Solomon's Sea and π

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Andrew Simoson (ajsimoso@king.edu) received a Ph.D. in mathematics from the University of Wyoming in 1979, and is now professor of mathematics at King College in Tennessee. Over the years, he collected, among other things, various whimsical explanations of π being three. The bulging folder led to some fun mathematics talks, and ultimately to this survey article.

Of the beginnings of π, it is often said that the biblical value is 3, for *I Kings 7:23* reads

> Solomon made a molten sea, ten cubits from the one brim to the other: it was round all about, and its height was five cubits: and a line of thirty cubits did compass it round about.

(Unless stated otherwise, Bible references are from the King James version.) This bronze basin is known as Solomon's Sea, and is illustrated in Figure 1. The text seems to say that for a circumference of 30, a circle's diameter is 10, implying that π is 3. To delineate this strand of the lore about π, we present various perspectives on this puzzle, presenting them roughly in order of increasing complexity, although not necessarily in order of increasing credibility.

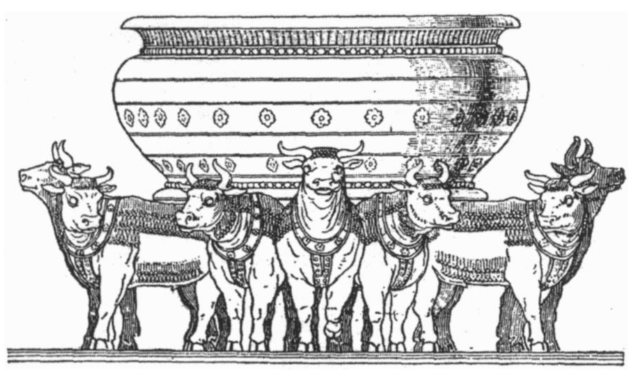


Figure 1. Solomon's Sea [**3**, plate 87].

Noise

We call those who measured Solomon's Sea, surveyors; those who wrote the historical books of the Bible, chroniclers; and those who copied or translated the books, scribes. It is possible that the surveyors measured wrongly, that the chroniclers recorded information imperfectly, or that later scribes transcribed erroneously.

The biblical books of *Kings* and *Chronicles* are parallel texts. Scholars have tried to harmonize the apparent noise of the differing passages, which of course include

Solomon's Sea. Payne [**19**] accounts for textual anomalies as instances of "accidental corruption by a later scribe," either through a "mistaken reading" of word form or through ambiguous, "unclear" numerical expressions; "rounding numbers" and "hyperbolically" inflating numbers so as to make a point; and "different methods of reckoning." As a simple example of this noise, Herzog [**12**] points out that the Greek Septuagint translation renders the Sea's circumference 33 cubits in *Kings*, while rendering it 30 cubits in *II Chronicles 4:2*.

As a more serious example, consider the disagreement on the capacity of Solomon's Sea, where *I Kings 7:26* puts it at 2000 baths (where a bath is somewhere between 4.5 and 10 gallons) while *II Chronicles 4:5* puts it at 3000 baths. Rabbinic scholars of the *Talmud, Erubin 14b*, dating to about 500 AD, explain this difference by rendering the 2000 baths of the *Kings* passage as liquid measure and the 3000 baths of the *Chronicles* passage as dry measure, and say that the dry measure would include a heap above the brim, being one third of the total measure. Wylie [**25**] attributes this difference in volume measurement to a confusion about the Sea's structure: he says that whereas the Sea probably had a hemispherical basin, the chronicler of *Chronicles* assumed that the Sea had a cylindrical basin, and so recorded its corresponding volume. A cylinder circumscribed about a hemisphere (where the two share the same base) has 1.5 times the volume of the hemisphere; since the 3000 baths of *Chronicles* is 1.5 that of the 2000 baths of *Kings*, the two passages are thereby reconciled.

A highly whimsical and more recent example of surveyor noise in the context of calculating π's value comes from Dudley [**8**]. He stumbled across a compilation of mid-nineteenth century approximations for π (taken from apparently sincere attempts at squaring the circle), as shown in Table 1. For a moment, imagine that π's value is linear in time and that the given data is indicative of π's variable value. The line of best fit through the data, $p(t)$, gives the approximate value of π in year t, where t is the Gregorian year:

$$p(t) = 3.1239827671 + 0.0000157082t. \tag{1}$$

Table 1. Novice attempts at the first five decimal digits of π, 1832–1879.

$t: \pi$	$t: \pi$	$t: \pi$	$t: \pi$	$t: \pi$
1832: 3.06250	1845: 3.16667	1855: 3.15532	1865: 3.16049	1874: 3.15208
1833: 3.20222	1846: 3.17480	1858: 3.20000	1866: 3.24000	1874: 3.14270
1833: 3.16483	1848: 3.20000	1859: 3.14159	1868: 3.14214	1874: 3.15300
1835: 3.20000	1848: 3.12500	1860: 3.12500	1868: 3.14159	1875: 3.14270
1836: 3.12500	1849: 3.14159	1860: 3.14241	1869: 3.12500	1875: 3.15333
1837: 3.23077	1850: 3.14159	1862: 3.14159	1871: 3.15470	1876: 3.13397
1841: 3.12019	1851: 3.14286	1862: 3.14214	1871: 3.15544	1878: 3.20000
1843: 3.04862	1853: 3.12381	1862: 3.20000	1872: 3.16667	1878: 3.13514
1844: 3.17778	1854: 3.17124	1863: 3.14063	1873: 3.14286	1879: 3.14286

Dudley makes some amusing extrapolations, such as when π would have value 3.2, or when it had value π, 3, or 0. For example, by (1), π's decimal expansion would be 3 in the year 7893 BC, perhaps the year when Adam and Eve found themselves in the Garden, or when early man first drew the sun and moon on cave walls and discovered that the proportion of circumference to diameter is close to three.

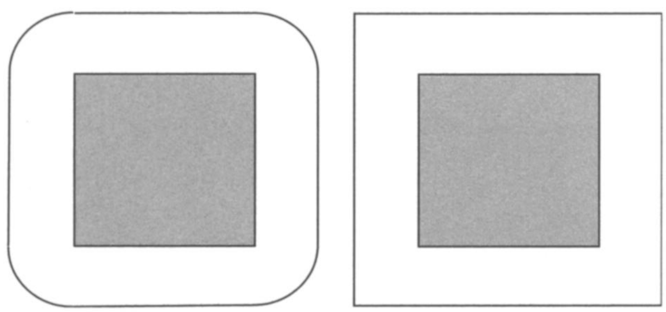
Tradition

The ancients had many different rules for π, some of whose natural interpretations implicitly define π as 3. Castellanos [**5**] and Gupta [**11**] cite various documents demonstrating that the ancient Babylonians, Egyptians, Indians, and Chinese had such a rule. Of course, we focus on the Jewish tradition of π being 3 because the Bible is readily available, and the image of a large bronze basin being the center of some controversy makes a great story. One such Hebrew rule is found in the *Mishnah*, a compilation of Jewish traditions, dating to the second century AD. In *Mishnah, Erubin 1:5,* we read

> Whatsoever is three handbreadths in circumference is one handbreadth in width.

(All *Mishnah* and *Talmud* references are taken from the Soncino Press, 1948 edition of the *Talmud* [**24**]. A reference of the form Xa or Xb is the *Talmud* page X commentary of a specific *Mishnah* book, a for left-hand page, and b for right-hand page.) Zuckermann, a nineteenth century German scholar, points out that the rabbis who compiled the *Mishnah* "were aware of more exact values [of π], but accepted the value of 3 as a workable number for religious purposes" [**10**, p. 23]. By way of illustration of religious purposes, consider the following passage from *Erubin 14b* from the *Talmud*, which is an expansive commentary on the *Mishnah*. The above *Mishnah* rule is given in the following equivalent form:

> But consider: By how much does a square exceed that of a circle? By a quarter.



(a) Round corners (b) Square corners

Figure 2. Levite gardens about the cities of refuge.

This rule is to be interpreted in the following way: Take a square of side length 2 and inscribe a circle within it; the area of this square is 4; removing $\frac{1}{4}$ of this area from 4 leaves 3, the approximate area of the circle and the implicit, practical Talmudic value of π. One of the early applications for this rule, and in fact an application which may have led to the formulation of this rule (see *Erubin 56b–57a*), is the problem described in *Numbers 35:4–5* in the time of Joshua: cities measuring 2000 cubits from north to south and 2000 cubits from east to west, with 1000 cubits outward from the walls roundabout, were to be given to the Levite tribe—were the corners to be round or square (see Figure 2)? The difference in area is worth discussing, at least if you were a Levite.

This same problem was a lively issue in resolving the problem of how far one is allowed to walk on the sabbath; *Erubin 4:8* says that [one could] travel within two thousand cubits in any direction as [though he was within] a circle [while] the Sages say: As [though he was within] a square, so that he wins the benefit of the corners.

Thus, in light of the above examples, even if the surveyors had measured the Sea as a $31\frac{1}{2}$-cubit circumference and a 10-cubit diameter, for example, it is possible that these values may have been adjusted to harmonize with the tradition of π being 3.

It is also possible that this tradition of implicitly identifying 3 with π arose from a practice of rounding to the nearest integer. And therefore, as Meeus [**16**] points out, if the diameter of the Sea lay between 9.5 and 10.5 cubits, and the circumference lay between 29.5 and 30.5 cubits, then the biblical value of π is between the bounds of 2.81 and 3.21, thereby resolving any measurement anomaly. However, *Exodus 37:1* gives the measurements of the ark of the covenant as $2\frac{1}{2}$ by $1\frac{1}{2}$ by $1\frac{1}{2}$ cubits. Adjusting Meeus's argument to round to the nearest half leaves the biblical value of π between 2.90 and 3.10, not nearly so satisfactory.

One shortcoming of these kinds of arguments is that since a great deal of thought and effort went into casting the Sea, one might expect those dimensions to be recorded accurately.

The hidden key

Perhaps a hidden key exists to unlock the meaning of this passage. Posamentier [**20**] relates the story of an 18th century Polish rabbi, Elijah of Vilnah, who observed in the Masoretic text, the Hebrew Bible, that the word "line" in the parallel *Kings* and *Chronicles* texts of this passage are spelled קוה and קו, respectively. (Others attribute this gematria reasoning to Rabbi Matityahu Hakohen Munk [**7**].) The extra ה is the key. How is it used? Take the ratio of the sums of the standard numeric values of the Hebrew letters (ק ≡ 100, ו ≡ 6, ה ≡ 5) for each of these words, obtaining $111/106$; multiply by 3—the apparent value of π—and obtain $\pi \approx 333/106 \approx 3.141509$, a value agreeing with π to four decimal digits. Stern [**22**] comes to the same conclusion independently by examining only the *Kings* passage, observing that the word "line" while written as קוה is pronounced only as קו since ה is silent.

A natural question with respect to this method is, why add, divide, and multiply the letters of the words? Perhaps an even more basic question is, why all the mystery in the first place? Furthermore, H. W. Guggenheimer, in his *Mathematical Reviews* note on [**22**], seriously doubts that the use of letters as numerals predates Alexandrian times; or if such is the case, the chronicler did not know the key. Moreover, even if this remarkable approximation to π is more than coincidence, this explanation does not resolve the obvious measurement discrepancy—the 30-cubit circumference and the 10-cubit diameter.

Finally, Deakin [**7**] points out that if the deity truly is at work in this phenomenon of scripture revealing an accurate approximation of π, a much better fraction not far from $333/106$ would most definitely have been selected instead. How so? The basic idea is from ancient Egypt, where the custom of dealing with a fractional quantity was to write it as the sum of unitary fractions, fractions with numerator 1 and denominator a positive integer. Thus an Egyptian would write $5/6$ as $1/2 + 1/3$. The notion of continued fractions is a natural development of this bias. That is, the number denoted as the sequence of positive integers (except that the first may be 0) $[a_0, a_1, a_2, \ldots]$ is

$$[a_0, a_1, a_2, a_3, \dots] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cdots}}}.$$

Each rational number can be expressed as a terminating sequence of integers $[a_0, a_1, \dots, a_n]$ for some n, while every irrational number, including π, has a unique infinite sequence. Its continued fraction, from any text on number theory, starts with

$$\pi = [3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, \dots],$$

so that π is the limit of the progression

$$3 \longrightarrow 3 + \frac{1}{7} \longrightarrow 3 + \frac{1}{7 + \frac{1}{15}} \longrightarrow 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1}}} \longrightarrow \cdots$$

written more familiarly as

$$3 \longrightarrow \frac{22}{7} \longrightarrow \frac{333}{106} \longrightarrow \frac{355}{113} \longrightarrow \frac{103993}{33102} \longrightarrow \cdots.$$

That is, God would most surely have selected $355/113$ rather than $333/106$ as representative of π for three reasons:

- The denominators 113 and 106 are very close; that is, the fraction $355/113$ is only marginally more complicated than $333/106$.
- The fraction $355/113$ to ten decimals is 3.141592920, giving 6-digit agreement with π rather than four, a hundred times better!
- The fraction $355/113$ is easily remembered—the digits of the fraction when following an *S* pattern from below form the sequence 113355.

The inside story

The *Talmud*, *Erubin 14a* maintains that the 30-cubit measurement was the inside circumference of the Sea. Such a measurement, when made compatible with $\pi \approx 3.14$ and a 10-cubit outside diameter, means that the thickness of the Sea is about four inches, the approximate width of a man's hand, which is how *I Kings 7:26* describes it. That is, if t is the thickness, then the inside diameter is $10 - 2t$ and so $30 = \pi(10 - 2t)$, which means that $t \approx 0.225$ cubits; since a cubit is approximately 18 inches, $t \approx 4$ inches. Rabbi Nehemiah, in the *Mishnat ha-Middot*, the earliest extant Hebrew work on geometry, dating to about 150 AD, outlines this same approach [**4**, pp. 75–76].

Measuring the inside circumference of a basin with a line is tricky however. One way to approximate this measure is to "walk" a cubit stick around the inside of the opening, so tracing out an inscribed 30-gon of sorts. Along these lines, Zuckermann proposed a dodecagonal shape for the Sea's opening [**10**, p. 51]; see Figure 4(d). Both of these models are in agreement with the *Talmud*'s conclusion in *Erubin 14a*.

A tradition that the *Talmud* may have used as justification for its explanation is described in *Mishnah*, *Kelim 18:1*:

The School of Shammai say: A chest should be measured on the inside [to determine its capacity]. And the School of Hillel say: On the outside.

Since the diameter measure is clearly an outside measurement from the *Kings* passage, and since there is some ambiguity in the measurement of the circumference, the *Talmud* adopted the former tradition rather than the latter for that measurement, even though the English translation, "a line of thirty cubits did compass it round about," suggests an outside measurement.

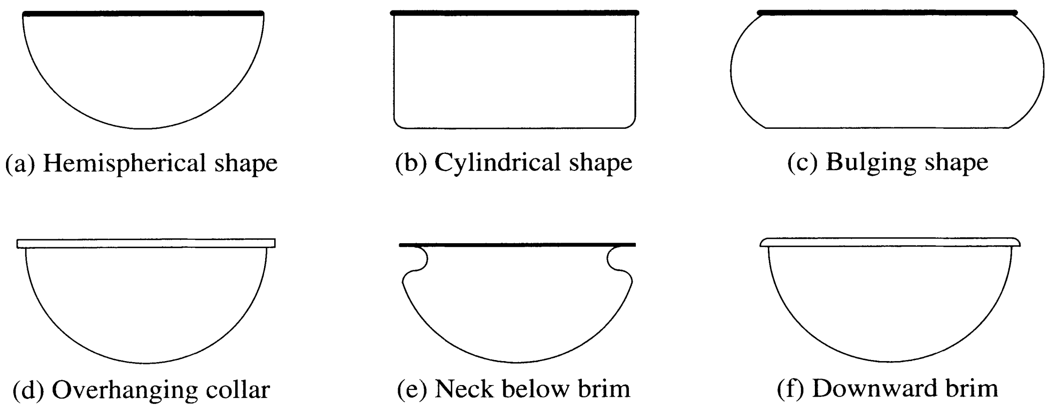


Figure 3. Possible sea profiles.

The protruding brim

A natural model for the Sea's shape is a hemispherical bowl whose girth is greatest at the brim so that the Sea has a somewhat circular profile as in Figure 3(a). Josephus says as much ([**13**], *Antiquities of the Jews*, V:5, p. 245). In *Erubin 14b*, Rami bar Ezekiel says that the Sea was square from its base to three cubits up, while round at the brim to two cubits down. Another interpretation is that the cross-sections from the base to the rim follow a homotopy of a square transforming into a circle as is done linearly in Figure 4(a); a more elegant rendering is the hourglass transformation of Figure 4(c); in these models the juncture to which Rami bar Ezekiel alludes is illustrated by Figure 4(b), the cross-sectional shape at height three cubits, above which the cross-sections are rounder and below which they are more square. Zuckermann interprets this passage literally, so that the top (two cubits) is cylindrical and the bottom (three cubits) is prismatic, as in Figure 4(d) [**10**, p. 51]. Zuidhof [**26**] proposes a cylindrical body, and thus a rectangular profile. Payne [**19**, p. 122] maintains that the Sea had a "considerable bulge to accommodate even (the) two thousand baths (of *I Kings 7:26*)." So the shape of the Sea is quite unresolved. But *I Kings 7:24* says that beneath the brim of the Sea were two rows of *knops*—grape-like, decorative knobs—forming a kind of collar, so that the upper part of the Sea's silhouette looked something like the upper part of Figure 3(d); perhaps the circumference measurement was taken just beneath this collar, as Steveson [**23**] and Zuidhof [**26**] suggest, or was taken as the measurement around the neck of Figure 3(e) or around the waist of Figure 4(c).

Another explanation is that the brim of the Sea overhung its crest as in Figure 3(f), so that the length of a cord strung "from one brim to the other" would be greater than the actual diameter. If this extra downward curve of the Sea's lip gives an extra four inches or so on each side, the measurement anomaly is resolved.

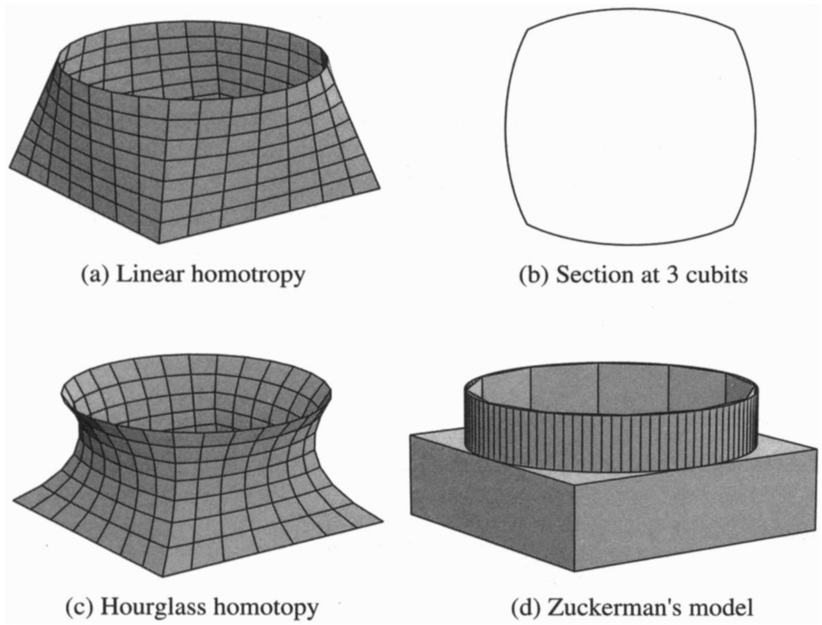


Figure 4. Some "square-round" models.

The premature conic

As suggested by Read [**21**], suppose that the brim's contour is oval shaped or an ellipse, so that the diameter—the major axis—is 10 cubits. To find the minor axis, $2b$, where the ellipse in parametric polar coordinates is $x = 5\sin\theta$ and $y = b\cos\theta$, write the integral expression for arc length, and equate it to a perimeter of 30 cubits, resulting in the equation

$$4\int_0^{\frac{\pi}{2}} \sqrt{25\cos^2(\theta) + b^2\sin^2(\theta)}\, d\theta = 30.$$

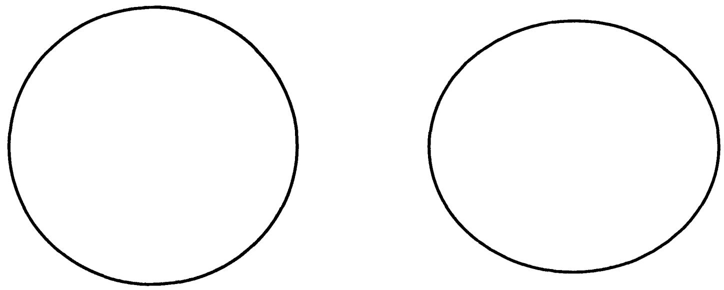
When solved, this gives $b \approx 4.54$. That is, the minor axis of such an ellipse is about an inch more than 9 cubits. To model the Sea's opening by other ovals, the integral formula in [**15**] may be useful.

Although ellipses were not defined until Menaechmus, around 350 BC, ovals were certainly familiar to the ancients. So if one wished to design a round object with perimeter 30, long diameter 10, and short diameter an integer, then the ellipse of Figure 5(b) (or an oval very close to it) is what will most likely be designed by trial and error.

Steveson [**23**] disagrees with this idea, saying that the twelve symmetrically placed oxen upon which the Sea sat *(I Kings 7:25)* supports a circular shape. Three of these oxen faced north, three west, three south, and three east in the counterclockwise convention. In such a tradition that each direction is of equal importance, an oval opening might be viewed as improper.

A family of curves that has a more proper four-fold symmetry in keeping with the four natural directions and the four oxen are the pseudo-circles,

$$|x|^p + |y|^p = r^p, \tag{2}$$



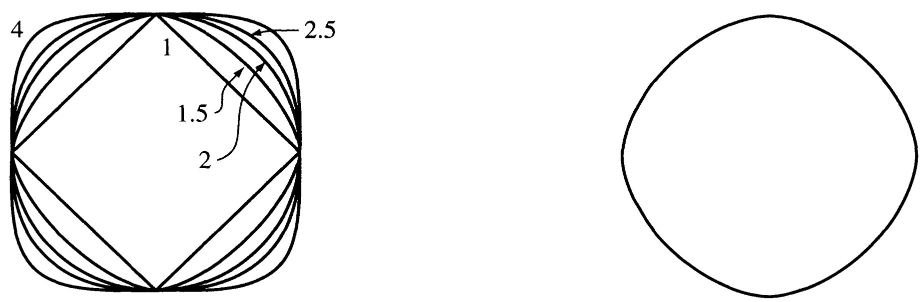
(a) Circle: $r = 5$ (b) Ellipse: $a = 5$, $b = 4.54$

Figure 5. The Sea from above.

where r is the pseudo-radius and p is any positive number. For example, Figure 6(a) shows the family members for $p = 1, 1.5, 2, 2.5, 4$. By the integral arc-length formula, the circumference $C(p)$ of a pseudo-circle is the intimidating looking integral

$$C(p) = 8r \int_0^{\left(\frac{1}{2}\right)^{\frac{1}{p}}} \sqrt{1 + z^{2p-2}(1 - z^p)^{\frac{2(1-p)}{p}}}\, dz,$$

where $z = x/r$. Using a computer algebra system to compute $C(p)/(2r)$ gives a ratio of 3.03 when $p = 1.7$. That is, Figure 6(b) is a good candidate for the shape of the Sea's rim.



(a) A family of pseudo-circles (b) With $p \approx 1.7$, circumference to long diameter: 3

Figure 6. Pseudo-circles.

The double standard

Since the cubit is approximately the length of a forearm from elbow to finger tip, about 1.5 feet, a simple-minded idea that resolves the ratio dilemma is for a taller craftsman to measure the circumference and a shorter craftsman to measure the diameter.

Is there any merit to this argument?

There were at least three different cubit lengths in use in biblical times. The second temple (dating to no earlier than 500 BC) housed a bureau of standards, as we would call it, within its eastern gate, referred to as the Castle of Ŝuŝan [**14**, p. 121]. *Mishnah, Kelim 17:9* describes the relationship between three of these units.

> And there were two (standard) cubits in the castle of Ŝuŝan, one on the northeastern corner, and the other on the southeastern corner. The one on the northeastern corner exceeded that of Moses by half a fingerbreadth, [while] the one on the southeastern

> corner exceeded the other by half a fingerbreadth, so that the latter exceeded that of Moses by a fingerbreadth. And why did they prescribe one large and one small? Only [for this reason]: that the craftsmen might take [material] according to the small [cubit] and return [their finished work] according to the large [cubit], so that they might not be guilty of trespass [of Temple property].

In conventional units, two standard cubits were used in the time of Solomon and the first temple, the cubit of Moses (M) of length 42.8 cm, and the large cubit (L) of length 44.6 cm; a third standard, the small cubit (S) of length 43.7 cm came much later, according to Kaufman [**14**].

This *Kelim* passage describes a curious measurement tradition in the days of the second temple. That is, temple craftsmen took materials of wood or stone from the temple in terms of the *profane* (ordinary) S cubit, worked with those materials outside the temple (as the sound of hammer and chisel was forbidden on the temple site), and returned the finished items in terms of the *holy* L cubit, installing them inside the temple. This measurement rule seems austere for craftsmen, because actual lengths of finished products are usually less than the actual lengths of the raw materials used. It looks like double jeopardy! Apparently, temple personnel held craftsmen to a very strict accounting. An editorial footnote for this passage of the *Talmud* summarizes this point:

> [these rules made] sure that they [the workmen] neither appropriated any material that belonged to the Temple nor received payment for labour they had not performed.

Kelim 17:10 goes on to point out that all measurements of the second temple itself were in terms of the S cubit except for the measurements of "the Golden Altar and the horns and the Circuit and the Base [of the Altar]." The editorial notes go on to say that these most holy and inner things of the temple appear to have been measured in terms of the M cubit.

In view of such measurement traditions in the days of the second temple, it is reasonable to imagine similar ones in the days of the first temple. In particular, since M was an older standard than L, profane or ordinary objects were probably measured with L while holy objects were probably measured with M. It is therefore possible that as a meaningful gesture, since this basin's function was to cleanse, rendering the profane into the holy, the engineers of the Sea may have ceremoniously designed the Sea so that the outside—the circumference—was in terms of L, and that the inside—the diameter—was in terms of M. Such a conjecture results in $\pi \approx 3.12$ (where $\pi(10)(42.8) \approx 30(44.6)$).

Furthermore, this value of π is independent of the stated cubit's lengths of 42.8 cm and 44.6 cm. Let l and m be the lengths of L and M respectively. Since each cubit is 24 fingerbreadths long, and since this *Kelim* passage asserts that the L cubit exceeds the M cubit by a fingerbreadth, then since M is an older unit than L, a natural interpretation is that $l = (25/24)m$. If so, a circumference of 30 of the L cubits and a diameter of 10 of the M cubits yields the relation

$$30\left(\frac{25}{24}\right)m = 10\pi m,$$

which gives $\pi \approx 3.125 = 25/8$, which is a Babylonian approximation for π in vogue during Solomon's day [**4**, pp. 21–22]. With such a close approximation to π, it is easy to wonder whether the L cubit was initially defined so that 3 of its cubits would encompass a circle of diameter 1 of the cubits of Moses.

Finally, there are mathematical variants of this double standard idea for making π evaluate to 3.

Andersen [**2**] and Chasse [**6**] suggest using non-Euclidean elliptical geometry. As such, a good model is the surface of a sphere or globe. Identify the rim of the Sea as the circle of latitude at 60°N on a globe, so that the diameter measure will follow a great circle arc across the north pole. The ratio of circumference to this curved diameter is indeed 3. This reasoning is reminiscent of Figure 3(f), wherein a line measuring the diameter may drape across a curved surface.

R. Euler [**9**] and Adler [**1**] use a pseudo-metric to measure circumference and diameter of the pseudo-circles (2) where the pseudo-distance $d(X, Y)$ between two points in the plane is

$$d(X, Y) = \left((x_1 - y_1)^p + (x_2 - y_2)^p\right)^{\frac{1}{p}},$$

with $X = (x_1, x_2)$, $Y = (y_1, y_2)$, and p a positive number. As they point out, the least possible value for the ratio of circumference to diameter for pseudo-circles is π. However if the circumference of the pseudo-circle is measured in the pseudo-metric while the diameter is measured with the usual Euclidean metric, then for the pseudo-circle with $p \approx 2.37$, the ratio of circumference to diameter taken along the line $y = x$ is 3.

Norwood [**17**] explains that by the Lorentz-Fitzgerald contraction principle of relativity theory, a circle of radius 1 meter spinning at 10 million rpm will make $\pi = 3$. He explains further in [**18**] that his measuring stick is stationary and outside the spinning system, so that the measuring stick's length remains invariant during the measurement process.

Concluding remarks

Which perspective is correct? Since the Sea is reported as broken and carted away in about 586 BC by the conquering Babylonians in *Jeremiah 52:17*, there are no irrefutable answers. Each of the arguments has some merit. And it may very well be that the true story lies in a combination of these perspectives. Whatever the resolution for this puzzle, what I find most interesting is that the chroniclers somehow decided that the diameter and girth measurements of Solomon's Sea were sufficiently striking to include in their narrative. It is almost as if they saw "as through a glass darkly" the abstract π, and could not but help to record in passing this particular instance of a most curious geometric relationship.

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The flight ended in Metapontum, where again disturbances are said to have arisen. Pythagoras took refuge in the shrine of the Muses and died there, either of natural causes—after enduring forty days without food—or, in a moving account, by his own hand.... In another version what Neanthes says about later Pythagoreans is transferred to Pythagoras himself: That as he was fleeing he came to a field full of beans, and stopped there instantly, in order not to traverse it, and said: "Better to be captured than to tread on [beans]!" And so, it is said, he was killed by his pursuers.... So, like his birth, Pythagoras' death is recounted with many variations.

—Christoph Riedweg, *Pythagoras: His Life, Teaching, and Influence*, p. 20.