A Nonlinear Adaptive Approach to Microjet-Based Flow Separation Control

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Boundary layer separation, a critical phenomenon in the operation of aerodynamic surfaces, limits the performance of compressor and turbine blades, fixed and rotary wings, as well as bluff bodies moving through a fluid. Flow separation leads to increased drag, decreased lift, and unpredictable vibrations due to unsteadiness. On these systems, effective control of separation could provide greater maneuverability and performance, and reduced vibration. Separated flow is a macro-scale phenomenon governed by complex flow interactions but can be controlled by micro-scale actuation. Recently, the emergence of closed loop methods has enhanced robustness. Modern processors enable the use of sophisticated adaptive control methods that achieve separation control with adaptive models. This paper considers control of flow separation over a NACA-0025 airfoil using microjet actuators. Experimental results are presented for a novel approach to Nonlinear Model Predictive Control, Adaptive Sampling Based Model Predictive Control (Adaptive SBMPC), which applies the Minimal Resource Allocation Network algorithm for nonlinear system identification and the Sampling Based Model Predictive Optimization algorithm to achieve effective nonlinear control. Through pressure data and flow characterization from wind tunnel experiments, effective and robust separation control is demonstrated. The methods computational efficiency is sufficient for successful real time experimental implementation.

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Nomenclature

Flow and Geometry Variables

\( \alpha \)  
Angle of attack (between chord and freestream)

\( \alpha_{sep} \)  
Angle of attack at separation

\( \rho \)  
Air density

\( b \)  
Airfoil span

\( c \)  
Chord length

\( \delta \)  
\( C_\mu \)  
characteristic height

\( C_L \)  
Lift coefficient

\( C_D \)  
Drag coefficient

\( C_\mu \)  
Momentum coefficient

\( D \)  
Drag

\( L \)  
Lift

\( p \)  
Static pressure

\( p_i \)  
Static pressure at transducer \( i \)

\( RE \)  
Reynolds number based on chord length

\( Q_v \)  
Microjet exit momentum

\( V_e \)  
Microjet exit velocity

\( V_\infty \)  
Freestream velocity

\( w_i \)  
Trapezoidal summation weights

\( Z_{lift} \)  
Lift-approximating function

Control System Variables

\( x_k \)  
State vector at time \( k \)

\( y_k \)  
Output vector at time \( k \)

\( u_k \)  
Input vector at time \( k \)

\( \phi \)  
Gaussian activation function
I. Introduction

For decades, flow separation has been of particular interest in engineering applications for which improved aerodynamic performance is desired. Control of flow separation, both active and passive, is a key component of existing aviation technology. Passive control methods, such as leading edge surface roughness\(^{1}\) or the inclusion of vortex generators\(^{2,11}\) rely on a stationary structural component to produce a reduction or delay in separation effects. These techniques typically produce an early transition to turbulence or streamwise vortical structures, each of which has been shown to mitigate separation\(^{6}\). The effectiveness of passive techniques is limited in comparison with active techniques, which rely on powered mechanisms to alter the flow, especially when conditions off the design point are considered\(^{2}\). These types of off-design conditions are common for aeronautical systems that are deployed in diverse environments, including micro-aerial vehicles and full-size aircraft that perform maneuvers. Although the potential benefits of active flow control (AFC) in terms of improved performance are notable (and the present work provides another example of these benefits), a number of challenges remain. Active control requires energy input, additional manufacturing costs and system complexity that are introduced by active control methods can be an impediment. Hence, although AFC has been examined in the laboratory for over a decade the AFC technologies are still at a lower technology readiness level (TRL) and need to go through significant testing and maturation process for translation to aircraft systems\(^{6}\).

The significant increase in capability and decrease in cost of computing hardware has led to the feasibility of active control techniques of increasing sophistication. Both open-loop and closed-loop control schemes have been applied to flow control problems and there is evidence that closed loop approaches offer significant improvement in performance over open-loop control\(^{8,9}\).

Several areas of flow control research, including suppression of cavity flow resonant tones and mitigation of flow separation, have advanced rapidly in recent years as new technologies have both created demand for and expanded the capabilities of such systems. The study of cavity flow control has demonstrated that mathematical techniques such as Proper Orthogonal Decomposition (POD) and model reduction are effective tools for controlling aerodynamic systems\(^{10,11}\). While these studies incorporated flow characterization and analysis efforts to produce nonlinear models that are effective for developing open-loop control strategies, it has been more practical to linearize these models for the sake of computational efficiency when implementing real-time closed loop control\(^{8,9}\).

Frequency domain methods have been applied to model and control the flow dynamics in flow control and related applications\(^{12,13,14}\). Nonlinear POD methods similar to those applied to cavity flow have also been used for flow separation\(^{15,16}\). Actuator development has also been key to the recent advances in the field of flow separation control. Both synthetic jet unsteady actuators\(^{17,18}\) and steady microjet actuators\(^{13,19}\) have
been effective in closed-loop separation control experiments. Because of the advantage of high momentum
capability with low power requirements, microjet arrays are used as actuators in this research. These
actuators are capable of both pulsed and steady blowing.

Nonlinear model predictive control (NMPC) has been proposed for flow control, but “a huge numerical
burden” is listed as a drawback of such methods. The method presented in this paper, Adaptive Sampling
Based Model Predictive Control (Adaptive SBMPC) is a new paradigm for NMPC that is computationally
efficient. Adaptive SBMPC differs from past adaptive, closed-loop approaches to flow control by identifying
a nonlinear model for control. Preliminary results using this method have been presented in a prior publica-
tion. This paper provides additional experimental results, Particle Image Velocimetry (PIV) visualization,
and more detailed analysis. The identification and control processing is executed in real time, demonstrating
the potential of this method for in-flight application.

The primary objective of this research is to maximize the lift coefficient \( C_L \) by means of delaying the
onset of separation as well as controlling flows with initially-separated boundary layers. The experimental
model is not instrumented with sensors capable of directly measuring the drag coefficient \( C_D \); for hardware
platforms configured to measure both lift and drag, the quantities \( C_D \) and \( C_L/C_D \) could be incorporated
into the cost function without necessitating any control system modifications. Given a discrete time series of
multiple sensor measurements located along an airfoil’s chord (plant outputs), the proposed research aims to
determine an optimal series of microjet pressure signals (plant inputs), maximizing the lift performance of
the airfoil. The control system should achieve this task for a broad range of steady or dynamically-prescribed
Reynolds numbers or angle of attack parameters. This is to be accomplished without direct measurement of
either parameter and without the modification of tuning parameters.

II. Control Method

As a means of solving Model Predictive Optimization problems without computing gradients, Sampling
Based Model Predictive Optimization (SBMPO) has been developed and implemented on both simulated
and experimental platforms. One key advantage of SBMPO is the ability to find globally optimal
solutions, while other methods that rely on linearization are likely to converge to locally minimum solutions.
A second advantage, computational efficiency enables the use of SBMPO to solve real time NMPC trajec-
tories. When SBMPO is combined with a neural network model of system behavior, the overall method for
identification and control is called Adaptive Sampling Based Model Predictive Control (Adaptive SBMPC).
SBMPO may be applied to solve the nonlinear optimization problem,

\[
\min_{\{u(k), \ldots, u(k+N-1)\}} \sum_{i=0}^{N-1} C (y(k+i+1) - r(k+i+1)) ,
\]
Figure 1. Sampling Based Model Predictive Control Summary. The algorithm discretizes the input space and makes model-based state predictions, \( \hat{x}_{k+j} \), in order to minimize a cost function.

subject to the nonlinear state space equations,

\[
\begin{align*}
    x(k+i) &= g(x(k+i-1), u(k+i-1)), \\
    y(k) &= h(x(k)), \\
\end{align*}
\]

and the constraints,

\[
\begin{align*}
    x(k+i) &\in X_{\text{free}} \quad \forall \ i \leq N, \\
    u(k+i) &\in U_{\text{free}} \quad \forall \ i \leq N, \\
\end{align*}
\]

where the cost function \( C(\cdot) \geq 0 \), \( r(k) \) is the reference input, and \( X_{\text{free}} \) and \( U_{\text{free}} \) respectively represent the states and inputs that do not violate any of the problem constraints. SBMPC is described in Fig. 1 and is easily applied to both linear and nonlinear models, combining techniques for sampling the input domain with an efficient graph search method such as A*.

A. Sampling the Input Domain

The field of path planning in robotics has seen recent innovations that have used sampling techniques. SBMPC involves the sampling of the space of allowable inputs. Halton sampling, in particular, is a method based on the low-discrepancy Halton sequences that has been shown to provide representative sample sets consisting of fewer points than sets generated using pseudo-random numbers or regular grids. Satisfaction of input constraints is automatic because it is the allowable inputs that are sampled. Also, since the inputs are propagated forward through the model, no inversion of the model is needed.

B. The Graph Search

Using the current state and input samples, several new nodes are computed by propagating the model and adding to a graph with tree connectivity, as illustrated in Fig. 2. The branchout factor \( B \), a tuning parameter of the algorithm, determines how many child nodes are generated when a particular parent node is expanded.
The uniform sampling density typically used for SBMPC may be transformed in order to achieve greater relative sampling density in a desired region of the input domain. In order to preserve input constraint satisfaction, the transformation must guarantee that no valid inputs are mapped to invalid inputs. Using a nonuniform sampling density can improve the performance of SBMPC by sampling large input changes more coarsely while sampling small input changes more finely. This finer sampling allows the trajectory to converge with small steady-state errors.

The Halton sequence algorithm, which is potentially used thousands of times at each time interval when the SBMPC routine is called, was a major contributor to the total runtime of SBMPC. During benchmark testing of C code on multiple machines, batches of 1 million elements of a Halton sequence were computed, and then precomputed batches of equivalent size were loaded from a binary file. The median CPU time required to compute the samples was 210 milliseconds, while the median time required to load the precomputed values was 1 millisecond; therefore, computing these samples offline and storing them in a binary file enables the run-time code to execute much faster while yielding identical results.

C. Nonlinear Modelling

Previous adaptive flow separation research has aimed to capture the nonlinear dynamics of the flow field by identifying an instantaneously linearized model that varies with time. The optimization of a linear model has the advantages of speed and simplicity over those that consider nonlinear models. The unmodeled dynamics, however, can cause such techniques to be suboptimal and even unstable.

1. Nonlinear POD Methods

Proper Orthogonal Decomposition (POD) techniques have been used to identify models in the form of polynomial difference equations that are sufficient for open loop control design. However, due to computational expense, closed loop flow control implementations have been limited to linear models.
Although SBMPO has the ability to optimize inputs to a POD model, POD models that are steady state in nature do not capture the transient behavior or hysteresis effects that are necessary to control separation under changing flow conditions. Global POD models are an extension of the POD technique that is well equipped to represent transient nonlinear behavior.

2. SBMPO Compatibility with More General Models

A primary advantage of SBMPO over alternative optimization methods is that this technique has no intrinsic preference for linear models over nonlinear models, and the algorithm does not need to compute closed-form gradients. This allows SBMPO to be applied to a more general class of systems than could be handled by previous methods. Even systems with strong nonlinearities or non-differentiable functions can be optimized using SBMPO.

3. Nonlinear Neural Network Methods

The use of an artificial neural network allows nonlinear models to be identified in a general manner by composing a function that computes future outputs based on past states. For this application the state vector,

\[ x_k = (y_{k-1}^T, y_{k-2}^T, ..., y_{k-n_y}^T, u_{k-1}^T, u_{k-2}^T, ..., u_{k-n_u}^T)^T, \]  

(6)

consists of \( n_u \) prior plant inputs and \( n_y \) outputs. The form,

\[ y_k = f(y_{k-1}, y_{k-2}, ..., y_{k-n_y}, u_{k-1}, u_{k-2}, ..., u_{k-n_u}) \equiv F(x_k), \]  

(7)

is known as a nonlinear autoregressive exogenous inputs (NARX) model form because it is analogous to the autoregressive exogenous inputs (ARX) form for linear models. If \( F(x_k) \) were a linear function, the equation would simplify to an ARX formulation,

\[ \hat{y}_k = Ax_k, \]  

(8)

with the matrix \( A \) of constant coefficients.
Figure 3. The layout of a sample RBF network consists of a hidden layer containing one hidden neuron for each distinct state pattern that the algorithm encounters. Collectively these pattern state vectors make up a basis with which all system dynamics are modeled. With enough of these hidden neurons the network is able to match arbitrarily complex nonlinear behavior.

While there are many ways to construct the nonlinear function \( F(x_k) \), neural network methods construct this function as a composition of simpler functions called neural units, which can be arranged in series and parallel to form a network. The network implemented in this research has a two-layer structure first proposed by Platt. This Resource Allocation Network (RAN) has for its first layer, a collection of neural units that respond in parallel when state \( x_k \) is nearby (according to the Euclidean norm) to a previously seen pattern. An extension of RAN, the Minimal Resource Allocation Network (MRAN) is used in this research. MRAN adds a pruning step that keeps the neural network size reasonable by discarding neural units with insignificant contribution to the output. The Gaussian function,

\[
\phi_i = \exp \left( -\frac{\|x_k - \mu_i\|^2}{\sigma_i^2} \right),
\] (9)

is selected for the first layer because it achieves the desired radial symmetry using a simple computation that has closed-form derivatives. The second layer combines these Gaussian outputs in a weighted sum, yielding the Radial Basis Function (RBF) representation,

\[
y = F(x_k) = a_0 + \sum_{i=1}^{N} a_i \phi_i.
\] (10)

For each of the \( N \) neural units, the MRAN algorithm must specify a basis vector \( \mu_i \), a Gaussian width \( \sigma_i \), and weight coefficients \( a_i \). The detailed specification of the MRAN algorithm is given in prior publications.
III. Experiments and Results

Results are presented for wind tunnel experiments with the goal of mitigating separation and therefore controlling lift. This section contains a description of the experimental setup in Subsection A, results of open-loop characterization tests in Subsection B and results of closed-loop control in Subsection C.

A. Flow Control Experimental Setup

Figure 4. Airfoil Schematic. This diagram includes microjet array and pressure transducer locations. The pressure transducers at four distinct locations gives enough information about the surface pressure distribution to approximate lift changes. The cross section schematic (lower) is also shown.

The wind tunnel experiments were carried out in the subsonic closed loop wind tunnel at the FAMU-FSU College of Engineering. This wind tunnel has a 12 inch x 12 inch square test section and is capable of air flow speeds from 3.5 m/s to 35 m/s. The NACA-0025 airfoil model of Fig. 4 is used in the following tests and is mounted to allow a variable pitch angle. The airfoil was constructed from ABS plastic and was sanded to a smooth finish. No boundary layer tripping mechanisms were added to the leading edge. Sensor and actuator placement is given in Table 1. The tunnel blockage varied with angle of attack, from 12.5%
at 15° AOA to 18% at 22° AOA. We caution the reader that a high levels of blockage can affect the flow. However, although the blockage is rather high at the high angle of attacks, similar behavior was observed for test conducted using a smaller airfoil (this larger model was subsequently used to allow for the use of more pressure sensors and higher fidelity flow characterization) Consequently, we fully expect that the trends observed and the conclusions drawn in the present study are not significantly impacted. This is supported by the plot shown in Fig. B.3 of an external reference[37], where, for the same airfoil, tests were conducted for Reynolds number $1.0 \times 10^5$, and Fig. 2 of Sarraf et al.[38] in which involves Reynolds numbers much higher than those considered in the present study (thus with a fully turbulent BL). As clearly seen in appendix A, the lift coefficient values measured for this model are within the range of prior published measurements.

At each pressure transducer location, P1 through P4, an Endevco unsteady pressure transducer (1 psi range) was mounted flush with the airfoil surface along the center span. At each microjet location, M1 and M2, an array of 40 equally-spaced microjets, covering 10 inches of the 11.5 inch airfoil span, was drilled into the channel, normal to the airfoil surface. Each microjet channel was supplied from both ends by plastic tubing to the output end of a solenoid valve (SensorTechnics HF Pro). The input end of the solenoid valve was attached to a steady pressure source supply between 0.5 psi and 10 psi. In each experiment, angle of attack was measured by taking a side view photograph and comparing pixel coordinates of the support pins to pixel coordinates of points on the tunnel wall.

<table>
<thead>
<tr>
<th>Component</th>
<th>M1</th>
<th>M2</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x/c</td>
<td>0.0</td>
<td>0.063</td>
<td>0.127</td>
<td>0.300</td>
<td>0.452</td>
<td>0.667</td>
</tr>
</tbody>
</table>

Table 1. Chord Locations of Microjet exits and Transducers.

During tests, data signals from the four pressure transducers were amplified using a Hendrick & Associates multichannel amplifier and acquired digitally using dSPACE hardware. The command signals to pressure transducers were sent from dSPACE and amplified using the op-amp buffer circuit of Fig. [3]. In open and closed loop testing, the single-input cases used only microjet array M2, and the two-input cases used both arrays M1 and M2, supplied independently by two solenoid valves.

Two-Component Particle Image Velocimetry (PIV) was used in order to quantitatively examine the flow field and provide evidence of separated flow reattachment. Because surface mounted unsteady pressure sensors were in operation during PIV acquisition, a seeding fluid which left little to no residue on the sensors was needed. For this application, Di-Ethyl-Hexyl-Sebacat (DEHS) was used with a Laskin type seeder; nominal particle diameter was $\sim 1 \mu m$. A 200 mJ, pulsed, Quantel® Nd:YAG laser illumined the seeded flow field of interest on the suction side of the airfoil. The laser sheet was formed by passing the beam through LaVision sheet forming optics and expanded by a cylindrical lens; the optics were adjusted to achieve a laser
Figure 5. Buffer Circuit Diagram. The buffer circuit amplifies an input voltage signal so that the output voltage has sufficient current to drive the solenoid valves.

sheet thickness of $\sim 1$ mm throughout the measurement domain. The spanwise location of the laser sheet was approximately at the airfoil centerline. Due to the unsteady pressure sensors installed at the airfoil’s true centerline, the laser sheet was offset by one pressure sensor diameter ($\sim 3$ mm) in order to minimize reflections.

Images were acquired at a rate of 15 Hz with a 5.5 megapixel sCMOS camera equipped with a 55 mm lens. Each PIV case consisted of the ensemble average of 1,000 instantaneous image pairs in order to adequately estimate mean and turbulent statistics of interest. Images were processed using LaVision’s DaVis 8.2 software. Velocity fields were correlated using a multipass algorithm with a final interrogation window size of 32 x 32 with a 75% overlap. The resulting velocity field spatial resolution is 0.9 mm x 0.9 mm.

B. Open-Loop Characterization

Open loop characterization of the flow separation control system relied on a frequency sweep input. The supply voltage was a 50% duty cycle square wave of increasing frequency. After making pressure measurements for a steady supply pressure, the frequency was increased from 0 Hz to 300 Hz in 6 Hz increments, and 3 seconds of data was taken at each frequency. Measurements consisted of signals from pressure transducers located at the microjet channel (at the edge of the airfoil span) and at the microjet exit (2 mm from the jet exit at the center of the airfoil span). The results of one case of these tests, performed on a bench top setup, are shown in Fig. 7.

The data was sampled at 2 kHz, and time and frequency domain analysis was performed on these measurements. From 0 to approximately 80 Hz, the pressure output frequency is identical to the voltage excitation frequency. Above 80 Hz, mode switching occurs due to the dynamics of the solenoid valve. The valve, which is designed for 30 Hz operation, switches primarily between the input frequency and $\frac{1}{2}$ of the input frequency.

Microjet channel pressure data is plotted in Fig. 6. Because of the narrow diameter of the microjets, there
Figure 6. Channel Pressures during Pulsed Actuation. The response of the actuator to frequency inputs is plotted. This plot displays the response measured at the microjet channel for several different source pressures $P_0$. 

$P_{RMS}$: MICROJET CHANNEL PRESSURE

INPUT FREQUENCY [Hz]

OUTPUT ROOT MEAN SQUARE PRESSURE [PSI]

INCREASING $P_0$

NOISE FLOOR
Figure 7. Exit Pressures during Pulsed Actuation. The pressures measured at the microjet exit display a similar trend to those measured within the channel; however, because of the low signal-to-noise ratio, the data for a 10 PSI source pressure is plotted.
Figure 8. $Z_{lift}$ response to actuation frequency at AOA = 19.8°. For various flow conditions, the majority of the lift improvement occurs from 0 to 80 Hz. This is consistent with findings from bandwidth tests, which indicated that 80 Hz was the highest frequency to which the valves are capable of responding.
Figure 9. $Z_{lift}$ response to actuation frequency at AOA = 22.0$^\circ$. Open loop results at a different angle of attack indicate a clearly nonlinear relationship between input frequency and $Z_{lift}$.
is great attenuation between pressure signals measured at the microjet channel and the microjet exit (see Fig. 4), the typical operating pressures (below 2 PSI) did not produce a measurable response in pressure measured at the exit. Because of this, a 10 PSI source was used, to produce the data plotted in Fig. 7 is provided here to allow a qualitative comparison in trends. This higher source pressure saturates the transducer connected to the microjet channel. The pressure data collected at the microjet exit (Fig. 7) displayed similar trends to the data collected at the microjet channel (Fig. 6).

Figure 10. Measured $c_p$ Values at Pressure Transducer Locations. The pressure transducers are positioned to resolve the suction peak, which grows in each case due to actuation frequency. Larger suction peaks generally correspond to increased overall lift.

In addition to bench top characterization, the open loop response of the overall system was measured in
the wind tunnel. The input for these tests was the solenoid valve voltage frequency, and the output was $Z_{lift}$, the lift based performance function computed from the transducer measurements at locations P1 through P4. This performance function, with a detailed derivation provided in Reese et al.\textsuperscript{39} is used for the purpose of detecting increases and decreases in lift. Although this function was shown in simulated and experimental cases to increase as lift increases and decrease as lift decreases, $Z_{lift}$ does not scale linearly with lift and therefore is not meant to be an exact, direct measurement of lift. As long as the direction of lift changes are accurately predicted, $Z_{lift}$ serves as an adequate feedback variable to manipulate and maximize lift, given that an appropriate nonlinear control technique is applied. This has been demonstrated in the present study as evidenced by the results shown in Figs. 11 through 15 and the accompanying discussion. The performance function uses the four pressure signals to approximate lift changes using a coarse Riemann-sum calculation, weighting each sensor reading based on sensor spacing and airfoil geometry. For the NACA-0025 with sensors located as indicated in Table 1, $Z_{lift}$ may be computed as

$$Z_{lift} = \frac{\sum_{i=1}^{N} w_i p_i}{\frac{1}{2} \rho V_{\infty}^2},$$  \hspace{1cm} (11)

with the weights $w_i$ given in Table 2. Some sample plots that represent the comparison of approximation $Z_{lift}$ with actual lift are given in Figs. 18, 19, and 20 of Appendix A. These and more detailed data are presented in Reese et al.\textsuperscript{39} For the angles of attack and Reynolds numbers considered, $Z_{lift}$ does increase and decrease in agreement with actual lift in both simulated and experimental data.

Table 2. The experiment parameter table gives the parameters describing the wind tunnel experiment configuration.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$p_1$ Weight</td>
<td>-1.000</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$p_2$ Weight</td>
<td>-0.9781</td>
</tr>
<tr>
<td>$w_3$</td>
<td>$p_3$ Weight</td>
<td>-0.9348</td>
</tr>
<tr>
<td>$w_4$</td>
<td>$p_4$ Weight</td>
<td>-0.9126</td>
</tr>
<tr>
<td>$c$</td>
<td>Chord Length</td>
<td>6 inches</td>
</tr>
<tr>
<td>$b$</td>
<td>Span Length</td>
<td>11.5 inches</td>
</tr>
<tr>
<td>$Q_v$</td>
<td>Average Microjet Momentum</td>
<td>$1.23 \times 10^{-4}$ kg m/s</td>
</tr>
<tr>
<td>$Q_v$</td>
<td>Average Microjet Velocity</td>
<td>5 m/s</td>
</tr>
</tbody>
</table>

Using the freestream velocity $V_{\infty}$, the pressures $p_i$ are normalized by $\frac{1}{2} \rho V_{\infty}^2$ to yield a dimensionless $Z_{lift}$. In the state vector of Eq. (6), each $y_k$ represents a $Z_{lift}$ measurement, and each $u_k$ represents a...
voltage frequency supplied to the solenoid valve for microjet channel M2. Changes in the command voltage frequency were observed to cause changes in the steady state value of $Z_{lift}$. After each 6 Hz frequency increment, data was collected after the system reached a steady state. The data for angles of attack 19.8° and 22.0° is presented in Figs. 8 and 9. The nonlinearity in the relationship between the actuator frequency and the output $Z_{lift}$ (as well as changing flow conditions) is what makes the closed-loop control of $Z_{lift}$ a formidable problem. Evidence of an actual change in lift as $Z_{lift}$ changes is given in the $c_p$ plots of Fig. 10. The cases shown in plots (a) and (c) of Fig. 10 match the PIV cases given in Section C. These discrete $c_p$ measurements suggest that the magnitude of the area under the curve increases when going from 0 Hz to 72 Hz, which corresponds to greater suction on the upper surface and higher lift. The data points plotted indicate that the distribution along the chord changes with actuation. They provide evidence that the actuators do have some control authority, but these are only the results of open-loop forcing. The value of the momentum coefficient $C_{\mu}$ was approximated as 7% for Reynolds Number 150,000 and 12% for Reynolds number 90,000 according to the equation:

$$C_{\mu} = \frac{\rho Q_v V_{\infty}^2}{0.5 \rho V_{\infty}^2 b \delta},$$

(12)

where $\rho$ is the air density, $Q_v$ and $V_v$ given in Table 2 represent the exit momentum and exit velocity from the microjet array averaged over the pulse phase, freestream velocity $V_\infty$ is 15 m/s for RE=150,000 and 9 m/s for RE=90,000, the span $b$ is 11.5 inches, and the characteristic height $\delta$ is taken to be the chord location of the microjet array, $\delta = 0.063 c$.

The main contribution of this paper is the control system that builds and adapts a model of this behavior caused by actuation and uses the model to implement closed-loop optimization. Further evidence includes the PIV measurements that are included in the following section provide definitive evidence of the effectiveness of the microjet actuators to mitigate separation. The possible mechanisms affecting the flow that could contribute to a reduction in separation include forcing a transition to turbulence, introducing streamwise vortices, and interacting with structures present in the separated flow. At each of the various actuation frequencies, one or more of these mechanisms affects the extent of flow separation. The out-of-family increase seen in the Re=144,000 data of Fig. 8 is possibly due to a shift in transition location.

C. Closed-Loop Control

Frequency sweeps as mentioned above were used to train the neural network and represent the input-output behavior of the system with a nonlinear model. Using a model initialized with sweep data, Adaptive SBMPC was applied to perform closed loop control. During the 30-second learning phase (not plotted), the identification algorithm was enabled, but the control algorithm was disabled. During the control phase,
both the controller and identification were enabled and produced successful tracking of the desired reference value of $Z_{lift}$. In each experiment, the reference value was commanded manually, and the control system automatically made the input adjustments necessary to drive the measured $Z_{lift}$ signal to match the desired reference signal. The tuning parameters selected for the identification and control algorithms are given respectively in Figures 5 and 4 of Appendix B. This type of scaling is commonly applied when using neural networks so that inverted matrices are numerically well-conditioned. Reference tracking capability is demonstrated for a variety of reference trajectories. Figs. 11 and 12 indicate with an arrow the time at which control was activated. In the subsequent figures, the controller is already active at time zero. When there are oscillations in the measured data curve that are not captured in the predicted data curve, notably at 49 seconds in Fig. 12 data, this represents unmodeled system behavior and unmodeled disturbances. In the case mentioned this likely represents a separation and reattachment behavior not captured by the neural network model. In addition to this, disturbances such as variation in the speed of the wind tunnel fan also contribute to perturbations from the desired reference signal. Fig. 13 contains oscillations after 35 seconds that are potentially explained by uncaptured model behavior and disturbances.

These results demonstrate the capability to not only maximize lift, but also control lift by commanding intermediate values. In Fig. 11 the commanded reference signal (dashed line) is constant, which means the control system was configured to increase and hold $Z_{lift}$ constant. The actual output signal (solid line) achieves the commanded increase-and-hold behavior. In Fig. 12 the commanded reference was manually stepped downwards, in order to demonstrate the ability of the control system to decrease $Z_{lift}$ when desired. Fig. 13 displays similar results and includes both upward and downward stepwise motion. For the case shown in Fig. 14 the neural network was trained at a Reynolds number of 125,000, but control is demonstrated for a Reynolds number of 150,000. Some prediction error, indicated by the departure of the dotted line from the solid line, occurs around 13 seconds, but the error is corrected within 5 seconds. This demonstrates the robustness achieved by online system identification.

Fig. 15 shows the control adjustment in reaction of the control system to changes in Reynolds number during operation. The system adjusts the neural network model at the same time as the updated control signal is being computed and executed. The adaptation of the neural network model is vital when flow conditions change. In Fig. 15 the model adaptation is what allows the neural net prediction signal (dotted curve) to adjust to match the measured signal (solid curve) even when the flow conditions change. SBMPC is then responsible for modifying the control input signal so that the prediction signal matches the reference signal (dashed curve). Because of the closed loop control system, it was possible to take a scenario where the flow was controlled and attached, change the wind tunnel speed so that the flow separates, and observe the neural network adaptation and controller reaction as the flow is automatically reattached. The 15-second
transient beginning at the 10 second mark is a model adaptation, not a transient on control. While the model has moderate error, the control system continues to operate, but may be less efficient until this adaptation is complete.
Figure 11. Closed Loop Case 1. The Reynolds number is 125,000, and the angle of attack is 20°. This closed loop case illustrates ability to follow a step reference signal subsequent to the activation of the SBMPC controller.
Figure 12. Closed Loop Case 2. The Reynolds number is 125,000, and the angle of attack is 20°. This closed loop case illustrates the ability to command intermediate values of $Z_{lift}$. 

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Figure 13. Closed Loop Case 3. The Reynolds number is 125,000, and the angle of attack is $20^\circ$. This closed loop case illustrates the saturation behavior of the controller: from time 0 to 5 seconds, $Z_{lift}$ is simply maximized when the specified reference is above the attainable actuation range. Tracking resumes when the reference value is decreased to an attainable value.
Figure 14. Closed Loop Case 4. The Reynolds number is 150,000, and the angle of attack is 20°. This closed loop case illustrates the ability to track ramp trajectories. During training, the tunnel Reynolds number was set to 125,000, ensuring that online model adaptation would be required when tested at a higher Reynolds number.
Figure 15. Closed Loop Case 5. The Reynolds number shifts from 150,000 to 140,000, and the angle of attack is 20°. This case demonstrates that the ability to adapt enables the control system to be robust to changes in the flow conditions.
In order to verify that the changes in $Z_{lift}$ actually correspond to a changing degree of separation in the flow, visualization experiments were performed to capture the velocity field surrounding the wing both with and without control enabled. In these experiments, the airfoil wind tunnel configuration was fixed to a particular angle of attack and tunnel velocity. The velocity field was then measured via PIV and averaged over a 1000-frame, 60-second window. For the purpose of visualization, the image processing results are displayed with a flipped y-axis so that the angle of attack is depicted as positive (the PIV experiments were performed with a negative angle of attack). The control system was then enabled to maximize $Z_{lift}$. The control system can be configured to maximize lift by prescribing a constant reference trajectory that is greater than the attainable range of $Z_{lift}$ values. In this case, the reference for $Z_{lift}$ was set to 0. A second series of PIV measurements of the same duration was collected to characterize the controlled flow. Contours of streamwise velocity $V_x$ are shown in Figs. 16 and 17 and display the uncontrolled and controlled flows for two different flow conditions (AOA 16°, Re 150,000, and 22°, Re 90,000, respectively) when the controller is set to maximize $Z_{lift}$. In the absence of blockage effects and for a larger PIV window, the value of $V_x/V_\infty$ would equal 1 far from the airfoil. In these figures, the separated region is outlined by zero-velocity contours shown in black. Based on averaged pressure data, the enabling of control increased average $Z_{lift}$ by 1.4 and 1.7 respectively. In both cases, the separation bubble is greatly reduced in size and further downstream. The flow fields indicate massive separation when uncontrolled, and while there is some separation with control, the size of the trailing edge separated region is less, indicating the limitations of the actuators’ control authority.

![Figure 16](image-url) Baseline and controlled $V$ for 16°, 15 m/s. The PIV data on the left was collected with the control system off, the data on the right was collected with the control system set to maximize $Z_{lift}$, and the tunnel speed corresponds to a Reynolds number of 150,000 based on chord length. The white dashed line indicates zero horizontal velocity $V_x$. 
IV. Conclusions

Closed loop control of separated flows has been demonstrated using the Adaptive SBMPC control system. Quantitative results using a pressure-based lift approximation and PIV flow imaging indicate the effectiveness of the control system to control flow behavior based on a nonlinear neural network model constructed from data. This model is adjusted in real time to represent changes in flow conditions while the controller is in operation. The closed loop experiments demonstrated successful tracking of desired values of $Z_{lft}$ by mitigating flow separation. The control system is able to increase or decrease lift in response to an external command, subject to the limitations of the actuator. Both the nonlinear input to output behavior of the system and the nonlinear control law are learned adaptively, so even when flow conditions were modified during a control experiment, the control system was successful in adjusting the inputs to meet the desired $Z_{lft}$ value. To date, only a few selected cases have been visualized. The nonlinear system identification and control techniques applied in this research is demonstrated for the first time with a hardware experiment. The control method makes few assumptions about the system being controlled, namely the order of its dynamics, and requires few tuning parameters, making it applicable to many other configurations of sensors and actuators beyond the unsteady pressure transducer and microjet configuration described in this paper. This implementation of Adaptive SBMPC is the first to demonstrate flow separation control using a model that is nonlinear and learned online. While there are potential benefits for the aviation industry from this technology in the future, the immediate application of this technology is more likely in lower risk applications such as ground based wind turbines and small unmanned aerial systems.
Appendices

A. \( Z_{\text{lift}} \) and Lift Comparison Data

Some plots of representative data are given here as evidence of the behavior of the four-pressure-transducer approximation \( Z_{\text{lift}} \) as actual lift varies. Simulated data plots are given in Figs. 18 and 19 and an experimental data plot is given in Fig. 20. These plots are provided to illustrate that the increasing and decreasing trends match well between lift coefficient and the variable that is used to estimate, \( Z_{\text{lift}} \). Additional cases may be found in Reese et al. 2014. For simulated cases, the plots contain a comparison (for many angle of attack configurations) of \( Z_{\text{lift}} \), computed using 4 pressure values taken from locations given in Table 1 with \( C_L \) and \( C_L/C_D \) computed by evaluating integrals over 160 data points over the upper and lower airfoil surfaces. The experimental case given provides a comparison between \( Z_{\text{lift}} \) as measured from the four pressure transducers and lift values measured with a force balance.

![SIMULATED LIFT AND L/D AT RE=90000](image)

Figure 18. Simulated Case 1.
Figure 19. Simulated Case 2.

Figure 20. Experimental Case 1. For RE = 95,000, computed $Z_{lift}$ values are plotted with measured $C_L$ values.
B. List of Tuning Parameter Choices

Tables 3 and 4 give the tuning parameters used in the experiment for each identification and control method.

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<th>Parameter</th>
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<td>E3_{MIN}</td>
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Table 4. SBMPC Parameter Choices

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Acknowledgments

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References


