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Quantum Nonlocality as an Axiom

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In the conventional approach to quantum mechanics, indeterminism is an axiom and nonlocality is a theorem. We consider inverting the logical order, making nonlocality an axiom and indeterminism a theorem. Nonlocal "superquantum" correlations, preserving relativistic causality, can violate the CHSH inequality more strongly than any quantum correlations.

What is the quantum principle? J. Wheeler named it the "Merlin principle" after the legendary magician who, when pursued, could change his form again and again. The more we pursue the quantum principle, the more it changes: from discreteness, to indeterminism, to sums over paths, to many worlds, and so on. By comparison, the relativity principle is easy to grasp. Relativity theory and quantum theory underlie all of physics, but we do not always know how to reconcile them. Here, we take nonlocality as the quantum principle, and we ask what nonlocality and relativistic causality together imply. It is a pleasure to dedicate this paper to Professor Fritz Rohrlich, who has contributed much to the juncture of quantum theory and relativity theory, including its most spectacular success, quantum electrodynamics, and who has written both on quantum paradoxes(1) and the logical structure of physical theory.(2)

Bell(3) proved that some predictions of quantum mechanics cannot be reproduced by any theory of local physical variables. Although Bell worked within nonrelativistic quantum theory, the definition of local variable is relativistic: a local variable can be influenced only by events in its backward light cone, not by events outside, and can influence events in its

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forward light cone only. Quantum mechanics, which does not allow us to transmit signals faster than light, preserves relativistic causality. But quantum mechanics does not always allow us to consider distant systems as separate, as Einstein assumed. The failure of Einstein separability violates, not the letter, but the spirit of special relativity, and left many physicists (including Bell) deeply unsettled. The quantum nonlocality manifest in the Aharonov–Bohm effect[4] also took many physicists by surprise. The trajectory of an electron through a region free of magnetic flux depends upon flux *outside* the region—something incomprehensible to a classical physicist. Today, quantum nonlocality seems as fundamental as ever. All fundamental interactions we know are gauge interactions with the AB effect (or its non-Abelian analogue) at their core. In addition, the nonlocal quantum correlations displayed by Bell are now known to be generic: *any* entangled quantum state of any number of systems yields nonlocal correlations.[5] Only product states do not.

Nonlocality, then, is an essential feature of quantum theory, but it often appears in a negative light. Here, we propose to show quantum nonlocality in a more positive light. What new possibilities does quantum nonlocality offer us? In particular, if we make nonlocality an axiom, what becomes of the logical structure of quantum theory? The special theory of relativity, we know, can be deduced in its entirety from two axioms: the equivalence of inertial reference frames, and the constancy of the speed of light. Aharonov[6] has proposed such a logical structure for quantum theory. Suppose we take, as axioms of quantum theory, relativistic causality and nonlocality. As an initial, immediate result, we deduce that quantum theory is not completely deterministic: otherwise these two axioms would be incompatible.[6] Then a "negative" aspect of quantum mechanics—indeterminacy and limits on measurements—appears as a consequence of a fundamental "positive" aspect: the possibility of nonlocal action.

Before proceeding, we must formulate our two axioms precisely. Relativistic causality is well defined, but quantum nonlocality appears in both nonlocal correlations and the AB effect. While these two effects are related,[7] they are not equivalent. In the AB effect, an isolated magnetic flux, inserted between two slits, shifts the interference pattern of electrons passing through the slits. Aharonov has shown[6] that a physical quantity, the *modular momentum* of the flux,[8] is indeterminate by exactly the amount required to keep us from seeing the nonlocal force. But otherwise, modular momentum is measurable. The approach of Aharonov[6] starts from quantum mechanics itself and emphasizes nonlocal *equations of motion*, such as apply to modular variables. In the present work we take a different approach, which does not start from quantum mechanics, and does not address the nonlocality of the AB effect directly. We discuss nonlocality with reference to nonlocal correlations, and without considering equations of motion. We apply our two axioms simply by asking which theories give rise to nonlocal correlations, in the sense of Bell's theorem,[3] while preserving causality. We find that quantum mechanics is only one of a class of theories consistent with our two axioms, and, in a certain sense, not even the most nonlocal theory. Thus, our result is completely independent of quantum mechanics or any particular model. It is more difficult to abstract the nonlocality considered by Aharonov, which refers to equations of motion of a given theory. The fact that we have chosen to base the axiom of nonlocality on nonlocal correlations does not imply that we consider them to be the essence of nonlocality. On the contrary, we suspect that the nonlocality of the AB effect and modular variables has deeper physical significance, just because it is connected to dynamics. However, in this paper we set ourselves an easier task.

We begin with the Clauser, Horne, Shimony, and Holt (CHSH) inequality,[9] a convenient form of Bell's inequality. The CHSH inequality concerns systems with two parts far from one another. Let A, A', B, and B' be physical variables taking values 1 and -1, with A and A' referring to measurements on one part of the system by a local observer, and B and B' referring to measurements on the other part. If $P_{AB}(a, b)$ denotes the joint probability of obtaining $A=a$ and $B=b$ when A and B are measured, the correlation $E(A, B)$ of A and B is defined as

$$E(A, B)=P_{AB}(1, 1)+P_{AB}(-1, -1)-P_{AB}(1, -1)-P_{AB}(-1, 1) \quad (1)$$

The CHSH inequality holds in any classical theory (that is, any theory of local hidden variables), and states that a particular combination of correlations lies between -2 and 2:

$$-2 \leqslant E(A, B)+E(A, B')+E(A', B)-E(A', B') \leqslant 2 \quad (2)$$

For a system in a pure state $|\psi\rangle$, quantum mechanics predicts the correlation of A and B as

$$E_Q(A, B)=\langle\psi| AB |\psi\rangle \quad (3)$$

where A and B correspond to self-adjoint operators. For certain choices of A, A', B, B', and $|\psi\rangle$, quantum correlations violate the CHSH inequality. Besides 2, two other numbers, $2\sqrt{2}$ and 4, are important bounds on the CHSH sum of correlations. If the four correlations in Eq. (2) were independent, the absolute value of the sum could be as much as 4. The sum of the quantum correlations, however, is less than 4: for quantum correlations, the CHSH sum of correlations is bounded[10] in absolute value by $2\sqrt{2}$. Where does this bound come from? It derives from the Hilbert space structure of quantum mechanics, but what does it mean? Rather than ask why quantum correlations violate the CHSH inequality, we might ask why they do not violate it *more*.

If we return to the two axioms proposed above, we may say that the axiom of nonlocality implies that quantum correlations should violate the CHSH inequality at least sometimes. It is natural to guess that the other axiom, relativistic causality, might imply that quantum correlations do not violate it maximally. Could it be that relativistic causality restricts the violation to $2\sqrt{2}$ instead of 4? If so, then the two axioms of nonlocality and relativistic causality determine the quantum violation of the CHSH inequality. To answer our question, we consider what restrictions relativistic causality imposes on joint probabilities. Relativistic causality forbids sending messages faster than light. Thus, if one observer measures the observable A, the probability for the outcomes $A=1$ and $A=-1$ must be independent of whether the other observer chooses to measure B or B'. The probability for the first observer to obtain $A=1$ when the second observer has measured B is

$$P_{AB}(1, ?) = P_{AB}(1, 1) + P_{AB}(1, -1) \tag{4}$$

$P_{AB}(1, ?)$ is a sum over the outcomes of the measurement of B, since the result for B is unknown. Relativistic causality then requires

$$P_{AB}(1, ?) = P_{AB'}(1, ?) \tag{5}$$

where B' is completely arbitrary. Analogous equations hold for $P_{AB}(-1, ?)$, $P_{A'B}(1, ?)$, and so forth. Note that relativistic causality does not imply, for example, that

$$P_{AB}(1, 1) = P_{AB'}(1, 1) \tag{6}$$

Joint probabilities can be different and still preserve relativistic causality. Since an observer can learn about joint probabilities only from communication with the other observer—by assumption, at subluminal speeds—there is no conflict with relativistic causality.

Do constraints of the form of Eq. (5) restrict the CHSH sum of correlations to be less than or equal to $2\sqrt{2}$, as in quantum mechanics? Actually, they do not. Suppose we have a set of joint probabilities

$$\begin{aligned} P_{AB}(1, 1) &= P_{AB}(-1, -1) = P_{AB'}(1, 1) = P_{AB'}(-1, -1) \\ &= P_{A'B}(1, 1) = P_{A'B}(-1, -1) = P_{A'B'}(1, -1) \\ &= P_{A'B'}(-1, 1) = 1/2 \end{aligned} \tag{7}$$

with all other joint probabilities equal to zero. Then the probability for either result of any local measurement equals 1/2, and it clearly obeys all constraints of the form of Eq. (5). However, the CHSH sum of correlations corresponding to these joint probabilities is 4. Thus, relativistic causality does not by itself constrain the maximum CHSH sum of quantum correlations to $2\sqrt{2}$.

As a concrete illustration, consider two spinors in a "superquantum" singlet (rotationally symmetric) state. We define a superquantum correlation function E for measurements of their spins along given axes. The correlation function depends only on the relative angle θ between axes. Rotational symmetry implies that the probabilities for results of a measurement on one particle are independent of a measurement on the other. Then for any pair of axes, the outcomes $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ must be equally likely, and similarly for $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$; since these four probabilities sum to 1, the probabilities for $|\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\rangle$ sum to 1/2. In any direction, the probability of $|\uparrow\rangle$ or $|\downarrow\rangle$ is 1/2 irrespective of a measurement on the other particle. Since measurements on one particle yield no information about measurements on the other, relativistic causality holds.

The correlation function then satisfies $E(\pi-\theta) = -E(\theta)$. Now let $E(\theta)$ have the following form (see Fig. 1):

(i) $E(\theta) = 1$ for $0 \leqslant \theta \leqslant \pi/6$;

(ii) $E(\theta)$ decreases linearly from 1 to -1 as θ increases from $\pi/6$ to $\pi/4$;

(iii) $E(\theta) = -1$ for $\pi/4 \leqslant \theta \leqslant \pi/3$;

(iv) $E(\theta)$ increases linearly from -1 to 1 as θ increases for $\pi/3$ to $2\pi/3$;

(v) $E(\theta) = 1$ for $2\pi/3 \leqslant \theta \leqslant 3\pi/4$;

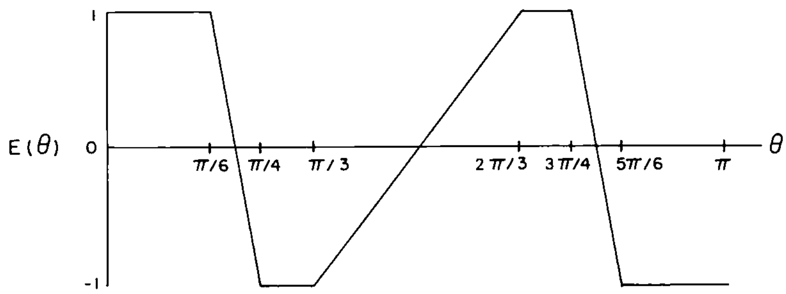

Fig. 1. The "superquantum" correlation function $E(\theta)$.

(vi) $E(\theta)$ decreases linearly from 1 to -1 as θ increases from $3\pi/4$ to $5\pi/6$;

(vii) $E(\theta) = -1$ for $5\pi/6 \leqslant \theta \leqslant \pi$.

Consider four measurements along axes defined by unit vectors $\hat{a}'$, $\hat{b}$, $\hat{a}$, and $\hat{b}'$ separated by successive angles of $\pi/12$ and lying in a plane. If we now apply the CHSH inequality Eq. (2) to these directions, we find that the sum of correlations

$$E(\hat{a}, \hat{b}) + E(\hat{a}', \hat{b}) + E(\hat{a}, \hat{b}') - E(\hat{a}', \hat{b}') = 3E(\pi/12) - E(\pi/4) = 4 \quad (8)$$

violates the CHSH inequality with the maximal value 4. Of course, the correlation function $E(\theta)$ is contrived, but it illustrates how a correlation function could satisfy relativistic causality and still violate the CHSH inequality with the maximal value 4.

The results of tests of violations of the CHSH inequality[11] are consistent with quantum predictions, but stronger violations have not been ruled out. Our analysis shows that stronger violations would not conflict with relativity theory. We emphasize that an experiment could test for such violations, which would disprove quantum mechanics without reference to any model.

What do we conclude from this analysis? We hope to find in quantum theory the logical simplicity that characterizes the special theory of relativity, which derives in its entirety from two axioms. Following Aharonov, we have proposed two axioms for quantum theory, nonlocality and relativistic causality, which together imply quantum indeterminacy. From our brief exercise with nonlocal correlations, however, we learn that our two axioms do not determine quantum theory: a theory that allows nonlocal correlations but preserves relativistic causality might not be quantum mechanics but a "superquantum" mechanics. Thus, we have identified a class of theories, to which quantum mechanics belongs, that yield nonlocal correlations while preserving causality. Perhaps quantum mechanics is not a correct theory. But if quantum mechanics is what we wish to derive, our two axioms are not enough. Still, our two axioms refer only to correlations, and not to the nonlocality of the AB effect. The conjecture of Aharonov[6] states that quantum mechanics is the only causal theory with nonlocal *equations of motion.* To test this conjecture we must identify the class of theories that are causal and obey a different axiom of nonlocality. We hope then to find a logically simple quantum theory.

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NOTE

After this work was completed, we learned that the result for the maximal violation of the CHSH inequality consistent with relativity was found also by B. Tsirelson (Cirel'son).[12]

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