W/O REFGAC</th>
<th>W PID</th>
<th>W PID and Kalman</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Radial Error. (mm)</td>
<td>0.02606</td>
<td>0.00225 (91%)</td>
<td>0.00139 (95%)</td>
</tr>
<tr>
<td>Max.-Min. (mm)</td>
<td>0.06690</td>
<td>0.02974 (56%)</td>
<td>0.01166 (83%)</td>
</tr>
<tr>
<td>Std. Dev. from Mean</td>
<td>0.01939</td>
<td>0.00483 (75%)</td>
<td>0.00253 (87%)</td>
</tr>
<tr>
<td>Std. Dev. from Trendline</td>
<td>0.00097</td>
<td>0.00374 (-2.86)</td>
<td>0.00091 (6.2%, 76%)</td>
</tr>
</tbody>
</table>

To compensate for the sensor delay, it is necessary to design an estimator to predict the actual plant output based on the delayed measurement. This is accomplished by using a Kalman estimator, as detailed below.

2.4. Design Kalman Filter for Prediction

To develop an n-step-ahead predictor, a Kalman filter can be designed based on an augmented SS model which includes delayed measurements as states. The Kalman filter can then be used to
reconstruct the current state, although it is not actually measured. The model of a one cycle delay of a quantity \( y \) is

\[
y_{1d}(k + 1) = y(k),
\]

where \( y_{1d} \) is \( y \) delayed by one cycle and is an additional state element that is added to the system model.\(^4\) The model for an \( n \) cycle delay can be obtained by adding the additional state elements \( y_{1d}, y_{2d}, y_{3d}, \ldots, y_{nd} \), defined by

\[
\begin{align*}
y_{2d}(k + 1) &= y_{1d}(k), \\
y_{3d}(k + 1) &= y_{2d}(k), \\
&\vdots \\
y_{nd}(k + 1) &= y_{(n-1)d}(k).
\end{align*}
\]

In particular for a system given by

\[
\begin{align*}
x(k + 1) &= \Phi(k)x(k) + \Gamma(k)u(k) + \Gamma(k)w(k), \\
y(k) &= H(k)x(k) + J(k)u(k) + v(k),
\end{align*}
\]

the augmented system model including an \( n \)-cycle sensor delay is

\[
\begin{bmatrix}
x(k + 1) \\
y_{1d}(k + 1) \\
y_{2d}(k + 1) \\
y_{3d}(k + 1) \\
&\vdots \\
y_{nd}(k + 1)
\end{bmatrix} =
\begin{bmatrix}
\Phi(k) & 0 & 0 & 0 & \cdots & 0 \\
H(k) & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 \\
&\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
x(k) \\
y_{1d}(k) \\
y_{2d}(k) \\
y_{3d}(k) \\
&\vdots \\
y_{nd}(k)
\end{bmatrix}
+ \begin{bmatrix}
\Gamma(k) \\
J(k) \\
0 \\
&\vdots \\
0
\end{bmatrix} u(k) + \begin{bmatrix}
\Gamma(k) \\
0 \\
0 \\
&\vdots \\
0
\end{bmatrix} w(k),
\]
\[ y_d(k) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ y_{1d}(k) \\ y_{2d}(k) \\ y_{3d}(k) \\ \vdots \\ y_{nd}(k) \end{bmatrix} + [0] u(k) + [0] w(k) + v(k). \] (42)

where \( y_d \) is the output \( y \) delayed by \( n \) cycles.

Define

\[
X(k) = \begin{bmatrix} x(k) \\ y_{1d}(k) \\ y_{2d}(k) \\ y_{3d}(k) \\ \vdots \\ y_{nd}(k) \end{bmatrix},
\] (43)

\[
A(k) = \begin{bmatrix} \Phi(k) & 0 & 0 & \cdots & 0 \\ H(k) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix},
\] (44)

\[
B(k) = \begin{bmatrix} \Gamma(k) \\ J(k) \\ 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\] (45)

\[
F(k) = \begin{bmatrix} \Gamma(k) \\ 0 \\ 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 \\ 0 \end{bmatrix},
\] (46)

\[
C(k) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \end{bmatrix}
\] (47)

\[
D(k) = [0].
\] (48)
Then, (41)-(42) are equivalent to
\[
\mathcal{X}(k+1) = \mathcal{A}(k)\mathcal{X}(k) + \mathcal{B}(k)u(k) + \mathcal{F}(k)w(k), \\
y_d(k) = \mathcal{C}(k)\mathcal{X}(k) + \mathcal{D}(k)u(k) + v(k).
\] (49)

For the laser measurement of the bar dimension there is a 2 second sensing delay and the sampling period for the Kalman filter is chosen to be 0.1 second. Hence, the delay is 20 sampling periods.

The dimension of the original plant is four. Hence the dimension of the plant plus delay is 24.

With known inputs \(u(k)\), process noise \(w(k)\), measurement noise \(v(k)\), and noise covariances
\[
E[w(k)w(k)'] = R_w, \quad E[v(k)v(k)'] = R_v, \quad E[w(k)v(k)'] = N_n, \quad k \geq 0,
\] (51)
a Kalman filter can be designed for the system described by (49)-(50).

If the system (49)-(50) is time invariant, i.e., \(A, B, C, D, \) and \(F\) are constant matrices, the Kalman filter algorithm reduces to the following steady-state filter:
\[
\hat{X}(k|k) = \hat{X}(k|k-1) + L_\infty(y_d(k) - C\hat{X}(k|k-1) - Du(k)), \\
\hat{X}(k+1|k) = A\hat{X}(k|k) + Bu(k).
\] (52)

The steady-state Kalman gain
\[
L_\infty = M_\infty C^TR_v^{-1},
\] (54)
where \(M_\infty\) is the unique, non-negative definite solution of the algebraic Riccati equation
\[
AM_\infty + M_\infty A^T + R_w - M_\infty C^TR_v^{-1}CM_\infty = 0.
\] (55)

Note that the gain \(L_\infty\) can be computed off-line.

The inputs to the Kalman filter are \(u(k)\) and \(y_d(k)\). The outputs are the state estimates \(\hat{X}(k|k)\).

Therefore, the estimate of the current measurement \(\hat{y}(k|k)\) can be obtained from
\[
\hat{y}(k|k) = H\hat{x}(k|k) + Ju(k).
\] (56)

As a result, \(\hat{y}(k|k)\) instead of \(y_{nd}(k)\) is used for control.
The model (32)-(33), developed for bar bending, is a time-varying system. In order to implement the Kalman filter in real-time, we update the system matrices every 5 second and assume that the system matrices are constants within the 5 second. Steady-state Kalman gains were computed off-line for corresponding constant system matrices and stored in vector form to be gain scheduled in real-time implementation.

2.5. Simulation and Experimental Response of Radial Error Feedback with PID and Kalman Compensation

The block-diagram of the control system with the Kalman filter is shown in Figure 10. Figure 11 compares the simulation results for the radial error feedback system with and without the use of the Kalman filter for prediction. It is clear that the response of the system is significantly improved by the use of the Kalman filter.

![Figure 10: Feedback of the Estimated Radial Error](image)

![Figure 11: Simulated Radial Error with and without the Kalman Filter](image)

Next, the Kalman filters were implemented on the experimental apparatus. There were two
different controls strategies used in REFGAC implementation. One was with PID control only and the other is with both PID control and a Kalman filter for prediction. Figures 7 and 12 show that the experimental results verify simulation results on these two kinds of REFGAC control schemes. Figure 13 shows the measurement of the radial error of uniformed bars machined with and without REFGAC. The accuracy of the uniform bar is greatly improved by using the PID control law in conjunction with the Kalman predictor.

**Figure 12**: Comparison of the Simulation and Experimental Results with PID Control and Kalman Filter Prediction

**Figure 13**: Experimental Radial Error With and Without the REFGAC

The fourth column of Table 2 shows the results of implementing REFGAC with PID control and
a Kalman filter on the experimental apparatus. By comparing the 2nd column to the 4th column, it is seen that the average radial error is reduced by 95% and the standard deviation from mean is reduced by 87% while the standard deviation from trendline is reduced by 6.2%. By comparing the 3rd column to the 4th column, it is seen that the standard deviation from the trendline is reduced by 76%, which means that the surface finish of the bar is significantly improved by using a Kalman filter for prediction.

3. Control Design and Implementation Based on Finite Element Modeling

For non-uniform bars, normal mode shapes cannot be obtained analytically. Numerical methods, such as finite element modeling (FEM) can be used to obtain the numerical approximation of the normal mode shapes. This section employs FEM to develop a state-space model of the bending in the bar turning process. The advantage of this modeling methodology is that it can be used for a non-uniform bar. To illustrate the numerical design process and compare it with the previous analytical modeling method, the PID control law and Kalman filter are designed based on this model and implemented on the experimental apparatus for uniform bars. A corresponding simulation was also performed for tapered bars.

3.1. Model Derivation

In FEM, a bar is divided into $n$ beam elements (Figure 14). It is assumed that the physical and mechanical properties (such as the cross section area, the mass density, the modulus of elasticity, and the moment of inertial, etc.) are constant within one element.
Figure 14: A Non-uniform Bar and One of Its Beam Elements

The following notation is used below:

- $E_i$ modulus of elasticity of $i$th beam element, $i = 1, 2, \ldots, n$
- $I_i$ moment of inertia of $i$th beam element,
- $A_i$ cross section area of $i$th beam element,
- $\rho_i$ mass density of $i$th beam element,
- $l_i$ length of $i$th beam element,
- $\phi_{ij}^{(2i-1)}$ $j$th deflectional mode shape at nodal point $i$, $j = 1, 2, \ldots, 2n$,
- $\phi_{ij}^{(2i)}$ $j$th rotational mode shape at nodal point $i$,
- $\omega_j$ $j$th natural frequency,
- $\delta_i$ transverse deflection at $i$th nodal point,
- $\theta_i$ transverse rotation at $i$th nodal point,
- $m_i$ mass matrix of $i$th beam element,
- $k_i$ stiffness matrix of $i$th beam element,
- $F_i$ transverse force at $i$th nodal point,
- $M_i$ bending moment at $i$th nodal point,
- $M$ global mass matrix,
- $K$ global stiffness matrix,
- $C$ global damping matrix,
- $F$ global force matrix.

The bar turning process is a forced vibration system. This section will derive the SS model of the bar turning process. It will be shown that the model is a function of the natural frequencies and mode shapes.
The displacement vector is \( X' = [\delta_1 \theta_1 \delta_2 \theta_2 \ldots \delta_i \theta_i \ldots \delta_n \theta_n] \). At time \( t_i \), the cutting force is at the \( i^{th} \) nodal point, the global force matrix is \( F' = [0 0 0 \ldots F_i 0 \ldots 0 0] \), and the equation of motion for the forced vibration of the whole cantilevered beam with modal damping can be written as

\[
\begin{bmatrix}
\ddot{\delta}_1 \\
\ddot{\theta}_1 \\
\ddot{\delta}_2 \\
\ddot{\theta}_2 \\
\vdots \\
\ddot{\delta}_i \\
\ddot{\theta}_i \\
\vdots \\
\ddot{\delta}_n \\
\ddot{\theta}_n
\end{bmatrix} + \begin{bmatrix}
\dot{\delta}_1 \\
\dot{\theta}_1 \\
\dot{\delta}_2 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\delta}_i \\
\dot{\theta}_i \\
\vdots \\
\dot{\delta}_n \\
\dot{\theta}_n
\end{bmatrix} + \begin{bmatrix}
\delta_1 \\
\theta_1 \\
\delta_2 \\
\theta_2 \\
\vdots \\
\delta_i \\
\theta_i \\
\vdots \\
\delta_n \\
\theta_n
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix} + \begin{bmatrix} F_i \\
0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}, \quad (57)
\]

where \( C = \alpha M + \beta K \), and \( \alpha \) and \( \beta \) are usually determined empirically. The displacement vector \( X \) under forced excitation may be written as the superposition of the product of mode shapes and time functions, i.e.,

\[
X^{(k)}(t) = \sum_{j=1}^{2n} \phi_j^{(k)} q_j(t), \quad k = 1, 2, 3, \ldots, 2n, \quad (58)
\]

where \( X^{(k)} \) is the \( k^{th} \) element of the displacement vector \( X \) and \( \phi_j^{(k)} \) is the \( k^{th} \) element of the \( j^{th} \) mode shape vector.

In bar turning, since the excitation, the cutting force, centers around the lower frequencies, the higher modes will not be excited and the forced response may be represented as the superposition of only a few of the lower frequency modes; for example, if three modes are used, then

\[
X^{(k)}(t) = \phi_1^{(k)} q_1(t) + \phi_2^{(k)} q_2(t) + \phi_3^{(k)} q_3(t). \quad (59)
\]

Define \( P = [\phi_1 \phi_2 \phi_3] \), and \( Q^T = [q_1(t) \ q_2(t) \ q_3(t)] \). Multiplying both sides of (57) by \( P^T \) yields

\[
P^T M P \dot{Q} + P^T C P \dot{Q} + P^T K P Q = P^T F. \quad (60)
\]

Since the three columns of modal matrix \( P \) are eigenvectors, \( P^T M P \) and \( P^T K P \) are \( 3 \times 3 \) diagonal matrices. Since \( C = \alpha M + \beta K \), it can be shown that \( P^T C P \) is also \( 3 \times 3 \) diagonal matrix.
Furthermore,\
\[ P^T K P = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix}, \] (61)

and if the eigenvectors are properly normalized, \( P^T M P = I \). Then, (60) becomes
\[
\begin{bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \ddot{q}_3(t) \end{bmatrix} + \begin{bmatrix} \alpha + \beta \omega_1^2 & 0 & 0 \\ 0 & \alpha + \beta \omega_2^2 & 0 \\ 0 & 0 & \alpha + \beta \omega_3^2 \end{bmatrix} \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \dot{q}_3(t) \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix} = \begin{bmatrix} \phi_1^{(2i-1)} F_i \\ \phi_2^{(2i-1)} F_i \\ \phi_3^{(2i-1)} F_i \end{bmatrix}. \] (62)

At time \( t_i \), we are interested in the deflection at the \( i \)th nodal point, i.e., \( \delta_i(t_i) \). From (59), it follows that
\[
\dot{\delta}_i(t) = \dot{X}^{(2i-1)}(t) = \phi_1^{(2i-1)} \dot{q}_1(t) + \phi_2^{(2i-1)} \dot{q}_2(t) + \phi_3^{(2i-1)} \dot{q}_3(t). \] (63)

From the previous section, \( F_i = k_f (d_c + \Delta d_d - \delta) \) where \( \delta = \delta_i(t) \). Define the states and inputs of the SS model of the bar bending as follows:
\[
x_1 = q_1, \quad \text{ (64)} \\
x_2 = q_2, \quad \text{ (65)} \\
x_3 = q_3, \quad \text{ (66)} \\
x_4 = \dot{q}_1, \quad \text{ (67)} \\
x_5 = \dot{q}_2, \quad \text{ (68)} \\
x_6 = \dot{q}_3, \quad \text{ (69)} \\
x_7 = \delta = \delta_i \quad \text{ (70)} \\
x_8 = \Delta d_d, \quad \text{ (71)} \\
x_9 = \Delta d_d, \quad \text{ (72)} \\
u_1 = \Delta d_c, \quad \text{ (73)} \\
u_2 = d_c. \quad \text{ (74)}
\]
For simplicity, define

\[ \phi_1^* = \phi_1^{(2i-1)}, \quad (75) \]
\[ \phi_2^* = \phi_2^{(2i-1)}, \quad (76) \]
\[ \phi_3^* = \phi_3^{(2i-1)}. \quad (77) \]

Then, the SS model at time instant \( t_i \) is

\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\Delta d_d \\
\Delta d_d
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-\omega_1^2 & 0 & 0 & -2\xi_1\omega_1 & 0 & 0 & -\phi_1^* k_f & \phi_1^* k_f & 0 \\
0 & -\omega_2^2 & 0 & 0 & -2\xi_2\omega_2 & 0 & -\phi_2^* k_f & \phi_2^* k_f & 0 \\
0 & 0 & -\omega_3^2 & 0 & 0 & -2\xi_3\omega_3 & -\phi_3^* k_f & \phi_3^* k_f & 0 \\
0 & 0 & 0 & \phi_1^* & \phi_2^* & \phi_3^* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\omega_d^2 & -2\xi_d\omega_d & 0
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
q_3 \\
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\delta \\
\Delta d_d \\
\Delta d_d
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta d_c \\
d_c \\
w_1 \\
w_2 \\
v
\end{bmatrix}, \quad (78)
\]
y_m = \begin{bmatrix} y_m \\ y_p \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \delta \\ \Delta d_c \\ \dot{\Delta d}_c \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_c \\ d_c \\ w_1 \\ w_2 \\ v \end{bmatrix}, \quad (79)

where 2\xi_j \omega_j = \alpha + \beta \omega_j^2 (j = 1, 2, 3). Since \phi_j^{(2i-1)} is in general different for each i, the SS model matrices vary at each time instant t_i.

3.2. Experimental Results for a Uniform Bar

Numerical mode shapes and corresponding natural frequencies are obtained using the FEM software Algor. In order to compare the results with that of the analytical approach, only the first mode shape and natural frequency were used. Equations (78) - (79) are then simplified to

\begin{equation}
\begin{bmatrix} \dot{q}_1 \\ \dot{\dot{q}}_1 \\ \dot{\delta} \\ \dot{\Delta d}_c \\ \dot{\Delta d}_d \end{bmatrix} = \begin{bmatrix} 0 & -\omega_1^2 & 2\xi_1 \omega_1 & -\phi_1^* k_f & \phi_1^* k_f & 0 & 0 \\ -\omega_1^2 & 0 & \phi_1^* & 0 & 0 & 0 & 0 \\ 0 & \phi_1^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_d^2 & -2\xi_d \omega_d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_d^2 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ \delta \\ \Delta d_c \\ \dot{\Delta d}_c \\ \dot{\Delta d}_d \end{bmatrix} = \begin{bmatrix} \Delta d_c \\ d_c \\ w_1 \\ w_2 \\ v \end{bmatrix}, \quad (80)
\end{equation}

\begin{equation}
\begin{bmatrix} y_m \\ y_p \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ \delta \\ \Delta d_c \\ \dot{\Delta d}_c \\ \dot{\Delta d}_d \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_c \\ d_c \\ w_1 \\ w_2 \\ v \end{bmatrix}, \quad (81)
\end{equation}

The natural frequency \omega_1 (= 1650.93 rad/sec) and the mode shapes \phi_1^{(2i-1)} were computed by Algor. Figure 15 compares the analytical mode shape (26) with the FEM mode shape \phi_1^{(2i-1)}. The mean and the standard deviation of the difference of the two mode shapes are 3.32e - 5 and
2.59e – 5; hence they are almost identical. The rest of the parameters in (80) take the values in Table 1.

![Figure 15: Mode Shapes from Analytical Modeling and Finite Element Modeling](image)

The PID controller, given by $K = 2.250$, $T_I = 1.667sec$, $T_D = 0.000sec$, and the Kalman filter were design based on the FEM SS model. Figure 16 shows the experimental results. The mean and the standard deviation of the difference of the two radial error curves are $-7.76e - 4$ and $0.0016$ respectively, so they are very similar and the FEM yields essentially the same improvement as the analytical modeling.

![Figure 16: Radial Errors with Analytical Modeling and Finite Element Modeling](image)
3.3. Simulation Results for Tapered Bar Turning Using Finite Element State Space Model

A simulation was performed to demonstrate the possible improvement of dimensional accuracy in tapered bar turning. Because the area of cross section of a tapered bar changes along workpiece length, analytical mode shapes are not available. Hence, FEM is used to obtain the numerical mode shapes and corresponding natural frequencies.

Figure 17 shows the dimension of the tapered bar used in the simulation. Figure 18 shows the first mode shape of the tapered bar. The corresponding natural frequency $\omega_1$ is $2911 \text{rad/sec}$. Other parameters take the values in Table 1. (Note that the values of the parameters need to be determined from the experimentally obtained deflection curve. Since the curve is not available, we simple assume some values in the simulation experiment.) Figure 19 shows the simulated radial error of the tapered bar machined without REFGAC. Figure 20 shows the simulated radial error of the tapered bar with PID control and with PID control plus a Kalman filter. Table 3 compares the simulation results for the tapered bars with REFGAC and without REFGAC. The average radial error is reduced by 96% with PID control and 98% with PID control and a Kalman filter for prediction. Because of the oscillations caused by the measurement delay, the standard deviation from trendline (surface finish) with PID control is $1.41 \times 10^{-5}$ time worse than that without REFGAC. It appears that the surface finish with PID control law and Kalman filter is 10% worse than that without REFGAC. However, since the difference is in the order of $10^{-5}$, it is reasonable to think that these two methods will produce bars with similar surface finish. The surface finish with PID

![Figure 17: Tapered Bar](image-url)
control plus a Kalman filter is improved by 54% compared to that with only PID control.

Table 3: Comparison of the Simulation Results With and Without REFGAC for Tapered Bars

<table>
<thead>
<tr>
<th></th>
<th>W/O REFGAC</th>
<th>W PID</th>
<th>W PID and Kalman</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Radial Error. (mm)</td>
<td>0.00548</td>
<td>0.00023 (96%)</td>
<td>0.00009 (98%)</td>
</tr>
<tr>
<td>Max.-Min. (mm)</td>
<td>0.02455</td>
<td>0.01002 (59%)</td>
<td>0.00742 (70%)</td>
</tr>
<tr>
<td>Std. Dev. from Mean</td>
<td>0.00635</td>
<td>0.00149 (77%)</td>
<td>0.00068 (89%)</td>
</tr>
<tr>
<td>Std. Dev. from Trendline</td>
<td>0.00057</td>
<td>0.00138 (-1.41)</td>
<td>0.00063 (-10%, 54%)</td>
</tr>
</tbody>
</table>

4. Conclusions

This research developed a radial error feedback geometrical adaptive control system for bar turning in CNC turning centers. Since the workpiece deflection due to the cutting force dominates the machining error budget, the REFGAC approach was designed to compensate for workpiece deflection. The part diameter is measured while it is in-cut by a non-contact laser sensor in a dry cutting environment. The laser sensor was easy to install and calibrate.

The analytical model based on classical beam vibration theory was derived for uniform bars. A PID control law was designed based on the model. Since the sensor cannot measure the cutting point in practice, both simulation and experimental results demonstrated the oscillatory response
caused by measurement delay. To compensate for the sensor delay, a Kalman filter was designed to implement prediction. The machining accuracy was improved by 95% with the PID compensation plus a Kalman filter for prediction.

A finite element model that is applicable for both uniform and non-uniform bars was also developed in this research. To compare it with the previous analytical modeling method, the PID control law and Kalman filter were designed based on this model and implemented on the experimental apparatus for uniform bars. The FEM yields essentially the same improvement as the analytical modeling. The possible improvement of dimensional accuracy in tapered bar turning
was also demonstrated via simulation.
References


