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ROBUST FAULT DETECTION AND DIAGNOSIS FOR PERMANENT MAGNET SYNCHRONOUS MOTORS

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ABSTRACT

Faults in engineering systems are difficult to avoid and may result in serious consequences. Effective fault detection and diagnosis (FDD) can improve system reliability and avoid expensive maintenance. FDD is especially important for some special applications, such as Navy ships operating in hostile environments. So far, FDD for nonlinear systems has not been fully explored. There is still a big gap between FDD theories and applications. This dissertation makes an effort to fill the gap by developing an integrated FDD system structure and a series of algorithms for FDD of Permanent Magnet Synchronous Motors (PMSM).

A fault model is proposed for the stator winding turn-to-turn fault of PMSM. The model provides a good compromise between computational complexity and model accuracy and is versatile for both the healthy and the fault condition. Simulation studies demonstrate a good correspondence with both the theoretical analysis and the experimental observations in the existing literature. The model is especially important for the design of model based FDD algorithms.

Based on the fault model, a series of algorithms are proposed for the fault detection and diagnosis of PMSM. Since the reliability of sensors is the basis of FDD and control systems, a nonlinear parity relation based algorithm is proposed for sensor fault detection. The algorithm can successfully detect single faults in the currents and speed sensors. To track the parameter variations, which are symptoms of system internal changes and faults, an adaptive synchronization based parameter estimation algorithm is proposed. Simulation and experimental studies demonstrate that the algorithm can estimate not only constant parameters but also slowly time varying and abruptly changing parameters in a fast manner. Besides the detection of PMSM internal faults, the algorithm can also provide accurate parameters for the sensor fault detection algorithm. Based on the fault data, a Particle Swarm Optimization based fault diagnosis approach is proposed to find the fault location and severity information of a stator winding turn-to-turn fault. Finally, the proposed algorithms are integrated into a general FDD system structure. The integration provides a FDD system with enhanced robustness to system parametric uncertainties, as shown by extensive simulation studies.
CHAPTER 1

INTRODUCTION

Automation is a significant component of modern engineering systems. While automation brings improved productivity, reduced labor cost, etc., it also increases system complexity. Complicated systems are often more susceptible to faults. Characterized by undesirable system dynamics, system faults can lead to serious consequences, such as plant shutdown, huge economic loss, and human casualties. Therefore, along with automation, system Fault Detection and Diagnosis (FDD) attracts increasing interest. Detection and diagnosis of abnormal events at early stages can prevent the occurrence of catastrophic failures.

In highly automated systems, it is not realistic to obtain control solutions that are robust with respect to any fault. Advances of Fault Tolerable Control (FTC) can further improve system reliability and availability. FTC provides control algorithms capable of maintaining stability and performance despite the occurrence of faults. FTC is based on significant apriori and postpriori knowledge about the faults. Fault detection and diagnosis often need to be performed to provide the requisite knowledge, and thus become essential steps in many FTC system designs. This dissertation studies the fault detection and diagnosis of Permanent Magnet Synchronous Motors (PMSM) using nonlinear model based approaches. However, the approach is general enough to be applied to many machines whose dynamics can be represented by a set of continuously differentiable dynamic equations.

Contributions of this work include several aspects: First, a fault model is proposed to simulate a PMSM operating under stator winding fault conditions. Second, two fault detection algorithms are put forward to detect both sensor faults and motor internal faults using nonlinear model based methods. Third, based on the proposed fault model, an intelligent optimization algorithm, Particle Swarm Optimization (PSO), is applied to perform fault diagnosis for systems with complex and
highly nonlinear model structure. The PSO based fault diagnosis method is capable of identifying both fault location and fault severity. Finally, an integrated FDD system structure is proposed by synthesizing the above algorithms. This FDD system is robust to structured uncertainties represented by PMSM model parameter variations.

This chapter is organized as follows: Section 1.1 provides background information, motivations and objectives about this dissertation. Contributions are summarized in section 1.2. Organization of the dissertation is introduced in Section 1.3.

1.1 Background

In this section, background knowledge about FDD is given first. Then, it is related to FDD issues for PMSM applied to high performance systems, e.g. a shipboard power system. Additionally, motivations and objectives of this work are emphasized.

1.1.1 Fault Detection and Diagnosis

Faults in engineering systems are difficult to avoid. In complex systems, any fault possesses the potential to impact the entire system’s behavior. In a manufacturing process, a simple fault may result in off specification products, higher operation costs, shutdown of production lines, and environmental damage, etc. In a continuously operated system, ignoring a small fault can lead to disastrous consequences. For instance, a position sensor fault in an automated guided vehicle (AGV) may bring the vehicle towards hazardous directions or cause a collision. Without taking these fault conditions into consideration, fully automated systems are not reliable. To improve reliability of automated systems Blanke, et al proposed the concept of fault tolerable and reconfigurable systems in [1][2]. Figure 1.1 illustrates the three layer architecture for a fault tolerant control (FTC) system. The bottom level comprises traditional control loop with sensors, actuators, signal processing, and controllers. The middle level consists of fault detection, diagnosis, and accommodation. Fault accommodation is performed through reconfiguration or other remedial actions initiated by the autonomous supervisor, which is at the top level. Though this architecture separates these functions into three independent layers, they are highly coupled. Located in the middle, FDD plays an important role by linking the other parts into an integrated
system. Thus FDD becomes an essential component in a robust FTC system.

Routinely, particularly more than five years ago, fault detection was implemented using hardware redundancy, i.e. installing multiple sensors, actuators, and other components into the original system. Many measurements from an identical component are then compared for consistency. Any discrepancy among these measurements indicates a fault in that component. According to Frank (1990) [3], Gertler (1998) [4], Patton, Frank and Clark (1989) [5], hardware redundancy methods have significant disadvantages, though being theoretically simple, and also due to the requirement of many safety and reliability standards. The redundant hardware results in a more expensive and complex system. Also, the increased size and weight of the extra hardware is not always acceptable in some special environments, such as marine and aerospace applications. Due to these drawbacks, hardware redundancy is most used to evaluate sensor conditions to improve sensing and control robustness. Frank, Patton, Clark and Angeli claimed in [3][5][6] that hardware redundancy is not capable of detecting anomalies inside monitored process. However, detection and diagnosis of anomalies, such as a system internal fault, is very important to obtain a robust FTC system.

Figure 1.1 A three layer fault tolerant control architecture
To overcome the shortcomings of hardware redundancy, recent FDD development has focused mainly on model based techniques. Model based approaches rely on analytical redundancy, i.e. explicit mathematical models of the monitored system, as opposed to physical redundancy. The main idea is to check the consistency between system measurements and the computed values based on an analytical model. As claimed by Frank (1990) [3] and Kinnaert (2003) [7], by replacing the extra hardware with mathematical models, model based methods are advantageous in many aspects, such as cost, weight, size, etc. As analytical modeling provides insights to system dynamics, model based approaches can be used to detect not only sensor faults but also problems inside of the system. Early detection of a system internal fault can greatly improve system reliability by avoiding disastrous failures. With the development of advanced modeling techniques, model based methods with analytical redundancy are becoming an attractive approach to FDD for FTC.

Model based fault detection and diagnosis have been extensively explored for systems represented by linear models. However, most practical engineering systems are nonlinear in nature. When a nonlinear system is described by a linear model, the model can only be used about some particular linearization point. If the nonlinear system’s operating points deviate from the linearization points, the linear model can no longer represent the system accurately. Fault detection and diagnosis based on an inaccurate model may lead to wrong detection results. Hence, nonlinear models with high accuracy are preferable to represent systems with large dynamic range. The acquisition of accurate nonlinear system models is challenging, especially when taking the fault conditions into account. FDD based on nonlinear models is an area requiring further explorations.

While analytical redundancy is more capable and advantageous to hardware redundancy in many ways, this method brings its own issues. One of the most important problems is robustness to modeling errors. Though desired, it is unrealistic to expect perfect modeling for complex systems. The causes for errors are diverse. For instance, disturbances of an unknown structure, various noise effects, and uncertain or time varying system parameters can cause significant deviation in component output from that of its model. As asserted by Gertler and Patton, et al in [4] and [5], robustness has been the most important requirement for successful fault detection and diagnosis since the initial development of model based approaches.
1.1.2 PMSM for High Performance Applications, and Related FDD Problem

Among the numerous types of electric machinery, each offers its individual abilities that suit them well for different applications. PMSM is an important category of electric machines, in which the rotor magnetization is created by permanent magnets attached to the rotor. PMSMs are referred to as “synchronous” because their rotor magnetic fields rotate at the same speed (synchronous speed) as their stator fields. Based on rotor construction, poles of PMSM can be placed in either nonsalient or salient layout. Nonsalient pole motors have a cylindrical rotor and thus a uniform airgap. A simplified cross section of a cylindrical rotor is shown in Figure 1.2 (a). Salient motors have a varying airgap length. An illustration of a 4-pole salient pole rotor is shown in Figure 1.2 (b). Salient pole machines are more difficult to model than nonsalient ones. From a control design point of view, PMSMs are the simplest form of AC machines, as the torque output can be adjusted flexibly by varying the stator currents as argued by Lyshevski [8].

PMSMs are attractive for industrial applications. Their high power density, which is defined as the amount of output power for a unit weight (power ($\text{watts}$) / weight), is a distinct advantage over other types of electric machines. A motor with higher power density often leads to desirably more compact designs. Furthermore, PMSMs are more advantageous over other electric motors in efficiency and robustness [9][10]. Even so, high power PMSM applications were not widespread.
until recently. This is due to availability, cost, and manufacturability of permanent magnet materials. With the development of the new magnet material Ndeodymium-Iron-Boron (Nd-Fe-B) in the 1980’s, there is an increasing trend in applications of high powered PMSMs, where high performance systems put higher priorities to power density and efficiency. In these applications, PMSM becomes a superior choice. For instance, the US Navy has listed PMSM among the most promising candidates for military ship based propulsion systems [10]. Figure 1.3 shows the structure where PMSM are used as ship propulsion motors.

High performance systems usually have demanding requirements on availability, reliability and survivability. Again, take the shipboard system application as an example. Designs of military ships need to consider frequent operations under hostile environments. Healthy and safe operations of devices onboard are extremely important because any unexpected fault or shutdown may result in loss of a ship and human lives. As a key component in the ship propulsion system, as shown in Figure 1.3, the reliable operation of PMSM is a primary concern for the entire ship system. FDD of PMSM is critical for the improvement of system reliability. It has become a very active area of research and development throughout the world [9][10][11].

Figure 1.3 Typical electric ship propulsion scheme
FDD of PMSM is a challenging problem and has not been fully explored. Literature shows there is still a gap between FDD theory and applications. This dissertation makes an effort to fill the gap by developing FDD algorithms for PMSM under practical operating conditions. The main contributions of the dissertation will be spelled out in section 1.2.

1.2 Contributions

According to the previous discussions, effective fault detection and diagnosis is very important for PMSMs’ high performance applications. PMSMs’ FDD with nonlinear models has not been fully explored. The objective of this dissertation is to develop a new FDD scheme for PMSM used in high performance applications that can accurately detect faults in real time and provide useful information about the detected faults. For this purpose, this dissertation developed a robust FDD scheme by integrating the proposed nonlinear model based FDD algorithms for PMSM, which can improve system’s reliability and availability by effective detection and diagnosis of sensor and internal faults. The robust FDD scheme can be summarized in Figure 1.4. This system consists of three modules, i.e. sensor fault detection, internal fault detection, and fault diagnosis. Three nonlinear model based algorithms are designed for these three modules respectively. The integration of the three function modules constitutes a robust FDD system structure. According to this FDD structure, specific contributions of this dissertation are summarized as follows:

- Propose a new nonlinear ordinary differential equation model in $abc$-phase frame using the magnetic circuit principle. The model is applicable to PMSM operating under both normal conditions and asymmetrical stator winding short fault conditions. The proposed model is used in later fault diagnosis.
- Develop an algorithm to detect sensor faults based on nonlinear parity relations. Disturbance decoupling is taken into consideration in the design. The algorithm can be used to detect and isolate different sensor faults.
- Propose a nonlinear synchronization based adaptive parameter estimation approach for PMSM. The identification can identify internal changes of PMSM. Experiment is successfully performed to verify this estimation algorithm.
Propose a diagnosis approach based on Particle Swarm Optimization for the diagnosis of PMSM stator winding fault. Both the fault location and fault severity can be identified with this intelligent diagnosis approach. The performances of the PSO algorithm in this application are demonstrated.

Develop a robust FDD system structure, which integrates the FDD algorithms proposed in this dissertation. This FDD system deals with sensor fault detection, internal fault detection, and fault diagnosis in one framework with improved robustness to system parametric uncertainties.

The contents in the above contributions provide an integrated architecture for PMSM FDD based on nonlinear system models as illustrated in Figure 1.4. Such a FDD system can successfully detect abnormal components at the early stage and perform diagnosis on PMSM internal anomalies. With this FDD system, PMSM’s reliability can be greatly improved. It is thus suitable for high performance applications.

1.3 Dissertation Outline

The rest of this dissertation is organized as follows:
**Chapter 2** provides a state-of-art review on model based fault detection and diagnosis methods, as well as their applications to electric machinery. Key issues are addressed about model based approaches’ practical applications.

**Chapter 3** proposes a versatile nonlinear model for PMSM operating under healthy and asymmetrical stator winding fault conditions. Simulation results demonstrate the effectiveness of the proposed model. The model is used later for fault diagnosis purpose.

**Chapter 4** presents two fault detection algorithms for PMSM. The first algorithm is used to detect sensor faults and the second is used to detect system internal changes. In addition to simulation studies, experiments are also performed to evaluate the parameter estimation algorithm.

**Chapter 5** introduces a fault diagnosis algorithm for PMSM stator winding turn-to-turn short fault. This is a subsequent step after an internal fault is detected. The fault diagnosis problem is formulated as a general nonlinear parameter identification problem. Then, intelligent optimization algorithm, PSO, is proposed to perform this diagnosis. Simulation results demonstrate the effectiveness of the proposed methods. Experimental results are provided for a healthy PMSM identification using PSO algorithm.

**Chapter 6** proposes an integrated FDD structure by synthesizing the algorithms proposed in Chapter 4 and Chapter 5. This FDD system has enhanced robustness to system parametric uncertainties.

**Chapter 7** summarizes the whole dissertation. Concluding remarks and recommendations for future work are presented.
CHAPTER 2

FAULT DETECTION AND DIAGNOSIS

This chapter first gives a brief introduction to some commonly used concepts and nomenclature in fault detection and diagnosis (FDD). Then, literature reviews are presented on the theories and applications of model based FDD, especially on its application to electric motors. This chapter ends with the challenging problems in FDD applications being summarized. The objective of this work is to develop a set of solutions to these problems.

2.1 Overview

2.1.1 Introduction

Since the emergence of machines, humans have been concerned about these machines’ conditions, such as availability and reliability. The primitive FDD were based on observations of machines’ operating conditions. Later on, measuring devices were introduced that provided more accurate information about important variables. But these devices were also prone to malfunction. Systematic development of automatic FDD did not come until the widespread applications of computers. Numerous methods for FDD have been published. However, there is still a large gap between the established theories and practical applications, especially for the FDD of nonlinear systems.

As defined by Iserman and Ulieru (1993), the main task of FDD can be subdivided into two parts [12]. First, to detect whether a fault appears in the system by analytical symptom generation; and second to further diagnose the fault based on the observed analytical symptoms. Fault diagnosis usually consists of identifying the fault’s type, location, and size. Just “What is fault diagnosis?” has not been standardized. For instance, Gertler (1998) defined the task of fault
diagnosis as fault isolation and fault identification. Therein, *fault isolation* specifies the sources associated with the detected fault, *fault identification* determines the size and general characteristics of that fault [4]. These terms are consistent with Isermann *et al* defined in [12]. In this dissertation, fault isolation and fault identification are integrated together as fault diagnosis. Although definitions are clearly given for different functions, the tasks of fault detection and fault diagnosis are interconnected to each other. In simple cases, a good design of symptoms (residuals) generation for fault detection purpose can greatly facilitate fault diagnosis in later stages.

A typical FDD framework in applications is shown in Figure 2.1. In this diagram, the operating status of the system is monitored by sensors. Components within this system are susceptible to various faults, disturbances, and noises. A FDD module takes measured data to detect and diagnose the faults. The diagnosis results are then fed back to the control system for corrective actions, such as shutdown for on demand maintenance, continuous operation with automatic reconfiguration, etc. Such a control system with effective FDD design can greatly improve a system’s reliability and availability.

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**Figure 2.1 Block diagram illustration of FDD**
2.1.2 FDD Terminologies

Before giving more details on FDD theories, it is helpful to introduce other important definitions for the technical terms used in this field. These terms are defined by the FDD initiators, e.g. Gertler, Patton, Frank, Clark, Isermann, etc., in their published books and some well recognized papers, see [4][5][13][14][15][16] for references.

A fault is defined as “an unpermitted deviation of at least one characteristic property of a variable from an acceptable behavior” according to Isermann [14]. Faults in a system may lead to degraded performance, malfunctions, or failures. Different from fault, the consequences of a failure are usually more serious, such as partial or complete system breakdown.

Based on its severity and timeliness, a fault can be classified into two types. A hard fault, sometimes called abrupt fault, means an abrupt complete loss in sensing signals or abrupt changes of other type functions. During simulation, hard faults are usually simulated with a step change in inputs or system parameters. A soft fault, sometimes called incipient fault, usually refers to slow changes of system’s parameters or drift of measured data caused by sensors problems. Incipient faults may indicate potential malfunctions. Considering the relatively small magnitude and slowly time varying properties, soft faults are more difficult to detect than hard faults.

Based on a fault’s influence on system models, faults can be categorized into two types, additive faults and multiplicative faults. According to Figure 2.2, an additive fault influences a variable \( Y \) by a simple addition of the fault \( f \). A multiplicative fault has more complicated impact upon a system by the product of the variable \( U \) with \( f \). The most typical additive faults are sensor offsets, whereas multiplicative faults are parameter changes within a system.

![Figure 2.2 Fault types [14]](image-url)
In medicine, *symptom* means a sign or an indication of disorder or disease. The same word is utilized in FDD to represent the characteristics resulting from a fault in the system. Symptoms of a fault are usually reflected as deviations of a signal from its normal value. In FDD, fault symptoms are usually evaluated by *residuals*. By definition, residual is a fault indicator constructed based on deviation between a system’s actual and its normal behaviors. Residuals caused by different faults need to be distinguishable from each other. Besides faults, many other factors, such as system operation changes, disturbances, sensor noises, etc., may also exert influence to residuals. These influences are undesirable because they may result in false alarms. False alarms can be avoided by *robust* FDD design. Since the research on FDD was initiated, robustness has been an important goal.

### 2.2 Model Based Fault Detection and Diagnosis

Study of model based FDD began in 1970’s. FDD is generally considered to have entered its modern era with the publications of Mehra and Peschon (1971), as well as Lainiotis and Park (1973) [17][18]. In these papers, the proposed FDD algorithms were applied to chemical plants and airplanes. In 1976, Willsky wrote the first major survey paper on model based FDD [19], especially on linear time invariant (LTI) systems. After that, Isermann reviewed FDD design based on modeling and estimation method in 1984 [20]. In 1988, Gertler summarized the main features of model based FDD and presented the robustness and sensitivity considerations in [21]. Meanwhile, Basseville presented both online and offline FDD algorithms in his survey paper with a particular emphasis on statistical methods for detection [22]. With the development of Artificial Intelligence (AI), a lot of AI based FDD designs were proposed. This topic was surveyed by Frank in 1990 [3]. In 1997, Frank and Köppen-Seliger did a followup survey on AI based FDD in [23]. This 1997 paper focused on fuzzy logic and introduced the concept of knowledge observer. In the mid 1990s, some nonlinear model based FDD methods were proposed, a survey on which was made by García and Frank [24].

In general, a model based FDD system consists of two modules: residual generator and residual evaluator, as shown in Figure 2.3. In this diagram, the residual generator uses system measurements as input to generate residuals as fault symptoms. Then, these residuals are sent to the residual evaluator to make diagnosis decisions.
According to selected FDD references, e.g. Gertler (1998) [4] and Patton, Frank and Clark (1989) [5], etc., in an effective FDD system, residuals should be designed insensitive to disturbances, noise, and other uncertainties, while remaining sensitive to faults. Therefore, an ideal residual should follow (2.1) for fault detection purpose:

\[ r(t) = 0 \text{ as } f(t) = 0 \text{ and } r(t) \neq 0 \text{ as } f(t) \neq 0 \]  

(2.1)

where \( f(t) \) represents a fault, and \( r(t) \) represents the residual designed for the detection of that fault. Equation (2.1) indicates that \( r(t) \) should be zero when the fault \( f(t) \) does not occur, and be nonzero otherwise.

In addition, to distinguish different faults, a set of residuals must be designed with each residual responding only to a specific subset of faults. This design is usually called \textit{structured residual}. Gertler introduced the design of structured residuals for LTI systems in [4]. For example, when a number of \( n \) residuals are designed to detect \( n \) faults in a system, a well structured residual set can be designed as (2.2).

\[ r_i(t) = 0 \text{ as } f_i(t) = 0 \text{ and } r_i(t) \neq 0 \text{ as } f_i(t) \neq 0 \text{ with } i = 1, 2, \ldots, n \]  

(2.2)

In (2.2), \( f_i(t) \) denotes the \( i^{th} \) fault, and \( r_i(t) \) denotes the \( i^{th} \) residual. This design means that residual \( r_i(t) \) only responds to \( f_i(t) \), and remain zero as long as this fault does not occur.
Residual generator plays an important role in model based FDD. In the past thirty years, many algorithms have been proposed for residual generation. These algorithms can usually be classified into three categories, which are parity relations, diagnostic observers, and parameter estimations. A review of these methods is given in the following subsections.

### 2.2.1 Fault detection with parity relations

Parity relation sometimes is also called consistency relation, or analytical redundancy relation. This approach was first introduced by Frank [3] and Geltler [4]. Parity relations directly check the consistency between the model and measured system outputs, thus is the most intuitive fault detection method. The main idea of this method is shown in Figure 2.4. $G_P$ represents the monitored system, and $G_M$ is the analytical model of that system. Residual $r$ based on parity relations can then be mathematically expressed in (2.3) with $u$ denoting the control signal input.

$$r = (G_P - G_M)u$$  \hspace{1cm} (2.3)

While being the most computationally efficient and simplest in implementation, the parity relation based approach assumes that the model parameters are known and constant. Thus, residuals based on this method are susceptible to model inaccuracies due to system uncertainties. To overcome the problem of imprecise models, robust parity relations were introduced in the mid 1980s, by Chow and Willsky [13], Lou, Willsky and Verghese [25].

![Figure 2.4 Parity relation based residual generation](image-url)
In [16], Isermann claimed that parity relation is the most straightforward way to detect additive faults. This method is very sensitive to abrupt additive faults. However, the method has difficulty identifying parameter changes caused by multiplicative faults. Since its computational effort is relatively small, parity relation is the predominant FDD algorithm for additive faults. Some representative work can be found in [26][27][28].

2.2.2 Fault detection with diagnostic observers

If a system is observable, observers can be used to estimate states of a system using measured data. The estimated states can then be used to observe the system outputs. Diagnostic observers utilize the errors between estimated and measured outputs as residuals. The schematic diagram of this approach is shown in Figure 2.5. In this figure, $y_e$ denotes the estimated system output, $y$ denotes the measured system output, and $r$ denotes the residual, which can be calculated according to (2.4).

$$ r = y - y_e $$

(2.4)

Diagnostic observer design takes advantages of widely used observer designs. This method is especially useful when dealing with the cases where states or outputs are not measurable. Similar to parity relation method, diagnostic observers also require accurate knowledge of system model.

![Figure 2.5 Diagnostic observer](image)

Figure 2.5 Diagnostic observer
parameters. The relationship between diagnostic observers and parity relation based designs was discussed by Mgni and Mouyon for linear systems [29]. Since accurate system models are often unavailable for practical systems, robust designs are important for diagnostic observers. Some related work can be found in [17][30][31][32].

2.2.3 Fault detection with parameter estimation

Both the parity relations and the diagnostic observers require an accurate model of the system. For some problems, the parameters of a system model may normally vary continuously in certain ranges. Since the above two methods cannot handle this problem, parameter estimation based fault detection is used for FDD. Parameter estimation is proved to be advantageous in this regard according to Gertler, Hofling, and Isermann [33][34]. Parameter estimation methods only require the structure of the system model to be known. The parameters of the model are estimated and compared to their values during normal operating conditions. This parameter estimation based fault detection method is illustrated in Figure 2.6. In this figure, $p_e$ represents the estimated parameter, $p$ represents $p_e$’s nominal value, and $r$ represents the residual as calculated in (2.5):

$$r = p - p_e$$ (2.5)

In [35], Isermann (1991) states that parameter estimation provides deeper insights into system dynamics and thus makes fault diagnosis easier. In many cases, changes in model parameters are directly related to system internal faults. Thus, the parameter estimation approach is especially suitable for the detection of the multiplicative faults according to Gertler (1998) and Isermann (1984, 2005) [4][14][20]. Also, parameter estimation allows the detection of very small changes of the system conditions, including slowly developing faults. In [33][34], Gertler (1995), Hofling and Isermann (1995) proved the mathematical equivalence between the parameter estimation and the parity relation with an accurate model. A disadvantage of this method is that a Persistent Excitation (PE) condition is usually required.
Since these three residual generation approaches have their pros and cons, a combination of two or more may provide a better solution.

### 2.2.4 Application Issues of Model Based FDD

The above methods are relatively mature for linear systems but not for nonlinear systems. Since practical systems are usually nonlinear in nature, the FDD techniques developed for linear systems cannot be extended to nonlinear systems directly. Linearization may be a possible way to apply linear system based FDD algorithms to nonlinear systems. But linearization of nonlinear models will lead to modeling errors since the linearized model is only valid around some linearization point [4]. If the system is operating away from the linearization point, the performance of the linear system based FDD algorithm cannot be guaranteed.

In recent FDD papers, researchers tend to utilize qualitative, instead of quantitative models in their study on nonlinear model based FDD. Representative papers in this aspect are found, e.g. Vinson, *et al* (1992) [36], Glass, *et al* (1995) [37], Goode, *et al* (1995) [38], etc. Development of quantitative models usually requires explicit description of systems’ dynamic behaviors. It is a difficult task especially for complex nonlinear systems. To avoid this problem, qualitative models
are used to describe system variables with a finite number of modalities and sets of rules, often fuzzy ones, as summarized by Staroswiecki (2000) [39]. Qualitative models can be built upon relatively simple understanding about a system, together with expert knowledge or experience. Techniques used in qualitative models based FDD include fuzzy logic, expert systems, statistical approaches, and some other AI algorithms. Venkatasubramanian wrote a good review on this subject in 2003 [40]. Qualitative models are generally an incomplete or fuzzy description of the actual system. The lack of accuracy makes the detection of small incipient faults difficult. This is a major concern for high performance applications. To improve FDD system performance especially in high performance applications, quantitative model based FDD techniques are still necessary.

FDD techniques based on quantitative models usually provide more accurate diagnosis but are very sensitive to modeling errors. Modeling errors may lead to false alarms. A simple way to decrease the false alarm rate is to increase the threshold for fault decision making. However, raising the threshold hinders the detection of small incipient faults. To improve robustness of FDD designs, several approaches have been proposed. These methods can be broadly classified into two categories, i.e. active and passive approaches. Active approaches are applied to the residual generator. The objective is to minimize the influence of the model impreciseness on residuals [4]. Passive approaches are applied to the residual evaluator. The objective is to minimize false alarms by using proper decision making logic from the noised residual [41].

Summarizing this section, a good FDD design should address both of the above two problems, i.e. system nonlinearity and robustness.

2.3 Fault Detection and Diagnosis for Electric Motors

Although many FDD algorithms have been proposed in the past decades, they are still inadequate to fulfill the needs in practical applications. Reliability has been the main concern of industries and special application areas, such as aerospace and military applications. Being key components in these applications, electric machines’ operating conditions are important to the whole systems’ reliability. As electric machines are becoming increasingly complicated, the FDD for electric machines are becoming even more difficult. There is a need for reliable FDD techniques for electric machines. The research in this area has attracted a lot of attention.
This section presents a review on FDD applications to electric machines. The needs for advanced FDD algorithms and the lack of FDD research on PMSM motivate the work presented in this dissertation.

2.3.1 Common Fault Modes of Electric Motors

There has been a need for FDD since the introduction of electric machines [42], because electric machines are frequently exposed to nonideal or even detrimental operating environments [43]. These circumstances include overload, insufficient lubrication, frequent motor starts/stops, inadequate cooling, etc. Under these conditions, electric motors are subjected to undesirable stresses, which put the motors under risk of faults or failures [44][45]. Due to the prevalence of electric motors and their significant positions in applications, many efforts have been made to improve a motors’ reliability. According to IEEE Standard 493-1997 [46], the most common faults and their statistical occurrences are listed in Table 1. This table is based on a survey on various motors in industrial applications. According to the table, most faults happen to bearings and windings. A 1985 statistical study by the Electric Power Research Institute (EPRI) provides similar results, i.e., bearing (41%), stator (37%), rotor (10%) and other (12%) [47].

<table>
<thead>
<tr>
<th></th>
<th>Number of Faults / Failures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Induction motor</td>
</tr>
<tr>
<td>Bearing</td>
<td>152</td>
</tr>
<tr>
<td>Windings</td>
<td>75</td>
</tr>
<tr>
<td>Rotor</td>
<td>8</td>
</tr>
<tr>
<td>Shaft</td>
<td>19</td>
</tr>
<tr>
<td>Brushes or slip rings</td>
<td>-</td>
</tr>
<tr>
<td>External devices</td>
<td>40</td>
</tr>
<tr>
<td>Others</td>
<td>10</td>
</tr>
</tbody>
</table>
The common internal fault modes in Table 2.1 can be mainly categorized into two groups: mechanical faults and electrical faults. Mechanical faults include bearing faults, air gap eccentricity, and faults caused by overheating, or cooling problems. Electrical faults include faults caused by winding insulation problems, and some rotor faults. Sensors are important components for a motors’ operation. As external devices, sensor faults present an important challenge in motor FDD investigation. Details about these faults are discussed as follows.

- **Mechanical faults**
  - *Bearing faults*: Bearings play important roles in all types of rotating machines. Bearing problems account for over 40% of all the fault modes. In many cases, the accuracy of the motor monitoring and control instruments is highly dependent on performance of motor bearings. Since bearing faults usually generate excessive vibrations, an undetected incipient bearing fault can lead to incorrect motor operation, extra maintenance downtime and cost, etc. The commonly used bearing fault detection methods are based on vibration, and stator current monitoring. In this regard, good references can be found in [48][49][50] by Schoen, Habetler, Eren, Devaney et al.

  - *Air gap eccentricity*: Machine eccentricity is defined as “the condition of unequal air gap that exists between the stator and rotor” according to Vas [51]. It is another important type of mechanical faults. Air gap eccentricity can be static or dynamic, which tend to coexist. Certain level of eccentricity, e.g. 10% [52], is acceptable, however, it can reach beyond this a certain level after long time operation. The mostly used detection approaches for this type of faults are stator core vibration and the stator current monitoring [52][53].

  - *Overheating/cooling problems*: The most common cause of overheating is improper ventilation or high ambient temperature. Additionally, overheating can also be a side effect of overcurrent operation. The maximum permitted temperature for ordinary industrial motors is about 40°C. Extremely high temperature operation is very detrimental to motor performance [54]. Diagnosis of the cause of motor overheating is more significant than the temperature monitoring itself.
Electrical faults:

- **Faults caused by winding insulation problems:** Winding faults (mainly in stator windings) are ranked the second most frequently occurring faults according to motor reliability surveys. As Nandi and Toliyat said in their review, winding faults usually relate to insulation breakdowns, which can be caused by moist, hot environments, leakage in cooling systems, electrical discharge, etc. [44]. Though starting with incipient turn-to-turn short, undetected small insulation failures can deteriorate and culminate rapidly. This may eventually lead to phase loss, phase to phase or phase to ground faults [44][55]. The popular tool for stator winding fault diagnosis is the stator current analysis in frequency domain.

- **Rotor faults** (Flux disturbance): For induction machines, the rotor fault mainly indicates broken bars, whereas for permanent magnet machines, it means magnet defects. These faults can be the results of pulsating load, or certain manufacturing problems. Rotor faults cause motor speed and torque fluctuation, vibrations, abnormal noises, etc. Vibration monitoring, and stator current analysis can be applied to detect the rotor faults.

Sensor faults

The fault modes discussed above are the most common problems occurring inside of machines. External components or devices are also susceptible to faults. Sensors are important external devices that are used to monitor the operating condition of motors for control and fault diagnosis purposes. Usually, the possibility of sensor faults is similar to that of the machines’ internal faults. Faults can happen to current sensors, voltage sensors, rotor speed and position encoders, torque transducers, and temperature sensors, etc. Detection of sensor faults is necessary and should be the first step in FDD system.

2.3.2 Variety of FDD Techniques for Electric Motors

FDD methods have been proposed for different types of motor faults, such as bearing faults, stator winding faults, rotor faults, and air gap eccentricity. These methods include motor current signature analysis (MCSA), vibration analysis, axial flux analysis, partial discharge analysis, and model based FDD. For each fault mode, the available FDD techniques are listed in Table 2.2. In
Table 2.2  Various motor FDD techniques and their applicability

<table>
<thead>
<tr>
<th></th>
<th>Bearing</th>
<th>Stator Winding</th>
<th>Rotor</th>
<th>Airgap Eccentricity</th>
<th>External (Sensors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor current signature analysis</td>
<td>✓</td>
<td>Partial</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Vibration analysis</td>
<td>✓</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Axial flux</td>
<td>×</td>
<td>Partial</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Partial discharge</td>
<td>×</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Model based FDD</td>
<td>?</td>
<td>✓</td>
<td>?</td>
<td>?</td>
<td>✓</td>
</tr>
</tbody>
</table>

this table, ‘✓’ denotes a technique listed in the first column can be used to detect the corresponding fault in the first row; ‘×’ denotes a technique cannot be used to detect that fault; ‘Partial’ denotes a technique can only detect part of the corresponding fault; and ‘?’ denotes a technique has the potential to detect the corresponding fault but has not been fully investigated. Table 2.2 is summarized based on selected references [48]-[68].

*Motor current signature analysis* (MCSA) is a widely used technique, which analyzes motor current waveforms using signal processing algorithms (FFT, wavelet etc.) and other AI techniques. Different currents are used for MCSA, such as sequence components, shaft currents, radio frequency components of neutral current, etc. [48][49][56][57][58]. Properly designed MCSA can provide early warning of machine deteriorations. However, the MCSA approach is mathematically complex, expensive to realize, and requires too much historical data.

*Vibration analysis* has been applied for the diagnosis of electric motors for decades. Theoretically, every fault in a rotating machine generates vibrations with different characteristics. So, sampled vibration signals can be used to compare with reference patterns to perform FDD. In practice, vibration analysis has been used for the detection of various mechanical faults, and some unbalanced electrical faults [50][53][59]. Riley, Lin, and Habetler, et al. noticed the linear relationship between current harmonics and vibration level [60]. Vibration analysis has the disadvantages of requiring a large amount of data and excessive sensors, being costly and susceptible to errors.

*Axial fluxes* can reflect any unbalance in the magnetic circuit [61]. Many methods, such as spectrum analysis, have been proposed for axial fluxes analysis to diagnose motor faults. This
technique can be utilized to detect electrical anomalies, e.g. rotor defects, unbalanced voltage supply, asymmetrical stator faults, etc. [62][63]. It is difficult to measure axial fluxes accurately. Special care is required to obtain reliable data.

*Partial discharges* (PD) are small electrical sparks that occur within the electric devices. Partial Discharge Analysis uses PD measurements to evaluate the integrity of this equipment. This approach is mostly used for the detection of winding insulation problems for large motors or generators. It is relatively cheap to implement PD analysis. However, Stone and Kapler claim in [64] that no single PD technique will work for all types of machines. Different types of sensor systems are necessary for different motors [65].

Apart from the above techniques, there are many other ways to perform FDD for electric motors, such as thermal monitoring, gas in oil analysis, motor circuit analysis, etc. These are beyond the scope of this dissertation, which focuses on FDD of PMSM stator winding short faults.

*Model based FDD* techniques perform fault detection based on system models. A good model usually means a good understanding of the system’s behavior, so model based FDD techniques are able to handle various faults, even those that cannot be detected using the above techniques. It is especially useful for the detection of small incipient faults and sensor faults. Although the initial model acquisition process is painful, it is worthy to do that considering the advantages of this method. Some papers have been published in this area, such as [57][66][67][68]. All of these papers are proposed for some specific electric machines types and specific faults. There is still a lack of research on the model based FDD of PMSM.

### 2.3.3 Model based FDD for PMSM vs. Other Motor Types

Table 2.3 summarizes some recently published papers on the model based FDD applied to electric machines. The success of model based FDD relies on the accuracy of system models. Besides the issue of model accuracy, the simulation of various fault conditions is also a major concern for model based FDD. In Table 2.3, the listed papers are also classified according to their applications with brief descriptions about their objectives and methods.
Table 2.3 Recent publications on model based FDD applied to electric motors

<table>
<thead>
<tr>
<th>REFERENCES (TIME)</th>
<th>OBJECTIVES / METHODS</th>
<th>APPLICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[70] (2004)</td>
<td>Equivalent circuit analysis for stator faults</td>
<td></td>
</tr>
<tr>
<td>[71] (2005)</td>
<td>Equivalent circuit analysis for rotor asymmetries</td>
<td></td>
</tr>
<tr>
<td>[72] (1995)</td>
<td>Knowledge-based FDD for various faults</td>
<td></td>
</tr>
<tr>
<td>[74] (2003)</td>
<td>Simulation and statistical approach for the detection of stator insulation faults</td>
<td></td>
</tr>
<tr>
<td>[75] (2005)</td>
<td>Coupled circuit modeling for rotor faults and estimation method for detection</td>
<td></td>
</tr>
<tr>
<td>[76] (1997)</td>
<td>Adaptive residual shaping for detection of additive and multiplicative faults</td>
<td></td>
</tr>
<tr>
<td>[77] (2005)</td>
<td>H∞ design for robust fault detection</td>
<td></td>
</tr>
<tr>
<td>[78] (1995)</td>
<td>FEM analysis, magnetic circuit model for rotor fault modeling</td>
<td>Reluctance machines</td>
</tr>
<tr>
<td>[66] (2000)</td>
<td>Parameter identification approach for the detection of cooling and lubrication problem</td>
<td></td>
</tr>
<tr>
<td>[79] (2005)</td>
<td>Stator fault simulations using FEM analysis</td>
<td>Brushless DC motors</td>
</tr>
<tr>
<td>[80][81][82]</td>
<td>Adaptive neuro-fuzzy inference systems (ANFIS) to diagnosis various faults based on lumped-parameter network model</td>
<td></td>
</tr>
<tr>
<td>(2002,2005)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Literature reveals that there are a very limited number of publications on the model based FDD for PMSM compared to other machine types. One reason for this is that the cost in production limited the application of PMSM. The advent of new magnetic materials and advancements of manufacturing techniques already has recently made PMSM affordable. Considering the advantages of PMSM, a trend of increasing PMSM’s application can be foreseen. Thus, research
on nonlinear model based FDD for the PMSM has become more important.

Existing publications for PMSM are mostly about abrupt faults on switching devices in motor drive systems. Analysis of PMSM operating under both normal and fault conditions, and FDD on PMSM’s internal faults have not been significantly explored. These are the problems to be addressed in this dissertation.

2.4 Summary

This chapter provides a literature review of FDD theory and application to electric motors. At first, basic concepts and general FDD procedures are introduced. Then, established fault detection methods, i.e. parity relations, diagnostic observers, and parameter estimation, are briefly reviewed and compared based on their advantages and disadvantages. After that, the main problems in FDD applications are summarized and a review is given on FDD’s application to electric motors. In the end, the review points out the need for advanced model based FDD algorithms for PMSM in high performance.

The first task to be done in model based FDD is to develop a suitable fault model. A good fault model not only can be used to detect faults but also can be used to identify the exact location and severity of the fault in the following fault diagnosis. In the next chapter, a fault model will be proposed for the detection of a stator winding fault in PMSMs.
CHAPTER 3

PMSM MODELING UNDER NORMAL AND STATOR FAULT CONDITIONS

For model based approaches, effective fault detection and diagnosis (FDD) relies on successful distinction of system behaviors under various operating modes. Fault detection is used to judge whether a fault happens or not, and fault diagnosis is used to obtain detailed information of the fault based on further analysis of the fault symptoms. Both of the two tasks rely on analytical models of the monitored system. Acquisition of an appropriate PMSM model is the first step of model based FDD. System modeling requires indepth analysis of system dynamics. This analysis usually provides insights on those properties that may be used as fault symptoms. The modeling process provides understanding of system operations and thus helps the selection of the most suitable FDD methods. To design a FDD system, two types of system models need to be developed. A healthy model represents system dynamics under normal operating conditions. Deviations of system performance from the performance described by the healthy model indicate the existence of faults. Therefore, the healthy model can be used to detect anomalies in the system. In order to perform fault diagnosis for an already detected fault, i.e. fault location and fault severity (size), a fault model is needed. This model should take the fault effects into account and properly describe system dynamics under these faults. Fault modeling and the corresponding fault analysis present challenging problems in the model based FDD.

This chapter first gives a review of the existing PMSM models under healthy operation conditions. In this work, two models are reviewed: one in $abc$-phase frame and the other in the rotating $dq$-reference frame. The three-dimensional AC model can be transformed into two-dimensional $dq$-domain using Park’s transformation as explained in section 3.1.2. Then, a new model is proposed for PMSM operating with stator winding short fault. The models’ dynamics of
both \(dq\)-axis healthy model and the \(abc\)-phase fault model are compared by MATLAB simulations to show the symptoms under stator winding fault conditions. In later chapters, the healthy model is used to detect a fault, and the fault model is used to perform further diagnosis.

### 3.1 PMSM Models for Healthy Operations

Since the introduction of PMSM, many models have been proposed for different applications. Initially, the models were developed to optimize the designs of PMSM [8]. Later, models appeared that were used to design motor control systems [86][87]. Recently, several new models were proposed to describe PMSM fault condition operations for FDD [76][79]. The above models are different in assumptions, fundamental principles, computational complexity, and modeling accuracy, etc. Differences in these aspects result in different applicability of different models. Due to the simplicity of the two axis \(dq\)-model, it becomes the most widely used model in PMSM engineering controller design. The \(dq\)-model offers significant convenience for control system design by transforming stationary symmetrical AC variables to DC ones in a rotating reference frame. This advantage can also be used in a fault detector design for PMSM. The fault detector is a key component of a FDD system.

This section starts with a review of the healthy PMSM model in \(abc\)-phase frame based on the fundamental principles. This review helps to understand the PMSM fault model developed in later sections. Using Park’s transformation, the \(abc\)-phase model can be transformed into the \(dq\)-model mentioned above. The \(dq\)-model’s structural similarity to the chaotic Lorenz system is illustrated with simulation results.

#### 3.1.1 PMSM Modeling in \(abc\)-Phase Frame

PMSM are brushless machines with sinusoidally distributed stator windings. The excitation flux of PMSM is produced by the permanent magnet rotor. Figure 3.1 and Kirchhoff’s law are used to develop the electric model of the PMSM seen in (3.1). The following assumptions are made in developing this model:

- The magnetic permeability of iron is considered to be infinite.
- The operation is far from magnetic saturation.
• The magnetic motive force and the flux profiles are considered sinusoidally distributed.
• Higher harmonics are neglected.

These assumptions are commonly acknowledged as appropriate for a lumped parameter model of the electric motors, as found in the electric motor references by Lyshevski [8] and Krishnan [86]. Figure 3.1 is a schematic illustration of a three-phase two-pole PMSM.

In the above figure, symbols \( v_s \) and \( i_s \) represent terminal voltages and currents in each phase; \( R_s \) and \( L_s \) represent the stator resistance and inductance respectively, \( N_s \) is the turn numbers of each stator winding.

Without magnetic saturation, the electrical model of the three-phase PMSM is obtained in its \( abc \)-phase frame as (3.1):

\[
v_{abc}^s = R_{abc}^s i_{abc}^s + \frac{d\Psi_{abc}^s}{dt}
\]  

(3.1)
In (3.1), \( \Psi^s \) denotes stator magnetic fluxes for the three phases. For conciseness, matrix expressions are used to denote three-phase variables, i.e.

\[
\begin{bmatrix}
\Psi^s_{abc}
\end{bmatrix} = \begin{bmatrix}
\Psi^s_a \\
\Psi^s_b \\
\Psi^s_c
\end{bmatrix}
\]

\[
R^s_{abc} = \begin{bmatrix}
R^s_a & 0 & 0 \\
0 & R^s_b & 0 \\
0 & 0 & R^s_c
\end{bmatrix}
\]

\[
i^s_{abc} = \begin{bmatrix}
i^s_a \\
i^s_b \\
i^s_c
\end{bmatrix}^T
\]

The model in (3.1) is based on the magnetic circuit of the motor in Figure 3.1. The stator phase voltages are composed of two parts: a resistive part representing the voltage drops across the stator resistance, and a magnetic part resulting from the changing of the stator magnetic flux linkage. This model generally applies to PMSMs with both symmetrical and asymmetrical phase windings. The winding asymmetry can be represented by variations in resistances matrix and inductances related to the magnetic flux.

For PMSM, the magnetic flux \( \Psi^s_{abc} \) in (3.2) are generated by two different sources: one from the flux created in the stator self inductances and the other from the flux created by the permanent magnet \( \Psi^s_{mabc} \) on the rotor side. Thus, \( \Psi^s_{abc} \) can be calculated according to the following equation.

\[
\Psi^s_{abc} = L_s i^s_{abc} + \Psi^s_{mabc}
\]

Stator inductance \( L_s \) in (3.3) is a 3x3 symmetric matrix as defined in (3.4). The diagonal elements are the self inductances of each winding, and the off diagonal elements are the mutual inductances between different phase windings.

\[
L_s = \begin{bmatrix}
L_{aa} & L_{ab} & L_{ac} \\
L_{ba} & L_{bb} & L_{bc} \\
L_{ca} & L_{cb} & L_{cc}
\end{bmatrix}
\]
expressed as in (3.5) [8][87]:

\[ L_{ia} = L_{ls} + \overline{L}_m - L_{\Delta m} \cos(2\theta_r) \]
\[ L_{bb} = L_{ls} + \overline{L}_m - L_{\Delta m} \cos \left( \frac{3}{2} \theta_r + \frac{2}{3} \pi \right) \]
\[ L_{cc} = L_{ls} + \overline{L}_m - L_{\Delta m} \cos \left( \frac{3}{2} \theta_r - \frac{2}{3} \pi \right) \]
\[ L_{ab} = L_{ba} = -\frac{1}{2} \overline{L}_m - L_{\Delta m} \cos \left( \frac{3}{2} \theta_r + \frac{2}{3} \pi \right) \]
\[ L_{ac} = L_{ca} = -\frac{1}{2} \overline{L}_m - L_{\Delta m} \cos \left( \frac{3}{2} \theta_r - \frac{2}{3} \pi \right) \]
\[ L_{bc} = L_{cb} = -\frac{1}{2} \overline{L}_m - L_{\Delta m} \cos(2\theta_r) \]

(3.5)

In (3.5), \( L_{ls} \), \( \overline{L}_m \) and \( L_{\Delta m} \) are positive constants denoting the stator leakage inductance, the average value of the winding magnetizing inductance, and the magnitude of the inductance variation due to the non-uniformity of the air gap, respectively. The equations in (3.5) are highly nonlinear, which may complicate the numerical solution of electric variables from the electric model. The coupling between the inductances and rotor position angle vanishes when the motor has a uniform air gap. This is the case for PMSMs with nonsalient pole structure, which will be used later in section 4.2 for experimental validation of the synchronization based parameter estimation algorithm. Since this type of PMSM has identical magnetic paths, \( L_{\Delta m} \) in (3.5) becomes 0. Thus, \( L_s \) in (3.4) and (3.5) becomes a constant as defined in (3.6) [8].

\[ L_s = \begin{bmatrix} \dot{e} \\ \dot{e} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} L_{ls} + \frac{1}{2} L_m & -\frac{1}{2} L_m & -\frac{1}{2} L_m \\ -\frac{1}{2} L_m & \frac{1}{2} L_{ls} + \frac{1}{2} L_m & -\frac{1}{2} L_m \\ -\frac{1}{2} L_m & -\frac{1}{2} L_m & \frac{1}{2} L_{ls} + \frac{1}{2} L_m \end{bmatrix} \]

(3.6)

The flux linkage generated by permanent magnet, \( \Psi_{mabc} \), relates to the rotor electrical angular position, \( \theta_r \). Assuming that the stator windings are placed evenly with a relative phase angle of 120° and the flux linkage distribution obeys the sinusoidal law, \( \Psi_{mabc} \) can then be expressed as periodic...
functions of \( \theta_r \) defined by (3.7). \( \Psi_m \) in (3.7) is the magnitude of the permanent magnet flux linkage.

\[
\begin{align*}
\dot{\Psi}_{m} &= \dot{\Psi}_{ma}(\theta_r) \varnothing \\
\dot{\Psi}_{mb}(\theta_r) &= \Psi_m \\
\dot{\Psi}_{mc}(\theta_r) &= \Psi_m \\
\end{align*}
\]

Now, voltage equation (3.1) can be rewritten as (3.8) using (3.3) and (3.7).

\[
\begin{align*}
v_{abc}^s &= r_{abc}^s i_{abc}^s + L_s \frac{d}{dt} i_{abc}^s + \frac{d\Psi_{mabc}}{dt}, \text{ with} \\
&= \dot{\varnothing} \cos \theta_r - \frac{2\pi}{3} \dot{\varnothing} + \dot{\psi} \frac{2\pi}{3} \dot{\varnothing} \\
\frac{d\Psi_{mabc}}{dt} &= \Psi_m \omega_r \\
\end{align*}
\]

In (3.8), the rotor position angle \( \theta_r \) and the electrical angular velocity \( \omega_r \) are two unknown variables that need to be calculated before solving the equations. To do this, PMSM’s mechanical dynamic equations are incorporated. According to the Newton’s law, the mechanical model can be described using (3.9) and (3.10).

\[
\begin{align*}
\frac{d\omega_r}{dt} &= \frac{1}{J} n_p \dot{\varnothing} - B_m \frac{\omega_r}{n_p} - T_e \varnothing \\
\frac{d\theta_r}{dt} &= \omega_r
\end{align*}
\]

In (3.9), \( n_p \) is the number of the pole pairs of the motor; \( J \) is the inertia of the rotor; \( B_m \) is the viscous friction coefficient; \( T_e \) is the electromagnetic torque and \( T_L \) is the load torque of the motor.

The electromagnetic torque, \( T_e \), generated by PMSM, can be derived from the coenergy, \( W_c \), of the magnetic system as in (3.11), according to Lyshevski [8], Krishnan [86], and Khorrami,
In (3.11), $W_{PM}$ is a constant representing the energy stored in the permanent magnet rotor of PMSM. By substituting (3.7) into (3.11), the torque can be expressed using phase currents and rotor angular position as in (3.12).

The PMSM transient response can now be obtained by solving the differential equations (3.8)–(3.12).

During the derivation of PMSMs’ electric model in $abc$-phase, no assumptions are made about the motor winding geometry. Thus, this model is also applicable when winding asymmetry exists. This property is important for the development of PMSMs’ fault model with asymmetrical stator windings.

The general applicability of the $abc$-phase model described above comes at the expense of model complexity. This complexity may bring difficulties for certain applications, such as controller designs. Fortunately, under the normal operating mode, this complexity can be greatly reduced by applying the well known Park’s transformation, which transforms the symmetrical...
sinusoidal $abc$-phase variables into DC variables in $dq\theta$-coordinates, creating the $dq$-axis healthy model. Thus, a much simpler model can be obtained. The simplified model is widely utilized and is very important for the FDD of PMSM.

### 3.1.2 Park’s Transformation and $dq$-model

Park’s $dq$-transformation is a coordinate transformation that converts the three-phase stationary variables into variables in a rotating coordinate system. In $dq$-transformation, the rotating coordinate is defined relative to a stationary reference angle. The stationary reference can be selected arbitrarily. For simplicity, this reference is usually selected at the location of the phase $a$ axis. Thus, the transformation angle has the same value as that of the rotor electrical position angle $\theta_r$, as illustrated in Figure 3.2.

![Figure 3.2. Park’s transformation](image)

Figure 3.2. Park’s transformation
Denoting the variables in the rotating reference frame as direct \((d)\), quadrature \((q)\) and zero \((0)\) sequences, \(u_{dq0}\), Park’s transformation can be represented by (3.13).

\[
\begin{align*}
\dot{e} &= \cos \theta \dot{e}_{d} + \frac{2}{3} \pi \dot{e}_{d} + \frac{2}{3} \pi \dot{e}_{q} \\
\dot{e}_{d} &= \frac{2}{3} \sin \theta \dot{e}_{q} \\
\dot{e}_{q} &= \frac{1}{2} \dot{e}_{d} + \frac{1}{2} \dot{e}_{q}
\end{align*}
\]

In (3.13), \(u\) can be any three-phase variable, i.e. voltage, current, impedance, flux, etc. When these variables are symmetrical three-phase variables, Park’s transformation produces zero components for \(u_0\). This zero sequence is then ignorable, and the transformation becomes a 3-to-2 transformation. Under asymmetrical circumstances, the zero sequence is nonzero, which is a symptom of fault conditions.

Under normal operations, the PMSM model can be simplified by applying this Park’s transformation. As a result, the electric model in the new two dimensional rotating reference frame are expressed in (3.14)–(3.15), which can also be derived directly from the equivalent circuit model shown in Figure 3.3.

![Figure 3.3](image-url)

Figure 3.3 \(dq\)-axis equivalent circuit model of PMSM
In (3.14) and (3.15), \(L_d\) and \(L_q\) are defined as \(dq\)-axis inductances. They can be calculated from the inductance constants in the \(abc\)-phase model, i.e.

\[
L_d = \frac{3}{2}(L_m - L_{\Delta m}) \\
L_q = \frac{3}{2}(L_m + L_{\Delta m})
\]  

Then, the electromagnetic torque in \(dq\) reference frame becomes:

\[
T_e = \frac{3}{4} P \frac{\dot{\psi}_m}{\psi_m} + \left( L_d - L_q \right) \dot{i}_q \dot{i}_d
\]  

Equations (3.9), and (3.14)~(3.17) complete the description of system dynamics for PMSM. In state space matrix format, this model can be rewritten as (3.18).

\[
\begin{align*}
\frac{di_d'}{dt} &= \frac{1}{L_d} \dot{\psi}_d' - R_i \dot{i}_d' + \omega L_q \dot{i}_q' \\
\frac{di_q'}{dt} &= \frac{1}{L_q} \dot{\psi}_q' - R_i \dot{i}_q' - \omega L_d \dot{i}_d' - \omega \psi_m \\
\end{align*}
\]  

For a nonsalient pole PMSM with \(L_d = L_q = L_{dq}\), (3.18) can be simplified as (3.19).
Obviously, this $dq$-model describes a highly coupled nonlinear 2-input 3-output system in $dq$ reference frame. $i_{d}^{r}$, $i_{q}^{r}$ and $\omega^{r}$ are state variables. $v_{d}^{r}$ and $v_{q}^{r}$ are control signals (inputs). Under steady state operation with balanced three-phase power input, the currents and voltages in $dq$ reference frame are constants, which make it a preferable model in motor control and FDD.

3.1.3 Nonlinearity of PMSM Model

The study on the dynamics of electric motors began long time ago. But the chaos phenomenon exhibited in induction motors was not introduced until 1989 by Kuroe and Hayashi [88]. After that, the chaotic behavior of PMSM was also investigated based on its $dq$-model. Literature has revealed the existence of Hopf bifurcations, limit cycles and even chaotic attractors in the system. These nonlinear properties usually lead to complex dynamical behaviors, detailed analysis of which were given by Dean, Hamill [89], and Zhong, Park, et al [90]. In [91] and [92], Harb, Hemati and Kwatny compared the PMSM’s $dq$-model with the chaotic Lorenz systems and demonstrated the similarity of these two systems. This similarity indicates PMSM is prone to similar chaotic behaviors to those found in Lorenz systems. Simulation results were provided to illustrate the nonlinear behavior of an uncontrolled PMSM. During simulation, the initial condition of the system was set to an unstable equilibrium point. Figure 3.4 shows the time responses of $i_{d}^{r}$, $i_{q}^{r}$ and $\omega^{r}$ and Figure 3.5 shows the system phase planes during the chaotic oscillations. Both the figures show typical nonlinear behaviors, not associated with linear systems.

Figure 3.4 Uncontrolled PMSM time response exhibiting nonlinear dynamics
Although the model of PMSM is highly nonlinear as shown in the above figures, most existing controller designs are still based on linear models. Considering that the time constant of PMSM’s mechanical system is much larger than that of the electrical system, the controller designs for the two systems are usually designed separately [95]. The performance of these designs are acceptable for regular applications but not sufficient for the high performance applications. Similar to the control problems, precise models are also critical for model based FDD designs.

### 3.1.4 dq-model’s Suitability Under Fault Conditions

Because of its simplicity, the dq-model is widely used in electric motor controller designs. But Park’s transformation, which is the basis of the dq-model, loses its advantage for PMSM with asymmetrical windings. Many kinds of faults will result in this asymmetry, such as the stator winding fault condition studied in this dissertation. To overcome the above problem posed by the dq-model, Dai, Keyhani, and Sebastian applied Finite Element Analysis (FEA) to build the model for FDD [79]. FEA provides accurate approximation of motor performances under fault conditions. However, FEA involves solving a large number of nonlinear equations, thus it is not computationally efficient and not suitable for online implementation [96]. There is still a need for suitable modeling approaches, which can balance accuracy and computational efficiency.
3.2 PMSM under Asymmetrical Stator Winding Fault

Table 2.1 shows that the stator winding fault is the most likely electrical fault in PMSM [47]. Stator winding faults are usually caused by insulation breakdown between coils in the same phase or different phases. Figure 3.6 compares the stator windings before and after a turn-to-turn short fault happened to an industrial motor.

While the winding short usually emerges locally as an incipient fault, it may propagate rapidly and result in the failure of the entire phase. This is due to the increased ohmic heating associated with the large current in the shorted portion of the winding. The excessive heating will lead to significant temperature increase and faster deterioration of the insulation system as stated by Stone Kapler [64], Wolbank, Loparo, and Wohrschimmel [74]. Therefore, an effective FDD system for this type of fault is highly important.

As an electric motor can be considered a transformer with a rotating secondary winding, the motor fault modeling is analogous to modeling transformers as described in [98][99]. The stator winding fault model proposed here is applicable for both normal and fault conditions. This attribute avoids the need of switching models from one operating mode to the other during the FDD simulation study. Details of the proposed model are introduced in the following section.

Figure 3.6 Motor stator winding turn-to-turn fault [97]
(a) Healthy stator windings  (b) Winding turn-to-turn short
3.2.1 General Model with Extra Fault Circuit

Figure 3.7 illustrates the magnetic circuit of a single pole PMSM with a turn-to-turn short fault. Without loss of generality, from now on, it is assumed that the stator winding fault occurs in phase \( b \) in the following model derivation. This is the case shown in Figure 3.7. The same procedure also applies when the fault occurs in other phases. In order to consider the turn-to-turn fault in PMSM model, the affected phase windings are partitioned into two portions: a healthy portion and a shorted portion. The shorted portion forms an extra circuit, which creates a stationary magnetic field. This new magnetic field modifies the original field by adding the fourth coupled magnetic circuit into the system. To represent the fault size and its location, two new parameters, \( \sigma \) and \( \theta_f \), are introduced. \( \sigma \) denotes the fault size, which is defined as the ratio of the shorted turns \( (N_f) \) to the total turn number \( (N_s) \). \( \theta_f \) is a fault location parameter, which is the angle between the faulted phase and the phase \( a \) axis. Hence, \( \theta_f \) can only take three different values, 0, \( 2\pi/3 \) or \(-2\pi/3\), corresponding to a stator winding fault in \( a \), \( b \), or \( c \) phase respectively. By defining the electrical quantities of the new circuit with the subscript \( f \) (fault), the new motor voltage equation are rewritten as (3.20) and (3.21). Some quantities are primed (’’) to signify a change under the fault condition from their values under the health operation.

\[
V^{'}_{abcf} = R_{abcf}^{'} \dot{i}_f + \frac{d\Psi^{'}_{abcf}}{dt} \tag{3.20}
\]

\[
V^{s}_{abcf} = \begin{bmatrix} \dot{\Psi}^s_{a} \\ \dot{\Psi}^s_{b} \\ \dot{\Psi}^s_{c} \\ \dot{\Psi}^s_{f} \end{bmatrix}, \quad v^{s}_{f} = 0
\]

\[
R_{abcf}^{s} = \begin{bmatrix} \dot{\Psi}^s_{a} & 0 & 0 & 0 \\ 0 & \dot{\Psi}^s_{b} & 0 & 0 \\ 0 & 0 & \dot{\Psi}^s_{c} & 0 \\ 0 & 0 & 0 & \dot{\Psi}^s_{f} \end{bmatrix}
\]

\[
i^{s}_{abcf} = \begin{bmatrix} \dot{i}^s_{a} \\ \dot{i}^s_{b} \\ \dot{i}^s_{c} \\ \dot{i}^s_{f} \end{bmatrix}, \quad v^{s} = 0
\]

\[
\Psi^{s}_{abcf} = \begin{bmatrix} \dot{\Psi}^s_{a} \\ \dot{\Psi}^s_{b} \\ \dot{\Psi}^s_{c} \\ \dot{\Psi}^s_{f} \end{bmatrix}
\]
Dai, Keyhani, and Sebastian showed that the demagnetization problem does not need to be considered during the modeling of stator winding faults for PMSM made of Nd-Fe-B material [79]. Hence, the PM flux linkage in the faulty phase can be partitioned into two parts, i.e. $\Psi'_b$ and $\Psi'_f$, which are proportional to fault size parameter $\sigma$. Including the fault location parameter $\theta_f$, the flux linkage established by the permanent magnet in (3.7) is modified as (3.22).

$$\Psi_{\text{mabc}} = \begin{bmatrix} \dot{\Psi}_{\text{ma}}(\theta_r, \theta_f) \bar{u} \\ \dot{\Psi}_{\text{mb}}(\theta_r, \theta_f) \bar{u} \\ \dot{\Psi}_{\text{mc}}(\theta_r, \theta_f) \bar{u} \\ \dot{\Psi}_{\text{mf}}(\theta_r, \theta_f) \bar{u} \end{bmatrix} = \begin{bmatrix} \dot{\Psi}_m \bar{u} \\ \dot{\Psi}_m \bar{u} \\ \dot{\Psi}_m \bar{u} \end{bmatrix} = \begin{bmatrix} \sin(\theta_r - \theta_f) \\ (1-\sigma)\sin\frac{\theta_r - 2\pi}{3} - \theta_f \\ \sigma\sin\frac{\theta_r + 2\pi}{3} - \theta_f \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{u} \\ \bar{u} \end{bmatrix}$$

(3.22)

The mechanical model in (3.9)–(3.12) still applies when the internal faults occur. Then,
equations (3.9)-(3.12) together with (3.19)-(3.22) can be used to describe the system operating under a turn-to-turn stator winding fault.

Now, the fundamental problem is how to calculate the new parameters in (3.21), specifically, the new resistances and inductances. Both become $4 \times 4$ matrices here. A simple proportional property of the windings resistances can be used to find the new resistances. If $N_f$ turns of the total $N_s$ turns are shorted in phase $b$ winding, the resistances can be calculated according to (3.23).

$$R'_b = (1 - \sigma) R_s, \quad R'_f = \sigma R_s \quad \text{with} \quad \sigma = \frac{N_f}{N_s} \quad (3.23)$$

The new magnetic system inductance, $L_s$, is composed of self and mutual inductances between the shorted portion and other healthy portions of the stator windings as in (3.24).

$$L_s = \begin{bmatrix}
\hat{e}L'_{aa} & \hat{e}L'_{ab} & \hat{e}L'_{ac} & \hat{e}L'_{af} \\
\hat{e}L'_{ba} & \hat{e}L'_{bb} & \hat{e}L'_{bc} & \hat{e}L'_{bf} \\
\hat{e}L'_{ca} & \hat{e}L'_{cb} & \hat{e}L'_{cc} & \hat{e}L'_{cf} \\
\hat{e}L'_{fa} & \hat{e}L'_{fb} & \hat{e}L'_{fc} & \hat{e}L'_{ff}
\end{bmatrix} \quad (3.24)$$

This problem has been discussed in several publications, e.g. [93][94][98][99]. For example, Bastard proposed a method for transformer fault studies in [99], which provides a good balance between model simplicity and accuracy. The principles of consistency, leakage and proportionality therein are applicable to PMSM. The consistency principle guarantees that when using the $4 \times 4$ inductance matrix for the fault free case simulation, one can obtain the same result as using the healthy model and the $3 \times 3$ matrix in (3.4). The leakage principle is usually represented by a small positive constant defined as leakage factor. This factor is important because it is directly related to the current developed in the faulted circuit winding. The proportionality principle is employed to describe the inductances’ relation to the number of turns in a coil. In our stator winding fault study, this principle expresses the relationship between the two parts of the partitioned winding. Actually, this principle is only strictly true when the leakage is zero. Fortunately, the principle is applicable because the leakage is practically close to zero.

According to the three principles, new inductance matrix during the stator winding short fault
can be obtained as follows. The elements in the new inductance matrix that are related to the faulted phase windings, \( L_{bb}' \), \( L_{ff} \), and \( L_{bf} \) are considered first. The remaining elements will be considered later. The three principles for \( L_{bb}' \), \( L_{ff} \), and \( L_{bf} \) are as follows:

- **Consistency:**

\[
L_{bb}' + 2L_{bf} + L_{ff} = L_{bb} \tag{3.25}
\]

- **Leakage:**

\[
\delta_{bf} = 1 - \frac{L_{bf}^2}{L_{bb} L_{ff}} \tag{3.26}
\]

- **Proportionality:**

\[
\frac{L_{bb}'}{L_{bf}} = \frac{1}{\frac{\delta_{bf}}{\sigma} + \frac{2(1 - \sigma)}{\sigma} \sqrt{1 - \delta_{bf}} + 1} \tag{3.27}
\]

Then, the inductances \( L_{bb}' \), \( L_{ff} \), and \( L_{bf} \) can be calculated according to (3.28)–(3.30).

\[
L_{bb}' = L_{bb} \frac{1}{\frac{\delta_{bf}}{\sigma} + \frac{2(1 - \sigma)}{\sigma} \sqrt{1 - \delta_{bf}} + 1} \tag{3.28}
\]

\[
L_{ff} = L_{bb} \frac{1}{\frac{\delta_{bf}}{\sigma} + \frac{2(1 - \sigma)}{\sigma} \sqrt{1 - \delta_{bf}} + 1} \tag{3.29}
\]

\[
L_{bf} = L_{bb} \frac{\sqrt{1 - \delta_{bf}}}{\frac{1 - \sigma}{\sigma} + \frac{\sigma}{1 - \sigma} + 2 \sqrt{1 - \delta_{bf}}} \tag{3.30}
\]

For the mutual inductances between the faulted phase winding and other phase windings, the consistency principle becomes:

\[
L_{bx}' + L_{fx} = L_{bx}, \quad \text{where } x = a, c \tag{3.31}
\]

Considering the proportionality principle, \( L_{ba}' \), \( L_{bc}' \), \( L_{fa} \), and \( L_{fc} \) can be calculated according to (3.32) and (3.33).
\[ L_{ba} = L_{ba}(1-\sigma); L_{bc} = L_{bc}(1-\sigma) \]  \hspace{1cm} (3.32)

\[ L_{ja} = L_{ja}\sigma; L_{fc} = L_{hc}\sigma \]  \hspace{1cm} (3.33)

The other elements in (3.24) are either symmetrical to (3.30), (3.32), and (3.33), or remain their original values as in (3.5). Similar to the healthy model, when the PMSM rotor has a salient pole structure, the new stator inductances obtained from these equations are closely related to the rotor angular position, \( \theta \). For non-salient pole motors, the stator winding inductances are decoupled from the rotor position. In this case, (3.28)~(3.33) can be rewritten with some inductance constants. The entire stator inductance matrix can be expressed in (3.34):

\[
L_s = \begin{pmatrix}
L_{ls} + \bar{L}_m & -\frac{1}{2}(1-\sigma)\bar{L}_m & \cdots \\
-\frac{1}{2}(1-\sigma)\bar{L}_m & (L_{ls} + \bar{L}_m)\frac{1}{\frac{3\delta^2}{\delta} - \sigma^2} & \cdots \\
-\frac{1}{2}(1-\sigma)\bar{L}_m & -\frac{1}{2}(1-\sigma)\bar{L}_m & \cdots \\
-\frac{1}{2}\sigma\bar{L}_m & (L_{ls} + \bar{L}_m)\frac{\sqrt{1-\delta_{bf}}}{\sigma + \frac{1-\sigma}{1-\sigma} + 2\sqrt{1-\delta_{bf}}} & \cdots \\
\cdots & -\frac{1}{2}(1-\sigma)\bar{L}_m & -\frac{1}{2}\sigma\bar{L}_m \\
\cdots & -\frac{1}{2}(1-\sigma)\bar{L}_m & (L_{ls} + \bar{L}_m)\frac{\sqrt{1-\delta_{bf}}}{\sigma + \frac{1-\sigma}{1-\sigma} + 2\sqrt{1-\delta_{bf}}} \\
\cdots & L_{ls} + \bar{L}_m & -\frac{1}{2}\sigma\bar{L}_m \\
\cdots & -\frac{1}{2}\sigma\bar{L}_m & (L_{ls} + \bar{L}_m)\frac{1}{\frac{3\delta^2}{\delta} - \sigma^2 + 2\frac{1-\sigma}{\sigma} \sqrt{1-\delta_{bf}} + 1} \\
\end{pmatrix} \tag{3.34}
\]

Based on the new motor parameters derived above, the PMSM dynamic response during the turn-to-turn winding fault can be simulated using the electrical equation (3.19) and the mechanical equations (3.9)~(3.12).
3.2.2 Simulation and Dynamics Analysis

To exercise the developed model, simulation studies were performed in MATLAB\(^\text{®}\). Although the proposed model is independent of the types of PMSM structure, i.e. salient or nonsalient, a PMSM with nonsalient poles was selected for simplicity. The nominal parameters of the simulated PMSM are listed in Table 3.1. In the simulated system, the PMSM is controlled by a PWM Voltage Source Inverter (VSI) as shown in Figure 3.8. Both healthy and an asymmetrical internal turn-to-turn short fault conditions were simulated. Some simulation results are shown in Figures 3.9~3.19.

3.2.2.1 Simulation results of healthy PMSM

The purpose of healthy PMSM simulation is to compare the proposed model with the conventional \(dq\)-model provided by SimPowerSystems blockset of MATLAB\(^\text{®}\). In this way, the validity and compatibility of the new model under the healthy operating condition can be assessed. Respectively, Figure 3.9 and Figure 3.10 compare the PMSM’s rotor electrical speed, \(\omega_r\), and the electromagnetic torque output, \(T_e\), between \(dq\)-model and the proposed model. Simulation results show a good match even during startup. The rotor speed spectrum is obtained by FFT.

<table>
<thead>
<tr>
<th>PMSM Parameters</th>
<th>Nominal Values (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pairs (n_p)</td>
<td>1</td>
</tr>
<tr>
<td>Stator resistance (R_s)</td>
<td>1.4 ((\Omega))</td>
</tr>
<tr>
<td>(q)-axis inductance (L_q)</td>
<td>7.3 (mH)</td>
</tr>
<tr>
<td>(d)-axis inductance (L_d)</td>
<td>7.3 (mH)</td>
</tr>
<tr>
<td>PM flux linkage (\psi_m)</td>
<td>0.1546 (Wb-turn)</td>
</tr>
<tr>
<td>Frictional coefficient (B_m)</td>
<td>0.001 (N.m/rad/sec)</td>
</tr>
<tr>
<td>Moment of inertia (J)</td>
<td>0.006 (kg.m(^2))</td>
</tr>
</tbody>
</table>
analysis as shown in Figure 3.11. The speed signal has a DC magnitude of 375.6 rad/sec and 0% total harmonic distortion (THD). Figure 3.12 displays the currents in the two segments of in the phase winding where the fault is to be applied later. As expected, the two currents’ curves match each other. Figure 3.13 shows balanced sine waveforms of the three-phase currents during healthy operations.

![Figure 3.8. Simulation setup for PMSM model validation](image)

![Figure 3.9. Simulation comparison of rotor speed responses under healthy condition](image)
Figure 3.10. Simulation comparison of torque responses under healthy condition

Figure 3.11. FFT analysis of simulated rotor speed signal under healthy condition
Simulation results show that the proposed model is acceptable for PMSM operating under healthy operating conditions. In the next section, the proposed model will be further tested under fault conditions to see if the model is also valid for fault situations.
3.2.2.2 Simulation results for PMSM with turn-to-turn faults

During simulation, an internal winding turn-to-turn short of 5% of the entire winding is introduced into phase $b$ at 0.15 sec. This fault can represent typical incipient stator winding faults caused by insulation breakdown. According to industrial experiences, such a fault will not impact motor operation substantially [54][81]. Rotor speed and torque output do not change much, except for some higher order harmonics due to the winding asymmetry. These expected behaviors can be observed from Figure 3.14 and Figure 3.15. The rotor speed responses with healthy and faulty stator windings are compared in Figure 3.14. The rotor speed under the fault only decreases slightly.

Figure 3.14 Simulation comparison of rotor speed responses under healthy and fault conditions
(a) Original view    (b) Zoomed view
compared to the healthy condition. Similarly, the torque output displayed in Figure 3.15 also shows a small decrease in the average value after the fault is introduced. According to these two figures, some harmonics are introduced to both $\omega_r$ and $T_e$ during the fault. A 2$^{nd}$ order harmonic in rotor speed can be observed in Figure 3.16 compared to the healthy condition in Figure 3.11. Similar phenomenon can be seen in the torque output shown in Figure 3.15. These phenomena are typical symptoms for PMSM operating under asymmetrical stator winding turn-to-turn faults.

Since there are no significant changes in torques and speeds under a small stator winding fault, the three-phase currents of the motor will not change dramatically either. However, these signals will become unbalanced due to the winding asymmetry. This asymmetry in currents will lead to harmonics in motor speeds and torques as shown before. The three-phase currents responses to a stator winding turn-to-turn fault condition are shown in Figure 3.17. After fault, the magnitude of the current in the faulty phase (phase b) becomes larger than that of the other two phases. Furthermore, harmonics are more severe than that of the other two phases.

![Figure 3.15](image)

Figure 3.15 Simulation comparison of electromagnetic torque responses under healthy and fault conditions
As mentioned before, fault current will result in the temperature increase in the stator, accelerate the aging of stator winding insulation, and eventually lead to failure of the entire phase. Thus, the response of the stator currents under fault is very important and needs to be considered during fault modeling. The responses of the shorted portion current, $i_f$, and the terminal current, $i_b'$,
are shown in Figure 3.18. The fault current developed in the shorted portion is much larger than that of the terminal current, and directions of the two currents are opposite. Therefore, the fault current will create a braking torque counteracting the overall torque output. This is the reason for average torque decrease during this kind of fault. Similar phenomena were also observed by Awadallah et al in [100] and [101].

To investigate the development of the incipient turn-to-turn stator winding faults, spectra of torque signals for different fault sizes, 5%, 10% and 20%, are presented in Figure 3.19. According to the figure, it can be seen that the fundamentals of torque output decrease with the development of the incipient winding fault, which verifies the previous analysis on the effect of the fault current. On the contrary, the harmonics increase with the development of the incipient fault. The simulation results here are consistent with experiment results in [82]. This symptom will be helpful for fault diagnosis if the motor torque can be measured accurately.

Figure 3.18  Simulation comparison of fault current and terminal current responses to a stator winding fault
3.3 Summary

Fault modeling is the fundamental step for the design of model based FDD. Accurate mathematical model can provide insights to fault conditions with reduced cost compared to experiments. The conventional $dq$-model is very helpful for PMSM operating under balanced conditions, but not beneficial for PMSM operating under unbalanced fault conditions. As one of common faults for electric machines, the stator winding turn-to-turn fault is modeled based on the coupled magnetic circuit principle. The model provides a good compromise between computational complexity and model accuracy and is versatile for both the healthy and the fault condition. Simulation results demonstrate a good coincidence with both the theoretical analysis and the experimental observations in existing literature. Several FDD techniques are proposed based on the models developed in this chapter, which will be introduced in the following chapters.

Figure 3.19 Spectra of the simulated electromagnetic torque at different operating conditions
CHAPTER 4
NONLINEAR FAULT DETECTION FOR PMSM

Successful operation of a model based FDD system depends on the quality of the measured data. If the sensor is not working correctly, the data provided to FDD system may lead to wrong decisions. Thus, sensor faults, which are typically additive faults, must be identified beforehand. In this chapter, a nonlinear parity equation based approach is proposed for the detection of PMSM sensor faults. Based on the data provided by these fault free sensors, a synchronization based adaptive parameter estimation algorithm is proposed for the detection of PMSM internal anomalies.

This chapter is organized as follows. In section 4.1, the proposed nonlinear algorithm for sensor fault detection is introduced. In section 4.2, a nonlinear adaptive parameter estimation algorithm is proposed for PMSM internal change detection. In this section, besides simulation verifications, experimental validation is also performed for the parameter estimation algorithm.

4.1 PMSM Sensor Fault Detection

Modern sensors can provide better measurement of signals with increasing robustness. However, faults still may occur to sensors under severe conditions, especially after long operation. The occurrence of sensor faults is at the same level as the system internal faults. Considering the importance of sensors to monitoring and control systems, the detection of sensor faults should be the first priority in a FDD system [4][16].
4.1.1 Background of Sensor Fault Detection

The sensor fault detection algorithm developed in this work is based on two assumptions. First, it is assumed there are no multiple sensor faults occurring at the same time. Second, it is assumed no system internal faults of any type accompany the sensor fault. Since for a practical system, the chance is rare for multiple faults occurring simultaneously, these assumptions are considered acceptable.

To formulate the sensor fault detection problem, the dynamic model of PMSM is briefly reviewed here. According to Chapter 3, the PMSM model under healthy operation can be described by three differential equations as in (3.18). \( i_d \), \( i_q \), and \( \omega_r \) are assumed to be measured directly using sensors. Note that \( i_d \) and \( i_q \) are not practically measurable. These two values are calculated from the measured phase currents \( i_{abc} \) and rotor angular position \( \theta_f \) by applying the Park’s transformation. Considering that \( i_d \) and \( i_q \) exist in control memory, they present less computational complexity. Therefore, \( i_d \) and \( i_q \) signals are selected in this fault detection design for simplicity. Instead of using a detected change in \( i_d \) and \( i_q \) signals as real fault symptoms in the two virtual sensors, it can be used as an indication of possible faults in the sensors for phase currents and rotor angular position measurement.

According to (3.18), the system model can be rewritten as (4.1). (4.2) gives the relationship between the actual signals and the measured signals.

\[
\begin{align*}
\frac{di_d^r}{dt} &= \frac{1}{L_d} \left[ v_d^r - R_d i_d^r + \omega_r L_q i_q^r \right] \\
\frac{di_q^r}{dt} &= \frac{1}{L_q} \left[ v_q^r - R_q i_q^r - \omega_r L_d i_d^r - \omega_r \Psi_{mag} \right] \\
d\omega_r &= \frac{3}{2} J n_p^2 \Psi_{mag} i_q^r + \frac{3}{2} J n_p^2 \left( L_d - L_q \right) i_d^r - \frac{B}{J} \omega_r - n_p \frac{1}{J} T_L \\
y_1 &= i_d^r + f_d \\
y_2 &= i_q^r + f_q \\
y_3 &= \omega_r + f_\omega
\end{align*}
\]

In (4.2), \( f_d \), \( f_q \), and \( f_\omega \) represent the effects of sensor faults. The sensor faults considered here refer to those faults that will result in DC offsets to the original signals. Symptoms of these kinds
of faults include complete losses of signals because of sensor failure, drifts in the sensed signals due to aging, etc. The sensor fault detection algorithm will determine whether there is a sensor fault based on the measured data, \(y_1, y_2,\) and \(y_3\). The details of the proposed algorithm are given in the next section.

**4.1.2 Nonlinear Parity Relation Based Sensor faults Detection**

According to the review of model based fault detection techniques in section 2.1.2, parity relation is a suitable method for the detection of additive faults. One disadvantage of this approach is its susceptibility to modeling errors caused by uncertainties. Thus, robust design is important to this approach. A nonlinear parity relation based sensor fault detection algorithm is proposed here, and its robust improvement will be discussed in Chapter 6.

For simplification, system states, control inputs, system outputs, faults and disturbances are renamed as \(x, u, y, f\) and \(d\) respectively in (4.3).

\[
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \omega_r \end{bmatrix}; \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}; \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}; \quad f = \begin{bmatrix} f_d \\ f_q \\ f_o \end{bmatrix}; \quad d = T_L
\] (4.3)

Then, the PMSM model is rewritten as (4.4) and (4.5).

\[
\begin{align*}
\dot{x}_1 &= f_1(x, u, d) = a_1x_1 + a_2x_2x_3 + a_3u_1 \\
\dot{x}_2 &= f_2(x, u, d) = a_4x_2 + a_5x_1x_3 + a_6x_3 + a_7u_2 , \quad \text{where} \\
\dot{x}_3 &= f_3(x, u, d) = a_8x_2 + a_9x_1x_3 + a_{10}x_3 + a_{11}d \\
\end{align*}
\] (4.4)

\[
\begin{align*}
a_1 &= \frac{-R_s}{L_d}; \quad a_2 = \frac{L_q}{L_d}; \quad a_3 = \frac{1}{L_d}; \quad a_4 = \frac{-R_s}{L_q}; \quad a_5 = \frac{-L_d}{L_q}; \quad a_6 = \frac{-\Psi_{mag}}{L_q}; \\
a_7 &= \frac{1}{L_q}; \quad a_8 = \frac{3}{2} \frac{1}{J} n_p^2 \Psi_{mag}; \quad a_9 = \frac{3}{2} \frac{1}{J} n_p^2 \left( L_d - L_q \right); \quad a_{10} = \frac{-B}{J}; \quad a_{11} = -n_p \frac{1}{J} \\
\end{align*}
\] (4.5)

\[
\begin{align*}
y_1 &= x_1 + f_d \\
y_2 &= x_2 + f_q \\
y_3 &= x_3 + f_o
\end{align*}
\]

By definition, a parity relation is a relation of system observations (expression of measurements) that holds in fault free conditions. This relation can be derived from the system
model by reorganizing the system variables or changing the model structure. Theoretically, a violation of the parity relation indicates a fault. This is the basic principle based on which the residuals are constructed for fault detection. If the parity relation is represented by (4.6), the residuals can be defined as in (4.7).

\[ c(u(t), y(t)) = 0, \quad \forall t, u, y, d, \quad \text{iff} \quad f = 0 \]

\[ r(t) = c(u(t), y(t)) \]  

(4.7)

In (4.6), \( c(u(t), y(t)) \) is the expression of parity relation. It is a function of the system inputs and outputs only. Since the system states \( x \) and disturbances \( d \) do not appear in the parity relation, the parity relation is robust to immeasurable states and disturbances. The parity relation holds for all fault free cases, which is expressed as \( f = 0 \). The design of parity relations is straightforward for linear systems but still a challenging work for nonlinear systems. So far, there is no general solution for the parity relation design of nonlinear systems [16].

Derivation of parity relations for a nonlinear system requires the decoupling (eliminating) of any variable that is not measurable. Mathematically, decoupling can be interpreted as elimination of these unwanted variables from the system equations. For the sensor fault detection problem here, some mathematical manipulation is needed to relate outputs \( y \) to states \( x \). By taking the derivatives of \( y \) in (4.5) and substituting \( \dot{x} \) in (4.4), the system model can be rewritten as (4.8).

\[ \dot{y}_1 = a_1x_1 + a_2x_2x_3 + a_4u_1; \]
\[ \dot{y}_2 = a_4x_2 + a_5x_3 + a_6x_3 + a_7u_2; \]
\[ \dot{y}_3 = a_8x_2 + a_9x_3 + a_{10}x_3 + a_{11}d; \]  

(4.8)

By replacing the states with outputs and faults according to (4.5), and moving all terms to the left hand side, the parity relations can be expressed in (4.9).

\[ \dot{y}_1 - a_1(y_1 - f_d) - a_2(y_2 - f_q)(y_3 - f_o) - a_4u_1 = 0; \]
\[ \dot{y}_2 - a_4(y_2 - f_q) - a_5(y_1 - f_d)(y_3 - f_o) - a_6(y_3 - f_o) - a_7u_2 = 0; \]
\[ \dot{y}_3 - a_8(y_2 - f_q) - a_9(y_1 - f_d)(y_2 - f_q) - a_{10}(y_3 - f_o) - a_{11}d = 0 \]  

(4.9)

For fault free cases \( (f_d,f_q,f_o) = 0 \), equation (4.9) becomes a relation between system inputs and
outputs. However, this is not a valid parity relation yet, because of the presence of system disturbance $d$. The system disturbance needs to be eliminated so that the residuals are insensitive to disturbances. Furthermore, a set of structured residuals with different fault sensitivities is needed to distinguish different sensor faults. Namely, for a particular fault, some of the residuals respond, while others do not. Hence, different combinations of residuals correspond to unique sensor faults. For a certain residual, the faults, that this residual does not respond to, should be decoupled at the same time as the disturbances.

For the sensor fault detection problem described earlier, three sensor faults need to be considered. Two candidate residual structures are shown in Table 4.1 and Table 4.2 respectively. Disturbance decoupling is taken into account together with the fault responses in both residual structures. Details of this decoupling are explicitly explained as follows.

Table 4.1  Candidate residual structure no. 1 for sensor fault detection

<table>
<thead>
<tr>
<th></th>
<th>$f_w$</th>
<th>$f_q$</th>
<th>$f_d$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$r_3$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2  Candidate residual structure no. 2 for sensor fault detection

<table>
<thead>
<tr>
<th></th>
<th>$f_w$</th>
<th>$f_q$</th>
<th>$f_d$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
In the above tables, a “1” denotes that the residual in the row responds to the corresponding fault or disturbance in the column, while a “0” denotes that the residual does not respond to that fault or disturbance. Combinations of the “1”s and “0”s in each column form fault codes for the detection of the corresponding faults. In a good residual structure, these codes for different faults should be different, thus different faults can be distinguished. Furthermore, none of the residuals should respond to disturbances, which can be seen from the “0”s in the column corresponding to \(d\) in the two tables. During the design, all the faults or disturbances corresponding to “0”s need to be decoupled from the corresponding residuals. Mathematically, this variable decoupling in residual designs can be considered as a variable elimination problem. In the established algebra theories, Gröbner Basis, introduced by Cox, et al [102] in 1960s, is a powerful tool for solving many problems in polynomial equations. Fundamental theories on ideals and the elimination method can be used for disturbance decoupling to obtain residuals. This elimination process is very complex and cannot be performed by hand. Commercial software package, such as Maple\textsuperscript{®} and Mathematica\textsuperscript{®}, can be used to perform elimination based on Gröbner Basis. Details about Gröbner Basis and elimination theory can be found in Appendix A.

According to Table 4.1, three residuals should be designed. Each residual only responds to one sensor fault, and remains insensitive to the other two faults and the disturbance. This structure is preferable, because it can detect multiple faults that happen simultaneously. However, this good feature comes with computational complexity, and is difficult to design for systems with complex models. In our case, the design in Table 4.1 requires the decoupling of three variables, \((d\) and two fault variables out of \(f_{dq(o)}\)), from (4.9) for each residual. For instance, to obtain the residual \(r_1\) in Table 4.1, variables \(f_w, f_q, \) and \(d\) need to be eliminated from (4.9). When trying to calculate the residual by eliminating the three variables using Mathematica\textsuperscript{®}, the software gives no satisfactory result. That means the residual structure in Table 4.1 is mathematically unachievable.

According to Table 4.2, only two residuals are needed to detect the three sensor faults. Each residual responds to two faults and remains insensitive to the rest. Thus, only two variables in (4.9) need to be decoupled. The design in Table 4.2 is chosen for this application. A shortcoming of the design in Table 4.2 is that it cannot detect simultaneous multiple faults. This problem needs to be investigated in the future. A detailed residual design procedure is presented as follows.

Take the residual \(r_1\) in Table 4.2 as an example. To derive \(r_1\), both variables \(f_w\) and \(d\) need to be
eliminated from (4.9). After the proper eliminations using Gröbner Basis and elimination theory, the two residuals in Table 4.2 are expressed in (4.10) and (4.11).

$$r_{1}^{in} = f(f_d, f_q) + f(u, y)$$

$$= \left( L_d \dot{y}_1 f_d + \frac{L_q}{L_d} \dot{y}_2 f_q - f_d^2 R_s - \frac{L_q}{L_d} R_s f_q^2 + \frac{\Psi_{mag} R_s}{L_d} f_d - u_1 f_d - \frac{L_q}{L_d} u_2 f_q + 2 R_s y_1 f_d + \frac{2L_q R_s}{L_d} y_2 f_q \right)$$

$$+ \left( -\Psi_{mag} \dot{y}_1 + \frac{\Psi_{mag} R_s}{L_d} u_1 - L_d \dot{y}_1 y_1 - \frac{\Psi_{mag} R_s}{L_d} y_1 + y_1 u_1 - R_s y_1^2 - \frac{L_q}{L_d} \dot{y}_2 y_2 + \frac{L_q}{L_d} u_2 y_2 - \frac{L_q R_s}{L_d} y_2^2 \right)$$

(4.10)

$$r_{2}^{in} = f(f_d, f_o) + f(u, y)$$

$$= \left( \left( f_d f_o \right)^2 - \frac{L_q}{L_d} \dot{y}_2 f_o + \frac{\Psi_{mag} f_o}{L_d} \dot{f}_o \right) - \frac{R_s^2}{L_d} f_d + \frac{1}{L_d} u_2 f_o + y_1 f_o^2 + 2 y_2 f_d f_o - 2 y_1 y_3 f_o$$

$$- \frac{2\Psi_{mag}}{L_d} y_3 f_o - y_2^2 f_d + \left( \frac{R_s}{L_q} \dot{y}_3 y_3 - \frac{R_s}{L_d L_q} y_1 y_3 \right) + \frac{L_q}{L_d} y_2 - \frac{1}{L_d} u_2 y_3 + \frac{\Psi_{mag} y_3^2 + y_1 y_3^2}{L_d}$$

(4.11)

In (4.10) and (4.11), the superscript “in” in residuals $r_1$ and $r_2$ denotes that these residuals are in their internal form. An internal residual shows how the residual depends on the faults according to [4][5]. $r_{1}^{in}$ and $r_{2}^{in}$ consist of two parts: one part is fault related ($f(f_d, f_q)$ and $f(f_d, f_o)$), and the other part is not fault related ($f(u, y)$). Then, the computational residuals can be obtained by omitting the fault related parts from (4.10) and (4.11) as expressed in (4.12) and (4.13). In the following equations, superscript $c$ denotes computational.

$$r_{1}^{c} = -\Psi_{mag} \dot{y}_1 + \frac{\Psi_{mag} R_s}{L_d} \dot{y}_1 y_1 - \frac{\Psi_{mag} R_s}{L_d} y_1 + y_1 u_1 - R_s y_1^2 - \frac{L_q}{L_d} \dot{y}_2 y_2 + \frac{L_q}{L_d} u_2 y_2 - \frac{L_q R_s}{L_d} y_2^2$$

(4.12)

$$r_{2}^{c} = \frac{R_s}{L_q} \dot{y}_3 - \frac{R_s}{L_d L_q} y_1 + \frac{R_s^2}{L_d L_q} y_1 + \frac{L_q}{L_d} \dot{y}_2 - \frac{1}{L_d} u_2 y_3 + \frac{\Psi_{mag} y_3^2 + y_1 y_3^2}{L_d}$$

(4.13)

Computational residuals in (4.12) and (4.13) are still not realizable because they contain not only system inputs and outputs but also their derivatives. Direct differentiation is not a good
solution because of its sensitivity to high frequency noises. Derivative elements in (4.12) and (4.13) need to be approximated to obtain implementable residuals. This process is called residual realization or residual implementation according to Gertler [4]. Since a computational residual may have multiple forms of residual realization, a relatively simple low order implementation is preferable. A simple way to do this is by defining new intermediate variables instead of calculating the derivatives directly. The procedure is as follows.

Take the computational residual $r_1^c$ as an example. First, the computational residual is expressed as a summation of two parts, $r_{11}^c$ and $r_{12}^c$, as in (4.14). Each part of $r_1^c$ contains a fewer number of derivatives than $r_1^c$ does.

$$r_1^c = r_{11}^c + r_{12}^c$$  \hspace{1cm} (4.14)

where

$$r_{11}^c = -\frac{L_q^2}{L_d} \dot{y}_2 y_2 + g_1(u, y); \quad r_{12}^c = -\Psi_{mag} \dot{y}_1 - L_d \dot{y}_1 y_1$$

$$g_1(u, y) = \frac{\Psi_{mag}}{L_d} u + \frac{\Psi_{mag} R_s}{L_d} y_1 + y_1 u_1 - R_s y_1^2 + \frac{L_q}{L_d} u_2 y_2 - \frac{L_q R_s}{L_d} y_2^2$$

For simplicity, all of the nonderivative terms in (4.14) are defined as $g_1(u, y)$. During practical applications, significant high frequency noise may exist, which may impact the above computational residuals. Instead of using these computational residuals, the implementable residuals, $r_{11}$ and $r_{12}$, are defined as the filtered computational residuals $r_{11}^c$ and $r_{12}^c$ as in (4.15).

$$\dot{r}_{11} + \alpha_r r_{11} = -\frac{L_q^2}{L_d} \dot{y}_2 y_2 + g_1(u, y)$$

$$\dot{r}_{12} + \beta_r r_{12} = -\Psi_{mag} \dot{y}_1 - L_d \dot{y}_1 y_1$$  \hspace{1cm} (4.15)

In (4.15), $\alpha_r$ and $\beta_r$ are positive constants whose values should be properly chosen to obtain the desirable filtering effect. To eliminate the derivative terms in (4.15), two intermediate variables, $z_{11}$ and $z_{12}$, are defined in (4.16).
\[ z_{11} = r_1 + \frac{L_q^2}{2L_d} y_2^2 \]  

(4.16)

\[ z_{12} = r_2 + \left( \psi_{mag} y_1 + \frac{L_d}{2} y_1^2 \right) \]

By differentiating (4.16) and plugging in the relations in (4.15), (4.17) can be obtained.

\[ \dot{z}_{11} = -\alpha_{1} \left( z_{11} - \frac{L_q^2}{2L_d} y_2^2 \right) + g_1(u, y) \]

\[ \dot{z}_{12} = -\beta_{1} r_{12} = -\beta_{1} \left( z_{12} - \psi_{mag} y_1 - \frac{L_d}{2} y_1^2 \right) \]

(4.17)

It can be seen in (4.17) that the derivative terms have been successfully removed. \( z_{11} \) and \( z_{12} \) can be easily implemented according to (4.17). Then, the final implementable residuals can be calculated according to (4.18).

\[ r_i = r_{11} + r_{12} \text{ with } r_{11} = z_{11} - \frac{L_q^2}{2L_d} y_2^2 \text{ and } r_{12} = z_{12} - \psi_{mag} y_1 - \frac{L_d}{2} y_1^2 \]

(4.18)

The same procedure can be applied to obtain the implementable residual \( r_2 \) from \( r_2^c \). The final residual is expressed in (4.18) and (4.19) with \( \alpha_2 \) and \( \beta_2 \) as positive constants.

\[ \dot{z}_{21} = -\alpha_{2} \left( z_{21} + \frac{R}{L_q} y_1 \right) + \frac{R}{L_d L_q} u_1 + \frac{R_{21}^2}{L_d L_q} y_1 + -\frac{1}{L_d} u_2 y_3 + \frac{\psi_{mag}}{L_d} y_3^2 + y_1 y_3^2 \]

\[ \dot{z}_{22} = -\beta_{2} \left( z_{22} + \frac{L_q}{L_d} y_2 \right) \]

(4.19)

\[ r_2 = r_{21} + r_{22} \text{ with } r_{21} = z_{21} + \frac{R}{L_q} y_1 \text{ and } r_{22} = z_{22} + \frac{L_q}{L_d} y_2 \]

(4.20)

As mentioned previously, the above implementable residuals derivation procedure is not unique. Other methods also apply as long as the implementable residuals can be successfully calculated. Though the current focus is PMSM sensor fault detection, the method introduced here can also be applied to other nonlinear residuals generation problems. Some simulation results are
presented in next section to demonstrate the effectiveness of the proposed algorithm.

### 4.1.3 Simulation and Discussion

During simulation, the nominal parameters of PMSM listed in Table 4.3 come from a practical PMSM, which is also used in experimental studies in a later part of this work.

Four scenarios are investigated to verify the performance of the above sensor fault detection scheme. One is the fault free case and the other three are faulty cases representing a fault in one of the three sensors respectively. The residual responses in the fault free case are shown in Figure 4.1

<table>
<thead>
<tr>
<th>Table 4.3 PMSM parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMSM Parameters</td>
</tr>
<tr>
<td>Power rating, $P_r$</td>
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<tr>
<td>Rated speed, $\omega_r$</td>
</tr>
<tr>
<td>Current at rated speed, $I_r$</td>
</tr>
<tr>
<td>Torque at rated speed, $T_r$</td>
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<tr>
<td>Voltage constant, $k_e$</td>
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<tr>
<td>Torque constant, $k_t$</td>
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<tr>
<td>Max bus voltage, $V_{DC}$</td>
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<td>Pole pairs, $n_p$</td>
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<tr>
<td>Stator resistance, $R_s$</td>
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<tr>
<td>q-axis inductance, $L_q$</td>
</tr>
<tr>
<td>d-axis inductance, $L_d$</td>
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<tr>
<td>Static friction, $T_f$</td>
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<tr>
<td>Damping coefficient, $B$</td>
</tr>
<tr>
<td>Moment of inertia, $J$</td>
</tr>
</tbody>
</table>
Figure 4.1. Residuals for sensor fault detection

(a) Fault free case residuals  
(b) Rotor speed sensor fault at 1.0 second ($f_\omega$)  
(c) Current sensor fault at 1.0 second ($f_\delta$)  
(d) Current sensor fault at 1.0 second ($f_\delta$)
Figure 4.1 – continued

(a). Fault free case residuals    (b). Rotor speed sensor fault at 1.0 second ($f_{\omega}$)
(c). Current sensor fault at 1.0 second ($f_{q}$)    (d). Current sensor fault at 1.0 second ($f_{d}$)
From this figure, it can be seen that both residuals stay approximately zero after the initial transients. As the disturbance torque has been decoupled from the residuals, the residuals should not be impacted by any load torque variation. The residual responses for the rest three faulty cases are shown in Figure 4.1 (b)–(d). During simulations, the sensor faults are simulated by adding offsets to the actual signals at 1.0 second. Figure 4.1 (b) shows the residual responses to a fault occurring in rotor speed sensor. In this case, residual \( r_1 \) stays at zero all the time, which is same as the fault free case. Residual \( r_2 \) presents an abrupt step response immediately after the fault occurs.

As describe earlier, this residual design realizes the first column of the residual structure described in Table 4.2. Similar responses can be observed in (c) and (d) of Figure 4.1 for the other two faulty scenarios. Residual responses in these two figures correspond to the second and third columns of Table 4.2 respectively. From (c) and (d), some dynamic transients can be observed in the post-fault residuals. These transients are related to the approximation during the residual implementation process. However, the transients will not affect the performance of this sensor detection algorithm. Since the results in Figure 4.1 are from simulations with an accurate system model, any deviation of the residuals from zero indicates a fault symptom. Hence, discussions on threshold setting are not necessary.

Note that, since the sensors for \( i_d \) and \( i_q \) are virtual, the above design only detects the abnormal signals in \( i_d \) and \( i_q \). \( i_d \) and \( i_q \) are usually calculated from phase currents \( i_{abc} \) and rotor position \( \theta_r \). Abnormal changes in \( i_d \) and \( i_q \) may indicate a fault appearing in the phase current sensors or the rotor position encoder, but this design will not provide more specific information. Furthermore, as the calculation of \( i_d \) and \( i_q \) are coupled, if a fault occurring in the measurement devices for \( i_{abc} \) and \( \theta_r \), faults in \( i_d \) and \( i_q \) will appear simultaneously. This is a multiple fault scenario. Detection of multiple faults is beyond the ability of the proposed algorithm. The purpose in this work is to propose a possible solution to nonlinear model based fault detection problem. The proposed algorithm can help the future design of more advanced algorithms, e.g. direct detection of faults in phase current sensors. As mentioned previously, more efforts are needed to develop general fault detection algorithms for complex nonlinear systems.
4.2 PMSM Internal Faults Detection

During practical operations, the parameters in PMSMs may change in response to variations in temperature, humidity, etc. The changes in parameters will influence the accuracy of a system model, which plays a key role in model based FDD designs. Besides these normal changes, there are possible abnormal changes caused by faults or failures. Most of these changes, either normal or abnormal, emerge as parametric variations in a system model. Thus, parameter estimation is a suitable method for the detection of these changes [4][14][20].

In this section, a synchronization based adaptive parameter estimation algorithm is proposed to detect PMSM internal changes. First, a general procedure is proposed for the synchronization based adaptive parameter estimation problems. Suitable designs of feedback and updating rules can provide accurate and stable estimation of the unknown parameters. These designs can be derived from Lyapunov stability theory and LaSalle’s invariance principle. The parameter estimation algorithm for PMSM is described as an application of this estimation design. Simulation and experimental studies demonstrate the effectiveness of the algorithm.

4.2.1 Synchronization Based Adaptive Parameter Estimation Algorithm

The synchronization based parameter estimation algorithm is inspired by the investigation of the structural similarity between an unforced Lorenz system and the nondimensionalized PMSM model. Detailed discussions about this similarity and the nondimensionalization procedure can be found in [91]-[93]. The chaotic Lorenz system is a set of differential equations that was developed for the modeling of an idealized thermosyphon. The chaos synchronization based on Lorenz systems has been extensively explored for various applications, such as control and encoded telecommunications, etc. For PMSM’s parameter estimation problem, the model is not an unforced Lorenz system anymore, since there are inputs, \( v_d \) and \( v_q \), to the system. Thus, the existing synchronization algorithms for the chaotic Lorenz system cannot be directly applied to the online parameter estimation problem here. Some modifications to the passive Lorenz system based synchronization algorithm are necessary.

For a general forced Lorenz system with inputs, the dynamic model can be expressed as a differential equation in (4.21).
\[ \dot{x} = F(p, x, u) \]  

(4.21)

In (4.21), \( F(p, x, u) \) denotes a vector of nonlinear functions, where \( p \) is an \( m \)-dimensional vector denoting the unknown parameters to be identified, \( x \) is an \( n \)-dimensional vector denoting system states, and \( u \) is the system input.

To estimate the unknown parameters, a new system is introduced as (4.22).

\[ \hat{x} = G(\hat{p}, \hat{x}, u) + U(\hat{p}, x, \hat{x}) \]  

(4.22)

In (4.22), \( \hat{x} \) and \( \hat{p} \) denote the estimations of the system states \( x \) and unknown parameters \( p \) respectively, \( U(\hat{p}, x, \hat{x}) \) is the feedback introduced to couple the two systems for synchronization purpose.

During the following stability analysis, the synchronization errors for the system states and the parameters are defined according to (4.23).

\[ e = \hat{x} - x \]

\[ \hat{p} = \hat{p} - p \]  

(4.23)

Based on (4.21) and (4.22), the synchronization dynamics can be expressed by the error derivatives as in (4.24).

\[ \dot{e} = G(\hat{p}, e + x, u) - F(\hat{p} - p, x, u) + U(\hat{p}, x, e + x) \]  

(4.24)

Accordingly, a Lyapunov function can be constructed as follows.

\[ V(e, \hat{p}) = e^T Ne + \hat{p}^T M \hat{p} \]  

(4.25)

In (4.25), \( N \) and \( M \) are \( n \times n \) and \( m \times m \) positive definite matrices respectively.

In a slowly time varying system, the model parameters can be treated as constants. For such system parameters, \( \hat{p} = \hat{p} - p = \dot{p} \) holds approximately. Since a properly designed parameter estimation algorithm can usually converge much faster than the speed of the parameters’ variations, the algorithm developed here is not only applicable to systems with constant parameters but also to slowly time varying systems. For these systems, the derivative of the Lyapunov function can be
expressed as (4.26) according to the above definitions.

\[
\begin{align*}
\dot{V}(e, \hat{p}) &= \left[ G(\hat{p}, e + x, u) - F(\hat{p} - \hat{p}, x, u) + U(\hat{p}, x, e + x) \right]^T N e + \\
&= e^T \left[ G(\hat{p}, e + x, u) - F(\hat{p} - \hat{p}, x, u) + U(\hat{p}, x, e + x) \right] + (\hat{p}^T M \hat{p} + \hat{p}^T M \hat{p})
\end{align*}
\] (4.26)

By proper selection of the feedback \( U(.) \) and the updating rules for \( \hat{p} \), a negative semidefinite Lyapunov derivative can be obtained as in (4.27).

\[
\dot{V} \leq 0, \quad \text{with} \quad \dot{V} = 0, \quad \text{iff} \quad e = 0 \text{ and } \hat{p} = 0
\] (4.27)

According to the LaSalle’s invariance principle in [105], starting from arbitrary initial values, the orbit will converge asymptotically to a compact set \( \Omega \) defined as \( \Omega = \{ \hat{x} = x, \hat{p} = p \} \), as \( t \to \infty \) [103]. This means the identified system (4.21) and the reference model (4.22) will be synchronized in both their states and the estimated parameters.

It is important to note that the LaSalle’s invariance principle only applies to time invariant systems strictly. As mentioned previously, the above design may also be valid for systems with slowly time varying parameters. This has been proved by both simulation and experimental results on a PMSM with such parameters. Furthermore, the above steps are not a strict proof because no specific expressions of the feedback and updating rules are given. It is a general procedure describing the design process of the synchronization based parameter estimation algorithm. For specific applications, stable designs can be obtained based on rigid Lyapunov stability analysis. As an example, a stable parameter estimation algorithm for nonsalient pole PMSM is developed as follows.

For a PMSM with nonsalient pole structure, the \( dq \)-axis inductances are equal, i.e. \( L_d = L_q = L \). The original PMSM dynamic model in (4.4) can be simplified to (4.28).

\[
\begin{align*}
\dot{x}_1 &= a_1 x_1 + x_2 x_3 + a_4 u_1 \\
\dot{x}_2 &= a_5 x_2 - x_1 x_3 - a_5 \Psi_{mag} x_3 + a_5 u_2 \\
\dot{x}_3 &= a_5 x_2 + a_{10} x_3 + a_{11} d \\
\text{where } a_1 &\triangleq -\frac{R_s}{L}; a_3 &\triangleq 1; a_8 &\triangleq \frac{1}{2} J; a_{10} &\triangleq -\frac{B}{J}; a_{11} &\triangleq -n_p \frac{1}{J}.
\end{align*}
\] (4.28)

In (4.28), parameters \( R_s \) and \( L \) are considered unknown, because these two parameters are most
susceptible to PMSM operating condition variations, or under stator winding fault conditions discussed earlier. To form an estimation problem with linear parameter structure, \( a_1 \) and \( a_3 \) are chosen as the parameters to be identified. Since \( a_1 \) and \( a_3 \) are functions of the unknown parameters \( R_s \) and \( L \), \( R_s \) and \( L \) can be calculated according to (4.29) after \( a_1 \) and \( a_3 \) have been successfully obtained.

\[
R_s = -\frac{a_1}{a_3} \quad \text{and} \quad L = \frac{1}{a_3} \quad (4.29)
\]

According to the procedure described before, the new system for the purpose of synchronization can be selected as (4.30).

\[
\begin{align*}
\dot{x}_1 &= a_m \dot{x}_1 + (\dot{a}_1 - a_{m1}) x_1 + x_2 \dot{x}_3 + \dot{a}_3 u_1 + U_1 \\
\dot{x}_2 &= a_m \dot{x}_2 + (\dot{a}_1 - a_{m1}) x_2 - x_1 \dot{x}_3 - \dot{a}_3 \Psi_{mag} x_3 + \dot{a}_3 u_2 + U_2 \\
\dot{x}_3 &= \dot{a}_3 \dot{x}_2 + a_{10} \dot{x}_3 + a_{11} d + U_3 
\end{align*}
\quad (4.30)
\]

In (4.30), \( \dot{a}_1 \) and \( \dot{a}_3 \) are the estimated parameters corresponding to \( a_1 \) and \( a_3 \) respectively, \( a_m \) is a constant. \( U_{123} \) is feedback introduced to cancel the unwanted terms during the Lyapunov analysis. By defining the parameters and states estimation errors as (4.31), the error dynamics during system synchronization can be expressed as (4.32).

\[
\begin{align*}
e_1 &= \dot{x}_1 - x_1, e_2 = \dot{x}_2 - x_2, e_3 = \dot{x}_3 - x_3 \\
\ddot{a}_1 &= \dot{a}_1 - a_1, \ddot{a}_3 = \dot{a}_3 - a_3 \\
\dot{e}_1 &= a_m e_1 + \ddot{a}_1 x_1 + x_2 e_3 + \dot{a}_3 u_1 + U_1 \\
\dot{e}_2 &= a_m e_2 + \ddot{a}_1 x_2 - x_1 e_3 - \dot{a}_3 \Psi_{mag} x_3 + \dot{a}_3 u_2 + U_2 \\
\dot{e}_3 &= a_8 e_2 + a_{10} e_3 + U_3 
\end{align*}
\quad (4.32)
\]

The Lyapunov function for this problem can be defined according to (4.33), where \( \gamma_1, \gamma_2 \) are positive constants.

\[
V(e_1, e_2, e_3, \ddot{a}_1, \ddot{a}_3) = \frac{1}{2} \left( e_1^2 + e_2^2 + e_3^2 + \frac{1}{\gamma_1} \ddot{a}_1^2 + \frac{1}{\gamma_2} \ddot{a}_3^2 \right) 
\quad (4.33)
\]

From (4.33), the Lyapunov derivative can be obtained based on the above definitions and
equations as in (4.34).

\[
\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + \frac{1}{\gamma_1} \hat{a}_1 \dot{\hat{a}}_1 + \frac{1}{\gamma_2} \hat{a}_3 \dot{\hat{a}}_3 \\
= a_m \left( e_1^2 + e_2^2 \right) + a_{10} e_3^2 + e_1 \left( \hat{a}_1 x_1 + x_2 e_3 + \hat{a}_3 u_1 + U_1 \right) \\
+ e_2 \left( \hat{a}_1 x_2 - x_i e_3 - \hat{a}_3 \Psi_{mag} x_3 + \hat{a}_2 u_2 + U_2 \right) + e_3 \left( a_8 e_2 + U_3 \right) + \frac{1}{\gamma_1} \hat{a}_1 \dot{\hat{a}}_1 + \frac{1}{\gamma_2} \hat{a}_3 \dot{\hat{a}}_3
\]  

(4.34)

In order to obtain a negative semidefinite \( \dot{V} \), the updating rules for the unknown parameters \( \hat{a}_1 \), \( \hat{a}_3 \) and the feedback signals \( U_{1,2,3} \) are properly selected according to (4.35) and (4.36) respectively.

\[
\dot{\hat{a}}_1 = -\gamma_1 \left( e_1 x_1 + e_2 x_2 \right) \\
\dot{\hat{a}}_3 = -\gamma_2 \left( e_1 u_1 - e_2 \Psi_{mag} x_3 + e_2 u_2 \right)
\]  

(4.35)

\[
U_1 = -e_3 x_2 \\
U_2 = e_3 x_1 \\
U_3 = -a_8 e_2
\]  

(4.36)

Therefore, the Lyapunov derivative in (4.34) becomes

\[
\dot{V} = a_m \left( e_1^2 + e_2^2 \right) + a_{10} e_3^2
\]  

(4.37)

Since \( a_{10} \) is a negative constant according to its definition, choosing \( a_m < 0 \) will result in a negative semidefinite Lyapunov derivative. According to LaSalle’s invariance principle, the parameter estimation based on adaptive laws (4.35), and feedback (4.36) will provide robust estimation with global convergence for the two unknown parameters.

4.2.2 Simulation Studies

The simulated system is the same as the system used for the PMSM model validation in Section 3.2.2. According to Figure 3.8, the PMSM is controlled by a PWM Voltage Source Inverter (VSI). The nominal parameters of PMSM are listed in Table 4.3.

In this simulation, two tests are performed to demonstrate the performance of the proposed algorithm. In the first test, the parameters to be estimated, \( R_s \) and \( L \), are set to be constants, i.e.,
their nominal values listed in Table 4.3. The objective of this test is to check the dynamic and stable responses so that the algorithm’s applicability to the estimation of time varying parameter can be evaluated. From the simulation results in Figures 4.2 and 4.3, it can be seen that the estimated parameters converge to the desired value in about 0.004 seconds. This convergence is significantly fast to make the slowly time varying parameter assumption for PMSM valid. This observation inspires us to test the algorithm’s performance for estimating slowly time varying parameters.

During the second test, the stator resistance $R_s$ is set to be slowly time varying and a small step change is added to the stator inductance $L$ at 0.04 second. Simulation results for this test are shown in Figure 4.4 and 4.5. From these two figures, it can be observed that the estimation algorithm can track slowly changing variables very well after the initial estimation transients. After the small step change, the system can converge very fast with acceptable dynamics in both the estimated parameters. This parameter tracking property is especially important for robust operation of the fault detection system to be discussed in Chapter 6.

![Figure 4.2. Identification of the simulated stator resistance $R_s$ in Test 1](image)
Figure 4.3. Identification of the simulated stator inductance $L$ in Test 1

Figure 4.4. Identification of the simulated stator resistance $R_s$ in Test 2
4.2.3 Experimental Studies

The experiments are conducted on a testbed, which is shown in Figure 4.6. In this testbed, two identical PMSMs are coupled together, one works as a servomotor fed from a closed loop drive, and the other works as its load. The system specifications are as listed in Table 4.3. For measurement and control purposes, the PMSM is equipped with speedometer, torque transducer and position encoder. All the components in Figure 4.6 are commercially available.
During experimental studies, the machine rotor speed, the rotor position, the electromagnetic torque, the stator phase currents and voltages are measured for estimation purposes. Some sampled data are displayed in Figures 4.7~4.11. Figures 4.7, 4.8, and 4.9 show the measured electromagnetic torque, rotor speed and rotor position angle, respectively. The stator current is shown in Figure 4.10. Since the machine current in this drive system is controlled by a high bandwidth PI-controller, the stator currents are nearly sinusoidal except for the switching harmonics. Instead, the applied voltages are highly nonsinusoidal as shown in Figure 4.11, because these stator voltages are formed by pulsed devices with rather high switching frequencies. To give an illustration of the measured data, only a small fraction of the voltage for phase $a$ is displayed in Figure 4.11.

With these measurements, the estimation algorithm can be tested. Since the measured signals are phase signals ($i_{abc}$ and $v_{abc}$), the $dq$-axis variables, $v_{dq}$ and $i_{dq}$, need to be calculated from the measurements first. Then, $v_{dq}$ are used as inputs to the synchronization model and the other measured data are used as the objectives to be synchronized. Though this estimation algorithm test is not running online, the experimental setup imitates the online application to a large extent. The only shortcoming is that the execution time of the estimation algorithm cannot be assessed to evaluate its online applicability. This experimental study is a necessary step to validate the algorithm’s performance before any real time hardware implementation.

![Experimental data – electromagnetic torque $T_e$](image-url)
Figure 4.8. Measured rotor speed $\omega_r$ during experimental studies

Figure 4.9. Measured rotor position angle $\theta_r$ during experimental studies
During experimental studies, the raw measured data are first used for parameter estimation. The estimated stator resistance $R_s$ can be found in Figure 4.13. Obviously, the performance is very
bad due to the unacceptable oscillations of the estimated parameter. To improve the performance, the raw stator voltage and current signals are filtered before being applied to the synchronization based estimation algorithm. The filtered three-phase voltage signals are shown in Figure 4.12. The estimated stator resistance based on the filtered data is compared with its nominal value as well as the estimation result based on the raw data. From Figure 4.13, it can be seen that estimation using the filtered data provides a much better estimation result. The synchronization processes for $i_d$, $i_q$, and $\omega_r$ are shown in Figures 4.14~4.16 respectively. The estimated signals are synchronized with the measurements so well that the difference between the two curves is insignificant.

From Figures 4.13 ~ 4.16, it can be seen that the model outputs track the references very well, but there are still some oscillations in the estimated parameter and some deviations from its nominal value. There are three possible reasons for this imperfect estimation. First, except for the estimated parameters, the rest parameters of PMSM are set to their nominal values, which are only valid under nominal operating conditions. However, it is very difficult to run the system in such a strict nominal operating condition. Deviations of operation conditions may result in model impreciseness, which will be reflected as estimation errors. Second, the reason lies in the model used during the estimation. As we know, it is very difficult to model a nonlinear physical system precisely. The $dq$-model model is a good representation for general PMSM operations. But for a specific PMSM under different environments, the model may be not accurate enough to model all the dynamics. This model inaccuracy will certainly lead to estimation errors. Third, the parameters oscillations are also possible during the experiment, e.g. due to motor temperature changes, etc. The oscillation around the nominal value may reflect this actual parameter fluctuations of during experiment.

Despite the inaccuracies mentioned above, the proposed algorithm can still indicate abrupt changes of the estimated parameters. This indication is very important to fault detection. Figure 4.17 gives an example of the stator resistance response to a stator winding fault. As demonstrated in simulation studies, the algorithm is able to detect this kind of fault by detecting the abrupt changes in the monitored parameters. After a fault is detected, fault diagnosis is necessary to obtain detail information about the fault based on the measurement. In the next chapter, a PSO based fault diagnosis algorithm will be introduced for the diagnosis of the stator winding turn-to-turn fault.
Figure 4.12. Comparison of measured and filter stator voltage signals

Figure 4.13. Estimations of the stator inductance $R_s$
Figure 4.14. Synchronization of $d$-axis current $i_d$

Figure 4.15. Synchronization of $q$-axis current $i_q$
Figure 4.16. Synchronization of rotor speed $\omega_r$

Figure 4.17. Stator resistance response to a stator winding fault
4.3. Summary

This chapter presents a series of algorithms for the nonlinear fault detection of PMSM. First, a nonlinear parity based sensor fault detection algorithm is proposed to detect the sensor faults for $i_d$, $i_q$ and $\omega_\alpha$. Simulation results show that the proposed algorithm can successfully detect single sensor fault related to the above signals but not simultaneous multiple sensor faults. Furthermore, there are no physical sensors for the direct detection of $i_d$ and $i_q$, a fault in a virtual sensor for $i_d$ and $i_q$ only indicates a fault with the phase current and rotor position sensors. After that, a nonlinear parameter estimation approach was proposed for the detection of PMSM internal parameters changes, which may represent symptoms of internal faults. Simulation and experimental studies show that the proposed algorithm can estimate not only constant parameters but also slowly time varying and abruptly changing parameters in a fast manner. This property is especially useful for online and practical applications. After a fault is detected, the fault data will be further analyzed to obtain more information about the fault. In next chapter, a particle swarm optimization based fault diagnosis algorithm will be introduced to find the fault location and severity of a detected stator winding turn-to-turn fault.
CHAPTER 5
PARTICLE SWARM OPTIMIZATION BASED
DIAGNOSIS FOR PMSM STATOR WINDING FAULT

In Chapter 4, several fault detection algorithms have been proposed for PMSM. The detection algorithms can detect both sensor faults and internal faults. After a fault is detected, more information is needed for maintenance or other upper level corrective control. Fault diagnosis is necessary to obtain this information. The objectives of fault diagnosis include identifying the location and severity of a fault. As described in Chapter 3, the PMSM model under the stator winding turn-to-turn fault condition is highly nonlinear, which makes the fault diagnosis a challenging problem. This chapter proposes an intelligent optimization, Particle Swarm Optimization (PSO), algorithm based approach for the diagnosis of PMSM stator winding fault. The algorithm can successfully identify the important parameters in the fault model for diagnosis purposes. The structure of this approach is very simple and can be extended to a broad class of parameter identification problems.

This chapter is organized as follows. First, a brief review is given on the proposed PMSM model under a stator winding fault condition. Based on this review, the fault diagnosis problem is formulated as a parameter identification problem. The difficulty in this identification is also analyzed based on the fault model structure with inherent high nonlinearities. To solve this problem, a PSO based fault diagnosis approach is proposed to avoid the formidable analysis required by other nonlinear identification algorithms. For better understanding, an overview is given for a broad class of intelligent optimization algorithms in Section 5.2, where the advantages of these algorithms are addressed. Then, a detailed introduction of the PSO algorithm is given. After that, the PSO based approach is presented for PMSM stator winding fault diagnosis. The method is applicable to parameter identification in general. Simulation results are provided in
Since it is difficult to simulate PMSM fault conditions experimentally, the PSO based identification approach is applied to identify model parameters for PMSM under healthy operations. This verifies the method’s effectiveness and proves its extendibility to a general class of identification problems.

5.1 Fault Diagnosis Problem Formulation

In Chapter 3, a PMSM model under stator winding turn-to-turn short fault was proposed based on the magnetic circuit principle. The model is briefly reviewed here since it will be used for the following fault diagnosis.

Comparing to the traditional PMSM model under healthy operations, the fault model contains two new parameters, $\sigma$ and $\theta_f$. $\sigma$ denotes the fault size, which is defined as the ratio of the shorted turns ($N_f$) to the total turn number $N_s$. $\theta_f$ denotes the fault location, which is the angle between the faulted phase and the phase $a$ axis. Thus, $\theta_f$ can only take one of the three values, $-2\pi/3$, 0, or $2\pi/3$, corresponding to a stator winding fault in phase $a$, $b$, or $c$ respectively.

Defining the new electrical quantities in the fault model with the subscript $f$ for fault, the motor voltage equation in matrix format can be expressed as (5.1).

$$
\begin{align*}
\mathbf{v}_{abcf}^s &= r_{abcf} \mathbf{i}_{abcf}^s + L_s \frac{d\mathbf{i}_{abcf}^s}{dt} + \frac{d\Psi_{mabcf}}{dt} \\
\text{where} & \\
\mathbf{v}_{abcf}^s &= \begin{bmatrix} v_{a}^s & v_{b}^s & v_{c}^s & v_{f}^s \end{bmatrix}^T, \quad \mathbf{v}_{f}^s = 0 \\
\mathbf{i}_{abcf}^s &= \begin{bmatrix} i_{a}^s & i_{b}^s & i_{c}^s & i_{f}^s \end{bmatrix}^T \\
\Psi_{mabcf} &= \begin{bmatrix} \Psi_{ma} & \Psi_{mb} & \Psi_{mc} & \Psi_{mf} \end{bmatrix}^T
\end{align*}
$$

Faulted permanent magnetic fluxes, $\Psi_{mabcf}$, can be expressed as a vector of periodic functions of the rotor position, $\theta$, fault location angle, $\theta_f$, as in (5.2), where constant $\Psi_m$ is the magnitude of the permanent magnet flux linkage.
Under the fault condition, the matrices, $R^r$ and $L_s$, become functions of $\sigma$ and $\theta_f$. For a fault in phase $b$, the resistance $R^r$ can be expressed in (5.3).

$$R_{abef}^r = \begin{bmatrix} eR_s^r & 0 & 0 & 0 \ 0 & (1-\sigma)R_b^r & 0 & 0 \ 0 & 0 & R_e^r & 0 \ 0 & 0 & 0 & \sigma R_f^r \end{bmatrix}$$ (5.3)

As shown in (3.30), calculation of inductance matrix $L_s$ is very complex. For simplification, the nonsalient pole PMSM with smooth airgap is taken as an example to illustrate the nature of this fault diagnosis problem. In this case, $L_s$ can be simplified with the inductance leakage neglected as described in (5.4). The general expression of $L_s$ can be found in (3.33).

$$L_s = \begin{bmatrix} L & (1-\sigma)M & M & \sigma M & \varnothing \ (1-\sigma)M & (1-\sigma)^2 L & (1-\sigma)M & (1-\sigma)\sigma M \ L & (1-\sigma)M & L & \sigma M \ \sigma M & (1-\sigma)\sigma M & \sigma M & \sigma^2 L \end{bmatrix}$$ (5.4)

In (5.4), $L$ and $M$ are abbreviated notations for PMSM inductance constants. Calculation of these constants can calculated according to (3.6) or (3.30).

During fault diagnosis, both the fault location and the fault severity (size) need to be identified. For the above fault model, the fault diagnosis problem can be viewed as a parameter identification problem. Specifically, two parameters need to be identified, the fault size $\sigma$ and the fault location $\theta_f$. This identification task is very challenging because the fault model in (5.1)–(5.4) contains nonlinearities in both the inputs and the parameters. This model structure is defined as nonlinear inputs and nonlinear parameters (non-LI and non-LP) and is considered one of the most
challenging identification problems according to Walter and Pronzato [108]. Therefore, the fault diagnosis of PMSM is difficult. Furthermore, the two parameters to be estimated are of different types, i.e. $\sigma$ is a continuous variable and $\theta_f$ is a discrete variable. Although the identification of linear systems is mature, the identification algorithms developed for linear systems cannot be applied to the identification problem described above. To solve this problem, a PSO based fault approach is proposed for PMSM stator winding fault diagnosis.

Background knowledge of the PSO based diagnosis approach is given in the next section. First, a general overview of intelligent optimization algorithms is introduced. Then, PSO, one of the latest intelligent optimization algorithms, is introduced and illustrated using some diagrams. The advantages of PSO are summarized, which make it suitable for the fault diagnosis of PMSM in this work.

5.2 Introduction to Particle Swarm Optimization Algorithm

5.2.1 Overview of Intelligent Optimization Using Stochastic Search

Here, intelligent optimization refers to a broad category of population based stochastic optimization algorithms, such as Differential Evolution (DE), Genetic Algorithms (GA), Particle Swarm Optimization (PSO), etc. Intelligent optimization algorithms are considered advantageous compared with classic optimization methods if the optimization problem is complex, stochastic, or highly nonlinear with multiple local optima. Specifically, the advantages of intelligent optimization lie in their intrinsic parallelism, ability to solve huge and complex problems, minimum requirement on domain specific knowledge, etc. Details about these advantages are discussed as follows.

First, these intelligent optimization algorithms are intrinsically parallel. Most classic algorithms are serial and can only explore the searching space in one direction at a time. This difference between classic algorithms and intelligent algorithms is illustrated in Figure 5.1. From this figure, it can be seen that if the solution discovered turns out to be suboptimal, there is nothing to do but abandon the current search and start over. Instead, the intelligent optimization algorithms can explore the solution space in multiple directions simultaneously. If one path does not work, they can easily eliminate that path and continue work on more promising ones. This provides a greater chance to find the optimal solution.
Due to the parallelism, intelligent optimization algorithms are particularly suitable for huge problems where the solution space is too vast to search exhaustively in reasonable time. For example, for the defensive islanding problem of a power system consisting of \( n \) transmission lines described in [109], the number of possible solutions is equal to \( 2^n \). Thus, the search space is huge even for a medium scale power system with 41 lines because of the existence of \( 2^{41} = 2.199*10^{12} \) possible solutions. The implicit parallelism of the intelligent optimization algorithms allow them to successfully find optimal or very good results in a short time after exploring a small region of the searching space.

Another notable strength of the intelligent optimization algorithms is that they perform well for complex problems with multiple local optima. Figure 5.2 illustrates the existence of local optima in a two-dimensional searching space. Most practical problems are much more complex than this example and may have a vast searching space that is impossible to search thoroughly. Many classic search algorithms can be trapped in local optima. Intelligent optimization algorithms have been proven to be effective at escaping from local optima and discovering the global optimum in complex searching spaces. It should be noted that, sometimes, there is no way to tell whether a solution is the global optimum or just a very good local optimum. However, even when the
intelligent optimization algorithms cannot find the global optima, it can usually find a good local optimum.

The last advantage worth mentioning here is that these intelligent optimization algorithms do not require detailed domain specific knowledge as other optimization algorithms do. This advantage is very important for their possible application to the fault diagnosis problem here. Instead of using domain specific information to guide the search, these algorithms make random changes to their candidate solutions and then use a fitness function, often multivalued, to determine whether those solutions are good or not.

As a new development of the intelligent optimization algorithms, Particle Swarm Optimization is simple in concept and highly computationally efficient. It has been proved to be a very powerful algorithm and has been applied to a lot of practical problems in the past years. For the above mentioned defensive islanding problem, the PSO algorithm can find efficient solutions by investigating just a very small percent of the candidate solutions, i.e. 400 instead of $2.199 \times 10^{12}$ solutions [109]. This work explores the application of PSO to the PMSM fault diagnosis problem. Details of the Particle Swarm Optimization are given next.
5.2.2 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a population based stochastic search algorithm. It was first introduced by Kennedy and Eberhart in 1995 [112]. Since then, it has been widely used to solve a broad range of optimization problems. The algorithm was presented as simulating animals’ social activities, e.g. insects, birds, etc. It attempts to mimic the natural process of group communication to share individual knowledge when such swarms flock, migrate, or hunt. If one member sees a desirable path to go, the rest of this swarm will follow quickly. In PSO, this behavior of animals is imitated by particles with certain positions and velocities in a searching space, wherein the population is called a swarm, and each member of the swarm is called a particle. Starting with a randomly initialized population, each particle in PSO flies through the searching space and remembers the best position it has seen. Members of a swarm communicate good positions to each other and dynamically adjust their own position and velocity based on these good positions. The velocity adjustment is based upon the historical behaviors of the particles themselves as well as their companions’. In this way, the particles tend to fly towards better and better searching areas over the searching process [112][115]. The searching procedure based on this concept can be described by (5.5).

\[ v_i^{k+1} = w v_i^k + c_1 rand_1 \times (pbest_i - x_i^k) + c_2 rand_2 \times (gbest - x_i^k) \]  \hspace{1cm} (5.5a)
\[ x_i^{k+1} = x_i^k + v_i^{k+1} \]  \hspace{1cm} (5.5b)

In (5.5), \( c_1 \) and \( c_2 \) are positive constants, defined as acceleration coefficients; \( w \) is the inertia weight factor; \( rand_1 \) and \( rand_2 \) are two random functions in the range of \([0,1]\); \( x_i \) represents the \( i^{th} \) particle and \( pbest_i \) the best previous position of \( x_i \); \( gbest \) is the best particle among the entire population; \( v_i \) is the rate of the position change (velocity) for particle \( x_i \). Velocity changes in (5.5a) comprise three parts, i.e. the momentum part, the cognitive part, and the social part. This combination provides a velocity getting closer to \( pbest \) and \( gbest \). Every particle’s current position is then evolved according to (5.5b), which produces a better position in the solution space. Figure 5.3 is a conceptual illustration of this searching process according to (5.5).
According to (5.5), several factors impact the performance of the PSO algorithm, i.e. the inertia weight factor $w$ and the two acceleration coefficients, $c_1$ and $c_2$. Since the introduction of the PSO method in 1995, a considerable amount of work have been done in improving the original version of PSO by varying these three factors. Using different or time varying inertia weight factor can balance the local and global search during the optimization process. The acceleration constants serve dual purposes in this algorithm. First, they control the relative influence toward $g_{best}$ and $p_{best_i}$ respectively. Second, the two acceleration coefficients combined form what is analogous to the step size of an adaptive algorithm. Acceleration coefficients closer to zero will produce fine searches of a region, while coefficients closer to one will result in lesser exploration and faster convergence. Setting the acceleration greater than one allows the particle to possibly overstep $g_{best}$ and $p_{best_i}$, resulting in a broader search. Further discussion about the proper parameter setting can be found in [113]-[116] and their references.

The implementation procedure of the PSO algorithm can be illustrated with the flowchart in Figure 5.4. According to Figure 5.2, the optimization process can be divided into six steps as follows.

(i) Initialization. During this step, the bounds of the each element are set for the position and velocity particles; the population is initialized randomly; the parameters in (5.5) and the terminal condition are all preset; and $p_{bests}$ are set to be this initial population.
(ii) Evaluation of the initial population. In this step, the cost for all particles in the initial population are evaluated according to a fitness (cost or objective) function; the global best particle, $g_{best}$, is chosen to be the particle with the best fitness.

(iii) Updating position and velocity. The position and the velocity particle are updated according to (5.5a) and (5.5b) respectively; check if the updated position and velocity particles are beyond their corresponding limits; if it happens, the over limit elements are set to the corresponding limits.

(iv) Evaluation of the updated population. Similar to (ii), the updated position particles are evaluated according to their fitness; the $p_{best}$ and $g_{best}$ particles will be updated if necessary.

(v) Check if the terminal condition is satisfied. This is done according to the preset terminal

![Flowchart of the PSO algorithm](image)

Figure 5.4 Flowchart of the PSO algorithm
condition, such as number of iterations reaching the maximum, quality of the best solution being satisfactory, etc. If the terminal condition has not been satisfied, the updating process (iii) will be repeated; otherwise, the optimization process ends.

(i) Output results. The best solution obtained during the optimization process, gbest, is output in this step.

From the updating rules and the flow chart of PSO, it can be seen that PSO is very simple in concept and easy in realization. Thus, PSO gains popularity in recent years. Since the original PSO can only be applied to continuous variables problems, a modification to the original PSO algorithm is necessary. An extension of PSO for binary variable problems was proposed by Kennedy, Eberhart, and Mohan, Al-kazemi [110][111]. But there is no publication, which clearly deals with these general discrete variable problems, where the variables may take more than two different values. Actually, the modification is quite simple. For discrete variables, after the new velocity is calculated from (5.5 a), it is used to update the position using (5.5 b). Since this updated position from (5.5 b) may not be a feasible solution, it needs to be rounded to the nearest feasible discrete value. This process will be further explained during the simulation studies for the PMSM fault diagnosis problem.

In the following sections, the PSO based parameter identification approach will be applied to general parameter identification and fault diagnosis of the stator winding fault of PMSM. Simulation and experiment results demonstrate the effectiveness of the proposed approach.

5.3 PSO Algorithm Applied to PMSM Identification and Stator Fault Diagnosis

Due to the advantages discussed previously, PSO has been successfully applied to many practical problems. It is especially helpful for those problems that cannot be solved using traditional optimization algorithms. This section first introduces a PSO based identification approach that can be applied to general parameter identification problems. Then, the application of the proposed approach for PMSM stator winding fault diagnosis is presented.
5.3.1 General Parameter Identification Using Particle Swarm Optimization

For a system with known model structure but unknown parameters, the parameter identification problem can be treated as an optimization problem. The basic idea is to compare the system output with the model output. The discrepancy between the system and model outputs is minimized by optimization based on a fitness function. The fitness function is defined as a measure of how well the model output fits the measured system output.

Generally, a system’s dynamics can be described using a differential equation such as (5.6):

\[ \dot{x} = f(p, x, u) \]
\[ y = g(p, x) \]  
(5.6)

In (5.6), functions \( f(p, x, u) \) and \( g(p, x) \) can be either linear or nonlinear. The parameter vector, \( p \), is the unknown to be identified. To identify \( p \), a model of the system is introduced as (5.7).

\[ \dot{\hat{x}} = f(\hat{p}, \hat{x}, u) \]
\[ \hat{y} = g(\hat{p}, \hat{x}) \]  
(5.7)

According to (5.7), the same system input, \( u \), is fed into the system model, and the model has the same model structure as (5.6), i.e. same \( f(p, x, u) \) and \( g(p, x) \). The parameter vector to be identified is denoted using \( \hat{p} \). Since the states and outputs of the model are calculated based on system input and the estimated parameters, \( \hat{x} \) and \( \hat{y} \) are used to denote the states and the output of the model. To evaluate the parameters to be identified, system output \( y \) is compared with model output \( \hat{y} \). The fitness function can be defined as the weighted quadratic function shown in (5.8).

\[ C(\hat{p}) = \int (y - \hat{y})^T W (y - \hat{y}) dt \]  
(5.8)

In (5.8), \( W \) is a positive definite matrix. Obviously, the fitness function is a function of the estimated parameter vector, \( \hat{p} \). The identification error will result in a nonzero \( C(\hat{p}) \). Thus, the fitness function can be used to guide the search for better estimation. Now, the identification problem has been transformed into an optimization problem. Since traditional algorithms usually have difficulties optimizing complex nonlinear systems with multiple local optima, the intelligent optimization algorithms mentioned before are a better choice for this problem.
The structure of the PSO based parameter identification approach can be illustrated using Figure 5.5. First, the system input, \( u \), is given to both the system to be identified and the system model. Then, the outputs from the system and its model are input to the performance evaluator, where the fitness will be calculated. The calculated fitness \( C(\hat{p}) \) is then input to the PSO based identifier to identify the unknown parameter vector \( \hat{p} \). For offline applications, the identification process will end after \( \hat{p} \) has been successfully identified. For online applications, after \( \hat{p} \) is successfully identified or the preset maximum iteration number has been reached, the estimated \( \hat{p} \) will be used to update the system model, and then the above process will be repeated.

### 5.3.2 PSO Based Diagnosis of PMSM Stator Fault

To apply the above PSO based identification approach to the fault diagnosis of PMSM stator winding turn-to-turn fault, the cost function needs to be defined for this problem specifically. In the defined cost function (5.11), the phase currents \( i_{abc} \) and the rotor speed \( \omega_r \) are the measurable system outputs, \( \hat{i}_{abc} \) and \( \hat{\omega}_r \) are the estimated values of \( i_{abc} \) and \( \omega_r \), which are calculated from the system model, \( \sigma \) and \( \theta_f \) are the unknown parameters to be identified for diagnosis purposes, \( n \) is the number of sampled data, and the weighting factor \( w \) is defined as \( w = \frac{|i_{abc}|}{|\omega_r|} \).

\[
C(\sigma, \theta_f) = \frac{1}{n} \sum_{k=1}^{n} \left[ (i_a(k) - \hat{i}_a(k))^2 + (i_b(k) - \hat{i}_b(k))^2 + (i_c(k) - \hat{i}_c(k))^2 + w(\omega_r(k) - \hat{\omega}_r(k))^2 \right]^{\frac{q}{2}} \quad (5.11)
\]

![Figure 5.5 Block diagram of the intelligent optimization based identification approach](image-url)
In the next section, simulation and experimental results are presented for PSO’s application to PMSM identification and diagnosis.

5.4 Simulation and Experimental Verification

The proposed PSO based fault diagnosis approach is evaluated under healthy and fault conditions. Both simulation and experimental studies are conducted for healthy operating conditions. Since it is difficult to do a fault test on a real motor, the fault diagnosis is only evaluated based on simulation. That is, this dissertation experimentally validates PSO based diagnostics for a healthy PMSM. Experimental validation for a faulted PMSM is left to future work. The simulations results are given as follows.

5.4.1 Parameter Identification of Healthy PMSM

In this section, both offline and online simulations are performed to evaluate the performance of the PSO based parameter identification. After that, the identification approach is also used to identify PMSM parameters based on a sample of experimental data from a practical PMSM testbed.

5.4.1.1 Simulation Studies

In the simulation setup, a three-phase nonsalient pole PMSM is controlled by a variable frequency drive system with cascaded PI-controllers (Figure 3.8). Nominal parameters of the simulated PMSM are from motor datasheet and can be found in Table 4.3. To evaluate the proposed approach’s ability for online applications, the stator resistance $R_s$, motor inertia $J$, and frictional coefficient $B_m$, are set to be time varying around their nominal values. According to the PMSM mechanical model (3.9), the changing inertia $J$ and viscous frictional coefficient $B_m$ can be treated as load torque disturbances. Thus the total disturbed torque $T_{ld}$ equals to the sum of the actual load torque $T_L$ and the disturbances caused by the inertia and the frictional coefficient variations as in (5.12).
\[ T_{ld} = T_L + \ddot{J} \frac{1}{n_p} \frac{d\omega_r}{dt} + \dot{B} \frac{1}{n_p} \omega_r \]  \hfill (5.12)

In (5.12), \( \ddot{J} \) and \( \dot{B} \) are defined as \( \ddot{J} = J - J_0 \) and \( \dot{B} = B - B_0 \) respectively, where \( J_0 \) and \( B_0 \) denote the nominal values of motor inertia and viscous friction coefficient. In this simulation, there are two parameters to be identified. One is the stator resistance \( R_s \), and the other is the disturbed load torque \( T_{ld} \). These two parameters are chosen here to demonstrate the PSO based algorithm’s capability in identifying multiple parameters simultaneously.

During simulations, each swarm generation contains 10 position particles. \( w, c_1, \) and \( c_2 \) in (5.1) are set to 0.8, 2, and 2 respectively. Figure 5.6 shows the optimization process of the proposed algorithm for a sample dataset. In this figure, (a) shows the optimization process of the fitness function (cost), (b) and (c) show the optimization process of the estimated parameters, \( R_s \) and \( T_{ld} \), respectively. Simulation studies show that the proposed approach converges here within 20 generations. This 20 generation upper bound was routinely observed. The efficiency and accuracy of the approach make it suitable for applications that require fast solutions.

Based on the observation from offline identification, the PSO based identifier can give good estimation within 20 iterations in this application to PMSM. To ensure the estimation accuracy, the maximum number of allowed iteration is set to 30 for each sample of data. During simulation, the sampling frequency is set to 100kHz. That means 3300 pairs of data are sampled within 0.033 second. Then the sample data set is input to the PSO based identifier to estimate the stator resistance \( R_s \) and the load torque \( T_{ld} \). After PSO completes its searching of 30 iterations, the identification process for this data set stops. Then, the estimated parameters will be used as the initial values for the estimation of next data set. The above process repeats again and again during the online applications.

Figure 5.7 and Figure 5.8 show the online identification results of the proposed algorithm. From these figures, it can be seen that the proposed approach can identify time varying parameters successfully. Figure 5.9 shows the entire optimization process of the PSO algorithm during online identification. For each run of the PSO based algorithm, the previously optimized parameters are used as the initial value. In doing so, the identification process for online applications tends to converge faster in terms of less number of iterations. This is especially true for systems with slowly time varying parameters. From Figure 5.9, it can be seen that the algorithm usually converges within 10 iterations because of a good initial guess. But from the figure we can also see
that the initial fitness for the runs of 5, 6, 7, and 8 are not as good as others. This is because the fitness’s sensitivity to parameter changes is different for different operating points. Sometimes, even a small change in the parameters will result in a large change in the fitness.

![Graphs showing PSO optimization process for one dataset](image)

**Figure 5.6** PSO optimization process for one dataset
(a) Fitness function  (b) Estimation of stator resistance $R_s$  (c) Estimation of load torque $T_L$
Figure 5.7 Identification of the stator resistance $R_s$

Figure 5.8 Identification of the load torque $T_{Ld}$
Note that the convergence speed of the identification for one data set is evaluated in terms of
the number of PSO iterations. The time required is not evaluated strictly during the online
application. In this online application, the simulation for next period does not begin until the 30
iterations for the previous period are completed. Simulations in Matlab and Simulink show that
the identification for one dataset can be done on the order of minutes. But this should not be used
as a precise evaluation on the required time. The computational speed is impacted by several
factors, such as implementation hardware setup, the sampling frequency, and the choice of
Ordinary Differential Equation (ODE) solvers, etc. As just mentioned, for continuous online
applications, tremendous computation time can be saved due to good guesses of initial values for
each run. Therefore, the computation time should not be a big issue for online implementations.

Besides comparing the actual and identified parameters, the PSO based identification approach
can also be evaluated by comparing the performances of the identified model with the original
system. This is important since any errors in the estimated parameters may result in deviations in
system dynamic responses. This comparison is shown in Figure 5.10. In this figure, (a) is the stator
phase a current $i_a$, and (b) is the rotor speed $\omega_r$. Since the performances of the two models match
each other quite well, it is very difficult to observe discrepancy from in Figure 5.10.
5.4.1.2 Experimental Studies

In order to further investigate the proposed identification approach, experimental studies are performed on the practical PMSM drive system as described in Section 4.2.3 (Figure 4.6). During the experimental study, the PSO based parameter estimation algorithm is tested with the measured data from the PMSM under normal operating conditions. The procedure of this experimental verification is similar to that of the simulation described previously, except that the real data are utilized. Experimental results are provided in Figure 5.11 and Figure 5.12. In Figure 5.11, raw data are used directly to identify the parameters of interest. In Figure 5.12, the measured raw data are filtered first to eliminate high order harmonics. From these figures, it can be seen that the proposed
algorithm works well for the raw data and that the performance can be further improved by prefiltering.

During the experimental study, the stator resistance $R_s$ is chosen to be identified with the rest parameters set to their nominal values given in the datasheet (Table 4.3). Experimental results show that the identified stator resistance $R_s$ is with acceptable accuracy and the $dq$-model with the identified parameters can give a good approximation to the experimental data. However, the identified parameters obtained from the experimental study are not identical to their ideal values as shown in the simulation studies. The first reason for this deviation is that the values in the datasheet are given at the nominal operating condition, which is different from the experimental environment. The second possible reason is that the $dq$-model used is not accurate enough to represent all the dynamics of the PMSM. The third reason is that the measured data is far from ideal, which can be clearly observed from the raw data in Figure 5.10. In general, the above experimental results are acceptable considering these three factors. The PSO based identification approach should be applicable for the purpose of fault diagnosis.

5.4.2 PSO based Approach for PMSM Stator Winding Fault Diagnosis

The target of this subsection is to demonstrate the performance of the PSO approach for the diagnosis of PMSM stator winding fault. Since it is very difficult to simulate a stator winding fault on a practical PMSM, simulation is the only way to verify the capability of the proposed approach at the present stage.

As mentioned before, the two parameters $\sigma$ and $\theta_f$, representing the ratio of the shorted turns to the total turn number and the angle between the faulted phase and the phase $a$ axis respectively, are directly related to the fault severity and location. Thus, the objective of the stator winding fault diagnosis is to identify these two parameters.

In our problem, $\theta_f$ can only take three different values, $-2\pi/3$, $0$, or $2\pi/3$. So a modification to the standard PSO algorithm is necessary. First, during the initialization process, the values corresponding to $\theta_f$ are randomly set to one of the three values. Second, during the calculation of its updated position, the new position particle is calculated based on the updated velocity particle, which is a continuous variable. This updated position particle is then rounded to its nearest feasible value ($-2\pi/3$, $0$, or $2\pi/3$). Thus the updating equations for this mixed variable optimization
problems are consistent with (5.5) with some special treatment to the discrete variables as just described.

Figure 5.11 Experimental results with raw data: estimation vs. measurement
(a) Phase $a$ current $i_a$  (b) Rotor speed $\omega_r$
During simulation, a 4.5% turn-to-turn fault is applied to stator phase $b$ winding at 0.3 second. The fault of this magnitude represents a typical incipient fault. If the proposed approach can successfully identify this small fault, then the approach can certainly identify larger faults. The phase currents responses are displayed in Figure 5.13. It can be seen that the original balanced three-phase current becomes slightly unbalanced after the fault. Figure 5.14~Figure 5.16 shows a typical optimization process of the PSO based fault diagnosis approach. Simulation studies show that the optimization process always converges within 25 iterations. From Figure 5.14, it can be seen that the estimation of $\theta_f$ converges to the ideal value even at the first iteration. This is because the model with ideal value provides much better fitness compared to the other two values. Thus,
the best solution of $\theta_i$ can always be found during the evaluation of the initial population. Figure 5.15 shows the estimation process of the fault size parameter $\sigma$. And the optimization process of fitness function plot is shown in Figure 5.16.

![Figure 5.13](image1.png)  
**Figure 5.13**  
PMSM phase current under a 4.5% stator fault in phase $b$

![Figure 5.14](image2.png)  
**Figure 5.14**  
Fault location estimation during a 4.5% stator fault in phase $b$
5.5 Summary

Because of the nature of the nonlinearity of the fault model, existing nonlinear theory based identification algorithms are difficult to apply to the fault diagnosis problem of PMSM stator winding fault. To solve this problem, the fault diagnosis is formulated as an identification of parameters that represent the fault severity and location. The identification is based on
optimization of a defined fitness function. As an efficient intelligent optimization algorithm, PSO is applied to the identification for fault diagnosis purpose. The proposed approach can be applied to the identification of a general class of nonlinear systems. Extensive simulation and experiment studies demonstrate the effectiveness of the proposed approach. Identification of a healthy PMSM is conducted as a partial evaluation. In the future, fault experimentation is still necessary for further validation.
CHAPTER 6
INTEGRATED FAULT DETECTION AND DIAGNOSIS SYSTEM

Based on the fault detection and diagnosis algorithms proposed in previous chapters, this chapter develops an integrated FDD system structure that can handle system uncertainty reflected as time varying model parameters. In Section 6.1, the robustness problem in FDD design is described and illustrated with an example. Then, the robust FDD system structure is developed in Section 6.2 by integrating the proposed algorithms. The execution procedure of this FDD system is explained therein.

6.1. Robustness Problem in FDD Design

An important issue in practical applications of model based FDD systems is their robustness to model inaccuracy. Discrepancies or errors between system models and reality are referred to as uncertainty. Generally speaking, uncertainties can be either structured or unstructured. Structured uncertainties are represented by model parameter variations, while unstructured uncertainties indicate that it is unknown how they impact the system model. Both types cause robustness problems and degrade the performance of fault detection. Only structured uncertainties are considered in this work.

According to the residual designs for sensor fault detection in (4.17)–(4.20), the two residuals are closely related to PMSM’s physical parameters, such as stator’s resistance and inductance, etc. In order to achieve the residual structure in Table 4.2 through disturbances and faults decoupling, residuals’ dependency on model parameters becomes unavoidable. During residual implementation, simpler lower order designs are preferable to reduce residuals’ dependency on model parameters. An appropriate residual implementation can achieve relatively better
robustness against structured parametric uncertainties. For this PMSM application, it is impossible to eliminate the parametric dependency completely. When the system parameters fluctuate with the operating condition changes, the residuals will not stay zero during the fault free conditions. Thus false alarms may be raised for a nonexistent fault. False alarms will decrease system availability and increase system operational cost by performing unnecessary maintainances or mistakenly replacing healthy components, etc. The system parameters fluctuations are common for PMSMs operating in practical operating conditions. For example, when PMSM is running in high temperature environment, the stator resistance \( R_s \) can increase to one or two times its nominal value [86]. Also, the stator inductances, usually given as \( dq \)-axis parameters, are susceptible to the magnetic saturation effect. Even though these uncertainties may not impact motor operations, their impacts on fault detection systems are not ignorable. Special attention is required to obtain robust fault detection designs.

To demonstrate the robustness problem in PMSM sensor fault detection, some simulation results are provided in Figures 6.1 and 6.2. The simulation setup is very similar to the one used in Section 4.1.3. The only difference is that small variations are introduced to the stator resistance \( R_s \). As illustrated in Figure 6.1, \( R_s \) fluctuates within \( \pm 5\% \) of its nominal value. The residual responses under this parameter uncertainty are shown in Figure 6.2. Compared with the ideal case results in Figure 4.1(a), the residuals obtained from (4.17)–(4.20) do not hold to zeros anymore because of the fluctuation of \( R_s \). Since \( R_s \) appears in these residual equations, this phenomenon is predictable. To avoid false alarms causing by nonzero fault free residuals, a robust residual design with least relevance to system parameters is preferable. However, as mentioned before, completely decoupling the model parameters from the residuals is impossible for this problem. Therefore, alternative methods need to be found to improve existing residual designs.

Robustness of sensor fault detection can be improved by either improving the designs of residuals or adjusting the predefined thresholds for residual evaluations. Since it is very difficult to predict the variation of the residuals during practical operations, adjusting the thresholds is not an easy job. If the threshold is too small, the fault detection algorithm will give more false alarms. On the other hand, if the threshold is too large, the fault detection algorithm may fail to detect incipient or small faults. Since the synchronization based parameter estimation algorithm proposed in Chapter 4 can successfully detect parameter changes online, its integration with the proposed
sensor fault detection algorithms should be able to provide a more robust FDD system design. This robust FDD structure is introduced in the next section.

Figure 6.1  Source of uncertainties: time varying PMSM stator resistance, $R_s$

Figure 6.2  Residuals of the original design at presence of parameter variations
6.2. Integrated FDD System Structure

To design a robust FDD system, the FDD algorithms proposed in Chapter 4 and Chapter 5 are integrated together. The integrated structure is shown in Figure 6.3. This structure includes various modules for sensor fault detection, internal fault detection, and fault diagnosis. Robustness to system parametric uncertainties is a significant advantage brought about by the integration.

During fault detection, the system input and output data are fed into the parameter identifier and the residual generator simultaneously. The parameter identifier can identify changes in system model parameters. The identified parameters are then input to the internal fault evaluator to determine whether an internal fault appears or not. If there is no change in the parameters, or the changes are resulted from uncertainties relating to normal operating condition variations, no fault will be reported to the fault diagnosis module. In this case, the estimated parameters will be used to update the residual generator. The residual generator produces residuals for sensor fault detection based on the system measurement and the updated parameters. The obtained residuals will be sent to the sensor faults evaluator to make a decision about sensor status based on the preset threshold. If the sensor fault evaluator determines that a sensor fault happens, the fault data will be sent to the fault diagnosis module. When a fault decision is made by internal fault evaluator about system

![Figure 6.3 General structure of robust fault detection system](image-url)
internal anomalies, the fault data will also be sent to the fault diagnosis module. Based on the fault data from both fault evaluators, the diagnosis module can perform further analysis to find out the exact information, such as fault location and fault severity. The modules of sensor fault detection, internal fault detection, and fault diagnosis form the overall fault detection and diagnosis system.

To demonstrate the performance of the proposed structure, the robust fault detection system is used to detect sensor fault under the same $R_s$ changes as Figure 6.1. Figure 6.4 shows the online identification of the stator resistance $R_s$ based on the parameter estimation algorithm in Section 4.2. Since the parameter identifier can give a good estimation of the changing parameters, the residuals calculated according to the updated parameters remain zeros for the fault free case. This can be observed from Figure 6.5.

### 6.3. Summary

To construct an integrated FDD system, the algorithms proposed in previously chapters are synthesized into a general FDD framework. In this FDD framework, the robustness of the sensor fault detection is improved by combining the synchronization based parameter estimation and the parity relation based sensor fault detection algorithms. The estimated system parameters are used to update the residual generator. Thus, the false alarms rate can be decreased and the sensor faults can be detected more accurately. Correspondingly, robust sensors also can provide accurate information for the parameter estimation process. There is still a lot of work to be done to improve the performance of the proposed FDD system structure. This is to be further discussed in the next chapter.
Figure 6.4 Online identification of stator resistance $R_s$

Figure 6.5 Robust fault free case residuals
CHAPTER 7
CONCLUSIONS AND FUTURE WORK

In this dissertation, a series of algorithms are proposed for the fault detection and diagnosis of PMSM. The studies include fault modeling, fault detection, and fault diagnosis, which are presented in Chapters 3, 4, and 5 respectively. In this chapter, first, the proposed model and algorithms are summarized with main results and conclusions highlighted. After that, possible research directions are suggested as future work based on the unsolved problems.

7.1 Conclusions

Electric machines are important components for industries and special applications. The continuous healthy operations of machines are critical for the reliability of the entire system. Effective FDD can detect and diagnose abnormal conditions in time, and thus reduce the chances of catastrophic failures. Due to its high efficiency, high power density, and robustness, PMSM has been widely used, especially in systems that have demanding requirements on reliability, such as navy shipboard power systems. For a Navy ship that may operate in extreme and hostile environments, effective FDD at the early stage is especially important for survivability. Today, the FDD of PMSM has not been fully explored and there is still a big gap between available theories and practical applications. This dissertation aims at designing an integrated FDD system for PMSM that can effectively detect sensor faults, system internal faults, and diagnose occurred or occurring faults. The contributions of this work include a stator winding turn-to-turn fault model, a parity relation based sensor fault detection algorithm, a synchronization based parameter estimation algorithm, and a PSO based fault diagnosis algorithm. The performances of the proposed model and algorithms have been demonstrated by both simulations and experiments studies. Specific results and conclusions from the proposed model and algorithms are summarized
separately in the following subsections.

7.1.1 PMSM Modeling under Stator Winding Fault

As one of the most common electric faults in PMSM, the stator winding fault needs to be detected in time to avoid larger losses. Since accurate mathematical modeling can provide more insights to PMSM dynamics during faults, fault modeling is very important to fault detection and diagnosis. Furthermore, fault models are indispensable for model based FDD algorithms, which have been proved to be advantageous in many applications. This dissertation proposes a new model consisting of four equations by applying the coupled magnetic circuit principle to the circuit model of the faulted PMSM. The proposed model is applicable to both healthy and stator winding fault conditions. Fault simulation results are consistent with observed fault symptoms. This fault model was used as a basis for the following fault detection and diagnosis algorithms. Analysis on PMSM models also revealed the inherent nonlinearity, which makes model based FDD a challenging problem.

7.1.2 Nonlinear Fault Detection for PMSM

Considering the importance of sensors to monitoring and control systems, the detection of sensor faults should be the first step in a FDD system. Based on the theory of nonlinear parity relation, a fault detection algorithm was proposed for the detection of faults in currents and speed sensors. The nonlinear parity relations were derived from PMSM’s differential equations. Based on these parity relations, the residuals were expressed in their computational form using algebraic elimination theory. Then the derivative terms in the computational residuals were eliminated to obtain the implementable residuals. Simulation for various sensor fault scenarios demonstrated the algorithm could successfully detect single sensor fault. This nonlinear parity relation method can be applicable to a lot of systems that can be represented using ordinary differential equations.

To detect PMSM internal anomalies, synchronization based adaptive parameter estimation algorithm was proposed. This algorithm performed estimation by synchronizing the PMSM model with a reference system. The general design procedure was described for the selection of feedback signals and the updating rules of the estimated parameters. The parameter estimation problem of PMSM is used as an example to illustrate the synchronization based design. Both simulation and
experimental results demonstrated the effectiveness of this proposed algorithm. Although the
design is only strictly proved for the estimation of time invariant parameters, simulation results
show that the algorithm is also valid for the estimations of slowly time varying and step changing
parameters. This property is very important for the proposed robust FDD structure.

To integrate the sensor and internal fault detection with fault diagnosis, a robust FDD structure
was proposed for online applications. In the proposed structure, the parameter estimation module
provides real time update to parameters used in the residual generator. This is important to the
robust operation of the residual generator for sensor fault detection. If a fault is identified, the fault
data will be input to the fault diagnosis module to get more detailed information for maintenance.
This robust FDD structure is generally applicable to other applications.

7.1.3 PSO Based Fault Diagnosis for PMSM

Fault diagnosis is a subsequent stage after the detection of abnormal conditions inside the
PMSM. Accurate information of fault location and severity is very helpful for post fault
maintenances. Since it is very difficult for the traditional parameter identification algorithms to be
applied to complex nonlinear systems, a PSO based parameter identification algorithm was
proposed by transforming identification problems into optimization problems. The PMSM stator
winding fault was taken as an example to explain how the PSO based algorithm works. Both
simulation and experimental results demonstrated that the proposed algorithm can successfully
identify the important parameters related to the stator winding faults. The proposed algorithm is
not system specific. It can be applied to a general class of identification and fault diagnosis
problems as long as a fitness function can be appropriately defined.

7.2 Future Work

Although a lot of important problems have been addressed in this dissertation, there is still a lot
of work to be done in the future. To be consistent with the organization of this dissertation, the
future work is divided into two categories, i.e. detailed fault modeling and algorithm development.
Both of these aspects need theoretical studies and experimental verification.
7.2.1 Detailed Fault Modeling

Though the most common electric fault in PMSM, the stator-winding turn-to-turn fault, has been modeled in this dissertation, the work on PMSM fault modeling is still inadequate.

First, during the development of the fault model, several assumption are made for simplicity, such as the assumptions on infinite magnetic permeability of iron, sinusoidal distribution of magnetic motive force and the flux, and the lack of consideration of magnetic saturation and higher order harmonics, etc. These assumptions will result in an imprecise model, which may be not good enough for model based FDD. Thus, advanced modeling techniques are needed to overcome this problem.

Second, this dissertation only considered the stator winding turn-to-turn fault. However, there are many other types of faults need to be considered, such as bearing faults, airgap eccentricity, shaft faults, etc. For a comprehensive FDD system, the possible faults should be considered to the maximum extent. Thus, a versatile model or separate models for these faults need to be developed in the future.

Since it is difficult to simulate a fault on a practical motor, the verification of the proposed fault model becomes a challenging task. To validate a fault model, the PMSM needs to be reconstructed in certain way so that the motor imitates the corresponding fault. For example, in order to test the PMSM operating with stator winding turn-to-turn fault, the stator winding has to be specially rewound. A schematic illustration of this reconstruction is shown in Figure 7.1. In this diagram, the winding of phase $b$ is equipped with several taps to mimic turn-to-turn faults of different sizes by shorting part of the winding intentionally. With this rebuilt PMSM, experiments can be performed to validate the proposed fault model.

7.2.2 Algorithm Developments

During the development of the FDD algorithms, several problems have been found challenging and left unsolved at the present stage. These problems are summarized in four aspects as follows.

First, the sensor fault detection algorithms can only detect anomalies in the immediate states of the $dq$-model, such as $i_d$ and $i_q$. Since these two virtual sensing signals are practically calculated from other measurements, the anomalies in $i_d$ and $i_q$ only indicate sensor faults with the phase
current sensors and the rotor position sensor. Thus, it cannot tell which specific sensor has a fault. In the future, more advanced algorithms need to be developed to detect the phase current sensor faults directly.

Second, the developed algorithms did not consider the signal availability problems. This is acceptable for this PMSM FDD problem, but may result in impractical designs if some data are not available because of the measurability and cost concerns. If a variable is not available, some advanced techniques, such as state observation and estimation, will need to be incorporated into the FDD algorithm design.

Third, the proposed sensor fault detection algorithm can only detect a single fault at a time and cannot deal with simultaneous multiple faults in the whole PMSM system. Although the simultaneous occurrence of multiple faults is rare, a well developed algorithm should be able to cover all the possibilities. The limitation of our algorithm comes from the mathematical restriction of decoupling more variables. Alternative solutions need to be developed in the future.

At last, the parameter estimation algorithm proposed in Section 4.2 can successfully estimate nonsalient pole PMSM parameters, i.e. $L = L_d = L_q$, and $R_s$. For salient pole PMSM with different $L_d$ and $L_q$, the estimation problem is much more difficult because of the coupling of the parameters, which is categorized into the parameter estimation problem with nonlinear parametric (non-LP) structure. So far, no general theories are available for this type of problems. In the future, more theoretical study needs to be performed to solve the estimation of multiple coupled parameter problem.
APPENDIX A

IDEALS, GRÖBNER BASIS, AND ELIMINATION THEORY

Some notation and a theorem from basic elimination theory (Cox, Little, and O'Shea 1996 [102]) are needed to describe the residual design procedure.

Let $k[x_1, x_2, \ldots, x_n]$ denotes the set of polynomials in variables $x_1, x_2, \ldots, x_n$ with coefficients in an arbitrary field $k$, e.g.

$$x_1 + x_2 x_4 + 2x_2 x_3^2 - 5x_1^3 x_4 \in k[x_1, x_2, x_3, x_4]$$ (A.1)

An important concept, ideal, is defined as follows.

**Definition 1. (Ideal).** Let $g_1, g_2, \ldots, g_s$ be polynomials in $k[x_1, x_2, \ldots, x_n]$. Then denote

$$I = \langle g_1, g_2, \ldots, g_s \rangle = \{ \sum_{j=1}^{s} h_j \cdot g_j : h_j \in k[x_1, x_2, \ldots, x_n] \}$$ (A.2)

$I$ is called the ideal generated by the polynomials $g_1, g_2, \ldots, g_s$.

This means that $I$ is the set of all linear combinations of the polynomials $g_i$ with polynomial coefficients $h_i$.

The main theorem used in the design is the well known elimination theorem:

**Definition 2. (Lexicographic order).** In mathematics, the lexicographical order, (also known as dictionary order or alphabetic order), is a natural order structure of the Cartesian product of two ordered sets. In algebra it is traditional to order terms in a polynomial, by ordering the monomials in the indeterminates. This is fundamental, in order to have a normal form. In practice one has an alphabet of indeterminates $X, Y, \ldots$, and orders all monomials formed from them by a variant of
lexicographical order. For example if one decides to order the alphabet by $X > Y > ...$ and also to look at higher terms first, that means ordering $... > X^3 > X^2 > X$ and also $X > Y^k$ for all $k$.

**Theorem. (Elimination Theorem).** Let $I \subseteq k[x_1, x_2, ..., x_n]$ be an ideal and let $G$ be a Gröbner basis of $I$ with respect to lexicographic order $x_1 > x_2 > ... > x_n$. Then, for every $0 \leq k \leq n$, the set $G_k = G \cap k[x_{k+1}, ..., x_n]$ is a Gröbner basis of the $k^{th}$ elimination ideal

$$I_k = I \cap k[x_{k+1}, ..., x_n]$$  \hspace{0.5cm} (A.3)

This means that all polynomials, where variables $x_1, x_2, ..., x_k$ has been eliminated, can be written as in Definition 1 where $g_1, g_2, ..., g_s$ are the polynomials in $G_k$. 

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APPENDIX B
LYAPUNOV STABILITY AND LASALLE’S INVARIANCE PRINCIPLE

B.1 Lyapunov’s stability theorem

Consider the autonomous system

\[ \dot{x} = f(x) \]  \hspace{1cm} (B.1)

where \( f : D \rightarrow \mathbb{R}^n \) is a locally Lipschitz map from a domain \( D \subseteq \mathbb{R}^n \) into \( \mathbb{R}^n \). Let \( x = 0 \) be an equilibrium point for (B.1) and \( D \subseteq \mathbb{R}^n \) be a domain containing \( x = 0 \). Let \( V : D \rightarrow \mathbb{R} \) be a continuously differentiable function such that

\[ V(0) = 0 \quad \text{and} \quad V(x) > 0 \quad \text{in} \quad D - \{0\} \]  \hspace{1cm} (B.2)
\[ \dot{V}(x) \leq 0 \quad \text{in} \quad D \]

Then, \( x = 0 \) is stable. Moreover, if

\[ \dot{V}(x) < 0 \quad \text{in} \quad D - \{0\} \]  \hspace{1cm} (B.3)

then \( x = 0 \) is asymptotically stable.

B.2 Lasalle’s invariance principle

Let \( x(t) \) be a solution of (B.1). A point \( q \) is said to be a positive limit point of \( x(t) \) if there is a sequence \( \{t_n\} \), with \( t_n \rightarrow \infty \) as \( n \rightarrow \infty \), such that \( x(t_n) \rightarrow q \) as \( n \rightarrow \infty \). The set of all positive limit
points of $x(t)$ is called the positive limit set of $x(t)$. A set $M$ is said to be an invariant set with respect to (B.1) if

$$x(0) \in M \Rightarrow x(t) \in M, \quad \forall t \in \mathbb{R}$$  \hspace{1cm} (B.4)

That is, if a solution belongs to $M$ at some time instant, then it belongs to $M$ for all past and future time.

Let $\Omega \subset D$ be a compact set that is positively invariant with respect to (B.1). Let $V : D \to \mathbb{R}$ be a continuously differentiable function such that $\dot{V}(x) \leq 0$ in $\Omega$. Let $E$ be the set of all points in $\Omega$ where $\dot{V}(x) = 0$. Let $M$ be the largest invariant set in $E$. Then every solution starting in $\Omega$ approaches $M$ as $t \to \infty$. 

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BIOGRAPHICAL SKETCH

Li Liu was born in Jilin, China. She got her BS degree in Mechanical Engineering from Tong Ji University, Shanghai, China in 1997; and MS degree in Mechanical Engineering from Shanghai Jiao Tong University, Shanghai, China in 2000. She then joined General Motors (Shanghai) as a Product Engineer and worked there for two years. In 2002, she entered Florida State University (FSU) to pursue her PhD degree. She is now a PhD candidate in Mechanical Engineering Department, FSU; and with Center for Advanced Power Systems (CAPS) as a Graduate Research Assistant. Her research interests include electrical machine modeling, fault detection and diagnosis, nonlinear system identification and control, etc.
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