Computer Modeling of the Trajectories of Blood Droplets and Bloodstain Pattern Analysis with a PC Computer

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COMPUTER MODELING BLOOD DROPLET TRAJECTORIES

Two different computer programs whose names are DROPLETS and TRAJECTORIES were created for the Bloodstain Analysis Course held in Ottawa in May 1989, by the RCMP and The departments of Physics and Mathematics of Carleton University. They are designed to run on an IBM PS/2 (AT) computer.

The first program called DROPLETS, was developed as an aid for teaching the physical laws that govern projectiles such as blood droplets. It simulates, on the video screen, the flight paths of individual blood droplets through the air before striking a vertical wall. This program takes into account the effect of air resistance on the path of different size droplets in addition to the gravitational force.

The computer calculates points along the path taken by the droplet and draws on the video screen a 3-dimensional plot of this path. See figure 1. When the calculated position of the path of the droplet coincides with vertical wall, the computer then calculates the impact position and the speed and direction of the velocity of the droplet just before impact. This direction, denoted by the angles alpha and gamma degrees, along with the impact speed, are called the "impact values" or "impact parameters" and they determine the characteristic shape of the spot produced by the blood droplet as it strikes the wall.

Figure 1 shows the computer display for five different sizes of droplets originating from a single point with all their directions and speeds the same. (i.e. 45 degrees above the horizontal and 800 cm/sec.) The effect of air resistance on the path of the drops is greater for smaller drops. This is apparent from the display where it is seen that the smaller droplets strike the wall at successively lower points. The top left window shows the drop # and the initial values for the direction and speed of the droplet. The top right window shows the calculated values of the parameters describing the impact of the droplet with the wall.

These "impact parameters" are important because the knowledge of these numbers enables one to calculate, using the Laws of Physics, the path of the droplet through the air before striking the wall. Thus if the investigator can examine the spot produced by the blood droplet and estimate reasonable values for the impact parameters it is possible to reconstruct, by calculation, the path followed by the droplet while travelling from the point of origin to the point of impact with the wall. The second computer program called TRAJECTORIES was created to perform such calculations.

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Figure 1. A slightly edited version of the computer display for the program DROPLETS showing 5 different droplets projected from the same point with the same speeds and the same directions but with different sizes. The droplet volumes are 20, 10, 5, 2.5 and 1.25 microliters for droplets 1 to 5 respectively.
The program TRAJECTORIES grew out of the Bloodstain Analysis Course held in Ottawa by the Royal Canadian Mounted Police and Carleton University in May 1982 and more recently in May 1989. This program accepts as input the six impact parameters for each single droplet stain. With these parameters and using the Laws of Physics the computer can reconstruct the trajectory which corresponds to the path through space for the droplet.

The six impact parameters are,

1. The droplet volume in microliters. (1 microliter of water = 1 milligram).
2. The glancing angle gamma in degrees.
3. The impact angle alpha in degrees.
4. The impact speed in centimeters per second.
5. The height in centimeters (Z-coordinate).
6. The horizontal position in centimeters (Y-coordinate).

We report on two experiments using blood supplied by the Canadian Red Cross for one and fresh blood donated by a volunteer for the other. A bloodstain pattern was produced for each of the two types of blood in the laboratory under controlled conditions. The bloodstain patterns were then analysed with the help of a PC computer.

A technique was developed to obtain the dry weight of each droplet used in the analysis. (10 in each experiment). The volume of the droplets were then calculated using the dry to wet ratios which were measured for the two different blood samples along with their densities. The volumes ranged from 0.05 to 4 microliters (0.05 to 4 milligrams approximately).

The angle alpha for each drop was determined from the shape of the droplet stain. The ratio of the width to the length of a stain was equated to the sin of the angle alpha.

The angle gamma is the angle between the vertical plumb line and the major axis of the stain.

Ten droplet stains were used in each experiment. (This is a limitation imposed by the present form of the program and can be increased for more practical usage.) They were divided into two groups of five stains. The five stains were chosen so as to lie in a vertical strip. This gave two strips, A and B, separated by a horizontal

The instructors were Professor Kenneth Hardy of the Mathematics department and Professors M.K. Sundaresan and Alfred L. Carter of the Physics department.
THE VERTICAL STRIP METHOD

It is understood that bloodstain analysts often do not have the luxury of dealing with bloodstain patterns that can be treated by this method. However when we were developing the course it became apparent to us that choosing the droplet stains that lie in a vertical strip about 1 inch wide, i.e., choosing droplet stains that are located one above the other takes advantage of one of the properties of projectiles and allows a mathematical procedure for averaging over the group of stains that is well suited for a computer.

To our knowledge the method is not taught elsewhere or mentioned in anywhere in the literature.

The property of projectiles mentioned above is the fact that the top view of the trajectory of any ballistic projectile, regardless of its speed, is a straight line extending from the source of the projectile to the point on the target hit by the projectile. This is a result of the nature of the two forces acting on the projectiles considered in this work. These forces are (1) gravity which pulls downward on the blood droplet and (2) air resistance which acts exactly in the opposite direction to the velocity of the blood droplet i.e. the droplet moves up and down in a vertical plane, there is no movement sideways.

This has the effect that the top view of the trajectories of a group of droplets belonging to the same vertical strip will appear to be one and the same line. By choosing two such strips A and B separated by a distance D centimeters the two lines as seen from the top view will intersect at the origin of the droplets. (The point of convergence.)

These two lines intersect with the wall containing the stains making the angles beta1 and beta2. (See diagram) The position of the intersection can be calculated from the value of D, beta1 and beta2. It should be noted that for a group of stains in strip A say, beta1 will be the average value of the individual betas for each droplet stain, same for strip B and beta2. The relationship between beta, alpha and gamma is given by the formula,

\[
\tan(\beta) = \frac{\tan(\alpha)}{\sin(\gamma)}.
\]

Note that when gamma = 90 degrees, beta and alpha are the same angle. The topview trajectories, it is again emphasized, do not depend on the individual speeds of the droplets. This is a great advantage for the analysis.
The Vertical Strip Method cont.

The following diagram shows the relationships between the strips A and B and the quantities D, beta1 and beta2 and the point of convergence.

The following show the experiment with the Red Cross Blood. The raw data shows the different values for the betas before the averaging. The averaging calculation yields a value for beta1 and beta2. The knowledge of the separation of strips A and B (called D above) combined with beta1 and beta2 gives a solution for the point of convergence.

The raw data showing that each of the drops 1 to 5 have a different beta2 and each of the drops 6 to 10 have a different value of beta1.

The view after the alpha values have been averaged to give an average value for the points beta1 and beta2.
FRESH BLOOD EXPERIMENT

Computer display for the raw data from the fresh blood experiment.

Same as above except slope angles and velocities adjusted.

The velocity distribution at the point of convergence.
RED CROSS BLOOD EXPERIMENT

Computer display for raw data. Red Cross blood experiment.

Red Cross blood experiment with alphas averaged and speeds adjusted.

Initial Velocity Distribution (Trials)

The velocity distribution at the point of convergence.
THE ANGLES ALPHa AND GAMMA DEFINED.

Gamma and Alpha for the case where $V_z = V_y$, $V_x = P V_y$.

Gamma $45^\circ$

Alpha $54.7^\circ$

(The blood spot is located in the Y-Z plane)

Same as above except $V_z$ is pointing downward.

Gamma $135^\circ$

Alpha $54.7^\circ$

$\Gamma = \arctan \left( \frac{V_y}{V_z} \right) \quad \alpha = \arctan \left( \frac{V_x}{\sqrt{V_y^2 + V_z^2}} \right)$