The Role of International Reserves in Sovereign Debt Restructuring under Fiscal Adjustment

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Abstract

Highly indebted developing economies commonly also hold large external reserves. This behavior seems puzzling given that governments borrow with an interest rate penalty to compensate lenders for default risk. Reducing debt to the same extent as reserves would reduce interest payments, however reserves also have insurance benefits during default crises. Moreover reserves can also be used to improve lenders recovery rates upon default, thus decreasing the interest rate penalty in non-defaulting times. A standard model of sovereign default risk, augmented with distortionary tax policies and debt restructuring, can quantitatively replicate the observed data patterns on debt and reserves holdings.

JEL classification: F32, F34, F41, E62

Keywords: Sovereign default, international reserves, distortionary taxation, external debt, sudden stops, debt renegotiation

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1 Introduction

International reserves are important policy tools in developing economies. A clear pattern from the data shows that many countries decide to hold large positions of international reserves while, at the same time, maintain high levels of external debts. This observation has intrigued economists that raised questions about the optimal management of international reserves. Holding reserves while keeping positive debt levels entails a cost when borrowing interest rates are larger than the interest rate earned on the foreign assets. For instance, by the end of 2005, governments of emerging economies held on average more that 15% of GDP in foreign reserves, while external debts were on excess of 33% of GDP. This behavior, not exclusive to a few Asian countries, seems to be more general and includes also Latin American countries (figure 1). Departing from this observation, Rodrik (2006) estimated that these countries incur annually in an average GDP loss of 1% for maintaining a choice of high debt and reserves. Others have however argued that this cost is outweighed by the benefit that reserves provide as an insurance instrument against the occurrence of financial crisis (Feldstein, 1999). This paper deals with the question of why developing economies hold simultaneously large amounts of debt and reserves, and what kind of financial crisis are more prone to induce such choice.

Figure 1: International reserves and external debt accumulation for Latin American countries in 2005 (source: *World Development Indicators*)
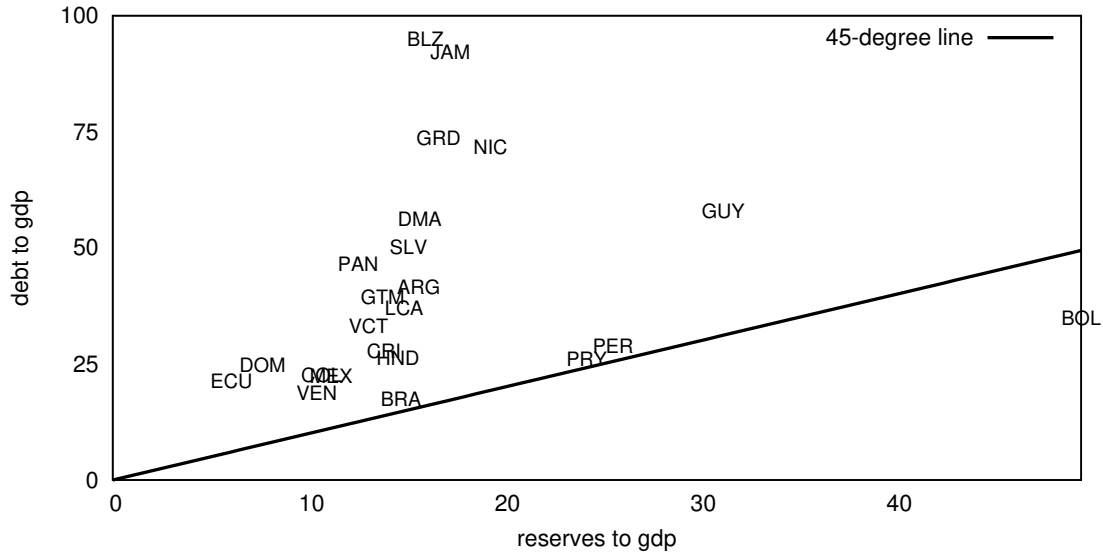
To analyze the above question, this paper adds four new features to the sovereign default model of Eaton and Gersovitz (1981). First, reserves accumulation is explicitly modeled by allowing the sovereign government to choose a portfolio of external debt and international

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reserves. Second, the government is assumed to raise revenue using distortionary taxation. Third, the model includes a sudden stop shock, defined as a complete impediment to borrow. That is, if the economy is hit by a sudden stop shock, the sovereign cannot borrow in the current period and has to repay its debt or default. This feature of the model intends to reinforce the role of financial crisis at generating positive demand of reserves for precautionary motives. Fourth, a renegotiation stage is assumed to occur after a sovereign defaults. By allowing for endogenous renegotiation between lenders and the government, the model adequately accounts for realistic debt reductions observed in the data. Additionally, if international reserves affect the sovereign value of regaining access to international markets, then different levels of reserves will imply different recovery rates for the lenders, thus providing another channel influencing demand for international reserves.

The model economy works as follows. A government from a small open economy chooses distortionary taxes, international reserves and external debt in order to finance public expenditure and maximize households welfare. Households supply labor to firms that produce final consumption goods subjected to productivity shocks. However, wages earned by households are taxed by the government. Due to limited commitment in international credit markets, lenders cannot oblige governments to repay and therefore supply credit with an interest rate spread that reflect the risk of default. If the government decides to default, domestic productivity suffers a loss and access to credit markets is temporarily barred until debt is restructured. Additionally, if the economy is hit by a sudden stop shock, international lenders loose faith in the government credit for exogenous reasons. If that happens, the government has two options: either adjust consumption by repaying all outstanding debt, or default and bear the associated losses.

Given a positive probability of default, the interest rate spread for debt will be positive. Therefore, an impatient government, desiring to shift consumption from the future to the present by taking debt, will face a large cost of carrying positive amounts of risk free interest paying reserves. Reducing simultaneously debt and reserves increases current consumption as the interest rate on reserves is lower than the interest rate on debt and the level of net debt doesn't change. Given this cost, it seems unreasonable for a government to hold any debt at all if reserves have no other purpose. However, a choice of no reserves makes the economy more exposed to sudden stops. If the economy is hit by a sudden stop, the government is forced to adjust consumption either by repaying debt or defaulting. In this case, reserves can be used to repay debt, helping the government to prevent a sharp adjustment in consumption and even a default event. The larger the costs associated with the impact of a financial crisis, the more the insurance provided by reserves[1]. On one hand, pro-cyclical taxation, especially

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[1]Recent empirical evidence finds that international reserves reduce the likelihood that a country is hit

recurrent in developing economies (Gavin and Perotti, 1997; Kaminsky, Reinhart, and Végh, 2005), aggravates such costs as higher taxes distorts output further more. On the other hand, if a government defaults, then current reserves are used during debt restructuring negotiations, thus affecting the recovery rate that lenders face. With reserves providing a limited benefit to the government would it be permanently in autarky, recovery rates will be positively related with the level of reserves. This mechanism provides an additional channel over which a positive level of reserves provides benefits to impatient governments: lenders will transmit an expected higher recovery rate to lower interest rate spreads. To summarize, reserves provide two main benefits to the government: an insurance benefit arising from a precautionary motif and a decrease in interest rate from the renegotiation channel.

Using the model outlined above, this paper goal is to explain non-trivial levels of reserve and debt to output ratios while verifying other features from the data at the business cycle frequency. The focus on a short-term analysis is related with the fact that international reserves seem to be actively managed by governments during crisis periods. Broner, Didier, Erce, and Schmukler (2011) documents dynamics related to debt and reserve accumulation around times of financial crisis in emerging market economies, namely, that gross capital inflows and outflows are correlated and both collapse around crisis. Additionally, this paper also documents a positive association of international reservers and recovery rates using a dataset compiled by Benjamin and Wright (2009) on default episodes and debt restructuring. These two empirical regularities are analyzed by computing a numerical solution for the model outlined above. The resulting simulations are then used to generate moments that are compared against Mexican data. The reasons for the choice of this country as a benchmark to evaluate the model relate to the fact that: Mexico is representative of emerging market economies in the sense that its debt and reserves dynamics resemble figure 1; and this economy experienced several financial crisis in the last 30 years.

Link with the literature

This paper is related with the literature of sovereign default of small open economies, where the classical work is Eaton and Gersovitz (1981). In that paper, the authors present a small open economy dynamic model with non-contingent debt and lack of full commitment that generates equilibrium default. Aguiar and Gopinath (2006) show the quantitative relevance of that model in replicating key business cycle statistics for emerging market economies, in particular, pro-cyclical dynamics of net capital inflows. In a related paper, Arellano (2008) uses a non-linear output loss in the event of a default (increasing in the

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negatively by global adverse effects, for example, Frankel and Saravelos (2012) or Bussière, Cheng, Chinn, and Lisack (2014).

endowment realization) to generate similar results but with a higher default rate that is in fact closer to what is seen in the dade. Using a model similar to Eaton and Gersovitz, Alfaro and Kanczuk (2009) study directly reserve and debt accumulation by small open economies, concluding that reserves play no role as insurance instruments. Two main reasons explain why they find such result. First, a proportional output loss is assumed as the penalty faced by economies that default. As a result, a substantially low discount factor has to be used in order to generate realistic debt to GDP holdings. With such a lower discount factor, savings becomes almost prohibited in the model. Second, the only risk faced by borrowers in their model is endowment fluctuations. In that case, reserves provide a bad hedge against that risk: market exclusion becomes more bearable when the government holds reserves, thus increasing the spread charged by lenders.

Alternatively, the model presented in this paper uses an output loss similar to Arellano (2008), and adds a sudden stop shock[2]. Ranciere and Jeanne (2006) provides an early model where reserves play a direct role in providing insurance against sudden stops when the country has positive debt holdings. In that paper, a sudden stop is modeled as a persistent event that, on top of the exclusion from markets, entails a output losses on its own. More recently, Bianchi, Hatchondo, and Martinez (2012) use the same idea to generate positive demand for reserves in a model of the Eaton and Gersovitz type with long-term debt. Contrary to those papers, the sudden stop shock version used in this paper does not impact the country in any other way, but in a temporary lack of credit access, similarly to the sudden stop shock proposed in Roch and Uhlig, 2012. Additionally, this paper includes endogenous renegotiation and distortionary taxation to the model. Yue (2010) studies the interaction between sovereign default and ex post debt renegotiation, concluding that recovery rates are decreasing the the level of debt. Detragiache (1996) and Aizenman and Marion (2004), using simple two-period models, have argued that costly taxation might play an important role in generating demand for reserves when the country faces default risk. Under such circumstances governments would want to prevent not just the direct effect of an output loss generated by a default event, but also the costs of raising revenues when taxation is particularly costly. More recently, Cuadra, Sanchez, and Sapriza (2010) underline that distortionary taxation becomes especially relevant in an environment of limited risk sharing due to the presence of default. In their model, tax rates increase when output is low, consistent with evidence that developing economies tend to maintain pro-cyclical fiscal policies (Gavin and Perotti, 1997; Kaminsky, Reinhart, and Végh, 2005; Vegh and Vuletin, 2012; Bauducco and Caprioli, 2014).

This paper contributes to the literature by providing a framework for studying the dynam-

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[2]A term first used in Calvo (1998).

ics of reserves, debt, and sovereign spreads. Moreover, the results points to the importance of distortionary taxation and debt restructuring for a realistic quantification of the demand for reserves. To my best knowledge, this paper is the first that can deliver realistic results regarding debt and reserves using a model with one-period debt.

Main Results

As a preview of the results presented in later sections, the model used in this paper is quantitatively able to replicate some data moments of the business cycle statistics, such as large debt holdings, default rates, negative co-movement of trade balance and output, and positive correlation between gross capital outflows and inflows. Additionally, strong reserve accumulation is also generated in the simulations. This result contrasts the findings from Alfaro and Kanczuk (2009). The reason why this emerges as a result is related with the fact that, under the proposed model, losses of default are painful enough to generate a large insurance role for reserves. Of key importance to this result is the extension of the baseline model to include debt restructuring and costly fiscal collection. Quantitative results from computing and calibrating the model indicate that the baseline model can generate 10% international reserves to GDP as seen in the data for Mexico. If fiscal distortions are shutdown from the model, then only 5% reserves to GDP are sustained in equilibrium, and, if renegotiation is not allowed, only 0.4% reserves to GDP are generated.

The remaining paper is outlined as follows. Section 2 identifies the main trends in the data and shows empirical relationships between reserves and relevant variables. Section 3 presents the model and defines the equilibrium. Section 4 justifies the functional forms, calibration and analyzes the simulation results. Section 5 concludes the paper. The numerical procedure and alternative model specifications are relegated into appendices in A.

2   Empirical Evidence

This paper is mostly concerned with the accumulation of international reserves and external debts.

For that purpose, international reserves are defined as external assets held by a country's government or central bank. According to the guidelines of the *International Financial Statistics* (IFS), compiled by the International Monetary Fund (IMF), these assets "comprise holdings of monetary gold, special drawing rights, reserves of IMF members held by the IMF, and holdings of foreign exchange", in which should be included, "foreign banknotes, bank deposits, treasury bills, short- and long-term government securities". As such, most assets considered as reserves are highly liquid and yielding interest close to the risk free

rate. It should be noted however that assets managed by sovereign wealth funds, typically yielding higher returns, are not considered international reserves. Different management principles dictate the dynamics of such funds, often characterized by the seek of higher yields or strategic value, and, for that reason, fall outside the scope of this study.

In its turn, external debts are defined by the IFS as "external obligations of public debtors, including the national government, political subdivisions (or an agency of either), and autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity"[3]. Because this paper focus on the implications of debt default on risk sharing, the debt considered is that owned directly or indirectly by a government, that is by an agent with the option to repudiate it.

Additional details, definitions and sources for all variables used in this paper can be found in appendix A.1.

What are the costs of holding international reserves?

Part of the reason why a choice of large debt and reserves is intriguing relates with the cost of maintaining such portfolio. As an example, consider a situation where the borrowing interest rate equals i and the savings i^*. In this case, $i \cdot ND$ is the cost of holding ND of debt without any reserves. An equal net debt holding can be achieved by borrowing $D = ND + R$ and, at the same time, saving R. The total interest cost of this choice would be $i \cdot (ND + R)$ and $i^* \cdot R$ would be the revenues. Thus, the cost difference between these two financing options amounts to $(i - i^*) \cdot R$, that is, if the borrowing interest rate is higher than the saving interest rate, holding debt and reserves carries a cost equal to the interest rate difference - or spread - times the level of reserves.

This example is closely related with the trends observed in emerging market economies. Figure 2 illustrates this fact by showing both levels of external debt and international reserves for a group of selected countries. External debt varies between 15 and 60% of GDP, while international reserves between 5 and 20%. With a positive spread between the interest rate on debt relative to reserves, such gap becomes costly. In fact, due to prevalent sovereign debt crisis in emerging market countries, spreads have been large as international investors take into account the risk of default when they lend to such countries (see figure 3). Spreads[4] are generally quite volatile and high, reaching magnitudes of 20% and larger, even in non-default

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[3]Due to data limitations from the *World Development Indicators,* public and privately owned external short-term debt are indistinguishable and will be included in the measures of debt used in this paper without distinction.

[4]Spreads are given by the secondary market rate, provided by JPMorgan's Emerging Markets Bond Index (EMBI). These spreads are measured by an index that includes sovereign and quasi-sovereign (guaranteed by the sovereign) instruments that satisfy certain liquidity criteria in their trading. All spreads are calculated as the premium paid by an emerging market economy over a U.S. government bond with comparable maturity.

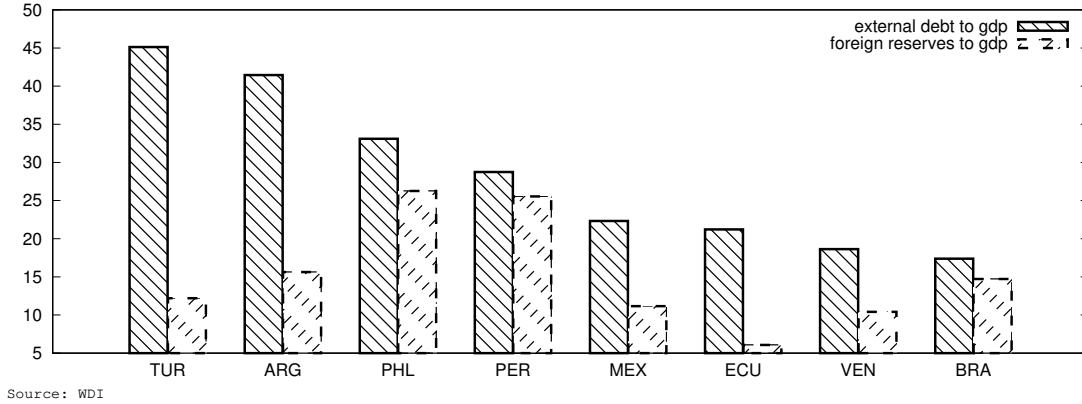Figure 2: Debt in selected emerging market economies in 2000

Figure 3: Spreads in selected emerging market economies
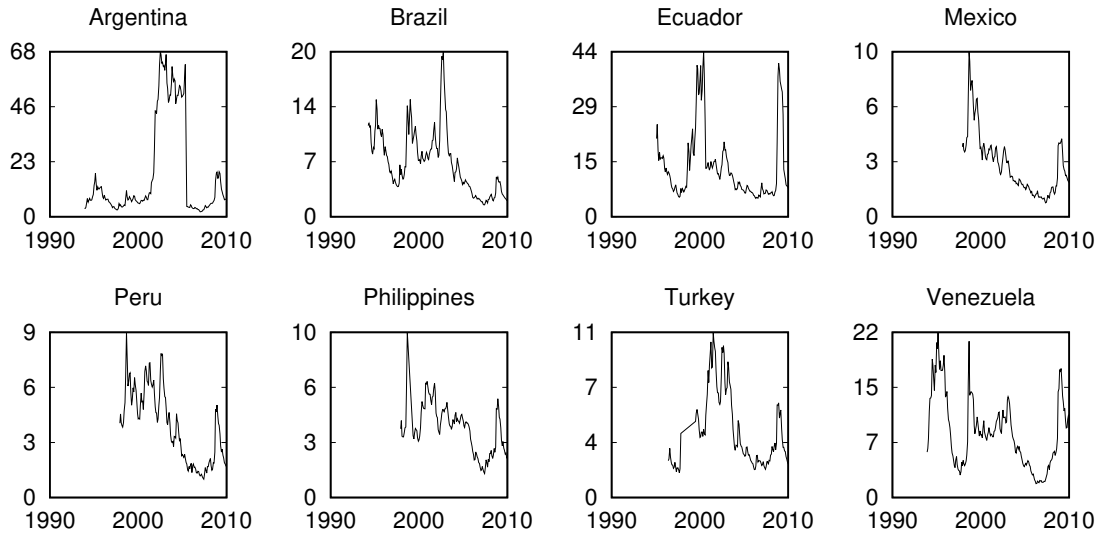
episode periods[5]. With these facts, a crude estimate on the annual cost of holding reserves can be built as being the simple product of the spreads and international reserves. Table 1 shows the calculations: costs can be substantial, oscillating on average between a range of 0.31 to 1.54% annual GDP[6]. Rodrik (2006), using different assumptions, also estimates substantial costs that can be larger than 1% of GDP.

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[5]The extreme interest rate spread spikes observed in Argentina and Ecuador coincide with default episodes.

[6]The range changes to 0.28-0.9% annual GDP if Argentina is excluded.

Table 1: Crude estimates of costs of holding reserves for selected countries (in % of annual GDP)

year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Argentine	0.59	0.84	5.90	6.08	6.70	4.11	0.51	0.57	1.20	1.87	0.98
Brazil	0.37	0.58	1.03	0.75	0.43	0.24	0.19	0.24	0.35	0.45	0.27
Ecuador	1.85	0.61	0.51	0.43	0.31	0.37	0.24	0.47	0.91	1.34	0.36
Mexico	0.19	0.22	0.21	0.20	0.16	0.13	0.10	0.09	0.20	0.34	0.21
Peru	0.92	1.09	1.05	0.72	0.64	0.35	0.30	0.36	0.66	0.74	0.48
Philippines	0.96	1.21	0.91	0.93	0.81	0.72	0.44	0.40	0.69	0.89	0.64
Turkey	0.43	0.90	0.93	0.74	0.34	0.29	0.26	0.25	0.38	0.45	0.26
Venezuela	1.18	0.91	1.36	2.50	1.21	0.85	0.44	0.48	1.13	1.32	0.81
mean	**0.81**	**0.79**	**1.49**	**1.54**	**1.32**	**0.88**	**0.31**	**0.36**	**0.69**	**0.93**	**0.50**

International reserves impact on interest rate spreads

Given non-negligible costs of holding reserves, one could argue that there must be counterweight benefits to rationalize the observed levels of reserves. This section explores potential benefits of holding international reserves through their influence on interest rate spreads. To evaluate empirically the relationship between spreads and international reserves, this study follows a long literature of regressing spreads on covariates[7]. All variables and sources are described in appendix A.1. Our evidence is based on large panel regressions controlling for country and time effects using annual data. Periods of default and market exclusion are not considered for the analysis. Table 2 reports the results from 3 different commonly used econometric specifications.

All three specifications are consistent at showing that reserves to GDP are negatively associated with spreads while controlling for other variables[8]. As for external debt to GDP, the coefficients across regressions have the opposite sign. These results maintain statistical significance even after controlling for country and time effects. The fixed effects column in table 2 shows that an increase of 10pp of reserves to GDP is associated with an average fall of spreads in the order of 36 basis points or 0.36%. At the same time a 10pp increase of debt to GDP or fall in real GDP of 10% is associated with an increase of spreads of 29 and 60 basis points respectively. The regression coefficients for the remaining controls have the expected signs, for example, countries under worse budget condition have larger spreads, or

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[7]For example Edwards (1984), Akitoby and Stratmann (2008) or Panizza, Sturzenegger, and Zettelmeyer (2009).

[8]To deal with potential sources of contemporaneous endogeneity with spreads, lagged variables were used for: reserves/GDP, debt/GDP, real GDP growth rate, revenues/GDP, expenditures/GDP, inflation and current account to GDP.

economies with better institutions, measured from a rule of law index, tend to be associated with lower spreads. These results update and are consistent with previous empirical studies. For the purpose of this paper, the main message is that international reserves seem to be negatively related with interest rates charged to countries that seek external financing. Such effect can be seen as a benefit to hold international reserves.

Table 2: Effects of international reserves on spreads (dependent variable: annual mean spread in basis points)

	Pooled OLS	Random Effects	Fixed effects
L.reserves to gdp (%)	-2.52**	-3.78**	-3.56*
	(0.67)	(1.38)	(1.85)
L.debt to gdp (%)	2.12**	2.49**	2.86**
	(0.44)	(0.68)	(0.8)
L.rgdp growth rate (%)	-1.71	-5.39*	-6.04**
	(3.48)	(2.75)	(2.8)
L.revenues to gdp (%)	-10.78**	-11.37	-9.69
	(3.61)	(7.43)	(9.89)
L.expenditures to gdp (%)	11.56**	12.16**	13.22**
	(3.86)	(5.43)	(5.87)
L.inflation (%)	9.17**	6.75**	6.69**
	(2.91)	(1.41)	(1.49)
L.current account to gdp (%)	1.50	-3.29	-5.84*
	(1.89)	(2.85)	(3.16)
Openness	-0.15	0.04	0.08
	(0.3)	(0.66)	(1.03)
Contagion	0.12	0.17**	0.20**
	(0.07)	(0.06)	(0.06)
Rule of law	-298.53**	-322.16**	-351.26**
	(43.71)	(64.44)	(78.19)
Urban population	-0.56	-0.21	9.84
	(0.83)	(1.71)	(9.17)
constant	454.16**	65.49	-539.27
	(107.82)	(145.51)	(610.42)
Time effects	Yes	Yes	Yes
Observations	304	304	304
R^2	0.59	0.56	0.36

International reserves and haircuts

Rational investors lending to a country should price the interest rate taking into account the probability of repayment and, in case of a default, the expected recovery rate of the

overdue debt. Previous empirical studies on the determinants of interest rate spreads, for example Edwards (1984) or Akitoby and Stratmann (2008), assume that such expected recovery rate is zero, implying that regression coefficients similar to the ones presented in table 2 could be directly mapped into default probabilities. However, with non-zero expected recovery rates, these coefficients may confound two different effects of the related variable: the probability of default and the expected recovery rate. This implies that a more complete analysis of the relationship of reserves on spreads should also take into account how recovery rates relate with reserves.

Figure 4: Haircuts on default episodes for middle income countries
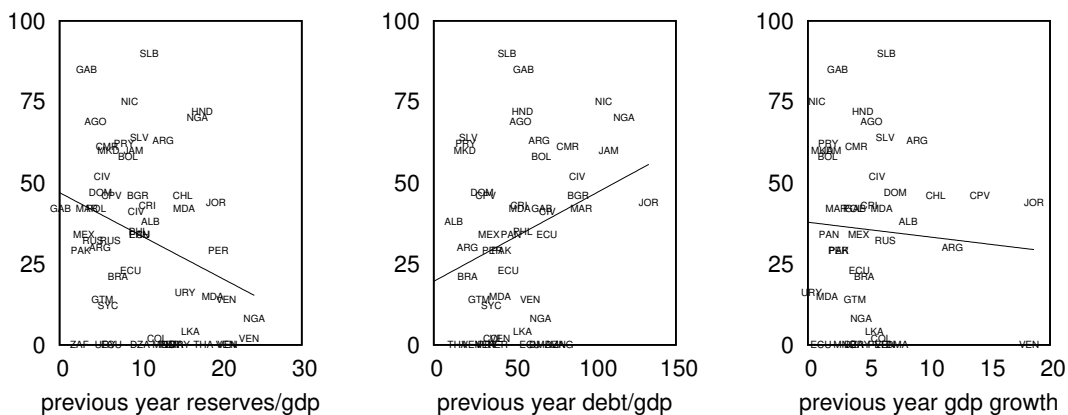
To better study how recovery rates or, using a different term, haircuts (defined as the complement of the recovery rate), I use a dataset on historical haircuts associated to sovereign default episodes compiled by Benjamin and Wright (2009). These estimates have become commonly used in the literature studying restructuring of sovereign defaulted debt[9]. Figure 4 shows how haircuts relate with lagged reserves, debt and GDP growth rate. The picture suggests a negative relationship between haircuts and reserves, that is, the higher the level the reserves, the larger is the recovery rate on the defaulted debt. At the same time, countries with larger debt to GDP tend to have large haircuts and GDP growth rate doesn't seem to matter much for haircuts. Simple multivariate regression techniques[10] presented in table 3 confirm that observation, showing a negative a statistically significant coefficient for lagged reserves even after controlling for lagged debt or output growth: a 10pp increase in lagged reserves to GDP is associated with 16pp fall in debt haircuts.

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[9]Examples include Yue (2010) and Erasmo (2008), where the first studies how haircuts change with the level of debt, and the second studies how haircuts are related with delays in restructuring.

[10]More elaborate regression techniques, exploring the panel structure of the data, were not used due to small number of default observations per country.

Table 3: Effects of international reserves on haircuts (dependent variable: haircut in % of overdue debt)

Haircut	OLS I	OLS II	OLS III
L.reserves to gdp (%)	-1.17**	-1.6**	-1.6**
L.debt to gdp (%)		0.30**	0.31**
L.rgdp growth rate (%)			0.46
Constant	44.76	34.68	32.84
Observations	58	51	51
R^2	0.06	0.23	0.23

3 Model

In light of the evidence presented in the previous sections, a model economy is introduced in this section where the optimal choice of reserves depends on the tradeoff between the cost of holding reserves, closely related with interest rate spread, and the benefits of holding reserves, linked to 3 different factors: smoothing the impact of a crisis on consumption, minimizing fiscal distortions on production, and change the negotiation position in default episodes. These last 2 benefits represent the main innovation that is introduced here relatively to previous models from the literature. In proceeding this way, the model allows for a better accounting of the observed dynamics regarding debt and reserves.

The model economy[11] builds up from the classical work of Eaton and Gersovitz (1981), with recent applications in, among others, Aguiar and Gopinath (2006) and Arellano (2008). A small open economy is populated with a representative household, a firm and a government. The household is 'hand-to-mouth', simply consuming any income net of taxes earned in each period - they never own any asset whatsoever. This assumption mutes domestic credit markets to highlight the role of external debt and international reserves markets. The firm buys labor to produce final goods with a production function that is subjected to diminishing returns and to a multiplicative technology shock. The government acts on behalf of the household by making decisions about the amount of debt and reserves to hold - available in international markets. It also taxes consumers in order to finance public expenditures which, for the sake of simplicity, are exogenous. Due to limited commitment in the enforcement of debt contracts, the government can default on its own debt. Under such scenario, access to international markets is temporarily shut down and a renegotiation process follows. While excluded, firms suffer a loss in productivity and lenders recover part of the repudiated debt

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[11]A simple version of this model with some implications is introduced in the appendix A.2.

by allowing the government to regain access to the market. Additionally, in any time period, lenders can loose confidence in debtors for exogenous reasons. A sudden stop then echoes as an impediment to the renewal of the government's loans.

3.1 Household

An infinite lived representative household values lifetime consumption and labor accordingly to:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{1}$$

where $\mathbb{E}$ is the expectation operator, β denotes the discount factor, and the period utility $u : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}$ is: continuous, differentiable and concave in both arguments; increasing in c and decreasing in h.

The household supplies labor to the firm at a wage rate w_t, taxed at a rate τ_t. No savings are allowed and profits π_t are transferred as *lump sum*. Income thus earned is used for consumption, yielding the following time t budget constrain:

$$c_t = (1 - \tau_t)w_t h_t + \pi_t \tag{2}$$

Optimal household behavior regarding consumption and hours supply can therefore be characterized by equation (2) and the following first order condition:

$$-\frac{u_h(c_t, h_t)}{u_c(c_t, h_t)} = (1 - \tau_t)w_t \tag{3}$$

3.2 Firm

The firm in the economy maximizes profits by using labor h_t in a production technology subjected to diminishing returns $f(h_t)$ and random productivity shocks z_t. The function $f : [0, 1] \to \mathbb{R}$ is continuous, differentiable, concave and satisfies the Inada conditions. The productivity shock z_t evolves accordingly to a Markov process. Profits in time t are given by

$$\pi_t = z_t f(h_t) - w_t h_t \tag{4}$$

Which imply the following labor demand condition:

$$w_t = z_t f_h(h_t) \tag{5}$$

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3.3 Government

Acting on the household's behalf, the government with a good credit history has the ability to borrow or save in international markets. Due to incompleteness of financial markets, the government can only borrow by selling non-contingent one period bonds D_t. At the same time, it can also choose to save by buying international reserves R_t. To finance public expenditures g_t, it can also tax wages. If the government decides to repudiate its own debt, credit history becomes bad with further exclusion from international credit markets. In this situation, firms productivity becomes $\tilde{z}_t = z_t - l(z_t)$, where $l(z_t)$ is a continuous loss function such that $0 \leq l(z_t) \leq z_t$. Putting all elements together, the government budget constraint in period t is given by:

$$\tau_t w_t h_t = g_t + D_t - q_t D_{t+1} - R_t + \bar{q} R_{t+1} \quad if \ good \ credit \ history \tag{6}$$

$$\tau_t w_t h_t = g_t - R_t + \bar{q} R_{t+1} \quad\quad\quad\quad\quad\quad\quad otherwise \tag{7}$$

where q_t is the price of new debt and $\bar{q}$ the risk-free price of new international reserves.

3.4 International investors

International investors provide debt and reserve assets to the government. However, the economy can suffer a random sudden stop shock denoted by s. Similarly to Roch and Uhlig (2012), s is interpreted as a "crisis" sunspot where, for extraneous reasons, the government looses access to international markets[12]. In this model the sudden shock realization is independent from all other variables, taking the value $s = 1$ if the economy is hit or $s = 0$ if it's not. If the economy is in a sudden stop, no international investor provides credit to the economy. This is equivalent as stating that the price of new debt equals zero. If instead the economy is not in a sudden stop, international investors price debt so that, in expectation, their profits are zero. In the event of a default, it is assumed that they recover an amount $\tilde{\varphi}(D, R, z) \in [0, D]$ of the arrears debt in the period following renegotiation. Note that under this case, the recovery rate schedule is given by $\tilde{\varphi}(D, R, z)/D$. Letting $\tilde{Z}(D, R, z)$ be an indicator function taking 1 if the government defaults and 0 otherwise, new debt becomes priced as:

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[12]For a model where sunspots can generate large shifts in the borrowing conditions see Cole and Kehoe (2000).

$$q(D_{t+1}, R_{t+1}, z_t, s_t) = \begin{cases} 0 & if \ s_t = 1 \\ q(D_{t+1}, R_{t+1}, z_t) & otherwise \end{cases} \quad (8)$$

where

$$
\begin{aligned}
q(D_{t+1}, R_{t+1}, z_t) =& \bar{q} \cdot \left\{ \int \left[1 - \tilde{Z}(D_{t+1}, R_{t+1}, z_{t+1})\right] dF(z_{t+1}, s_{t+1} | z_t, s_t) \right. \\
& \left. + \int \tilde{Z}(D_{t+1}, R_{t+1}, z_{t+1}) \cdot \bar{q} \frac{\tilde{\varphi}(D_{t+1}, R_{t+1}, z_{t+1})}{D_{t+1}} dF(z_{t+1}, s_{t+1} | z_t, s_t) \right\} \quad (9)
\end{aligned}
$$

and it is assumed that (z_t, s_t) evolve according to the transition probability given by $F(z_{t+1}, s_{t+1} | z_t, s_t)$. From the definition of (8), the price schedule is bounded by $q_t \in [0, \bar{q}]$, in other words, interest rate on borrowing is always equal or larger than the risk free rate.

3.5 Timing

The events characterizing this model can be structured with the following order. At the beginning of period t, a government with good credit history:

1. Starts period t with debt and reserves levels of D_t and R_t respectively.

2. Sudden stop and productivity shocks are realized: s_t and z_t.

3. The government decides whether or not to default.

    (a) If the government decides not to default:

        i. Choses D_{t+1}, and R_{t+1} at prices $q(D_{t+1}, R_{t+1}, z_t, s_t)$. The remaining variables $(c_t, h_t, \tau_t, w_t, \pi_t)$ are determined by the model's agents[13].

        ii. Advances to period $t + 1$ with a good credit history and debt and reserves levels of D_{t+1} and R_{t+1}.

    (b) If the government decides do default:

        i. The country enters in financial autarky. The government still chooses R_{t+1} and the remaining variables $(c_t, h_t, \tau_t, w_t, \pi_t)$ are determined by the model's agents. At the same time, the government negotiates how to restructure its debts with lenders, agreeing to pay $\tilde{\varphi}(D_t, R_t, z_t)$ next period.

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[13]Note that when D_{t+1} and R_{t+1} are chosen, τ_t is uniquely determined from equations (3), (5), (6) and (7).

ii. Advances to period $t+1$ still with bad credit history. The government pays $\varphi(D_t, R_t, z_t)$ and decides on R_{t+2}. The remaining variables are determined $(c_t, h_t, \tau_t, w_t, \pi_t)$ by the model agents.

iii. Advances to period $t+2$ with a good credit history, no debt and reserves level of R_{t+2}.

Given that the focus of this paper is not in the study of delays in debt restructuring, it is assumed in the model that restructuring is exogenously resolved in the period that follows a default. Given this framework, the government must negotiate a debt restructuring with lenders when it defaults and, in the next period, transfer the agreed amount. Despite being restrictive, this environment generates results that are similar to the ones obtained in models of renegotiation such as Yue (2010) or Erasmo (2008) where agents choose to renegotiate very quickly, even though they are specifically allowed to delay repayments or renegotiations.

For convenience, the timing of the model is also schematized in figure 5.

Figure 5: Sequence of events of the model
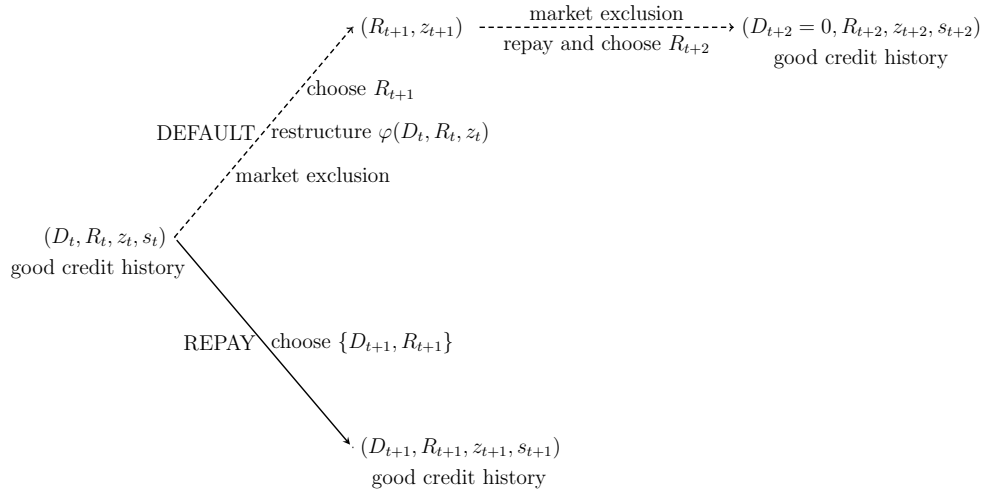
3.6 Recursive formulation of the problem

The government problem consists in maximizing consumers utility given by (1) subject to all the constraints summarized in equations (2) to (8). Let v^{rep} be the value for a government who repays its debt, v^{def} the value of a government who defaults and (D, R, z, s) state

variables. Then the previous problem can be represented recursively as:

$$v^{rep}(D, R, z, s) = \max_{D', R'} \left\{ u(c, h) + \beta E_{z,s} \left[\max \left\{ v^{rep}(D', R', z', s'), v^{def}(D', R', z') \right\} \right] \right\} \quad (10)$$

$$st$$

$$c = zf(h) - g - D + qD' + R - \bar{q}R' \quad (11)$$

$$-\frac{u_h(c, h)}{u_c(c, h)} = zf_h(h) - \frac{g + D - qD' - R + \bar{q}R'}{h} \quad (12)$$

The first constraint (11), resulting from combining equations (2), and (4)-(6), is a resources constraint: private and public consumption equate to the sum of domestic production plus net external inflows, whether positives or negatives. The second constraint (12), which combines (2)-(6), is a labor market equilibrium condition. That is, it represents the set of competitive allocations (c, h) such that both consumers and firms are optimizing given prices and taxes. Note also that, under this representation, tax rate is no longer explicitly present in the equations. This is because they are determined by the next period choice of debt and reserves. As such, the government's problem collapses into choosing the level of reserves and debt for next period subject to both resources and labor market constraints. Note however that given an allocation (c^*, h^*, D'^*, R'^*) that is a solution for (10), tax rates can be recovered from equations (3) and (5):

$$(1 - \tau) = -\frac{u_h(c^*, h^*)}{u_c(c^*, h^*)} \cdot \frac{1}{zf_h(h^*)}$$

If, instead, the government decides not to repay, it spends one additional period in financial autarky that will be used to reach an agreement with its creditors. Given these elements, the government's value of default can be defined as:

$$v^{def}(D, R, z) = \max_{R'} \left\{ u(c, h) + E_z \left[\max_{R''} \left\{ \beta u(c') + \beta E_{z'} v^{rep}(0, R'', z'', 0) \right\} \right] \right\} \quad (13)$$

$$st$$

$$c = \tilde{z}(z)f(h) - g + R - \bar{q}R'$$

$$\frac{u_h(c, h)}{u_c(c, h)} = \tilde{z}(z)f_h(h) - \frac{g - R + \bar{q}R'}{h}$$

$$c' = \tilde{z}(z')f(h') - g + R' - \bar{q}R'' - \tilde{\varphi}(D, R, z)$$

$$\frac{u_h(c', h')}{u_c(c', h')} = \tilde{z}(z')f'_h(c', h') - \frac{g - R' + \bar{q}R''}{h'}$$

$$R' \leq R$$

17

The first and second pair of equations represent the resources constraints in the first and second period, before the government is readmitted in international credit markets. In the first period of default, the government suffers a productivity loss $\tilde{z}(z)$ and agrees to repay $\tilde{\varphi}(D, R, z)$ to his creditors in the next period. Here, I assume that in order to regain access to international markets, the government has first to repay his agreed debt. Note that the last constrain imposes that the government cannot increase reserves while in autarky. This assumption prevents the government from accumulating too much reserves in the period preceding restructuring. A more complete model with a longer and uncertain period of market exclusion would not require to have such constraint.

3.7 Renegotiation

As in Yue (2010), if the government defaults and becomes excluded of international markets, then renegotiation follows immediately where borrowers and lenders bargain over a recovery amount in exchange for access to international credit markets. If negotiations fail, the government becomes forever excluded of international credit markets. Let $v^{aut}(R, z)$ be the value of permanent autarky and $v^{rep}(0, R, z, s)$ the value of being in the market with zero debt. For a government owing D, holding R and with productivity z, reaching an agreement for a recovery amount φ has a value given by:

$$
\begin{aligned}
\Lambda^G(\varphi; D, R, z) &= \max_{R' < R} \left\{ u(c, h) + E_{z,s} \left[\max_{R'' < R'} \left\{ u(c', h') + \beta E_{z', s'} \left[v^{rep}(0, R'', z'', s'') \right] \right\} \right] \right\} \\
&\quad - v^{aut}(R, z)
\end{aligned}
$$

$$st$$

$$c = \tilde{z}(z) f(h) - g + R - \bar{q} R'$$
$$\frac{u_h(c, h)}{u_c(c, h)} = \tilde{z}(z) f_h(h) - \frac{g - R + \bar{q} R'}{h}$$
$$c' = \tilde{z}(z') f(h') - g + R' - \bar{q} R'' - \varphi$$
$$\frac{u_h(c', h')}{u_c(c', h')} = \tilde{z}(z') f_h'(h') - \frac{g - R' + \bar{q} R''}{h'}$$
$$R' \leq R$$

where $z - \tilde{z} = l(z) \geq 0$ represents the output loss of exclusion and the outside option for the government - permanent autarky - has a value of:

$$v^{aut}(R, z) = \max_{R'} \left\{ u(c, h) + \beta E_z \left[v^{aut}(R', z') \right] \right\} \tag{14}$$

$$st$$

$$c = \tilde{z}(z) f(h) - g + R - \bar{q} R' \tag{15}$$

$$\frac{u_h(c, h)}{u_c(c, h)} = \tilde{z}(z) f_h(h) - \frac{g - R + \bar{q} R'}{h} \tag{16}$$

Similarly, lenders obtain an agreement value given by:

$$\Lambda^L(\varphi) = \bar{q}\varphi$$

This model features a renegotiation stage that is resolved endogenously in a Nash bargaining problem. For an η bargaining power for the debtor, the recovery amount φ that solves such problem is given by:

$$\tilde{\varphi}(D, R, z) = \arg \max_{0 \leq \varphi \leq D} \left\{ \left[\Lambda^G(\varphi; D, R, z) \right]^\eta \cdot \left[\Lambda^L(\varphi) \right]^{1-\eta} \right\} \tag{17}$$

3.8   Recursive Equilibrium

All elements are now available to define a stationary recursive equilibrium in this model economy. The equilibrium notion is of a Markov Perfect Equilibrium, that is policy actions about debt, reserves, default, and negotiation, depend only on pre-determined relevant variables.

Definition 1. A recursive equilibrium is a set of:

   i) Value functions: $v^{def}(D, R, z, s)$ and $v^{aut}(R, z)$

  ii) Debt price function: $q(D', R', z, s)$

 iii) Debt recovery function: $\tilde{\varphi}(D, R, z)$

Such that

 a) Given the debt price function $q(D', R', z, s)$ and the debt recovery function $\tilde{\varphi}(D, R, z)$, the value function $v^{rep}(D, R, z, s)$ solves the government problem (10)

 b) Given the value function $v^{rep}(D, R, z, s)$ and the debt recovery function $\tilde{\varphi}(D, R, z)$, the debt price function $q(D', R', z, s)$ is consistent with the lenders zero profit condition in (8)

c) Given the value functions of repayment $v^{rep}(D, R, z, s)$, autarky $v^{aut}(R, z)$ and the debt price function $q(D', R', z, s)$, the debt recovery function $\tilde{\varphi}(D, R, z)$ solves the debt renegotiation problem (17)

4 Calibration and quantitative analysis

To analyze the quantitative properties of the model introduced in the preceding section, functional forms are chosen and a calibration is proposed for the numerical computation. The model is used to evaluate the role of international reserves when the debt choice is endogenous and willingness-to-pay incentives becomes a function of the sovereign's choices.

4.1 Functional forms

The numerical implementation of the model uses a utility function of the form proposed by Greenwood, Hercowitz, and Huffman (1988):

$$u(c, h) = \frac{1}{1 - \sigma} \cdot \left(c - \Gamma \frac{h^{1+\gamma}}{1 + \gamma} \right)^{1-\sigma} \tag{18}$$

This utility function has the advantage of shutting down the wealth effect on labor supply, therefore shocks in the productivity process have an output response of the same signal[14].

Regarding the output level in the event of a default, Arellano (2008) showed that a non-linear function that induces a disproportionally larger loss if the country defaults in an expansion is important to allow for a large default probability. Also, large output contractions at defaults, followed by recoveries (coinciding with credit market re-access) have been documented, for example, in Yeyati and Panizza (2011). Protracted losses in output are explained with disruptions of credit flows to the private sector that prevents normal production (Mendoza and Yue, 2012). As such, a similar functional form is assumed as appropriate for the productivity under default of the model economy:

$$\tilde{z}(z) = \begin{cases} z & if \ z \geq \hat{z} \\ \hat{z} & otherwise \end{cases} \tag{19}$$

The sudden stop shock is added in order to induce the government to hold reserves as insurance against exogenous shut downs of credit markets. This idea was firstly introduced in Ranciere and Jeanne (2006) and applied more recently in Bianchi, Hatchondo, and Martinez

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[14]This utility function has a long tradition in literature studying business cycles in small open economies, for instance, Mendoza (1991), Neumeyer and Perri (2005) or Aguiar and Gopinath (2007).

([2012](#)). In those models, governments hold reserves as a buffer not just against rollover risks, but also against direct output costs that comes along with a sudden stop. This paper assumes a milder version of a sudden stop shock that does not impact the economy in any other way but in the momentarily exclusion from credit markets. Additionally, the sudden stop shock is modeled as being independent and identically distributed in every period.

Finally, as commonly used in the literature, the productivity process is modeled as a log-normal $AR(1)$, with

$$\log z' = \rho_z \log z + \epsilon', \quad \epsilon' \sim N(0, \sigma_z)$$

This process is discretized into a 21 state Markov chain using Tauchen ([1986](#)) method, with bounds given by $\log z \in \left[-3.5 \cdot \sigma_z/\sqrt{1-\rho_z^2}; 3.5 \cdot \sigma_z/\sqrt{1-\rho_z^2}\right]$.

The model is numerically solved using value function iteration. A detailed explanation of the algorithm and numerical methods used can be found in the appendix [A.3](#).

4.2   Parameters and calibration

The model is computed at a quarterly frequency. Then, the solution is used to evaluate the model's ability to generate large government's choices of debt and reserves while allowing the economy to match other features of the data such as default rates and cyclical properties of consumption, trade balance, or interest rates. For this exercise, Mexico is used as reference for the parameter choice. As a representative country from the set of emerging economies, Mexico has an additional advantage of having available data at a quarterly frequency for a period ranging 1981 to 2012[15]. Moreover, the Mexican economy experienced a sovereign default episode in 1983 (after a collapse world commodity prices) and a near default in 1994 (when the country was rescued by IMF and the U.S. Treasury). Mexico has been displaying also strong dynamics in the accumulation of both debt and reserves: in the last decade the government more than doubled its holdings reserve to about 10% of GDP while keeping a debt level to GDP ratio of more than 20%.

All the data referring to Mexico are seasonally adjusted quarterly real series obtained from OECD, except for external debt and international reserves that are taken from the World Development Indicators at yearly frequency. Output and private consumption are in logs and the trade balance is presented as a percentage of GDP. Following the methodology proposed by Mendoza, Razin, and Tesar ([1994](#)), an effective tax rate is computed for the

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[15]A quarterly times series on Mexico is publicly available byNeumeyer and Perri ([2005](#)) for 1981 to 2001 and, for the remaining period, by OECD .

Mexican economy[16]. The interest rate spreads corresponds to the EMBI for 1994–2007 and all other series are from 1980 to 2010. All series are filtered with a Hodrick–Prescott filter.

Table 4: Parameter values

Parameters		Value	Target
Risk aversion	σ	2	Standard in the literature
Inverse Frish elasticity	γ	2	Standard in the literature
Risk free debt price	$\bar{q}$	1/1.017	US interest rate
Probability of sudden stop	ω	0.025	Ranciere and Jeanne (2006)
Output elasticity of labor	α	0.5	Labour income share in GDP (Mexico)
Productivity shock persistency	ρ_z	0.95	Output volatility and autocorrelation (Mexico)
Productivity shock volatility	σ_z	0.007	Output volatility and autocorrelation (Mexico)
Discount factor	β	0.93	Debt/GDP and volatility of trade balance (Mexico)
Productivity cost	$\hat{z}$	0.96	Debt/GDP and volatility of trade balance (Mexico)
Bargaining power	η	0.2	Benjamin and Wright (2009)
Government spending	g	0.116	Average government spending to GDP (Mexico)
Disutility of labor	Γ	4.66	Average hours of 1/3

Table 4 lists the parameters used in the baseline solution of the model. The table is divided in a first set of parameters taken directly from the data or the literature, and a second set that uses the model simulated moments to infer the parameter values. The two first parameters refer to utility function of the household. The parameter σ, the risk aversion coefficient, is set to 2 which is a standard value used in the quantitative macroeconomics literature. The parameter $1/\gamma$ is related with the empirical evidence on the Frish wage elasticity. Given the range of estimates available in the literature (for example in Greenwood, Hercowitz, and Huffman 1988), a value of 0.5 is picked for the elasticity, that is $\gamma = 2$.

In Ranciere and Jeanne (2006), a sudden stop corresponds to episodes when capital inflows to GDP, measured by the current account, falls by more than 5 percent of GDP relative to previous year. They then verify that on average 1 sudden stop occurs every 10 years for a set of developing countries. This estimate gives a 2.5% probability of being hit by a sudden stop, a value that is also used to set ω.

As common in the literature of sovereign default studies, the risk free bond price $\bar{q}$ is set to 1/1.017, intended to capture the historical average quarterly1.7% interest rate of a five-year US treasury bond. As for the parameter α, governing the output elasticity with

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[16]Appendix A.1 shows how these estimates are computed and specifies data sources.

respect to output, its value is taken directly from OECD estimates on labor income share which averages to 0.5.

The remaining parameters are jointly calibrated to match certain moments of the data. An observed average 20% of public consumption to GDP is targeted setting $g = 0.0116$. Due to lack of enough data on haircuts for Mexico's default episodes, the haircut average of 42% present Benjamin and Wright (2009) dataset is target instead. This is calibrated with parameter value of $\eta = 0.2$. Both parameters governing the productivity shock are calibrated simultaneously as output dynamics are not directly inherited from the productivity z due to endogenous labor supply. The targeted data moments are the standard deviation and autocorrelation of GDP given by $stdev(GDP) = 0.026$ and $corr(GDP_t, GDP_{t-1}) = 0.8$ with correspondent parameter values of $\sigma_z = 0.007$ and $\rho_z = 0.95$. The last 2 parameters $\{\beta, \hat{z}\}$ are simultaneously calibrated to target the following data moments: mean debt to GDP and the standard deviation of the trade balance. Debt to GDP is targeted to be 32% in the simulations and the the standard deviation of the trade balance to 1.4%.

4.3 Simulation results

This section compares the quantitative predictions of the model against observed data. To that end, a model using table 4 parameters is computed and simulated by averaging the moments of interest for 1000 sample economies, each one running for 500 periods, where the first 300 are discarded to reduce the influence of initial conditions. These moments are computed for 44 periods before a default episode and at least 16 periods after a market exclusion. Table 5 reports the results of the exercise. The standard deviations referred in the table are expressed in percentage points; y and c refer to the log of output and consumption of a de-trended series; $D^{recover}/D$ is the recovery rate faced by lenders at a default; the default rate is an annual rate; trade balance TB/Y is defined as the difference between output and consumption relative to output; D/Y and R/Y are, respectively, the external debt and international reserve level expressed as a percentage of output; G/Y refers to government spending to GDP; the annual interest spread is given by $i^{spread} = (q/\bar{q})^4 - 1$.

Table 5: Business cycle statistics for the benchmark model and data

	Data	Model
$mean(D/Y)$	32	34
$stdev(TB/Y)$	1.4	1.3
$stdev(y)$	2.6	2.6
$mean(D^{recover}/D)$	58	55
$mean(G/Y)$	20	20
$mean(R/Y)$	9	10
$mean(i^{spread})$	3.0	1.4
$stdev(i^{spread})$	1.5	1.3
$stdev(c)/stdev(y)$	1.2	1.4
$corr(y,c)$	0.94	0.96
$corr(y,i^{spread})$	-0.56	-0.26
$corr(TB/Y,y)$	-0.66	-0.66
$corr(\tau,y)$	-0.49	-0.81
$default\, rate$	2.0	2.8

The first impression of the results is that the model can achieve some success at explaining key features of the data. The model delivers a mean debt to output ratio of 34% while 10% for reserves to output. Large values of debt to output are sustained in equilibrium due to intense losses in the event of a default. In the present model, default losses are associated with exogenous losses of productivity given by the function (19) and an endogenous fiscal adjustment that distorts production severely by reducing labor supply. This is in contrast with Arellano (2008) that, to target the default rate observed in Argentina, uses a milder and completely exogenous output loss, ending up with a model that generates a counterfactually low level of debt to output. When that is the case, lenders became weary to lend to a country that suffers mildly if it decides to default. Such is not the case in the current framework. Figure 6, characterizing numerically the typical default event in the simulations, shows a fall of output of around 15%, accounted by a 4% slump in productivity and a 11% reduction in hours. From this contraction of output, only 2% are directly caused by exogenous output loss given by function (19). As the government advances into default, a debt restructuring negotiation leads to an agreement with international investors that implies repaying part of the overdue debt. To finance such repayment, the government has to raise taxes that distort labor even further, hence generating the described large fall in output.

Similarly to the result regarding debt, reserves to output also matches the data with an average level of 10% in the ergodic distribution. In this model reserves play an insurance role: if the country defaults without any reserves, then consumption has to painfully adjust by bearing all the losses of default. However, since the probability of default is an endogenous choice for the country, that risk can be reduced by decreasing the debt it chooses to hold. Additionally, lower debt maps into lower interest rates. This channel would make reserves less important as an insurance instrument. Note that this is essentially the result in Alfaro and Kanczuk (2009): international reserves cannot be sustained in equilibrium[17]. The positive reserve holdings observed in the simulation are related with the additional risk that the country cannot roll-over debt if hit by a sudden stop. In such scenario, the government finds it optimal to use reserves to repay debt, avoiding the negative impacts of a default. If the sudden shock carried further costs to the borrower, then reserves would play an even more prominent role as an insurance mechanism in this model. In fact, such costs are present in this model as, to repay the outstanding debt, the government has to raise distortionary taxes with the effects that are in everything similar to the ones described before.

Other features of the data captured by this model refer to the negative correlation between the trade balance and output, and the default rate. Part of the model's larger than in the data default rate is accounted by the presence of the sudden stop shock. As shown in Roch and Uhlig (2012), adding a sudden stop shock with the characteristics presented in the model, widens the borrowing risky region as now, for certain shock realizations, the borrower is not able to roll over his debt. This effect increases the number of defaults in the economy. The result of a negative correlation between trade balance and output of $corr(tb, y) = -0.66$ is also obtained in similar papers, for example, Aguiar and Gopinath, 2006 or Yue, 2010. Net capital outflows occur in recessions due to an increase in interest rates as probability of default increases in recessions. This effect constrains borrowing endogenously.

Regarding to the mean interest rate spread, the model misses out the data target. The presence of renegotiation in the model explains why spreads are on average low. This can be seen from the second term of the right hand side of equation (9) that is non-negative. That is, the presence of a positive recovery rate in the event of a default becomes priced in new lending with lower spreads[18]. Other authors have overcome this issue by assuming a time-varying lender's discount factor (Arellano, 2008) or a richer specification of the productivity loss (Chatterjee and Eyigungor, 2012). However, for the current investigation, such anomaly

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[17]Several features in Alfaro and Kanczuk (2009) model explain their striking result: in addition to using a very simple model, without the features presented in the current paper, the authors assume a quarterly discount factor of 0.5 and a proportional output loss function.

[18]Spreads that are lower than what is observed in the data are usually present in models that display endogenous negotiation, for instance, Yue, 2010 or Erasmo, 2008.

is not corrected due to computational limitations.

One last relevant moment from table 5 is the negative correlation between output and the tax rate. To capture the fact that fiscal policy in emerging market economies is often pro-cyclical, the model features an inelastic government expenditure with endogenous taxation. Cuadra, Sanchez, and Sapriza (2010) show that a similar model can generate realistic pro-cyclical fiscal policies. As output declines, the government with constrained credit market access has to raises taxes to compensate the smaller base of taxation. The same idea is also used in this paper to generate distortions in production that are negatively correlated with output, thus generating more demand for reserves for insurance purposes. One current limitation of the current model is that government expenditure doesn't co-move with output. The main consequence of that assumption is an overestimation of negative correlation of the tax rate with output. However, as shown in Cuadra, Sanchez, and Sapriza (2010), making the government expenditure endogenous would still maintain a negative correlation of taxes and output - at the cost of adding on the computational burden - without changing qualitatively any of the mechanisms present in this paper.

4.4 Macroeconomic dynamics around an average default episode

For all default episodes meeting the sampling criteria explained in the previous section, a time window of 12 periods is collected before and after a default occurs. Using this data sample, a period-by-period average of each variable of interest is computed and the date of default normalized to 0. These statistics are used to characterize, in figure 6, the dynamics of the average default episode that the model outlined produces.

The top two panels show the dynamic of output and productivity around a default. The pattern of productivity is the standard in models of sovereign default. At $t = 0$, the level of productivity z becomes 2.5% below average[19]. Note that the depicted z doesn't include default costs. At period $t = 0$ and $t = 1$ the level of productivity once those exogenous costs captured by function (19) are taken into account becomes 4.5%. This translates into a fall of output relative to trend of about 5% in $t = 0$ and 16% in $t = 1$. The relatively mild drop of output in period $t = 0$, despite the exogenous productivity loss, is related with the fact that, by defaulting, the sovereign is releasing resources to public and private consumption and so tax pressure is reduced. However in period $t = 1$, due to the agreement reached in the debt restructuring, the sovereign has to transfer the agreed overdue debt back to investors, increasing taxes in the process.

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[19]This magnitude corresponds to a 1.2 standard deviation of the underlying shock as $stdev(\log z) = \frac{\sigma_z}{\sqrt{1-\rho_z^2}} = 2.2\%$.

Both consumption and hours suffers in this joint process of debt restructuring and fiscal adjustment with the first falling 35% and the later 25% at the trough of the crisis. As already mentioned in the previous section, these large movements in consumption and hours are in part inflated by the fact that the government expenditure is exogenous in this model[20]. Both aggregates quickly recover once the market exclusion is over, that is, from period $t = 2, 3, ...$ until the next default.

Figure 6: Average default episode generated in the model's simulations
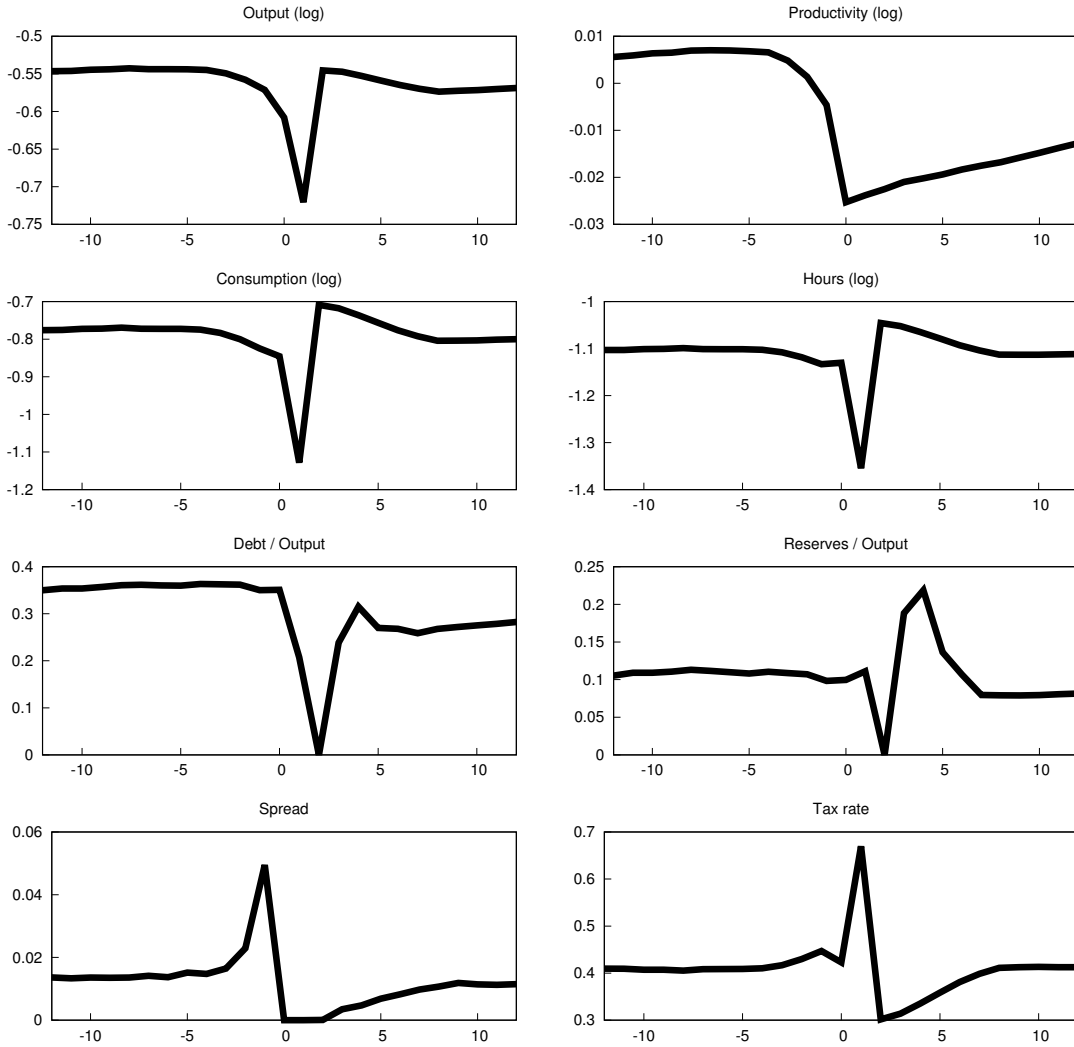
Also worth noting is the dynamics of reserves to output. Up until period $t = -1$, predicting an eminent default, the government maintains his levels of reserves more or less

constant. These reserves are then used in period $t = 1$ to restructure debt. After period $t = 2$, both debt and reserves recover quickly as the cost of holding reserves, given by the interest spread (bottom left panel), is low for low levels of debt which gets reseted after regaining market access. However, as debt comes back to normal levels in the following periods, so does the spreads and demand for reserves falls back to a level close to 10% of output.

Finally, the right bottom panel in figure 6 shows an average increase of 25pp in the tax rate during a default episode. While direct tax evidence about labor taxes is difficult to collect for emerging market economies, using the methodology of Mendoza, Razin, and Tesar (1994), it is possible to construct series on average effective tax rates for OECD countries that experienced default or near-default crisis in the past: Mexico, Portugal, Spain, Greece. Inspection of those series confirms that taxes tend to rise around crisis periods, although not in the same magnitude of what is being generated in the simulation. From equation (6), it is easy to see that tax rates in this model map into Chari, Kehoe, and McGrattan (2007) definition of a labor wedge. Regarding to financial crisis, Cho and Doblas-Madrid (2013) finds that labor wedges are major contributors for the observed movements of output. This implies that in a more loose interpretation of the above model, tax rates can be regarded as a set of unknown distortions in the labor market induced by the government that are observed in increasing labor wedges.

4.5 Understanding the mechanism

To better grasp the intuition[21] of the results presented in the previous sections, figure 7 depicts the policy functions of the model when the economy is not hit by a sudden stop in the current period, that is, new debt level $D'(D, R, z)$, reserve holdings $R'(D, R, z)$ and net debt, simply given by $D'(D, R, z) - R'(D, R, z)$.

The left panel of the figure shows the policy functions for next period reserves and debt for fixed current reserves level and productivity shock realization, when current debt changes, that is: $R'(D, \bar{R}, \bar{z})$ and $D'(D, \bar{R}, \bar{z})$ for $D \in [0, D^{max}]$. Similarly, the central and right panel shows the same functions when current reserves or current productivity are allowed to vary. The first observation is that reserves and debt are correlated and tend to move together. In particular the right panel shows that gross capital flows, defined as $D' + R'$ are pro-cyclical and tend to collapse in recessions, consistent with empirical findings of Broner, Didier, Erce, and Schmukler (2013). Additionally, the two panels at the left shows that lows levels of net debt, represented by the triangles, are associated with large levels of both debt and reserves.

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[21]A simple two-period model developed in the appendix A.2 can be useful to understand the intuition behind the full model.

This is because debt implies a low probability of default and therefore low spreads. But with low spreads, the cost of holding reserves is also low, and the sovereign has an incentive to accumulate large levels of both debt and reserves. This is also the reason why in figure 6 reserves and debt tend to recuperate quickly after a default.

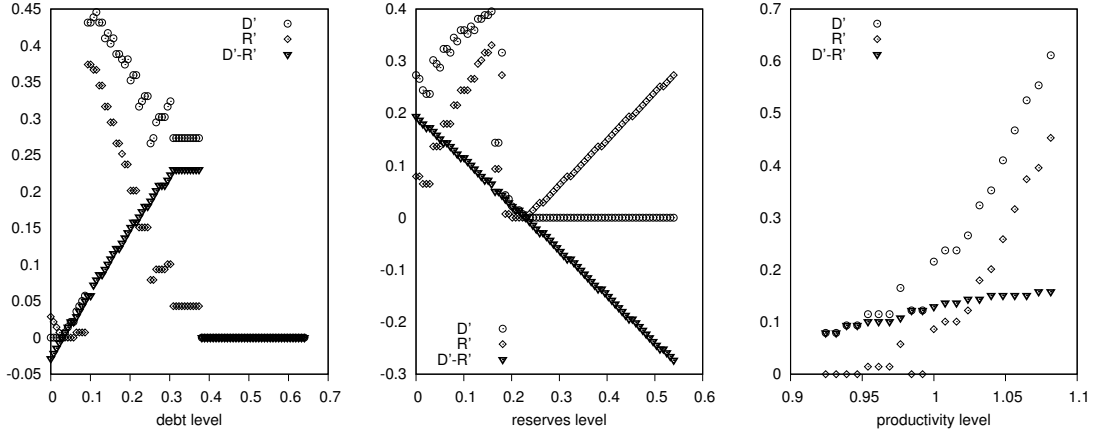Figure 7: Policy functions for new debt, reserves and net debt

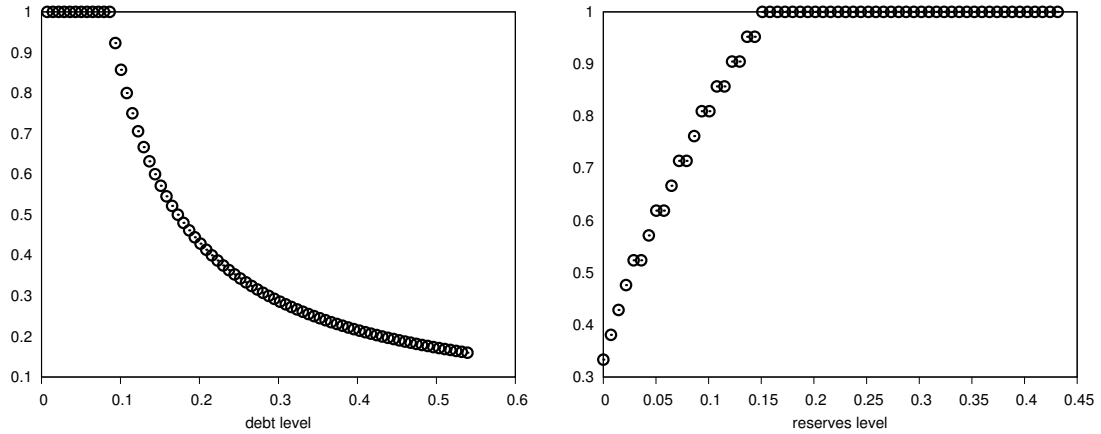Figure 8: Recovery rate schedules for overdue debt

Figure 8 plots the equilibrium debt recovery rate schedule as a function of debt and reserves. Similar to the findings in Yue (2010), debt recovery rate is decreasing with debt level with a threshold of debt before which recovery rate is 100%. A close inspection of problem (17) should clarify why the recovery rate schedule has that shape: in the bargaining problem, the settlement argument from φ is independent of the amount defaulted D, so the recovery rate schedule as a function of D becomes $0 \leq Rec(D) = \max\{1, \varphi/D\} \leq 1$.

Intuitively, after defaulting, D becomes 'sunk' and agents only bargain about future values that don't depend on D. That is not the case with R since a marginal unit of reserves impacts differently the value of renegotiation and the outside value for the sovereign which amounts to permanent exclusion. This means that different level of reserves implies different bargaining positions and, therefore, different debt reschedule outcomes. Note that in financial autarky, the sovereign doesn't have any additional purpose than to smooth consumption in subsequent periods. However, if the decision on restructuring prevails, anticipating that transfers will be made to lenders, the sovereign will have an additional incentive to save on top of the previous consumption smoothing motif. But this must mean that a marginal unit of reserves must be more valuable under restructuring than under permanent autarky, that is equivalent to say that the total surplus of the Nash bargaining problem is increasing in R. This argument justifies why the function of the right panel of the figure is increasing. Intuitively, reserves have more value for a government seeking credit market access than for a government staying in permanent autarky.

Table 6: Effects of international reserves on haircut (dependent variable: haircut in % of overdue debt)

Haircut	OLS Data	OLS Model
L.reserves to gdp (%)	-1.6**	-1.4**
L.debt to gdp (%)	0.31**	0.98**
L.rgdp growth rate (%)	0.46	0.14
Constant	32.84	24.52
Observations	51	51
R^2	0.23	0.49

The shapes depicted in figure 8 are consistent with the evidence presented in section 2, figure 4. To better assess how consistent are the debt recovery rate schedules with the data, the same statistics presented in table 3 are constructed using using data simulated from the model. The results are presented in table 6 where, for convenience, part of table 3 is replicated. Overall the model performs well at capturing the correlations found in the data, that is, each additional percentage point of lagged reserves to GDP is associated with an average fall in the haircut of 1.4pp in the model, while the data generates 1.6pp fall.

Figure 9 shows the debt price schedules for a country with high and low endowment (left panel) and high and low reserves (right and left panel). The plot in the left shows that prices are decreasing with debt levels as previously noted in the literature. More debt is associated with a higher default probability and lower recovery rate, which translates into

higher spreads. Note that the debt price also increases with productivity, thus explaining why the correlation spread/output in table 5 is negative. When a government receives a bad shock realization in productivity, an increase in the net debt increases the country's ex ante default incentive. A higher default probability and a lower debt recovery rate generate a higher sovereign debt spread, and thus a negative correlation exists between spreads and output. Additionally both panels shows also that more reserves are associated with larger debt prices. Keeping the debt level constant, larger reserves implies lower net debt, thus decreasing default probability and, as explained above, a higher debt recovery rate. Both effects implies that reserves are positively correlated with spreads, also consistent with regression evidence presented in table 2 from section 2.

Figure 9: Price schedule for new debt as a function of current debt and productivity
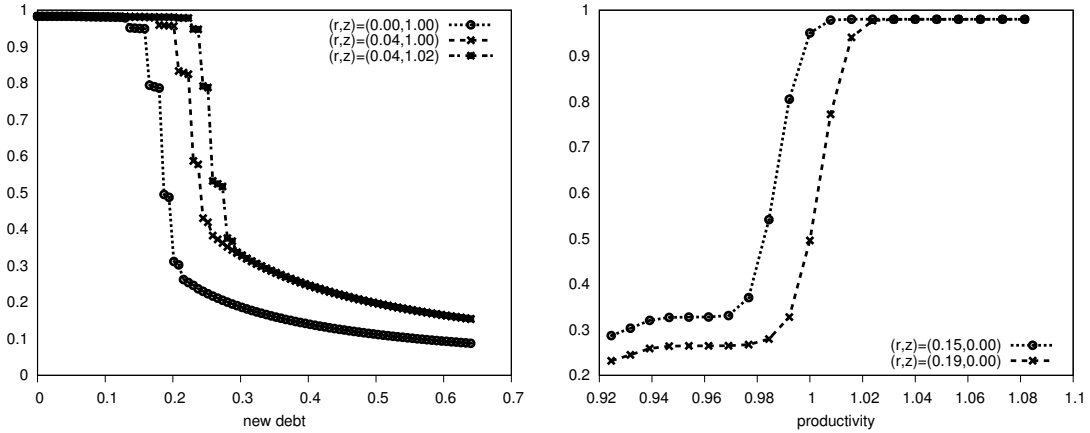
4.6   Sensitive Analysis

The model presented in section 3 adds many non-standard features to the more traditional models of sovereign default. To try to understand how these features change the results regarding the simultaneous holdings of debt and reserves, the baseline model is recomputed shutting down or changing some features to highlight the mechanisms that are driving the results. In this exercise, the following features are changed:

- Assume that government expenditure are zero: $G = 0$;

- Assume that there's lump sum tax policies instead of distortionary: equation (2) becomes

$$c_t = w_t h_t + \pi_t + T_t$$

where T_t are lump-sum taxes;

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- Assume a constant relative risk aversion utility function (CRRA): equation (18) becomes

$$u(c, h) = \frac{c^{1-\sigma}}{1 - \sigma} - \Gamma \cdot \frac{h^{1+\gamma}}{1 + \gamma};$$

- Assume there's no sudden stops in the economy: $\omega = 0$;

- Assume borrowers have all bargaining power: $\gamma = 1$ thus $\tilde{\varphi}(D, R, z) = 0 \ \forall D, R, z$, that is, renegotiation is shutdown.

To maintain some comparability between the baseline and the alternative models, only the volatility of output is targeted by re-calibrating the standard deviation of the innovation of the productivity process σ_z. The computed alternative model is simulated to generate all moments of table 5. However, because the main interest is on the impact of debt and reserve holdings, only the simulated distributions of the two variables are presented here[22] by showing the kernel density estimates of debt and reserves for a bandwidth choice varying between 0.01 and 0.03. Figure 10 shows the results where the panels at the left represent the distribution of external debt, while the ones at the right of external reserves; and the solid lines are for the baseline distributions, while the dashed lines for the alternative distributions.

The first 4 panels of the picture show very similar results: reducing the level of fiscal distortions in the model implies an higher level of debt to GDP, but reserves become less important for the sovereign. Reserves play an important role in the baseline economy as fiscal adjustments are costly and affect production. Eliminating such distortions reduces the need for reserves to 3-5%. The next 2 panels study the impact of using a CRRA utility function instead of the GHH function adopted in the baseline model. Qualitatively, this alternative doesn't change much, however, using a CRRA utility function decreases reserves to about 5% of output. This can be explained by the presence of wealth effects that dampens the fluctuations in output generated by productivity shocks. That is, with this utility function labor is not necessarily co-moving with output and this extra degree of adjustment reduces demand for reserves.

The panels relative to sudden stops show that the presence of such shock is not essential to generate realistic demand for reserves. As long as the probability of default remains positive, the important movements of fiscal consolidation and debt restructuring requires some kind of insurance that can be provided by reserves. Finally the last 2 panel show the most striking results from this exercise: shutting down renegotiation reduces abruptly the levels of debt and reserve holding. Under this specification, the model resembles Alfaro and Kanczuk (2009) in the sense that reserves plays no role in the optimal decision of the

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[22]Complete results for the simulated moments can be found in the appendix A.4.

sovereign.

Figure 10: Distribution approximations of debt and reserves for different model specifications
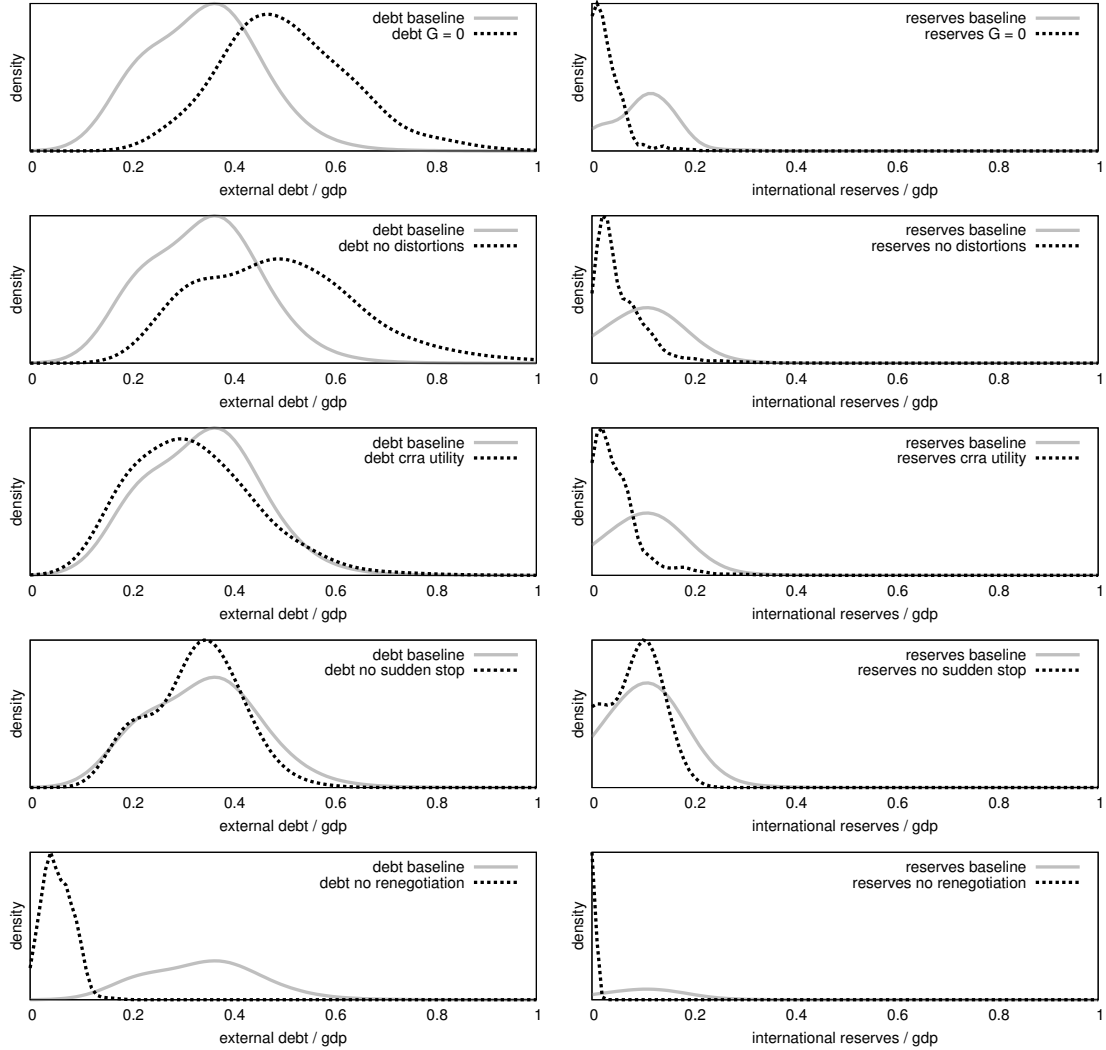
This result confirms the findings in the previous sections that showed that a substantial part of the cost associated with a default, resides on debt restructuring. Table 7 summarizes the findings from the exercise.

Table 7: Simulation results to alternative specification of the model

mean	Baseline	$G = 0$	No Distortions	CRRA	No Sudden Stop	No Renegotiation
D/Y	34%	51%	49%	32%	33%	5.4%
R/Y	10%	3%	5%	5%	7%	0.3%

To conclude this section, it is worth mention that part of the simulation results may be unrelated with the model solution, but to deficiencies in the numerical computation of the

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model. A visual inspection of figure 7 shows that the policy functions display certain "lumpy" areas that present due to issues in the numerical precision of the computation. Hatchondo, Martinez, and Sapriza (2010) studied the quantitative properties of models of sovereign default and conclude that precision is quite relevant for models that feature high sensitivity of the bond price to different bond levels. They advise researchers to either use a very fine grid point for the endowment process or to make the problem continuous by interpolating the value functions. Neither of those two approaches were used in this paper as the current model is already harder to compute than most common models: reserves, being an extra state variable, expands dramatically the problem's state space; each allocation requires the solution of a non-linear equation for the labor market; high order interpolation is not easily implementable as the problem has some defined regions of the state space whose value is negative infinite, thus, attempts to interpolate such functions leads to the so called Runge's phenomena. Future work will attempt to fix these computational issues.

5 Conclusion

This paper studied the extent to which reserves can have an important role as insurance instruments in a simple model that delivers realistic prediction regarding other dimensions of the data. Holding positive amounts of reserves is costly as current consumption is reduced when interest rate spreads are positive. However, reserves can be beneficial if the economy experiences a financial crisis that forces consumption to adjust abruptly.

A model of equilibrium default augmented with reserve accumulation and sudden stop shocks is computed and simulated in order to evaluate if the benefits of holding reserves can compensate the costs. The model is calibrated to generate simulated moments that are similar to the ones observed for the Mexican economy. A positive and non-trivial average reserve to output can be generated while keeping quantitative realistic moments in dimensions such as debt to output, pro-cyclical trade balance, and co-movements of consumption and output. Reserves play an insurance role by allowing consumption to be smoothed if the economy defaults or is hit by a sudden stop shock. Default carries substantial adjustments when the government raises taxes in order to accommodate inelastic spendings and transfers of restructured debt back to lenders. These large penalties of default turn out to be fundamental in generating realistic demand for reserves: without distortionary taxation the demand for reserves drops from 10% of GDP to 5%, and without renegotiation they fall further to close to zero. Moreover, endogenous renegotiation generates a debt recovery rate that is increasing with reserves and, in turn, affects the government's ex-ante incentive to default. In equilibrium, debt is priced to taking into account the risk of default and restruc-

turing. As such, the model predicts that interest rates and haircuts decrease with reserves, as observed in the data.

It should be noted that the results obtained abstain from many factors that, most likely, also influence the demand for reserves. Notably, this paper abstracts from the typical exchange rate management explanations (Obstfeld, Shambaugh, and Taylor, 2010). However, recent developments in the literature have started to interact exchange-rate policy with optimal default (Na, Schmitt-Grohe, Uribe, and Yue, 2014), thus providing an analytical framework to integrate international reserves in the analysis. This is a matter to be studied in future research.

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A   Appendix

A.1   Data Sources

The evidence present in section 2 uses the following variable:

Debt to GDP: from the *World Development Indicators* as the sum of long-term external debt stocks, public and publicly guaranteed in current US\$ (DT.DOD.DPPG.CD) with short-term external debt stocks in current US\$ (DT.DOD.DSTC.CD) divided by GDP in current US\$ (NY.GDP.MKTP.CD) at annual frequency.

Reserves to GDP: from the *World Development Indicators* as total reserves (includes gold) incurrent US\$ (FI.RES.TOTL.CD) divided by GDP in current US\$ (NY.GDP.MKTP.CD) at annual frequency.

Spreads: computed as an arithmetic, market-capitalization-weighted average of bond spreads over US treasury bonds of comparable duration from data collected in *J.P.Morgan's Emerging Markets Bond Index Plus (EMBI+)* at monthly frequency.

RGDP growth rate: from the *World Development Indicators* as the annual percentage growth rate of GDP at market prices based on constant local currency (NY.GDP.MKTP.KD.ZG).

Revenues to GDP: from the *IMF World Economic Outlook* as the general government revenue as a percent of GDP (GGR_NGDP) at annual frequency.

Expenditures to GDP: from the *IMF World Economic Outlook* as the general government total expenditure as a percent of GDP (GGX_NGDP) at annual frequency.

Inflation: from the *IMF World Economic Outlook* as the annual percentages of average consumer prices on year-on-year changes (PCPIPCH).

Current account to GDP: from the *IMF World Economic Outlook* as the annual current account balance as a percent of GDP (BCA_NGDPD).

Openness: from the *World Development Indicators* as the sum of exports of goods and services as % of GDP (NE.EXP.GNFS.ZS) with imports of goods and services as % of GDP (NE.IMP.GNFS.ZS) at annual frequency.

Contagion: constructed as the average of the *EMBI+* spreads across all countries, excluding the country of the observation, in a specific region (Latin America, Africa, Europe, Asia).

Rule of law: index from constructed by the *World Bank, Worldwide Governance Indicators* measuring perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, the police and the courts, as well as the likelihood of crime and violence.

Urban population: from the *World Development Indicators* as the urban population a percentage of total population (SP.URB.TOTL.IN.ZS).

Haircut: estimates taken from Benjamin and Wright (2009) on per dollar creditor losses on historical sovereign default episodes.

Section 4 uses data from Mexico to construct business cycle statistics. Data is taken, at quarterly frequency, from Neumeyer and Perri (2005) for 1981 to 2001 and from *OECD* for 2002 to 2012. Debt and reserves uses instead data from the *World Development Indicators* at

an annual frequency. Moments are computed using de-trended series using an HP-filter with a 1600 parameter. Consumption and income are expressed in logs. The default frequency is taken from historical Mexican defaults identified by Reinhart and Rogoff (2009) in the last 2 centuries. Data for tax rates uses *OECD Revenue Statistics - Comparative tables* following Mendoza, Razin, and Tesar (1994) methodology to compute effective average tax rates on labor. In particular, the following variables are used:

GOS: Gross operating surplus and gross mixed income from *OECD National Accounts*

W: Compensation of employees from *OECD National Accounts*

1100: Income, profit and capital gains taxes of individuals from *OECD Revenue Statistics*

2000: Social security contributions from *OECD Revenue Statistics*

2200: Social security contributions of employers from *OECD Revenue Statistics*

3000: Payroll taxes from *OECD Revenue Statistics*

Using these variables, the two effective average tax rates are given by:

$$
\begin{aligned}
\tau^h &= \frac{1100}{GOS + W} \\
\tau^l &= \frac{\tau^h \cdot W + 2000 + 3000}{W + 2200}
\end{aligned}
$$

Where τ^h is the personal income tax and τ^l is the labor income tax, used to compute the correlation with output.

A.2   Two Period Model of Debt and Reserves

To better understand the intuition of the results presented in the main text a simple two-period model is introduced. Due to its designed simplicity, this model isn't adequate to derive quantitative conclusions about the dynamics of the sovereign incentives to repay debt. However, different aspects of the role of reserves in a model allowing for debt default can be studied sequentially by assuming environments with different degrees of market completeness or commitment to repay debt.

The model environment involves an agent that lives for two periods and that is endowed with a certain income y_1 in the first period and an uncertain income y_2 in the second period that can only take two outcomes $y^h > y^l$. Assume that p is the probability of receiving y^h.

Lifetime utility from consuming c_1 and c_2 is given by:

$$W = c_1 + \beta Eu(c_2) = c_1 + \beta p \cdot u(c_2^h) + \beta(1-p) \cdot u(c_2^l) \tag{20}$$

where $u(c)$ is an utility function with $u' > 0$ and $u'' < 0$, and β is the discount coefficient. Note that the agent is risk neutral in the first period while risk averse in the second. This choice intends to capture, with a very simple specification, two distinct forces of sovereign default models: agents wish to bring consumption from the future into the present - captured by the coefficient β; agents wish to to smooth consumption emerging from uncertainty - captured by the uncertain endowment and the second period utility function.

Complete Markets

For the complete markets case, it is assumed that the agent can issue debt contracts with a risk-neutral external investor. Moreover, the agent can commit to repay his debts to the external investor. Under this assumptions, the agent budget constraint becomes:

$$c_1 = y_1 + q^h D^h + q^l D^l \tag{21}$$

$$c_2^i = y^i - D^i, \ \ i = h, l \tag{22}$$

where q^h and q^l are prices for debt contracts if, respectively, the high or low income state of the world realize in period 2.

The typical solution for the maximization of (20), subject to (21) and (22) by choosing $\{D^h, D^l\}$ is given by:

$$q^h = \beta p u'(c_2^h)$$

$$q^l = \beta(1-p)u'(c_2^l)$$

And, if the borrower is small compared to the external investor, then:

$$\bar{q} = \beta u'(c_2) \tag{23}$$

where $\bar{q} = q^h + q^l$ is the risk free debt price (or equivalently the external investor discount rate). Equations (23), (21) and (22) characterize the optimal allocation $\{c_1^*, c_2^*\}$ as function of $\beta, \bar{q}, y^h, y^l, p$. Note that under complete markets consumption in the second period is certain even when income isn't. This occurs because debt contracts are contingent.

Incomplete Markets with Full Commitment

Suppose now that the agent only has access to non contingent debt contracts. His be-comes:

$$c_1 = y_1 + \bar{q}D \tag{24}$$

$$c_2^i = y^i - D, \quad i = h, l \tag{25}$$

Note that now debt D has to be re-payed whether the agent receives a high or low endowment. First order condition now implies

$$\bar{q} = \beta E u'(c_2^i) = \beta \left[p u'(c_2^h) + (1-p) u'(c_2^l) \right] \tag{26}$$

The solution of equations (24), (25) and (26), gives now an optimal allocation $\{c_1^*, c_2^{h*}, c_2^{l*}\}$ where $c_2^{h*} > c_2^{l*}$. That is, under incomplete markets the agent is not able to smooth con-sumption in the second period.

Incomplete Markets without Full Commitment

With an option to repudiate debt, an agent with debt D decides to default in period 2 if the associated consumption is larger than what he would under repayment. Assuming that the losses of default imply an income given by:

$$\tilde{y}(y) = \begin{cases} y^{d,h} & if \ y = y^h \\ y^{d,l} & otherwise \end{cases}$$

Then, for some level of debt D:

$$c^{def} > c^{rep} \Rightarrow c^{h,rep} = y^h - D > y^{d,h} = c^{h,rep} \tag{27}$$

$$c^{l,rep} = y^l - D < y^{d,l} = c^{l,rep} \tag{28}$$

This default choice implies that the price one unit of un-contingent debt is $q = \bar{q}p$, so that the consumer budget constraint becomes:

$$c_1 = y_1 + qD \tag{29}$$

$$c_2^h = y^h - D \tag{30}$$

$$c_2^l = y^{d,l} \tag{31}$$

The first order condition and some algebra implies that $\bar{q} = \beta u'(c_2^h)$, that together with (29), (30) and (31) implies an allocation of consumption $\{c_1^*, c_2^{h*}, c_2^{l*}\}$ where $c_2^{h*} > c_2^{l*}$. Similarly to the previous case, is not necessary that consumption can be completely smoothed in the second period, irrespectively of the contingency that default allows.

Incomplete Markets with Reserves and without Full Commitment

Now the agent sees his contract choice expanded by reserves. The budget constraint is now given by:

$$c_1 = y_1 + qD - \bar{q}R$$
$$c_2^h = y^h + R - D$$
$$c_2^l = y^{d,l} + R$$

And the first order conditions by:

$$D: \quad q = \beta p u'(c_2^h) \Leftrightarrow \bar{q} = \beta u'(c_2^h) \tag{32}$$
$$R: \quad \bar{q} = \beta \left[p u'(c_2^h) + (1-p)u'(c_2^l) \right] \Rightarrow u'(c_2^h) = u'(c_2^l) \tag{33}$$

Implying an optimal allocation $\{c_1^*, c_2^*\}$. This result can be summarized in the following claim.

Claim 1. (consumption smoothing) Under incomplete markets with reserves and without full commitment, $c_2^h = c_2^l = c_2$ is a solution for a given $\tilde{y}(y)$ and β.

Proof. For a given D choose $\tilde{y}(y)$ such that (27) and (28) hold. Given (32), such D can be supported for a specific β. The rest of the proof follows directly from the first order conditions. $\qquad\square$

This result shows that reserves and the ability to default allows the agent to perfectly smooth consumption in the second period. In this sense, reserves are useful to complete the market.

Claim 2. (demand for reserves and loss of default) Under incomplete markets with reserves and without full commitment $\partial R / \partial y^{l,d} < 0$ for a given $\tilde{y}(y)$ and β

Proof. This result follows from applying the implicit function theorem to (33) and claim 1:

$$\bar{q} = \beta \left[pu'(c_2^h) + (1-p)u'(c_2^l) \right]$$
$$\Leftrightarrow \quad \bar{q} = \beta \left[\bar{q}/\beta + (1-p)u'(y^{d,l} + R) \right]$$
$$\Rightarrow \quad \frac{\partial R}{\partial y^{d,l}} = -1 < 0$$

$\square$

The result from claim 2 says that the demand for reserves is increasing with the output loss. This gives the intuition why the full model adds counter-cyclical distortionary taxation: increasing taxes when the economy is in default further depresses the economy thus increasing the usefulness of reserves in such events.

Reserves and Renegotiation

Consider that if the defaults in the second period, lenders make a one-shot offer of debt restructuring by granting market access to the agent. That option grants the following utility to the lender:

$$W^d = u(c_2) + A$$

where A is meant to capture in this simple framework all future benefits of regaining access to international credit markets. The offer made an offer of a debt restructuring $\tilde{D}$ such that the agent is indifferent in accepting and rejecting the offer, that is:

$$u(y + R) = u(y + R - \tilde{D}) + A$$

That equation implicitly defines a recovery schedule $\tilde{D}(y, R, A) \leq D$ where D is the defaulted debt. The following result follows:

Claim 3. (recovery rate is increasing with reserves) $\partial \left(\tilde{D}/D \right) /\partial R \geq 0$

Proof. This follows immediately from the fact that $u' < 0$ and $y + R \geq y + R - \tilde{D}$. $\square$

This result indicates that an agent is willing to transfer larger amounts of reserves to lenders if there are gains of regaining market access.

A.3   Numerical computation

The computational algorithm is coded in Fortran and is similar to the one used in Arellano (2008) or Yue (2010). For error tolerance ϵ_v and ϵ_φ, the algorithm steps follow the below sequence:

1. Discretize space of D, R and the shock process z

2. Solve first for the value of permanent autarky $v^{aut}(R, z)$ using value function iteration:

   (a) guess $v^{aut,0}(R, z)$

   (b) using $v^{aut,0}(R, z)$ solve for the maximization (14) to get $v^{aut,1}(R, z)$, using a non-linear equation solver to determine the labor equilibrium in (15) and (16) and a grid search method over the space of R

   (c) evaluate $||v^{aut,1}(R, z) - v^{aut,0}(R, z)||$; if it's larger than ϵ_v iterate on (a) using $v^{aut,0}(R, z) := v^{aut,1}(R, z)$ until converge

3. Guess the the recovery function $\varphi^0(D, R, z)$, and the value function $v^{rep,0}(D, R, z, s)$

4. Use $\varphi^0(D, R, z)$ to update the value function to $v^{rep,1}(D, R, z, s)$ using a grid search method over the space of (D, R):

   (a) labor equilibrium in (11) and (12) is obtained using a non-linear equation solver

   (b) $v^{def}(D, R, z)$ by maximizing (13) using $\varphi^0(D, R, z)$ and $v^{rep,0}(D, R, z, s)$

   (c) debt price function is computed from the definition (9) using the functions $\varphi^0(D, R, z)$, $v^{rep,0}(D, R, z, s)$ and $v^{def}(D, R, z)$

5. Evaluate $||v^{rep,1}(D, R, z, s) - v^{rep,0}(D, R, z, s)||$; if it's larger than ϵ_v iterate on (4) using $v^{rep,0}(D, R, z, s) := v^{rep,1}(D, R, z, s)$ until converge

6. Using $v^{rep,1}(D, R, z, s)$, compute $\varphi^1(D, R, z)$ by solving (17) using a grid search method over the space of D

7. Evaluate $|\varphi^1(D, R, z) - \varphi^0(D, R, z)|$; if it's larger than ϵ_v iterate on (3) using $\varphi^0(D, R, z) := \varphi^1(D, R, z)$ until converge.

In the benchmark model, the state space of D and R is $[0; 0.64]$ with 90 equally spaced grid-points, that is, both debt and reserves share the same state space. The productivity process $\log z$ is discretized with 21 grid points using the method proposed in Tauchen (1986) with bounds given by a margin of 3.5 unconditional standard deviations with respect to the mean. The maximum error allowed is $\epsilon_\nu = 10^{-6}$. The non-linear equation solver to determine labor equilibrium uses Brent's method.

A.4 Simulation results for alternative specifications of the model

This section shows the simulation results from using alternative specifications of the model using the parameter values from table 4.

	Data	Baseline	$G = 0$	No Dist	CRRA	No SS	No Ren
$mean(D/Y)$	32	34	51	49	32	33	5
$stdev(TB/Y)$	1.4	1.3	2.6	2.8	1.8	1.3	0.9
$stdev(y)$	2.6	2.6	2.6	2.6	2.6	2.6	2.6
$mean(D^{recover}/D)$	58	55	50	56	56	53	0
$mean(G/Y)$	20	20	0	20	20	20	20
$mean(R/Y)$	9	10	3	5	5	7	0
$mean(i^{spread})$	3.0	1.4	2.4	2.5	2.7	0.4	7.7
$stdev(i^{spread})$	1.5	1.3	2.4	2.9	2.9	1	16.8
$stdev(c)/stdev(y)$	1.2	1.4	1.9	1.7	1.5	1.4	1.2
$corr(y, c)$	0.94	0.96	0.92	0.83	0.91	0.97	0.97
$corr(y, i^{spread})$	-0.56	-0.26	-0.45	-0.36	-0.42	-0.37	-0.49
$corr(TB/Y, y)$	-0.66	-0.66	-0.69	-0.4	-0.46	-0.69	-0.56
$corr(\tau, y)$	-0.49	-0.81	-0.69	-	0.76	-0.82	-0.78
$default\, rate$	2.0	2.8	4.1	4.8	5.2	0.8	8.2