DYNAMIC MODELING AND MOTION PLANNING
FOR ROBOTIC SKID-STEERED VEHICLES

By
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Dedicated to my mother Meena Gupta, father Trilok Chand Gupta, and sister Nidhi Gupta for their conditional love and support throughout my life.
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ABSTRACT

Skid-steered robots are commonly used in outdoor applications due to their mechanical simplicity, high maneuverability, and robustness. The maneuverability of these robots allows them to perform turning maneuvers ranging from point turns to straight line motion under ideal conditions (e.g., flat terrain and powerful actuators). However, sloped terrain, terrain with high friction, or actuator torque and power limitations can limit the achievable turning radii. The aim of this research is to analyze and experimentally verify the dynamic and power models for skid-steered autonomous ground vehicles equipped with non-ideal (i.e., torque and power limited) actuators and moving on sloped terrains. In particular it investigates the ability of the proposed models to predict motor torques (including motor saturation), power requirement, and minimum turn radius as a function of terrain slope, vehicle heading, payload, terrain parameters and actuator characteristics.

The experimental results show that the model is able to predict motor torques for the full range of turning radii on flat ground, i.e., from point turns to straight line motion. In addition, it is shown that the proposed model is able to predict motor torques (including motor saturation) and minimum turn radius as a function of terrain slope, vehicle heading, payload, terrain parameters and actuator characteristics. This makes the model usable for curvilinear motion planning tasks on sloped surfaces.

The research uses these results along with Sampling Based Model Predictive Optimization to develop an effective methodology for generating dynamically feasible, energy efficient trajectories for skid-steered autonomous ground vehicles (AGVs) and compares the resultant trajectories with those based on the standard distance optimal trajectories. The simulated and experimental results consider an AGV moving at a constant forward velocity on both wood and asphalt surfaces under various loads. They show that a small increase in the distance of a trajectory over the distance optimal trajectory can result in a dramatic savings in the AGV’s energy consumption. They also
show that it is not difficult for distance optimal planning to produce trajectories that violate the motor torque constraints for skid-steered AGVs, which can result in poor navigation performance.

In addition, the research motivates and provides a methodology that integrates the robot’s dynamic model and actuator limitations, and the terrain models with SBMPO to exploit the vehicle momentum as a way to successfully traverse the difficult terrains such as steep hills, mud, or stiff vegetation patches. These scenarios are particularly critical for smaller robots with torque and power limited actuators, which as experimentally shown in this research can easily fail to accomplish their tasks in these environments. In particular, the experimental results showing the efficacy of the proposed methodology are presented for a vegetation patch and a steep hill. Finally, a discussion of the necessary perception work to fully automate the process is included.

Further, for walking and running robots, analysis of the power consumption is particularly important for trajectory planning tasks as it enables motion plans that minimize energy consumption and do not violate power limitations of the robot actuators. The research here is motivated by the hypothesis that for certain regimes of operation (i.e., certain gait parameters), legged robots from the RHex family behave in a similar fashion to skid-steered robots while in general curvilinear motion. Hence, using the experience gained from skid-steered wheeled vehicles, presents models of the inner and outer side torques and power requirements for the XRL hexapedal robot. In addition, the applicability of the power model to energy efficient motion planning is illustrated for a walking gait on a vinyl surface.
CHAPTER 1

INTRODUCTION

1.1 Motivation And Approach

Autonomous ground vehicles (AGVs) have played a major role in space exploration and have found increasing use in agricultural work. In the future, they are expected to perform a variety of challenging tasks in unstructured and dynamic outdoor environments such as search and rescue, construction, mining, and reconnaissance and surveillance. Skid-steered vehicles have been the vehicles of choice for many of these applications since they are simple and mechanically robust and have high maneuverability. A skid-steered vehicle can be either tracked or wheeled and is characterized by two features. First, the vehicle steering depends on controlling the relative velocities of the left and right side wheels or tracks. Second, all wheels or tracks remain parallel to the longitudinal axis of the vehicle and vehicle turning requires slippage of the wheels or tracks. However, as any real platform, skid-steered vehicles have some important limitations. As mentioned earlier, these platforms must slip and/or skid in order to turn, which in turn makes them less predictable than, for example, differentially driven vehicles. Also, while performing sharp turns, the required motor torques increase significantly when compared to straight line motion, which can lead to actuator saturation. This results in degraded performance.

The primary focus of this research is to develop 3D dynamic and power models for skid-steered vehicles and show their application in motion planning. In unstructured and dynamic environments it is possible to develop trajectories that violate the torque constraints of the actuators. For example, this may occur due to high friction between the tires and surface when making a sharp turn or the gravity effects associated with climbing a steep hill. These trajectories can lead to vehicle stall.
The research aims to investigate dynamic and power models for skid-steered vehicles as a function of terrain slope, vehicle heading, payload, terrain parameters and actuator characteristics. The proposed models should then be able to predict the motor torques (including motor saturation), power requirement, and minimum turn radius for the given environmental conditions, leading to efficient trajectories that do not violate the actuator constraints.

1.2 Background and Literature Review

1.2.1 Dynamic Modeling of a Skid-Steered Wheeled Vehicle

There has been significant prior work on dynamic modeling of skid-steered robots [1, 2, 3, 4, 5]. In [3] and [4] a thorough study of tracked vehicle dynamics during turning on firm ground was conducted. Wong’s dynamic model is based on the relationship between shear stress and shear displacement and was shown to be able to predict sprocket torques significantly more accurately than previous models based on Coulomb’s friction, in which shear stress is assumed to be constant for all turning radii [6]. In [4] the model is validated using four speed groups, with each group spanning speeds in a 5 km/h range. Wong’s model validation showed that although sprocket torques depend on speed, their variation with speed is negligible within each speed group. Though the study provided by Wong provided detailed analysis of vehicle dynamics, it was limited to tracked vehicles and did not consider actuator limitations or sloped surfaces.

Even though, the model of [4] was intended for hard surfaces, the research of [1] simulated Wong’s original model and tested its validity on soft ground using previously published experimental data. However, the model was validated for only a small set of turn radii (10 m – 30 m) and on flat ground. The model of [4] was extended in [5] for wheeled skid-steered vehicles and validated its performance on a vinyl surface for turn radii ($R$) exceeding the vehicle hull (i.e., $R > \frac{B}{2}$, where $B$ is the vehicle width).
The research of [2] and [7] developed dynamic models for skid-steered vehicles. However, they both were based on Coulomb friction, which limits the models to only small turning radii. Also, the effect of sloped surfaces was not discussed.

Regarding robot dynamic models and its application on hills, an experimental model to determine braking forces on hills with the aim of predicting stopping distances has been proposed [8]. However, the work was limited to linear motion. Perhaps the closest work to our current research was developed in [9] and [10], where trafficability and mobility for rovers on sloped terrain were analyzed using a terramechanics-based approach. However, the studies of [10] were developed for all-wheel steered vehicles moving on soft ground, which require a different wheel-terrain interaction model. Therefore, the research of this dissertation and the research of [10] are complementary. In addition, the work of [10] did not explicitly discuss the dependence of vehicle torques on turn radius.

1.2.2 Motion Planning for a Skid-Steered Wheeled Vehicle

Energy Efficient Motion Planning. An AGV has a finite energy supply stored in batteries and/or fuel, which limits its operational endurance. That’s why to enable an AGV to carry out more extensive missions without recharging or refueling, energy conservation is highly important. Some of this conservation may be accomplished by using hybrid power technologies [11]. However, once the power system on an AGV is chosen, substantial energy conservation may be achieved via energy efficient motion planning.

It is common to base motion planning on minimization of the distance traveled and most motion planning algorithms focus on this problem [12], [13], [14]. However, as this dissertation highlights, this approach can lead to trajectories that have unnecessary energy consumption and/or violate the torque constraints of a vehicle motor, causing poor trajectory tracking.

Despite its practical significance, to date there is very little published research in the field of energy efficient motion planning for mobile robots. In [15] the concept of vehicle velocity profile
is used to save energy for a mobile robot working in environments cluttered with moving obstacles. That energy-savings strategy avoids frequent acceleration and deceleration because of the high-energy consumption, but the work was limited to only straight line motion and did not consider more general curvilinear motion. In [16] a more comprehensive approach to energy efficient motion planning for mobile robots is presented. Power models of the robot motors are developed based on a combination of analytical motor models and experimental data. However, the constraints of a dynamic model (e.g., the minimum turn radius constraints considered here) are not considered. Furthermore, the power model is not based upon a dynamic model; hence, the power model is not valid when the vehicle load or the terrain surface changes, and developing a new model requires a completely new set of experiments.

**Momentum Based Motion Planning.** In many of the unstructured environments such as agriculture, mining, military applications to urban, polar and space exploration, tasks can be addressed by developing very mechanically robust and usually heavy and energy inefficient robots. On the other hand, a more difficult yet more beneficial approach, consists of developing smaller, more energy efficient and smarter robots. However, in order to achieve similar operation standards as the bulkier robotic counterparts, smaller robots need to be able to exploit their dynamics while executing tasks that can easily lead to saturation of their power and torque limited actuators.

The exploitation of robot dynamics has been demonstrated in present years mainly on bio-inspired robots, which while limited in size and actuation, can achieve outstanding locomotion tasks like running on horizontal and vertical surfaces, hopping, and flying [17], [18], [19]. However, the manipulation of dynamics by these type of robots has been typically embedded in their mechanical designs through the usage of compliant limbs and simple open-loop controllers and not at the higher motion planning level. Agile motion has also been achieved in wheeled mobile robots at the low level through manipulation of internal mass and inertial properties during locomotion [20].

At the high motion planning level, relevant work has been conducted for underactuated robots.
In particular, complex motions that incorporate joint and torque limits have been proposed for multi-link manipulators [21]. Additional related work includes the motion planning for autonomous door opening with a mobile manipulator [22], [23]. While that work considered dynamic manipulation of the door, the proposed motion planner disallowed states where the door required more force than the robot could exert from a given configuration, which is a main difference with the approach proposed in this dissertation. Here, we are interested in regimes of operation where the robot actuators cannot provide sufficient torque to accelerate the robot for some periods of the mission (e.g., when faced with the mobility challenges described before).

One approach to deal with actuator limitations consists of the wise usage of momentum. This approach is commonly used by high jump athletes and weight lifters [24]. However, little research has been conducted on the usage of momentum to traverse the type of mobility challenges in the previously mentioned terrains. A related problem was presented in [25], where motion planning was employed to climb a steep hill.

1.2.3 Sampling Based Model Predictive Optimization

Sampling Based Model Predictive Optimization (SBMPO) is a sampling-based algorithm for motion planning. It is based on sampling the inputs to a kinematic or dynamic model and integrating the model to build a tree, as illustrated by Fig. 1.1. A* optimization is used to find the optimal tree path, defining the optimal trajectory. SBMPO has been demonstrated as an effective and efficient trajectory planning technique for autonomous underwater vehicles (AUVs) [26], ground-based mobile robots [27], [28] and robotic manipulators [29]. The efficiency of SBMPO is closely linked to the development of an appropriate optimistic A* heuristic. Fig. 1.2 shows the block diagram of a trajectory planning strategy that uses SBMPO. The models, cost evaluation, and heuristic are supplied by the user. The inputs to either the kinematic model or dynamic model are sampled and that model is the one integrated by SBMPO. The remaining models (including the obstacle map) are represented as constraints that enable unacceptable states to be eliminated. It
should be noted that in the SBMPO algorithm, a graph is created from start to goal and each vertex on the graph keeps track of the states of the system, the control input, and cost associated with the state. For detailed information, please refer to [30] and [31].

![Graph Tree](image)

Figure 1.1: Portion of a graph tree resulting from the SBMPO sampling process and model integration.

The following are the main steps of SBMPO [30], [31]:

1. **Select a node with highest priority in the queue:** The nodes are collected in a Open List, which ranks the potential expansion by their priority or low cost associated with the node. The Open List is implemented as a heap so that the highest priority node that has not been expanded is on top. If the selected node is the goal SBMPO terminates, otherwise go to step 2. Note that the node representing the start will have the highest initial priority.

2. **Sample input space:** Generate a sample of the input to the system that satisfies the model constraints. The input sample and current state are passed to the system model, and the system is integrated to determine the next state of the system. It should be noted that the current state is the state of the selected node.

3. **Add new node to the graph:** Use an implicit grid [32] to check if the graph already contains a node close to the new state of the system. If such a node exists, only add an edge from the current node (i.e., the selected node) to the node whose state is similar to the new state. Otherwise, add a node whose state is the next state.
4. **Evaluate new node cost**: Use an A*-heuristic to evaluate the cost of the generated vertices based on the desired objective. Add a new node to the priority queue based on the nodes cost.

5. **Repeat 2-4 for B number of successors**: Repeat steps 2-4 for B number of successors where B is defined by the user. B is also known as "Branch-out factor".

6. **Repeat 1-5 until one of the stopping criteria is true**: Steps 1-5 will be repeated until the goal is found or the maximum number of allowable iterations is achieved.

### 1.3 Research Objective

Extending the work in [5], this dissertation develop 3D dynamic and power models for skid-steered wheeled vehicles and show their application for the motion planning task. Challenging tasks on outdoor terrains such as search and rescue, construction, and mining to cite a few, require highly versatile and maneuverable vehicles able to navigate on different terrains and perform
maneuvers that involve movements from sharp turns to linear motion. For this reason along with
the mechanical robustness and simplicity, skid-steered wheeled vehicles have been the vehicles
of choice for many of these applications. Mobile robots have also followed the same trend and
skid-steered wheeled mobile robots are today one of the most commonly employed mobility plat-
forms. However, as any real platform, skid-steered wheeled mobile robots have some important
limitations. First, as indicated by their name, these platforms must slip and/or skid in order to turn,
which in turn makes them less predictable than, for example, differentially driven robots. Second,
while performing sharp turns, the required motor torques increase significantly when compared to
straight line motion, which can lead to actuator saturation. This results in degraded performance
(e.g., the robot cannot track the desired turn radius or a desired velocity profile).

Figs. 1.3 and 1.4 illustrate this phenomenon. The FSU-Bot shown in Fig. 1.3 was placed
on a 10° inclined hill and was commanded to perform a turn of 0.4m radius at a constant speed
of 0.2m/s. However, as shown in Fig. 1.4, the vehicle was not able to track the commanded
velocities due to actuator saturation. Notice that during the first 5 seconds the actual angular
velocity is almost zero as the saturated torques were not enough to rotate the vehicle. Furthermore,
as shown in Fig. 1.4, after time \( t = 6s \), the robot velocity drastically overshot the commanded
velocity because the actuators came out of saturation and the velocity tracking error had grown
significantly while in saturation. It is important to note that this situation caused the robot to
lose control and it constitutes therefore an important motivation for developing and employing
robot models that capture the dynamics of turning. In addition, a dynamic model, such as the one
proposed in this dissertation can then be employed by motion planners to generate trajectories that
either avoid actuator saturation or exploit momentum, resulting in predictable robot motion.

In summary the proposed research work on the following contributions for the development of
dynamic and power models, and motion planning task of a skid-steered wheeled vehicle:

1. Development and enhancement of the dynamic model presented in [5]: First, the model
will be extended and validated to handle the full range of turn radii. It is important to note
that the robotic platform used in [5] was 12 kg heavier than the robot used for the dynamic model verification and was not capable of performing turn radii smaller than the vehicle hull. Second, the model will be extended and validated to handle various payload and curvilinear motion on hills.

2. **Development and experimental verification of dynamically feasible, energy efficient motion planning using SBMPO:** Using the dynamic model discussed above and extending
it to a power model as proposed in [33], a cost function (required by SBMPO) for the minimum energy motion planning on sloped terrains will be developed. Thus, combining the minimum energy cost function with dynamic constraints and a simple extended kinematic model, a comparison will be made between distance optimal motion planning and energy efficient motion planning. The tool used for the motion planning task will be SBMPO.

3. **Formulation of and proposed solution to the problem of mobility challenge traversal:**

The problem will be solved with the aim to minimize the traveling time, considering actuator constraints, and desired goal velocity. The proposed solution will be developed by integrating high level motion planning with simple yet accurate enough physics-based models, which allows for the removal of the many typically ad-hoc heuristics employed by motion planners. In addition, the research will experimentally demonstrate the effectiveness of the proposed solution on a robotic platform on stiff vegetation patches and steep hills. The dissertation will also propose a preliminary methodology to obtain the terrain parameters required for such a task and point out perception challenges that should be addressed to fully automate the process and increase the system robustness.

### 1.4 Outline of Dissertation

The dissertation is organized in six chapters. Chapter 2 presents and discuss the proposed dynamic and power model for a skid-steered wheeled vehicle. Chapter 3 discusses the various motion planning tasks based on the usage of dynamic and power models developed in Chapter 2. Chapter 4 first presents and discusses the results of experiments for dynamic model verification. It then presents simulation and experimental results that compare dynamically feasible, energy efficient motion trajectories with more standard distance optimal trajectories for several combinations of loads, surfaces, and speeds. Chapter 4 also discuss experimental results on momentum based motion planning on steep hill and stiff vegetation. Chapter 5 discusses the extension of the work
presented in Chapter 3 on a biologically inspired hexapedal legged robot (XRL). Finally, Chapter 6 summarizes the dissertation and discusses future work.
CHAPTER 2

MODELING OF A SKID-STEERED WHEELED VEHICLE

This chapter describes a simple kinematic model required for the development of the dynamic model of a skid-steered wheeled vehicle. It then presents the dynamic and power models of a skid-steered wheeled vehicle. The present work is the extension of the model presented in [5]. In this research the model is first extended to handle the full range of turn radii. It is important to note that the robotic platform used in [5] was 12kg heavier than the modified FSU-Bot used in this research and was not capable of performing turn radii smaller than the vehicle hull. Second, the model is extended to handle curvilinear motion on hills.

Figure 2.1: A skid-steered vehicle (the FSU-Bot) moving on a flat asphalt surface while being tracked by a Vicon motion capture system.
Figure 2.2: A skid-steered wheeled vehicle performing a circular turn at constant velocity.

## 2.1 Kinematic Model

As shown in Fig. 2.2, consider a skid-steered wheeled vehicle moving at constant velocity (i.e., $v_y$ and $\dot{\Psi}$ are constant) about an instantaneous center of rotation (I.C.R.). The local coordinate frame, which is attached to the body center of gravity (CG), is denoted by $x - y$, where $x$ is the lateral coordinate and $y$ is the longitudinal coordinate. When a skid-steered wheeled vehicle rotates, the inner side wheels of the vehicle experience longitudinal skidding or slipping depending on the turn radius, while the outer side wheels experience longitudinal slipping. The lateral sliding velocity $v_x$ is relatively small [34], [35], [36] and, hence, is neglected ($v_x = 0$). For vehicles that are symmetric about the $x$ and $y$ axes, an ideal symmetric experimental kinematic model of a skid-steered wheeled vehicle [34] is given by

$$
\begin{bmatrix}
v_y \\
\dot{\Psi}
\end{bmatrix} = \frac{r}{\alpha B} \begin{bmatrix}
\frac{\alpha B}{2} & \frac{\alpha B}{2} \\
-1 & 1
\end{bmatrix} \begin{bmatrix}
\omega_l \\
\omega_r
\end{bmatrix},
$$

where $v_y$ is the vehicle velocity in the forward direction, $\dot{\Psi}$ is the vehicle angular velocity, $B$ is the vehicle width, $r$ is the wheel radius, $\alpha$ is a terrain-dependent parameter, and $\omega_l$ and $\omega_r$ are the
angular velocities of the left and right wheels, respectively. Therefore, the kinematic model of
a skid-steered wheeled vehicle of width $B$ is equivalent to the kinematic model of a differential-
steered wheeled vehicle of width $\alpha B$.

The kinematic model of (2.1) is used in the development of the dynamic model described in
the next section. In addition, for vehicle control the inverse of this model is used to determine the
desired wheel velocities $\omega_l$ and $\omega_r$, given $v_y$ and $\dot{\psi}$.

2.2 Dynamic Modeling

The dynamic model was originally presented in [5] and is based on the experimentally verified
‘exponential friction model’ [4], [3] (i.e., the shear stress varies exponentially with respect to shear
displacement) to model the interaction of a skid-steered wheeled vehicle with the ground. The
model not only gives more accurate predictions of the applied motor torques but is also valid for
all turning radii in comparison to models developed using Coulomb friction (in which the shear
stress is assumed to be constant) [7], [2], which are valid only for large turning radii. It should
be noted that the exponential friction model is accurate only for small accelerations [5]. However,
for energy efficient motion planning, high accelerations and decelerations are generally minimized
[15], [16], [37]. Thus, the model is appropriate for the motion planning task considered in this
dissertation.

Following [5], the dynamic model for a skid-steered wheeled vehicle performing a turning
maneuver (see Fig. 2.3) can be expressed in terms of wheel states and is given by

$$M \ddot{q} + C(q, \dot{q}) + G(q) = \tau,$$  \hspace{1cm} (2.2)

where $M$ is the mass matrix, $C(q, \dot{q})$ is the resistive term, $G(q)$ is the gravitational term, $q = [$\theta_i \ \theta_o]^T$ is the angular position of the inner and outer side wheels respectively, $\dot{q} = [$\omega_i \ \omega_o]^T$ is the angular velocity of the inner and outer wheels, and $\tau = [$\tau_i \ \tau_o]^T$ is the torque of the inner and
outer motors. In this work the vehicle is assumed to be moving at constant velocity, which yields $M\ddot{q} = 0$. Also, note that if the vehicle is moving counterclockwise about an I.C.R., then the inner wheel is the left wheel and the outer wheel is the right wheel. The converse is true if the vehicle is moving clockwise about an I.C.R. (i.e., inner = right and outer = left).

In order to capture the ground-wheel interaction it is necessary to model the term $C(q, \dot{q})$ of Eq. (2.2). Here, we base our analysis on the model initially proposed in [3] for tracked vehicles and extended on [5] for skid-steered wheeled vehicles moving on flat ground. However, in this work, the $C(q, \dot{q})$ model is extended to capture the frictional forces involved in the full range of possible turn radii and also the effect of terrain slope and vehicle heading.

The modeling of the terrain ground interaction is based on the relationship between shear stress $\tau_{ss}$ and shear displacement $j$ given by [3]

$$\tau_{ss} = p\mu(1 - e^{-j/K}), \quad (2.3)$$

where $p$ is the normal pressure, $\mu$ is the coefficient of friction, and $K$ is the shear deformation modulus. It is then possible to compute the longitudinal frictional forces on each side of the
vehicle by integrating (2.3) over the contact patch of each wheel (shaded areas in Fig. 2.3).

Assuming that the vehicle turns with constant angular velocity $\Omega_z$ about a center of turn $O$ with turn radius $R'$ as illustrated in Fig. 2.3, it is possible to compute the shear displacement for the outer and inner vehicle sides along the $X$ and $Y$ directions. The vehicle of Fig. 2.3 is assumed to have wheels of radius $r$, wheel base $L$, track width $B$, and wheel contact patches of size $p_l \times b$. The contact patches were measured using the Tekscan pressure measurement system [38], which employs an array of pressure sensors placed under the tire to generate a profile of its imprint on the surface. Although the actual shape of the contact patch is elliptical, it can be closely approximated by a rectangular shape. In addition, since the tire pressure was maintained constant at a high value of 20 psi, it is assumed that the size of the contact patches remain constant for all experiments. For generality sake, the vehicle is assumed to have an off centered center of gravity (CG) located at $(C_x, C_y)$ and a center of turn shifted by an amount $S_0$ from the CG. Table. 2.1 lists the key parameters involved in the robot model.

Following [3] and the conventions of Fig. 2.3, the shear displacements for the inner side (front and rear) wheels at a point $(x_i, y_i)$ are given by

$$jX_{if} = (R'' - \frac{B}{2} - C_x + x_i) \cos \left(\frac{(n_a - y_i)\Omega_z}{rw_i}\right) - \left(\frac{l}{2} - C_y - S_0\right) + y_i \cos \left(\frac{(n_a - y_i)\Omega_z}{rw_i}\right), \quad (2.4)$$

$$jY_{if} = ((R'' - \frac{B}{2} - C_x + x_i) \sin \left(\frac{(n_a - y_i)\Omega_z}{rw_i}\right) - \left(\frac{l}{2} - C_y - S_0\right) + y_i \cos \left(\frac{(n_a - y_i)\Omega_z}{rw_i}\right), \quad (2.5)$$

$$j_{if} = \sqrt{jX_{if}^2 + jY_{if}^2}, \quad (2.6)$$

where $n_a = \frac{l}{2} - C_y - S_0$, $R'' = R' \cos \beta$ and $l = 2(\frac{L}{2} + \frac{p_l}{2})$.

$$jX_{ir} = (R'' - \frac{B}{2} - C_x + x_i) \left\{ \cos \left(\frac{(n_b - y_i)\Omega_z}{rw_i}\right) - 1 \right\} - y_i \sin \left(\frac{(n_b - y_i)\Omega_z}{rw_i}\right), \quad (2.7)$$
Table 2.1: FSU-Bot parameters for vehicle model

<table>
<thead>
<tr>
<th>Vehicle</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of Vehicle (N)</td>
<td>(W)</td>
</tr>
<tr>
<td>Weight of Visual Odemetry (N)</td>
<td>(W_{vo})</td>
</tr>
<tr>
<td>Track width (m)</td>
<td>(B)</td>
</tr>
<tr>
<td>Wheel base (m)</td>
<td>(L)</td>
</tr>
<tr>
<td>Contact patch length (m)</td>
<td>(p_l)</td>
</tr>
<tr>
<td>Contact patch width (m)</td>
<td>(b)</td>
</tr>
<tr>
<td>CG offset in x direction (m)</td>
<td>(C_x)</td>
</tr>
<tr>
<td>CG offset in y direction (m)</td>
<td>(C_y)</td>
</tr>
<tr>
<td>CG height (m)</td>
<td>(h)</td>
</tr>
<tr>
<td>Radius of tire (m)</td>
<td>(r)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque constant (Nm/A)</td>
<td>(K_t)</td>
</tr>
<tr>
<td>Speed constant (rad/sV)</td>
<td>(K_n)</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>(g_r)</td>
</tr>
<tr>
<td>Efficiency</td>
<td>(\eta)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor controller</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage (V)</td>
<td>(V_{cc})</td>
</tr>
<tr>
<td>Maximum pulse width modulation</td>
<td>(PWM_{max})</td>
</tr>
<tr>
<td>Voltage drop (V)</td>
<td>(V_{drop})</td>
</tr>
</tbody>
</table>

\[
j_{ir} = (R'' - \frac{B}{2} - C_x + x_i) \sin \left( \frac{(n_b - y_i)\Omega_z}{rw_i} \right) - \left( -\frac{l}{2} - C_y - S_0 + p_l \right) + y_i \cos \left( \frac{(n_b - y_i)\Omega_z}{rw_i} \right), \tag{2.8}
\]

\[
j_{ir} = \sqrt{jX_{ir}^2 + jY_{ir}^2}, \tag{2.9}
\]

where \(n_b = -\frac{l}{2} - C_y - S_0 + p_l\).

In a similar way, the shear displacements for the outer side (front and rear) wheels at a point \((x_o, y_o)\) are given by

\[
jX_{of} = (R'' + \frac{B}{2} - C_x + x_o) \left\{ \cos \left( \frac{(n_a - y_o)\Omega_z}{rw_0} \right) - 1 \right\} - y_o \sin \left( \frac{(n_a - y_o)\Omega_z}{rw_0} \right), \tag{2.10}
\]
\[ jY_{of} = (R'' + \frac{B}{2} - C_x + x_o) \sin \left( \frac{(n_a - y_o)\Omega_z}{rw_0} \right) - \left( \frac{l}{2} - C_y - S_0 \right) + y_o \cos \left( \frac{(n_a - y_o)\Omega_z}{rw_0} \right), \quad (2.11) \]

\[ j_{of} = \sqrt{jX_{of}^2 + jY_{of}^2}. \quad (2.12) \]

\[ jX_{or} = (R'' + \frac{B}{2} - C_x + x_o) \left\{ \cos \left( \frac{n_b\Omega_z}{rw_0} \right) - 1 \right\} - y_o \sin \left( \frac{n_b\Omega_z}{rw_0} \right), \quad (2.13) \]

\[ jY_{or} = (R'' + \frac{B}{2} - C_x + x_o) \sin \left( \frac{(n_b - y_o)\Omega_z}{rw_o} \right) - (- \frac{l}{2} - C_y + S_0 + p_l) + y_1 \cos \left( \frac{(n_b - y_o)\Omega_z}{rw_o} \right), \quad (2.14) \]

\[ j_{or} = \sqrt{jX_{or}^2 + jY_{or}^2}. \quad (2.15) \]

The longitudinal frictional forces for the inner \((F_i)\) and outer \((F_o)\) sides are obtained by integrating (2.3) over the contact patch area of each wheel. To capture experimentally observed asymmetries, different coefficients of friction \(\mu_i\) and \(\mu_o\) have been assumed for each side of the vehicle. (In the future, we expect to gain a better understanding of the need for these two different coefficients.) \(F_i\) and \(F_o\) are then given by

\[ F_i = \int_{n_a - p_l}^{n_a} \int_{-\frac{b}{2}}^{\frac{b}{2}} p_{if} \mu_i(1 - e^{-j_{if}/K}) \sin(\pi + \gamma_i) dx_i dy_i \]

\[ + \int_{n_b - p_l}^{n_b} \int_{-\frac{b}{2}}^{\frac{b}{2}} p_{ir} \mu_i(1 - e^{-j_{ir}/K}) \sin(\pi + \gamma_i) dx_i dy_i, \quad (2.16) \]

\[ F_o = \int_{n_a - p_l}^{n_a} \int_{-\frac{b}{2}}^{\frac{b}{2}} p_{of} \mu_o(1 - e^{-j_{of}/K}) \sin(\pi + \gamma_o) dx_o dy_o \]

\[ + \int_{n_b - p_l}^{n_b} \int_{-\frac{b}{2}}^{\frac{b}{2}} p_{or} \mu_o(1 - e^{-j_{or}/K}) \sin(\pi + \gamma_o) dx_o dy_o, \quad (2.17) \]

where \(\gamma_i\) and \(\gamma_o\) are the angles between the resultant sliding velocities of the inner and outer wheels and the lateral direction of the vehicle. Following [3], the inner and outer sliding velocities are given by

\[ V_{ji} = (V_{jxi}, V_{jyi}), \quad (2.18) \]
where $V_{jx_i} = -y_i \Omega_z$ and $V_{jy_i} = (R'' - B \frac{B}{2} - C_x + x_i) \Omega_z - rw_i$. Similarly,

$$V_{jxo} = (V_{jxo}, V_{jyo}), \quad (2.19)$$

where $V_{jxo} = -y_o \Omega_z$ and $V_{jyo} = (R'' - C_x + B \frac{B}{2} + x_o) \Omega_z - rw_o$.

Then, $\gamma_i$ and $\gamma_o$ can be computed as

$$\gamma_i = \arctan \left( \frac{V_{jyi}}{V_{jxi}} \right), \quad (2.20)$$

$$\gamma_o = \arctan \left( \frac{V_{jyo}}{V_{jxo}} \right). \quad (2.21)$$

The normal pressures on the wheels vary as a function of the terrain slope and the vehicle heading. Referring to Fig. 2.4, it is possible to perform a moment balance about the “inner and outer wheel lines” (i.e., the line connecting the center of the inner wheels and the corresponding line for the outer wheels) and obtain the following normal forces on the inner ($N_i$) and outer ($N_o$) sides as

$$N_i = \frac{W \cos \theta (\frac{B}{2} - C_x) + Wh \sin \theta \sin \psi - \frac{Whv^2}{gR'} \cos \beta}{B}, \quad (2.22)$$
\[
N_o = \frac{W \cos \theta \left( \frac{B}{2} + C_x \right) - Wh \sin \theta \sin \psi + \frac{Whv^2}{gR'} \cos \beta}{B},
\]

where \( \theta \) and \( \psi \) represent the slope of the hill and the vehicle heading, \( W \) is the vehicle weight, which as shown in Fig. 2.4 has a longitudinal component \( W_{l_b} \) and a lateral component \( W_{l_t} \), \( h \) is the height of the CG, \( R' \) is the distance from \( O \) to the CG, and \( \beta \) represents the angle between the vector going from the center of turn \( O \) to the CG and a vector going from \( O \) to \( O_v \) (refer to Fig. 2.3 for easy visualization of this parameter). Similarly, performing a moment balance about the front and rear axles, we get

\[
N_f = \frac{W \cos \theta \left( \frac{l}{2} + C_y \right) - Wh \sin \theta \cos \psi - \frac{Whv^2}{gR'} \sin \beta}{l},
\]

\[
N_r = \frac{W \cos \theta \left( \frac{l}{2} - C_y \right) + Wh \sin \theta \cos \psi + \frac{Whv^2}{gR'} \sin \beta}{l}.
\]

Assuming symmetry, the normal pressures on each wheel are then estimated using

\[
p_{of} = \frac{N_f}{\frac{W \cos \theta}{b} p_l},
\]

\[
p_{or} = \frac{N_r}{\frac{W \cos \theta}{b} p_l},
\]

\[
p_{if} = \frac{N_f}{\frac{W \cos \theta}{b} p_l},
\]

\[
p_{ir} = \frac{N_r}{\frac{W \cos \theta}{b} p_l}.
\]

The resistive torque \( C(q, \dot{q}) \), has an additional component due to the rolling resistance and friction in the motor bearings and different components that make up the driving system. The friction in the driving system \([\tau_{ires} \ \tau_{ores}]^T\) is experimentally determined by elevating the robot so that its wheels lose contact with the ground and measuring the motor torques while maintaining a nominal vehicle speed of 0.2 m/s. The rolling resistance forces \([R_i \ R_o]^T\) are determined by
measuring the motor torques while the vehicle is moving in a straight line at the nominal speed
and then subtracting the driving system friction $[\tau_{ires} \; \tau_{ores}]^T$. Once the rolling resistances are
determined, it is possible to estimate the coefficients of rolling resistances for the inner and outer
sides as

$$
\mu_{ri} = \frac{R_i}{N_i},
$$

(2.30)

$$
\mu_{ro} = \frac{R_o}{N_o}.
$$

(2.31)

Finally, the total resistance term $C(q, \dot{q})$ can be expressed by

$$
C(q, \dot{q}) = r \left[ \begin{array}{c}
F_i + R_i \\
F_o + R_o
\end{array} \right] + \left[ \begin{array}{c}
\tau_{ires} \\
\tau_{ores}
\end{array} \right],
$$

(2.32)

where $r$ is the wheel radius.

When traversing slopes, it is also necessary to overcome the gravitational term $G(q)$, which
can be derived from Fig. 2.4 by performing a force and moment balance as

$$
G(q) = \frac{r W \sin \theta \cos \psi}{B} \left[ \frac{B}{2} - C_x, \frac{B}{2} + C_x \right]^T.
$$

(2.33)

### 2.2.1 Actuator Saturation

As shown in Fig. 1.4, when non ideal actuators are pushed to their saturation limits, uncon-
trollable vehicle motion could result. These situations can be avoided by incorporating motor
and motor controller models into the vehicle dynamic model. In this research, the FSU-Bot uses
two Maxon motor controllers (one for each vehicle side) controlled in current mode. In order to
protect the motors, maximum conservative current levels of $I_{lim} = 5.46 A$ are set for both motor
controllers. The maximum output voltage from the motor controller is given by

$$
V_{max} = PWM_{max} \cdot V_{ce} - V_{drop},
$$

(2.34)
where $PWM_{\text{max}}$ is the maximum achievable pulse width modulation (PWM) value, which as stated by the manufacturer is of 0.90. $V_{cc}$ is the power supply voltage and $V_{\text{drop}}$ is a voltage drop on the motor controller.

In addition, from the speed-torque curve of the motor (reflected at the wheel), it is possible to derive that the maximum output torque $\tau$ at a given operating wheel speed $w$ is given by

$$\tau = 2\tau_s \left[1 - \frac{w}{w_{nl}}\right],$$

(2.35)

where $w_{nl}$ corresponds to the no load speed of the wheel and $\tau_s$ is the stall torque at the wheel; these variables are computed using

$$w_{nl} = \frac{k_n V_{\text{max}}}{g_r},$$

(2.36)

$$\tau_s = \frac{k_n V_{\text{max}} g_r \bar{\tau}_n}{w_{nl}},$$

(2.37)

where $g_r$ is the gear ratio and $\bar{\tau}_n$, and $w_{nl}$ are the stall torque and no load speed of the motor at a nominal voltage of 12V. Due to the current limitations imposed by the motor controllers, the maximum output torque is computed as

$$\tau_{\text{max}} = \min(\tau, I_{iam} K_t g_r),$$

(2.38)

where $K_t$ is the torque constant of the motor.

### 2.3 Power Modeling

In this section, the power model used in the study and its integration with respect to time to get the energy consumption of an AGV for a particular path is described. The model was originally presented in [33] and based on the experimentally verified 'exponential friction model' in [4] and [3] (i.e., the shear stress varies exponentially with respect to shear displacement) for the
modeling of sliding friction of a skid steered robot. This gives more accurate prediction of torque as compared to the Coulombs friction model (in which the shear stress is assumed to be constant). Dynamic and power models developed using the exponential friction model give accurate predictions of torque over a wide range of turning radii in comparison to models developed using Coulomb friction as in [2] and [7]. The exponential friction model is more accurate for small or no accelerations only [5] and for energy efficient motion planning high accelerations and decelerations are always avoided [15], [16], [37]. Thus, the model is apt for energy efficient motion planning task.

The power model in [33] is developed via the development of separate power models for the left and right side motors. The power models for the left and right side motors are described in terms of the mechanical power consumption associated with the rotation of the left and right wheels and the electrical power consumption due to the electrical resistance of the left and right motors. These two power models are combined to describe the power model for the entire vehicle. Particular attention is given to the inner motor (i.e., the left side motor in case when vehicle is turning counterclockwise (CCW) and right side motor in case when vehicle is turning clockwise (CW)) because even though the velocity of inner motor is always positive, as the turning radius decreases from infinity, the inner motor first consumes power, then generates power, and finally consumes power again.

The equivalent circuit describing the left and right side drive system and motors can be seen in Fig. 2.5. The circuit includes a battery, a motor controller, a motor and the motor electrical resistance $R_e$ for each side of the motor. In Fig. 2.5 $\omega_l$ and $\omega_r$ are the angular velocities of the left and right wheels, $U_l$ and $U_r$ are the output voltages of the left and right motor controllers, and $i_l$ and $i_r$ are the currents of the left and right circuits. The vehicle is assumed to be turning CCW and have turning radius larger than half the width of the vehicle so that $\omega_l$ and $\omega_r$ are always positive.

The relationship between the motor and motor controller is shown in Fig. 2.6. The motor controller regulates the current of the motor to achieve the desired velocity of each corresponding
Figure 2.5: The circuit diagram for the left and right side of a skid-steered wheeled vehicle.

Figure 2.6: The closed-loop control system for the left and right side of a skid-steered wheeled vehicle.

Wheel. When a motor controller provides current to a motor, the motor rotates and consumes power. On the contrary, when an external torque rotates a motor, the motor works as a generator, and the corresponding current in Fig. 2.5 will be reversed.

The power model of a DC motor is given by,

\[ P_m = \omega_m \tau + R_m i_m^2, \tag{2.39} \]

where the first term on right side is the mechanical power consumption, which includes the power to compensate the left and right sliding frictions and the moment of resistance along with the power to accelerate the motor, and the second term is the electrical power consumption due to the motor resistance dissipated as heat. Using (2.39), the power consumption for the left and right motors \( P_l \) and \( P_r \) having efficiency \( \eta_l \) and \( \eta_r \) respectively, can be expressed as

\[ P_l = \frac{\tau_l \omega_l}{\eta_l} + (i_l)^2 R_e, \tag{2.40} \]

and

\[ P_r = \frac{\tau_r \omega_r}{\eta} + (i_r)^2 R_e. \tag{2.41} \]
The current $i_m$ flowing in the motor may be expressed in terms of the torque $\tau_m$ as

$$i_m = \frac{\tau_m}{K_T g_m \eta_m}, \quad (2.42)$$

where $K_T$ is the torque constant, $g_m$ is the gear ratio, and $\eta_m$ is the mechanical efficiency of the motor. Hence $P_l$ and $P_r$ can be computed as a function of torque as

$$P_l = \frac{\tau_l \omega_l}{\eta_l} + \left( \frac{\tau_l}{K_T g_r \eta_l} \right)^2 R_e, \quad (2.43)$$

and

$$P_r = \frac{\tau_r \omega_r}{\eta_r} + \left( \frac{\tau_r}{K_T g_r \eta_r} \right)^2 R_e. \quad (2.44)$$

Let $P$ denote the power that must be supplied by the motor drivers to the motors to enable the motion of a skid-steered wheeled vehicle and define the operator $\sigma : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$\sigma(Q) = \begin{cases} Q : & Q \geq 0 \\ 0 : & Q < 0. \end{cases} \quad (2.45)$$

Then the entire power model of a skid-steered wheeled vehicle is

$$P = \sigma(P_r) + \sigma(P_l). \quad (2.46)$$

Typically, one might expect to write $P = P_r + P_l$. However, since it turns out that $P_l$ can be negative and that this generated power does not charge the battery in our research vehicle, the more general form (2.46) is used. To enable the battery to be charged requires modifications of the motor controller, which was beyond the scope of this research.

Since on a sloped terrain for a given linear velocity, the dynamic model yields that the torque generated by the left or right motor is a function of the turning radius and the heading angle (as seen in Fig. 2.7), the power for the motor can be correlated to the turning radius and the heading angle as well. Fig. 2.8 gives the simulation analysis of the power consumption for the left and right
motors corresponding to the turning radius of the vehicle. As can be seen from this figure, (2.43) and (2.44) are able to accurately predict the power consumption of the left and right side motors.

![Figure 2.7: Inner and outer torque curves for modified FSU-Bot moving on a wooden surface for different turning radii.](image1)

![Figure 2.8: Total power curve for modified FSU-Bot moving on a wooden surface for different turning radii.](image2)

The energy consumption $E$ for the vehicle in the time interval $[t_o, t_f]$ may be computed by the time integration of the power consumption, such that

$$E = \int_{t_o}^{t_f} Pdt.$$  \hspace{1cm} (2.47)
CHAPTER 3

MOTION PLANNING FOR A SKID-STEERED WHEELED VEHICLE

This chapter describes dynamically feasible, energy efficient motion planning and momentum based motion planning tasks for skid-steered wheeled vehicles. The motion planning tasks are performed using SBMPO in conjunction with dynamic and power models of the vehicle. The chapter also describes and discusses the cost functions used for both motion planning tasks.

3.1 Dynamically Feasible, Energy Efficient Motion Planning

The dynamically feasible, energy efficient motion planning task is performed using the vehicle’s dynamic model in conjunction with SBMPO. The energy costs within SBMPO are computed using the power model. To avoid integrating the underlying power model on-line, the integration is performed off-line and essentially produces a 3 dimensional curve that describes power consumption as a function of turning radius and initial heading angle for a given slope of the terrain (see Fig. 2.8).

3.1.1 Model used in SBMPO

Referring to the upper right hand block in Fig. 1.2, this section discusses the simple kinematic model used by SBMPO. Since, in this study \( v_y \) is kept constant, sampling \( \dot{\psi} \) and using a simple kinematic model provides the vehicle position in the global space by SBMPO. It is assumed that the vehicle states are sampled with fixed time period \( T \). Then, the position \( [X_G, Y_G] \) and orientation
\( \theta_G \) of the vehicle in the global space is given by

\[
\begin{align*}
\theta_{G,k} &= \theta_{G,k-1} + \psi_k T , \\
X_{G,k} &= X_{G,k-1} + v_y \cos(\theta_{G,k}) T , \\
Y_{G,k} &= Y_{G,k-1} + v_y \sin(\theta_{G,k}) T ,
\end{align*}
\]

(3.1)

where \( k \) is the time index for the current state. SBMPO uses this model to evaluate the vehicle cost for distance optimal motion planning and position in the global space for the \( \alpha \) corresponding to the surface under consideration.

### 3.1.2 Dynamic Constraints

Referring again to the upper right hand block in Fig. 1.2, this section describes the minimum turn radius constraints derived from the dynamic model. For a given surface, (constant) vehicle speed, and vehicle load, the dynamic model (2.2) can be used to generate curves that show the torque of the inner and outer motors as a function of the turn radius [5]. Examples of these curves for the FSU-Bot of Fig. 2.1 are the analytical curves shown in Fig. 3.1. Since the motors have torque limitations, illustrated by the solid horizontal line in Fig. 3.1, any turn radius less than the radius corresponding to the intersection of that line and the analytical curve (for the outer side motor) is unachievable. This turn radius is denoted the minimum turn radius (MTR) and is the radius corresponding to the small square in Fig. 3.1. Fig. 3.2 provides an example of a terrain dependent minimum turn radius (MTR) curve for a wooden sloped surface. This results in the constraint

\[
R \geq MTR .
\]

(3.2)

Note that the MTR will increase as the surface friction or payload increases. Also, for a given skid-steered wheeled vehicle, it is possible for the MTR to be zero. For example, this can occur if
Figure 3.1: Torque vs. Turn Radius curve for the FSU-Bot moving at a constant forward velocity of 0.2 m/s with no payload on a flat wooden surface. The minimum turn radius (MTR) for no payload is 1.5 m.

the motors are sufficiently powerful and the vehicle load is sufficiently small.

For implementation of the MTR constraint in SBMPO, the turn radii corresponding to the sampled nodes were computed and the nodes corresponding to the infeasible turn radii were rejected. The turn radius between any two nodes (see Fig. 1.1) is assumed to be constant at the value it has at the child node. (This is an approximation, which is needed for real time implementation.) Let \( k \) denote the current time, corresponding to the parent node, such that \( k + 1 \) is the time corresponding to any of the child nodes. Consider the child node corresponding to the \( i^{th} \) sample (of \( n \) samples). It has angular velocity denoted by \( \dot{\psi}_{k+1,i} \) and linear velocity \( v_y \) (since it is assumed that the linear velocity is \( v_y \) throughout the trajectory). The turn radius between these two nodes, denoted by \( R_{k\rightarrow k+1,i} \), is then given by

\[
R_{k\rightarrow k+1,i} = \frac{v_y}{\dot{\psi}_{k+1,i}}.
\] (3.3)
3.1.3 Cost Evaluation and Heuristic for Energy Efficient Planning

The elements of Fig. 1.2 that must be chosen for energy efficient motion planning are the cost function and the corresponding heuristic. An expression for the energy consumption $E$ has already been given by (2.47) in terms of $P(\tau)$. However, since $\tau$ depends on the vehicle turning radius $R$ and orientation (in case of sloped terrain), and both vary continuously, it is not generally feasible to perform the integration of (2.47) in real time.

Referring to (2.47), let $k = 0$ correspond to $t_o$, the time of the start node, and $k = N$ correspond to $t_f$, the time of the current node. Then the energy cost $E$ for the trajectory from the start node to the current node can be approximated by

$$E = \sum_{k=0}^{N} P_k T, \quad (3.4)$$

where $T$ is the constant sample time and $P_k$ is the power needed to move from the node at time $k$ to the node at time $k + 1$, assuming that, following the derivation of (3.3), for angular velocity $\dot{\psi}_{k+1}$ and linear velocity $v_y$ the radius of curvature is given by $R_{k\rightarrow k+1} = v_y/\dot{\psi}_{k+1}$. For efficient implementation with SBMPO, tables displaying power $P$ vs. turn radius $R$ for various payloads

![Figure 3.2: Terrain dependent minimum turn radii predictions for different heading and hill inclination angles on a sloped wooden surface for the modified FSU-Bot with no payload.](image)
and hill inclinations were generated using the dynamic and power model respectively described in Chapter 2. These tables are then used to determine the power terms $P_k$ in (3.4) as a function of the turn radius $R_{k\rightarrow k+1}$ for a given payload and the hill inclination.

The heuristic cost is computed as follows. Let $D$ denote the straight line distance from the current node to the goal node and let $t$ denote the time needed to move the distance $D$ at the linear velocity $v_y$, such that $t = \frac{D}{v_y}$. Then, an optimistic heuristic $H$ is given by

$$H = P_\infty t + mgh,$$

(3.5)

where $P_\infty$ denotes the power consumption corresponding to linear motion (i.e., $R = \infty$) on flat ground, $m$ is the mass of the vehicle, $g$ is the gravity constant, and $h$ is the vertical height of the goal from the vehicle’s current position in space. In most of the current study the robot is moving on a flat surface, in which case $h = 0$.

### 3.1.4 Tuning of SBMPO

As with all motion planning algorithms, SBMPO has parameters that require tuning to obtain efficient use of the algorithm. The primary tuning parameters are the dimension of the implicit grid in Step 3 of Chapter 1 Section 1.2.3 and the branchout factor, defined in Step 5 of that section. Each tuning parameter is non-linearly dependent on the other and can have a significant effect on the computation time. For the implementation of SBMPO used for the dynamically feasible, energy efficient motion planning task, trial and error revealed that a grid size of $0.005m$ and branch out factor of 20 tended to give minimal computation times. Hence, these parameters were used in each of the simulations of Chapter 4 Section 4.2.1.
3.2 Momentum Based Motion Planning

The proposed solution to the momentum based traversal of mobility challenges is formulated by utilizing sampling-based model predictive optimization (SBMPO) discussed in Chapter 1 Section 1.2.3. The following are the main steps of SBMPO discussed in Section 1.2.3 in the context of momentum based planning.

1. **Select a node with highest priority:** The nodes are collected in an Open List, which ranks them based on their priority for expansion. If the selected node is the goal SBMPO terminates, otherwise go to step 2.

2. **Sample control space:** Generate a sample of the control space. Since the vehicle model (2.2) is invertible, it was decided to sample the vehicle acceleration $\ddot{x}$. However, it would also be possible to sample the applied joint torques.

3. **Generate Neighbor Nodes:** Integrate the system model with the control samples to determine the neighbors of the current node. Here we assume that the time $t \in [NT, NT + T]$, where $N$ is some positive integer and $T$ is the sampling time. During this interval the desired acceleration $\ddot{x}_d(t)$ is held constant at its sampled value $\ddot{x}_d$. In this step each new sampled acceleration $\ddot{x}_d$ is validated with the forward vehicle dynamic model (2.2), by computing the required traction force $F_d$ to achieve the desired acceleration. That is:

$$F_d = M\ddot{x}_d(NT) + C(x_d(NT), \dot{x}_d(NT)) + G(x_d(NT))$$  \hspace{1cm} (3.6)

if $F_d > F_{max}$, with $F_{max}$ given by (3.17) then the desired acceleration is modified by

$$\ddot{x}_d \leftarrow M^{-1}[F_{max} - C(x_d(NT), \dot{x}_d(NT)) - G(x_d(NT))]$$  \hspace{1cm} (3.7)
Finally, the acceleration is integrated to find the new vehicle velocity and position as follows:

\[
\dot{x}_d(t) = \dot{x}_d(NT) + \ddot{x}_d(t) t, \quad t \in [NT, NT + T), \quad (3.8)
\]

\[
x_d(t) = x_d(NT) + \dot{x}_d(NT) t + \frac{1}{2} \ddot{x}_d(t) t^2, \quad t \in [NT, NT + T). \quad (3.9)
\]

4. **Add new node to the graph.**

5. **Evaluate new node cost:** Use an A*-like heuristic to evaluate the cost of generated nodes based on the desired objective and add it to the priority queue based on the node’s cost. In the current work a minimum time heuristic inspired by [39] and detailed below is employed. This “heuristic” is a rigorous bound on the chosen cost and plays a key role in the efficient computation of trajectories that end with zero velocity.

Consider a system described by

\[
\ddot{x} = u; \quad x(0) = x_0, \quad \dot{x}(0) = v_0; \quad -a \leq u \leq b. \quad (3.10)
\]

The state space description of (3.10) is given by

\[
\dot{x}_1 = x_2, \quad \dot{x}_2 = u; \quad x_1(0) = x_0 \triangleq x_{1,0}, \quad x_2(0) = v_0 \triangleq x_{2,0}. \quad (3.11)
\]

It is desired to find the minimum time needed to transfer the system form the original state \((x_{1,0}, x_{2,0})\) to the final state \((x_{1,f}, 0)\). Since the solution for transferring the system from \((x_{1,0}, x_{2,0})\) to the origin \((0, 0)\) is easily extended to the more general case by a simple change of variable, for ease of exposition we assume below that \(x_{1,f} = 0\).
Generalizing the results of [39], it is possible to show that the minimum time is the solution \( t_f \) of

\[
\frac{t_f^2}{a} - \frac{2x_{2,0}}{a} t_f = \frac{x_{2,0}^2 + 2(a + b)x_{1,0}}{ab} \quad \text{if } x_{1,0} + \frac{x_{2,0}}{2b} < 0,
\]

\[
\frac{t_f^2}{b} + \frac{2x_{2,0}}{b} t_f = \frac{x_{2,0}^2 - 2(a + b)x_{1,0}}{ab} \quad \text{if } x_{1,0} + \frac{x_{2,0}}{2a} > 0.
\] (3.12)

The minimum time \( (t_f) \) computed using (3.12) corresponds to a “bang-bang” optimal controller illustrated in Fig. 3.3, which shows switching curves that take the system to the origin using either the minimum or maximum control input (i.e., \( u = -a \) or \( u = b \)). For example, if \((x_{1,0}, x_{2,0})\) corresponds to point \( p_1 \) in Fig. 3.3, then the control input should be \( u = -a \) until the system reaches point \( p_2 \) on the switching curve corresponding to \( u = b \). At this point the control input is switched to \( u = b \), which will take the system to the origin.

It is also important to note that the “heuristic” defined by \( t_f \) is optimistic even when the real system cannot implement the optimal bang-bang control due to actuator limitations at some points of the state space.

![Figure 3.3: Minimum time “heuristic”.](image)

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6. Repeat 2-5 for B (“Branch-out factor”) number of successors.

7. Repeat 1-6 until one of the stopping criteria is true: Steps 1-6 will be repeated until the goal is found or the maximum number of allowable iterations is achieved.

3.2.1 Motor Modeling

In order to exploit the vehicle dynamics, it is necessary to embed into the equations of motion a proper model of the motors, gears, and motor controller. Fig. 3.4 shows a speed-torque curve for the motor reflected at the robot wheels. That is, \( w \) is the wheel speed \([\text{rad/s}]\) and \( \tau \) is the torque at the wheel \([\text{Nm}]\). The operating line \( \bar{\bar{V}}_m \) corresponds to the maximum voltage at the output of the motor controller, which is 14 V. Variable \( \bar{\tau} \) is the maximum output torque at the operating speed \( w \) and is given by

\[
\bar{\tau} = 2\tau_s \left[ 1 - \frac{w}{w_{nl}} \right],
\]

where \( r \) is the wheel radius, \( w_{nl} \) corresponds to the no load speed of the wheel and \( \tau_s \) is the stall torque at the wheel, and are computed using

\[
w_{nl} = \frac{k_n \bar{\bar{V}}_m}{g_r}, \tag{3.14}
\]

\[
\tau_s = \frac{k_n \bar{\bar{V}}_m g_r \bar{\bar{\tau}}^m_s}{\bar{\bar{w}}^m_{nl}}, \tag{3.15}
\]

where \( g_r \) is the gear ratio and \( \bar{\bar{\tau}}^m_s \), and \( \bar{\bar{w}}^m_{nl} \) are the stall torque and no load speed of the motor at a nominal voltage of 12 V. The variables \( \tau_a \) and \( \tau_b \) represent torque loads due to two different terrains. For the motor state depicted in Fig. 3.4, the motor would be able to accelerate while the vehicle is on terrain \( a \) but it would be forced to decelerate on terrain \( b \).

As mention in Chapter 4, the control system allows the inclusion of current (torque) limitation in the motor controllers (\( \tau_{mc} \)). Therefore, the maximum wheel torque at an operating wheel speed
\( \tau_{\text{max}} = \min(\tau, \tau_{mc}), \) \hspace{1cm} (3.16)

which in terms of traction force can be expressed as:

\( F_{\text{max}} = \min\left(\tau, \frac{\tau_{mc}}{r}\right). \) \hspace{1cm} (3.17)

Notice that, we are assuming that the surface can generate that tractive force. In the future, more detailed wheel-terrain interaction models will be considered to cope with surfaces where traction is limited.

### 3.2.2 Surface Modeling

Surface modeling of mobility challenges is difficult for two main reasons: First, due to the actuator limitations, experiments conducted to estimate the terrain model parameters can lead to robot immobilization and task completion failure. Second, for robots operating in the field, it is desirable to keep to a minimum the number of required experiments to determine the model parameters. These two challenges are addressed here by employing reduced order friction models and by following an experimental methodology to estimate the terrain parameters while dealing with actuator limitations.
The general form of the friction models is given by

\[ C(x, \dot{x}) = b(x)\dot{x} + R_r(x), \]  

(3.18)

where \( b \) is a damping term and \( R_r \) corresponds to the rolling resistance. The frictional term \( C \) captures the combined effect of the friction in the drive system and the friction due to the terrain. It is important to clarify that this friction model is only used here as a first approximation as it models reasonably well the situation were the robot is always traversing the same part of terrain. However, we are aware that the friction experienced by the robot is dependent on the density of the vegetation, which changes with the direction of traversal. Current work involves the improvement of this model to account for this uncertainty.

It is also important to mention that the modeling of benign (wooden) surface and a challenging (vegetation) surface is conducted using two different approaches. In the case of the wooden surface, which the robot can traverse without difficulty, both the damping and rolling resistance can be determined by running the robot over the surface at different constant speeds while monitoring the actuator torques. For the vegetation surface, the situation is more complicated because due to the actuator limitation, the robot is always forced to decelerate while traversing this terrain, which can lead to robot immobilization if the robot has little momentum when it comes in contact with the vegetation.

In the field, it is expected that robots will learn the properties of these difficult terrains by first traversing at high speeds terrains of similar characteristics. Then, when faced with new perceptually similar challenging terrains, the robot will employ its past observations as initial guesses of the terrain parameters.

In addition to estimating the model parameters, it is important to approximate the transitions between two different terrains. Here, we employ linear functions of position for both parameters \( b \) and \( R_r \). This transition model was selected because it represents the physical situation in which the
robot has the front set of wheels on the vegetation and the rear wheels on the wood. The complete surface model is illustrated in Fig. 3.5 and is expressed by

\[
R_r = \begin{cases} 
R_{rw} & \text{if } x < -l \\
R_{rv} + \frac{R_{rw} - R_{rw}}{2l}(x - l) & \text{if } -l \leq x < l \\
R_{rv} & \text{if } l \leq x < d - l \\
R_{rv} + \frac{R_{rw} - R_{rw}}{2l}(x - d) & \text{if } d - l \leq x < d + l \\
R_{rw} & \text{if } x \geq d + l,
\end{cases} 
\]

(3.19)

\[
b = \begin{cases} 
b_w & \text{if } x < -l \\
b_v + \frac{b_w - b_w}{2l}(x - l) & \text{if } -l \leq x < l \\
b_v & \text{if } l \leq x < d - l \\
b_v + \frac{b_w - b_w}{2l}(x - d) & \text{if } d - l \leq x < d + l \\
b_w & \text{if } x \geq d + l,
\end{cases} 
\]

(3.20)

Figure 3.5: Terrains and transitions models.
where $2l$ is the terrain transition size, which was set equal to the robot length, and $d$ is the length of the vegetation patch.

Finally, in the case of steep hills, the gravity term needs to be included. For the positions $x$ when all the wheels are on the hill, this term is given by

$$G(x) = \frac{W}{2} \sin(\theta),$$

(3.21)

where $W$ is the robot weight and $\theta$ is the hill inclination. For the transitions regions from level to inclined ground that occur at the base and top of the hill, the gravitational term is modeled in a similar fashion to (3.19).
This chapter discusses the experimental results for the dynamic model verification and motion planning results on wood and asphalt surfaces. The chapter compares distance optimal trajectories with the corresponding energy efficient trajectories on two different terrains, plywood\(^1\) (hereafter simply called “wood”) and asphalt. It also discusses the experimental validation of the proposed momentum-based motion planning on steep hill and stiff vegetation.

---

\(^{1}\)A modular wooden surface was used because it can be attached to a variable slope in the authors’ lab for future experiments that focus on sloped surfaces.
rate of $1 \text{KHz}$ and can be commanded with a linear velocity and turning radius. In order to verify the power model current and voltage sensors are installed at the output of the two motor controller systems to enable recording of the voltage and current for the right and the left motor. Also, the motor controllers can be configured either via hardware or software to limit the maximum current. Currently the system is set to a maximum of $5.46\text{A}$, which corresponds to a maximum torque of $4.63\text{Nm}$. However, since the FSU-Bot is not particularly rugged for vegetation traversal or steep hill climbing, the current (or torque) is limited via software. The robot also carry a visual odometry system that comprises of a bumblebee stereo vision camera and a stand alone computer system. The top plate of the robot was modified to accommodate steel bars of $2\text{kg}$ each as shown in Fig. 4.2. The added weight corresponds to the robot payload and the payloads in the experiments had increments of $4\text{kg}$ each; i.e., a $2\text{kg}$ bar was added on each side of the vehicle, equidistant from the center. The new modification helped to retain the bars securely without significantly affecting the vehicle’s center of mass in the horizontal plane, which made the use of the dynamic model in the experiments more convenient since the center of mass of the vehicle did not have to be recomputed. The total mass of the robot including the modified top plate without payload and the visual odometry system is $23.2\text{kg}$. However, for the dynamic model verification and uphill motions on wooden surface, modified FSU-Bot was considered. The modified FSU-Bot as shown in Fig. 1.3 do not carry visual odometry and the modified top plate. This reduces the weight of the robot

Figure 4.2: The payload carrying plate design for the FSU-Bot used in the study.
to 17.5 kg. All experiments were performed at a constant tire pressure of 20 psi. Table 2.1 lists the key parameters of the FSU-Bot used in the research.

4.1 Dynamic Model Verification

For the dynamic model verification, the speed of the robot was chosen to be constant at 0.2 m/s for the indoor runs, motivated by the findings of [40], which showed that the torques for the outer and inner sides do not vary significantly as a function of speed for the low speed range considered in this study. In addition, due to the low speeds, here we assume that the shift $S_o$ of the center of turn due to centrifugal force is negligible. All the experiments that involve curvilinear motion correspond to the left hand turns in the uphill direction.

Also, as discussed in Section 2.2, in order to compute the frictional forces, it is necessary to know the actual turn radius of the vehicle ($R'$) as a function of the commanded turn radius ($R_{com}$). Motivated by the work of [5] and [34], we employ an empirical expansion factor $\alpha$ such that $R' = \alpha R_{com}$.

To estimate the expansion factor, a group of experiments were conducted in which the robot was commanded to follow a set of different turn radii. The actual turn radius was then measured using an external high speed camera. As shown in Fig. 4.3, $\alpha$ varies as a function of the slope and as a function of the turn radius. It is important to notice that as expected, the expansion factor approaches 1.0 as the turn radius increases.

4.1.1 Experimental Results on Flat Ground

The first experimental runs were performed on a flat wood surface used to estimate the terrain dependent parameters of the dynamic model. The modified FSU-Bot was commanded to move at 0.2 m/s for all the turn radii $R$ in the set $\{0.02, 0.05, 0.10, 0.15, 0.20, 0.30, 0.40, 0.50, 0.70, 1.0, 1.5, 2.0, 3.0, 5.0, 7.0, 10.0, 15.0, 20.0, 30.0, 40.0, 50.0, 70.0, 100.0, 200.0, 500.0, 700.0, 1000.0\}$ m. A to-
Figure 4.3: Expansion factor as a function of slope and turn radius for the modified FSU-Bot on wooden surface.

Figure 4.4: Experimental and analytical torques for modified FSU-Bot for different turn radii (bars represent standard deviation) on flat wooden surface.

A total of 5 runs were performed for each turn radius and the motor torques for the outer and inner vehicle sides were measured by monitoring the current through the motors. The terrain dependent parameters, $\mu_o$, $\mu_i$, and $K$, were estimated by fitting in the least squares sense the proposed model to the experimental torques, yielding $\mu_o = 0.72$, $\mu_i = 0.53$, $K = 0.0006$. Table 4.1 shows the estimated terrain dependent surface parameters of the FSU-Bot.

Fig. 4.4 shows the resultant experimental and analytical curves of the inner and outer side torques. The turn radii $R \leq 0.20$ correspond to turn radii inside the vehicle hull. Notice that the
model performs good predictions for the outer side torques for the complete range of turn radii and for the inner side torque for large turn radii $R \geq \frac{B}{2}$. However, it does a poor job of predicting inner torques for $R < \frac{B}{2}$. This led to modifying the dynamic model so that

$$
\begin{bmatrix}
\tau_i \\
\tau_o
\end{bmatrix} = \begin{bmatrix}
 r(F_i + R_i) + \tau_{ires} \\
 r(F_o + R_o) + \tau_{ores}
\end{bmatrix}
$$

if $R \geq \frac{B}{2}$.

$$
\begin{bmatrix}
\tau_i \\
\tau_o
\end{bmatrix} = \begin{bmatrix}
 -r(F_o + R_i) + \tau_{ires} \\
 r(F_o + R_o) + \tau_{ores}
\end{bmatrix}
$$

if $R < \frac{B}{2}$.

This model modification was motivated by the fact that as the turn radius gets smaller ($R < \frac{B}{2}$), the vehicle behaves similarly to a force couple and therefore the torques on the outer and inner sides are of similar magnitude and opposite signs. It is important to add that due to the differences in rolling resistance and gear friction of the outer and inner sides, the torque magnitudes do not necessarily match. These differences are accounted for in the extended model by $R_i$, $R_o$, $\tau_{ires}$, and $\tau_{ores}$.

Fig. 4.5 shows the resultant experimental and analytical curves of the inner and outer side
Table 4.1: Surface parameters for the FSU-Bot

<table>
<thead>
<tr>
<th>Surface Parameters</th>
<th>FSU-Bot without Visual Odometry (modified)</th>
<th>FSU-Bot with Visual Odometry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wooden Surface</td>
<td>Wooden Surface</td>
</tr>
<tr>
<td>Expansion factor</td>
<td>$\alpha_{\text{wood}}$</td>
<td>$\alpha_{\text{wood}}$</td>
</tr>
<tr>
<td>Outer wheel coeff.</td>
<td>$\mu_o$</td>
<td>$\mu_o$</td>
</tr>
<tr>
<td>Inner wheel coeff.</td>
<td>$\mu_i$</td>
<td>$\mu_i$</td>
</tr>
<tr>
<td>Shear def.</td>
<td>$K$</td>
<td>$K$</td>
</tr>
<tr>
<td></td>
<td>1.44</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>0.72</td>
<td>0.8806</td>
</tr>
<tr>
<td></td>
<td>0.53</td>
<td>0.5795</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0013</td>
</tr>
<tr>
<td>Asphalt Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansion factor</td>
<td>$\alpha_{\text{asphalt}}$</td>
<td></td>
</tr>
<tr>
<td>Outer wheel coeff.</td>
<td>$\mu_o$</td>
<td></td>
</tr>
<tr>
<td>Inner wheel coeff.</td>
<td>$\mu_i$</td>
<td></td>
</tr>
<tr>
<td>Shear def.</td>
<td>$K$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.2165</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.8269</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0016</td>
<td></td>
</tr>
</tbody>
</table>

Torques using the extended model (4.2). The Root Mean Square Error (RMSE) corresponding to Fig. 4.5 for the outer and inner sides are 0.2186 N and 0.1362 N respectively.

4.1.2 Experimental Results on Slopes

Figure 4.6: Experimental and analytical torques for straight line motion on a 10° slope with an initial heading angle of 60°.
After obtaining surface parameters experiments for validation of the model on a slope employing the same wood platform were performed. First, experimental results are presented for linear motion on the hill with an arbitrarily heading. Second, we evaluate the model performance under steady circular motion. In all experiments, the heading angle $\psi$ is measured positive counterclockwise as illustrated in Fig. 2.4.

![Figure 4.7: Experimental and analytical torques for straight line motion on a 15° slope with an initial heading angle of 0°.](image)

Figs. 4.8 and 4.9 present the torque prediction for curvilinear motion. Fig. 4.8 corresponds to the vehicle making a 7m radius turn on a 10° inclined hill with an initial heading of 0°. The corresponding RMSE for the inner and outer sides are 0.3066 Nm and 0.1941 Nm. Fig. 4.9 corresponds
to the vehicle making a 1 m turn radius on a 15° inclined hill and −45° initial heading. Notice that in Fig. 4.9 the model captures the saturation of the outer side actuator.

4.1.3 Effect of Payload on Dynamic Model

This section discusses the experiment results for the effect of payload on the dynamics of a skid-steered wheeled vehicle. In order to study the effect of payload FSU-Bot with visual odometry and modified top plate is used on a different set of wooden surface, which has more compliance and
larger area compared to the one used in Sections 4.1.1 and 4.1.2. This led to recalculate the surface parameters for the robot on flat wooden surface.

Table 4.2: Minimum turn radius (MTR) constraint depending on terrain type and payload

<table>
<thead>
<tr>
<th>Terrain</th>
<th>Surface Parameters ((\mu_o, \mu_i, K))</th>
<th>Payload ((Kg))</th>
<th>MTR ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>((0.8806, 0.5795, 0.0013))</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>3.0</td>
</tr>
<tr>
<td>Asphalt</td>
<td>((1.2165, 0.8269, 0.0016))</td>
<td>0</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The FSU-Bot was commanded to move at \(0.2m/s\) constant forward velocity on each surface for various turn radii to estimate the terrain dependent surface parameters for the dynamic model discussed in Chapter 2 Section 2.2. A total of 3 runs were performed for each turn radius and the motor torques for the outer and inner vehicle sides were measured by monitoring the current through the motors. The robot with no payload was commanded to move on both the wood and asphalt surfaces. The terrain dependent parameters \((\mu_o, \mu_i, K)\) were estimated by fitting (in the least squares sense) the proposed model to the experimental torques (shown in Fig. 3.1), yielding \(\mu_o = 0.8806, \mu_i = 0.5795,\) and \(K = 0.0013\) for the flat wooden surface. It should be emphasized that the surface parameters obtained here were estimated for the no payload case. The solid black line in Fig. 3.1 represents the maximum torque limit of the vehicle \((4.63Nm)\) of one of the vehicle’s motors. A Vicon motion capture system was used to track the vehicle motion and estimate the expansion factor as discussed in Section 3.1.1. For the FSU-Bot moving on the flat wooden surface at \(0.2m/s\) with no payload the expansion factor was estimated to be \(\alpha = 1.44\). The estimated surface parameters for asphalt were \(\mu_o = 1.2165, \mu_i = 0.8269,\) and \(K = 0.0016\) with
Figure 4.10: Torque vs. Turn Radius curve for the FSU-Bot moving at a constant forward velocity of $0.2m/s$ with $4kg$ payload on a flat wooden surface. The minimum turn radius ($MTR$) for a $4kg$ payload is $2.0m$. (The surface parameters were estimated for no payload, demonstrating good generalization of the model to payload changes.)

An expansion factor $\alpha = 1.39$. Table 4.1 shows the estimated terrain dependent surface parameters of the FSU-Bot.

Later, using the surface parameters and the expansion factor corresponding to motion with no payload on a wooden surface, the model was used to predict the torques and minimum turn radius dynamic constraints for $4kg$, $8kg$ and $12kg$ payloads. For both the wood and asphalt surfaces, Table 4.2 summarizes the minimum turn radius dynamic constraints for the FSU-Bot with various payloads. As shown in Figs. 4.10–4.12, the model was able to accurately predict the effect of payload on the vehicle torques and the MTR, which is shown by the $x$-axis value corresponding to the point where the maximum torque line intersects the outer torque prediction curve (shown in Figs. 4.10–4.12). For the FSU-Bot with no payload moving on the wooden surface with a constant forward velocity of $0.2m/s$ the predicted MTR is $1.5m$. As shown in Fig. 3.1, the experimental results verified this value. With the increase in payload, the vehicle’s MTR increases. For example, for a $4kg$ payload the predicted and experimental MTR was $2m$ (see Fig. 4.10), whereas for a $12kg$ payload it turns out to be $3m$ (see Fig. 4.12).
Figure 4.11: Torque vs. Turn Radius curve for the FSU-Bot moving at a constant forward velocity of $0.2\, m/s$ with $8\, kg$ payload on a flat wooden surface. The minimum turn radius ($MTR$) for an $8\, kg$ payload is $2.5\, m$. (The surface parameters were estimated for no payload, demonstrating good generalization of the model to payload changes.)

Figure 4.12: Torque vs. Turn Radius curve for the FSU-Bot moving at a constant forward velocity of $0.2\, m/s$ with a $12\, kg$ payload on a flat wooden surface. The minimum turn radius ($MTR$) for $12\, kg$ payload is $3.0\, m$. (The surface parameters were estimated for no payload, demonstrating good generalization of the model to payload changes.)
4.2 Dynamically Feasible, Energy Efficient Motion Planning

4.2.1 Motion Planning Simulation Results for a Flat Wooden Surface

Simulations were performed to compare the distance and energy requirements of dynamically feasible, energy efficient trajectories with those of the corresponding distance optimal trajectories on a flat wooden surface. Below, “energy efficient motion planning” refers to motion planning that both optimizes an energy cost function and enforces the MTR constraint. Hence, an “energy efficient trajectory” is also dynamically feasible (i.e., it meets the MTR constraint). The simpler language is used for ease of presentation.

In this research distance optimal trajectories were computed using SBMPO along with the kinematic model discussed in Section 3.1.1. The cost function used for distance optimal motion planning was the distance traveled and the heuristic was the Euclidean distance. Since traditional distance optimal motion planning does not exploit the dynamics of the vehicle, no dynamic constraints (i.e., MTR constraints), were considered for those trajectories, such that it was assumed that the vehicle can provide all the actuation required to negotiate a given trajectory’s turns. Hence, the trajectories were not all necessarily dynamically feasible, i.e., they may violate the MTR constraints (and often did). However, even in this case, the energy predictions are indicative of the energy consumption of a vehicle that has more powerful motors. These results are important since it appears that skid steered vehicles are sometimes designed with extremely powerful motors for which MTR constraints may not exist under many circumstances (i.e., loads and terrain surfaces).

The simulations involved various scenarios, characterized by the initial position and orientation of the vehicle, the location of the goal, and the obstacle configuration (if any). However, due to space limitations only three of them are discussed in detail. Statistics based on all of the performed simulations are described at the end of this section. In each simulation, the robot was assumed to move at a constant forward velocity of 0.2 m/s on a 7 m × 7 m wooden surface. As shown in Figs. 4.13–4.15, the goal region is represented by a solid (blue) circle centered at the goal location and
having a radius of $0.3m$. Table 4.3 shows the computed results for total energy and path length and their comparison.

![Figure 4.13: Simulated motion planning results for Scenario 1 with $0^\circ$ initial heading with respect to the $x$-axis, start at $(0.5, 0.5)m$, goal at $(6.5, 6.5)m$, and a constant forward velocity of $0.2m/s$ on a flat wooden surface. The distance optimal trajectory did not change with the variation in payload and always made an initial sharp turn to orient the vehicle towards the goal and minimize the distance. The energy optimal trajectory avoided sharp turns to avoid unnecessary energy consumption. As the payload increased the energy optimal trajectory made wider turns.](image)

The planning is first performed for an obstacle free environment (Scenario 1) using both distance and energy optimization for various payloads. For Scenario 1, the robot was assumed to move from start position at $(0.5, 0.5)m$ with an initial heading direction of $0^\circ$ with respect to the $x$-axis to the goal region centered at $(6.5, 6.5)m$. As shown in Fig. 4.13, for distance optimal motion planning the vehicle made a sharp turn in the beginning and then moved in a straight line towards the goal. For no payload energy efficient motion planning, the vehicle avoided sharp turns and moved along an arc towards the goal. However, as the payload increased, the vehicle followed a wider turn. The predicted total energy consumption for the distance optimal trajectory was $361J$, whereas for the no payload energy efficient trajectory it was $461J$. In order to achieve the energy savings for no payload, the vehicle moved $8.6m$ for the energy efficient trajectory as compared to $8.2m$ distance for the distance optimal trajectory. As expected, with the increase in payload the energy consumption also increases for energy efficient motion planning (e.g. $564J$ for the $4kg$...
Figure 4.14: Simulated motion planning results for Scenario 2 with $0^\circ$ initial heading with respect to the $x$-axis, start at $(0.5, 0.5)m$, goal at $(6.5, 6.5)m$, no payload, and a constant forward velocity of $0.2 m/s$ on a flat wooden surface. The distance optimal trajectory made sharp turns to minimize the distance to reach the goal. In contrast, the energy optimal trajectory made wide turns to avoid the high energy consumption associated with the sharp turns.

payload and $910J$ for the $8kg$ payload).

Figure 4.15: Simulated motion planning results for Scenario 3 with $90^\circ$ initial heading with respect to the $x$-axis, start at $(2.0, 0.5)m$, goal at $(6.0, 6.5)m$, no payload, and a constant forward velocity of $0.2 m/s$ on a flat wooden surface. As in Fig. 4.14, the distance optimal trajectory made sharp turns to minimize the distance to reach the goal. In contrast, the energy optimal trajectory made wide turns to avoid the high energy consumption associated with the sharp turns.
Motion planning results for Scenarios 2 and 3 involved obstacles and are shown respectively in Figs. 4.14 and 4.15. For Scenario 2, the robot start position was \((0.5, 0.5)m\) with the goal region centered at \((6.5, 6.5)m\). However, for Scenario 3, the robot start position was \((2.0, 0.5)m\) with the goal region centered at \((6.0, 6.5)m\). The corresponding results of Table 4.3 reveal that for a small increase in distance, the energy efficient motion planning is able to dramatically decrease the energy consumption. For example, for Scenario 2 of Fig. 4.14, a 6.7% increase in distance led to a 33.1% decrease in energy consumption, while for Scenario 3 of Fig. 4.14, a 1.4% increase in distance led to a 47.7% decrease in energy consumption.

Table 4.3: Simulation results for motion planning on a flat wooden surface

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Planning</th>
<th>Computation Time (s)</th>
<th>Path Length (m)</th>
<th>Path Difference (%)</th>
<th>Energy (J)</th>
<th>Energy Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>Min. Dist.</td>
<td>0.5554</td>
<td>8.9</td>
<td>6.71</td>
<td>1276</td>
<td>33.07</td>
</tr>
<tr>
<td></td>
<td>Min. Energy</td>
<td>0.8798</td>
<td>9.5</td>
<td></td>
<td>854</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Min. Dist.</td>
<td>0.5664</td>
<td>7.3</td>
<td>1.35</td>
<td>935</td>
<td>47.70</td>
</tr>
<tr>
<td></td>
<td>Min. Energy</td>
<td>0.186</td>
<td>7.4</td>
<td></td>
<td>489</td>
<td></td>
</tr>
</tbody>
</table>

*The path and energy differences are computed with respect to minimum distance motion planning.

Trajectories were designed for the robot’s movement on the \(7m \times 7m\) wooden surface for a total of 14 scenarios. The payload was varied from \(0kg\) to \(12kg\) in increments of \(4kg\). The start and goal location of the robot were varied randomly with randomly placed obstacles, varying in number from 2 to 8. The orientation of the robot was selected as either \(0^\circ\) or \(90^\circ\) with respect to x-axis. In each of the simulation results, the distance optimal motion planning violated the MTR constraint. (It is certainly possible for the distance optimal planning to not violate the MTR constraint, for example if the vehicle is initially pointed towards the goal, in which case it will move in a trajectory with very large radii of curvature; however, the experiments cited here did not involve this situation.) In contrast energy efficient motion planning enforced the MTR constraint and re-
duced the energy consumption by an average of 38.85% while increasing the distance traveled by an average of only 4.3%. Furthermore, the average computational time for computing a distance optimal trajectory was in the range $[0.007, 0.566] \text{sec}$ and averaged $0.171 \text{sec}$, while the computational times for the energy efficient trajectories were in the range $[0.007, 1.635] \text{sec}$ and averaged $0.217 \text{sec}$. Although the energy efficient trajectories had the higher average computational time, for a given scenario they were not always more computationally expensive. The higher times appeared to be due to the enforcement of the MTR constraints, which sometimes forced SBMPO to search the input space more in order to find an input leading to a feasible trajectory. However, the MTR constraint can also cause the vehicle to find a trajectory that avoids obstacles, instead of weaving through them, which can lead to smaller computational times.

### 4.2.2 Experimental Results for a Wooden Surface

This section describes and discusses the results of experiments conducted on a $7m \times 7m$ flat wooden surface. Each of the energy efficient trajectories and each of the distance optimal trajectories, corresponded to the FSU-Bot moving at a constant forward velocity of $0.2m/s$. The experimental results discussed here and below in Section 4.2.3 (for movement on asphalt) highlight the importance of the MTR constraint, since it cannot be ignored as it was in the simulation results of the previous section. In particular, they provide concrete illustration of the difficulties in attempting to track trajectories designed without taking into account this constraint. They also give insight into the accuracy of the energy consumption predicted by the power model.

The tracking results for the robot are shown in Figs. 4.16–4.18 for Scenario 2, and in Figs. 4.19–4.21 for Scenario 3. Figs. 4.16 and 4.19 show the position tracking of the robot using a Vicon motion capture system for both distance optimal and energy efficient trajectories. Figs. 4.18 and 4.21 show the velocity tracking and torque measurement of the robot while executing the energy

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2This dissertation has supplementary downloadable material (Motion_planning.zip) showing the motion planning results for movement on both wood and asphalt surfaces as presented in this dissertation.
Figure 4.16: Motion planning results for Scenario 2 with 0° initial heading with respect to the x-axis, start at (0.5, 0.5)m, goal at (6.5, 6.5)m, and no payload (E represents energy optimal trajectories and D represents distance optimal trajectory.) The experiments were conducted on a flat wooden surface with a commanded constant forward velocity of 0.2m/s. The robot was not able to track the distance optimal trajectory and headed away from the goal. For the energy optimal trajectory, the tracking was good and the robot was able to reach the goal.

Figure 4.17: Velocity tracking and torque measurement results for the distance optimal trajectory shown in Fig. 4.16. The vehicle was not able to track the velocity due to torque saturation.

efficient trajectories of Scenarios 2 and 3, respectively. As shown in these figures, the robot was able to properly track the energy efficient trajectories. The experimental results are shown in Table 4.4 along with the simulated results. For distance optimal trajectories, the robot trajectory tracking
was significantly off. As shown in Figs. 4.17 and 4.20, most of the time the robot actuators were saturated, causing poor velocity and position tracking as seen in Fig. 4.16 for Scenario 2 and Fig. 4.19 for Scenario 3. In Table 4.4, some cells are labeled as “N.A.”, indicating that the vehicle did not complete the mission by either hitting an obstacle or moving significantly off course.

![Figure 4.18: Velocity tracking and torque measurement results for the energy optimal trajectory shown in Fig. 4.16. The vehicle was able to avoid saturating the two actuators and had good velocity tracking.](image)

The experiments were further extended to study the effect of payload on the motion planning task. In Scenario 4, the robot was first commanded with a no payload, energy efficient trajectory. As can be seen from Fig. 4.22, the robot was able to closely track the trajectory with an actual energy consumption of $348J$ and a predicted energy consumption of $471J$. The robot was then commanded with the same trajectory but while carrying a payload of $8kg$. The position tracking results are shown in Fig. 4.22 with velocity tracking and torque measurements in Fig. 4.23. As can be seen from Fig. 4.23 the actuators of the robot were saturated, causing poor velocity tracking and leading to collision with an obstacle. However, when the robot having an $8kg$ payload was commanded with an $8kg$ energy efficient trajectory, it was able to avoid obstacles and reach the goal. It should be noted that the small error in the final position of the robot could be due to an error in the initial orientation of the robot.
Figure 4.19: Motion planning results for Scenario 3 with $90^\circ$ initial heading with respect to the $x$-axis, start at $(2.0, 0.5)m$, goal at $(6.0, 6.5)m$, and no payload ($E$ represents energy optimal trajectories and $D$ represents distance optimal trajectory.) The experiment was conducted on a flat wooden surface with a commanded constant forward velocity of $0.2m/s$. The robot was not able to track the distance optimal trajectory and headed away from the goal. For the energy optimal trajectory, the tracking was good and the robot was able to reach the goal.

Figure 4.20: Velocity tracking and torque measurement results for the distance optimal trajectory shown in Fig. 4.19. The vehicle was not able to track the velocity due to torque saturation.

Fig. 4.24 shows the percentage error in the energy prediction for 17 combinations of payloads and obstacle configurations for energy efficient motion planning on wood and asphalt. For each scenario, the error was averaged over three experimental runs. As can be seen in the Fig. 4.24, the maximum error was $-35.25\%$, the minimum error was $-5.90\%$, and the mean error was $-19.88\%$.
Figure 4.21: Velocity tracking and torque measurement results for the energy optimal trajectory shown in Fig. 4.19. The vehicle was able to avoid saturating the two actuators and had good velocity tracking.

Figure 4.22: Motion planning results for Scenario 4 with $90^\circ$ initial heading with respect to the $x$-axis, start at $(0.5, 0.5)m$, goal at $(6.5, 6.5)m$, and various payloads ($E$ represents energy optimal trajectories). The experiment was conducted on a flat wooden surface with a commanded constant forward velocity of $0.2m/s$. The robot carried an $8kg$ payload but the commanded energy optimal trajectory assumed no payload and the robot was not able to track the trajectory and even hit an obstacle.

for the different scenarios. This over-prediction of energy consumption is partly due to the fact that the predicted torques tend to be greater than the experimental torques as evidenced in Figs. 3.1–4.12, leading to an over-prediction of power. Integration of the small error in power over time leads to significant errors in the prediction of energy. The higher errors seen for the wooden
surface are likely due to the an-isotropic nature of this surface, which contrasts with the relatively isotropic nature of the asphalt surface. (Experimental results for the asphalt surface are discussed in Section 4.2.3.) It should be emphasized that the dynamic model described in Section II-B assumes an isotropic surface since the surface parameters ($\mu_o$, $\mu_i$, and $K$) are constant. To account for anisotropy, these parameters must vary according to the vehicle orientation with respect to the surface. This makes the model (and its use) more complex and in practice, would also require a perception system that can detect the grain direction or other directional properties of the surface.

Figure 4.23: Velocity tracking and torque measurement results with the robot carrying an 8 kg payload but commanded to follow the 0 kg payload energy optimal trajectory shown in Fig. 4.22. The vehicle was not able to track the velocity due to torque saturation.

Table 4.4 shows the computational times for both distance optimal and energy efficient motion planning. As can be seen from the table, the computational times for energy efficient motion planning ranged from 0.01 sec to 1.64 sec and were comparable with the corresponding times (when they exist) for the distance optimal trajectories.

**4.2.3 Experimental Results for an Asphalt Surface**

The results of Section 4.2.2 were later extended to an outdoor asphalt surface with the robot moving at the higher speed of $0.6m/s$. First, the dynamic model for FSU-Bot was verified for
Table 4.4: Experimental results for motion planning

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Planning</th>
<th>Computation Time (s)</th>
<th>Path Length (m)</th>
<th>Predicted Energy (J)</th>
<th>Actual Energy (J)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>Min. Dist.</td>
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<td>8.9</td>
<td>1276</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>Min. Energy</td>
<td>0.88</td>
<td>9.5</td>
<td>854</td>
<td>765.17</td>
<td>-11.61</td>
</tr>
<tr>
<td>3.</td>
<td>Min. Dist.</td>
<td>0.57</td>
<td>8.9</td>
<td>935</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>Min. Energy</td>
<td>0.19</td>
<td>7.3</td>
<td>489</td>
<td>387</td>
<td>-26.34</td>
</tr>
<tr>
<td></td>
<td>Payload</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assumed</td>
<td>Actual</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0kg</td>
<td>0kg</td>
<td>0.01</td>
<td>8.5</td>
<td>471</td>
<td>348</td>
</tr>
<tr>
<td></td>
<td>0kg</td>
<td>8kg</td>
<td>0.01</td>
<td>8.5</td>
<td>471</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>8kg</td>
<td>8kg</td>
<td>1.64</td>
<td>9.4</td>
<td>1122</td>
<td>927</td>
</tr>
<tr>
<td>5.</td>
<td>Min. Dist.</td>
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<td>419</td>
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<td>11.76</td>
<td>511</td>
<td>N.A.</td>
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<td>0.02</td>
<td>12.48</td>
<td>749</td>
<td>622</td>
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</tbody>
</table>

*N.A. = Not applicable if the vehicle hits an obstacle or headed away from the goal.
Figure 4.24: The graph of the error \( \frac{E_{\text{act}} - E_{\text{pred}}}{E_{\text{act}}} \), where \( E_{\text{act}} \) and \( E_{\text{pred}} \) are respectively the actual and predicted energy consumption, for the robot executing various energy optimal trajectories on the wood and asphalt surfaces.

Figure 4.25: Torque vs. Turn Radius curve for the FSU-Bot with no payload on a flat asphalt surface running at a constant forward velocity of 0.6 m/s. The expansion factor and surface parameters were maintained at the same values as those identified with no payload and a constant forward velocity of 0.2 m/s. The minimum turn radius (\( MTR \)) for no payload is 2.0 m.

A constant forward velocity of 0.6 m/s and the results are shown in Fig 4.25. It should be noted that the surface parameters were maintained at the same values identified for the robot running at a constant forward velocity of 0.2 m/s on the asphalt surface. Then, simulation and experimental motion planning was performed for the FSU-Bot running at a constant forward velocity of 0.6 m/s.
The motion planning results for Scenario 5 and Scenario 6 are shown in Figs. 4.26–4.30 and the computational time comparison is shown in Table 4.4. The high quality of the MTR and energy prediction results here strongly suggests that the dynamic and power model have low dependence on the vehicle speed.

![Figure 4.26: Motion planning results for Scenario 5 with 0° initial heading with respect to the x-axis, start at (0.5, 0.5)m, goal at (8.5, 8.5)m, and no payload (\(E\) represents energy optimal trajectories and \(D\) represents distance optimal trajectory.) The experiment was conducted on an asphalt surface with a commanded constant forward velocity of 0.6\(m/s\). The robot was not able to track the distance optimal trajectory and actually hit an obstacle. For the energy optimal trajectory, the robot was able to track the trajectory and reach the goal.](image)

Fig. 4.26 shows the position tracking of the FSU-Bot while following the energy efficient and distance optimal trajectories for Scenario 5 at a constant forward velocity of 0.6\(m/s\). As shown in this figure, the robot was able to track the energy efficient trajectory. In contrast, while tracking the distance optimal trajectory, the left actuator saturated during sharp turns (as shown in Fig. 4.27), causing the robot to hit an obstacle. Fig. 4.28 shows the velocity tracking and torque measurements for the FSU-Bot while tracking the energy efficient trajectory of Scenario 5. The total actual energy for the energy efficient trajectory was 439\(J\) as compared to the total predicted energy of 507\(J\) with an error of \(-15.55\%\). Since the robot hit an obstacle while tracking the distance optimal trajectory of Scenario 5 (see Fig. 4.26), the total actual energy measurement was not applicable here.
Figure 4.27: Velocity tracking and torque measurement results for the distance optimal trajectory shown in Fig. 4.26. The vehicle was not able to track the velocity due to torque saturation.

Figure 4.28: Velocity tracking and torque measurement results for the energy optimal trajectory shown in Fig. 4.26. The vehicle was able to avoid saturating the two actuators and had good velocity tracking.

Experiments were also conducted to test the effect of payload while tracking the energy efficient trajectory on the asphalt surface. First, the robot was commanded to follow the energy efficient trajectory with no payload for Scenario 6, as shown in Fig. 4.29. The total actual energy was $419J$ in comparison to the predicted energy of $511J$ with an error of $-22.0\%$. Then, the robot was commanded with the same trajectory but having an actual payload of $8kg$. As can be seen in
Figure 4.29: Motion planning results for Scenario 6 with $0^\circ$ initial heading with respect to the $x$-axis, start at $(0.5, 0.5)m$, goal at $(8.5, 8.5)m$, and various payload ($E$ represents energy optimal trajectories). The experiment was conducted on an asphalt surface with a commanded constant forward velocity of $0.6m/s$. The robot carried an $8kg$ payload but was commanded to follow a $0kg$ payload energy optimal trajectory and hit an obstacle. Fig. 4.29, the robot was not able to track the trajectory and eventually hit an obstacle. The velocity tracking and torque measurement for this scenario can be seen in Fig. 4.30. However, when the robot having an $8kg$ payload was commanded with an $8kg$ payload trajectory, it was able to track the trajectory (shown in Fig. 4.29). The total actual energy was $622J$ in comparison to the total predicted energy of $649J$ with an error of $-20.4\%$.

4.2.4 Motion Planning Simulation Results for a Sloped Wooden Surface

Simulation experiments for motion planning were performed via SBMPO using distance optimization and energy optimization. The planning is performed for one uncluttered and three cluttered environment scenarios. The robot was assumed to move at a constant linear speed of $0.2m/s$ on an inclined wooden surface with $\phi = 10^\circ$ and the kinematic model in (3.1) was used as the SBMPO model of Fig. 1.2. In all three scenarios the start point was $(0, 0, 0)m$ and the goal region was a hemisphere (in solid blue color) centered at $(5, 5, 0.86)m$ and having a radius of $0.5m$ (as shown in Fig. 4.31). Table 4.5 shows the computed results for total energy and path length and their comparison.
Figure 4.30: Velocity tracking and torque measurement results with the robot carrying an 8kg payload but commanded to follow the 0kg payload energy optimal trajectory shown in Fig. 4.29. The vehicle was not able to track the velocity due to torque saturation.

Figure 4.31: Simulated wooden platform with goal at (5, 5, 0.86)m.

The planning is first performed for an uncluttered environment using both distance and energy optimization. The vehicle in both the cases is oriented towards the goal with $\theta_{in} = 45^\circ$ ($\theta_{in}$ represents initial orientation of the vehicle). As shown in Fig. 4.32, as expected, for both cases the vehicle moves in the same straight line path towards the goal and hence the total energy consumption and the path length are identical. However, distance and energy optimization do not yield identical trajectories if the initial orientation of the vehicle is not towards the goal as shown in Fig. 4.33, where $\theta_{in} = 0^\circ$. As expected distance optimization provided a shorter path trajectory than energy optimization while energy optimization provided a smaller energy trajectory than distance.
Figure 4.32: Simulated motion planning results for no obstacle and with an initial orientation of $45^\circ$ w.r.t. x-axis at $(0, 0, 0)m$.

Figure 4.33: Simulated motion planning results for no obstacle and with an initial orientation of $0^\circ$ w.r.t. x-axis at $(0, 0, 0)m$.

optimization. Referring to Fig. 4.33 the reason for this is that distance optimization yielded a sharp turn near the origin to enable the robot to align itself towards the goal and thus move in a straight path to reach the goal. For energy optimization sharp turns are avoided when possible since they consume a lot of energy. (As Fig. 2.5 reveals, the power consumption for a skid-steered wheeled vehicle increases exponentially with decreasing turn radius.) Hence, the trajectory corresponds to an initially large radius path to enable the robot to align itself towards the goal.

For motion planning in cluttered environments obstacles are shown by the varied size grey color hemispheres at different locations as seen in Figs. 4.34 - 4.36. The results shown in Table 4.5 reveal that for small increases in distance, energy efficient motion planning is able to dramatically
Figure 4.34: Simulated motion planning results for Scenario 1 with an initial orientation of 45° w.r.t. x-axis at (0, 0, 0)m.

Figure 4.35: Simulated motion planning results for Scenario 2 with an initial orientation of 45° w.r.t. x-axis at (0, 0, 0)m.

Figure 4.36: Simulated motion planning results for Scenario 3 with an initial orientation of 0° w.r.t. x-axis at (0, 0, 0)m.
decrease the energy consumption. For example, for Scenario 1 shown in Fig. 4.34 with \( \theta_{in} = 45^\circ \), a 1.2% increase in distance led to a 43.8% decrease in energy consumption, while for Scenario 2 shown in Fig. 4.35 with \( \theta_{in} = 45^\circ \), a 7.5% increase in distance led to a 34.9% decrease in energy consumption. The energy optimal trajectories are substantially smoother with fewer number of turns and large turning radii as compared to the corresponding distance optimal trajectories. Similar trends have been observed for initial vehicle orientations (given by \( \theta_{in} = 0^\circ \) and \( \theta_{in} = 90^\circ \)) and test scenarios, as shown in Figs. 4.36 - 4.39.

Figure 4.37: Simulated motion planning results for Scenario 2 with an initial orientation of \( 0^\circ \) w.r.t. x-axis at \((0, 0, 0)m\).

Figure 4.38: Simulated motion planning results for Scenario 3 with an initial orientation of \( 90^\circ \) w.r.t. x-axis at \((0, 0, 0)m\).
Figure 4.39: Simulated motion planning results for Scenario 2 with an initial orientation of $90^\circ$ w.r.t. x-axis at $(0, 0, 0)m$.

Table 4.5: Simulated motion planning results for energy efficient and distance optimal motion planning.

<table>
<thead>
<tr>
<th>Orientation [Deg]</th>
<th>Obstacle Case</th>
<th>Cost Function</th>
<th>Energy [J]</th>
<th>Distance [m]</th>
<th>Energy Difference %</th>
<th>Distance Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$45^\circ$</td>
<td>No obstacle</td>
<td>Min. Distance Min. Energy</td>
<td>551 551</td>
<td>6.72 6.72</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>No obstacle</td>
<td>Min. Distance Min. Energy</td>
<td>872 605</td>
<td>6.71 6.85</td>
<td>30.62</td>
<td>2.08</td>
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<td>Scenario 1</td>
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</tr>
<tr>
<td>$45^\circ$</td>
<td>Scenario 2</td>
<td>Min. Distance Min. Energy</td>
<td>1015 661</td>
<td>7.18 7.72</td>
<td>34.87</td>
<td>7.52</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>Scenario 3</td>
<td>Min. Distance Min. Energy</td>
<td>1295 651</td>
<td>7.43 7.49</td>
<td>49.73</td>
<td>0.80</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>Scenario 2</td>
<td>Min. Distance Min. Energy</td>
<td>1270 649</td>
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<td>48.89</td>
<td>7.39</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>Scenario 3</td>
<td>Min. Distance Min. Energy</td>
<td>1226 653</td>
<td>7.44 7.48</td>
<td>46.74</td>
<td>0.54</td>
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<tr>
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<td>1271 719</td>
<td>7.18 7.22</td>
<td>43.43</td>
<td>0.56</td>
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</table>
4.3 Momentum Based Motion Planning

Two mobility challenges have been considered in the study so far: stiff vegetation and steep hill\(^3\). The vegetation patch depicted in Fig. 4.40 consists of a wooden platform and a 0.91 \(\times\) 0.91 \(m^2\) patch of artificial plants. The starting point of the vegetation patch was considered as the origin of the reference frame used to define the start and goal locations. To get the surface parameters, the following experiment was conducted. Starting from the wooden surface, the robot was allowed to accelerate to a relatively high speed of about 1.0 \(m/s\), which gave it enough momentum to traverse the vegetation. During this process, actuator torques, and robot velocity were measured. Fig. 4.41 shows the velocity profile of this maneuver, which clearly shows that during the traversal of the patch, the robot is forced to decelerate. There is also a noticeable high variance in the velocity caused by the non homogeneity of the terrain. Due to all this uncertainty and the fact that in the field, experiments should be kept to a minimum, it was decided to simplify the model of the vegetation and include only a high rolling resistance term, which was estimated by

\[ R_{rv} = F - M\ddot{x}, \]  

where \(\ddot{x}\) corresponds to the average deceleration of the robot while traversing the vegetation, and \(F\) is the applied tractive force. The estimated resistance obtained from the experiment of Fig. 4.41 was 27.59 \(N\).

Initially, it was demonstrated that the terrain patch was indeed a mobility challenge for the torque limited FSU-Bot by conducting experiments that lead to task completion failure (i.e., the robot becomes immobilized in the terrain). Finally, experiments for the robot’s ability to manipulate its momentum as a way to traverse the terrain were performed for different start and/or goal positions.

\(^3\) This dissertation has supplementary downloadable material (Motion_planning.zip) showing the momentum based motion planning results for movement on both steep hill and stiff vegetation as presented in this dissertation.
locations. The objective was to reach the specified goal location at zero velocity with minimum time.

The second mobility challenge was a steep hill. As shown in Fig. 4.42, the robot was faced with a wooden steep hill of length $1.22m$ having a variable slope from $0^\circ$ to $16^\circ$ range. However for the experiment a fixed slope of $16^\circ$ was considered. The bottom edge of the hill was considered as the origin (see Fig. 4.42). The estimated surface parameters for the hill surface were $R_{rw} = 7.86N$ and $b_w = 3.36Ns/m$. Similar to the previous scenario, the experiment comprised of two stages. In the first part, the robot’s inability to reach the top of the hill quasi-statically was demonstrated. In the second part, experiments for the robot’s ability to reach a position at the top of the hill with
zero velocity were performed for different initial positions.

Before proceeding with the experimental validation of the proposed approach. It is important to clarify some aspects about the “Branch out” factor used by SBMPO. For the problem considered in this paper, it is possible to argue that during each iteration of SBMPO, it would suffice to consider only three deterministic inputs each time. That is, given the current vehicle state, one can use the dynamic model in (2.2) to compute the maximum, minimum and ‘0’ acceleration control inputs and let SBMPO generate the resulting trajectory based on these. In effect, this approach is effective for these type of scenarios, and should be used whenever possible due to its low computational time. However, for more general motion, where in addition to exploiting the momentum, the vehicle should avoid obstacles, it would be required to increase the number of control samples. Motivated by this reasoning, a branch out factor of 10 was selected for the following scenarios. In all experiments, the sampling time \( T \) was set to 0.05s, which results in velocity profiles at 20Hz. However, since the robot is controlled at 1kHz, linear interpolation between consecutive samples was employed.

Initially, motion planning was performed using the robot kinematic model only (i.e., the planner had no information about the ground interaction, robot inertia, and motor model) for both stiff vegetation and steep hill. In both experiments the robot was located close to the beginning of
the mobility challenge and was commanded to follow a velocity of $1.0 \text{m/s}$ for the vegetation and $1.5 \text{m/s}$ for the hill (the commanded velocities could have been the same values. Notice that in both cases the maximum velocity achieved by the robot was below $0.6 \text{m/s}$). However, as shown in the position and velocity profiles of Figs. 4.43 and 4.44, the robot became immobilized in the middle of the mobility challenges. The included torque profiles clearly show that the torques are saturated and the robot didn’t have enough momentum to traverse the mobility challenges. It is important to clarify that the maximum torque was set to $25\%$ for the stiff vegetation and $40\%$ for the steep hill. This was done to guarantee that the terrains constitute indeed mobility challenges for the FSU-Bot.

In Scenario 1, the motivational experiment shown in Fig. 4.43 was repeated for the stiff vegetation patch. However, in this case the robot exploits its dynamics and finds a trajectory from the initial state $(x_0, \dot{x}_0) = (-0.23\text{m}, \text{0.0m/s})$ to the final state $(x_f, \dot{x}_f) = (1.8\text{m}, \text{0.0m/s})$, which

![Graphs showing position, velocity, and torque profiles](image)

**Figure 4.43:** Position, velocity and torque profiles, corresponding to the robot immobilization on the stiff vegetation (shaded area represents the location of the vegetation patch).
requires the robot to back up to gather the required momentum to traverse the vegetation and reach the goal in minimum time and at zero velocity. Fig. 4.45 compares the desired and actual position and velocity profiles.

It is interesting to point out that as predicted by the dynamics, and clearly shown in the velocity profile of Fig. 4.45, the slopes of the acceleration ($t < 1.6s$) and deceleration ($t < 2.3s$) regions are not symmetric. This is due to the fact that while the robot is decelerating, the rolling resistance is helping the motors. Fig. 4.45 also shows the torque profiles, which illustrate the effectiveness of the manipulation of the vehicle momentum as a strategy to traverse the patch. Notice that for most of the region where the vehicle is traversing the patch, the wheel torques are saturated and the vehicle is decelerating (the observed variability in the torque correspond to the discrete location of the vegetation stems).

In Scenario 2, the robot was confronted with the vegetation patch but its initial state was set to $(-1.06m,$

![Graphs showing position, velocity, and torque profiles.](image)

Figure 4.44: Position, velocity and torque profiles, corresponding to the robot immobilization on the steep hill (shaded area represents the location of the steep hill).
Figure 4.45: Position, velocity and torque profiles, corresponding to the successful traversal of the vegetation patch of the motivational experiment (shaded area in the position profile represents the vegetation patch).

Figure 4.46: Position, velocity, and torque profiles, corresponding to Scenario 2.

0m/s), which corresponds to the distance that the motion planner asked the robot to back up in the first scenario. The goal was kept at (1.8m, 0.0m/s). As expected, the generated minimum

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Figure 4.47: Position, velocity and torque profiles, corresponding to the successful traversal of the steep hill of the motivational experiment (the shaded region in the position profile represents the steep hill).

time trajectory shown in Fig. 4.46 does not require the robot to back up and the robot successfully completed its mission.

Scenario 3, corresponds to the steep hill experiment of the motivational experiment shown in Fig. 4.44. The hill inclination remained at $10^\circ$ and the goal state was $(1.8m, 0.0m/s)$. As shown in the trajectory profiles of Fig. 4.47, the robot backed up the necessary distance to gather the required momentum to conquer the hill and reach the goal state in minimum time.

Scenario 4, corresponds to a steep hill scenario with an inclination of $16^\circ$ and serves to motivate important future research in the area of robust integration of perception and control. Two different experiments were conducted in this setup. Initially, accurate perception (i.e., correct hill inclination and correct initial and goal states) was assumed and as shown in the profiles of Fig. 4.48, the robot successfully conquered the hill by gathering the required momentum to reach the goal state $(1.8m, 0.0 m/s)$ in minimum time. However, in a second experiment on the same setup, an
erroneous estimate of the hill inclination was given to the planner. That is, the robot was told that the slope was 10° when the actual value stayed at 16°. Notice from the trajectories of Fig. 4.48 that the robot failed to complete its mission due to the introduced perceptual error. An additional experiment where the inclination of the hill was overestimated by 4° was conducted and in that case the robot accomplished the mission successfully.

Scenario 5, illustrates an experiment in which a perception error of 1.03 m was introduced in the initial location of the robot with respect to the vegetation patch. As shown in the position and velocity profiles of Fig. 4.49, the robot failed to complete its mission as it did not have the required momentum when it became in contact with the patch.

Finally, Table 4.6 summarize the motion planning results. It is important to emphasize that efficient computations of the minimum time trajectories were possible thanks to the properly selected minimum time “heuristic”.

Figure 4.48: Comparison of position and velocity profiles on a steep hill with correct and erroneous information about the hill inclination.
Figure 4.49: Position and velocity profiles on a stiff vegetation patch with erroneous information about the initial position of the robot with respect to the vegetation (the dark shaded region represents the location where the robot believes the patch is located and the light shaded region represents the actual location of the patch).

Table 4.6: Motion planning results (Scenarios 1b and 2b correspond to trajectories using only maximum, minimum, and ‘0’ accelerations. Scenario 4b corresponds to the experiment with erroneous perception of the hill inclination)

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<th>Computation time[s]</th>
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</tr>
<tr>
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</tr>
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<tr>
<td>3</td>
<td>20</td>
<td>10</td>
<td></td>
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<tr>
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<td>20</td>
<td>10</td>
<td>0.032</td>
</tr>
<tr>
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</tr>
<tr>
<td>5</td>
<td>20</td>
<td>10</td>
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CHAPTER 5

MODELING AND MOTION PLANNING FOR A RHEX-TYPE ROBOT

This chapter describes the dynamic and power models and motion planning task for Rhex-type robot. The present work is motivated by the hypothesis that for certain regimes of operation (i.e., certain gait parameters), legged robots from the RHex family behave in a similar fashion to skid-steered vehicles while in general curvilinear motion. As discussed and validated in Chapter 2 and Chapter 4 respectively, dynamic and power models for skid-steered vehicles rely on the terramechanics of wheel-terrain interactions. Reduced order models that relax some of the assumptions of the terramechanics models have also been derived for skid-steered vehicles [40]. The work on skid-steered vehicles can also be extended to the dynamic and power modeling of the Rhex-type robots. While the ground contact patterns for legged robots and in particular the RHex-type robots considered here [41], are characterized by discrete rather than continuous contact, some work has been done to experimentally develop kinematic models of these systems [41]. There has been some related work focused on reducing power (or specific resistance) of these systems, but not specifically on modeling the power [42].

The chapter first describes the experimental platform, X-Rhex Lite (XRL), used in the research and present an approximated kinematic model for the same. Then it presents the dynamic and power model for XRL. The chapter also discusses the application of the proposed models for energy efficient motion planning.
5.1 Robotic Platform and Approximated Kinematics

The XRL (X-RHex Light) experimental platform [43, 44] employed in this research and shown in Fig. 5.1 is a very versatile, slightly lighter version of the X-RHex robot, capable of performing diverse gaits, including walking, jogging, running and pronking. The robot has 6 compliant C-shaped legs independently actuated by 50 Watt Brushless Maxon motors with a gear ratio of 18:1.

![Figure 5.1: XRL robot utilized as the experimental platform for the research.](image)

For this dissertation an alternating tripod gait for the XRL locomotion has been considered. Referring to Fig. 5.2, the alternating tripods correspond to a left tripod formed by legs 1,3, and 5 and a right tripod formed by legs 2,4, and 6.

![Figure 5.2: Left and right tripods for XRL.](image)

Each of the tripods follows at the low level a desired trajectory generated by a Bueheler clock [41], which characterizes the gait by four parameters: the frequency $f [Hz]$, nominal duty factor $d_f$, nominal leg offset $\phi_0 [rad]$ and the nominal leg sweep angle $\phi_s [rad]$. These parameters are grouped into the control vector $u = [f, d_f, \phi_0, \phi_s]^T$, where the frequency and duty factor are expressed in terms of the cycle time $t_c$ and stance time $t_s$ by $f = \frac{1}{t_c}$ and $d_f = \frac{t_s}{t_c}$.

In order to turn while in motion, perturbations on the gait parameters for the inner (1,2,3) and outer (4,5,6) legs are introduced (hereafter we assume left hand turns). These perturbations are
functions of a turn gain $t_g$ and some extra constants $(\alpha, \beta, \gamma)$. The perturbed gait parameters for the inner legs are expressed by

$$\phi_{i_o}^i = \phi_o - t_g \alpha, \quad (5.1)$$

$$\phi_{i_s}^i = \phi_s - t_g \beta, \quad (5.2)$$

$$d_{f_i}^i = d_{f} - t_g \gamma. \quad (5.3)$$

The outer legs parameters $\phi_{o_o}^o, \phi_{o_s}^o$, and $d_{f}^o$ are computed similarly but with opposite signs for the perturbations. For all experiments performed in this work, $\alpha = 0.05, \beta = 0.7, \text{ and } \gamma = 0.0$.

The robot kinematics were derived by conducting experiments for all the turning gains in the set $\{0.0, 0.04, 0.1, 0.2, 0.4, 0.5, 0.6\}$ and frequencies in the set $\{0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1\}$ Hz using a walking gait parameterized by $u = [f, t_g, 0.65, -0.314, 0.785]^T$. Robot forward velocity and turn radius were measured by tracking a set of LEDs mounted on the robot with a high-speed digital camera (Casio Exilim EX-F1). Figs. 5.3 and 5.4 summarize the approximated kinematics for the walking gait on a vinyl surface. In particular, the robot forward velocity $v [m/s]$ and the robot turn radius $\rho [m]$ are approximated by

$$v = 0.16f, \quad (5.4)$$

$$\rho = 13.92e^{-5.78t_g} + 0.764. \quad (5.5)$$

It is important to note that, as can be seen from Figs. 5.3 and 5.4, no clear dependence of the velocity on the turn gain and of the turn radius on the frequency was observed and that is the reason why (5.4), and (5.5) depend only on one variable. To generate these models, the experimental data set was split using 80% of the data for model fitting and 20% for validation, which yielded average prediction errors of 4.1% for the forward velocity and 17.7% for the turn radius.
5.2 Torque and Power Model

During the experiments described in Chapter 5 Section 5.1, motor torques $\tau$ for each leg were monitored. Then, the average torques per cycle for the inner and outer legs were estimated using

$$\bar{\tau}_i = \tau_1(t) + \tau_2(t) + \tau_3(t), \quad t \in [0, tc],$$  \hspace{1cm} (5.6)

$$\bar{\tau}_o = \tau_4(t) + \tau_5(t) + \tau_6(t), \quad t \in [0, tc],$$  \hspace{1cm} (5.7)

where $\bar{x}$ is the average of $x$. Fig. 5.5 shows that the torques follow clear exponential trends as a function of turn radius, which is qualitatively similar to the torque vs turn radius curves observed for skid-steered robots [5], [40]. Notice that as the turn radius increases, the torques tend to a small
value which is associated with the rolling resistance (error bars represent torque standard deviation for the different speeds).

The validation turn radii $\rho$ are in the set $\{5.14, 11.81\}m$. For these $\rho$, the fitted models yield average prediction errors of 3.81% for the outer side and 6.04% for the inner side. For turn radii of less than 2.14$m$, the model does not capture the behavior of the inner side torque, perhaps due to the fact that for these sharper turns roll motion is increased, making the gait less stable and modifying the steering dynamics significantly.

Power for the inner and outer sides was computed by adding the mechanical power and the electrical losses as

$$
P_i = \sum_{j=1}^{3} (|\tau_j(t)w_j(t)| + I_j^2(t)R), \quad t \in [0, tc], \tag{5.8}$$

$$
P_o = \sum_{j=4}^{6} (|\tau_j(t)w_j(t)| + I_j^2(t)R), \quad t \in [0, tc], \tag{5.9}$$

where $w_j$ is the angular velocity of leg $j$, $I_j$ is the current through the motor corresponding to leg $j$ and $R$ is the phase to phase electrical resistance of each of the six motors. Motivated by insight gained from power modeling of skid-steered robots [40], we look for a model of similar shape.
Figure 5.6: Inner side power curves (only two turn radii (1.28 m and $\infty$) are shown for ease of visualization).

Table 5.1: Power model parameters for XRL.

<table>
<thead>
<tr>
<th>INNER SIDE</th>
<th>OUTER SIDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i(\rho)$</td>
<td>$b_i(\rho)$</td>
</tr>
<tr>
<td>$240.6e^{-1.26\rho} + 88.82$</td>
<td>$-1.40e^{-0.07\rho} - 4.35$</td>
</tr>
<tr>
<td>$a_o(\rho)$</td>
<td>$b_o(\rho)$</td>
</tr>
<tr>
<td>106.9</td>
<td>$9.21e^{-0.34\rho} - 6.12$</td>
</tr>
</tbody>
</table>

where power is related to velocity and turn radius by

$$P = a(\rho)v + b(\rho),$$

(5.10)

where $a$ and $b$ are coefficients dependent on the turn radius. Experimental curves shown in Figs. 5.6 and 5.7 relating power and speed for different turn radii were obtained for both the inner and outer robot sides. Straight lines were then fitted to data corresponding to each turn radius and their intercepts and slopes were used to estimate the power model parameters $a$ and $b$ for both robot sides. These parameters are summarized in Table 5.1. The power model was then evaluated at the test speeds $v$ in the set $\{0.096, 0.128, 0.16\}$ m/s, yielding average errors over all turn radii of 8.69% and 9.19% for the outer and inner side respectively.
Figure 5.7: Outer side power curves (only three turn radii are shown for ease of visualization).

Figure 5.8: XRL power surface.

5.3 Application of the Proposed Model to Energy Efficient Motion Planning

Using the power models developed in Section 5.2, it is possible to generate the power surface illustrated in Fig. 5.8, which corresponds to the total power consumption of the robot. This power surface was then integrated with SBMPO. The outcome of this integration is the simulation result shown in Fig. 5.9, which compares paths obtained by optimizing a distance cost function and an energy cost function. Notice that energy efficient motion planning saved 33.5% in energy with only a 13.1% increase in distance traveled.
Figure 5.9: Energy efficient vs. distance optimal trajectory planning for XRL.
This dissertation considered autonomous skid-steered wheeled vehicles. First, a terrain dependent dynamic model for skid-steered platforms was developed and validated on slopes and then motion planning task was performed for various cost functions as objectives. It was experimentally shown that the proposed model predicts motor torques as a function of turn radius for a full spectrum of turn radii (from point turn to linear motion). The model was also validated on a wooden surface for different inclinations and different turn radii. In addition, it was shown that the model was able to predict actuator saturation as a function of the terrain and turn radius, which led to the proposal of the $MTR$ curves (a set of curves that can be incorporated with motion planners in order to obtain predictable vehicle trajectories).

After having experimentally verified 3D dynamic for skid-steered vehicles, the work concentrated on the efficient computation and evaluation of trajectories that are dynamically feasible (i.e., they obey a minimum turn radius (MTR) constraint) and energy efficient. These trajectories were compared in both simulation and experiments with more standard distance optimal trajectories. The results relied upon dynamic and power models for skid-steered wheeled vehicles, discussed respectively in Sections 2.2 and 2.3, and their use with Sampling Based Model Predictive Optimization (SBMPO), the motion planning algorithm of Section 1.2.3. The key contributions are the validation of the models for different payloads and speeds, their integration with the planner and the insights that have been developed in performing the simulations and experiments. It should be emphasized that although SBMPO was used in this research, it certainly seems possible to approach the planning using alternative kino-dynamic motion planning methodologies. The key is the use of the power model to compute energy costs.
An interesting and useful experimental result, revealed in Section 4.1, is that the surface parameters appearing in the dynamic model of the vehicle, i.e., the coefficients of friction for the inner and outer wheels ($\mu_i$ and $\mu_o$) and the shear deformation modulus $K$ can be experimentally identified when the vehicle is not carrying a payload, but will yield fairly accurate predictions of torque when the vehicle is carrying a substantial payload. Hence, these parameters do not have to be re-identified when the vehicle’s payload increases.

The simulation results of Section 4.2.1 reveal particular insights for skid-steered wheeled vehicles that have motors that are so powerful that there may not be any MTR constraints. In particular, they show that distance optimal trajectories, if they can be followed by the robot, can be very energy consuming, primarily because they often require the vehicle to move in a trajectory for which segments have very small turn radii. In contrast the dynamically feasible, energy efficient trajectories tend to require the vehicle to have a slight increase (an average of 4.3% in the simulations) in distance and time traveled, but lead to a dramatic decrease (an average of 38.85% in the simulations) in energy consumed. These results revealed that when energy conservation for skid-steered autonomous ground vehicles is important, for example, to increase a mission’s endurance, energy efficient motion planning can be critical.

The computational results of Sections 4.2.2 and 4.2.3 show that there is no simple relationship between the computational times for distance optimal and energy efficient planning. However, on average the energy efficient planning took more time, probably because of enforcement of the MTR constraints.

Because most of the distance optimal trajectories violated the corresponding MTR constraint, the experimental results of Section 4.2.2 particularly highlighted the importance of enforcing these constraints in order to enable successful trajectory tracking. Without this enforcement, it is easy to develop trajectories that the vehicle simply cannot follow. It is pertinent to emphasize that in our control system, the company-specified current (i.e., torque) limits of each of the vehicle’s two motors is enforced by each motor’s controller. However, it is possible to violate these current limits
at the risk of burning out one or more of the motors. Hence, if the current limit was not rigorously enforced, the vehicle may have been able to track the distance optimal trajectories, albeit at great risk. The contention here is that this would be very poor practice.

The experimental results of Section 4.2.2 also reveal that for anisotropic surfaces (e.g., the modular plywood surface used in many of the experiments) it is difficult to accurately predict energy consumption. However, this should not prevent the development of energy efficient trajectories since the power consumption still increases with decreasing turn radius.

Initial simulation results were shown in Section 4.2.4 for energy efficient motion planning on a slope. A focus of future work will be energy efficient planning in environments with variable and slippery slopes. In such scenarios, it is expected that the current kinematic model will have to be enhanced with more detailed slip models such as those of [45].

The experimentally verified 3D dynamic model for the skid-steered vehicles was then also used for momentum based motion planning using the minimum time cost function. In this work, a strategy to exploit momentum as a way to deal with mobility challenges typically encountered by mobile robots in the field was proposed. The presented methodology integrates dynamic models of the robot, actuators, and terrain with an efficient motion planner. In particular, the approach was demonstrated in scenarios in which the robot faces patches of stiff vegetation and steep hills. In addition to overcoming the mobility challenge, the robot completed its mission in minimum time and with a desired velocity (i.e., zero velocity) at the goal.

Later, motivated by the hypothesis that for certain regimes of operation (i.e., certain gait parameters), legged robots from the RHex family behave in a similar fashion to skid-steered robots while in general curvilinear motion, power modeling of the XRL (X-Rhex Lite), a hexapedal robot was presented. The work also proposed and presented initial simulation results for its application to energy efficient motion planning for the XRL. Preliminary experimental results for a walking gait show significant similarities in the torque and power vs. turn radius trends for the XRL and skid-steered wheeled vehicles. The observed similarities, suggest that at least to some extent the
frictional forces involved in the curvilinear motion of both platforms share some commonalities and should be exploited in the future to obtain an analytical model of the dynamics of turning for RHex-type robots.

In addition, initial motion planning results using the obtained power model for a walking gait show that trajectories obtained using energy optimization tend to be smoother than the trajectories obtained using distance optimization since the power curves show that more power is needed for turns with small radius. Future work will involve the verification of these observations for various gaits and surfaces, and the experimental validation of the energy estimates obtained using SBMPO.

In the near future the proposed 3D dynamic model for skid-steered vehicles will be validated on different surfaces. In addition, experiments including downhill and crossed slope turns will be performed. Future work will also involve experiments at higher speeds where it is expected that centrifugal effects will not be negligible. It is worth mentioning that for that type of regimes of operation with high centrifugal forces, it would be important to include low level control strategies as the ones proposed in [20] to manipulate internal mass and inertial properties of the vehicle during motion.

Another important area of future work is rapid re-planning of energy efficient trajectories. As is well known, in a real world scenario a robot must update its plans to take into account new environmental information and differences between its current state and predicted state. It is possible to develop and implement an incremental version of SBMPO that will enable this rapid replanning and this work is underway.

For momentum based motion planning future work involves the extension of the proposed approach to a generalized 2 dimensional motion. Additionally, the proposed approach will be evaluated in outdoor mobility challenges that demand more detailed wheel-interaction models. This includes slippery hills, water bodies, and sand and mud patches. As pointed out in Section 3.2.2, it is required to integrate the current research work with perception to achieve full autonomy in the traversal of these challenging scenarios. Concretely, the perception system should be able to detect
the presence of mobility challenges and characterize them through association with previously experienced and perceptually similar terrains. Another big challenge, motivated by the experiments of Scenarios 4 and 5 in Section 4.3, consists in the development of principled ways to translate the perceptual data to the motion planner and low level controllers in a way that preserves robustness against perception errors and model uncertainty but still avoids over conservative solutions. Finally, online learning techniques will be exploited to adapt the vehicle and friction models as the robot navigates over the different mobility challenges.
BIBLIOGRAPHY


BIOGRAPHICAL SKETCH

Nikhil Gupta received his B.Tech. degree in mechanical engineering from Y.M.C.A. Institute of Technology, Haryana, India in 2008 and M.S. degree from North Dakota State University, Fargo, ND, in 2010. During the master’s degree he worked on the design and development of decentralized distributed control system for an all wheel drive all wheel steered robotic vehicle. He is currently working towards his Ph.D. degree at the Department of Mechanical Engineering, Florida State University, Tallahassee, FL. His research interests are primarily in the areas of dynamic system modeling, intelligent control, autonomous mobile wheeled and legged robotics, kinodynamic motion planning, and mechatronics.