Cost Optimal Control of Microgrids
Having Solar Power and Energy Storage

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Abstract—Solar power availability is intermittent and must be accompanied by an energy storage system (ESS). Hence, a strategy is needed to combine the use of grid power, solar power, and ESS power to minimize the total cost of energy. A solution to this problem is proposed here as Advanced Optimal Resource Allocation (AORA). This control scheme uses the prediction of solar power availability, the real-time price of energy, and levelized costs of energy for both solar power and ESS to optimally combine the use of the power sources. Simulation results are presented for a 24 h prediction window and compared with a baseline power usage scheme that is based on a fixed charging and discharging schedule for the ESS. The comparison shows that AORA results in a substantial cost savings over the baseline scheme.

I. INTRODUCTION

Over the last decade, solar power technology has increased in efficiency and affordability. However, it still faces a penetration problem due to the intermittent nature of solar power and its imperfect alignment with the load [1]. Maximizing penetration levels and effective utilization of solar power in the electric power grid requires a more sophisticated approach that dynamically leverages other distributed energy resources (DER) such as the electric grid and energy storage systems to economically, reliably, and safely meet the load demands. Due to the intermittent characteristics of sustainable energy sources such as solar power and accompanying constraints on real and reactive power, cost optimal usage of DER is a challenging optimization and control problem.

Past research has used off-line day-ahead scheduling with stochastic programming to formulate energy management as a deterministic problem based on scenarios generated by Monte Carlo simulations [2], [3], [4], [5]. These approaches are computationally intensive due to the number of scenarios that must be simulated. There have also been attempts to design real-time energy management systems in micro-grids to optimize the long-term cost [6]. Yu et. al formulates the problem as a real-time N-follower Stackelberg game and shows that the approach can efficiently optimize the demand response based on current data and fluctuating real-time price. Soares et. al proposed a multi-period AC optimal power flow taking into account the uncertainties of the solar and wind resources to ensure reliable solutions for the distribution system operators [7]. These approaches do not take into account the future availability of each of the DERs and the real-time pricing forecast of energy.

This paper presents an optimal control scheme for local solar power generation integrated with an energy storage system (ESS) and the electric grid. The proposed scheme incorporates predictions in loads, available power resources, and real-time grid pricing.

The rest of the paper is structured as follows. Section II introduces Sampling-Based Model Predictive Control, which is used for optimal predictive control. Section III discusses the proposed control scheme, Advanced Optimal Resource Allocation (AORA). Section IV presents the Micro-grid system used in the study. Section V discusses the results. Finally, Section VI presents conclusions.

II. SAMPLING-BASED MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) uses a predictive model to optimize a cost function while enforcing constraints on the system inputs and outputs and is widely applied in industrial process control [8]. Typically, industrial MPC is implemented for linear models, but the use of nonlinear models allows for better performance over a wider operating range [9].

Sampling-Based MPC (SBMPC) [10], [11] is an MPC method that uses a receding horizon along with an optimization algorithm — Sampling-Based Model Predictive Optimization (SBMPO) — that samples the input of the predictive model to compute a graph tree with nodes and branches as shown in Fig. 1. The number of branches of the tree emanating from a node is called the branchout factor. SBMPO searches the tree using LPA* [12], a variant of the well-known A* algorithm [13]. A*-type algorithms depend upon the use of a heuristic (the cost to move from the current node to the goal node) to provide the algorithms with computational speed. If the heuristic is optimistic, i.e., a lower bound for the actual cost, the algorithm is guaranteed to find the optimal path in the graph.

Fig. 2 shows the primary components of the algorithm and their interrelationships. The propagation model is used to generate the graph via input sampling, while the edge cost evaluation (the cost to move from one node to another) and heuristic (an estimate of the cost to move from the current
node to the goal node) are based upon the cost function to be optimized. The heuristic is key to algorithm speedup. To guarantee optimality the heuristic should be optimistic, i.e., a rigorous lower bound on the cost-to-goal. However, for speed the heuristic should also be a reasonable estimate of the cost-to-goal.

The following are the main steps of SBMPO:

1) **Select a node with highest priority:** The nodes are collected in an Open List, which ranks them based on their priority for expansion. The first node in the Open List is the start node. The expanded node is then moved to the Closed List.

2) **Sample control space:** Generate a set of samples of the control space that satisfy the input constraints.

3) **Generate neighbor nodes:** Propogate the system model with the control samples to determine the neighbors of the current node.

4) **Validate the generated nodes:** Check for any violation by the nodes using the constraints definitions. Invalid nodes are removed from the list.

5) **Add new nodes to the tree.**

6) **Evaluate new node cost:** Use an A*-like heuristic to evaluate the cost of the generated nodes based on the desired objective and add it to the priority queue based on the node’s cost.

7) **Repeat 2-5 for B (“branchout factor”) number of successors.**

8) **Repeat 1-6 for time steps defined by the prediction horizon.**

9) **Repeat 1-7 until the stopping criteria is true.**

Besides computational speed, an advantage of SBMPO for this problem is that it applies easily to nonlinear systems and the resolution completeness is not based upon the type of model used. (It will be seen in Section III that due to the battery, the SBMPO propagation model for this system is nonlinear.) An additional advantage is that it seamlessly applies to optimization problems for which some of the inputs are continuous and some are discrete, i.e., the mixed integer problem [14]. In this paper, the inputs are inherently continuous, but if the results are extended to load control, one of the inputs will be discrete.

### III. Advanced Optimal Resource Allocation

Advanced Optimal Resource Allocation (AORA), shown in Fig. 3, is an optimal source allocation control scheme that considers various power sources (solar, battery and grid) to meet the load requirement while minimizing the total cost of energy. In order to optimally select the combination of these power sources, accomplished here using SBMPC, the approach uses a forecasted load profile, the predicted availability of solar power, and a predicted real-time grid pricing (RTP) signal. Note that AORA enables PV power to be sold to the utility and the energy storage to both charge and discharge.

To enforce realistic physical and practical limitations, a variety of constraints were implemented within the AORA scheme. Since, it is assumed that the algorithm must utilize all available solar power ($P_{PV}$), the grid power ($P_G$) is simply a function of the battery power ($P_{ES}$), which is sampled as the input sampling for SBMPC, and the predicted load ($P_{LOAD}$) as
\[ P_G(t_k) = P_{LOAD}(t_k) - P_{PV}(t_k) - P^*_E(t_k), \]  

where \( P^*_E(t_k) \) is computed using a simple battery model as shown in Table I.

### Table I

**Nonlinear battery model used in the study [SOMEONE NEEDS TO DEFINE]**

<table>
<thead>
<tr>
<th>Case</th>
<th>( SOC_E(t_k) )</th>
<th>( P^*_E(t_k) )</th>
<th>( P^*_E(t_k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( &lt; SOC_E )</td>
<td>( &gt; 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>2.</td>
<td>( &lt; SOC_E )</td>
<td>( \leq 0 )</td>
<td>( P^*_E(t_k) )</td>
</tr>
<tr>
<td>3.</td>
<td>( \geq SOC_E )</td>
<td>( &gt; 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>4.</td>
<td>( \geq SOC_E )</td>
<td>( \geq 0 )</td>
<td>( P^*_E(t_k) )</td>
</tr>
<tr>
<td>5.</td>
<td>( SOC_E \leq SOC_E \leq \infty )</td>
<td>(</td>
<td>P^*_E(t_k)</td>
</tr>
</tbody>
</table>

The cost-function was chosen to be the actual dollar cost over the prediction horizon and is given as,

\[ J = \sum_{t_k=0}^{t_n-1} (r_G(t_k)P_G(t_k) + r_{PV}P_{PV}(t_k) + r_{ES}P_{ES}(t_k)) \Delta t \]

\[ = \sum_{t_k=0}^{t_n-1} J(t_{k+1}), \]

where \( \Delta t = t_k - t_{k-1} \) (for all k) and

\[ r_G(t_k) = \begin{cases} w \bar{r}_G(t_k) & \text{if } P_G(t_k) < 0 \& flag_{ES} = 1, \\ \bar{r}_G(t_k) & \text{if } P_G(t_k) < 0 \& flag_{ES} = 0, \\ \bar{r}_G(t_k) & \text{if } flag_{ES} = 1. \end{cases} \]

**NEED TO DEFINE flag_{ES}**. Here, \( J(t_{k+1}) \) is the cost incurred from time \( t_k \) to \( t_{k+1} \); \( P_G, P_{PV}, \) and \( P_{ES} \) represent real power from the grid, solar, and battery, respectively; and \( r_G, r_{PV}, \) and \( r_{ES} \) represent the price rates (in $/kW) associated with the grid, PV, and battery, respectively. The weight, \( w \), is chosen on the interval \( 0 < w < 1 \) so as to incorporate differential pricing that distinguishes the sell-to-grid and buy-from-grid rates. This helps the optimization algorithm to prioritize charging the battery over immediately selling stored energy at RTP.

Fig. 4 shows the propagation model used in SBMPO (see Fig. 2) for the optimal dispatch in AORA (Fig. 3). Unlike previous uses of SBMPO, the output of the propagation model is the edge cost.

A major benefit of \( A^* \)-based graph-search methods is the computational efficiency afforded by the use of a heuristic, an estimate of the cost from the current node to the goal. For AORA, a heuristic was defined based on the optimistic assumption that the predicted load at any time step can be fulfilled entirely by the cheapest power source for that time interval. Illustrated in Fig. 5, for the horizon from \( t_0 \) to \( t_n \), the heuristic expression \( H \) represents a summation of the lowest cost terms associated with each power source (highlighted by (red) boxes on the time line in the figure). In any instance,

\[ H = C_G(1) + C_G(2) + C_G(3) + C_E(4) + C_{PV}(5) + \cdots + C_{PV}(n) \]

**IV. Micro-Grid System**

For the solar power system and the energy storage system, constraints enforced by SBMPO (see Fig. 2) were derived from the physical parameters of the hardware in consideration. The physical bounds (constraints) and price rates associated with the solar power systems are summarized in the Table II and the physical constraints are given by

\[ Q_{PV} \leq Q_{PV}(t_k) \leq \bar{Q}_{PV}, \]

\[ P_{PV}^2(t_k) + \bar{Q}_{PV}^2(t_k) \leq S_{PV}^2, \]

where

\[ \bar{Q}_{PV} = P_{PV} \tan(\arccos(pf_{PV})), \]

\[ Q_{PV} = -P_{PV} \tan(\arccos(pf_{PV})). \]

### Table II

**Specifications for Solar Power System Used in the Study**

<table>
<thead>
<tr>
<th>PV System Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panels Rating</td>
<td>875 (kW)</td>
</tr>
<tr>
<td>Inverter Rating (( S_{PV} ))</td>
<td>900 (kVA)</td>
</tr>
<tr>
<td>Power Factor (( pf_{PV} )) Range</td>
<td>0.8-1.0</td>
</tr>
<tr>
<td>Maximum Reactive Power (( Q_{PV} ))</td>
<td>540 (kVAR)</td>
</tr>
<tr>
<td>Minimum Reactive Power (( \bar{Q}_{PV} ))</td>
<td>-540 (kVAR)</td>
</tr>
<tr>
<td>LCOE* (( Q_{PV} ))</td>
<td>2.510653965 (2/GWh))</td>
</tr>
</tbody>
</table>

*LCOE: Levelized cost of energy
For the energy storage system constraints and price rates are summarized in Table III and the constraints are given by

\[ Q_{ES} = P_{ES} \tan(\arccos(p_{f_{ES}})), \]  
\[ Q_{ES} = -P_{ES} \tan(\arccos(p_{f_{ES}})), \]  
\[ Q_{ES} \leq Q_{ES}(t_k) \leq Q_{ES}. \]

(9)

(10)

(11)

<p>| TABLE III |
| Specifications for energy storage system used in the study |</p>
<table>
<thead>
<tr>
<th>ESS parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery Rating</td>
<td>750 (kW)</td>
</tr>
<tr>
<td>Inverter Rating (S_{ES})</td>
<td>750 (kV A)</td>
</tr>
<tr>
<td>Maximum State-of-Charge (SOC_{ES})</td>
<td>2190 (kWh)</td>
</tr>
<tr>
<td>Minimum State-of-Charge (SOC_{ES})</td>
<td>219 (kWh)</td>
</tr>
<tr>
<td>Power Factor (p_{f_{ES}}) Range</td>
<td>0.8-1.0</td>
</tr>
<tr>
<td>Maximum Reactive Power (Q_{PV})</td>
<td>450 (kVAR)</td>
</tr>
<tr>
<td>Minimum Reactive Power (Q_{PV})</td>
<td>-450 (kVAR)</td>
</tr>
<tr>
<td>LCOE* (r_{PV})</td>
<td>2.510637965 (¢/kWh))</td>
</tr>
</tbody>
</table>

* LCOE: Levelized cost of energy

To perform the simulations, data related to the distribution network, solar generation profiles, load profiles, and an example feeder were obtained from the available open source databases. The load profile for the distribution network used in this study was obtained from the SUNGRIN project database, which has network model data and load profiles from various feeders located around the state of Florida [15]. Fig. 6 presents an actual load profile with the maximum, minimum, and average power requirement for a 24 h window of a feeder network used in the study. It is important to note that this study used the actual solar irradiance and load profile values obtained from the SUNGRIN project database as the predicted values. The results will degrade when imperfect knowledge exists.

Fig. 6. Actual load profile with the maximum, minimum, and average power requirement for a 24 h window of a feeder network used in the study.

As evidenced by the parameters of Tables II and III, the solar power system and ESS were sized for a large-scale (e.g., commercial, industrial or campus scale) micro-grid application. Using the analysis of [16], r_{ES}, the constant levelized cost of energy (LCOE) for the ESS was derived as

\[ r_{ES} = \frac{ES_{total}}{(Cycles)(ES_{Cap})(DoD)(\eta_r)}, \]

(12)

where \( ES_{total} \) is the total price of the ESS, \( Cycles \) is the total number of cycles under warranty at depth-of-discharge, \( ES_{Cap} \) is the total energy capacity of the ESS, \( DoD \) is the desired depth-of-discharge of the ESS, and \( \eta_r \) is the round trip efficiency of the system.

A modified real-time price (RTP) data set was produced using publicly available New York Independent System Operator’s (NYISO) wholesale location-based marginal pricing [17]. The modification of the price consists of the addition of an RTP supplier charge made by the hypothetical utility company, plus the biasing of an off-peak or on-peak rate determined by using the energy tariff currently available for the TOU program used by the City of Tallahassee. [18]. The reason for such modification is to preserve the fluctuating nature of the NYISO price data while placing the values in the range of the TOU prices used by the City of Tallahassee, and, therefore more appropriate to North Florida load patterns and markets. The modified RTP values for the 24 h prediction window with a step size of 15 min is shown in Fig. 7.

Fig. 7. 24 h real-time pricing (RTP) data for grid power, solar power, and battery power for a step size of 15 min used in the study. [THE LEGEND, LABELS, AND AXIS NUMBERS ARE ALL TOO SMALL TO READ.]

V. SIMULATION RESULTS

The performance of AORA and the baseline power usage scheme were simulated for the micro-grid system described in Section IV. For the baseline scheme, the ESS charging period was from 11 AM to 2 PM and the discharging period was from 7 PM to 11 PM. The results were generated for a 24 h prediction window with a control horizon of 1 h. The time step size for the control was set to \( \Delta t = 15 \) min. The power profile generated by AORA (see below) took only 0.6 sec to compute. [IS THIS THE CORRECT TIME?]

The power profile for the baseline scheme is shown in Fig. 8, resulting in a cost of $1021.40. The power profile for AORA is shown in Fig. 9, resulting in a cost of $634.98, which provide a cost savings of 34% over the baseline scheme. Note that it is seen in Fig. 9 that for AORA the ESS charged or discharged intermittently throughout the 24 h period based on the cost economics. This is in contrast to the fixed ESS discharge time of the baseline as seen in Fig. 8.
The main computational tool used by AORA is Sampling-Based Model Predictive Optimization, an algorithm that uses graph search, works seamlessly with nonlinear models, and uses a heuristic (i.e., cost-to-goal estimate) to speed up computations. The AORA simulation results were generated assuming models for power profile prediction and real-time grid pricing are available. The results were then compared with a base test case based on fixed charging and discharging schedules for ESS. The comparison shows that AORA saved about 34% in comparison with the base test case.

Ongoing work includes the development of longer term simulations and the relaxing of the assumptions of perfect knowledge of future load power requirements, PV power, and real time grid pricing. In fact, this research motivates additional research on the development of models to enable more accurate prediction of these variables. In addition, ongoing research is extending the AORA approach to not only compute the power distribution, but to also control the loads.

VI. CONCLUSION

Solar power capacity has increased dramatically over the past decade. However, the market still has limited penetration in terms of utilization. The work in this paper proposes a cost optimal control scheme, called Advanced Optimal Resource Allocation (AORA), for power dispatch using grid, solar power systems, and energy storage systems (ESS). The aim is to increase the solar power penetration and reduce the overall cost of energy.

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REFERENCES